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INVESTIGATION INTO CAVITY FLOW NATURAL CONVECTION FOR AL_2O_3-WATER NANOFLUIDS NUMERICALLY

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ABSTRACT

Numerical simulations have been carried out on natural convection heat transfer in a rectangular cavity. The effect of distilled water-Al_2O_3 nanofluid on heat transfer in a rectangular cavity heated vertically on sidewalls is analysed. The effective properties of distilled water-Al_2O_3 nanofluid were calculated from the correlations obtained from literature. The simulation was carried under different volume fraction concentration of nanoparticles as well as different correlations for effective properties of Al_2O_3-water nanofluid. The results indicate that in general adding Al_2O_3 nanoparticles into pure water improves its heat transfer performance; however, there is an optimum nanoparticle volume fraction which maximises the heat transfer rate for each condition. It is investigated and discussed the influence of uncertainty of available correlations for effective properties of distilled water-Al_2O_3 nanofluid. The effects of aspect ratios of the cavity on heat transfer were also analysed. The influence of pertinent parameters such as Rayleigh number and Nusselt number on the heat transfer characteristics of natural convection is also investigated.

INTRODUCTION

The poor physical properties of the conventional heat transfer fluids such as water, ethylene glycol and mineral oils, are the major problems in improving the performance of engineering equipment. The conventional heat transfer fluids have a limitation to the effectiveness of heat removal from the systems whose temperature control relies on natural convection. The effective thermal conductivity of the conventional heat transfer fluids can be improved by suspending nano-sized solid particles into them which enhances the heat transfer characteristics of the base fluid. These particles are called nanoparticles and the resultant mixture is named nanofluid. Therefore, a nanofluid is a suspension of ultrafine particles in a conventional (base) fluid which enhances the heat transfer characteristics of the base fluid.

Free or natural convection is convection caused by temperature difference within the fluid. Natural convection heat transfer in enclosures is preferred in several situations by heat transfer designers to avoid the use of mechanical equipment for the coolant circulation, due to power consumption, excessive operating noise, or reliability. It can be used in engineering applications such as microelectronic cooling.

NOMENCLATURE

AR	[-]	Aspect ratio
c_p	[J/kgK]	Specific heat at constant pressure
d_p	[m]	Nanoparticle diameter
d_f	[m]	Equivalent diameter of the base fluid molecule
Gr	[-]	Grashoff number
H	[m]	Height of the enclosure
L	[m]	Width of the enclosure
M	[-]	Molecular weight of the base fluid
N	[1/mol]	Avogadro number = 6.022x1023
Nu	[-]	Nusselt number
k	[W/mK]	Thermal conductivity
k_B	[J/K]	Boltzmann's constant = 1.38068x10-23
Ra	[-]	Rayleigh number
Re	[-]	Nanoparticle Reynolds number
Pr	[m]	Prandtl number
T	[-]	Temperature
u_B	[m/s]	Nanoparticle mean Brownian velocity

Special characters

α	[m2/s]	Thermal diffusivity of nanofluid
β	[1/K]	Effective coefficient of thermal expansion
μ	[Pas]	Dynamic viscosity of the base fluid
ρ	[kg/m3]	Mass density of nanofluid
τ_D	[t]	Time required for distance d_p moving at velocity u_B
φ	[-]	Nanoparticle volume fraction

Subscripts

bf	Base fluid
c	Cooled sidewall of the enclosure
eff	Effective
f_r	Freezing point of the base fluid
h	Heated sidewall of the enclosure
nf	Nanofluid
opt	Optimum value
p	Nanoparticle
h	reference state for thermophysical properties

Natural convection in the rectangular cavity has been studied extensively in the past [1], and comprehensive reviews of both experimental and theoretical results have done by S. Ostrach [2] and I. Catton [3]. Oztop and Abu-Nada [4] studied numerically the effects of a partial heater on natural convection using different types and concentrations of nanoparticles. They found that the heat transfer is dependent on the types and on the volume fractions of nanoparticles in suspension. Studies on natural convection using nanofluids were also done by Putra et al. [5], Wang and Mujumdar [6], Abu-Nada [7], and Abu-Nada and Oztop [8]. The results have indicated that the heat transfer rate depend on the nanoparticles volume fraction in the suspension, the shape of particles, the dimensions of particles and the thermal properties of particle materials.

