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OPTIMUM ARRANGEMENT OF HEATERS IN A THREE-DIMENSIONAL RADIANT FURNACE USING THE GENETIC ALGORITHM

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ABSTRACT

The optimum number and locations of heaters over the heater surface of a cubic radiant oven are optimized by using the genetic algorithm of searching. The objective of the problem is to produce the uniform temperature and heat flux distributions over the design surface. The wall surfaces of the oven are subdivided into surface elements and the net radiation method is used to calculate the radiative heat exchange between surface elements. The optimization procedure in the present study is based on the micro genetic algorithm. The procedure consists of the basic genetic operations, selection and crossover, and another recommended operation to use, elitism. The performance and accuracy of the method is evaluated by two numerical examples.

INTRODUCTION

In designing industrial furnaces, the heater arrangement dramatically affects the efficiency and uniformity with which the materials are heated. The design problem is concerned with the determination of uniform heat flux and temperature distributions over the part of the enclosure, namely the design surface. The design surface may be a specific part of a product which is annealed, cured, coated or baked through a heating operation in a radiant furnace.

In recent years some efforts have been made to find the strength of radiative heaters over the heater surface to meet the design goal. Fedrov et al. [1] used the gradient-based optimization techniques to determine optimal radiant heater settings in an industrial oven. The optimum design of radiant enclosures containing a transparent medium was reported by Daun et al. [2]. Sarvari et al. [3-5] reported a methodology for designing radiant enclosures containing absorbing-emitting media to find the appropriate heater settings. The procedure was extended by Bayat et al. [6] to design of radiant enclosures with diffuse-spectral surfaces. Daun et al. [7] and Mehraban et al. [8] presented an inverse radiation design problem for finding the transient heater settings to produce the transient conditions over the products in radiant furnaces. Daun et al. [9,10] used the gradient-based optimization approach to geometry design of a two-dimensional radiant enclosure containing specular surfaces.

NOMENCLATURE

A	[m^2]	Area,
E_b	[W/m^2]	Emissive power
F	[-]	Configuration factor
f	[-]	Objective function
J	[W/m^2]	Radiosity
q	[W/m^2]	Heat flux
S	[m]	Distance between two areas

Special characters

ε	[-]	Emissivity
θ	[rad]	Angle measured from normal to surface

Subscripts

d	[-]	Desired, Design
e	[-]	Estimated
h	[-]	Heater

Genetic algorithm (GA) represented by Holland [11], is a kind of stochastic search and optimization technique inspired by the biological mechanisms of reproduction. The GA employs the Darwinian survival of the fittest theory to yield the best or better characters among the old population, and performs a random information exchange to create superior offspring. Comprehensive discussions of the GA are given in Goldberg [12]. The GA searches the global optima without the dependence of initial guesses. However, the GA is sometimes very poor in terms of convergence performance, especially when the searching has reached a local region near the global optimum. To improve the convergence performance and enhance the searching capability, Krishnakumar [13] developed a version of the GA, known as micro genetic algorithm (*m*GA), which allows for a very small population size (typically 5-8). The *m*GA was applied to design of the laser systems by Carroll [14]. Yang and Liu [15] used the *m*GA to identify the material properties of a printed circuit board and components mounted on it. The application of the GA for optimization of heating systems has been paid much attention for the past years and there exists a considerable body of knowledge surrounding the subject. Sarvari [16] presented the utilization of the GA to optimum geometry design of a radiant furnace with transparent medium. The application of the GA for optimum placement of heaters in a two-dimensional radiant furnace was investigated by Sarvari [17]. A comprehensive review on application of the GA in heat transfer problems has been reported by Gosselin et al. [18]

In this paper, the application of the *m*GA to find the optimal number and locations of radiant heaters over the heater surface located at the top surface of a radiant oven with diffuse-gray walls and containing transparent media is demonstrated. The desired uniform temperature and heat flux distributions over the design surface located at the bottom surface are satisfied by a set of heaters with equal specified powers. All the wall surfaces of the enclosure are subdivided into small-area surface elements, and the net radiation method is used to determine the radiation exchange between surface elements. The *m*GA is used for minimizing an objective function which is defined as the sum of square deviations between desired and estimated heat fluxes over the design surface. The performance and the accuracy of the optimization approach is examined by two example problem. The results show that the accuracy of the method is good for engineering applications.

