SASEC2015
Third Southern African Solar Energy Conference
11-13 May 2015
Kruger National Park, South Africa

Can Nanofluids Improve Solar Collector Efficiency?

Michelle MacDevette *and B. Daya Reddy
Centre for Research in Computational and Applied Mechanics, UCT
email: michelle.macdevette@uct.ac.za

Abstract

Direct solar collectors are designed to absorb solar energy into a carrier fluid which then transports the heat away to some storage area. To increase the efficiency of the solar collectors the design is changed to that of a direct solar collector and nanoparticles are suspended in the heat transfer fluid. A two dimensional model of a static nanofluid in a solar collector is presented in this paper. Two equations are integral in describing the problem mathematically; the radiative transfer equation accounts for the attenuation of solar radiation through the depth of the collector and the heat equation describes the distribution of heat in the collector. The source term that accounts for the volumetric heat release is derived under the assumption of independent scattering. Taking advantage of the small aspect ratio the full model is approximated by the one-dimensional limit. Furthermore, a decaying exponential function is used to approximate the source term which allows for an exact analytical solution to be presented. In power generation applications high operating temperatures and maximum energy absorption are favourable. Therefore, the collector efficiency is measured in terms of the Carnot number and total solar energy absorption. For a fixed collector height, varying nanoparticle concentrations and exposure times are investigated to maximise collector efficiency.

1 Introduction

The majority of solar thermal technologies today use receivers with absorbing surfaces to convert solar energy into thermal energy. This energy is transferred to a carrier fluid which then advects the heat away to some storage area. Although the receivers are efficient at solar-to-thermal conversion, a large amount of energy is lost when transferred to the fluid. A recent development involves directly absorbing the solar energy within the fluid: this is the so-called direct absorption solar collector (DASC). In this paper we investigate increasing the performance of a DASC by using a high performance heat transfer fluid; the nanofluid. Conventional fluids such as water (which is one of the most efficient fluids so far tested) absorbs only 13% of the incident solar energy [1]. Clearly this needs to be improved upon.

One of the most exciting and heavily studied materials in the field of heat transfer is the nanofluid. This is a fluid containing a small fraction of nanoparticles (particles with diameter $D < 100$ nm (10^{-7} m)). Due to the high surface-to-volume ratio of the nanoparticles the heat transfer rate between the base fluid and particles is high. So even with very low concentrations of nanoparticles, these fluids have been shown to have greatly enhanced heat transfer properties.

A large body of work dealing with the mathematical modelling of DASC with nanofluids as the working fluids can be found in the literature. In general, the models involve two equations; the heat equation that accounts for the temperature profile and the radiative transport equation (RTE) which is used to define the heat generation term within the solar collector. Due to the complexity of the RTE the majority of the work involves numerical analyses. Furthermore, due to the complexity in modelling the properties of the nanofluid a number of assumptions are made. For example, scattering and absorption of solar radiation due to the base fluid is often neglected. So if the nanoparticles were removed, there would be no solar radiation absorption.

Veeraragavan *et. al.*[2] studied the steady state regime with uniform flow through the DASC in order to report optimal efficiencies and receiver design. Although the optical properties of the base fluid were neglected, they showed performance could be optimised by varying the receiver length. Lenert and Wang [3] did an experimental study of the unsteady, stationary DASC, where they assumed the thermal physical properties did not vary with particle concentration, i.e. the nanofluid and base fluid possessed the same thermal properties. Under their

*MM acknowledges the support of the Claude Leon Foundation.

model they showed nanofluids increase solar receiver efficiencies by 35%. Otanicar *et. al.*[4] did a numerical study of a steady state receiver under a plug flow regime where they showed a performance increase of 5%. Tyagi *et. al.*[5] did a similar study, but included the effect of the base fluid in the optical properties of the nanofluid, which resulted in a 10% performance increase. All these models make assumptions regarding the thermalphyiscal or optical properties of the nanofluid and then go on to solve the problem numerically.

