

## THERMODYNAMIC APPROACH ON THE CONDENSATION RISK IN BUILT ENVIRONMENT

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### ABSTRACT

Worldwide energy consumption in residential and public buildings represents almost one third of primary energy. It is one of the larger contributors to fossil fuel use and the carbon dioxide production. Therefore, many countries are currently developing projects for the implementation of measures aimed at energy efficiency for built environment aiming targets of consumption reduction of 15% to 30%.

However, less careful implementation of some measures for energy conservation is creating problems of water condensation in both, the newly constructed and the old buildings. Those are problems that created a bad perception regarding energy conservation measures, and also resulted in serious quality problems in built ambient.

The main goal of this work is to evaluate the condensation risk in built environment and the influence on the energy consumption of the “inside” thermal insulation placement. A single-zone time-dependent mathematical model is developed based on specific parameters characterizing buildings’ use for commercial purposes.

Numerical solutions are determined for commercial buildings over the course of a year. Suitable low cost strategies for energy conservation are developed to avoid condensation and mould growth for a typical classroom at the Federal University of Paraná, Curitiba, PR, Brazil, and to prevent further degradation in problem buildings.

### INTRODUCTION

A large body of already published works answer “outside” the challenging question of optimizing buildings’ thermal retrofit in a temperate climate zone: on which side of the external wall to place the thermal insulation? Arguments supporting that it should be “outside” are that (1) the thermal gradient becomes less abrupt preventing the condensation of indoor water vapor within the wall, (2) the “outside” placement of thermal insulation prevents large amounts of solar energy to be stored within the walls during the summer and (3) it also

guarantees the thermal insulation is not discontinued when the building has inner partition walls ending in a façade wall [1].

We may also account for arguments for the thermal insulation placement on the inner walls’ surface such as that when renovating historical buildings this would avoid troubles with changing the facades and the “inside” thermal insulation placement makes unnecessary weather-resisting layers on the outer surfaces.

### NOMENCLATURE

$\dot{m}$	[kg/s]	Mass flow rate
$\dot{W}$	[W]	Work transfer interaction
$\dot{Q}$	[W]	Heat transfer interaction
H	[m]	Height
L	[m]	Length
M	[kg/kg]	Grains moisture content, dry basis
P	[N/m <sup>2</sup> ]	Pressure
q	[W/m <sup>2</sup> ]	Heat unitary flux
T	[°C]	Temperature
u	[J/kg]	Specific internal energy
V	[m <sup>3</sup> ]	Volume
W	[m]	Width
x	[m]	Cartesian axis direction
y	[m]	Cartesian axis direction
z	[m]	Cartesian axis direction

#### Special characters

$\alpha$	[-]	Absorptivity
$\delta$	[m]	Thickness
$\epsilon$	[-]	Emissivity
$\phi$	[-]	Relative humidity
$\lambda$	[W/(mK)]	Thermal conductivity
$\omega$	[kg/kg]	Specific humidity

#### Subscripts & Superscripts

0	Ambient or reference
a	Air
c	convection
gen	Internally generated
liq	Liquid
sat	Saturation
vap	Vapor
w	Water

The main advantage of the “outside” placement of thermal insulation in residential buildings which are not permanently air conditioned is that heating up the inside air when the occupants arrive in the afternoon is less energy consuming than continuously maintaining the whole building mass at high temperature in the winter. Favorable effects also appear in commercial buildings during the summer time. Occupation occurs during the day and mild temperatures during the night could help cooling down the building. Therefore, it sounds more opportune to place thermal insulation “outside”, provided a night-time ventilation of the interior space is possible. Economic benefits in connection with the thermal insulation placement “outside” walls were carefully examined by Dylewski and Adamczyk [2], and the optimization of thermal insulation thickness was presented by Kaynakli [3].

The main goal of this work is to evaluate the influence on the energy consumption of the “inside” thermal insulation placement and to optimize the thermal insulation placement for avoiding condensation and mould growth for a typical classroom at the Federal University of Paraná, Curitiba, PR, Brazil, and to prevent further degradation in problem buildings.

A single-zone time-dependent mathematical model based on specific parameters characterizing buildings’ use for commercial purposes is presented. Numerical solutions are determined for commercial buildings when the control variables considered are of the  $f$ 's (“mass furniture factor”) numerical values, the ventilated atmospheric air mass-flow and the Curitiba’s local climatic characteristics.

For a better approach of some important influences such as the seasonal variation in the condensation risk and formation of mould, and also to capture effects of the atmospheric air temperature’s continuous variation, the numerical computation

was performed based on the average hourly statistics for the dry bulb temperature and the relative humidity according to the Climate Design Data 2009 ASHRAE Handbook.

## BUILDINGS’ THERMAL BEHAVIOR

Rather than mild conditions, temperate climate might represent a double problem since there is excessive heat in some summer days and occasionally extreme cold during the winter time. Accordingly, various strategies are required to optimize the energy consumption in passively conditioned buildings.

### Physical modeling

This study considers two physical solutions for retrofitting in buildings with envelopes of high thermal transmittance. Firstly, we consider for commercial buildings the “inside” placement of thermal insulation. When considering the vertical walls isolated “inside”, the floor also receives wooden boards and the ceiling receives an insulating layer.

