BUBBLE FORMATION IN VISCOUS LIQUIDS CONTAINED IN CONES AND CYLINDERS

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ABSTRACT

In this work we consider the problem of growth and detachment of gas bubbles in axi-symmetric reservoirs filled with viscous liquids whose walls are very close to the gas injection orifice, in such a manner that the walls affects the bubble shapes and its maximum volume of growing. Using the Stokes equations for slow viscous flow, we have studied two cases of interest: a) the case where the walls make a vertical inverted cone, and b) the case of a cylindrical wall concentric to the injection orifice. In both cases the fluid flow equations were solved numerically by using the Boundary Element Method (BEM), and the results are given in terms of the bubble shapes, their maximum volumes and other properties of interest for different values of the Bond and Capillary numbers. We present a qualitative comparison with the experimental bubbles obtained at constant gas flow rates, in the air-glycerin and air-silicon oil systems. This comparison allows us to conclude that the numerical solutions describe very well this phenomenon. Our results also show that by changing the cone angle or the cylinder radius it is possible to obtain an efficient method to control the shape and size of the produced bubbles.

INTRODUCTION

Bubble generation by means of gas injection through orifices on flat plates is typical in theoretical and experimental studies. In cases where liquids have very low viscosity a lot of work has been made [1-5]. Moreover, applications of results derived from the inviscid approximation are useful for many industrial processes such as clean of metals, chemical reactors, because, commonly, operation conditions involve liquids of very low viscosity. On the other hand, in the case of highly viscous liquids, the presence of bubbles occurs, for example, in processes of liquid polymers, flows of lava and, in the recovery of oil in production pipelines [6-9]. The latter case is that we are most interested and motivated because the technique of gas lift (gas injection within the pipe) involves the formation of bubbles in confined spaces and with a narrow possibility of changing the volume gas flow rate Q injected into the crude oil [7]. In general, a fundamental study of bubbles in viscous liquids in semi-infinite systems (in the absence of walls) is well advanced [11, 14]

In the case of viscous liquids, the presence of walls close to the bubbles is very important because viscous stresses importantly affect their growth,, detachment and coalescence. To appreciate these effects, a theoretical-numeric treatment of the formation of bubbles in viscous liquids and vessels with conical and cylinder geometries is presented. Conical vessels yield a very simple, axy-symmetric system where walls could affect the bubble formation. This geometry can also be seen as a system in which the bottom, where the injection orifice is located, inclines symmetrically. Thus the bubble will grow from the apex of the inverted cone. We assume that the angle from cone α is measured from the horizontal up to the surface of the cone. See Fig. 1.

NOMENCLATURE

Ca	Capillary number
Bo	Bond number
Re	Reynolds number
α	Cone inclination angle
R	Cylinder radius
a	Injection tube radius
θ	Contact angle
Q	Gas flow rate
ρ	Density
μ	Viscosity
p	Pressure
v	Velocity field
i	Normal vector
σ	Surface tension
g	Gravity
τ	Stress tensor
Γ	Bubble deformation parameter
x_{CM}	Mass center
V_{0f}	Critical dimensionless volume
V_0	initial volume

Another case of interest and the one where gradually, the influence of the walls on the bubble growth occurs when the radius R of a cylinder initially large (semi-infinite systems) it is reduced to reach values close of the injection tube a (i.e., $R/a \to 1$); see Fig. 2. In connection with the cones, this case corresponds formally to have conical containers with $\alpha = 90^\circ$.

In the case of viscous liquids, the main dimensionless parameters are the capillary number, Ca, and the Bond number, Bo, respectively

$$Ca = \frac{\mu Q}{\sigma a^2}, \qquad Bo = \frac{\rho g a^2}{\sigma}. \tag{1}$$

In this work we assume the presence of a viscous liquid and that the gas injected gas, to a constant flow rate Q, is not viscous. This leads to very low values of the Reynolds number, Re [11, 14].

