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NATURAL CONVECTIVE HEAT TRANSFER FROM A RECESSED NARROW VERTICAL FLAT PLATE WITH A UNIFORM HEAT FLUX AT THE SURFACE

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ABSTRACT

The natural convective heat transfer rate from a relatively narrow heated vertical plate with a uniform heat flux over its surface has been numerically studied. This heated plate is embedded in a large plane adiabatic surface, the surface of the heated plate being somewhat "below" that of the adiabatic surface, i.e., the heated plate surface is recessed in the adiabatic surface. The main aim of the present work was to study how the recessing of the heated plate into the surface affects the mean heat transfer rate from the plate. It has been assumed that the flow is steady and laminar and that the fluid properties are constant except for the density change with temperature which gives rise to the buoyancy forces, this having been treated by using the Boussinesq approach. It has also been assumed that the flow is symmetrical about the vertical centre-plane of the plate. The solution has been obtained by numerically solving the full three-dimensional form of the governing equations, these equations being written in dimensionless form. The solution was obtained using a commercial finite element method based code, FIDAP. The solution has the Rayleigh number based on the plate height and the surface heat flux, the dimensionless plate width, the dimensionless depth that the heated plate is recessed into the adiabatic surface, and the Prandtl number as parameters. Results have only been obtained for Pr = 0.7. A relatively wide range of values of the other input parameters have been considered and the effects of these parameters on the mean Nusselt number have been determined and used to study the effects that arise due to the plate being recessed and that arise due to the fact that the plate is relatively narrow.

INTRODUCTION

Two-dimensional natural convective heat transfer from a vertical plate with a uniform heat flux at the surface has been quite extensively studied. However, when the width of the plate is relatively small compared to its height, the heat transfer rate can be considerably greater than that predicted by these two-dimensional flow results. The increase in the heat transfer rate from narrow plates relative to that from wide plates under the same conditions results from the fact that fluid flow is induced inwards near the edges of the plate and the flow near the edge of the plate is thus three-dimensional. The increase in the heat transfer rate, therefore, is often said to be due to "three-dimensional effects" or "edge effects". Situations that can be approximately modelled as narrow vertical plates occur in a number of practical cases so there exists a need to be able to predict the Nusselt number for such narrow plates. In some such cases of practical interest the heated plate is sometimes recessed by a small amount into the adiabatic plane surface in which it is mounted. In the present study, therefore, a heated flat plate over which there is a uniform heat flux, embedded in a large plane adiabatic surface has been considered, the surface of the heated plate being somewhat "below" that of the adiabatic surface, i.e., the heated plate surface is recessed in the adiabatic surface. The flow situation considered is thus as shown in Figs. 1 and 2. Figure 1 shows the overall situation considered while Fig. 2 shows a vertical cross-section through the system. The width of the plate, w, is assumed to be of the same order of magnitude as the vertical height of the plate, l, and the depth to which the plate is recessed, d , has been assumed to be relatively small compared to the plate height, l. The value of d in Fig. 2 is approximately 0.15 which is greater than the largest value of d for which results have here been obtained this being 0.1.

Experimental studies of natural convective heat transfer from narrow vertical plates are described in References 1 and 2. However, the conditions at the edges of the plates used in these studies was very different from those assumed in the present study and the results obtained in the present study cannot therefore be quantitatively compared with those obtained in