In the commercial building unit, the following assumptions were made: occupation from 8 AM to 18 PM, constant ventilation rate of 0.03 m<sup>3</sup>/s during occupation, constant ventilation rate of 0.03 m<sup>3</sup>/s after occupation in the winter time, intense ventilation rate of 2.00 m<sup>3</sup>/s after occupation in the summer time and temperature set points of minimum 18°C (winter) and maximum 25°C (summer).

Geometry represented in Figure 1 considered a single-zone built environment within a commercial building at latitude -25° (south) with a 30% glazed northern façade and the main external dimensions  $L \times H \times W$ .

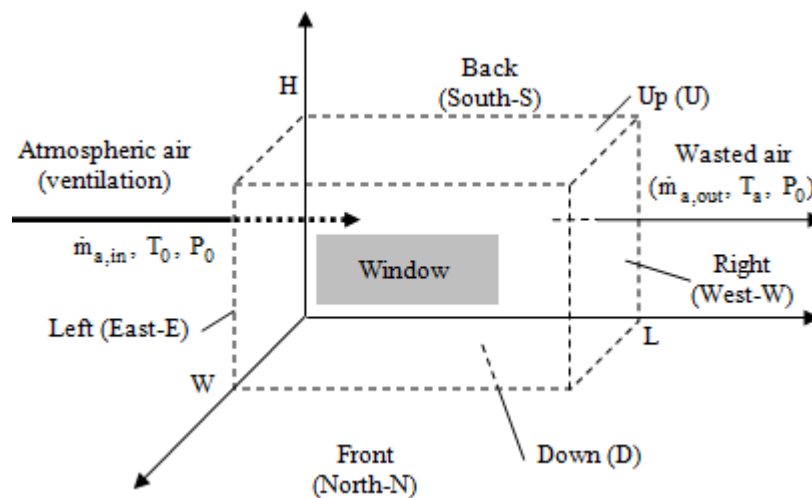


Figure 1 Schematic view of the commercial building’s geometry

### Mathematical modeling

The mathematical model was derived from the dry air and water vapor mass conservation and from the energy

conservation applied to the control volume in Figure 1 when assuming one inlet port and one outlet port.

The mathematical problem consisted of computing the

temperature and specific and relative humidity variations of the moist air inside the control volume during some specific summer (excessive hot) and winter (excessive cold) periods of time.

Next, the well stirred tank model is employed in connection to the following additional hypothesis:

- instantaneous thermodynamic equilibrium between the dry air and the vaporized moisture;
- all flows are assumed to have no pressure losses;
- no mass exchange through the walls, floor and ceiling;
- heat transfer occurs only through the walls along perpendicular direction;
- no kinetic or potential energy variations;
- dry air and water vapor are assumed perfect gases;
- Dalton model is assumed for calculating thermodynamic properties of moist air's components.

The mathematical model was employed in this study focusing on determining the best physical thermal insulation placement for retrofitting commercial buildings in temperate climate and guarantees the maximum comfort.

## NUMERICAL PROCEDURE

Based on the previously formulated hypothesis the mathematical model consists of the dry air and water mass conservation Eqs. (1) and (2), and by Eq. (3) representing the energy conservation within the control volume in Figure 1.

$$\frac{dm_a}{dt} = \sum_{\text{inlet ports}} \dot{m}_{a,in} - \sum_{\text{outlet ports}} \dot{m}_{a,out} = \dot{m}_{a,in} - \dot{m}_{a,out} \quad (1)$$

$$\begin{aligned} \frac{dm_w}{dt} &= \sum_{\text{inlet ports}} \dot{m}_{w,in} + \dot{m}_{w,gen} - \sum_{\text{outlet ports}} \dot{m}_{w,out} = \\ &= (\dot{m}_{w,in} + \dot{m}_{w,gen}) - (\dot{m}_{w,out,vap} + \dot{m}_{w,out,liq}) \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{dU}{dt} &= \sum_j \dot{Q}_j - \sum_k \dot{W}_k + \sum_{\text{inlet ports}} \dot{m}_{in} h_{in} + \\ &+ \dot{m}_{w,gen} h_{w,gen} - \sum_{\text{outlet ports}} \dot{m}_{out} h_{out} \end{aligned} \quad (3)$$

In Eqs. (1) and (2) the mass flow rates of dry air and water vapor,  $\dot{m}_{a,in} = P_{a,0} \dot{V}_{vent} / (R_a T_0)$  and  $\dot{m}_{w,in} = P_{w,0} \dot{V}_{vent} / (R_w T_0)$ , are calculated based on the volumetric mass flow rate  $\dot{V}_{vent}$  of the atmospheric air provided by the ventilating system at temperature  $T_0$ , pressure  $P_0$ , relative humidity  $\phi_0$  and specific humidity  $\omega_0$ . Dalton model for mixtures of perfect gases is employed to determine the partial pressures of water vapor and dry air into the atmospheric air,  $P_{w,0} = \phi_0 P_{sat}(T_0)$  and

$P_{a,0} = P_0 - P_{w,0}$  with  $P_0$  representing the constant atmospheric pressure.