We model the formation of bubbles using the Stokes equations, valid for slow viscous fluid flows. Stokes and continuity equations are solved numerically using the boundary elements method (BEM). Subsequently, the velocity field and pressure resulting from the numerical solution are incorporated into the equation of the free surface, leading to the evolution of the surface of the bubble in the presence of walls very close to the gas injection tube

Another parameter influencing the growth of the bubbles in inviscid and viscous liquids is the contact angle θ [9]. Inclusion of this quantity leads to consider wetting properties of the bottom. Numeric codes similar to those developed for systems with flat bottoms by Higuera [9, 10] have been used in the development of the present studies. Following it, we will also show that the maximum volume of the bubbles is described adequately, by using the angle of inclination of the cone walls and the capillary and Bond numbers. As a starting point, in the following section the governing equations and the boundary conditions on the bubble surface will be formulated for the present case.

EQUATIONS FOR THE FORMATION OF BUBBLES IN VISCOUS LIQUIDS

In the case here considered, an incompressible gas, with insignificant density and viscosity, is injected at a constant flow rate, Q, into a stagnant viscous liquid of dynamic viscosity μ. Injection is through a circular orifice of radius a and wall thickness b', located at the bottom of the hollow cone or cylinder. We suppose that $f_i(\mathbf{x}, t) = 0$ is the free surface of the it bubble, with $f_i > 0$ in the liquid. We also assume that inertia on the viscous liquid is neglected ($Re = \frac{\rho Q}{\mu a} \ll 1$), thus the continuity and Stokes non dimensional equations are, respectively,

$$\nabla \cdot \mathbf{v} = 0 \tag{2}$$
$$0 = -\nabla p - Bo\mathbf{i} + \nabla^2 \mathbf{v} \tag{3}$$

where p is the pressure, $\mathbf{i}$ is the unitary vector on the bubble surface, pointing to the outer normal direction and $\mathbf{v}$ is the velocity vector. The boundary conditions for the i-th bubble have the form

$$\frac{Df_i}{Dt} = 0, \tag{4}$$

$$-p\mathbf{n}_i + \tau' \cdot n_i = (\nabla \cdot \mathbf{n}_i - p_{g_i})n_i \tag{5}$$

The Eq. (4) states that the surface of each bubble is a smooth surface [15] and the equation (5) specifies the balance of the stress acting on the bubble surface.

The quantity D/Dt=∂/∂t+$\mathbf{v}\cdot\nabla$ in (4) is the material derivative in the points on the surface of the bubbles; $\mathbf{n}_i = \nabla f_i/|\nabla f_i|$ is a unit vector normal to the surface f_i; τ' is the viscous stress tensor, given by the Navier-Poisson law [15] and p_{g_i} is the dimensionless pressure of the gas in the i-th bubble. This pressure is determined by conditions on the bubble dynamics and its volume after departure, V_i (with i=1,2,3,...), which do not change with time. Finally, we assume that the dimensionless gas flow rate, of the initial bubble is $dV_0/dt=Ca=$ constant; i.e., dimensionless gas flow rate that forms the bubble is exactly the capillary number.

The adherence condition is also satisfied on the cone mouth $z = ma$ on the cylinder wall $r = R$ and in infinite pressure is $p+Bmr=constant$. Distance and time are scaled with the size of the gas outlet orifice a and with the viscous time $\mu a/\sigma$, respectively.

For the inverted cone containing a viscous liquid the wall of the cone obeys the equation. $z = m\sqrt{x^2 + y^2} = mr$, where $z=r=0$ is the apex of the cone and $m = tan\,\alpha$ is their slope. An important condition for the growing of the bubble that departure from the gas outlet orifice is the angle of contact. This angle is measured from the horizontal floor, the same where the take off orifice is located, towards the bubble surface. This new parameter is important for the growth and development of the bubbles, in further works the effect of this parameter on the dynamic of growth will be analyzed, however this work only takes into account this parameter to consider the contact lines fix, with a $\theta = 45^\circ$.
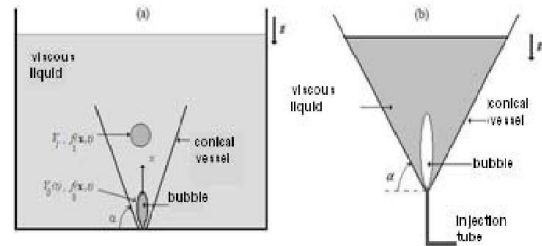