What lacks in the literature is an analytical study of the full problem. This is due to the complexity of modelling the source term in the heat equation and modelling the nanofluid where the thermalphysical properties vary according to the nanoparticle concentration. Here we consider the unsteady problem of a stationary nanofluid in a DASC, with varying thermalphysical properties. Furthermore, the model presented will also incorporate the correct optical properties for the nanofluid. The model is set up accordingly and an analytical solution by separation of variables is presented. To conclude the analysis we will look at performance of the receiver in terms of efficiencies. Two aspects are considered when analysing the performance of solar collectors; high operating temperatures and high energy storage capabilities. In power generation applications the heat transfer fluid is pumped into a heat exchanger or is used to drive turbines, therefore a higher temperature results in higher efficiencies and so is favoured. Of course, since the objective is to harness the energy of the sun, the total energy stored by the DASC will also be optimised.

In the next section the mathematical model is laid out followed by a full discussion on how the source term is modelled. Section 4 deals with the non-dimensionalization of the problem which is then solved analytically in §5. A discussion of the results follows in §6 with special attention given to definition and reporting of DASC efficiencies before the paper is concluded in §7.

2 Mathematical model

A rectangular direct solar collector containing a static nanofluid placed on a flat surface is modelled. The collector has dimensions $H \times L$, so the length is described by $x \in [0, L]$ and the height $y \in [0, H]$, where $y = 0$ is the top. The bottom of the collect is assumed to be insulated, while the top is fitted with a transparent, thin film so the fluid is exposed to direct sunlight. This allows the solar flux to pass into the fluid resulting in volumetric heating of the fluid, which is modelled as a source term. There is also heat loss at the top due to convection which is modelled using a Newton cooling condition. The unsteady, two-dimensional thermal problem is governed by the heat equation

$$\rho c \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + s, \quad y \in [0, H], x \in [0, L] \quad (1)$$

where T is the temperature and s is the heat generation term. The thermophysical properties of the nanofluid, such as density ρ, thermal conductivity k and heat capacity c, are assumed temperature independent but depend on the volume fraction ϕ:

$$\rho = \rho_{\text{np}}\phi + \rho_{\text{bf}}(1-\phi), \quad (2)$$

$$\chi = \rho c = c_{\text{np}}\rho_{\text{np}}\phi + c_{\text{bf}}\rho_{\text{bf}}(1-\phi), \quad (3)$$

$$k = \frac{k_{\text{bf}}}{(1-\phi^{1/3})^2}\left[(1-\phi) + \phi\frac{\rho_{\text{np}}c_{\text{np}}}{\rho_{\text{bf}}c_{\text{bf}}}\right] \times \frac{n-1}{2(n+1)}\left[\frac{1+\phi^{1/3}}{2} - \frac{1}{n+1}\right]^{-1}, \quad (4)$$

where subscripts bf, nf represent the base fluid and nanoparticle constituents and $n = 2.233$ is a fixed constant [6, 7].

Initially the fluid has a uniform temperature $T(x, y, t = 0) = T_{\text{in}}$ and is kept constant at the left boundary $T(x = 0, y, t) = T_{\text{in}}$. The bottom is insulated $kT_y(y = 0) = 0$ while the top and right boundaries are subject to a Newton cooling condition

$$y = 0 \qquad kT_y = h_t\left(T - T_{amb}\right), \quad (5)$$

$$x = 0 \qquad kT_x = h_r\left(T - T_{amb}\right) \quad (6)$$

where T_{amb} is the ambient temperature of the surrounding air, and h_t,h_r are the convective heat transfer coefficients [5] at the top and right boundaries.

3 Source term

The fluid heats up due to the incident solar radiation. This is incorporated into the model via the source term s whose heat release profile is dependent on the attenuation of the solar radiation through the depth of the collector. Since the collector is flat the radiation intensity within the fluid varies only in y and is described by the one-dimensional radiative transfer equation

$$\frac{dI_\lambda}{dy} = -K_{e,\lambda} I_\lambda, \quad (7)$$

where I_λ is the transmitted light intensity and $K_{e,\lambda}$ is the spectral extinction coefficient. The boundary condition at the top of the receiver ($y = 0$) is approximated by Plank's black body relation given by