Terms  $\dot{m}_{w,gen}$  and  $\dot{m}_{w,out,liq}$  in Eq. (2) account respectively for the water vapor internal generation by human respiration/transpiration or any other process that may occur inside the control volume and respectively for the water liquefaction processes that may happen when the air temperature reaches values below the Dew point temperature.

Based on Dalton's model for perfect gases mixtures the mass derivatives in Eqs. (1) and (2) might be evaluated based on the perfect gas state equation

$$\frac{dm_a}{dt} = \frac{d}{dt} \left( \frac{P_a LHW}{R_a T_a} \right) = \frac{LHW}{R_a T_a} \left( \frac{dP_a}{dt} - \frac{P_a}{T_a} \frac{dT_a}{dt} \right) \quad (4)$$

and

$$\frac{dm_w}{dt} = \frac{d}{dt} \left( \frac{P_w LHW}{R_w T_w} \right) = \frac{LHW}{R_w T_w} \left( \frac{dP_w}{dt} - \frac{P_w}{T_w} \frac{dT_w}{dt} \right) \quad (5)$$

where  $L$ ,  $H$  and  $W$  are respectively the length, the height and the width of the control volume (Fig. 1),  $T_a = T_w$  is the instantaneous bulk temperature of the moist air, and  $R_a$  and  $R_w$  are the constants of dry air and water vapor.

Now, the mass flow rates of dry air and water vapor leaving the control volume are given by

$$\dot{m}_{a,out} = P_{a,0} \dot{V}_v / (R_a T_0) - (dm_a / dt) \quad (6)$$

with  $\dot{m}_{w,out,vap} = \omega \dot{m}_{a,out}$  and

$$\begin{aligned} \dot{m}_{w,out,liq} &= P_{w,0} \dot{V}_{vent} / (R_w T_0) + \\ &+ \dot{m}_{w,gen} - \omega \dot{m}_{a,out} - (dm_w / dt) \end{aligned} \quad (7)$$

Variation of the control volume internal energy,  $U = U_a + U_w$ , in Eq. (3) is written down by considering separately the dry air's internal energy,  $U_a = m_a u_a$ , and the water vapor internal energy,  $U_w = m_w u_w$

$$\begin{aligned} \frac{d(m_a u_a + m_w u_w)}{dt} &= m_a \frac{du_a}{dT_a} \frac{dT_a}{dt} + \\ &+ u_a \frac{dm_a}{dt} + m_w \frac{du_w}{dT_a} \frac{dT_a}{dt} + u_w \frac{dm_w}{dt} \end{aligned} \quad (8)$$

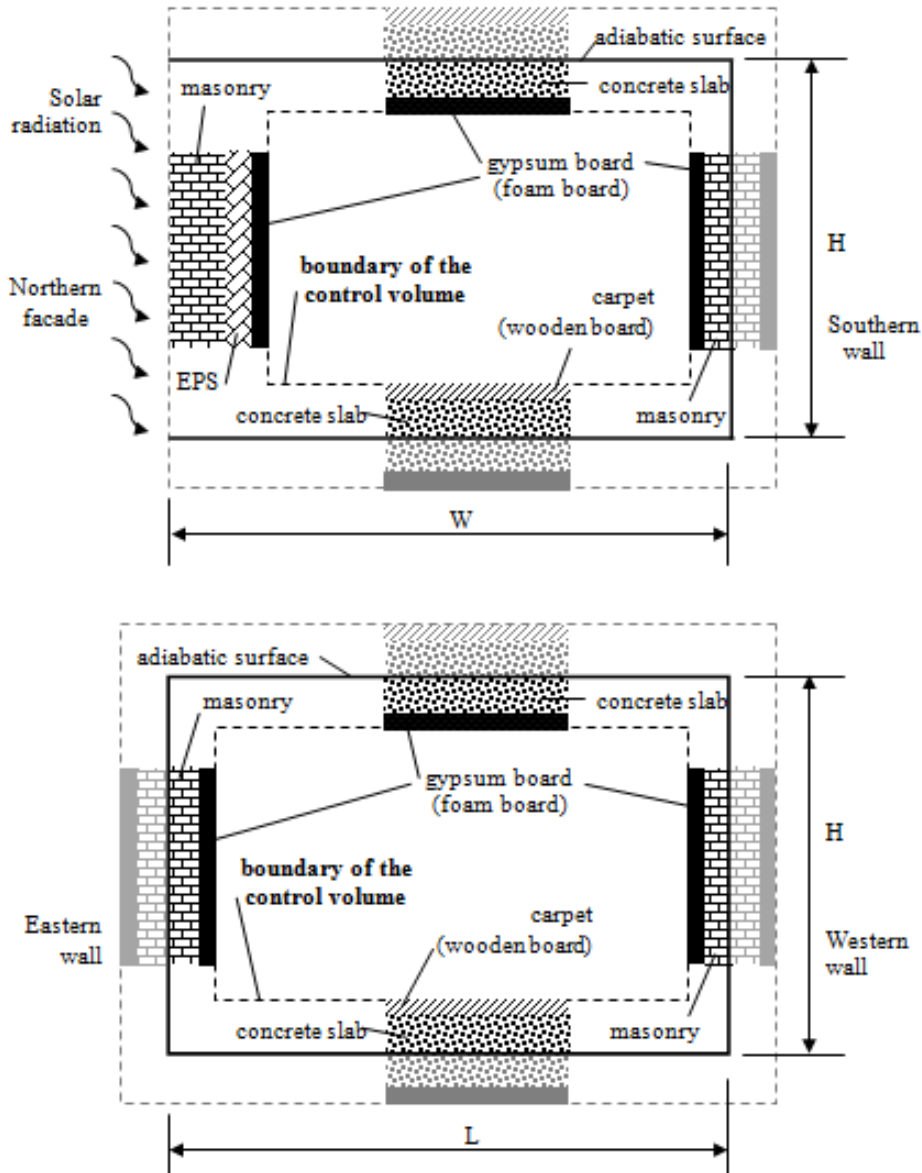
where  $m_a$ ,  $m_w$  represent the dry air and water vapor instantaneous mass inside the control volume, while  $u_a$ ,  $u_w$  are the specific internal energies. Temperature derivative  $dT_a / dt$  is then calculated by substituting Eq. (8) into Eq. (3) while assuming no mechanical interaction between the control