Figure 1 Schematic of a conical vessel containing a viscous liquid. The cone walls are inclined an angle α respect to the horizontal plane
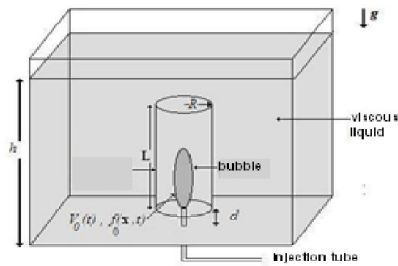
Figure 2 Formation of bubbles within a cylinder of radius R and height L. The reservoir containing the viscous liquid has a height $h>L$ and the basis of the cylinder is at a height d on the container.

NUMERICAL SOLUTION

The solution of equations is performed numerically by using BEM [12,13] this solution give $\mathbf{v}$ and p and such fields will be include in Eqs. (4, 5) to found the bubble shapes, f_i, with the Runge-Kutta second order method [9]. This last method was developed by using 60 and 120 nodes, tests with different node numbers were performed and it was found that this amount of nodes is the optimum one. The present work shows formation surveys from one bubble.
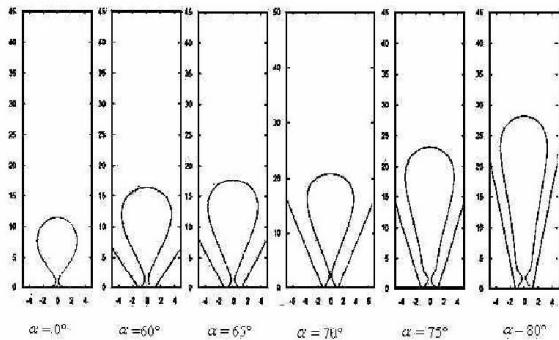

Figure 3 Dimensionless 2D profiles of the bubbles at the critical volume (maximum volume reached before the bubble departure) for different values of the angle of the cone. In this case the bubbles grow at constant flow. The calculations were made for Ca=10 and Bo=0.2.

Figure 3 show bubbles formed in cones at different angles of inclination and the Fig 4 bubbles formed in cylinders of different radius are shown. The volume of bubbles in cones and cylinders increases as a function of inclination angle of the walls or the cylinder radius.
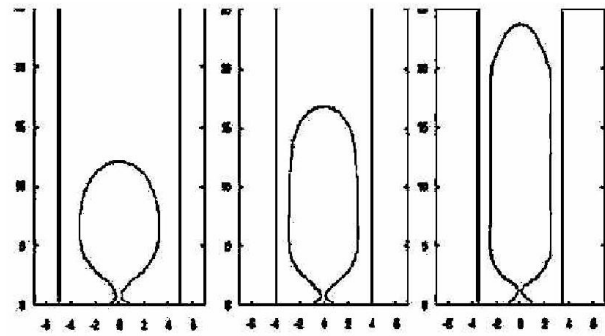

Figure 4 Dimensionless 2D profiles of bubbles formed in cylinders of different radii. From left to right, the dimensionless radii are R/a=5, 4 and 3.5. The dimensionless numbers are Ca=10 and Bo=0.2.

As the gas outlet orifice coincides with the cone walls at the bottom, it can be stated that if the inclination angle approaches to the straight angle the growth of the critical volume of the bubble will increase more, approaching the infinite when the angle is 90°, as observed in Fig. 5.
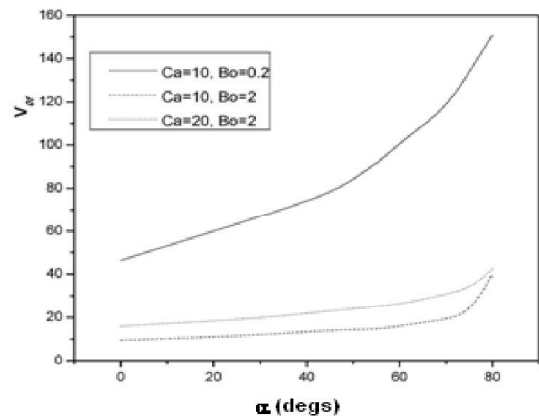

Figure 5 Dimensionless plot of the critical volume of the bubbles in cones. Calculations were made for Ca=10, Bo=0.2 (continuous line); Ca=10 y Bo=2 (long dash) and Ca=20 and Bo=2 (short dash).