Eastman et al. [9] observed that water-Al_2O_3 nanofluid and water-CuO nanofluid with 5% nanoparticle volume fractions, increased the thermal conductivity by 29% and 60%, respectively.

Jung et al. [10] found that the heat transfer coefficient increased 32% by dispersing 1.8% nanoparticles in a micro-rectangular channel with water-Al_2O_3 nanofluid. A theoretical study on a heated cavity also reported by Hwang et al. [11] which showed that the heat transfer coefficient of water-Al_2O_3 nanofluids reduced when there was an increase in size of nanoparticles and a decrease in average temperature. Particle concentration and tube size dependence of viscosities of water-Al_2O_3 nanofluid flowing through micro and mini-tubes was conducted by Jang et al. [12].

Lin and Violi [13] studied thermal conductivity variation on natural convection heat transfer of nanofluids in a rectangular cavity. Their results showed the effect of non-uniform particle diameter and temperature on thermal conductivity, where decreasing the Prandtl number resulted in amplifying the effects of nanoparticles due to increased effective thermal diffusivity.

Abu-Nada et al. [14] investigated numerically on side-heated cavities filled with water-Al_2O_3 nanofluid (dp=47 nm) as well as water-CuO nanofluid (dp=29 nm). The effective thermal conductivity was evaluated through the empirical correlation proposed by Chon et al. [15] whereas the effective dynamic viscosity was calculated by a pair of correlations based on the experimental data of Nguyen et al. [16]. They found that for the convectional dominated regime, the average Nusselt number decreased with increasing the nanoparticle volume fraction. Nguyen et al. measured dynamic viscosities for water-Al_2O_3 nanofluid (dp=47 nm) and they find that the results are higher than those measured for water-Al_2O_3 nanofluid (dp=36 nm) which is in contrast with the other works. Moreover, as the data relative to dp=36 nm are in substantial good agreement with the results obtained by Chavalier et al. [17] for dp=35 nm, it follows that the data for water-Al_2O_3 nanofluid (dp=47 nm) tend to overestimate the actual viscosity values. The Nguyen et al. viscosity values for water-CuO nanofluid (dp=29 nm) are also larger than those available in the literature for nanofluids containing nanoparticles of similar size, which is the case of the data reported by Masuda et al. [18] for dp=27 nm, Pak and Cho [19] for dp=27 nm, and Wang et al. [20] for dp=28 nm.

The aim of this study is to investigate numerically, natural convection heat transfer in a rectangular cavity filled with water-Al_2O_3 nanofluids. The cavity is heated at constant temperature in one vertical side and the opposite vertical side is also cooled at constant temperature. All other sides including the top and bottom surfaces are kept adiabatic.

THE PROBLEM EQUATIONS

Fig. 1 shows the Schematic of enclosure for simulation. In order to simulate the cavity flow for nanofluids, it needs to use some correlations to define the effective properties of the nanofluid for the software. They consist of density, specific heat capacity, coefficient of thermal expansion, thermal conductivity and viscosity. The first three of the mentioned properties have limited correlations to apply which they do not be a matter for the simulation, but there are a lot of choices for effective thermal conductivity and viscosity of nanofluids. Following are the correlations which have been chosen for this work which they include different correlation for effective thermal conductivity and viscosity.