volume and its surroundings

$$\frac{dT_a}{dt} = \frac{-u_a \frac{dm_a}{dt} - u_w \frac{dm_w}{dt} + \sum_j \dot{Q}_j}{m_a (du_a/dT_a) + m_w (du_w/dT_a)} + \frac{\sum_{\substack{\text{inlet} \\ \text{ports}}} \dot{m}_{in} h_{in} + \dot{m}_{w,gen} h_{w,gen} - \sum_{\substack{\text{outlet} \\ \text{ports}}} \dot{m}_{out} h_{out}}{m_a (du_a/dT_a) + m_w (du_w/dT_a)} \quad (9)$$

Terms  $\dot{Q}_j$  in Eq. (9) account for the solar radiation  $\dot{Q}_{sun}$  and the internally generated heat flux  $\dot{Q}_{in}$ , and also for the heat transfer interactions between the internal air and the windows ( $\dot{Q}_{win}^{c,r}$ ), the walls ( $\dot{Q}_N^{c,r}$ ,  $\dot{Q}_S^{c,r}$ ,  $\dot{Q}_E^{c,r}$ ,  $\dot{Q}_W^{c,r}$ ), the ceiling ( $\dot{Q}_C^{c,r}$ ) and the floor surface,  $\dot{Q}_F^{c,r}$ .

Superscripts “c” and “r” indicate the heat convection or the heat radiation transfer mechanism.



**Figure 2** The physical structure of the control volume's surroundings when the expanded polystyrene (EPS) thermal insulation is placed on the “inner” surface of the northern facade

To approach terms describing the heat transfer interactions between the control volume and the windows, the walls, the ceiling and the floor surfaces we employed the well known formulae  $\dot{Q}_j^{c,r} = q_{j,i}^{c,r} A_j$  where  $q_{j,i}^{c,r}$  represent the unitary heat fluxes crossing the middle line of the inner wall's layer

$$q_{j,i}^c \cong \frac{T_{j,i} - T_a}{1/h_{j,i} + \delta_{j,i}/(2\lambda_{j,i})} \quad (10a)$$

and the unitary fluxes transferred by thermal radiation are evaluated based on

$$q_{j,i}^r \cong C_{\text{rad}}^{\text{rad}} \sigma (\alpha_a T_{j,i}^4 - \varepsilon_a T_a^4) \quad (10b)$$

with  $j \in \{\text{win}, N, S, E, W, C, F\}$  and  $A_j \in \{A_{\text{win}}, HL-A_{\text{win}}, HL, HW, HW, WL, WL\}$  respectively. In Eqs. (10a) and (10b)  $h_i$ ,  $\delta_i$ ,  $\lambda_i$ ,  $\alpha_a$  and  $\varepsilon_a$  represent respectively the inner surface film coefficient, layer thickness and thermal conductivity, and the water vapor absorptivity and emissivity. The Stefan-Boltzmann Constant and temperature at the middle line of the inner layer are given by  $\sigma = 5.669 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$  and  $T_i$  respectively.

Solution of the ODE Eqs. (4), (5) and (9) is performed based on the following additional boundary conditions:

- adiabatic surfaces in the middle line of the S, E, W, C and F walls (Fig. 2) yielding

$$q_{j,i} + q_{j,m} + \delta_{j,i} \rho_{j,i} c_{j,i} \frac{dT_{j,i}}{dt} = 0 \quad (11)$$

which represents the energy conservation law applied to the wall's inner layer (Fig. 3) and

$$q_{j,m} - \frac{1}{2} \delta_{j,m} \rho_{j,m} c_{j,m} \frac{dT_{j,m}}{dt} = 0 \quad (12)$$

for the energy conservation law applied for the inner half of the wall's middle layer (Fig. 3).

The unitary heat flux  $q_{j,m}$  calculates with

$$q_{j,m} = \frac{T_{j,i} - T_{j,m}}{\delta_{j,i}/(2\lambda_{j,i}) + \delta_{j,m}/(2\lambda_{j,m})} \quad (13)$$

where  $\delta_m$  and  $\lambda_m$  represent respectively the thickness and thermal conductivity of the wall's middle layer. Temperature in the middle line of the middle layer is given by  $T_m$ .