$$I_{b,\lambda} = \frac{2c^2 h S_{att}}{\lambda^5\left(\exp\left[\frac{hc}{\lambda k_B T_{sun}}\right] - 1\right)}, \quad (8)$$

where c is the speed of light, h is Planck's constant, S_{att} is the attenuation constant, λ is the light wavelength, k_B is the Boltzmann constant and T_{sun} is the temperature of the sun [2, 5, 8]. Equation (7) may immediately be

integrated, and applying (8) gives $I_\lambda = I_{b,\lambda} e^{-K_{e,\lambda} y}$. It is clear the intensity of transmitted light decreases with increasing y, i.e. decreases with depth of the solar collector as expected. The volumetric heat release (or source term) may be expressed as the divergence of the spectral flux, I_λ, integrated over all wavelengths [2, 5]

$$s = -\frac{d}{dy}\int_0^\infty I_{b,\lambda} e^{-K_{e,\lambda} y} d\lambda = -\frac{dP}{dy}. \tag{9}$$

A common approach to estimating the extinction coefficient is to neglect absorption and scattering due to the base fluid [2, 5]. However, provided $\phi < 0.006$ we may assume independent scattering [9] and so the effects of absorption and scattering of the nanofluid's constituents may be added. We now define the spectral extinction coefficient, $K_{e,\lambda}$, as

$$K_{e,\lambda} = K^{\text{bf}}_{a,\lambda} + K^{\text{bf}}_{s,\lambda} + K^{\text{np}}_{a,\lambda} + K^{\text{np}}_{s,\lambda}. \tag{10}$$

The subscripts a,s represent absorption and scattering. For the base fluid constituent we assume scattering to be negligible. However, attenuation due to absorption is considered and so the coefficients corresponding to the base fluid may be expressed as

$$K^{\text{bf}}_{a,\lambda} = \frac{4\pi\kappa_{\text{bf}}}{\lambda}, \qquad K^{\text{bf}}_{s,\lambda} = 0 \tag{11}$$

where κ_{bf} is the index of absorption. For the nanoparticle constituent we assume light is scattered under the Rayleigh regime, i.e. the particle size is significantly smaller than the spectral wavelength. The size parameter of the particles is defined by $\alpha = \frac{\pi D}{\lambda}$ where D is the particle diameter. For the Rayleigh scattering assumption to be valid we require $\alpha \ll 1$ and $|m|\alpha \ll 1$ where m is the normalized refractive index of the particles to the base fluid $m = (n_{\text{np}} + i\kappa_{\text{np}})/n_{\text{bf}}$. Since the optical properties of graphite are not strong functions of wavelength we can take $n_{\text{np}} = 2.72, \kappa_{\text{np}} = 1.31$ as a constant [2]. Similarly the refractive index of water is approximated to be $n_{\text{bf}} = 1.336$. The extinction coefficient of the nanoparticles may be written as

$$K^{\text{np}}_{e,\lambda} = \frac{3\phi Q^{\text{np}}_{e,\lambda}(\alpha, m)}{2D} \tag{12}$$

where the extinction efficiency in the Rayleigh regime comprises of an absorption and scattering component $Q^{\text{np}}_{e,\lambda} = Q^{\text{np}}_{a,\lambda} + Q^{\text{np}}_{s,\lambda}$. These are defined as

$$Q^{\text{np}}_{a,\lambda} = 4\alpha \text{Im}\left(\frac{m^2-1}{m^2+2} B\right) \tag{13}$$

$$B = \left[1 + \frac{\alpha^2}{15}\left(\frac{m^2-1}{m^2+2}\right)\frac{m^4+27m^2+38}{2m^2+3}\right] \tag{14}$$

$$Q^{\text{np}}_{s,\lambda} = \frac{8}{3}\alpha^4 \text{Re}\left|\frac{m^2-1}{m^2+2}\right|^2 \tag{15}$$