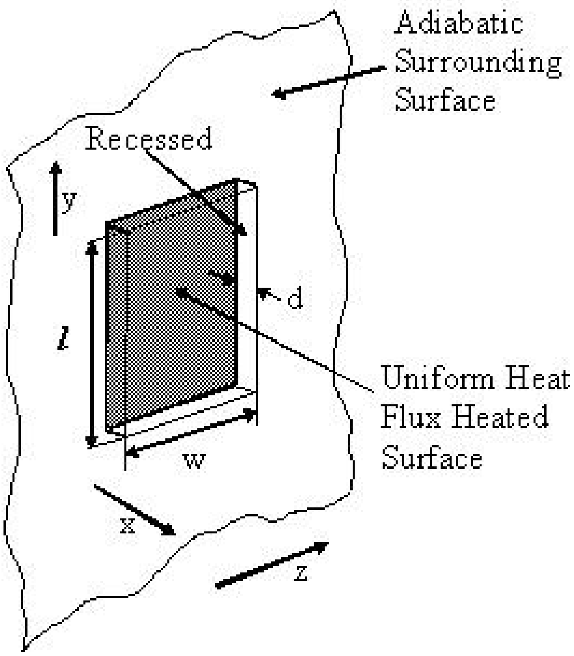

Figure 1 Situation Considered

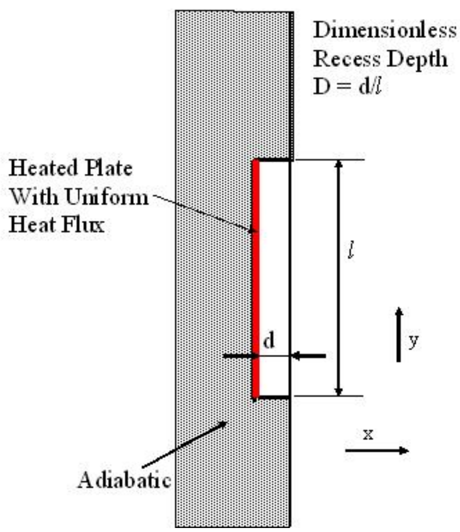

Figure 2 Centre-Plane Section Through Plate Arrangement Considered

these experimental studies. A study of natural convective heat transfer from narrow vertical plates based on the use of the boundary layer equations is described in Refs. 3 and 4. An experimental study of the free convection heat transfer rates from flush mounted and protruding microelectronic chips was undertaken by Park and Bergles [5]. In their study, the microelectronic chips were simulated by thin foil heaters. It was found that the Nusselt number increased as the chip width decreased. Even with the widest of their heaters, the heat transfer coefficients were higher than the values predicted by two-dimensional theory. References 6, 7 and 8 describe numerical studies of natural convective heat transfer from narrow vertical plates. However a limited range of the governing parameters was covered in the first two of these studies making it difficult to draw general conclusions from the results. The present study is an extension of that described in Ref. 8. The effect of the conditions at the edge of the plate was not considered in any of these studies, i.e., none of these studies have considered a recessed plate.

NOMENCLATURE

D	[-]	Dimensionless recess depth of plate, d / l
d	[m]	Depth of plate below adiabatic surrounding surface
g	[m/s^2]	Gravitational acceleration
k	[W/mK]	Thermal conductivity of fluid
l	[m]	Height of heated plate
Nu	[-]	Mean Nusselt number based on l and on $(T_{Hm} - T_F)$
P	[-]	Dimensionless pressure
p	[Pa]	Pressure
p_F	[Pa]	Pressure in undisturbed fluid
Pr	[-]	Prandtl number
q'	[W/m^2]	Heat transfer rate over surface
Ra^*	[-]	Heat Flux Rayleigh number based on l and on q'
T	[K]	Temperature
T_F	[K]	Temperature of undisturbed fluid
T_{Hm}	[K]	Mean temperature of heated plate
U_X	[-]	Dimensionless velocity component in X direction
u_x	[m/s]	Velocity component in x direction
u_r	[m/s]	Reference velocity
U_Y	[-]	Dimensionless velocity component in Y direction
u_y	[m/s]	Velocity component in y direction
U_Z	[-]	Dimensionless velocity component in Z direction
u_z	[m/s]	Velocity component in z direction
W	[-]	Dimensionless width of plate, w / l
w	[m]	Width of plate.
X	[-]	Dimensionless horizontal coordinate normal to plate
x	[m]	Horizontal coordinate normal to plate
Y	[-]	Dimensionless vertical coordinate
y	[m]	Vertical coordinate
Z	[-]	Dimensionless horizontal coordinate in plane of plate
z	[m]	Horizontal coordinate in plane of plate

Special characters

α	[m^2/s]	Thermal diffusivity
β	[K^{-1}]	Bulk expansion coefficient
θ	[-]	Dimensionless temperature
θ_{Hm}	[-]	Mean dimensionless temperature of heated plate
ν	[m^2/s]	Kinematic viscosity

