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Nu-Ra CORRELATIONS FOR STEADY STATE LAMINAR NATURAL CONVECTION IN 2D PARALLELOGRAMMIC ENCLOSURES

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ABSTRACT

Nu-Ra correlations are proposed to evaluate the steady-state natural convection heat transfer taking place in air-filled enclosures of parallelogrammic section. The thermal conditions and dimensions of the treated cavities lead to values of the Rayleigh number reaching 1.4×10^7. The considered cavities are formed by two vertical active, hot and cold, walls of the same height H at a horizontal distance between them $L=H$, connected by a closing adiabatic channel. The cold and hot walls are maintained isotherm at temperature T_c and T_h respectively. The upper and lower walls of the channel are inclined at an angle α with respect to the horizontal. This angle is either positive (hot wall below the level of the cold one) or negative, giving rise respectively to a conducting or insulating cavity in the convective sense of the term. A detailed steady-state 2D numerical study is carried out with the finite volume method. The numerical approach provides the thermal and dynamical fields of the convective flow. We present the dependence of the mean Nusselt number as a function of the inclination angle α and of the Rayleigh number Ra. Correlations of the type *Nu-Ra*, suitable for $-45° < \alpha < +45°$, are proposed for the range $1{,}7 \times 10^3 < Ra < 1{,}4 \times 10^7$. They make it possible to properly size thermal systems to be used in several sectors of engineering.

KEYWORDS

2D steady laminar natural convection, engineering applications, parallelogrammic air-filled cavity, numerical simulation, *Nu-Ra* correlations, heat transfer.

NOMENCLATURE

A	[-]	Aspect ratio
a	[m²s⁻¹]	Thermal diffusivity of the air
C_p	[J.kg⁻¹K⁻¹]	Constant pressure specific heat
g	[ms⁻²]	Acceleration of the gravity
$\underline{\underline{\overline{h_{\alpha,c}}}}$	[Wm⁻²K⁻¹]	Calculated mean convection coefficient over the hot wall for an angle α (Wm⁻²K⁻¹)
H	[m]	Height of the cavity
$k(\alpha),n$	[-]	Coefficient and exponent of $\underline{\underline{\overline{Nu_\alpha}}} = k(\alpha)Ra^n$
$\underline{\underline{\overline{Nu_\alpha}}}$	[-]	Correlated mean Nusselt number for an angle α
$\underline{\underline{\overline{Nu_{\alpha,c}}}}$	[-]	Calculated mean Nusselt number for an angle α
p	[Pa]	Pressure
p^*	[-]	Dimensionless pressure
Pr	[-]	Prandtl number
Ra	[-]	Rayleigh number
T	[K]	Local temperature
T_c	[K]	Temperature of the cold wall
T_h	[K]	Temperature of the hot wall
T^*	[-]	dimensionless temperature
u, v	[m s⁻¹]	fluid velocity components
u^*, v^*	[-]	Dimensionless flow velocity components
W	[m]	Depth of the cavity
x, y	[m]	Cartesian coordinates
x^*, y^*	[-]	Dimensionless Cartesian coordinates
Greek symbols		
α	[°]	Inclination angle of the cavity
β	[K⁻¹]	Volumetric expansion coefficient of the air
ΔT	[K]	Temperature difference $\Delta T = T_h - T_c$
$\nabla u^*, \nabla v^*$	[m⁻¹s⁻¹]	Laplacian expressions of u^*, v^*
$\delta(\alpha)$	[%]	Deviation between $\underline{\underline{\overline{Nu_\alpha}}}$ and $\underline{\underline{\overline{Nu_{\alpha,c}}}}$
φ	[Wm⁻²]	Convective heat flux at the hot wall
λ	[Wm⁻¹K⁻¹]	Thermal conductivity of the air
μ	[Pa.s]	Dynamic viscosity of the air
ρ	[kg.m⁻³]	Density of the air

INTRODUCTION

The main purpose of the present study is to make available to engineers correlations of the type $Nu_\alpha = k(\alpha)Ra^n$ that allow evaluation of the heat exchanges by natural convection occurring in air-filled cavities of parallelogrammic section. The work covers a wide range of Rayleigh numbers for the range $1.7 \times 10^3 < Ra < 1.4 \times 10^7$ and inclination angles of the cavity $+45° < \alpha < -45°$. In this cavity the hot and the cold walls are always vertical and parallel, they have identical height H and the distance between them is L. Both walls are maintained isothermal at temperatures T_h and T_c respectively. They are called "active" because their difference of temperatures produces the flow inside of the cavity. In our study the temperature difference $\Delta T= T_h-T_c$ is fixed equal to 15K. The four other walls of the cavity, called "passive", constitute the channel of the cavity and they are thermally insulating boundaries. We consider in the present work an aspect ratio $A=L/H=1$.

