New Drag-Coefficient Charts for Settling Spherical and Disk Particles in Shear Thinning Fluids

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ABSTRACT

Settlement of solid particles in shear-thinning non-Newtonian fluids is of great importance and has many applications in petroleum operations and chemical processes. In general, the drag coefficient (C_d) of a solid particle depends mainly on the particle Reynolds number (R_{ep}), which is mainly a function of fluid rheology, particle shape and size, and particle slip velocity. A non-Newtonian fluid is broadly defined as one for which the relationship between shear stress and shear rate is not constant (i.e., shear stress is changing with shear rate). For shear-thinning fluids, the apparent viscosity is decreasing with increasing shear rate.

For approximate calculations, most engineers used the Newtonian model to predict the drag coefficient of a solid particle in non-Newtonian fluid, because there is no distinct chart for the drag coefficient which includes the effect of changing fluid rheology and particle sphericity. But this approach gives poor and inaccurate results for all flow regimes around the particle.

In this study, the effects of fluid rheology, and particle shape and size on settling velocity measurement have been studied experimentally, from which, new drag coefficient charts have been developed for spherical and disk particles when they settled down through various shear-thinning fluids. The flow behavior indices (n) for these fluids ranged from 0.6 to 0.92, and the predicted drag coefficients ranged from 0.1 to 10000 to cover particle Reynolds number from 0.001 to 100.

For spherical particles, the new charts showed that the drag coefficient at a given particle Reynolds number is increased as the flow behavior index is decreased (i.e., the fluid became more non-Newtonian), and the drag coefficient is decreased as the particle Reynolds number is increased.

But for disk particles, two settling paths were observed. The first one when n is less than 0.8, the particles were settled-down edge-wise, and the other when n is greater than 0.8, the particles were settled-down base-wise. For both paths, the drag coefficient at a given particle Reynolds number is increased as the flow behavior index is decreased.

Finally, a new empirical correlation has been developed which can be used to determine easily and directly the settling velocity, the particle Reynolds number, and the drag coefficient of spherical and disk particles when settles down in shear-thinning fluids.

INTRODUCTION

The settlement (or movement) of solid particles in non-Newtonian fluids has many applications in petroleum and chemical engineering. In drilling oil or gas wells, it is required to have an efficient lifting program to remove the solid particles (or formation cuttings) out of the hole, where these particles are moving through a non-Newtonian drilling fluid, and therefore preventing many drilling problems.

It has been shown from the literatures that the drag coefficient is inversely related to the particle Reynolds number, and depending on their range, different relations are developed which are limited only to Newtonian fluids. Inadequately, most engineers used these relations to predict the drag coefficient (or the settling velocity) of solid particles in non-Newtonian fluids, which give inaccurate results for all flow regimes around the particle.

The main purpose of this study is to develop new drag coefficient charts for spherical and disk particles when they settles down through various shear-thinning non-Newtonian fluids. Numerous experiments have been done to study the common effects of the rheological properties and the particles shapes and sizes on the settling velocity measurements.

PREVIOUS WORK

Numerous authors had extensive contributions to study the mechanism of settling solid particles in non-Newtonian fluids. Some authors [1-7] stated that the equations developed for Newtonian fluids could be used for non-Newtonian fluids

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by simply replacing the viscosity term of Newtonian fluid with an apparent viscosity of non-Newtonian fluid.

However, Shah [9] in 1982 clearly stated that this approach is not accurate, and he presented a method whereby particle settling velocity in non-Newtonian fluids can be properly estimated. He proposed plotting (C_d^{2-n}) versus (R_{ep}) to show the dependency of (C_d) on the fluid flow behavior index (n). He clearly concluded that the drag coefficient depends on both (n) and (R_{ep}).

Chhabra [1,3] in 1990 attempted to obtain a unified model to predict the drag coefficient of a falling sphere in power-law fluids. He conclude that the standard curve available for Newtonian fluids provides adequate representation for power-law fluids without any dependence on flow behavior index, within the following ranges of $1.0 < (R_{ep}) < 1000$ and $0.535 < n < 1.0$.

Vassilios [7] in 2003, proposed an equation to predict the terminal settling velocity of solid spheres falling in non-Newtonian shear thinning fluids. He reached to his equation by comparing used equations with his experimental data. He concluded that the terminal velocity of solid sphere through stagnant shear thinning fluids can be predicted with acceptable accuracy from the correlations used for Newtonian fluids. His conclusion was based on the fact that his experimental data fell along the standard Newtonian drag curve.