SOLUTION PROCEDURE

The flow has been assumed to be steady and laminar and it has been assumed that the fluid properties are constant except for the density change with temperature which gives rise to the buoyancy forces, this having been treated by using the Boussinesq approach. It has also been assumed that the flow is symmetrical about the vertical centre-plane of the plate. The solution has been obtained by numerically solving the full three-dimensional form of the governing equations, these equations being written in terms of dimensionless variables using the height, l , of the heated plate as the length scale and $q'l / k$ as the temperature scale, q' being the uniform heat flux over the surface of the plate. Defining the following reference velocity:

$$u_r = \frac{\alpha}{l}\sqrt{Ra^* Pr} \tag{1}$$

where Pr is the Prandtl number and Ra^* is the heat flux Rayleigh number based on l, i.e.,:

$$Ra^* = \frac{\beta g q' l^4}{k\nu\alpha} \tag{2}$$

the following dimensionless variables have then been defined:

$$X = \frac{x}{l}, Y = \frac{y}{l}, Z = \frac{z}{l}, U_X = \frac{u_x}{u_r}, U_Y = \frac{u_y}{u_r}, U_Z = \frac{u_z}{u_r},$$

$$P = \frac{(p - p_F)l}{\mu u_r}, \theta = \frac{T - T_F}{q' l / k} \tag{3}$$

where T is the temperature and T_F is the fluid temperature far from the plate. The *X* coordinate is measured in the horizontal direction normal to the plate, the *Y*-coordinate is measured in the vertically upward direction and the *Z*-coordinate is measured in the horizontal direction in the plane of the plate.

In terms of these dimensionless variables, the governing equations are:

$$\frac{\partial U_X}{\partial X} + \frac{\partial U_Y}{\partial Y} + \frac{\partial U_Z}{\partial Z} = 0 \tag{4}$$

$$U_X \frac{\partial U_X}{\partial X} + U_Y \frac{\partial U_X}{\partial Y} + U_Z \frac{\partial U_X}{\partial Z} = \sqrt{\frac{Pr}{Ra^*}}\left(-\frac{\partial P}{\partial X} + \frac{\partial^2 U_X}{\partial X^2} + \frac{\partial^2 U_X}{\partial Y^2} + \frac{\partial^2 U_X}{\partial Z^2}\right) \tag{5}$$

$$U_X \frac{\partial U_Y}{\partial X} + U_Y \frac{\partial U_Y}{\partial X} + U_Z \frac{\partial U_Y}{\partial X} = \sqrt{\frac{Pr}{Ra^*}}\left(-\frac{\partial P}{\partial Y} + \frac{\partial^2 U_Y}{\partial X^2} + \frac{\partial^2 U_Y}{\partial Y^2} + \frac{\partial^2 U_Y}{\partial Z^2}\right) + \theta \tag{6}$$

$$U_X \frac{\partial U_Z}{\partial X} + U_Y \frac{\partial U_Z}{\partial Y} + U_Z \frac{\partial U_Z}{\partial Z} = \sqrt{\frac{Pr}{Ra^*}}\left(-\frac{\partial P}{\partial Z} + \frac{\partial^2 U_Z}{\partial X^2} + \frac{\partial^2 U_Z}{\partial Y^2} + \frac{\partial^2 U_Z}{\partial Z^2}\right) \tag{7}$$

$$U_X \frac{\partial \theta}{\partial X} + U_Y \frac{\partial \theta}{\partial Y} + U_Z \frac{\partial \theta}{\partial Z} = \frac{1}{\sqrt{Ra^* Pr}}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2}\right) \tag{8}$$

Because the flow has been assumed to be symmetrical about the vertical centre-line of the plate, the solution domain used in obtaining the solution is as shown in Fig. 3.

