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OPTIMAL DESIGN OF STATIONARY FLAT-PLATE SOLAR COLLECTORS: DETERMINISTIC AND PROBABILISTIC APPROACHES

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ABSTRACT

Because of the soaring energy prices, many countries have shown an increased interest in the utilization of solar energy. The optimization of the solar energy collector design plays a critical role in the efficient collection of solar energy. Flat-plate collectors can be designed in applications that require energy delivery at moderate temperatures (up to 100˚C above ambient temperature). These collectors use both beam and diffuse solar radiation, and do not need to track the sun. They are simple to manufacture and install with relatively low maintenance cost which make this kind of solar collectors more popular. The design of a flat-plate solar collector embraces many relationships among the collector parameters, field parameters and solar radiation data at any given location. The shading decreases the incident energy on collector plane of the field. The multi-objective optimum design of stationary flat-plate solar collectors is presented in this work. The clear day solar beam radiation and diffuse radiation at the location of the solar collector (Miami) are estimated. The maximization of the annual average incident solar energy, maximization of the lowest month incident solar energy and minimization of the cost are considered as objectives.. The game theory methodology is used for the solution of the three objective problems to find the best compromise solution. The sensitivity analysis with respect to the design variables and the solar constant are conducted to find the relative influence of the parameters on the design. The multi-objective optimum design of stationary flat-plate solar collectors under probabilistic uncertainty is also considered. The three objectives stated earlier are considered in the optimization problem. The solar constant, altitude, typical day of each month and most of the design variables have been treated as probabilistic variables following normal distribution. The game theory methodology is used for the solution of the three objective constrained optimization problems to find a balanced solution. A parametric study is conducted with respect to changes in the standard deviation of the mean values of design variables and probability of constraint satisfaction. This work represents a novel application of the multi-objective optimization strategy, including probabilistic approach, for the solution of the solar collector design problem. The present study is expected to help designers in creating optimized solar collectors based on any specified requirements.

INTRODUCTION

Solar energy is by far the Earth's most important available energy source that could be utilized. It is estimated that thirty minutes of solar radiation falling on earth is equal to the world energy demand for one year [1]. But until now solar energy has remained as the most expensive energy among the various alternative energies. The improvement of the efficiency as well as reducing the cost of solar energy system is a current topic of interest to many investigators. Several investigations focus on research related to new materials for thermal or photovoltaic solar panels in order to improve the efficiency [2-4]. Some researchers [5-8] focus on the design optimization based on the existing materials of solar panels in order to obtain maximum efficiency and reduced cost for the whole mechanical system. Flat-plate collectors are most economical and popular among the various types of solar collector systems since they are permanently fixed in positions, involve simple construction, and require little maintenance. The design of a solar energy system is generally concerned with obtaining maximum energy, maximum efficiency or minimum cost. The flat plate solar collectors are used in many different applications, such as air conditioning, industrial processes, domestic water heating and space-heating. Figure 1 shows a multi-row flat-plate solar collector. An increase in the number of rows and the size of solar collector can increase the total solar panel area but it also increases the shading area (darker area shown in Figure 1) which will decrease the total amount of solar radiation. Hence, there are multiple schemes for the optimal deployment of solar collectors in any specific area depending on the objective chosen. Weinstock and Appelbaum [6] formulated different optimization objectives including maximum incident energy on the collector plate, minimum field area for a given incident energy, and maximum energy per unit collector area. Hu and Rao [9] extended the model and applied a multi-objective scheme to consider different objective functions using a deterministic formulation.

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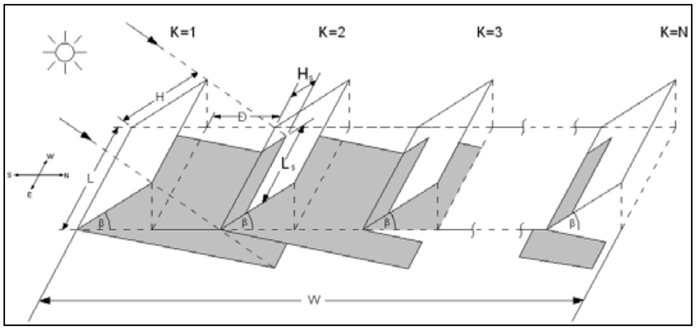


Figure 1 Multi-row flat plate collector in a given location

While most of the studies related to the analysis and design of solar collectors considered only deterministic approaches, this work presents the multi-objective optimization of flat plate solar collectors using both deterministic and probabilistic approaches. The probabilistic approach is more realistic because, in practice, most of the parameters influencing the performance of solar collectors are uncertain. For example, the incident solar radiation measured in terms of the solar constant at any location varies from instant to instant. Only for convenience and simplicity, an average value of solar radiation is defined not only for each day but also for a typical day of the month/year. Available statistical analysis of solar radiation measurements acknowledges this variability / uncertainty. In addition, in the design and manufacture (or construction) of solar collectors, design parameters such as the width and length of a flat plate solar collector, distance between any two rows of solar collectors and the inclined angle of solar flat plate collectors are to be specified using tolerances, such as $\bar{x} \pm \Delta x$ where $\bar{x}$ is the mean value and Δx is the tolerance ($\Delta x = 3\sigma_x$ if the manufacturing / construction process follows normal distribution) of a design parameter. In the probabilistic optimization, many of the design variables as well as the solar constant, altitude and typical day of each month are treated as probabilistic variables following normal distribution.