- solar radiation gains on the northern façade N and also possible losses or gains due to the temperature differences between the external wall surface and the surrounding atmospheric air at  $T_0$  are accounted for applying the energy conservation for each one of the three layers, namely the inner layer

$$q_{N,i} + q_{N,m} + \delta_{N,i} \rho_{N,i} c_{N,i} \frac{dT_{N,i}}{dt} = 0 \quad (14)$$

the middle layer

$$-q_{N,m} + q_{N,e} + \delta_{N,m} \rho_{N,m} c_{N,m} \frac{dT_{N,m}}{dt} = 0 \quad (15)$$

and the external layer where some solar radiation gain may occur

$$q_{\text{sun}} - q_{N,e} + \frac{T_{N,e} - T_0}{1/h_{N,e} + \delta_{N,e}/(2\lambda_{N,e})} + \delta_{N,e} \rho_{N,e} c_{N,e} \frac{dT_{N,e}}{dt} = 0 \quad (16)$$

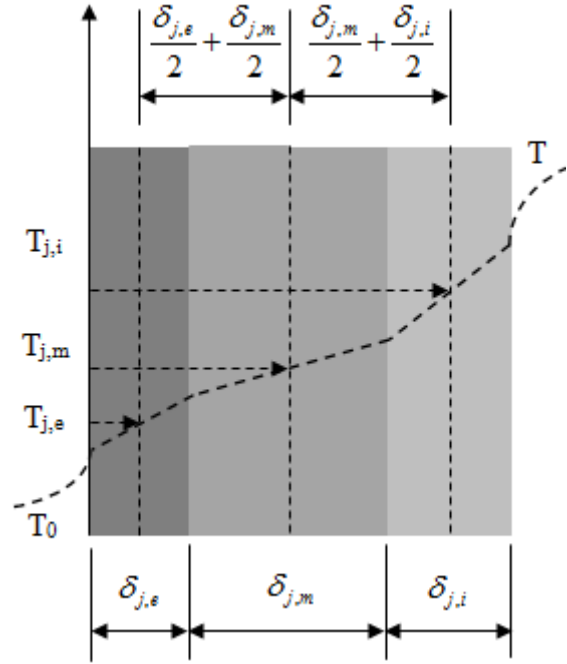


Figure 3 Middle lines of the northern wall layers

Unitary fluxes  $q_{N,m}$  and  $q_{N,e}$  are written down as

$$q_{N,m} = \frac{T_{N,i} - T_{N,m}}{\delta_{N,i}/(2\lambda_{N,i}) + \delta_{N,m}/(2\lambda_{N,m})} \quad (17)$$

$$q_{N,e} = \frac{T_{N,m} - T_{N,e}}{\delta_{N,m}/(2\lambda_{N,m}) + \delta_{N,e}/(2\lambda_{N,e})} \quad (18)$$

- to reduce the numerical effort to solve the mathematical model we assumed that there is no influence of solar radiation on the window temperature, thus energy conservation is written

$$q_{\text{win},i} + q_{\text{win},e} + \delta_{\text{win}} \rho_{\text{win}} c_{\text{win}} \frac{dT_{\text{win},i}}{dt} = 0 \quad (19)$$

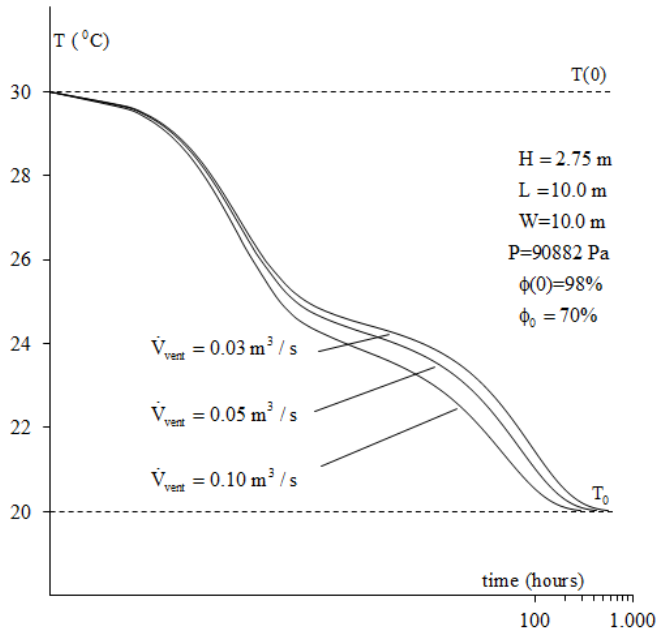
Based on the Dalton's Law for perfect gases mixtures we have  $dP_a = -dP_w$  and by differentiating the definition of specific humidity  $\omega = m_w/m_a = 0.622 P_w/P_a$  and rearranging the terms we calculate the two derivatives  $dP_w/dt = -dT_a/dt$

$$\frac{dP_w}{dt} = -\frac{dP_a}{dt} = \frac{P_a^2}{0.622 m_a P_0} \left( \frac{dm_w}{dt} - \omega \frac{dm_a}{dt} \right) \quad (20)$$

