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JET FORMATION AND CYCLONE-ANTYCLONE ASYMMETRY IN DECAYING ROTATING TURBULENCE

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ABSTRACT

The organization of a sequence of alternating intense and elongated eastward-westward bands i.e. zonal jets in the atmosphere of giant planets and in the Earth's oceans have been widely investigated, nevertheless, dynamical processes associated with jets formation, maintenance and role as material barriers still remain unclear. Also, the formation of these persistent, latitudinally aligned structures has found to be closely related with a prevalence of anticyclones in the evolution of an initially symmetric vorticity distribution. In this work we use laboratory experiments to focus on the role of rotation rate and fluid thickness on the dynamics of these structures in a turbulent shallow-water systems. In particular we consider the evolution of quasi-2D decaying turbulence generated placing an electro-magnetic cell on a rotating table. The initial flow is forced at small scales and evolves towards the organization of zonal circulations due to potential vorticity mixing and homogenization. Flow measurements have been performed in a Lagrangian frame of reference by using image analysis. Zonal velocity profiles and energy spectra typically associated with a banded flow configuration have been recovered. The asymmetry index of the vorticity distribution, quantified via the skewness of the relative vorticity field, indicates a cyclone-anticyclone symmetry breaking confirmed by the obtained value of the non-dimensional parameter RoB^{-1}.

INTRODUCTION

Freely decaying shallow water turbulence on a sphere is found to be characterized at latest stage by the emergence of robust jets and by the breaking of cyclone-anticyclone asymmetry [1][2][3]. Zonal jets spontaneously organize due to the interaction of turbulence with the latitudinal variation of the Coriolis parameter which modify the inverse cascade process channeling energy towards zonal modes [4] but may also originate from a baroclinic dynamics i.e. via the non-linear interaction between the eddies and the mean flow [5]. Rhines prediction (1975) for unforced and barotropic flows, indicates that unless dissipation or the limited size of the field domain arrests the inverse cascade at a certain scale, the energy is transferred towards slowest modes and accumulates in correspondence of the Rhines wavenumber k_{Rh}, where it is channeled into zonal jets and Rossby waves [6]. In this sense this scale is said to arrest the inverse energy cascade and the resulting flow is anisotropic and characterized by a a steep ($\sim k^{-5}$) power law spectrum for $k>k_{Rh}$. The concept of cascade arrest has been recently revisited by Sukoriansky et al. [7] considering a continuously forced flow and the effect of a large scale energy sink. Rhines.

The generation of zonal flows can be also described in terms of mixing and homogenization of the potential vorticity PV [8]. As a matter of fact, in the inviscid limit PV is materially conserved and the presence of a β plane induces a background PV, $\beta y/H$ (y is the distance along the meridional direction). Vortices will mix this PV distribution creating a step-like profile formed by regions of constant PV connected by jumps: eastward (narrower) jets forms between regions characterised by different (constant) values of PV, in correspondence of which lie westward (broader) jets. The corresponding PV meridional distribution is referred to a so-called staircase model [8]. According to this model, eastward jets are associated to the maximum horizontal gradient of PV and act as transport barrier inhibiting lateral displacement of fluid particles. This behaviour has been recovered in numerical simulations [9] and laboratory experiments [10][11].

In this work we use laboratory experiments to investigate the self-organization of these structures in freely decaying shallow-water turbulence. In particular the initial distribution of vorticity has been generated via the Lorentz force [12] in an electromagnetic cell placed on a rotating tank, the background planetary differential rotation have been simulated through a depth variation produced by the parabolic profile assumed by the fluid free surface under rotation and flow measurements have been performed using image analysis [13].