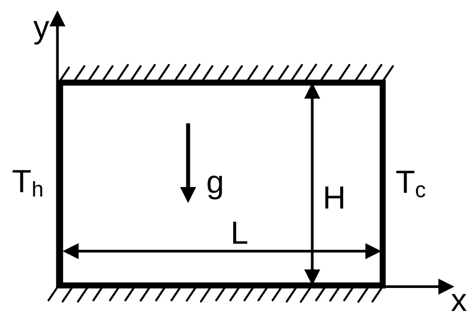

Figure 1 Schematic of enclosure

The effective density of nanofluid was determined analytically based on the physical principle of the mixture rule as recommended by Pak and Cho [19].

$$\rho_{eff} = (1-\varphi)\rho_{bf} + \varphi\rho_p \qquad (1)$$

The effective specific heat capacity at constant pressure for nanofluids can be calculated as suggested by Jang and Choi [21].

$$c_{p_{eff}} = (1-\varphi)c_{p_{bf}} + \varphi c_p \qquad (2)$$

The effective coefficient of thermal expansion of the nanofluid also can be determined as recommended by Hwang et al. [11]

$$\beta_{eff} = (1-\varphi)\beta_{bf} + \varphi\beta_p \qquad (3)$$

The selected effective thermal conductivities

• Chon et al. [15]

$$\frac{k_{eff}}{k_f}=1+64.7\phi^{0.7460}\left(\frac{d_f}{d_p}\right)^{0.3690}\left(\frac{k_p}{k_f}\right)^{0.7476}\mathrm{Pr}^{0.9955}\mathrm{Re}^{1.2321} \quad (4)$$

where

$$\mathrm{Re}_{dp}=\frac{\rho_{bf}k_BT}{3\pi\mu_{bf}^2 l_{bf}}, \qquad l_{bf}=0.738nm,$$

$$20°C\le T\le 50°C, \qquad 1\%\le\varphi\le 9\%,$$

$$13nm\le d_p\le 131nm$$

• Prasher et al. [22]

$$\frac{k_{eff}}{k_{bf}}=\left(1+A\mathrm{Re}^m\,\mathrm{Pr}^{0.333}\varphi\right)\left[\frac{k_p+2k_{bf}+2\varphi_p\left(k_p-k_{bf}\right)}{k_p+2k_{bf}-\varphi_p\left(k_p-k_{bf}\right)}\right] \quad (5)$$

where

$$A=4x10^4, \qquad m=2.5, \qquad \mathrm{Re}=\frac{1}{v}\sqrt{\frac{18k_BT}{\pi\rho_p d_p}}$$

$$20°C\le T\le 50°C, \qquad 1\%\le\varphi\le 9\%,$$

• Jang and Choi [23]

$$k_{eff}=k_{bf}\left(1-\varphi_p\right)+\beta k_p\phi+3C_1\frac{d_{bf}}{d_p}k_{bf}\,\mathrm{Re}_{d_p}^2\,\mathrm{Pr}_{bf}\,\varphi_p \quad (6)$$

where

$$\mathrm{Re}_{dp}=\frac{\rho_{bf}k_BT}{3\pi\mu_{bf}^2 l_{bf}}, \qquad C_1=18x10^6, \qquad \beta_1=0.01,$$

$$l_{bf}=0.738nm, \qquad d_{bf}=0.384nm$$

$$20°C\le T\le 50°C, \qquad 1\%\le\varphi\le 9\%,$$

The selected effective dynamic viscosities

• Maiga et al. [24]

$$\frac{\mu_{nf}}{\mu_{bf}}=1+7.3\phi_p+123\phi_p^2 \quad (7)$$

• Buongiorno [25]

$$\frac{\mu_{nf}}{\mu_{bf}}=1+39.11\phi_p+533.9\phi_p^2 \quad (8)$$

• Nguyen et al. [26]

$$\frac{\mu_{nf}}{\mu_{bf}}=1+0.025\phi+0.015\phi^2 \quad (9)$$

Table 1 Thermo-physical properties of water-Al_2O_3 nanofluids, [4, 27]

Physical properties	Water	Al_2O_3
ρ [kg m^{-3}]	998.377	3970
c_p [J kg^{-1} K^{-1}]	4182.11	765
k [W m^{-1} K^{-1}]	0.60304	40
μ x 10^{-3} [Pas]	1.004	-
β x 10^{-5} [K^{-1}]	21	0.85

COMPUTATIONAL PROCEDURES

ANSYS-FLUENT 14.5 is chosen for computational analysis. The effective parameters were determined by using the correlations presented above and the values introduced in the FLUENT. The effective thermal conductivity proposed by Chon et al. [15] is combined with effective dynamic viscosities proposed by Maiga et al. [24], Buongiorno [25] and Nguyen et al. [26]. Also the effective thermal conductivity proposed by Prasher et al. [22] and the effective thermal conductivity proposed by Jang and Choi [23] both are combined with effective dynamic viscosities mentioned above. The total number of combinations is 9.