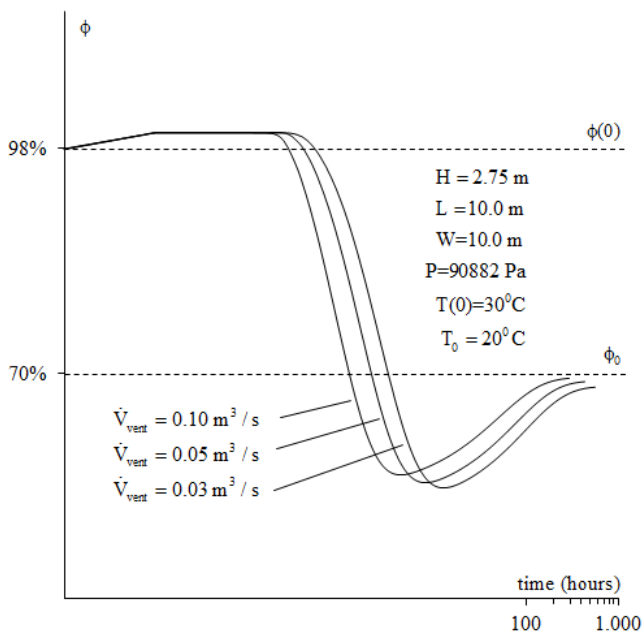
Equations (4), (5), (9), (11), (12), (14), (15), (16) (19) and (20) represent one ODE system that has been integrate based on the fourth order Runge-Kutta technique to determine variation of the dry air and water vapor mass, temperature and partial pressure into the control volume, and temperature variation of the walls, ceiling, floor and windows as previously defined.

## NUMERICAL RESULTS

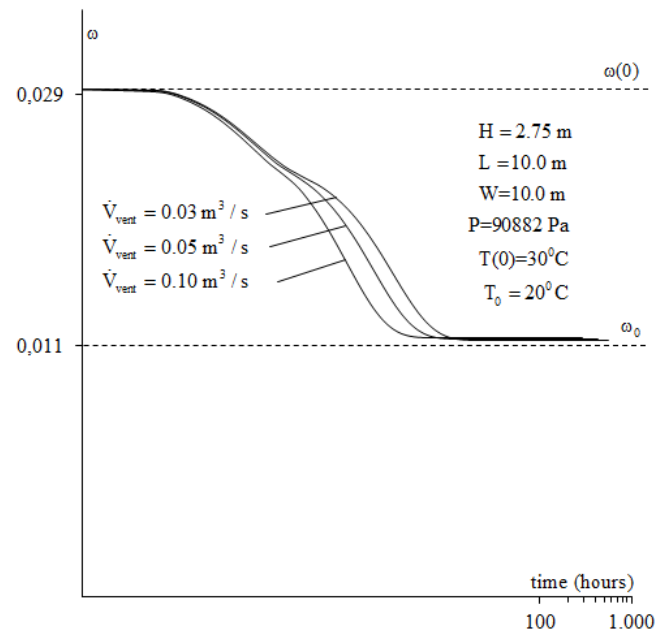
Since our first task was to calibrate the mathematical model, the first numerical integration was performed assuming: (1) constant ventilation rates, (2) no internal heat generation and (3) constant thermodynamic state of the external moist air ( $P_0$ ,  $T_0$ ,  $\phi_0$ ). In these conditions the volume  $H \times W \times L$  represents a reservoir constantly fed by the ventilation system with outside moist air, whose thermodynamic state does not change ( $P_0$ ,  $T_0$ ,  $\phi_0$ ). Since the internal air pressure is constant, occur simultaneously some air inlet and outlet. The thermodynamic state of the outlet air is the result of mixing between internal and the ventilated moist air.



**Figure 4** Time variation of the internal air temperature showing the convergence ( $T \rightarrow T_0$ ) for the well stirred tank model



**Figure 5** Time variation of the internal air relative humidity to show the convergence ( $\phi \rightarrow \phi_0$ ) for the well stirred tank model