A sensitivity analysis of the optimal design usually leads to the identification of the most and least influential parameters of the design. Since most design parameters are subject to variation due to random uncertainties, and manufacturing / installation errors; the sensitivity analysis results help in identifying the parameters that need to be controlled tightly. In this work, sensitivity analysis is conducted with respect to changes in the standard deviation of the design variables and the satisfaction level of the probabilistic constraints. This work is expected to help in a more realistic analysis and design of flat plate solar collectors. The optimization strategy presented and the results are expected to help designers to create optimized solar collectors depending on the specific requirements of the customers.

NOMENCLATURE

A	[m]	*Altitude*
c_i	[-]	Weight of objective function i, i = 1, 2, 3
D	[m]	Diatance between two collector rows
f_i	[-]	Objective function i, i = 1, 2, 3
F_{iU}	[-]	Worst value in game theory approach
FC	[-]	Pareto optimal objective
G_{sc}	[-]	Solar constant
H	[m]	Height (or mean value of height) of flat plate solar collector
h_S	[m]	Height of shaded area
K	[-]	Number of rows
N_d	[-]	Typical day for calculating the incident solar energy in each month
P, P	[-]	Probability of constraint satisfaction
Q	[W/m^2]	Solar constant
S	[-]	Supercriterion
W	[m]	Width of land

Special characters

β	[-]	Tilt angle, the angle between the plane of the surface in question and the horizontal
θ	[-]	Angle of incidence; the angle between the beam radiation on a surface and the normal to the surface
$\overline{x}$	[-]	Mean value of x
σ_x	[-]	Standard deviation of x

FORMULATION OF THE DETERMINISTIC OPTIMIZATION PROBLEM

Three objectives – the maximization of annual average incident energy, the Maximization of average incident solar energy for the lowest month, and minimization of cost – are considered for optimization. The detailed formulation of these objectives are indicated below.

Maximization of annual average incident solar energy

The problem of optimization of the solar collector design is to obtain maximum incident energy on a given horizontal and fixed flat-plate collector of dimensions L × W (length × width). The solar collector system (Figure 1) includes K rows of solar collectors with distance D between two neighboring rows and each collector is of length L and height H and inclined at an angle β with respect to the horizontal line. The design vector of the problem is:

$$\vec{X} = \begin{Bmatrix} H \\ L \\ D \\ \beta \\ K \end{Bmatrix} \quad (1)$$

The average incident solar energy of the field (for maximization) is given by:

$$Q = H\,L[q_b + q_d + (K-1)(q_b^{sh} + q_d^{sh})] \quad (3)$$

and the objective function for minimization is taken as:

$$f_1(\vec{Y}) = -Q = -H\,L[q_b + q_d + (K-1)(q_b^{sh} + q_d^{sh})] \quad (4)$$

The expressions for computing q_b, q_d, q_b^{sh}, q_d^{sh} are given in Appendix A. The constraints of the optimization problem can be stated as follows.

The total width (length) of the collectors must be less than or equal to the maximum width (length) of the available land:

$$K\,H\cos\beta + (K-1)\,D - W \le 0 \quad (5)$$

along with

$$L_{\min} \le L \le L_{\max} \quad (6)$$

The distance between two adjacent collector rows (spacing) must be larger than the minimum distance specified by the relevant standards:

$$D \ge D_{\min} \quad (7)$$

The height of the collector above the ground may have a limitation based on the installation and maintenance requirements:

$$H\,\sin\beta \le Y_{h\max} \quad (8)$$

The collector tilt angle is required to vary in the range of 0° to 90°:

$$0^o \le \beta \le 90^o \quad (9)$$

The number of rows should be more than two but less than a specified maximum number, $K_{\max}$:

$$2 \le K \le K_{\max} \quad (10)$$

Maximization of average incident solar energy for the lowest month

In general, the incident solar energy is more in summer than in winter; however, consumers need more heat energy in winter. It is therefore necessary to consider the maximization of incident solar energy for the lowest month as the objective function. The design variables and constraints will be same as in the case of the optimization problem formulated in section 2.2. The lowest incident solar energy month from the twelve month information can readily be found to evaluate the value of the objective function. Then the objective function for minimization is given by

$$f_2 = \text{- lowest incident monthly solar energy} \quad (17)$$

Minimization of cost

Another important objective function in the design of a solar collector is to minimize the cost. The design variables are same as those indicated in Eq. (7). The objective function (cost) to be minimized can be expressed as:

$$f_3 = Cost = s_1\,LW + s_2\,L\,H\,K \quad (18)$$

where s_1 is the unit cost of the land and s_2 is the unit cost of the collector. Note that additional cost components such as those associated with piping, heat exchanger, pump and backup energy could be added to the objective function if necessary.

The following additional constraints are considered while minimizing f_3.