A constant nanoparticles volume fraction of 1% in the cavity is examined and the results are presented in the Figs. 2, 4 and 6 while a constant nanoparticles volume fraction of 9% in the cavity is also analysed and the results displayed in the Figs. 3, 5 and 7. The size of the cavity is 10 mm x 10 mm for aspect ratio (AR=1) and 10 mm x 5 mm for aspect ratio (AR=2). The heated surface is maintained at temperature of 50 °C whereas the cooled surface is maintained at temperature of 20 °C.

RESULTS AND DISCUSSION

Fig. 2 shows the distribution of Static Temperature in the middle of the cavity along width of the cavity. The effective thermal conductivity proposed by Chon et al Eq. (4) and the effective dynamic viscosities proposed by Maiga et al. Eq. (7), Buongiorno Eq. (8) and Nguyen et al. Eq. (9) were used to produce the graphics in the Figs. 2 and 3. From Fig. 2, it can be seen that when the nanoparticles volume fraction in the cavity is low (about 1%), the correlations for effective properties of distilled water-Al_2O_3 nanofluid offer certainty in results whereas when the nanoparticles volume fraction in the cavity is high (as shown in the Fig. 3), the results produced by correlations for effective properties of distilled water-Al_2O_3 nanofluid are uncertainty.

From Eq. (4) for calculating the effective thermal conductivity, it can be concluded that the effective thermal conductivity depends on nanoparticle volume fraction in the cavity and on the film temperature and for the effective viscosities depend only on nanoparticle volume fraction in the cavity.

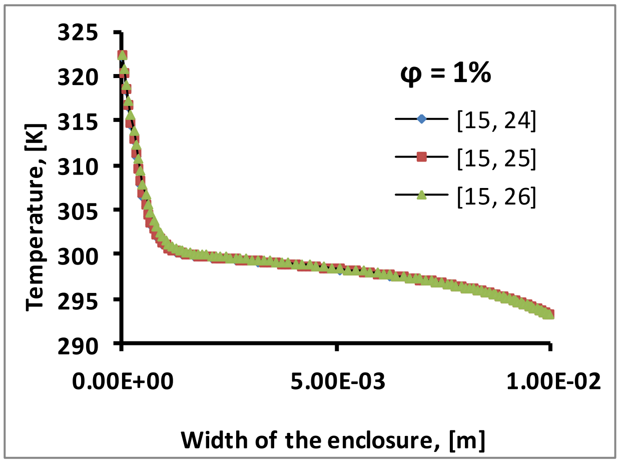

Figure 2 Distribution of Static Temperature at middle of the cavity with Chon et al. [15] thermal conductivity and three different viscosity [24-26] when φ=1%

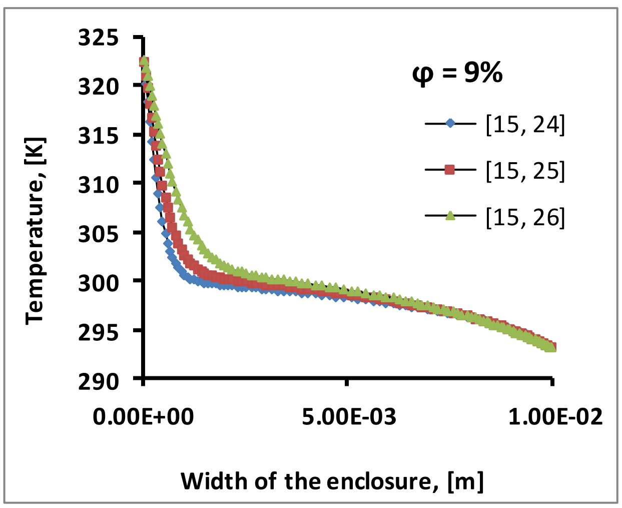

Figure 3 Distribution of Static Temperature at middle of the cavity with Chon et al. [15] thermal conductivity and three different viscosity [24-26] when φ=9%