El-fadili [12] in 2005, plotted his experimental data as $\sqrt{C_d^{2-n} \cdot R_{ep}^2}$ vs. (R_{ep}) on a log-log paper, rather than the conventional plot of (C_d) vs. (R_{ep}). He proposed a unified model to predict the drag coefficient of a single particle settling in Newtonian and shear thinning fluids. He concluded that his model is good for all shear thinning fluids, where the particle Reynolds number range is; $0.001 < (R_{ep}) < 1000$.

Aswad [13] in 2008, presented a new and simple approach for estimating the settling velocity of solid particles in shear thinning fluids. His approach includes three steps; the first one is to measure the rheological properties of the given non-Newtonian fluid, (i.e., to measure shear stresses at different shear rates). The second step is to select the best rheological model that fits this fluid. The third step is to calculate the settling velocity using the appropriate equation, which includes the effect of fluid rheology.

NON-NEWTONIAN FLUIDS

Non-Newtonian fluids (such as; polymers, paints, drilling mud, cement slurries, surfactants, blood, jelly, and starch) are those fluids which do not show linear relationship between shear stress and shear rate, (i.e., their shear stress or viscosity is changing with shear rate). Practically, non-Newtonian fluids are classified into three major groups; time-independent fluids, time-dependent fluids, and visco-elastic fluids. The time–independent fluids include three types; Pseudo-plastic fluids (also called shear-thinning fluids), Dilatant fluids (also called shear-thickening fluids), and Bingham plastic fluids. On other hand, the time-dependent fluids include only thixotropic and rheopectic fluids.

Fluid viscosity is defined as the internal resistance of fluid to flow, which is equal to the ratio of shear stress to shear rate. For non-Newtonian fluids, the measured viscosity is the "effective or apparent" viscosity. For shear-thinning fluids, the apparent viscosity decreases as the shear rate is increased, while for shear-thickening fluids the apparent viscosity increases as the shear rate is increased.

SETTLING VELOCITY

The settling velocity of a solid particle is defined as the rate at which the particle settles in still fluid. It is a function of particle size and shape (or particle sphericity), particle density, fluid viscosity (or fluid rheology), and fluid density. Since most solid particles have irregular shapes, then all particles will be related to that of a sphere by using the concept of "equivalent diameter, d_p" or the concept of "sphericity".

Generally, there are three forces acting on a solid particle falling through a given fluid, which are; gravity force, buoyancy force due to particle being submerged in fluid, and viscous drag force resulting from fluid resistance to particle movement. When the particle is moving at constant speed, the upward forces are equal to the downward forces. Thus;

The drag force + The buoyancy force = The gravity force

$$C_d \cdot \left(\frac{1}{2} \cdot \rho_f \cdot V_s^2 \right) \cdot \frac{\pi}{4} \cdot d_p^2 +$$

$$\frac{\pi}{6} \cdot d_p^3 \cdot \rho_f \cdot g = \frac{\pi}{6} \cdot d_p^3 \cdot \rho_p \cdot g \tag{1}$$

From which, the general slip velocity equation is given by equation (2);

$$V_s = \left[\frac{4 \cdot \left(\rho_p - \rho_f \right) \cdot d_p \cdot g}{3 \cdot \rho_f \cdot C_d} \right]^{1/2} \tag{2}$$

Where;

V_s = slip velocity, cm/sec.
ρ_p = density of the solid particle, g/cm.
ρ_f = fluid density, g/cm.
C_d = drag coefficient, dimensionless.
d_p = diameter of sphere, or equivalent diameter of a particle having the same volume as a sphere, cm.
g = gravity constant, cm/sec^2.

To calculate the slip velocity of a given particle, the equivalent diameter and the drag coefficient should be estimated first.

DRAG COEFFICIENT

The drag coefficient (C_d) of a solid particle is a dimensionless quantity that is used to quantify the drag or resistance of the particle when it is moving down through a given fluid. It is used in the drag force equation, where a lower drag coefficient indicates that the particle will have less hydrodynamic drag. The drag coefficient can be estimated from equation (3);

$$C_d = \left[\frac{4 \bullet \left(\rho_p - \rho_f \right) \bullet d_p \bullet g}{3 \bullet \rho_f \bullet v_s^2} \right] \quad (3)$$

To calculate the drag coefficient of a given particle, the equivalent diameter and the slip velocity should be estimated first.