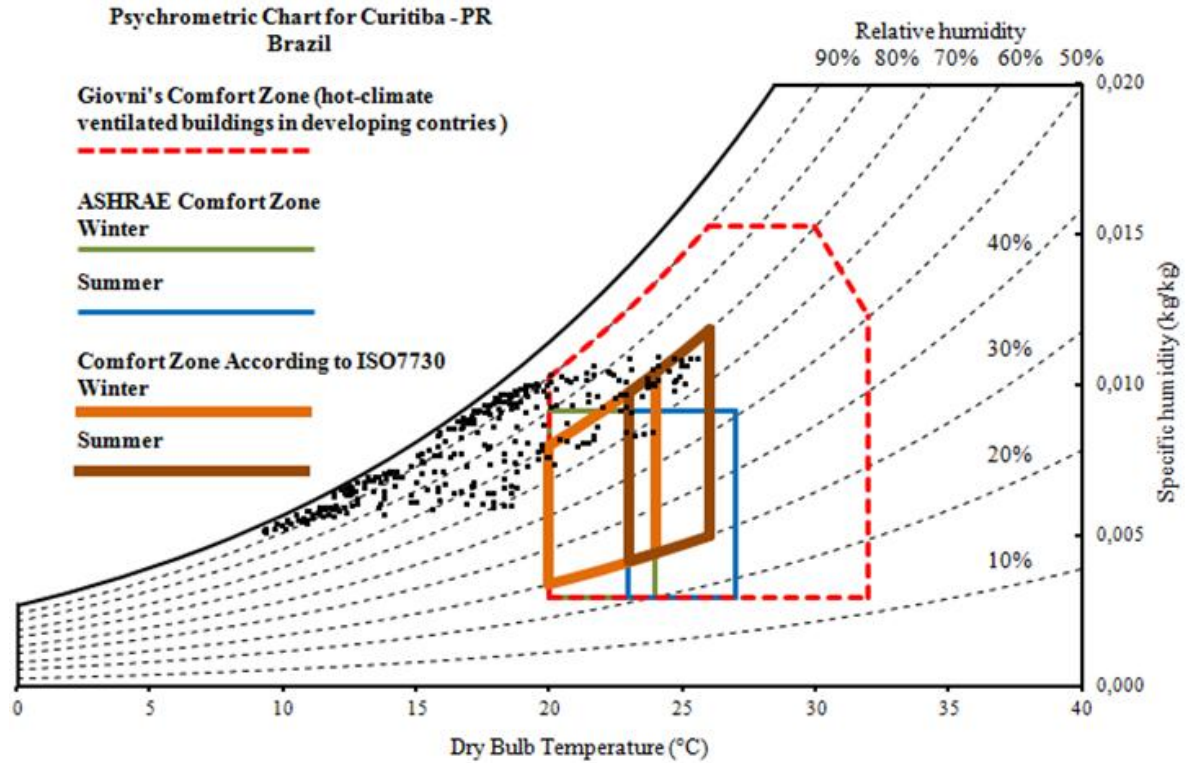
**Figure 6** Time variation of the internal air specific humidity to show the convergence ( $\omega \rightarrow \omega_0$ ) for the well stirred tank model

Numerical results shown in Figs (4) – (6) correctly predict that temperature, relative humidity and specific humidity of the internal air are given by  $T \rightarrow T_0$ ,  $\phi \rightarrow \phi_0$  and  $\omega \rightarrow \omega_0$  when the physical system reaches the permanent functioning regime and the air initially contained into the  $H \times W \times L$  volume is completely replaced by external moist air.

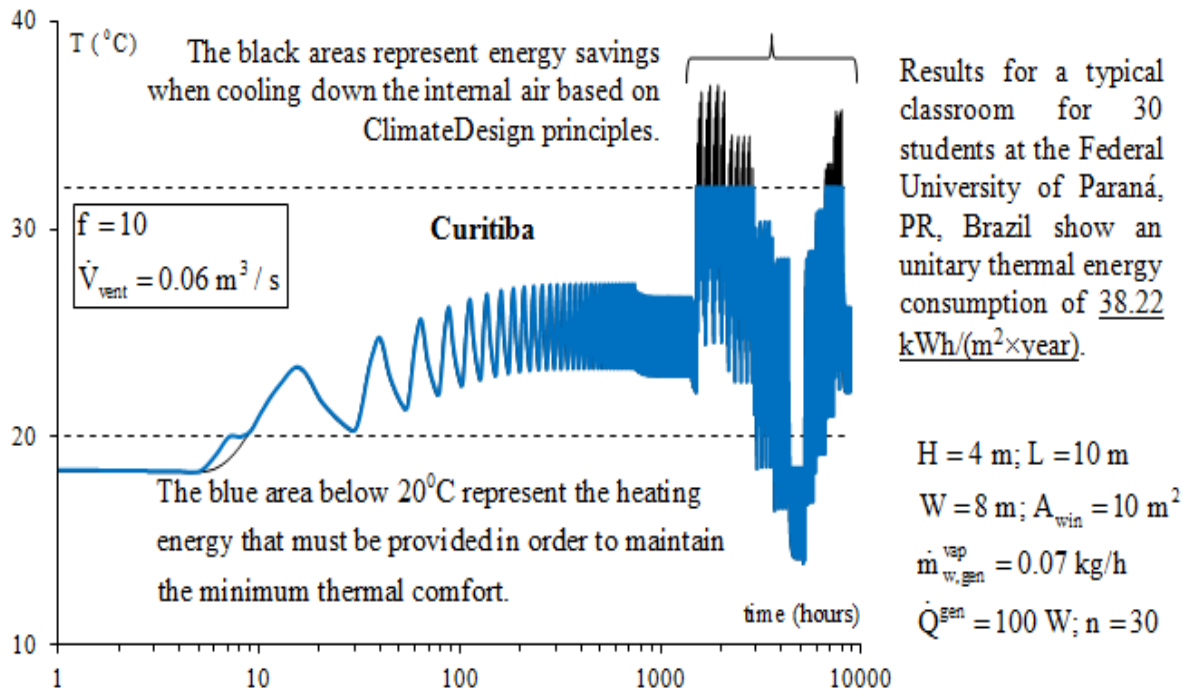
Aiming to qualitative assessment of the problems related to the condensation and mould growth risk in public buildings in Curitiba, Paraná, Brazil, in Figure 7 are represented into the Psychrometric Chart the hourly averaged values of the dry bulb temperatures and relative humidity according to the Climate Design Data 2009 ASHRAE Handbook. It can be seen that the large majority of this data belongs to the region where heating is necessary for maintaining the thermal comfort. This suggests that in fact the main focus of air conditioning in Curitiba should be the air heating in order to maintain comfort in built spaces such as a typical classroom at the Federal University of Paraná's Polytechnic Center.