Figs. 4 and 5 show the distribution of Static Temperature in the middle of the cavity along the width of the cavity for different nanoparticle volume fraction in the cavity. The effective thermal conductivity proposed by Prasher et al. Eq. (5) and the effective dynamic viscosities proposed by Maiga et al. Eq. (7), Buongiorno Eq. (8) and Nguyen et al. Eq. (9) are also used for plotting the graphs. Nevertheless the difference in equations for effective thermal conductivity, the behaviour is similar to the trend encountered in Figs. 2 and 3. The uncertainty occurs when nanoparticle volume fraction in the cavity increases as is shown in Fig. 5. Also the effective thermal conductivity depends on nanoparticle volume fraction in the cavity and on the film temperature whereas for the effective viscosities depend only on nanoparticle volume fraction in the cavity.

As in the previous cases, the Figs. 6 and 7 show the distribution of Static Temperature in the middle of the cavity along width of the cavity for different nanoparticle volume fraction in the cavity. The effective thermal conductivity proposed by Jang and Choi Eq. (6) is combined with effective dynamic viscosities mentioned above namely the effective dynamic viscosity proposed by Maiga et al. Eq. (7), Buongiorno Eq. (8) and Nguyen et al. Eq. (9). According Jang and Choi, the effective thermal conductivity depends on nanoparticle volume fraction in the cavity and on the film temperature whereas for the effective viscosities depend only on nanoparticle volume fraction in the cavity. Also, the uncertainty occurs when nanoparticle volume fraction in the cavity increases as is shown in Fig. 7. When nanoparticles volume fraction is nil, the effective thermal conductivity is equal to thermal conductivity of the base fluid and the effective viscosity is equal to viscosity of the base fluid.

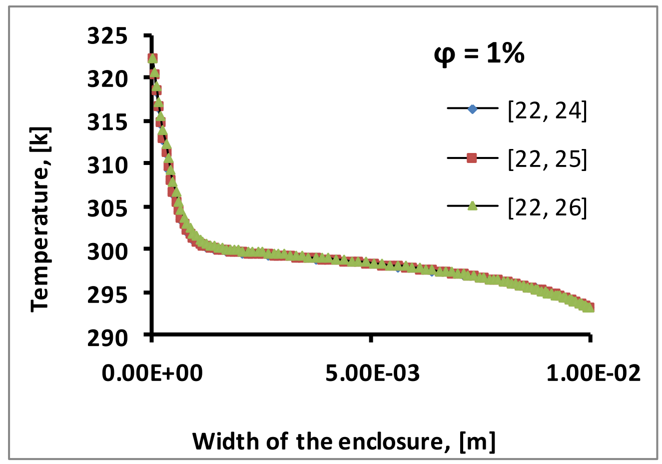

Figure 4 Distribution of Static Temperature at middle of the cavity with Prasher et al. [22] thermal conductivity and three different viscosity [24-26] when φ=1%

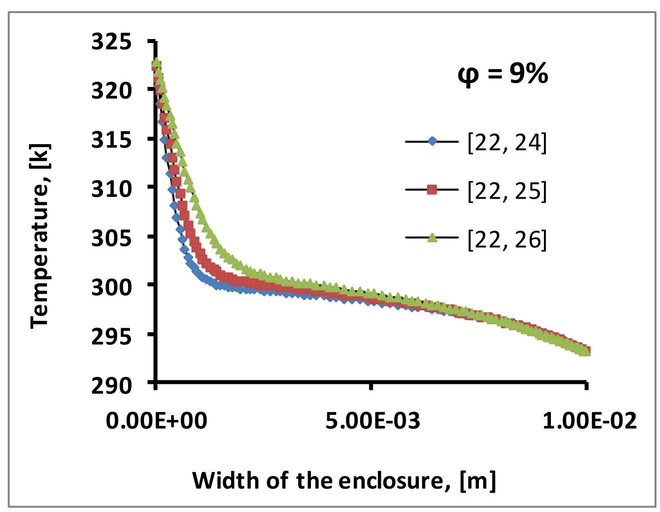

Figure 5 Distribution of Static Temperature at middle of the cavity with Prasher et al. [22] thermal conductivity and three different viscosity [24-26] when φ=9%