By exploiting the small size of α these expressions may be simplified by neglecting the higher order terms

$$Q^{\text{np}}_{a,\lambda} = 4\alpha K_1 + \mathcal{O}(\alpha^2), \quad Q^{\text{np}}_{s,\lambda} = 0 + \mathcal{O}(\alpha^4) \tag{16}$$

where $K_1 = \text{Im}\left(\frac{m^2-1}{m^2+2}\right)$. Now the extinction coefficient of the nanoparticles may be written as

$$K^{\text{np}}_{e,\lambda} = \frac{6\phi\pi K_1}{\lambda}. \tag{17}$$

The full extinction coefficient that accounts for the combined effects of absorption and scattering of both constituents of the nanofluid is

$$K_{e,\lambda} = \frac{6\phi\pi K_1}{\lambda} + \frac{4\pi\kappa_{\text{bf}}}{\lambda}. \tag{18}$$

Substituting (18) into (9) we may write the volumetric heat release as

$$s = -\frac{2hc^2 S_{att} C_0}{C_1^5}\psi_4\left(1 + \frac{C_0}{C_1} y\right). \tag{19}$$

where ψ_4 is the polygamma function of order 4 and $C_0 = 4\pi\kappa_{\text{bf}} + 6\pi K_1\phi$, $C_1 = hc/(k_B T_{sun})$. The value S_{att} is chosen such that $P = G_s = 1000W/m^2$ at the surface of the solar collector [5], where G_s is the incident solar flux on the collector plane. In the following section the problem will be solved analytically. To ensure equation (1) is integrable we approximate the source term by an exponentially decaying function of the form

$$s = s_0 e^{-s_1 y}, \tag{20}$$

where the constants s_0, s_1 are chosen such that there is good agreement between (20) and (19).

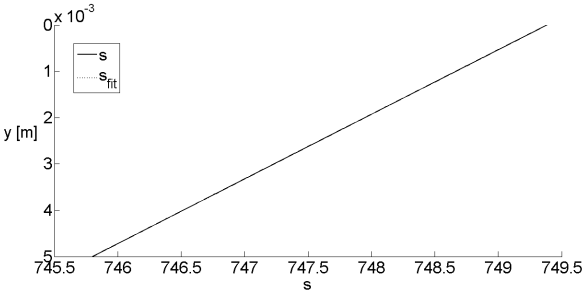

Figure 1: *Source term predicted by current model* (19) *(solid) and the exponential approximation 20 (dotted) for the base fluid, $\phi = 0$, is non-zero.*

Fig. 1 shows excellent agreement between the full source term (19) and the exponential approximation (20) for the base fluid ($\phi = 0$). This plot also highlights the quantitative difference between the current model and the popular model by Veeraragavan *et. al.*[2]; in their model the base fluid does not contribute to the extinction coefficient; therefore, when $\phi = 0$ the source term is zero, while the current model has a non-zero, linear source term. Fig. 3 compares the current model (19), that of Veeraragavan *et. al.*[2] and the exponential approximation (20) for $\phi = 0.0001, 0.001, 0.006$. Fig. 3 is a close up of the upper left hand corner of Fig. 2. For larger ϕ values the absorption and scattering due

to nanoparticles is more significant compared to that of the base fluid. So we expect the difference between the current model and that of Veeraragavan *et. al.*to decrease with increasing ϕ. The exponential function approximates the source terms well with a slight deviation near the change of gradients. For the purposes of this research approximation shows good agreement.

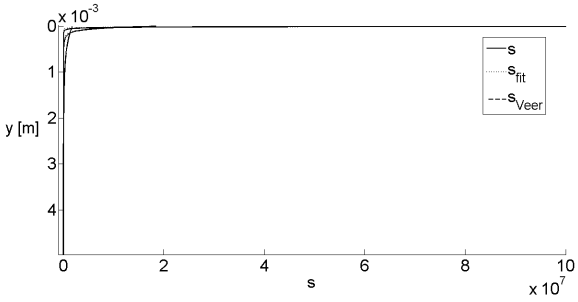

Figure 2: *Source term predicted by current model* (19) *(solid), the exponential approximation 20 (dotted) and Veeraragavan et. al.[2] (dashed) for* $\phi = 0.0001, 0.001, 0.006$.