The daily average incident solar energy in any month should be at least 60% of the daily optimum value found in the case of the problem described in section 2.2:

$$Q - 60\% \times H\,L[q_b + q_d + (K-1)(q_b^{sh} + q_d^{sh})] \le 0 \quad (19)$$

The average incident solar energy for the lowest month should be at least 60% of the daily optimum value found in the case of the problem describe in section 2.3:

$$Q_{\min} - 60\% \times H\,L[q_b + q_d + (K-1)(q_b^{sh} + q_d^{sh})]_{\min} \le 0 \quad (20)$$

The dimensions of the solar collector are bounded as:

$$b_1 \le \frac{W}{L} \le b_2 \quad (21)$$

where b_1 and b_2 are constants.

Formulation of the probabilistic optimization problem

A general deterministic optimization problem can be stated in the following general form:

Minimize or maximize f($\vec{X}$) with respect to $\vec{X}$, subject to

$$g_j(X) \le 0, \qquad j = 1,2,....m \quad (1)$$

$$a_k \le x_k \le b_k, \qquad k = 1,2,....l \quad (2)$$

where x_k is the k^{th} component of the design vector $\vec{X}$, and a_k and b_k are the lower and upper bounds on the design variable x_k, respectively.

When some of the parameters involved in the objective function and / or constraints vary about their respective mean values, the optimization problem needs to be formulated as a probabilistic programming problem. A probabilistic nonlinear programming problem can be stated as:

Find X which minimizes f(Y) (3)

subject to

$$P[g_j \le 0] \ge p_j, \qquad j = 1,2,...,m \quad (4)$$

where Y is the vector of N random variables $y_1, y_2, ..., y_N$ that might include the decision variables $x_1, , x_2, \ldots, x_l$. The case when X is deterministic can be obtained as a special case of the present formulation. Equations (4) denote that the probability of realizing $g_j(Y)$ less than or equal to zero must be greater than or equal to the specified minimum probability p_j. The problem stated in Eqs. (3) and (4) can be converted into an equivalent deterministic programming problem as Eqs.(5) and (6) by applying the chance constrained programming technique [5].

Find Y which minimizes

$$F(Y) = k_1 \overline{\psi} + k_2 \sigma_\psi \quad (5)$$

subject to

$$\overline{g}_j + \phi_j(p_j) \left[\sum_{i=1}^{N} \left(\frac{\partial g_j}{\partial y_i}\bigg|_{\bar{Y}} \right)^2 \sigma_{y_i}^2 \right]^{1/2} \geq 0, \quad j = 1,2,...,m \quad (6)$$

where k_1 and k_2 indicate the relative importances of $\overline{\psi}$ and σ_ψ for minimization with

$$f(Y) \cong f(\overline{Y}) - \sum_{i=1}^{N} \left(\frac{\partial f}{\partial y_i}\bigg|_{\bar{Y}} \right) \overline{y}_i + \sum_{i=1}^{N} \left(\frac{\partial f}{\partial y_i}\bigg|_{\bar{Y}} \right) y_i \equiv \psi(Y) \quad (21)$$

and

$$Var(\psi) = \sigma_\psi^2 = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial y_i}\bigg|_{\bar{Y}} \right)^2 \sigma_{y_i}^2 \quad (23)$$

Uncertainty of parameters

As stated earlier, the design variables, H, L, D and β are random due to the manufacturing tolerances used during production / construction of flat plate solar collectors. In any given city or location, the altitude (A) may vary, for example, when some solar panels are installed near a beach while others are installed on the roof of a skyscraper. Thus it becomes necessary to treat the altitude as a random variable. The solar constant (G_{sc}) denotes the amount of Sun's incoming radiation per unit area, measured on the outer surface of Earth's atmosphere in a plane perpendicular to the rays. The solar constant includes all types of solar radiation, not just the visible light. Several investigators measured the solar constant since 1884 and found that the solar constant varies between 1318 and 1548 W/m². At present, the value of the solar constant, as measured by satellites, is found to be roughly 1367 watts per square meter (W/m²), although the value fluctuates by about 6.9% during the year (from 1412 W/m² in early January to 1321 W/m² in early July) due to the variations in the distance between Earth and Sun. In fact, the value of the solar constant was found to vary by few parts per thousand from day to day [12]. Thus the solar constant is considered to be a random variable.

Instead of computing the incident solar energy for each day of the month and then finding the average, a typical day of each month is chosen in this work in order to reduce the computational effort during optimization. Klein [13] used both numerical and experimental methods to find a typical day which would represent the average radiation of each month as indicated in Table 1.

Table 1 Recommended Average Days of Months [21]

Jan.	Feb.	Mar.	Apr.	May	Jun.
17	16	16	15	15	11
Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
17	16	15	15	14	10

In this work, the fifteenth day of each month (the mean value of the average days indicated in Table 1) as the typical day for calculating value of the incident solar energy per day in any month, with 1% ~ 5% of the mean value chosen as the standard deviation of the incident solar energy in the formulation of the probabilistic optimization problem.

The conversion of probabilistic objective and constraint functions to equivalent deterministic form requires the partial derivatives of f and g_j with respect to the random variables y_k (k = 1, 2, …, N) as indicated in Eqs. (23) and (33). These derivatives are computed numerically using a finite difference scheme in this work.

