HEFAT2010
7th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics
19-21 July 2010
Antalya, Turkey

INFLUENCE OF GEOMETRICAL AND THERMAL PARAMETERS ON THE THERMAL COMPORTMENT OF A PIN-ON-DISK SYSTEM

Alilat N.*[1], Roseiro L.[2], Lozano J.-A.3, Guenoun S.[1], Guttiérez F.[3], Laraqi N.[1], García de María J.-M.[3] Baïri A.[1]
Author for correspondence * nalilat@u-paris10.fr
[1] : Université Paris Ouest, Laboratoire Thermique, Interfaces, Environnement LTIE-GTE
50, rue de Sèvres. F-92410 Ville d'Avray. France.
[2] : Departamento de Engenharia Mecânica. Instituto Superior de Engenharia de Coimbra
R. Pedro Nunes. Pt-3030 Coimbra, Portugal
[3] : Universidad Politécnica de Madrid, Ronda deValencia,3. E-28012 Madrid. Spain

ABSTRACT
Strong temperature gradients are often the cause of malfunctions taking place in mechanical systems which associate two rubbing solids. This work presents the thermal behaviour of a system consisting on a rotating disk in rubbing contact with a pin. Immersed in an environment characterized by a surface conductance h_∞ and a temperature T_∞, the disk is subjected to localised heat flux generated by the friction with the pin, eccentric with respect to the rotating axis of the disk. Several parameters intervene decisively on the local heat transfer and therefore on the temperature of the contact surface between the two solids in friction. In addition to the conductance, other parameters as the angular velocity of the disk, the frictional heat flux or the pin diameter and its off-center with respect to the disk rotation axis, play a major role in the thermal exchange. The present work examines the influence of such parameters on the thermal solution.
An analytical expression is proposed for the calculation of the 3D disk's temperature. The presented thermal cartographies make possible to locate the zones of the system undergoing the greatest temperature gradients and thus the associated spots of mechanical rupture. Results are compared with other analytical solutions found in the specialized literature.

INTRODUCTION

Temperature plays a major role in the sizing and thermal behaviour of systems with rotating elements. Friction between two solids converts mechanical energy into heat causing a local temperature increase. The thermal gradients are at the origin of the surface physical-chemical changes. This sometimes involves significant alterations, reversible or not, of the properties of the constituent materials or their total loss.

Several numerical and experimental studies treating this problem have been published in the specialised literature. We refer to [1, 2] who treat the coupling between two semi-infinite solids in contact on a rough surface of circular shape. The authors proposed solutions to calculate the temperature of a single solid subjected to a uniform heat source. In [1] the arithmetic mean of the surface temperature of both solids is adopted as the representative temperature of the interface. The case of a circular heat source in rectilinear motion on the surface of an isolated semi-infinite medium is treated in [3]. Other numerical and analytical studies covering the subject [4-10] have been published recently. The authors studied the heat flux division which is generated in the contact and distributed towards the two solids in contact. They carried out a parametric study and estimated the mean surface temperature and mean fluxes. The present work treats the case of a disk in rotation, subjected to a heat flux density on a portion of the upper face, generated by the friction of a pin. An expression making it possible to calculate the 3D temperature of the disk is proposed, followed by a study of sensitivity to some geometrical and physical parameters influencing the thermal behaviour of the system. The treated case can be applied among others to piece machining, operation of guide blades and bearings or to the vehicles brake systems.

NOMENCLATURE

a	[m]	Radius of the heat source
b	[m]	Radius of the disk
Bi	[1]	Biot number $Bi = h_d b / \lambda_d$
d	[m]	Thickness of the disk

e	[m]	Eccentricity
h_d	[W/m^{2}K]	Convection coefficient on the rubbing faceon the lower face
h_i	[W/m^{2}K]	Convection coefficient on the lower face
J_m	[1]	Bessel function of the first kind of order m
Pe	[1]	Peclet number. $Pe = ea\omega / \alpha_d$
q_d	[W/m^2]	Heat flux density
T_d	[K]	Disk temperature
T_∞	[K]	Ambient temperature $T_\infty = 273K$
T^*	[1]	dimensionless disk temperature $T^* = T_d / (q_d a / \lambda_d)$
r	[m]	radial coordinate
z	[m]	axial coordinate
Greek symbols		
α_d	[m^2/s]	Thermal diffusivity of the disk
λ_d	[W/m K]	Thermal conductivity of the disk
θ	[rad]	Angular coordinate
ω	[rad/s]	Angular velocity