Capillary numbers *Ca=10, 20* and a Bond numbers *Bo=0.2*, 2 were used in Fig. 5, the inclination angle values were α= *0, 45°, 50°, 55°, 60°, 65°, 70°, 75°* and *80°*. Calculations with higher capillary numbers were performed and results were similar to those shown in Fig. 5, nevertheless the values of volumes are higher for low *Bo* and high α. This effect on the size bubbles can be understood as a consequence of an effective friction generated by the walls of the cone, which allows to the bubble to grow more. By contrast, the volume V_{0f} only weakly increases when $Bo > 1$, except for very large angles, for which growth of V_{0f} is very intense. To our knowledge, this is a new

way to generate bubbles of increasing size, with the consequent increase of the angle, especially in the limit $Ca/Bo \gg 1$.

Figure 6 show a dimensionless plot of the critical volume for bubbles in cylinder with different radii. As can be seen, the final volume of the bubble increases with decreasing R. This volume is almost constant and equal to the volume of a bubble growing in semi-infinite means without walls, when $R/a > 6$.
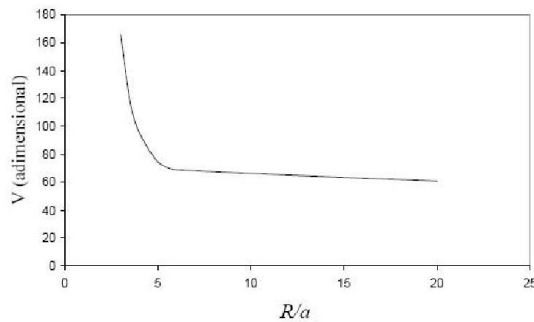

Figure 6 Dimensionless plot of the critical volume of the bubbles in tubes as a function of dimensionless radius R/a. The plot was made for Ca=10 and Bo=0.2.

To quantify the effect of the injected gas flow rate on the maximum size of the bubbles, we show in Fig. 7 the dimensionless volume V_{0f} as a function of capillary number for R/a=3.5 and Bo=0.2. The volume increases linearly with the capillary number, which grows faster than that found in a semi-infinite region or into a wide open cone.
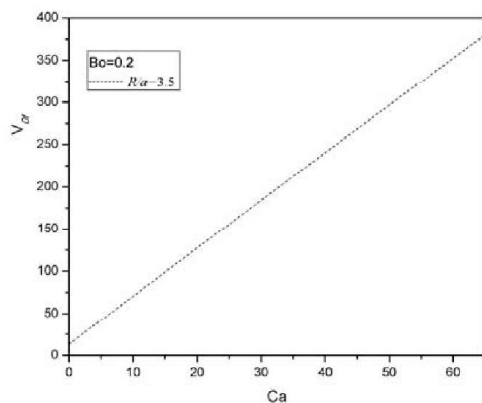

Figure 7 Dimensionless plot of the critical volume of bubbles in tubes as a function of capillary number, Ca. In this case R/a–3.5 and Bo–0.2.