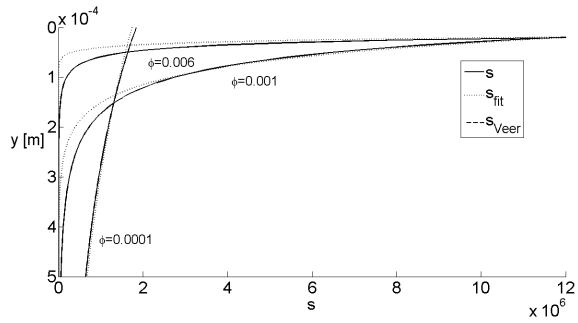

Figure 3: *Close up of Fig. 3 at top of solar collector* $y = 0$.

4 Non-dimensionalization

To simplify the problem we first introduce the dimensionless quantities

$$\hat{T} = \frac{T - T_{\text{in}}}{\Delta T}, \quad \hat{x} = \frac{x}{L}, \quad \hat{y} = \frac{y}{H}, \quad \hat{t} = \frac{t}{\tau} \tag{21}$$

and immediately drop the hat notation. Choosing the standard diffusion time scale $\tau = H^2/\alpha_{\text{bf}}$ the heat equation becomes

$$\chi' \frac{\partial T}{\partial t} = k' \left(h_0^2 \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{s_0 H^2}{k_{\text{bf}} \Delta T} e^{-s_1 y H} \tag{22}$$

where $\chi' = \chi/\chi_{\text{bf}} = (\rho c)/(\rho c)_{\text{bf}}$, $k' = k/k_{\text{bf}}$ and $h_0 = H/L$. Since the temperature variation is driven by the source term we choose the temperature scale $\Delta T = s_0 H^2/k_{\text{bf}}$. This leads to the non-dimensional form of the model

$$\chi' \frac{\partial T}{\partial t} = k' \left(h_0^2 \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + e^{-s_1 y H} \quad y \in (0,1), x \in (0,1), \tag{23}$$

with initial $T(t=0) = 0$ and boundary conditions

$$T(0,y) = 0, \quad k' \left.\frac{\partial T}{\partial x}\right|_{x=1} = \text{Nu}_r \left(T + T^*\right)$$
$$k' \left.\frac{\partial T}{\partial y}\right|_{y=1} = 0, \quad k' \left.\frac{\partial T}{\partial y}\right|_{y=0} = \text{Nu}_t \left(T + T^*\right) \tag{24}$$

where $\text{Nu}_r = h_r L/k_{\text{bf}}$, $\text{Nu}_t = h_t H/k_{\text{bf}}$, $T^* = T_{\text{in}} - T_{\text{amb}}/\Delta T$. We can take advantage of the large aspect ratio of solar collectors, i.e. $h_0^2 \ll 1$, to simply the model. This allows the full model to be approximated, with error $\mathcal{O}(h_0^2)$, with the unsteady one-dimensional limit. The governing equations become

$$\chi' \frac{\partial T}{\partial t} = k' \frac{\partial^2 T}{\partial y^2} + e^{-s_1 y H} \quad y \in (0,1), t \in (t_0, t_e) \tag{25}$$

with initial $T(t=0) = 0$ and boundary conditions

$$\left.\frac{\partial T}{\partial y}\right|_{y=1} = 0, \quad k' \left.\frac{\partial T}{\partial y}\right|_{y=0} = \text{Nu}_t \left(T + T^*\right). \tag{26}$$

5 Exact solution

An exact solution can be obtained by separation of variables. This solution method requires the system to be homogeneous, so we make the substitution

$$v(y,t) = T(y,t) - w(y), \tag{27}$$

such that,

$$w_y(1) = 0, \quad k' w_y(0,t) = \text{Nu}_t (w(0,t) + T^*). \tag{28}$$

Substitution of (27) into the heat equation (25) gives

$$v_t = \alpha' (w_{yy} + v_{yy}) + \frac{1}{\chi'} e^{-s_1 y H}, \tag{29}$$

where $\alpha' = \alpha/\alpha_{\text{bf}}$. To ensure a homogeneous problem for $v(y,t)$ we impose $\alpha' w_{yy} + \frac{1}{\chi'} e^{-s_1 y H} = 0$. Applying the boundary conditions we find