Natural convection in closed cavities has been extensively studied for the last decades, which is a proof of its interest. Many of these studies as [1] treated the problem in steady state. Some other works as [2-3] drew attention to the importance of the radiative exchange in this type of cavities. Other geometries have been studied, like the triangular ones treated in [4]. The survey in [5] discuss the rectangular cavities which hot wall is partially heated and examine the heat exchange in the presence of nanofluids. The definition of the Nusselt number for parallelogram-shape enclosures has been revised in [6] to properly interpret the specific convective and conductive phenomena that characterize them. The numerical study [7] considers different *Ra* numbers and several angles of inclination. That study revealed the influence of the Prandtl number and the aspect ratio on the convective transfer and on the evolution of the mean Nusselt number according to the tilt angle. For cubical cavities several aspects are treated by a great number of studies as [8-12].

The present numerical work comes to complement the previous studies in parallelogrammic cavities. We present the variations of the mean Nusselt number as a function of the inclination angle and of the Rayleigh number and we confirm the convective diode effect of this type of cavities. The treatment of several numerical results permitted to find correlations of the *Nu-Ra* type for laminar convection that can be useful in many engineering applications entailing small thermal power exchanges in fields such as solar energy, materials, building or even electronics.

THE TREATED ENCLOSURE

We quantify in this study the convection heat exchanges occurring at the hot wall of air-filled enclosures of parallelogrammic section. The thermal boundary conditions applied in conjunction with the cavity dimensions lead us to consider the convective flow which settles in the median plan of the cavity as 2D. This was verified in parallels numerical and experimental studies done in our laboratory which show that, as far as the total heat exchange is concerned, the deviations of 2D numerical calculations from those in 3D are about 8% on average. The simplified 2D model treated in the present work is shown in Fig 1.

The hot and cold walls, bounding the channel at both ends, remain always vertical and parallel. Both have the same height H and depth W and the horizontal distance between the two walls is equal to the height H (aspect ratio $A=L/H=1$). They are maintained isothermal at T_h and T_c respectively. The channel of the cavity is thermally adiabatic. The top and bottom passive insulated walls are inclined at an angle α with respect to the horizontal plane. In this study, this angle that can be either positive (hot wall below the level of the cold one) or negative is set to the particular values varying between –45° and +45° in steps of 15° being α=0° a particular case (cavities of square section).

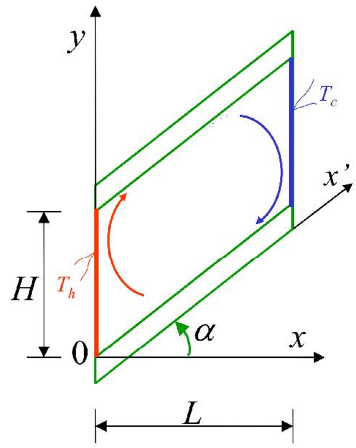

Figure1 The treated cavity

GOVERNING EQUATIONS. BOUNDARY CONDITIONS

1. Continuity equation

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{1}$$

The dimensionless velocity components defined as

$$x^*,y^*=\frac{x,y}{H} \qquad u^*,v^*=\frac{u,v}{a/H} \tag{2}$$

a is the thermal diffusivity.