NOMENCLATURE

Bu	[-]	Burger number
f	[rad/s]	Coriolis parameter
Fr	[-]	Froude number
H	[m]	Fluid depth
H_c	[m]	Fluid depth in correspondence of the tank centre (pole)
H_0	[m]	Fluid depth at rest (non rotating)
k_{Rh}	[1/m]	Rhines wavenumber
k_{Ro}	[1/m]	Rossby wavenumber
k_f	[1/m]	Forcing scale
L_{Rh}	[m]	Rhines scale
L_{Ro}	[m]	Rossby radius of deformation
q	[1/ms]	Quasi-geostrophic potential vorticity
r	[m]	Radial distance from the tank centre
r_m	[m]	Mid-latitude reference radius
Ro	[-]	Rossby number
s	[1/m]	Rotation parameter
u_{rms}	[m/s]	Velocity rms
U_r	[m/s]	Radial average of zonal velocity
X	[m]	Cartesian axis direction
Y	[m]	Cartesian axis direction
Z	[m]	Cartesian axis direction

Special characters

β	[1/ms]	Longitudinal gradient of the Coriolis parameter
γ	[1/m^{2}s]	Coefficient of the second order term of the Coriolis parameter
Ω	[rad/s]	Tank angular velocity
τ_D	[s]	Effective decay time-scale
τ_E	[s]	Ekman time-scale
ε	[-]	Initial Rossby number

Subscripts

D	Decay
E	Ekman
C	Centre
m	Mid latitude
0	Ambient or reference

PHYSICAL AND EXPERIMENTAL MODEL

Shallow water equations (SWE) describe the evolution of a thin layer of an incompressible fluid in response to gravitational and rotational accelerations [14]. A set of non-dimensional equations can be obtained introducing the characteristic scales of the flow U, L, H respectively for velocity, length and height [1]:

$$\begin{aligned} &\partial_t \boldsymbol{v}' + \boldsymbol{v}' \cdot \nabla \boldsymbol{v}' = - BR_o^{-1} \nabla h' - R_o^{-1} \boldsymbol{k} \times \boldsymbol{v}' \\ &\partial_t h' + \boldsymbol{v}' \cdot \nabla h' = -(1 + h') \nabla \cdot \boldsymbol{v}' \end{aligned} \tag{1}$$

where $\boldsymbol{v}' = \boldsymbol{v}/U$ is the non-dimensional velocity and h' is the non-dimensional characteristic deviation from the mean fluid height. The two non-dimensional parameters relevant for these flows are the Rossby number:

$$R_o = \frac{U}{2\Omega L}$$

where Ω is the rotation rate, and the Burger number:

$$Bu = \frac{R_o^2}{F^2} = \left(\frac{L_{Ro}}{L} \right)^2$$

where $F = U/\sqrt{gH}$ is the Froude number and the length scale $L_{Ro} = \sqrt{gH}/2\Omega$, the Rossby radius of deformation, accounts for the presence of a free surface.

In the non-dissipative case SWE conserve mass and energy, the potential vorticity $q = (\zeta + f)/h$, where ζ is the relative vorticity and f the planetary vorticity *i.e.* the Coriolis parameter, is invariant too.

Numerical simulations of SWE in spherical geometry [1][2][3] have been strongly motivated by the dynamics on the atmospheres of giant planets, this model in fact can represent the effects of both fluid stratification and the differential rotation (*i.e.* f variation with latitude). To this concern using a spherical domain allows to investigate differences between polar and equatorial regions that cannot be highlighted if a planar domain , *i.e.* a constant β value, is considered.

Rotating turbulence is often referred to β-plane turbulence even when the variation of f with latitude cannot be considered linear and the so-called β-plane approximation holds. The length scale defined by the rotation *i.e.* the Rhines scale $L_{rh} = 1/k_{Rh}\,; k_{Rh} = (\beta/U)^{0.5}$ represent the scale at which the inverse cascade is slowed and preferentially transferred towards zonal modes. Thus, persistent, latitudinally aligned structures called zonal jets spontaneously organize due to the interaction of turbulence with the latitudinal variation of f. The formation of these features has found to be closely related with a prevalence of anticyclones in the evolution of an initially symmetric vorticity distribution [1][2].