$$w(y) = -\frac{1}{(s_1 H)^2 k'} e^{-s_1 y H} + ay + b \tag{30}$$

where $a = -\frac{1}{s_1 H k'} e^{-s_1 H}, b = \left(\frac{1}{s_1 H k'} + a\right) \frac{k'}{\text{Nu}_t} - T^* + \frac{\alpha}{(s_1 H)^2 \chi}$ and the problem is reduced to solving

$$v_t = \alpha' v_{yy}, \quad k' v_y(0,t) = \text{Nu}_t v(0), \quad v_y(1) = 0. \tag{31}$$

Letting $v(y,t) = Y(y)T(t)$ leads to two ordinary differential equations

$$T_t = -\alpha' \lambda^2 T, \quad Y_{yy} = Y, \tag{32}$$

where λ is a constant. The above has a solution $T(t) = A_0 e^{-\alpha' \lambda^2 t}$, and $Y(y) = C_1 \sin(\lambda y) + C_2 \cos(\lambda y)$, where $C_1 = C_2 \text{Nu}_t/k'\lambda$ and, to avoid a trivial solution, the eigenvalues must satisfy

$$\frac{\text{Nu}_t}{k'} - \lambda \tan(\lambda) = 0. \tag{33}$$

The general solution to (31) is

$$v(y,t) = \sum_{n=1}^{\infty} C_n e^{-t\alpha'\lambda^2} \left(\frac{\mathrm{Nu}_t}{k'\lambda} \sin(\lambda y) + \cos(\lambda y) \right), \quad (34)$$

where the coefficients are found by applying the initial condition $v(y,t=0) = -w(y)$

$$C_n = -\frac{\int_0^1 w \left(\frac{\mathrm{Nu}_t}{k'\lambda} \sin(\lambda y) + \cos(\lambda y) \right) dy}{\int_0^1 \left(\frac{\mathrm{Nu}_t}{k'\lambda} \sin(\lambda y) + \cos(\lambda y) \right)^2 dy}. \quad (35)$$

This concludes the separable solution to (25) where the summation truncated at $N = 7$ provided a good approximation.

6 Results and Discussion

The parameter values used are for those of a water based nanofluid. For solar applications the nanoparticles should be low-cost, but highly absorptive, such as graphite [10] which is used here. For the analysis the device height is fixed at $H = 0.005$ m and taking the convective heat transfer coefficient $h_t = 6.43$ [5], results in $\mathrm{Nu}_r = 0.054$. Considering a solar collector with length $L = 0.05$ m, we expect a reasonable model error of the order $\mathcal{O}(h_0^2) = 1\%$.

Figure 4 shows the temperature profiles of the unsteady one-dimensional model for various times $t = 20$ min and $t = 3, 6, 8, 10$ hours (dashed lines) with $\phi = 0.001$. The temperature reaches steady state after approximately 10 hours, this is shown as a solid line in the far right plot. For small times the temperature near the top of the collector is higher than at the bottom. However, as time progresses the temperature profile under goes an inversion. This is because the heat loss at the top of the receiver begins to balance the heat absorption, while the rest of the fluid continues to rise in temperature. Consequently, the change in temperature at the top is slower than at the bottom. The profile eventually settles to that of steady state. However, practically, there are rarely so many solar hours for the collector to reach steady state temperatures. So the performance of the collector will have to be maximised during the unsteady phase.

A comparison between Fig 5 and 6 shows that increasing the volume fraction has a large effect on the temperature profile for fixed H. The base fluid has a smoother temperature distribution across the receiver, however the bulk temperature (~ 303K) is significantly lower than that of the nanofluid with $\phi = 0.006$ (~ 468K). Therefore increasing ϕ will result in higher temperatures.

For practical reasons we will only consider results for $t \leq 6$ hours, and will now analyse the performance of the solar collector.