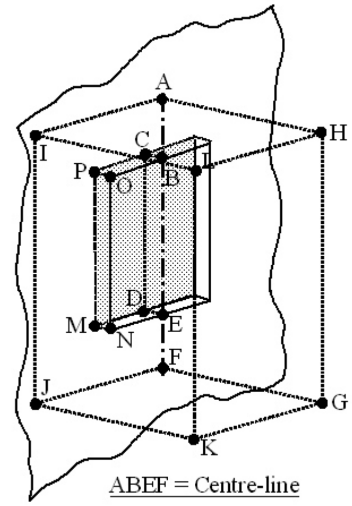

Figure 3 Solution Domain Used

The boundary conditions on the solution are (see Fig. 3):

FEHG: $U_X = 0, U_Y = 0, U_Z = 0, \frac{\partial T}{\partial Y} = -1$

AFEDMI except for FEHG: $U_X = 0, U_Y = 0, U_Z = 0, \frac{\partial T}{\partial X} = 0$

IJLM: $U_Y = 0, U_X = 0, T = 0$

DCLM: $U_X = 0, U_Z = 0, T = 0$

BCLJ: $U_Y = 0, U_Z = 0, T = 0$

ABCDEF: $U_Z = 0, \frac{\partial U_Y}{\partial Z} = 0, \frac{\partial U_X}{\partial Z} = 0, \frac{\partial T}{\partial Z} = 0$

The mean heat transfer rate from the heated plate has been expressed in terms of the following mean Nusselt number:

$$Nu = \frac{q' l}{k(T_{Hm} - T_F)} = \frac{1}{\theta_{Hm}} \tag{9}$$

where T_{Hm} and θ_{Hm} are the mean temperature and the mean dimensionless temperature of the heated surface respectively.

The dimensionless governing equations subject to the boundary conditions discussed above have been numerically solved using the commercial finite-element solver, FIDAP. Extensive grid- and convergence criterion independence testing was undertaken. This indicated that the heat transfer results presented here are to within 1% independent of the number of grid points and of the convergence-criterion used. The effect of the distance of the outer surfaces of the solution domain (i.e., surfaces HGKL, ILKJ, FGKJ, and AHLI in Fig. 3) from the heated surface was also examined and the distances used in obtaining the results discussed here were chosen to ensure that the heat transfer results were independent of this positioning to within one per cent.

RESULTS

The solution has the following parameters:

- The heat flux based Rayleigh number, Ra^*,
- The dimensionless plate width, W ,
- The dimensionless "depth" to which the plate is recessed, D.
- The Prandtl number, Pr,

Results have only been obtained for Pr = 0.7. Ra^* values between 10^3 and 10^8, W values between 0.2 and 1.6, and D values between 0.1 and 0 have been considered.

The effect of recessing the plate is illustrated by the results given in Fig. 4 which shows the variation of the mean Nusselt number with dimensionless recess depth, D , of the heated plate for two values of the dimensionless plate width for a Rayleigh number, Ra^*, of 10^5. It will be seen that the Nusselt number decreases as D increases, the decrease in the Nusselt number values of the range of D values considered being approximately 41% for W =0.3. It will also be noted that while the Nusselt number values for W = 0.3 are larger than those for W = 1.6 at low values of D, at the larger values of D considered the Nusselt number values for W = 0.3 are lower than those for W = 1.6. These effects are further illustrated by the results given in Figs. 5, 6, and 7. These figures show the variation of the mean Nusselt number with dimensionless plate width, W , for various values of D for Ra^* values of 10^5 (Fig. 5), 10^6 (Fig. 6), and 10^7 (Fig. 7). For comparison the variation of the mean Nusselt number with dimensionless plate width, W, for various values of Ra^* for a heated plate that is not recessed in the surrounding adiabatic surface is shown in Fig. 8.

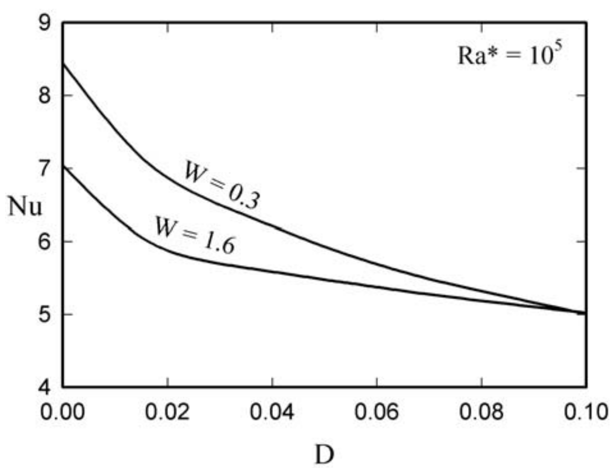

Figure 4 Variation of mean Nusselt number with dimensionless recess depth for two dimensionless plate widths for $Ra^* = 10^5$.