Curitiba City, the capital of the Paraná state in southern Brazil, is located by the following geographical coordinates: latitude  $25^\circ 31' S$ , longitude  $49^\circ 10' W$ , and elevation 908 m above the sea level. Climate of Curitiba is characterized as maritime temperate climate or subtropical highland climate (*Cfb*) according to the Köppen classification.

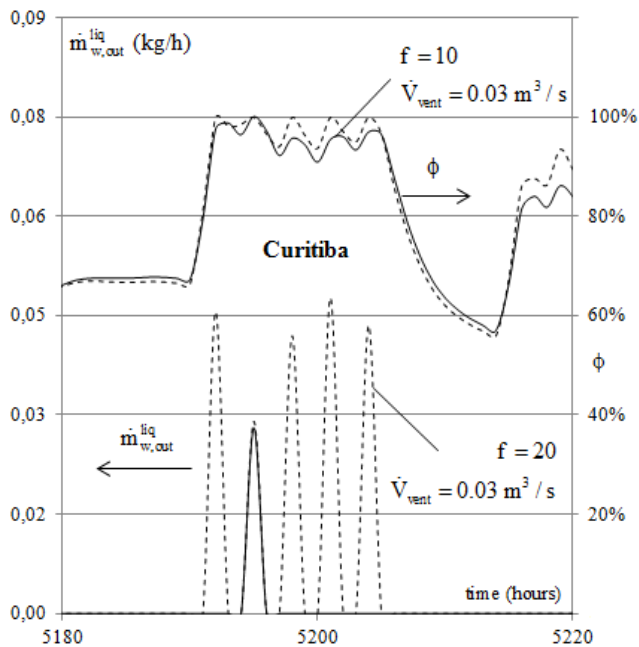
Sited into a temperate zone, the Curitiba City is located on a plateau where the flat terrain with flooded areas contribute to its mild and damp winter with an average minimum temperature of  $7^\circ C$  in the coldest month that sometimes falls below  $0^\circ C$  on the coldest nights. Snowfall very happily enjoyed and well remembered by the Curitiba's population occurred in 1889, 1892, 1912, 1928, 1942, 1955, 1957, and 1962 and for the last time in 1975. Average temperature is around  $18^\circ C$ , but it can get above  $30^\circ C$  on hottest days during the summer time.



**Figure 7** To maintain thermal comfort into a typical classroom in Curitiba the air heating should be considered (ensemble of black dots indicates the variation of the atmospheric air condition along the year based on the average hourly statistics for the dry bulb temperature and relative humidity according to the Climate Design Data 2009 ASHRAE Handbook)



**Figure 8** Annual time variation of internal air temperature for a typical classroom at the Federal University of Paraná, Curitiba, Paraná, Brazil



**Figure 9** Liquid water formation into a typical classroom at the Federal University of Paraná, Curitiba, PR, Brazil

During fall and winter in Curitiba often occur very cold days. Especially in the morning, attending early classes at 07:30 AM in a classroom at 15°C is associated with a very nasty thermal sensation. There are situations that it is necessary the use of heating systems whose operation should alleviate the situation very quickly.

The blue area well below 20°C in Fig. 8 represents the condensation and mould growth risk area. It is then recommended the use of some electrical heaters continuously controlled by thermostats for maintaining optimum temperature without excessive consumption of electricity.

The presented mathematical model calculates that, when the energy savings for the both, heating and cooling of the internal air are taken into account, 38.22 kWh/ (m<sup>2</sup>×year) is the minimum necessary consumption to maintain the thermal comfort into the typical classroom considered for Curitiba, PR-Brazil.

Figure 9 shows the numerical results calculated based on the mathematical model presented in this paper when quantitatively evaluated the potential for liquid water production by condensation of atmospheric air moisture. It is worth noting that in function of the f's ("mass furniture factor") numerical values, and depending on the ventilated atmospheric air mass-flow, significant amounts of liquid water separates from the air.

## CONCLUSION

A mathematical model to study time variation of temperature, relative humidity and specific humidity of internal air into a building has been developed and validated qualitatively when considering the well stirred tank model.

Based on this mathematical model studies were developed to evaluate the condensation and mould growth risk in new buildings for preventing further degradation in problem buildings.

The mathematical model presented captures well the physical behavior of the moist air, inclusively for extreme conditions when water condensation occurs.

Risk for liquid water formation into a typical classroom at the Federal University of Paraná, Curitiba, PR, Brazil has been identified when computation was performed based on ASHRAE Climate Data 2009.

When wisely employing the local climatic characteristics in Curitiba, there is a quite interesting potential for saving energy by cooling down the internal air by ventilation and evaporative cooling or by heating up the internal air by ventilation when hotter atmospheric air is available. When the energy savings for the both, heating and cooling of the internal air are taken into account, the presented mathematical model calculates that 38.22 kWh/ (m<sup>2</sup>×year) is the minimum necessary consumption to maintain the thermal comfort into the typical classroom considered for Curitiba, PR-Brazil.

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