The experiments here discussed are aimed to study the formation and the behaviour of zonal jets in a freely decaying quasi-2D turbulent rotating flow. The experimental setup (**Figure 1**) consists of a rotating tank where turbulence is generated by electromagnetically forcing a thin layer of an electrolyte solution, and the variation of Coriolis parameter have been simulated by the parabolic profile assumed by the free surface of the rotating fluid [15]. The resulting flow is then aimed to simulate the dynamics in correspondence to the polar region of a rotating sphere (indicatively $0^0 < \phi < 45^0$)

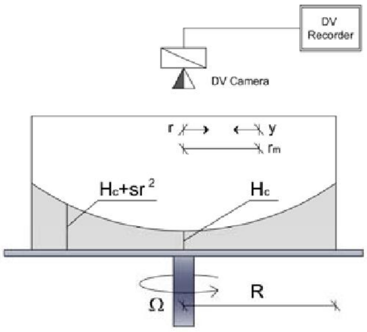

Figure 1 Experimental apparatus

In particular the test section is a plexiglass square tank (L = W =3.9 10^{-1} m) in which a cylinder (D = 3.6 10^{-1} m) is inserted to obtain a domain with circular boundary. The tank is filled with a layer of saline solution (water and NaCl (250 mg/l)) of height $H_0=2.7\cdot10^{-2}$ m.

The flow is generated by a superposition of electric and magnetic fields: a constant voltage $\Delta V=12$ Volts is applied at two electrodes placed in the tank (outside of the circular boundary) while the magnetic field is generated by a square array of permanent magnets placed on a metallic plate below the bottom surface of the tank. The combined effect of electric and magnetic forcing induces the formation of opposite signed eddies, cyclonic or anticyclonic according to the phase of the resulting Lorentz force, and whose initial horizontal length scales are related to the distance between magnets, ($2.5\cdot10^{-2}$ m here).

The tank is placed on a rotating table and rotation tends to suppress any motion along the direction of the rotation axis; experiments have been performed considering counter-clockwise rotation in order to simulate flows in the northern hemisphere. The dynamics associated with the Coriolis parameter in the polar region is captured by a quadratic variation of it in *r*, the meridional distance from the pole, assuming the pole as the reference point $r=0$ (this is the so-called β plane). The parabolic free surface assumed by the fluid under rotation has used to model in laboratory the variation of the Coriolis parameters with latitude [11][15]. The corresponding expression for *q* and β are [16]:

$$q=\frac{\zeta+f_0}{H_c+sr^2} \tag{2}$$

$$\beta=\frac{2sr_m f_0}{H_c+sr_m^2} \tag{3}$$

where $f_0=2\Omega$ (constant in laboratory), Ω is the tank rotation rate, $r_m=R/2$ is a 'mid-latitude' reference point ($r=r_m-y$; *R* is the tank radius, see **Figure 1**) H_c indicates the fluid thickness at $r=0$, $s=\Omega^2/2g$. In this model, the point of maximum depression of the fluid surface represents the pole, while the periphery of the domain correspond to the lower latitudes.

To perform flow measurements, the fluid surface is seeded with styrene particles (mean size ~ 250 μm), the forcing is turned on and the tank is brought up to the desired rotation. The voltage is then turned off after 10 minutes and the corresponding flow-*i.e.* the beginning of the decay- represents the time instant t = 0 for our experiments. A standard video camera (acquisition frequency = 25 Hz, resolution: 720x570 pixels), co-rotating with the system and whose optical axis is parallel to the rotation axis, is used to acquire flow images which are then digitized and post-processed using an image-processing technique [13][17]. This method allows to reconstruct the velocity field evolution in a Lagrangian framework, in these experiments at least 5000 tracers trajectories have been reconstructed. After the interpolation of velocity vectors over a regular 65x65 grid, the time evolution of Eulerian instantaneous velocity fields is obtained and derived quantities are evaluated.

TRENDS AND RESULTS

We presents results of an experiments performed with the following parameters:

- $\Omega=2.47$ rads^{-1}
- $s=0.31$ m^{-1}
- $H_c=1.87\cdot10^{-2}$ m
- $\beta=0.13\cdot10^{-2}$ m^{-1}s^{-1}
- $\tau_E=16.1$ s
- $\tau_D=5$ s
- $Bu=0.83$

We remind that the Ekman time scale τ_E represents the decay time scale for a fluid layer of constant depth, and in this case it has been evaluated considering $H=H_0$. The effective decay time scale τ_D, evaluated by the decay rate of the total energy, indicates a faster onset of dissipation effects due to both the bottom effects and the variation of the fluid height [15].

Velocity and vorticity field

In **Figures 2–4**, we show snapshots of the flow evolution in terms of reconstructed instantaneous velocity and relative vorticity fields. These plots clearly show that starting from an uniform distribution of positive/negative eddies (**Figure 2**) jets form and during the decay the flow field tends towards a banded configuration (**Figures 3-4**).