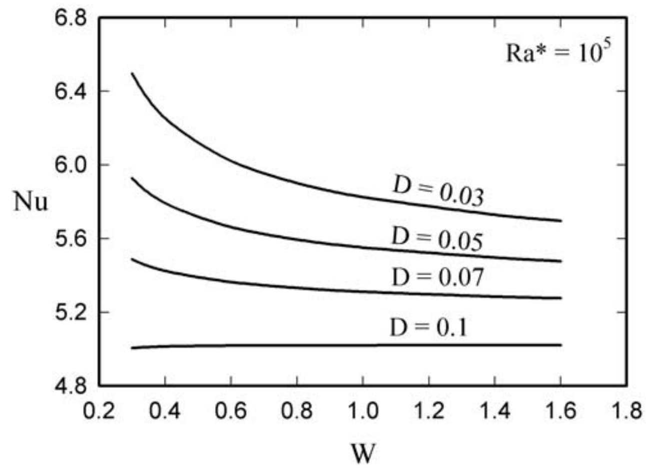

Figure 5 Variation of mean Nusselt number with dimensionless plate width for various values of D for $Ra^* = 10^5$.

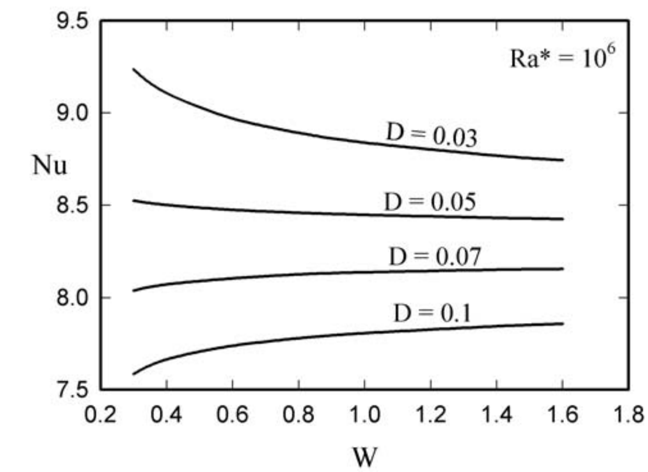

Figure 6 Variation of mean Nusselt number with dimensionless plate width for various values of D for $Ra^* = 10^6$.

It will be seen that when the heated surface is not recessed the Nusselt number always increases with decreasing dimensionless width, W. This is due to the induced inflow over the vertical edges of the plate which increases the local Nusselt number values in the region of these vertical edges. As the dimensionless width of the plate decreases these edge effects become increasingly important leading to the observed increase in mean Nusselt number with decreasing W. When the heated surface is recessed it will be seen that the Nusselt numbers are always lower than those existing under the same conditions when D =0. It will also be seen that, particularly at the larger values of Ra^* and at the larger values of D considered, the Nusselt number decreases with decreasing W.

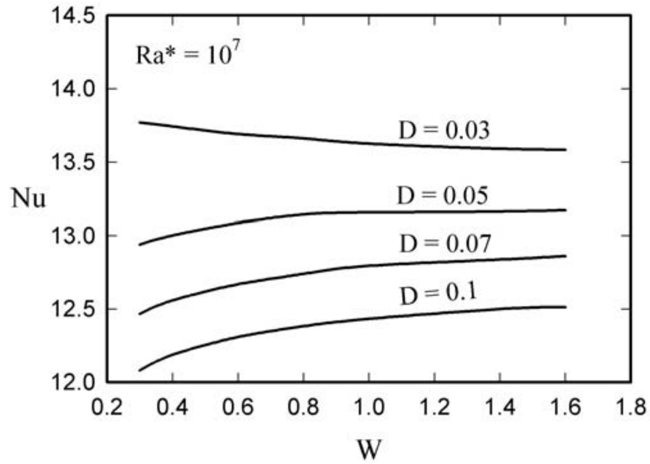

Figure 7 Variation of mean Nusselt number with dimensionless plate width for various values of *D* for $Ra^* = 10^7$.