The drag coefficient is always a function of particle Reynolds number, which is a function of slip velocity, fluid rheology, and particle shape and size. It has been shown from the literatures that the drag coefficient is inversely related to the particle Reynolds number, and depending on their range, different relations between (C_d) and (R_{ep}) are developed which are limited only to Newtonian fluids. Inadequately, most engineers used these relations to predict the drag coefficient (or the slip velocity) of solid particles in non-Newtonian fluids, which give inaccurate results for all flow regimes around the particle.

PARTICLE REYNOLDS NUMBER

The particle Reynolds number, (or the Reynolds number around the particle), is a dimensionless group which reflects the type of flow regime around a solid particle when settles in a given fluid, and is defined by;

$$R_{ep} = \frac{\rho_f \bullet d_p \bullet v_s}{\mu_{app}} \quad (4)$$

Where;
  R_{ep} = particle Reynolds number, dimensionless.
  μ_{app} = apparent fluid viscosity, (dyne-sec)/cm^2
Practically the values for Reynolds number ranged from 0.001 to 10000, which cover three distinct flow regimes around the particles; laminar, transition, and turbulent.

EXPERIMENTAL DATA

In this study, four non-Newtonian shear-thinning fluids (designated as: F1, F2, F3, and F4) have been used in order to investigate experimentally the common effects of the rheological properties and particles shapes and sizes on the drag coefficient charts.

The rheological properties of these fluids have been measured using Chandler-3500 rotational viscometer, which is a direct reading 12-speeds (i.e.; 1, 2, 3, 6, 10, 20, 30, 60, 100, 200, 300, and 600 rpm) viscometer. For this viscometer, the apparent viscosity [in (dyne-sec)/cm^2] can be determined from equation (5).

$$\mu_{app} = \frac{K_1 \bullet K_2 \bullet \theta}{\gamma} \quad (5)$$

Where;
  K_1 = viscometer constant = 386.0
  K_2 = constant = 0.01323
  θ = viscometer dial reading, dyne/ cm^2
  γ = shear rate, sec^{-1}
The flow behavior index (n) for fluids F1, F2, F3, and F4 are equal to 0.60, 0.74, 0.86, and 0.92, respectively.

For spherical particles study, seven steel balls (of diameters: 1, 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0 mm) have been used to measure experimentally their settling velocities in the four tested fluids. For disk particles study, eight particles (of different sizes) have been used to measure experimentally their settling velocities in the same fluids. The experimental data are available upon request from the author.

RESULTS AND ANALYSIS

1- From the rheological properties, the apparent viscosities for each tested fluid have been calculated using equation (5). Figures 1, 2, 3, and 4 show the plots of fluid viscosity vs. shear rate for fluids F1, F2, F3, and F4, respectively. These figures confirmed the shear-thinning behavior of these fluids.

2- For spherical and disk particles, the settling (or slip) velocities through the tested fluids have been estimated experimentally. Sample of these results are given in tables 1 and 2 for fluid F1.

3- It was observed that, for fluids with low values of n (such as F1 and F2), the disk particles were settled down edge-wise, since these fluids possess high resistance to particle movement. But for fluids with high values of n (such as F3 and F4), the disk particles were settled down base-wise.

4- For spherical and disk particles, the drag coefficients and particle Reynolds number have been calculated using equation (3) and (4), respectively, for each tested fluid. Some results of these calculations are given in tables 1 and 2 for fluid F1.

 (i) For spherical particles, the drag coefficients are plotted vs. particle Reynolds numbers, for each tested fluid, on log-log paper, as shown in figure 5. It is clear from this figure that there is symmetry and harmony between the plotted points for each fluid, noting that each fluid has different flow behavior index (n) value. It is also clear from this figure that the drag coefficient decreases as the particle Reynolds number increases and as the (n) value increases. On other hand, for a given flow regime around the particle, the drag coefficient increases as the flow behavior index decreases, (as the fluid become more non-Newtonian).

 (ii) Also for disk particles, the drag coefficients are plotted vs. Particle Reynolds numbers, for each tested fluid, on log-log paper, as shown in figure 6. It is clear from this figure that there is symmetry only between the plotted points for each tested fluid. It is also clear from this figure that the drag coefficient decreases as the particle Reynolds number increases and as the (n) value increases. On other hand, for a given flow regime around the particle, the drag coefficient increases as the flow behavior index decreases, (as the fluid become more non-Newtonian).