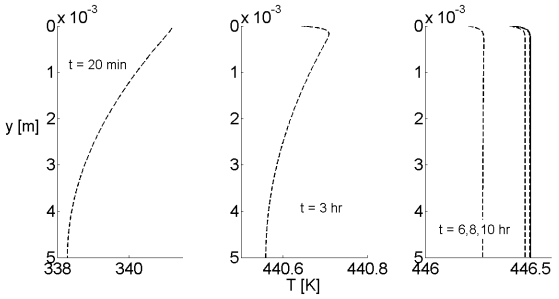

Figure 4: *Temperature profiles at various times with $\phi = 0.001$, $H = 0.005$. The dashed lines correspond to the unsteady solution while the solid line is the steady state solution.*

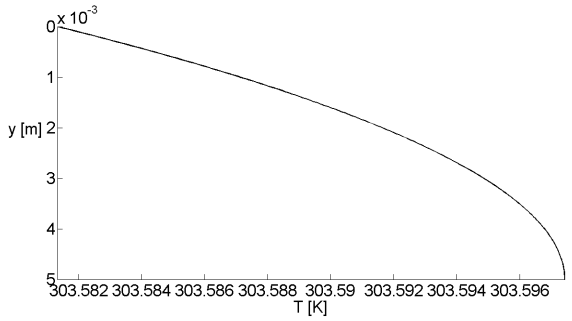

Figure 5: *The steady state temperature of the base fluid, $\phi = 0$.*

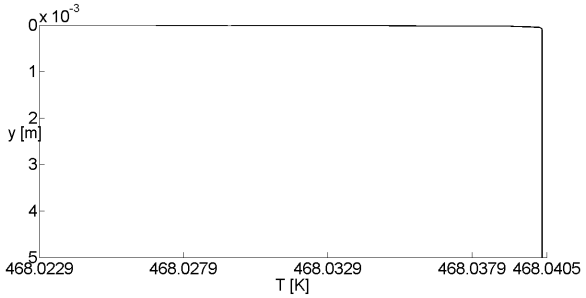

Figure 6: *The steady state temperature of the nanofluid with $\phi = 0.006$.*

6.1 Collector efficiency

The performance may be measured in two ways. First, the conversion from solar radiation to thermal energy needs to be optimised. This is quantified by the ratio of stored thermal energy to available incident solar energy [3, 5]. The total energy stored by a stationary fluid at time $t = t_e$ is

$$E_s = \frac{mc(\bar{T} - T_{\mathrm{in}})}{t_e}, \quad (36)$$

$m = \rho HA$ is the fluid mass and A is the surface area of the receiver [3]. The the average temperature $\bar{T}$ is defined by an energy balance [7]

$$\int_0^H cm(\bar{T} - T_{\mathrm{in}})dy = \int_0^L cm(T - T_{\mathrm{in}})dy. \quad (37)$$

Since $c, m, \bar{T}$ are constant in space we may write

$$\bar{T} = \frac{1}{H} \int_0^H cm(T - T_{\mathrm{in}})dy + T_{\mathrm{in}}. \quad (38)$$

The available solar incident energy is defined as $E_i = AG_s$, therefore

$$\eta_{rec} = \frac{E_s}{E_i} = \frac{\rho_{nf} Hc(\bar{T} - T_{\text{in}})}{t_e G_s}. \tag{39}$$

The second measure of performance involves with the receiver temperature. To measure this efficiency we make use of the Carnot number

$$\eta_{car} = 1 - \frac{T_{\text{amb}}}{\bar{T}}. \tag{40}$$

Now the total solar collector efficiency may be written as a product of (39) and (40)

$$\eta_{sys} = \eta_{rec}\eta_{car}. \tag{41}$$

Since we are considering a fixed collector height, the performance will only depend on the time exposed to solar radiation t_e and nanoparticle concentration ϕ. Figure 7 shows a comparison between the receiver, Carnot and total efficiencies against exposure times for varying volume fractions.