In addition, is analysed the movement of the fluid inside the cavity. In natural convection inside the cavity, when one surface is heated up at constant temperature and another surface (opposite surface) is cooled down also at constant temperature, and if there is difference in density inside the cavity, the fluid moves inside the cavity. Because of movement of the fluid, isotherms and streamlines are formed inside the cavity.

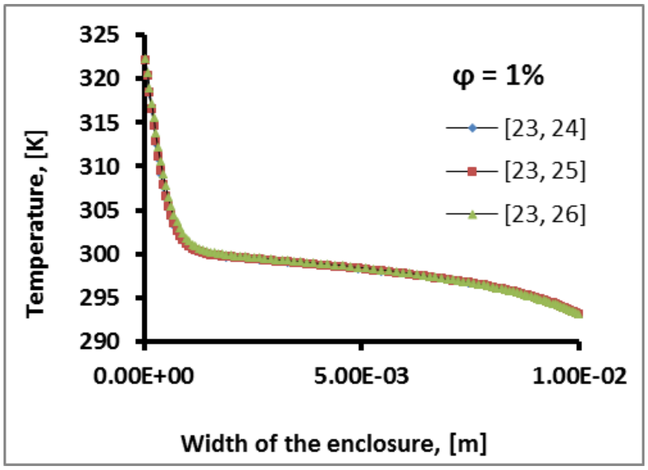

Figure 6 Distribution of Static Temperature at middle of the cavity with Jang and Choi [23] thermal conductivity and three different viscosity [24-26] when φ=1%

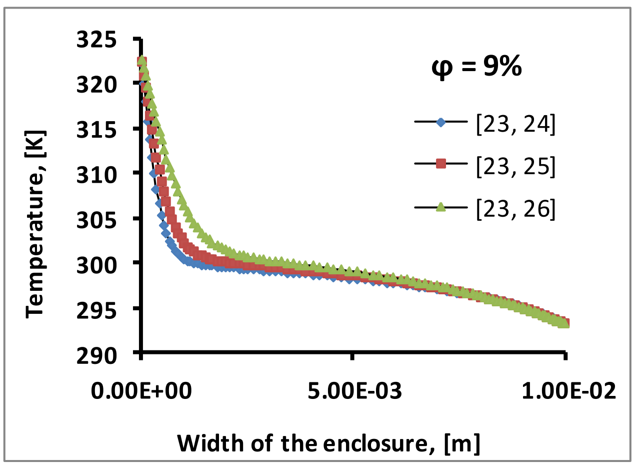

Figure 7 Distribution of Static Temperature at middle of the cavity with Jang and Choi [23] thermal conductivity and three different viscosity [24-26] when φ=9%

Figs. 8 and 9 display (a) isotherms and (b) streamlines. Inside the cavity the flow rotates in the clockwise direction indicating that the fluid filling the enclosure is moving up along the left heated wall, the top along insulated wall, dawn along the cooled right wall and finally horizontally to the heated wall along insulated bottom wall. The $Ra=10^5$ and AR=1. The nanoparticle volume fraction inside the cavity is 1%. In Fig. 9, also the $Ra=10^5$ and AR=1, but the nanoparticle volume fraction inside the cavity is 9% and due to high concentration of nanoparticles, the viscosity is higher that will suppress the flow but small compared to a favourable effect driven by the presence of the high thermal conductivity. The effect of viscosity has less impact since nanoparticles increase temperature inside the cavity and consequently increase the strength of the flow and the average rate of heat transfer. The high concentration of solid nanoparticles leads to high energy which accelerates the flow inside the cavity. Rising nanoparticles volume fraction in the cavity, enhances thermal conductivity and existing temperature, but reduces velocity of the nanofluid because of increasing solid concentration; therefore, nanofluid cannot move freely like base fluid.