2. Momentum equations

$$\begin{cases} u^*\dfrac{\partial u^*}{\partial x^*}+v^*\dfrac{\partial u^*}{\partial y^*}=Pr\nabla u^* \\ u^*\dfrac{\partial v^*}{\partial x^*}+v^*\dfrac{\partial v^*}{\partial y^*}=-\dfrac{H^3 g}{a^2}\left(\dfrac{\partial p^*}{\partial y^*}+1\right)+Ra\,Pr\,T^*+Pr\nabla v^* \end{cases} \tag{3}$$

The Rayleigh number Ra is based on the height H of the active walls:

$$Ra=\frac{g\beta(T_h-T_c)H^3\rho}{\mu a} \tag{4}$$

while the Prandtl number Pr, the dimensionless temperature T^* and the dimensionless pressure p^* are defined as

$$Pr=\frac{\mu C_p}{\lambda},\quad T^*=\frac{T-T_c}{T_h-T_c},\quad p^*=\frac{p}{\rho g H} \tag{5}$$

3. Energy equation

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \nabla T^* \quad (6)$$

Boundary and initial conditions

The entire cavity is assumed to be at a uniform temperature and the velocities are nil everywhere. The air is assumed to be isotropic a nd its properties are evaluated at the mean temperature between the hot temperature T_h. and the cold one T_c. Thermal and dynamic boundary conditions of the treated problem are:

- Adherence of the fluid to the walls of the enclosure (no-slip condition):

$$u^* = v^* = 0 \quad (7)$$

- Adiabatic upper and lower walls (thermal gradient of the top and bottom passive walls is zero):

$$\left(\frac{\partial T}{\partial n^*}\right)_{(0 \le x^* \le 1; y^* = x^* \tan\alpha);(0 \le x \le 1); y^* = 1 + x^* \tan\alpha)} = 0 \quad (8)$$

Where n^* is the dimensionless outgoing normal to the passive walls

- Isothermal cold wall

$$\left[(T^*)_{x^*=1}\right] = 0 \quad (9)$$

- Isothermal hot wall

$$\left[(T^*)_{x^*=0}\right] = 1 \quad (10)$$

The numerical calculations are carried out by means of the control volume formulation in accordance with the SIMPLE algorithm. The mesh used is constructed with quadrilateral elements following the shape of the cavity. The mesh size is finer near the active walls in order to capture the viscous effects and the heat exchange in the boundary layer. The number of mesh elements increases with Ra (attributable to complex flows) and with α. The convective heat transfer calculated for steady state conditions all along the hot wall allows the determination of the mean convection coefficient

$$\overline{\underline{\underline{h_{\alpha,c}}}} = \frac{\varphi}{\Delta T} \quad (11)$$

leading to the calculation of the mean steady-state Nusselt number over the hot wall for the studied inclination angle α

$$\overline{\underline{\underline{Nu_{\alpha,c}}}} = \frac{\overline{\underline{\underline{h_{\alpha,c}}}} H}{\lambda} \quad (12)$$

RESULTS

Numerical simulations have been performed for several values of Ra varying between 1.7×10^3 and 1.4×10^7 and seven specific angles α=0, ±15°, ±30°, and ±45°. The variation of $\overline{\underline{\underline{Nu_{\alpha,c}}}}$ with the inclination angle α is represented in Fig. 2 for different values of Ra.

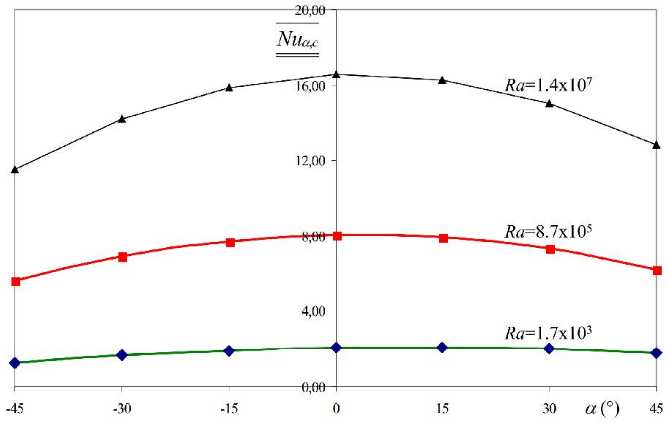

Figure 2 Variation of $\overline{\underline{\underline{Nu_{\alpha,c}}}}$ versus the inclination angle α for different Rayleigh numbers.

We observe for all the presented Ra numbers that the mean Nusselt number $\overline{\underline{\underline{Nu_{\alpha,c}}}}$ increases with α from -45° up to a limit value, here about 15º, beyond which it diminishes. These results coincide with other studies as [7] who also found out a maximum value of $\overline{\underline{\underline{Nu_{\alpha,c}}}}$ around α=15°.