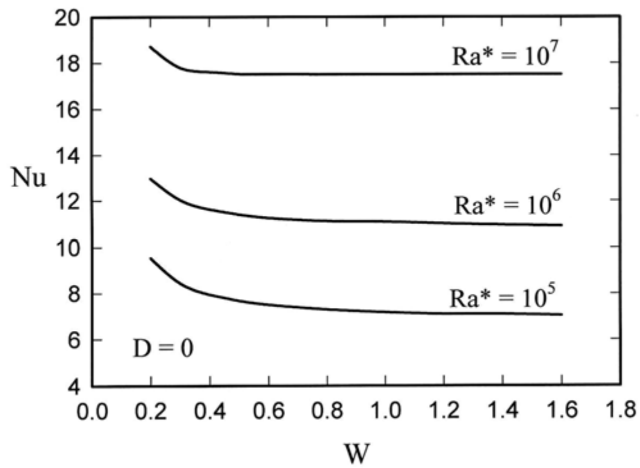

Figure 8 Variation of mean Nusselt number with dimensionless plate width for various values of Ra^* for a non-recessed heated plate.

These effects result from the fact that the recessed edges divert the flow away from the plate near the vertical edges and near the leading and trailing edges of the plate thus leading to a decrease in the Nusselt number values in these regions. Further, as the plate width is decreased, the effect of the decrease in local Nusselt number in the region of the vertical edges becomes more significant leading to a decrease in the overall average Nusselt number, i.e., leading to the observed decrease in mean Nusselt number with decreasing *W*. Furthermore, as the Rayleigh number increases, the boundary layer thickness decreases leading to an increase in the effect of the recessed edges and thus leading to the observed decrease in mean Nusselt number with decreasing *W* at higher values of Ra^*. The effect of Ra^* on the mean Nusselt number is illustrated by the results given in Figs. 9 and 10 which show variations of the mean Nusselt number with *W* for various values of Ra^* for two values of *D*. It will be seen that when $D = 0.03$, *Nu* decreases with increasing *W* at the two lower values

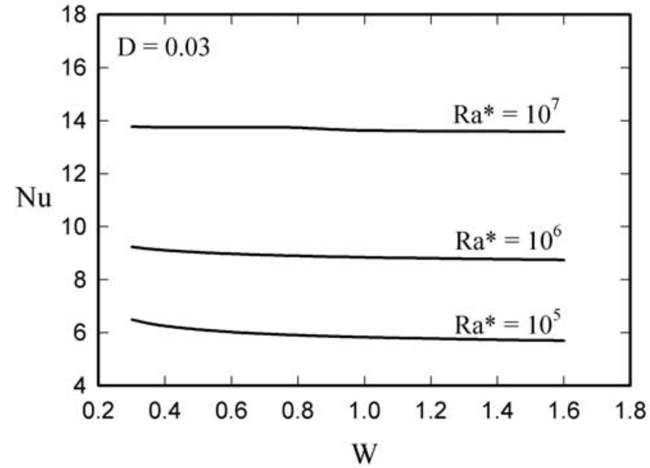

Figure 9 Variation of mean Nusselt number with dimensionless plate width for various values of Ra^* for a dimensionless recess depth, *D,* of 0.03. for a non-recessed heated plate.

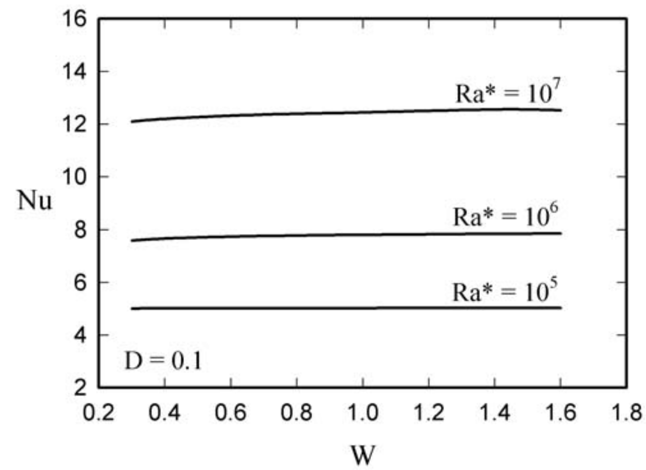

Figure 10 Variation of mean Nusselt number with dimensionless plate width for various values of Ra^* for a dimensionless recess depth, *D*, of 0.03. for a non-recessed heated plate.