THE PHYSICAL PROBLEM

The system treated is represented in Fig. 1. The disk of radius *b*, thickness *d* and rotating at angular velocity ω is subjected to a uniform heat flux q_d. This flux is caused by the contact between the disk and a pin of radius *a*, supposed semi-infinite, off-centred a distance *e* with respect to the disk axis. The rest of the upper face is subjected to an imposed surface conductance h_∞ and to ambient temperature T_∞, taken to be 0 ºC in the present work. These ambient thermal conditions are the same for the lateral and lower surfaces of the disk. The stationary thermal solution for this problem is related to the resolution of the 3D heat equation in cylindrical coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_d}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T_d}{\partial \theta^2}+\frac{\partial^2 T_d}{\partial z^2}-\frac{\omega}{\alpha_d}\frac{\partial T_d}{\partial \theta}=0 \quad (1)$$

with periodic conditions

$$\left(T_d\right)_{r,-\pi,z}=\left(T_d\right)_{r,\pi,z} \quad (2)$$

and

$$\left(\frac{\partial T_d}{\partial \theta}\right)_{r,-\pi,z}=\left(\frac{\partial T_d}{\partial \theta}\right)_{r,\pi,z} \quad (3)$$

The boundary conditions are:

- On the lateral surface

$$\left(\frac{\partial T_d}{\partial z}\right)_{b,\theta,z}=0 \quad (4)$$

- On the lower face

$$-\lambda_d\left(\frac{\partial T_d}{\partial z}\right)_{r,\theta,d}=-h_i T_d \quad (5)$$

- On the upper face

$$-\lambda_d\left(\frac{\partial T_d}{\partial z}\right)_{r,\theta,0}=\begin{cases}-h_d T_d \text{ (elsewhere)}\\ q_d \text{ (at the contact)}\end{cases} \quad (6)$$

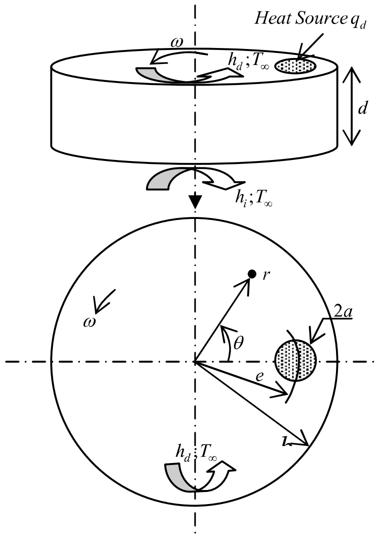

Figure 1 Studied system

The hypothesis of a uniform heat flux entering the disk through the contact is associated with the assumption of the integral cooling of the disk rubbing face (including the own area receiving the heat flux q_d).

This assumption is realistic as long as the ratio of areas pin-to-disk is small and the heat convection coefficient with the environment h_d does not exceed a few thousands of [W/m^{2}K]. This fact is confirmed in [11] concerning the study of an infinite cylinder receiving a heat flux on a portion of its periphery and cooled by convection on the remaining surfaces. The authors show that the adopted approximation remains valid as long as $h_d\sqrt{2\alpha_d/\omega}/\lambda_d \le 1$. This condition is largely checked for many practical applications like machining, pin-disk tribological devices and the disk brake system. If such conditions are not met, the solution appeared as the integral equation of Fredholm, that can be solved by successive analytical iterations or by decomposition in Fourier series.

In the adverse case of a disk in steel $\left(\alpha_d = 5.10^{-6} m^2.s^{-1}, \lambda_d = 20\, W.m^{-1}.K^{-1}\right)$ which would be rotating at small angular speed $\omega = 1\, s^{-1}$ and subjected to a high conductance $h_d = 5000\, W.m^{-2}.K^{-1}$, de hypothesis is satisfied because $h_d\sqrt{2\alpha_d/\omega}/\lambda_d$ ~0.8.

ANALYTIC SOLUTION AND VALIDATION

To solve the equation (1) with the conditions (2) to (6), we use the integral Fourier transform adapted to the problems showing a spatial periodicity (hereafter denoted by θ)

$$\tilde{T}_d = \frac{K_m}{2\pi}\int_{-\pi}^{\pi} T_d e^{-im\theta} d\theta \tag{7}$$

with $K_0 = 1$ or $K_{m\neq 0} = 2$ and i the imaginary unit, and the finite Hankel transform adapted to cylindrical geometries (hereafter denoted by r)

$$\tilde{\tilde{T}}_d = \int_0^b r\,\tilde{T}_d\, J_m(\beta_n r)\,dr \tag{8}$$

In (8), J_m represents the Bessel function of the first kind of order m, and β_n the root of the transcendental equation