DESCRIPTION OF PROBLEM

Consider a cubic radiant oven as depicted in figure 1. The product surface (design surface) is placed on the bottom wall, whereas the heaters are placed on the heater surface at the top of the oven. All the walls are gray-diffuse and the medium is transparent. The heat is transferred by a number of equal-power heaters which are arranged symmetrically over the heater surface. All the surface elements on the side walls and the heater surface are kept insulated, except the elements corresponding to heaters which are kept at a uniform heat flux of q_h. The aim of the design problem is to optimize the number and the locations of the heaters to produce a uniform heat flux distribution over the temperature-specified design surface.

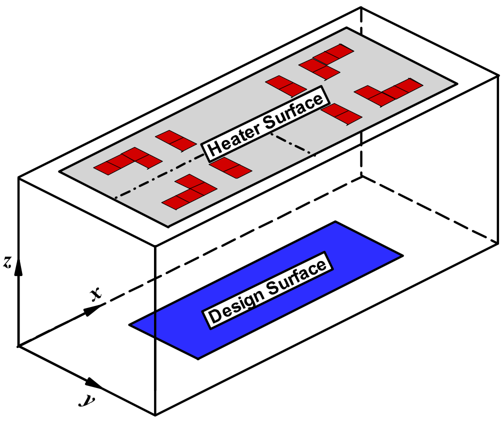


Figure 1 The schematic shape of the radiant oven with a heater surface on the top wall and a design surface over the bottom wall

DIRECT PROBLEM

The direct problem consists of the calculation of the radiation exchange between the surface elements on the wall surfaces of the radiant oven involving two types of boundary conditions; the surfaces with specified temperature and the surfaces with specified heat flux.

The net radiation method is used to solve the radiation exchange in the radiant oven. In this method, the boundary is subdivided into surface elements. Then the equation of radiation exchange for surface elements with specified temperature (emissive power) can be described by the following equation:

$$\sum_{j=1}^{K}\left[\delta_{kj}-(1-\varepsilon_k)F_{k-j}\right]J_j=\varepsilon_k E_{b,k}\ ,\ 1\le k\le K_1 \tag{1a}$$

whereas the radiative transfer equation for other surface elements with specified heat flux is given by

$$\sum_{j=1}^{K}\left(\delta_{kj}-F_{k-j}\right)J_j=q_k\ ,\ \ K_1+1\le k\le K \tag{1b}$$

where K_1 is the number of surface elements with specified temperature, K is the total number of surface elements, and δ_{kj} is the Kronicker delta which is defined as

$$\delta_{kj}=\begin{cases}1 & k=j\\ 0 & k\ne j\end{cases} \tag{2}$$

The set of equations (1) is solved to calculate the outgoing heat fluxes, $J_j, j=1,\ldots,K$, then the unknown boundary condition (emissive power or heat flux) is determined by the following equation:

$$\frac{1-\varepsilon_k}{\varepsilon_k}q_k+J_k=E_{b,k}\ \ ,\ 1\le k\le K \tag{3}$$

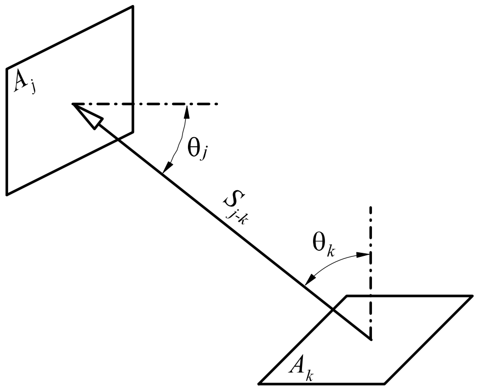
If the walls of the radiant oven are subdivided into small differential surface elements (figure 2a), then the configuration factors, F_{k-j}, may be calculated by the following equation [19]