5- The drag coefficients raised to power (2-n) have been also calculated for each tested fluid, using spherical and disk particles. Some results of these calculations are also given in tables 1 and 2 for fluid F1.

 (i) For spherical particles, the drag coefficients raised to power (2-n) as suggested by Shah [9] are plotted vs. Reynolds numbers, for each tested fluid, on log-log paper, as shown in figure 7. It is clear from this figure that same linear

relations as in figure 5 are obtained, but with different slopes and intercepts.

(ii) Also for disk particles, the drag coefficients raised to power (2-n) as suggested by Shah [9] are plotted vs. Reynolds numbers, for each tested fluid, on log-log paper, as shown in figure 8. It is clear from this figure that same linear relations as in figure 6 are obtained, but with different slopes and intercepts.

EMPIRICAL CORRELATION

Since figures 5 and 6 show linear relations between drag coefficient and particle Reynolds number for all fluids, therefore, the following general equation has been adopted, which represents a straight line equation for log-log plots;

$$C_d = \frac{a}{R_{ep}^b} \qquad (6)$$

Where;

a = the intercept of the straight line at (R_{ep} = 1.0)
b = the slope of the straight line, (it is negative slope). Equation 6 can be used for both spherical and disk particles. It is clear from figures 5 and 6 that the plotted straight lines have almost same slopes, but with different intercepts.

DETERMINATION OF PARAMETERS

The values of the parameters V_s, R_{ep}, and C_d can be determined easily and directly by using equation 6 together with figures 5 and 6, through the following steps;

1- For any fluid rheology, it is possible to generate the proper straight line that represents the corresponding n-value. Then the value of the intercept b can be determined easily at (R_{ep} = 1.0).

2- By knowing the values of the constants (a) and (b), one can use equation 6 to determine the slip velocity of any solid particle that settles through a given shear-thinning fluid.

3- Using the slip velocity value, it is possible to determine the particle Reynolds number, and the drag coefficient, which means it is possible to estimate the type of flow regime around the particle and how much drag force affecting the falling particle.

4- Steps from 1 to 3 can be repeated in the same manner using figures 7 and 8.

CONCLUSIONS

1- New drag coefficient charts have been developed for spherical and disk particles when settles down through various shear-thinning fluids, which include the effects of fluid rheology, particle shape and size, and particle settling velocity, and cover flow behavior indices from 0.60 to 0.92 for laminar to transient flow regimes around the particles. These charts show linear relations between C_d and $R_{e\,p}$, which have almost the same slopes but with different intercepts.

2- These C_d charts show that, for both spherical and disk particles, the drag coefficient at a given particle Reynolds number is increased as the flow behavior index is decreased (i.e., as the fluid becomes more non-Newtonian), which means higher resistance to particle movement. And, for a given fluid rheology, the drag coefficient is decreased as the particle Reynolds number is increased, which means less resistance to

particle movement (i.e., faster slip velocity). See figures 5 & 6.

3- Shah's Approach confirms the same linear relations between C_d and $R_{e\,p}$ as before, and shows also the same effects of fluid rheology and particle Reynolds number on the drag coefficient. See figures 7 & 8.

4- For fluids with low values of n, (as F1 and F2), it was observed that the disk particles were settled down edge-wise, since these fluids possess high resistance to particle movement. But for fluids with high values of n, (as F3 and F4), the disk particles were settled down base-wise.

5- An empirical correlation (equation 6) has been presented for spherical and disk particles when settles down in shear-thinning fluids, which can be used to determine easily and directly the settling velocity, the particle Reynolds number, and the drag coefficient of these particles.