From these results of the mean Nusselt numbers obtained by simulation, the exponents n that satisfy the correlation $\overline{\underline{\underline{Nu_{\alpha}}}} = k(\alpha) Ra^n$ were prospected for all the treated inclination angles. We found that the value of n is comprise between 0.26 and 0.28 with an average value around 0.266. The value n=0.26 has been retained to find the corresponding $k(\alpha)$ values represented in Fig. 3.

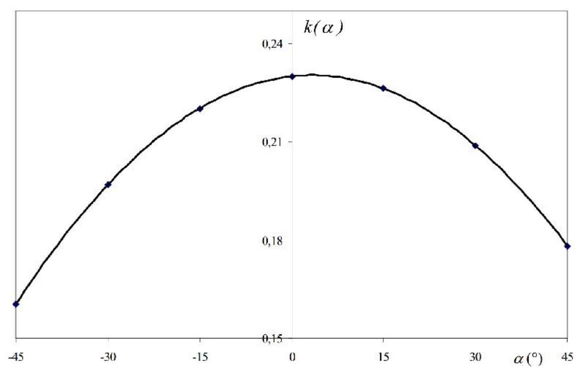

Figure 3. Coefficients $k(\alpha)$ for different angles α for $\overline{\underline{\underline{Nu_{\alpha}}}} = k(\alpha) Ra^{0.26}$

A quadratic-polynomial equation

$$\overline{\overline{Nu_\alpha}} = \left[0.23 + 10^{-5}\alpha(20 - 3\alpha)\right]Ra^{0.26}$$

valid for

$$\begin{cases} 1.7\text{x}10^3 \le Ra \le 1.4\text{x}10^7 \\ -45° \le \alpha \le +45° \end{cases} \qquad (16)$$

is found to provide a good fit, with a regression coefficient R^2=0.9952.

The comparison between the values calculated with this correlation and the values $\overline{\overline{Nu_{\alpha,c}}}$ obtained by simulation has been made for all the studied configurations (different combinations *α-Ra*). The mean deviations, calculated as

$$\bar{\delta}(\alpha) = \frac{\overline{\overline{Nu_\alpha}} - \overline{\overline{Nu_{\alpha,c}}}}{\overline{\overline{Nu_\alpha}}}100 \qquad (17)$$

are represented in Fig. 4. The maximum deviations are found for higher angles, being equal to 8.3% and 7.8% for α=-45° and +45° respectively. That corresponds to more complex flows. For such configurations, solutions are more laborious to attain, being the computing time up to 6 times more than for moderate inclinations. The average mean deviation for all treated angles is small, around 6%.

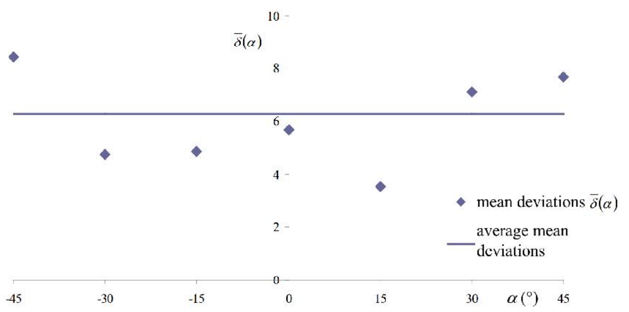

Figure 4 Values of mean deviations $\bar{\delta}(\alpha)$ and average mean deviations

CONCLUSIONS

We present in this work the natural convection flow taking place inside a 2D parallelogrammic cavity. The inclination angle plays an important role in the convection heat transfer and the convective diode effect is clearly confirmed. Radiation is always present and can become important. The correlations *Nu-Ra* proposed in the present work cover a wide range of Rayleigh numbers $1.7\text{x}10^3 \le Ra \le 1.4\text{x}10^7$ and apply to natural convection in parallelogrammic cavities with slight to high inclinations. These correlations make it possible to calculate the heat exchanges by natural convection of interest in several engineering applications. The range of Rayleigh number considered is adequate to study real systems covering a wide variety of geometrical and physical conditions. The peculiarities of parallelogrammic cavities can be advantageously applied to different engineering sectors such as building, solar energy uses or electronics, among others.

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