Solution of Multi-objective optimization problem

The problem of multi-objective optimization of the solar collector design can be stated in the standard form: Minimize the objective functions

$$f_1(\vec{X}), f_2(\vec{X}),..., f_k(\vec{X}) \quad (22)$$

with respect to the design vector

$$\vec{X} = \{x_1 \quad x_2 \quad ... \quad x_l\}^T \quad (23)$$

subject to the constraints

$$g_j(\vec{X}) \leq 0, j = 1,2,...m \quad (24)$$

$$a_k \leq x_k \leq b_k, k = 1,2,...,l \quad (25)$$

The problem stated in Eqs.(34)-(36) is solved using a modified game theory approach in this work. For this, the single objective constrained optimization problems are solved using nonlinear programming techniques (sequential quadratic programming). The omputational procedure for implementing the modified cooperative game theory can be stated as follows:

(1) Normalize the objectives so that no objective due to its magnitude will be favored. The following normalization

procedure gives zero as optimum value and one as the worst value of ith objective function:

$$f_{ni}(X) = \frac{f_i(X) - f_i(X_i^*)}{F_{iu} - f_i(X_i^*)} \quad (26)$$

where F_{iu} is the worst value, and $f_i(X_i^*)$ is the optimum value of the ith objective.

(2) Formulate a supercriterion S as the product of deviations of all objective functions from their respective worst values:

$$S = \prod_{j=1}^{k} \{1 - f_{ni}(X)\} \quad (27)$$

(3) Formulate a Pareto optimal objective FC using a weighted sum method as:

$$FC = \sum_{i=1}^{n} c_i f_{ni}(X) \quad (28)$$

where the sum of the weights c_i is equal to one.

(4)Since FC has to be minimized and S has to be maximized, a new objective is constructed as (for minimization):

$$OBJ = FC - S \quad (29)$$

subject to all the constraints. The minimization of OBJ gives the compromise Pareto optimal solution of the multi-objective optimization problem.

2. Illustrative example and numerical results

A numerical example is considered to illustrate the game theory approach for the multi-objective optimum design of stationary flat plate collectors. The following data are assumed:
L_{min} = 15m, L_{max} = 30m, H_{min} = 0.5m, H_{max} = 2m, W_{min} = 15m, W_{max} = 30m, $Y_{h\ max}$ = 2m, D_{min} = 0.8m, β_{min} = 50, β_{max} = 200, K_{min} = 50, K_{max} = 200, s_1 = 100 \$/m^2, s_2 = 100 \$/m^2.
The solar collector is assumed to be installed in a specific location, Miami (USA), where the latitude is 25.4°N and the altitude is 5m, and the solar collector is assumed to face the equator (south). The starting design vector is chosen as:

$$\vec{X} = [H \quad L \quad D \quad \beta \quad K]^T = [1.5 \quad 27 \quad 0.9 \quad 40 \quad 80]^T$$

Table 2 INITIAL DESIGN AND DETERMINISTIC SINGLE OBJECTIVE OPTIMIZATION RESULTS

Parameter	H	L	D	β	K	f_1	f_2	f_3
Unit	m	m	m	Deg	#	10^6×W	10^6×W	10^6×\$
Initial	1.80	27.00	0.90	40.00	80.00	-1.06	-0.89	0.88
Min f1	2.00	30.00	0.80	35.36	83.00	-1.37	-1.10	1.10
Min f2	2.00	30.00	0.80	53.43	101.00	-1.34	-1.11	1.20
Min f3	2.00	21.66	0.85	30.00	67.00	-0.84	-0.68	0.66

4.1 Deterministic optimization

The initial design and the results of deterministic optimization are shown in Tables 1 and 2(a) – (c). It can be seen that the height of the collector (H) reached its upper bound in all single as well as multi-objective optimizations. The length of the collector (L) attained its upper bound and the distance between the collector rows (D) attained its lower bound in the case of minimizations of f_1, f_2 and multi-objective optimization. The relative weights of the objective functions f_1, f_2 and f_3 at the compromise solution achieved by the game theory are 0.8, 0.1 and 0.1, respectively. This indicates that the first objective function (annual solar energy) dominates the compromise solution as per the supercriterion used. The multi-objective (compromise) solution corresponds to a value of f_1 that is 0.17% worse than the best value and 63.14% better than the worst possible value, a value of f_2 that is 1.52% worse than the best value and 60.34% better than the worst possible value, and a value of f_3 that is 62.38% worse than the best value and 12.00% better than the worst possible value.