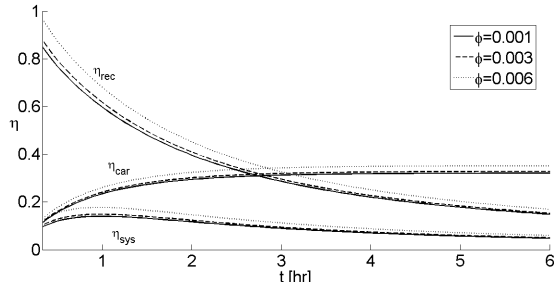

Figure 7: *Efficiencies plotted against time for varying ϕ values.*

Since the fluid is stationary we expect larger ϕ values to correspond to higher efficiencies, as shown in the plot. However, under this model the nanoparticle concentration is restricted at $\phi = 0.006$. Furthermore, the plot shows a maximum efficiency can be achieved at approximately $t = 1$ hour for all ϕ values. This suggests that the fluid should be circulated on an hourly basis. Comparing a nanofluid with $\phi = 0.006$ to the base fluid at $t = 1$ hour we note the receiver performance increases considerably from $\eta_{rec}^{\text{bf}} = 0.04$ to $\eta_{rec}^{\text{nf}} = 0.68$. And since the bulk temperature of the nanofluid base fluid ($\sim$ 408 K) is significantly higher than that of the base fluid ($\sim$ 303 K) we see a large difference in the respective Carnot numbers; $\eta_{car}^{\text{bf}} \approx 0$, $\eta_{car}^{\text{nf}} \approx 0.26$.

7 Conclusion

A two-dimensional model describing a flat, stationary solar collector subject to incident solar radiation is presented. The heat generation profile accounting for the attenuation of solar radiation through the collector is described by the radiative transport equation. By assuming independent scattering and absorption, the extinction coefficient is defined as a summation of four components; scattering and absorption by each constituent of the nanofluid. The full expression for the source term is then approximated by an exponentially decaying function so the governing equation can easily be integrated, allowing for an exact analytical solution by separation of variables. The model is further simplified by taking advantage of the small aspect ratio, which leads to a one-dimensional model. The performance of the solar collector is defined in terms of the working fluid temperature by means of the Carnot number and a comparison of total energy stored to the available solar incident energy.

It is shown that higher efficiencies may be achieved with higher particle volume fractions. This is due to the higher operating temperatures of the nanofluid as well as greater solar radiation absorption capabilities. However, under this model we are restricted to $\phi < 0.006$ due to the independent scattering assumption [2, 9]. Furthermore, the variation of the efficiencies against time suggests there is an optimal exposure time; it is best to circulate the fluid on an hourly basis. Note, this is for a particular configuration, we will have to adjust the model parameters to obtain circulation times for various nanofluids and device properties.

The direct relationship between volume fraction and efficiency may not be so obvious when considering non-stationary fluids. A worthwhile future extension to this work is to incorporate a non-uniform flow into the model, which will then become a two-dimensional convection-diffusion model. Then the maximum efficiency won't necessarily correspond to maximum particle concentration. This is because as ϕ increases the viscosity also increases, thus requiring more energy to move the fluid. So there will be a trade off between higher particle volume fraction and energy required to move the fluid.

[1] Otanicar, Todd P., Patrick E. Phelan, and Jay S. Golden. Solar Energy 83.7 (2009): 969-977.

[2] Veeraragavan, Ananthanarayanan, et al. International Journal of Heat and Mass Transfer 55.4 (2012): 556-564.

[3] Lenert, Andrej, and Evelyn N. Wang. Solar Energy 86.1 (2012): 253-265.

[4] Otanicar, Todd P., et al. Journal of renewable and sustainable energy 2.3 (2010): 033102.

[5] Tyagi, Himanshu, Patrick Phelan, and Ravi Prasher. Journal of solar energy engineering 131.4 (2009): 041004.

[6] Myers, T. G., M. M. MacDevette, and H. Ribera. Journal of nanoparticle research 15.7 (2013): 1-11.

[7] MacDevette, M. M., T. G. Myers, and B. Wetton. Microfluidics and Nanofluidics (2013): 1-12.

[8] Saidur, R., et al. International Journal of Heat and Mass Transfer 55.21 (2012): 5899-5907.

[9] Brewster, M. Q., and C. L. Tien. Journal of Heat Transfer 104.4 (1982): 573-579.

[10] Taylor, Robert A., et al. Nanoscale research letters 6.1 (2011): 1-11.