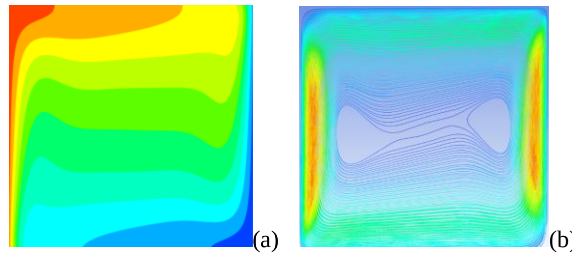

Figure 8 (a) Isotherms and (b) streamlines of water-Al_2O_3 nanofluids, $Ra=10^5$, AR=1 and φ=1%

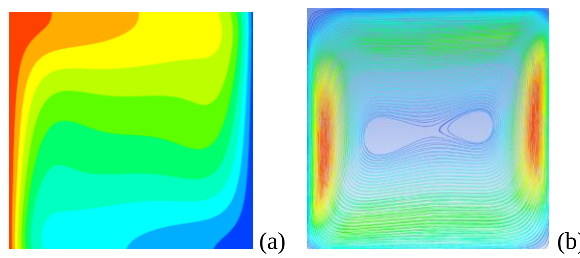

Figure 9 (a) Isotherms and (b) streamlines of water-Al_2O_3 nanofluids, $Ra=10^5$, AR=1 and φ=9

The influence of cavity aspect ratio is also investigated in this work which the simulation results offer in Figs. 10 and 11.

For the high aspect ratio which is considered the tall cavity, there exist a greater inequality between base fluid and nanoparticles. The distance between hot and cold surfaces in this tall cavity is very small. Cold surface can takes heat from hot surface very rapidly. The rate of heat transfer becomes more effective than the other values of aspect ratios.

Figs. 9 and 10 show the isotherms and streamlines for the same $Ra=10^4$ and AR=2, but different nanoparticle volume fractions. From Fig. 9, it can be seen that the nanoparticle volume fraction in the cavity is 1% whereas Fig. 10 the nanoparticle volume fraction in the cavity is 9%. Also, for Fig. 10, it can be seen that the flow strength and the temperature isotherms are influenced by the presence of nanoparticles.

The variation of local Nusselt number along the heated surface using water-Al_2O_3 nanofluid for different nanoparticles volume fraction (φ=1% and 9%), different values of $Ra=10^5$ for a square cavity AR=1, and $Ra=10^4$ and for a rectangular cavity AR=2 displayed in Fig. 11. It can be seen that when AR increases the value of Nusselt number also increases. It can be explained that the gradient of temperature between heated surface and the cooled surface is low. This can be seen from the figure for both AR=2. Also, for both cases, it is predicted that an increase in nanoparticle volume fraction of water-Al_2O_3 nanofluid results in reduction the local Nusselt number. The locations where enhancement is taking place are clearly demonstrated by looking the graph and it follows:

- For AR=1 and φ=1%, the optimal value of local Nusselt number occurs at 3.24E-04, 19.5. From this point, the value of local Nusselt number decreases until Y=1;
- For AR=1 and φ=9%, the optimal value of local Nusselt number occurs at 4.95E-04, 10.95. From this point, the value of local Nusselt number decreases until Y=1;
- For AR=2 and φ=1%, the optimal value of local Nusselt number occurs at 2.70E-04, 2.13E+01. From this point, the value of local Nusselt number decreases until Y=1; and
- For AR=2 and φ=9%, the optimal value of local Nusselt number occurs at 4.95E-04, 1.25E+01. From this point, the value of local Nusselt number decreases until Y=1.