The jets can be clearly represented by performing zonal averaging. To this aim the velocity field was computed as a function of *r*, the distance from the axis of rotation, and ϑ the polar angle (i.e. the longitude). An average is then performed over ϑ yielding U_r, which is shown in **Figure 5**.

From the velocity and vorticity distribution, we estimate a *rms* velocity $u_{rms}=0.1\cdot10^{-2}$ ms^{-1} and an initial Rossby number $\varepsilon\sim0.4$.

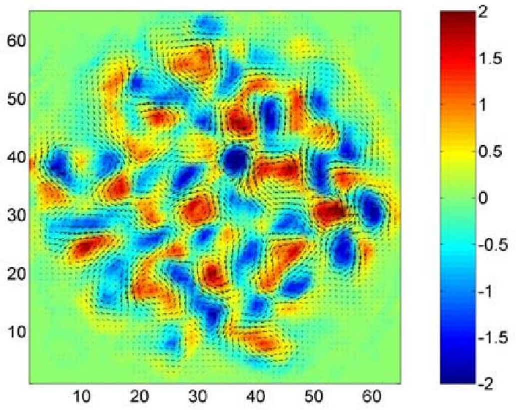

Figure 1 Velocity and vorticity fields at the beginning of the decay ($t=0$ s)

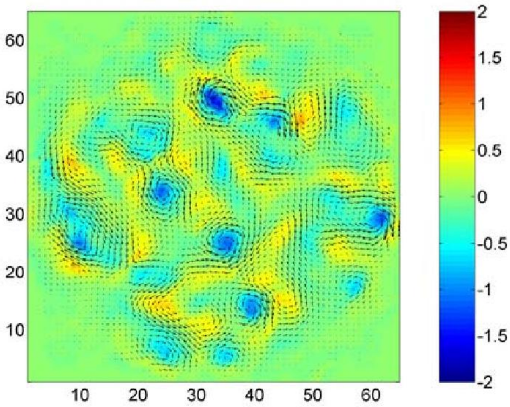

Figure 3 Velocity and vorticity fields at $t = 6.2$ s

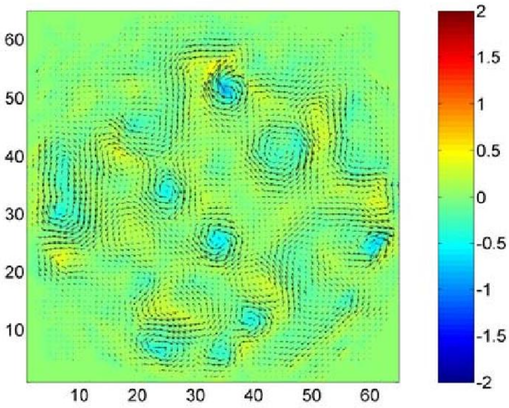

Figure 4 Velocity and vorticity fields at $t = 12.2$ s

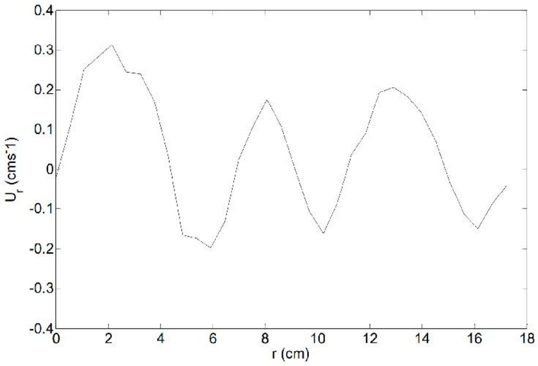

Figure 5 Radial distribution of azimuthally averaged zonal velocity at $t = 6.2$ s

Energy spectra

Further insight into the flow dynamics can be obtained by spectral analysis; energy spectra evaluation is based on a two-dimensional Fourier transform in Cartesian coordinates. The energy spectrum $E(k)$ with $k = \sqrt{x^2 + y^2}$ is calculated by averaging both over fixed time intervals and over direction in wave number space. The spectra (**Figure 6**) are respectively evaluated in three different time: when the forcing is still active; at t = 6.2 s after the forcing has stopped; at t = 12.2s after the forcing has stopped.