Table 3 INITIAL DESIGN AND DETERMINISTIC MULTI-OBJECTIVE OPTIMIZATION RESULTS

Parameter	H	L	D	β	K
Unit	m	m	m	Deg	#
Initial design	1.8	27	0.9	40	80
Multi-objective optimization	2	30	0.8	30	79

Table 4 INITIAL DESIGN AND DETERMINISTIC MULTI-OBJECTIVE OPTIMIZATION RESULTS (WEIGHT OF EACH SINGLE OBJECTIVE FUNCTIONS)

Weights of objectives	c_1	c_2	c_3
Unit	-	-	-
Initial Design	0.3333	0.3333	0.3334
Multi-objective optimization	0.8	0.1	0.1

Table 5 INITIAL DESIGN AND DETERMINIST MULTI-OBJECTIVE OPTIMIZATION RESULTS (OBJECTIVE FUNCTION VALUES)

Item	f_1 w/o standard deviation	f_2 w/o standard deviation	f_3 w/o standard deviation	Obj (FC-S)
Unit	10^6×W	10^6×W	10^6×\$	-
Initial Design	-1.057	-0.8942	0.8786	0.377
Multi-objective optimum	-1.3665	-1.0966	1.0758	-0.1439

4.2 Probabilistic optimization

Single objective optimization: Minimization of $(\overline{f_1} + \sigma_{f_1})$

Figures 2 (a) – (c) give the values of $(\overline{f_i} + \sigma_{f_i})$, (i = 1, 2, 3) at optimum solutions found by minimizing $(\overline{f_1} + \sigma_{f_1})$. It was noticed that the variations of the mean values of objective functions have almost the same trends as exhibited by Figs. 2 (a), (b) and (c), respectively. This is because the mean $\overline{f_i}$ of objective i (i = 1, 2, 3) is much greater than its standard deviation, σ_{f_i} (i = 1, 2, 3). With an increase in the level of constraint satisfaction probability, the absolute values of the objective functions decrease because of tighter constraints. It can be seen from Figs. 2 (a) – (c) that with increasing values of the constraint satisfaction level up to 99% (at constant standard deviations of random variables), the change in the objective function values is gradual and is less than or equal to about 10%, whereas at the 99.99997% constraint satisfaction level, the optimum values suddenly shoot to a newer levels and as a result, the objective function values lie far away from the base values. A larger value of the standard deviation of all random variables makes the absolute value of each objective function decrease rapidly.

Single objective optimization: Minimization of $(\overline{f_2} + \sigma_{f_2})$

In this case, the minimization of $(\overline{f_2} + \sigma_{f_2})$ is carried for different values of probabilities specified for constraint satisfaction using several values of the coefficient of variation for all the random variables. It is observed that the minimizations of $(\overline{f_1} + \sigma_{f_1})$ and $(\overline{f_2} + \sigma_{f_2})$ exhibited similar behaviors. The primary reason for this behavior can be attributed to the low altitude of the specific location (Miami) used in the numerical computations. For example, the variation in the monthly average temperature between summer months and winter months is not as significant as for locations such as Chicago. The lowest temperate generally recorded in the month of January in Miami is between 60 and 76 °F. This is almost 90% of the yearly average temperature. Gong et al. [13] presented the output in winter and the yearly average output of a photovoltaic system in Carbondale, Illinois based on different designs. The energy output in winter is only 50-70% of the yearly average energy output depending on different design. Wei et al. [14] compared the solar radiation in two cities of China, Kunming (latitude 25.04° N) and Beijing (latitude 39.55°N). They concluded that the solar collectors in Kunming have much less seasonal fluctuation in yearly energy output compared to those in Beijing. The present approach and methodology can be applied to any location using the corresponding solar data.

Single objective optimization: Minimization of $(\overline{f_3} + \sigma_{f_3})$

Figures 3 (a) – (c) present the values of $(\overline{f_i} + \sigma_{f_i})$ (i = 1, 2, 3) for different values of probability of constraint satisfaction at different values for the coefficient of variation of the random variables. It has been observed that the variations of the mean values of different objectives are found to be similar to those of Figs. 3 (a) - (c). This is because the mean value is very high compared to the standard deviation for any objective function i.
When the probability of constraint satisfaction is 50%, all the values of $\overline{f_i}$, (i = 1, 2, 3), remain the same irrespective of the standard deviation of the design variables. However, as indicated in Figs. 3 (a) to (c), at any particular value of constraint satisfaction including 50%, curves corresponding to different values of the standard deviations of the random variables lead to different values of σ_{f_i}, (i = 1, 2, 3) with larger standard deviations resulting in larger values.
With increasing values of probability of constraint satisfaction, the absolute values of the objective functions increased because of tighter constraints. Larger values of standard deviation of the random variables resulted in a rapid increase in the absolute values of each of the objectives. The mean values of the objective functions, $\overline{f_1}$ and $\overline{f_2}$, varied between -40% and 0% of the respective baseline values and exhibit similar nature whereas the value of $\overline{f_3}$, is found to have a conflicting behavior and varied between 0 and 40% of the baseline value.

Multi-objective optimization using game theory

The results of multi-objective optimization (using game theory) obtained with the coefficient of variation of the random variables varying from 1% to 5% are given in Table 3 and with the probability of constraint satisfaction varying from 50% to 99.99997% are given in Table 4. It is observed that with a coefficient of variation of random variables equal to 1%, the optimum values of design variables D (distance between two neighboring rows of collectors) attained its lower bound value and the design variables H (height of collector) and L (length of solar collector) attained their upper bound values when the probability of constraint satisfaction is 50%. As the probability of constraint satisfaction increased from 50% to 99.99997%, the optimum values of these design variables deviated gradually from their respective bounds. The optimum value of the tilt angle of the collector (the design variable β) increased gradually from 30º to 34º and the optimum number of rows of collectors (the design variable K) essentially remained constant at 79 as the probability of constraint satisfaction increased from 50% to 99% and slightly reduced to 78 as the probability of constraint satisfaction finally increased to 99.99997%.