$$F_{k-j}=\frac{\cos\theta_j\cos\theta_k}{\pi S_{k-j}^2}A_j \tag{4}$$

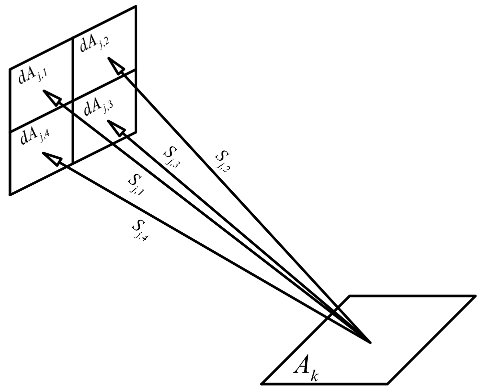
Although the relation presented by Eq. (4) is a good approximation for non-adjacent surface elements, but it creates more deviation from exact value for adjacent surface elements. In order to overcome this problem, the target surface element may be divided into four sub-elements, $dA_{j,l}$, $l=1,\ldots,4$. Then, the differential configuration factor between the origin surface element and each sub-element is calculated by

$$dF_{k-j,l}=\frac{\cos\theta_{j,l}\cos\theta_{k,l}}{\pi S_{k-j,l}^2}dA_{j,l} \tag{5}$$

Finally, the configuration factor F_{k-j} can be determined by applying a summation over four differential configuration factors (see figure 2b)



(a)



(b)

Figure 2 Radiant interchange between (a) two surfaces, (b) a surface with another surface subdivided into four small differential areas

MICRO GENETIC ALGORITHM

The mathematical formulation of the optimization problem is based on the minimization of an objective function which is defined as

$$f = -\sum_{n=1}^{N} (q_{d,n} - q_{e,n})^2 \tag{7}$$

The genetic algorithm is a technique used to automate the process of searching for an optimal solution. Because it searches from a population of points, the probability of the searches getting trapped in a local minimum is limited. GAs start searching by randomly sampling within an optimization solution space, and then use stochastic operators to direct a process based on the objective function values. A solution to a problem is an individual and the group of individuals is a population. Each individual is represented by a binary string called a chromosome, which encodes all the parameters of interest corresponding to that individual. The fitness of any particular individual corresponds to the value of its objective function. Genetic operators control the evaluation of successive generations. The three basic genetic operators are reproduction, crossover and mutation. The probability of a given solution's being chosen for reproduction is proportional to the fitness of that solution. Crossover implies that parts of two randomly chosen chromosomes will be swapped to create a new individual. Mutation involves randomly changing a chromosome in a solution to look for new points in the solution space. A genetic algorithm starts by generating a number of possible solutions to a problem, evaluates them and applies the basic genetic operators to the initial population according to the individual fitness of each individual. This process generates a new population with higher average fitness than the previous one, which in turn will be evaluated. The cycle is repeated for the number of generations set by the user, which is dependent on problem complexity.

There are several different versions of genetic algorithms. The micro genetic algorithm is one of the most widely used GAs. This algorithm produces fewer individuals in each generation and the individuals of each generation are usually created through two operations: selection and crossover. To maintain the genetic diversity in population, *m*GA uses a restart strategy. That is, once the current population converges, a new population would be generated, which has the same population size and consists of the best individual from the previously converged generation and other new randomly generated ones. This procedure would be continued until the global optimum is found (or the predesignated number of generations is reached). The reader can consult Ref. [14] for more information about *m*GA.

For the problem considered here, the locations of the equal-power heaters are considered as the design variables. The number of design variables is equal to the number of surface elements (or groups of surface elements) over a quarter of heater surface. Hence, the length of each gene is equal to one and the length of each chromosome is equal to the number of surface elements (or groups of surface elements) over a quarter of the heater surface. Each heater on the heater surface may have two status; on or off or, the value of each design parameter may be 0 or 1. For example, a value of 0 for the 3rd gene and a value of 1 for 6th gene mean that the heaters over the third and sixth element surfaces (and three corresponding symmetrical located heaters) are turned-off and turned-on, respectively.

RESULTS AND DISCUSSION

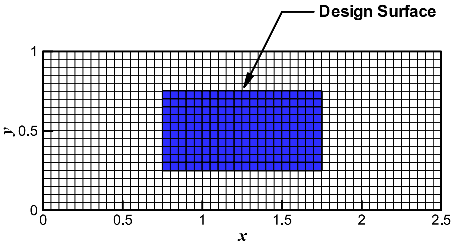
In order to investigate the performance and accuracy of the inverse design, we now consider some example problems. Two criteria for measuring the accuracy of the inverse estimation are the relative error and the root mean square error which are defined as