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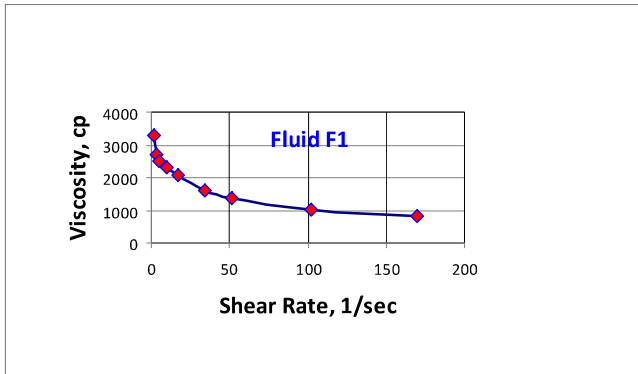

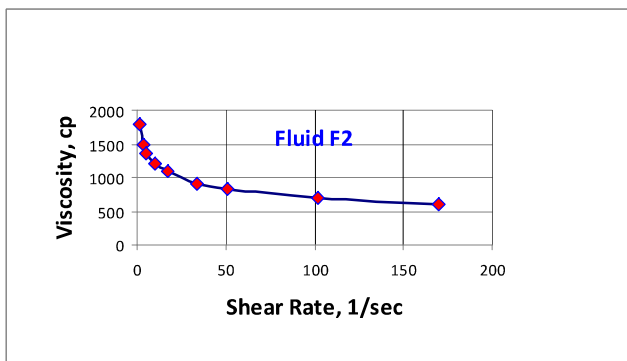**Figure (1)** Apparent Viscosity vs. Shear Rate for Fluid F1

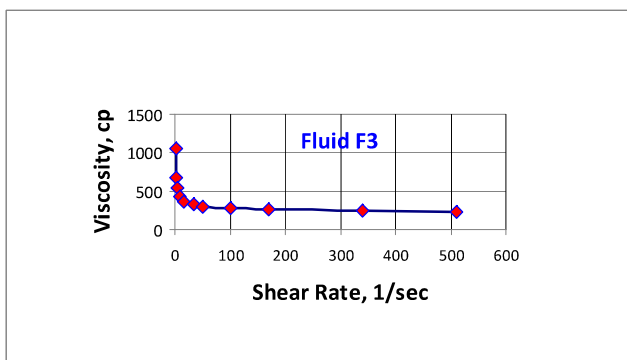**Figure (2)** Apparent Viscosity vs. Shear Rate for Fluid F2

Figure (3) Apparent Viscosity vs. Shear Rate for Fluid F3
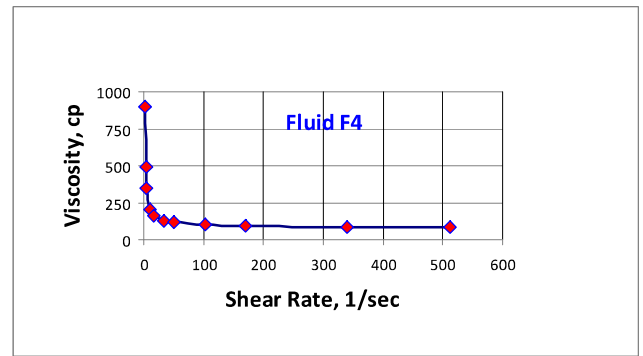

Figure (4) Apparent Viscosity vs. Shear Rate for Fluid F4
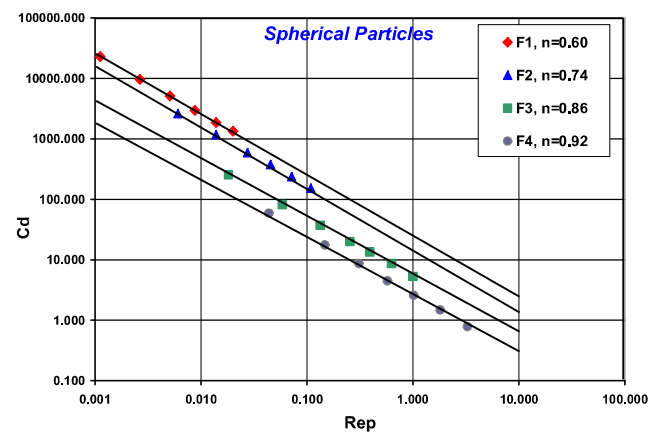

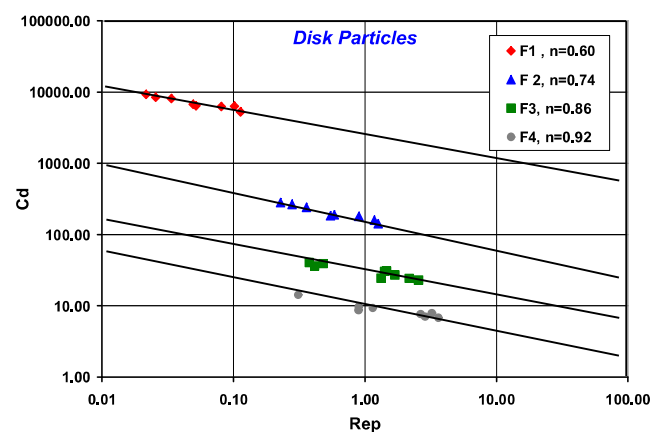**Figure (5)** Drag-Coefficient Chart for Spherical Particles in Shear-Thinning Fluids