Table 6 EFFECT OF VARIABILITY ON MULTI-OBJECTIVE OPTIMIZATION
(PROBABILITY OF CONSTRAINT SATISFACTION: 95%)

c.v. of uncertain parameters	Optimal design parameters (mean values)					Optimal objective function (FC-S), at $\overrightarrow{X^*}$	value† of $\vec{f}=\begin{Bmatrix}\bar{f}_1\\ \bar{f}_2\\ \bar{f}_3\end{Bmatrix}$ at $\overrightarrow{X^*}$
	H (m)	L (m)	D (m)	β (deg)	N		
0.01	1.9676	29.5145	0.8134	31.3127	79	-0.1321	$\begin{Bmatrix}-1.3080\\ -1.0566\\ 1.0361\end{Bmatrix}$
0.02	1.9363	29.0444	0.8272	32.6980	78	-0.1247	$\begin{Bmatrix}-1.2524\\ -1.0187\\ 0.9988\end{Bmatrix}$
0.03	1.9059	28.5891	0.8415	34.1617	78	-0.1251	$\begin{Bmatrix}-1.1994\\ -0.9828\\ 0.9637\end{Bmatrix}$
0.04	1.8765	28.1479	0.8563	35.7100	78	-0.1385	$\begin{Bmatrix}-1.1487\\ -0.9487\\ 0.9306\end{Bmatrix}$
0.05	1.8480	27.7200	0.8717	37.3499	78	-0.1440	$\begin{Bmatrix}-1.1001\\ -0.9162\\ 0.8994\end{Bmatrix}$

†: Units: 10^6 W for f_1, 10^6 W for f_2, 10^6 \$ for f_3.

The annual average incident solar energy decreased from 1.3665 × 10^6 W to 1.1974 × 10^6 W and the average incident solar energy for the lowest month also decreased from 1.0966 × 10^6 W to 0.9814 × 10^6 W and the cost of the solar collector decreased from \$ 1.0758 × 10^6 to \$ 0.9624 × 10^6 as the probability of constraint satisfaction increased from 50% to 99.99997%. The maximum value of game theory objective, -(FC - S), also decreased from 0.1407 to 0.1320 as the probability of constraint satisfaction varied from 50% to 99.99997%.

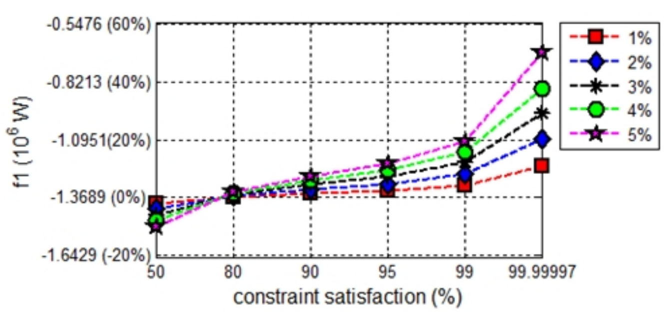
An observation on the variation of the weights (c_1, c_2, c_3) used in the Pareto optimal solutions indicated that f_1 is dominant in all the multi-objective optimization problems. The weight of the objective function of f_1 is found to drop from 80% to 71% with an increase in either the standard deviation of the random variables or the probability of constraint satisfaction. The increase in the weight of the second objective function (f_2) is found to be very small (10% to 11%). In the case of f_3, the increase in the weight (c_3) is relatively large (10% to 16%).

Figures 4 (a) – (c) give the values of $\bar{f}_i+\sigma_{f_i}$, (i = 1, 2, 3) at the optimum solutions obtained by multi-objective optimization. It has been observed that the mean values of the three objective functions exhibited trends similar to those of corresponding Figures 4(a) – (c). The reason is that the mean value is much larger than the standard deviation for each objective function.

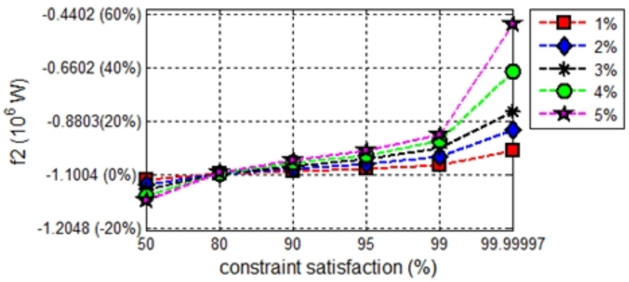
Table 7 CONSTRAINT SATISFACTION PROBABILITY ON MULTI-OBJECTIVE OPTIMIZATION
(COEFFICIENT OF VARIATION (c.v.) OF UNCERTAIN PARAMETERS: 0.01)