In Fig. 8 it is plotted the center of mass of bubbles during their growth in cones with several angles; the angles are those indicated in Fig. 5. It can be seen how the curves evolve as a function of time. Notice that the evolution of curves changes with α.
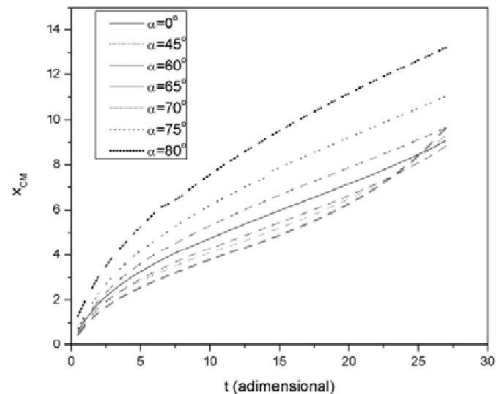

Figure 8 Evolution of the center of mass dimensionless, x_{CM}, as a function of dimensionless time for different values of . The calculations were made for Ca=10 and Bo=0.2.

The motion of the center of mass indicates a faster growth of bubbles at higher angles, this is a strong effect of the inclined walls, not only producing a faster growth of bubbles, but directly affecting their form, elongating expanding them in the higher part, according to the angle being worked on.
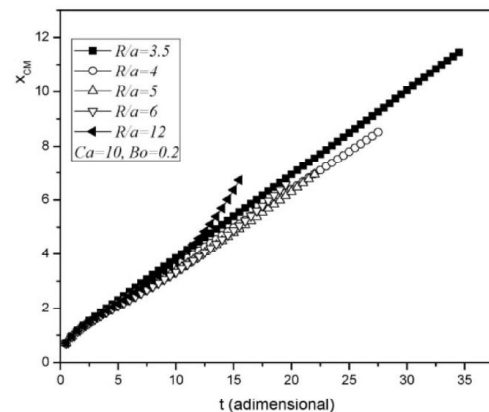

Figure 9 Graphical the evolution of the center of mass, x_{cm}, as a function of time for bubbles in cylindrical tubes with dimensionless radii, R/a=3.5, 6, 5, 4 and 3.5.

As in the case of the conical vessels, in Fig. 9 the center of mass, x_{cm}, as a function of time it is plotted for some representative values R/a. As in the case of small cone angle, the existence of walls reflects the existence of an intense frictional force whose effect is to make the bubble grow by raising its center of mass at constant speed. As the radius of the tube gets bigger, this behavior tends to be more weak and bubbles behave as in the case of semi-infinite reservoirs, for which the viscous drag force limits the speed of raising of the

center of mass of the bubble because the buoyancy force is greater, what follows a rapid increase in the size of the bubble along the axis of the tube.
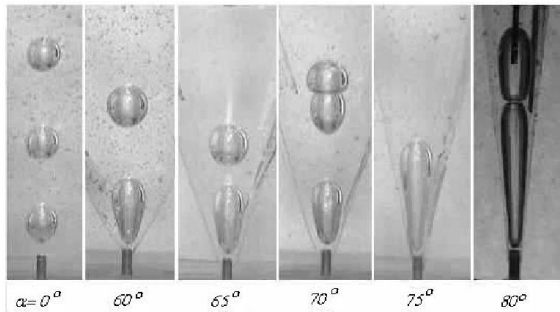

Figure 10 Pictures from the experiments with cones of different values of α.

Figure 10 shows bubble shapes similar to those obtained in the numerical calculations. It is obvious that the walls affect also the critical volume of the bubbles. In the Fig. 10 bubbles have higher volumes for lower angles and constant flow rate. Changes of volumes are significant when $\alpha > 60^o$.

For bubbles growing in cylindrical tubes no images are shown in this work but the shapes obtained are similar to those shown numerically.

CONCLUSIONS

As a main conclusion we found that it is possible to solve the continuity and Stokes equations for the movement of a viscous liquid, due to the gas injection from an orifice, in conical or cylindrical vessels. The method of solution is similar to other one recently published where a liquid is in a semi-infinite spatial region [11] which also corresponds to a cone with horizontal walls ($\alpha = 0$). The effect of imposing inclined walls and adherence condition is very strong on the shapes and sizes of the bubbles. Cases here considered, were those where the Bond and capillary number were relatively small, and the angle of contact from the bubble is 45°. It is briefly stated that our numeric results agree quite well with experimental measurements performed by colleges in our group.

ACKNOWLEDGMENTS

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