$$J'_m(\beta_n b) = 0 \tag{9}$$

Equation (1) then becomes

$$\frac{d^2\tilde{\tilde{T}}_d}{dz^2} - \left[\beta_n^2 + \frac{i\,m\,\omega}{\alpha_d}\right]\tilde{\tilde{T}}_d = 0 \tag{10}$$

with the associated boundary conditions

$$\begin{cases} -\lambda_d\left(\dfrac{d\tilde{\tilde{T}}_d}{dz}\right)_{z=0} = \tilde{\tilde{q}}_d - h_d\,\tilde{\tilde{T}}_d \\ -\lambda_d\left(\dfrac{d\tilde{\tilde{T}}_d}{dz}\right)_{z=d} = h_i\,\tilde{\tilde{T}}_d \end{cases} \tag{11}$$

where

$$\tilde{\tilde{q}}_d = q_d\, a\, K_m \frac{J_1(\beta_n a)\,J_m(\beta_n e)}{\beta_n} \tag{12}$$

The tranformed expression of (9) becomes

$$\tilde{\tilde{T}}_d(z) = E(z) + \tilde{\tilde{q}}_d F(z) \tag{13}$$

where

$$E(z) = \left[\tilde{\tilde{T}}_d(z)\right]_{m=0,n=0} = \frac{q_d a^2\left(-h_i z + \lambda_d + d h_i\right)}{2\left[\lambda_d\left(h_i + h_d\right) + d h_i h_d\right]} \tag{14}$$

represents the mean temperature at a given section of constant z, and

$$F(z) = \frac{\left(\lambda_d\gamma + h_i\right)}{\cos(\gamma d)\left[\lambda_d\gamma + h_i\tanh(\gamma d)\right]} \times \frac{\lambda_d\gamma\cosh\left[\gamma(d-z)\right] + h_i\sinh\left[\gamma(d-z)\right]}{\left(\lambda_d\gamma\right)^2\tanh(\gamma d) + h_i\lambda_d\gamma + h_i h_d\tanh(\gamma d)} \tag{15}$$

represents the fluctuating part of the temperature, with

$$\gamma = \rho\exp(i\varphi)\,;\ \rho = \left[\beta_n^4 + \left(m\omega/a\right)^2\right]^{1/4}\ ;\ \rho = \tan^{-1}\left(m\omega/a\beta_n^2\right)\Big/2 \tag{16}$$

The application of the respective inverse Hankel and Fourier transforms lead to the expression of the 3D temperature of the disk. There appear the geometrical and physical parameters of the treated problem

$$T_d(r,\theta,z) = \frac{1}{2b^2}E(z) + \sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\left[\frac{\tilde{\tilde{q}}_d\beta_n^2 J_m(\beta_n r)}{\left[(\beta_n b)^2 - m^2\right]J_m^2(\beta_n b)}\Re_e\left\{F(z)\exp(im\theta)\right\}\right] \tag{17}$$

In the case of a semi-infinite disk, this equation can be written as

$$T_{d\to\infty}(r,\theta,z) = \frac{q_d a^2}{h_d b^2} + \frac{2q_d a}{b}\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{K_m(\beta_n b)J_1(\beta_n a)J_m(\beta_n e)J_m(\beta_n r)}{\left[(\beta_n b)^2 - m^2\right]J_m^2(\beta_n b)\,\delta} \times\cos\left[m\theta - \psi - z\rho\sin(\varphi)\right]\exp\left[-z\rho\cos(\varphi)\right] \tag{18}$$

where

$$\begin{cases} \delta = \left[\lambda_d^2\rho^2 + h_d^2 + 2\lambda_d\rho h_d\cos(\varphi)\right]^{1/2} \\ \psi = \tan^{-1}\left[\dfrac{\lambda_d\rho\sin(\varphi)}{\lambda_d\rho\cos(\varphi) + h_d}\right] \end{cases} \tag{19}$$

This expression is valid whatever are the geometrical or physical parameters of the problem.

The bulk temperature, $T_{bulk} = q_d\, a^2/h_d\, b^2$, which is the first term of Eq. (18)], corresponds to the thermal equilibrium between the heat flux entering the disk $q_d\,\pi a^2$ and the convective flux exchanged with the ambient, $h_d\,\pi b^2 T_d$.

RESULTS AND INTERPRETATION

We discuss here the results obtained for the particular geometrical and physical data given in Table 1.

The analytical solution was used to calculate the dimensionless mean temperature of contact as a function of the Peclet number. The results are presented in Table 2. where the results obtained in [3] are also given for the sake of comparison.