c.v. of uncertain parameters	Optimal design parameters (mean values)					Optimal objective function (FC-S), at $\overrightarrow{X^*}$	value† of $\vec{f}=\begin{Bmatrix}\bar{f}_1\\ \bar{f}_2\\ \bar{f}_3\end{Bmatrix}$ at $\overrightarrow{X^*}$
	H (m)	L (m)	D (m)	β (deg)	N		
0.50	2.0000	30.0000	0.8000	30.0000	79	-0.1407	$\begin{Bmatrix}-1.3665\\ -1.0966\\ 1.0758\end{Bmatrix}$
0.80	1.9833	29.7492	0.8068	30.6640	79	-0.1345	$\begin{Bmatrix}-1.3362\\ -1.0758\\ 1.0552\end{Bmatrix}$
0.90	1.9747	29.6203	0.8104	31.0170	79	-0.1339	$\begin{Bmatrix}-1.3207\\ -1.0652\\ 1.0447\end{Bmatrix}$
0.95	1.9676	29.5145	0.8134	31.3127	79	-0.1321	$\begin{Bmatrix}-1.3080\\ -1.0566\\ 1.0361\end{Bmatrix}$
0.99	1.9545	29.3172	0.8191	31.8796	79	-0.1261	$\begin{Bmatrix}-1.2846\\ -1.0406\\ 1.0204\end{Bmatrix}$
0.9999997	1.9048	28.5714	0.8421	34.2212	78	-0.1320	$\begin{Bmatrix}-1.1974\\ -0.9814\\ 0.9624\end{Bmatrix}$

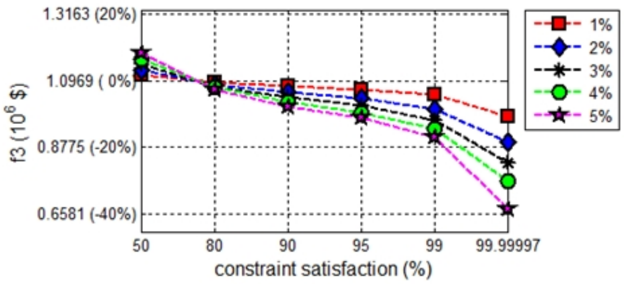
In Figs. 4 (a) to (c), when the probability of satisfaction is 50%, different objective functions have different standard deviations and hence the curves start at different points on the vertical axis. Although a 50% constraint satisfaction with relatively larger standard deviation can result in superior energy output, it may not be suitable for practical applications. With an increase in the probability of constraint satisfaction, the absolute values of the objective functions decrease because of tighter constraints.



(a) plots of f1 (mean + standard deviation)

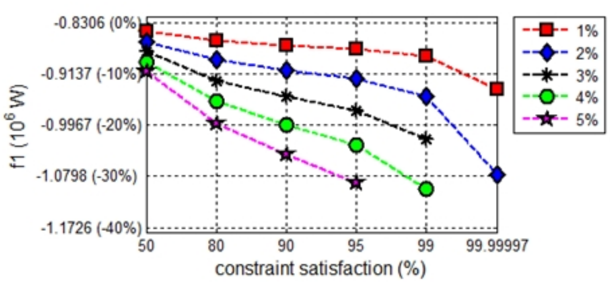


(b) plots of f2 (mean + standard deviation)

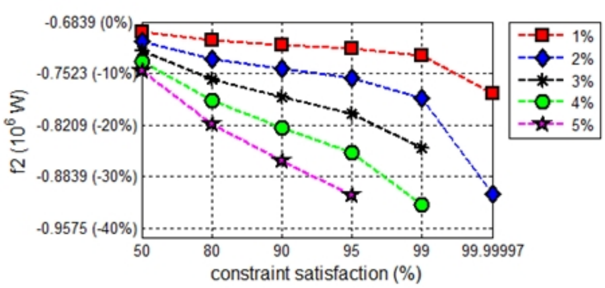


(c) plots of f3 (mean + standard deviation)

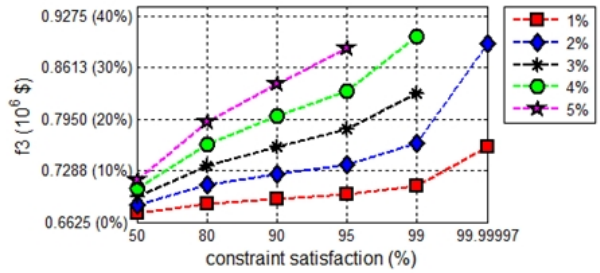
Figure 2. PROBABILISTIC SINGLE OBJECTIVE OPTIMIZATION; MINIMIZATION OF $\overline{f_1} + \sigma_{f_1}$



(a) plots of f1 (mean + standard deviation)

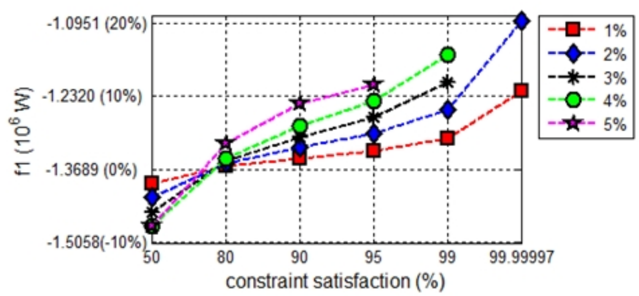


(b) plots of f2 (mean + standard deviation)