of Ra^*. However, when $D = 0.1$ it will be seen that *Nu* increases with increasing *W* at the all values of Ra^*. The effect of the recessed edges on the dimensionless temperature distribution over the surface of the heated plate is illustrated by the results given in Fig. 11 which shows dimensionless temperature distributions for a given *W* and *D* for various values of Ra^*. Regions of high dimensionless surface temperature are regions of low local Nusselt number and the effect of the top and side edges on the local Nusselt distribution will be clearly noted.

The results all therefore indicate that recessing the heated surface below the surrounding surface decreases the Nusselt number and in most cases eliminates the Nusselt number increase with decreasing plate width that occurs with a flush mounted heated plate.

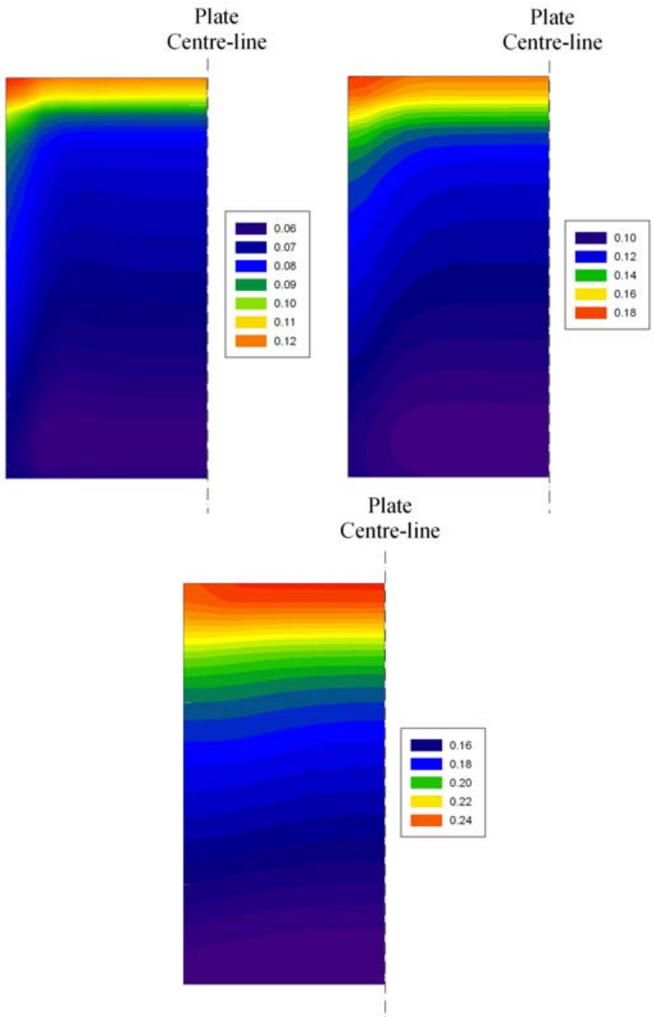

Figure 11 Variation of dimensionless temperature over heated surface for $B = 0.1$ and $W = 1.0$ for $Ra^* = 10^7$ (top left), $Ra^* = 10^6$ (top right), and $Ra^* = 10^5$ (bottom).

CONCLUSIONS

The results of the present study indicate that:

1. Under all of the conditions covered in the study, both the dimensionless plate width, W, and the dimensionless depth that the heated surface was recessed, D, had an influence on the mean Nusselt number.
2. Recessing the heated surface decreases the mean Nusselt number, the decrease in the mean Nusselt number increasing as D increases.
3. At values of D and Ra^* considered, the mean Nusselt number was found to decrease with decreasing dimensionless plate width W. By contrast, the Nusselt number always increases with decreasing W for a non-recessed heated surface.

ACKNOWLEDGEMENTS

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