$$E_{rel,n} = \left(q_{d,n} - q_{e,n}\right) / q_{d,n} \times 100 \tag{8a}$$

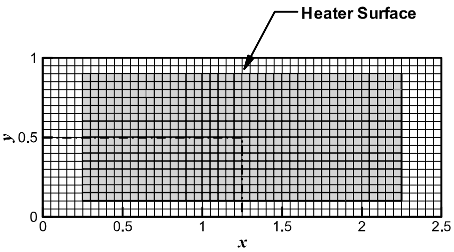
$$E_{rms} = \left\{ \frac{1}{N} \sum_{n=1}^{N} \left[\left(q_{d,n} - q_{e,n}\right) / q_{d,n} \times 100 \right]^2 \right\}^{1/2} \tag{8b}$$

The relative error measures the deviation between desired and estimated values on each surface element of the design surface, whereas the root mean square error measures the deviation between desired and estimated values over entire extent of the design surface.

Example Problem 1. Consider a cubic enclosure as is shown in figure 1. The dimensions of the design surface and the heater surface are shown in figure 3a and 3b, respectively. The boundary surface of the $2.5m \times 1m \times 1m$ enclosure is subdivided into 20000 square surface elements of $0.05m \times 0.05m$ area. The number of surface elements over the design surface and the heater surface are equal to 200 and 640, respectively. All the element surfaces on the side walls are kept insulated. The emissivity of the design surface and the heater surface is $\varepsilon_d = \varepsilon_h = 0.9$, and the value of emissivity for other walls is equal to $\varepsilon = 0.05$. The bottom wall, including the design surface, is maintained at a uniform emissive power of $E_b = 1.0 W/m^2$.



(a)



(b)

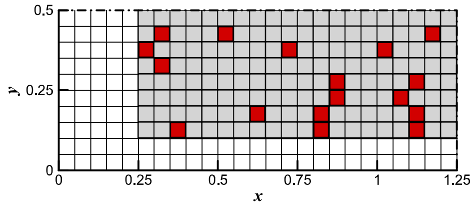
Figure 3 The location and dimensions of (a) the design surface (b) the heater surface

The goal of the design problem is to find the number and the locations of the surface elements (or groups of surface elements) with equal power of $q_h = 5W/m^2$ over the insulated heater surface to produce a uniform heat flux of $q_d = -2W/m^2$ over the design surface.

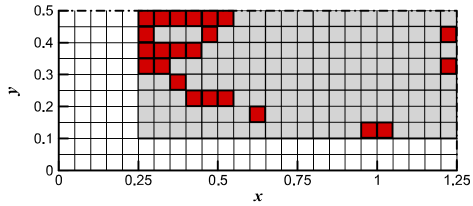
The number of variables to be optimized is equal to the number of surface elements over a quarter of the heater surface which is equal to 160. Each variable can take a value of 0 or 1. Zero means an insulated surface element ($q_h = 0$), whereas 1 means a uniform heat flux of $q_h = 5W/m^2$ over the surface element.

The optimum arrangements of heaters over a quarter of heater surface for two cases with different values of emissivity over the design surface, $\varepsilon_d = 0.5, 0.9$, are shown in Figs. 4a,b.

The distribution of relative error for estimated heat fluxes over the design surface for two cases are shown in Figs. 5a,b. Table 1 shows the corresponding values of maximum relative error, root mean square error, and the number of heaters over the heater surface for two values of emissivity over the design surface. The results show that the desired heat flux over the design surface is well recovered in an acceptable range of error.

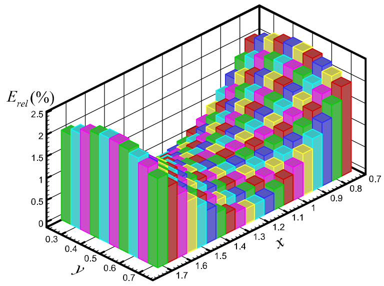


(a)

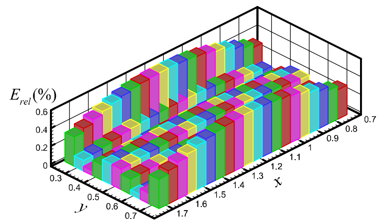


(b)

Figure 4 The location of heaters over a quarter of heater surface for (a) $\varepsilon_d = 0.5$ and (b) $\varepsilon_d = 0.9$



(a)



(b)