Variable	Numerical value	Units
b	0.1	[m]
a/b	0.1	[-]
e/b	0.7	[-]
Pe	100	[-]
q_d	10^5	[W/m^2]
α_d	10^{-5}	[m^2/s]
λ_d	20	[W/m K]
Bi	0.05	[-]

Table 1 Physical and geometrical data used

	$\sqrt{\pi}\bar{T}_c/(q_d a/\lambda_d)$	
Pe	Reference [3]	Present study ($a/b = 0.05$, $e/b = 0.78$)
0	0.4789	0.4769
1	0.3610	0.3613
2	0.3017	0.3037
4	0.2383	0.2388
8	0.1800	0.1791
16	0.1320	0.1313
32	0.0952	0.0959
64	0.0680	0.0704

Table 2 Comparison of the dimensionless mean temperatures of the present study with those in [3]

The comparison points up a good concordance between the values of the dimensionless mean temperature obtained in both works. Deviations are less than 4% and can be partially attributed to the effect of curvature

Fig 2. shows the thermal map of the disk surface obtained from (18). It is clearly visible the thermal trace starting at the contact point and extending in the disk rotation sense.

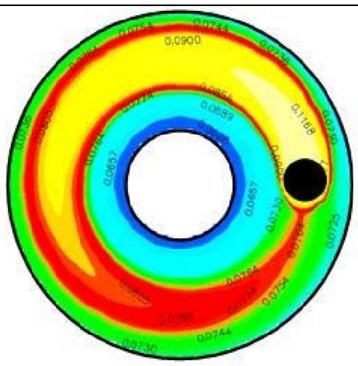

Figure 2 Isothermal lines for $Pe = 20$; $Bi = 1$; $a/b = 0.1$; $e/b = 0.7$

The angular and radial variations of the surface dimensionless temperature $T^*(e,\theta,0)$ and $T^*(r,0,0)$ are represented in Fig.3 and Fig. 4 respectively for different values of the Peclet number and the same constant heat flux q_d in all cases.

The set of curves show the increase of temperature with the angular velocity of the disk and the appearance of the flash temperature described in [1-2]. The rise in temperature when passing the contact zone is weakened for higher speeds. In dynamic mode, the maximum temperatures are always located at the exit of the contact. This zone is where the angular temperature gradient turns over, having important consequences in terms of mechanical performance (strong thermal stresses, cracking). The zone near to the entrance of the contact is equally affected.

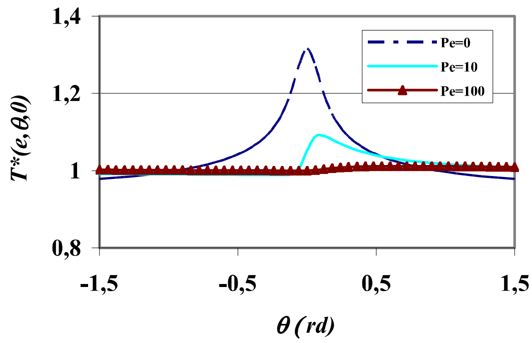

Figure 3 Influence of the rotating speed on the dimensionless disk temperature. Angular variation

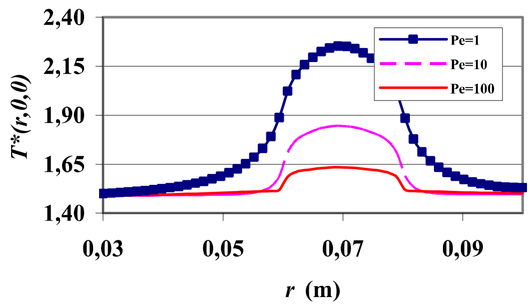

Figure 4 Influence of the rotating speed on the dimensionless disk temperature. Radial variation

Fig. 5 shows that, from a certain depth, the temperature becomes uniform and is not influenced anymore by the cyclic appearance of the heat source, which is in agreement with the values of the skin thickness.

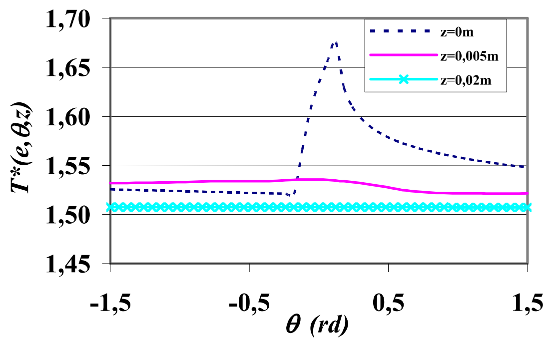

Figure 5 Angular variation of the dimensionless disk temperature at different depths z

The influence of the thermal surface conductance on the dimensionless temperature of the rubbing face of the disk is given in Fig. 6. It diminishes with increasing values of the Biot number, but the differences between the maximum and minimum values of temperature are independent of *Bi*, showing the same angular profile.