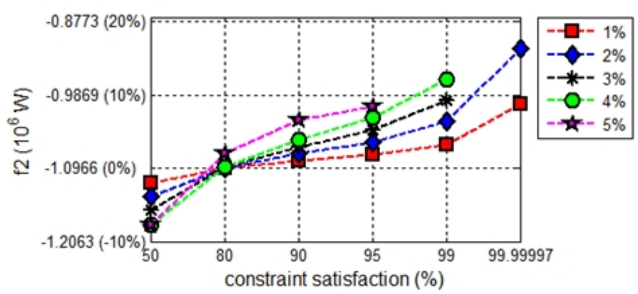


(c) plots of f3 (mean + standard deviation)

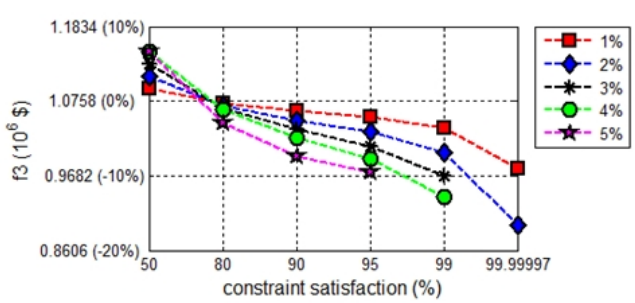
Figure 3. PROBABILISTIC SINGLE OBJECTIVE OPTIMIZATION; MINIMIZATION OF $\overline{f_3} + \sigma_{f_3}$



(a) plots of f1 (mean + standard deviation)



(b) plots of f2 (mean + standard deviation)



(c) plots of f3 (mean + standard deviation)

Figure 4. PROBABILISTIC MULTI-OBJECTIVE OPTIMIZATION (GAME THEORY); VARIATION OF OBJECTIVE FUNCTIONS

3. Conclusion

The multi-objective optimum design of flat-plate solar collectors is presented with a consideration of solar radiation with shading effect. Three objectives, namely, the maximization of the annual average incident solar energy, the maximization of the lowest month incident solar energy and minimization of the cost, are considered. Game theory methodology is used for the solution of the three objective constrained optimization problems to find a balanced solution. The solution represents the best compromise in terms of the super-criterion selected. Numerical results are obtained at a specific location (Miami, USA). Most of the design variables and the altitude, solar constant and typical day of each month are treated as random variables following normal distribution in the probabilistic approach example. When the probability of constraint satisfaction is 50%, all the design variables remain the same as the determinist optimum solution irrespective of standard deviation values of the design variables. The standard deviation of each of the random parameters is varied from 1% to 5% of the respective mean values to find the influence of uncertainty on different objective functions. The numerical results are given to show the influence of the level of probability of constraint satisfaction and the coefficient of variation of the random variables. It is observed that the absolute value of each objective function is decreased with an increase in either the probability of constraint satisfaction or the coefficient of variation of the random variables. Better objective function values can be obtained with a lower value of probability of constraint satisfaction, but it might not be suitable (safe) for practical applications. A relatively higher constraint satisfaction (like 99.9997%) would result in worse objective function values. The results of the present study help designers in producing optimum solar collectors based on customer requirements. As seen from the present results, there is a trade-off between the absolute values of the various objectives and the probability of constraint satisfaction. From practical point of view, an increase in the overall objective implies improvement in a combination of annual energy output, winter energy output and cost of manufacture. With a higher probability of constraint satisfaction, the manufacture has to sacrifice the energy values as well as the profit if the costs of raw and processed materials are relatively large.

Acknowledgment

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ANNEX A
CALCULATION OF SOLAR RADIATION

Since the atmospheric condition and air mass always change, the scattering and absorbing radiation also vary with time; thus it is difficult to accurately estimate the amount of solar radiation. It is therefore necessary to define a standard "clear" sky and calculate the hourly radiation that would be received on a horizontal surface under these standard conditions at a given location. Hottel [8] provided a method of estimating the beam radiation transmitted through clear atmosphere, which takes into account the zenith angle. The transmittance of the standard atmosphere for beam radiation can be determined for any zenith angle and any altitude up to 5 km. The clear-sky beam radiation is given by:

$$G_{cnb} = G_{on}\tau_b$$

where $G_{on} = G_{sc}(1 + 0.033\cos\frac{360n}{365})$ (30)

and G_{sc} is the solar constant (a value of 1367 W/m^2 (Iqbal [15]) is used for G_{sc} in this work). The clear-sky horizontal beam radiation can be determined as

$$G_b = G_{on}\tau_b \cos\theta_z \quad (31)$$

Liu and Jordan [16] developed an empirical relationship between the transmission coefficient for beam and diffuse radiation for clear days:

$$\tau_d = 0.271 - 0.294\tau_b \quad (32)$$

$$G_d = G_{on}\tau_d \cos\theta_z \quad (33)$$

The shaded and un-shaded irradiation per unit area (Eq.(22)) are:

$$Q = H\,L[q_b + q_d + (K-1)(q_b^{sh} + q_d^{sh})] \quad (34)$$

where the yearly beam and diffuse irradiations per unit area of an unshade collector (first row) and shaded collectors (rows 2 through K) can be computed using the procedure indicated by Hu and Rao [4].