Figure (6) Drag-Coefficient Chart for Disk Particles in Shear-Thinning Fluids
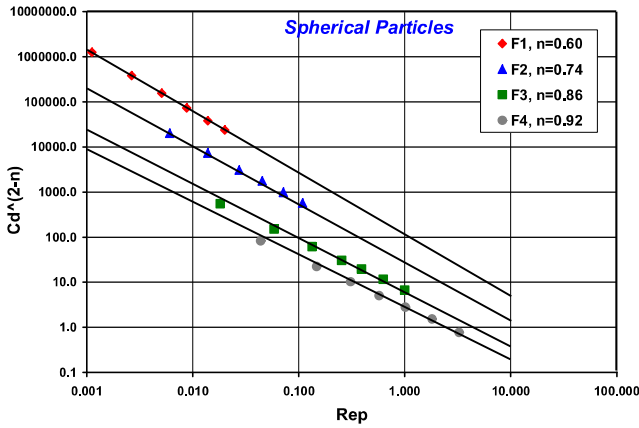
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Figure (7) Modified Drag-Coefficient Chart for Spherical Particles in Shear-Thinning Fluids, Shah Approach
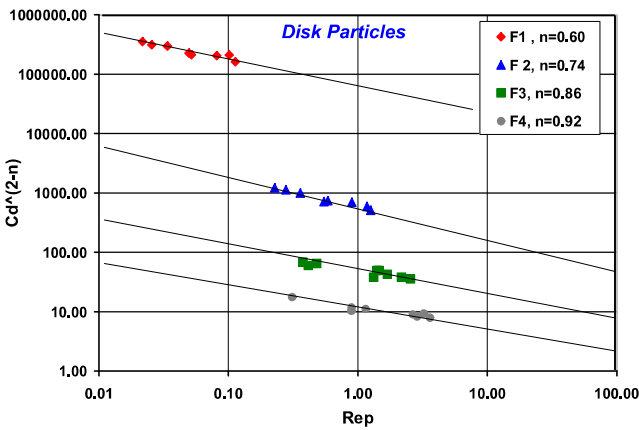

Figure (8) Modified Drag-Coefficient Chart for Disk Particles In Shear-Thinning Fluids, Shah Approach

Table (1) Experimental Data for Fluid F1, Using Spherical Particles

D_P mm	T, sec	V_S, cm/sec	R_{ep}	C_d	$(C_d)^{2-n}$
4.0	6.995	1.615	0.02007	1334.95	23749.31
3.5	8.855	1.276	0.01387	1871.86	38122.79
3.0	12.045	0.938	0.00874	2968.68	72709.36
2.5	17.280	0.654	0.00508	5091.63	154739.07
2.0	26.620	0.424	0.00264	9666.64	379652.31
1.5	47.185	0.239	0.00112	22778.69	1260492.76
1.0	---	---	---	---	---

L = 11.30 cm, ρ_p = 7.850 g/cm^3, ρ_f = 1.025 g/cm^3

Table (2) Experimental Data for Fluid F1, Using Disk Particles

D_P mm	ρ_p g/cm^3	T, sec	V_S, cm/sec	R_{ep}	C_d	$(C_d)^{2-n}$
1.240	3.6219	21.205	0.6649	0.02561	8411.00	312454.43
1.990	2.7073	17.650	0.7989	0.04938	6694.09	226969.88
1.992	2.5746	16.820	0.8383	0.05187	6405.00	213366.77
1.588	2.8907	20.640	0.6831	0.03370	8101.35	296469.68
1.148	3.3410	23.220	0.6072	0.02165	9201.33	354315.98
1.894	6.6821	10.240	1.3770	0.08101	6211.00	204374.23
2.094	6.7876	9.035	1.5606	0.10151	6322.62	209534.71
1.992	7.3919	7.720	1.8264	0.11301	5251.74	161593.77

L = 14.10 cm, ρ_f = 1.025 g/cm^3,