The wavenumbers k_f, k_{Rh} and k_{Ro} represent the mean wave number estimation respectively for the forcing scale, the Rhines scale and the Rossby radius of deformation, evaluated considering the *rms* velocity and vorticity field at $t = 0$. The following values have been recovered and shown in correspondence of the *k* axis on **Figure 6:**

- $k_f = 2.50 \cdot 10^{-2}$ m
- $k_{Rh} = 1.20 \cdot 10^{-2}$ m
- $k_{Ro} = 0.85 \cdot 10^{-2}$ m

Both the spectra evaluated in decaying conditions show a peak in correspondence of the Rhines wavenumber and the slope approximates the theoretical k^{5} scaling.

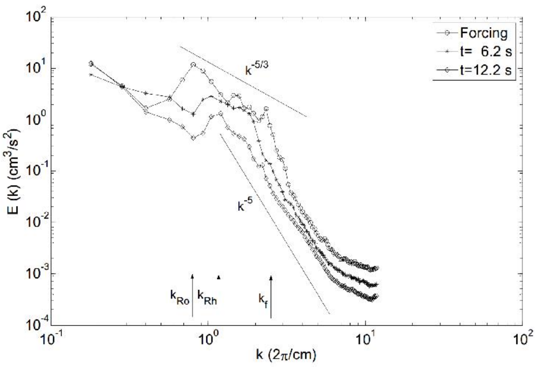

Figure 6 Energy spectra calculated at different time instants

Asymmetry index and cyclone-anticyclone asymmetry

Considering a rotating fluid bounded by a free surface and integrating the full set of SWE, asymmetry develops being the free surface variations comparable with the mean fluid layer thickness. The asymmetry index is usually quantified via the skewness of the relative vorticity field ζ *i.e.* $A(t) = \langle \zeta^3 \rangle / \langle \zeta^2 \rangle^{3/2}$ [1][2].

According to [1] this effect can be represented considering the parameter $RoBu^{-1}$, in particular a value of $RoBu^{-1} \geq 0.13 \pm 0.04$ leads to cyclone-anticyclone symmetry breaking. In agreement with this evidence, in our experiment we found $RoBu^{-1} = 0.48$ and a prevalence of anticyclonic vorticity.

From the analysis of the results (not showed here) obtained using the same set-up and varying experimental parameters *i.e.* the fluid height and the tank rotation velocity, we gather that asymmetry generally favours anticyclones and that the vorticity distribution become increasingly asymmetric as Froude and Rossby numbers increase.

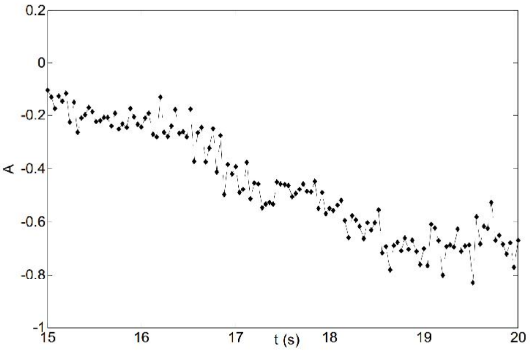

Figure 7 Asymmetry index time evolution

CONCLUSION

The present work features three important aspects of decaying shallow water turbulence in a rotating environment: 1) the experiment has shown the formation of an alternation of bands characterized by positive/negative mean zonal velocities. These structures forms due to the latitudinal variation of the Coriolis parameter f, in this case simulated by the topographic slope of the free surface in the rotating fluid; 2) the energy spectra develop a peak in correspondence to a wave number close to the theoretical estimate of k_{Rh}, in agreement with theoretical predictions for this type of flows, the estimated slope approximates k^{-5} scaling; 3) the negative asymmetry index and the recovered value for the parameter $RoBu^{-1}$ indicate the emergence of a state characterised by a prevalence of anticyclones from a symmetric initial condition i.e. the breaking of cyclone–anticyclone symmetry.

We are actually performing new experiments considering a wider scale separation among the characteristic scales of the flow in order to distinguish their different role in the energy/enstrophy cascades, and forced turbulence in order to verify if under this regime steady jets form.

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