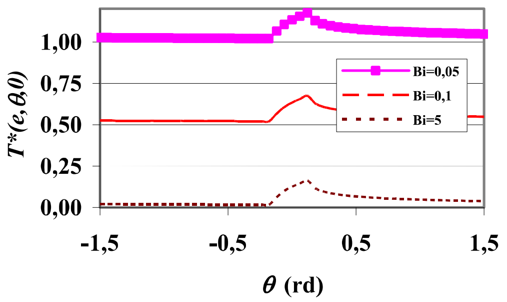

Figure 6 Influence of the heat convection coefficient on the dimensionless disk temperature

Finally the influence of the thermal conductivity of the disk on the temperature $T^*(e,\theta,0)$ is illustrated in Fig. 7 and 8. The profiles shown there are physically coherent and concordant with other published results.

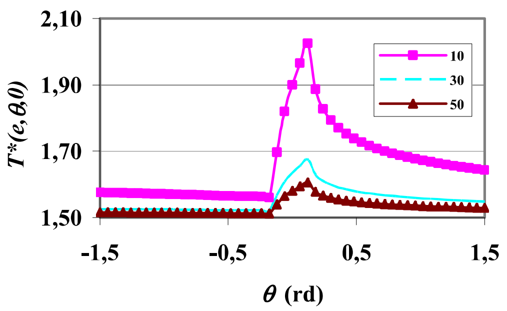

Figure 7 Influence of the thermal conductivity on the dimensionless disk temperature

CONCLUSIONS

An explicit analytical 3D model has been implemented to deal with the thermal problem of a rotating disk receiving heat on one of its faces from an off-centred heat source, to calculate the temperature distribution and to study the influence of the intervenient physical and geometrical parameters. This model is easy to implement and is solved by using the integral transforms of Fourier and Hankel. High thermal gradients in the proximity of the contact with the pin are clearly present on different thermal cartographies. These cartographies make easy to identify the zones with strong temperature variations that often cause fissures and cracks which are observable in brake systems. This model allows, in very short computing times, to analyse the influence of different physical parameters such as the convection coefficient, the conductivity of the material as well as the rotational speed on the temperature of the disk.

REFERENCES

[1] Block H., Flash temperature concept, *Wear*, 6, 1963, pp. 483-493.

[2] Desruisseaux N.R., Zerkle R.D., Temperature in Semi-Infinite and Cylindrical Bodies Subjected to Moving Heat Sources and Surface Cooling, *ASME Journal of Heat Transfer*, 1970, pp. 456-464.

[3] Tian X., and Kennedy F. E., Maximum and Average Flash Temperatures in Sliding Contacts, *ASME Journal of Tribology*, 1994, vol. 116, pp. 167–174.

[4] Laraqi N., Bauzin J.G., Alilat N., *Congrès français de thermique SFT*, Nantes 29-31 mai 2001, proceedings pp. 701-706.

[5] Laraqi N., An Exact Explicit Analytical Solution for Steady-State Temperature of a Half Space Subjected to a Circular Moving Heat Source, *ASME Journal of Tribol*ogy, 2003 vol. 125, pp. 859–862.

[6] Bourouga B., Goizet V., Bardon J.P., Modèle prédictif de la résistance thermique de contact dynamique adaptée au cas de l'interface pièce-outil de forgeage, *Int. J. Heat Mass Transfer*, 2003, pp. 565-576.

[7] Amara M., Timchenko V., El Ganaoui M., Leonardi E., de Vahl Davis G., A 3D computational model of heat transfer coupled to phase change in multilayer materials with random thermal contact resistance, *International. Journal of Thermal Sciences*, 2009, 48, pp. 421–427.

[8] Laraqi N., An exact explicit analytical solution for steady-state temperature of a half space subjected to a circularmoving heat source, *ASME Journal of Tribology*, 2003, vol. 125, pp 859–862.

[9] Laraqi N., Thermal impedance and transient temperature due to a spot of heat on a half-space, *International Journal of Thermal Sciences*, March 2010, vol. 49, Issue 3, pp 529-533.

[10] Baïri A., Laraqi N., Heat transfer across a solid–solid interface obtained by machining in a lathe, *Journal of Materials Processing Technology*, 2005, vol. 169, Issue 1, pp 89-93.

[11] Gecim B., Winer W.O., Steady Temperature in a Rotating Cylinder Subject to Surface Heating and Convective Cooling, *ASME Tribology Transactions*, 1984, 106, pp. 120-127.