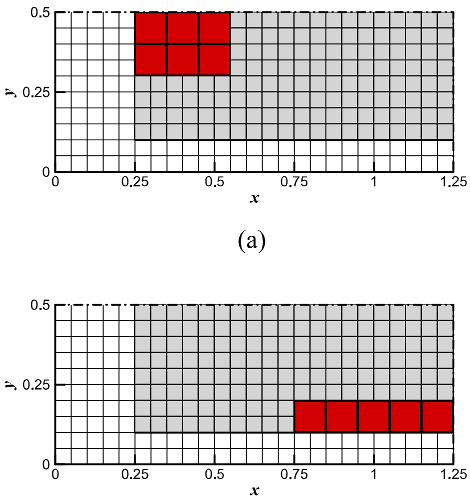
Figure 5 the distribution of relative error over the design surface for (a) $\varepsilon_d = 0.5$ and (b) $\varepsilon_d = 0.9$

Table 1- The maximum relative error, root mean square error and the number of heaters over the heater surface for two different values of emissivity over the design surface

ε_d	$E_{rel,\max}$	E_{rms}	N_h
0.5	2.40	1.36	92
0.9	0.57	0.28	68

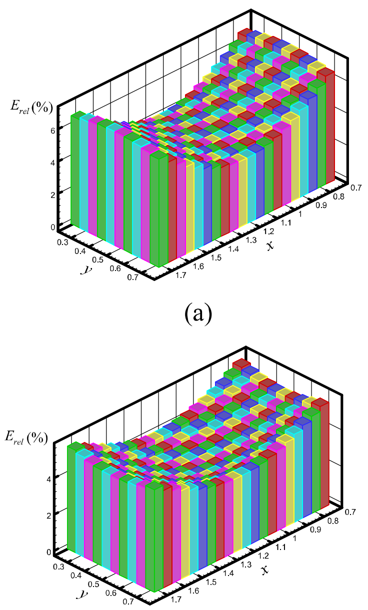
Example Problem 2. There are cases where the designer is more likely to group the heaters into bundles with higher powers, or due to constraint of design problem, the designer prefer to use larger heaters with higher powers. In these cases, the available locations of heaters are decreased and the degree of freedom for placement of heaters is decreased accordingly.

We now consider an alternative case, where the heaters are grouped into bundles of four elements with power of $q_h = 20W/m^2$. In this case, the number of locations (or the number of variables) over a quarter of the heater surface reduces to 40. All the radiative properties are the same as those in example problem 1.



(a) (b)

Figure 6 The location of heaters over a quarter of heater surface for (a) $\varepsilon_d = 0.5$ and (b) $\varepsilon_d = 0.9$



(a) (b)

Figure 7 the distribution of relative error over the design surface for (a) $\varepsilon_d = 0.5$ and (b) $\varepsilon_d = 0.9$

The optimum number and locations of heaters over a quarter of heater surface for two cases with different values of emissivitiy over the design surface are shown in Figs. 6a,b. The distribution of relative error over the design surface for two cases are shown in Figs. 6a,b. The results are summarised in Table 2. As seen, both the maximum relative error and the root mean square error are larger than those values in example problem 1. In fact, increasing the values of errors is the result of decrease in degree of freedom for locating the heaters over the heater surface.

Table 2- The maximum relative error, root mean square error and the number of heaters over the heater surface for two different values of emissivity over the design surface

ε_d	$E_{rel,\max}$	E_{rms}	N_h
0.5	7.07	5.39	24
0.9	5.62	4.33	20

CONCLUSION

This paper overviewed a genetic algorithm based approach for preliminary design of the radiative enclosures. The method can be applied to optimize the numbers and positions of the heaters in a three-dimensional radiative enclosure. The primary objective of the work was to recover a specified heat flux distribution over a specified temperature surface, namely the design surface. A floating-point encoded micro genetic algorithm was used to solve the optimization problem. The individuals of the population are the number and possible locations of the heaters in the furnace. The objective function was expressed by the sum of square residuals between estimated and desired heat fluxes over the design surface. The problem of radiation heat transfer was solved by the net radiation method. Two example problems were solved to show the performance and accuracy of the inverse method. The results implied that the accuracy of the method is decreased by decreasing the emissivity of the design surface. However, the desired uniform heat flux over the design surface was successfully recovered by the inverse approach in an acceptable range of error.

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