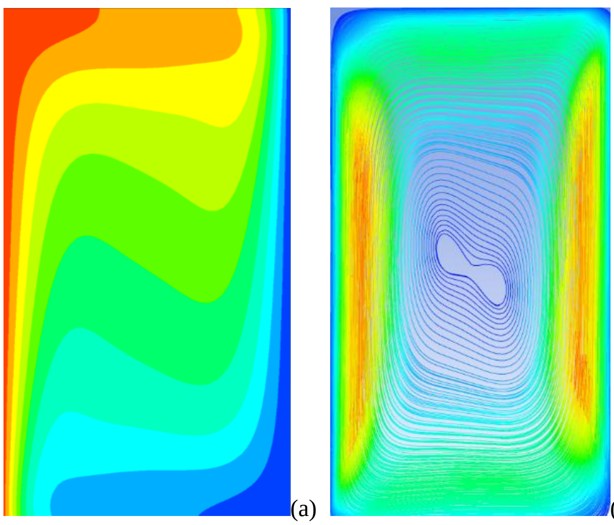

Figure 10 (a) Isotherms and (b) streamlines of water- Al_2O_3 nanofluids, $Ra=10^4$, AR=2 and φ=1%

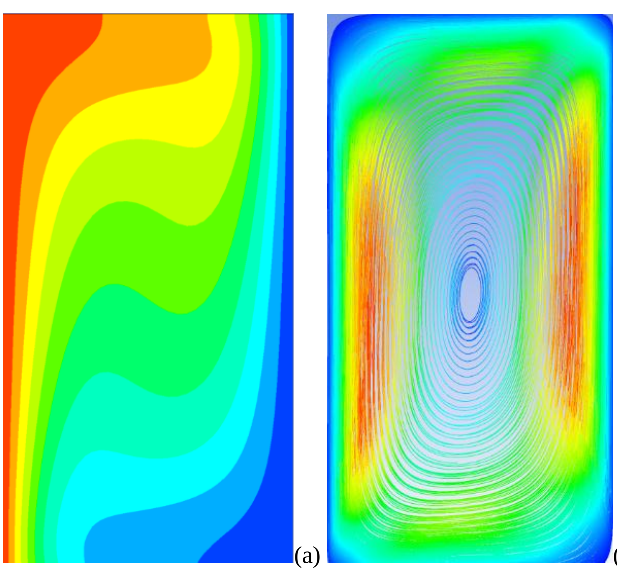

Figure 11 (a) Isotherms and (b) streamlines of water-Al_2O_3 nanofluids, $Ra=10^4$, AR=2 and φ=9%

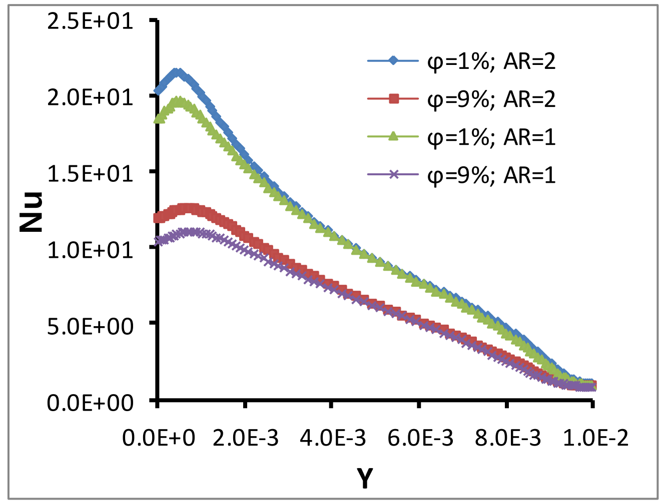

Figure 12 Variation of local Nusselt number along the heated surface, using water-Al_2O_3 nanofluid at $Ra=10^5$ for AR=1 and $Ra=10^4$ for AR=2

CONCLUSION

This paper analyses the effect of distilled water-Al_2O_3 nanofluid on natural convection heat transfer in a rectangular cavity heated vertically on two opposite side walls. The simulation was carried under different volume fraction concentration of nanoparticles as well as different correlations for effective properties of distilled water-Al_2O_3 nanofluid. The main conclusions achieved in this work are:

1. The results provided by available correlations for effective properties of distilled water-Al_2O_3 nanofluid, offer certainty at low nanoparticles volume fraction in the cavity and when nanoparticles volume fraction is high, the results become uncertainty. It can be seen from distribution of Static Temperature at middle of the cavity along width of the cavity Figs. 3, 5 and 7. In the figures, the temperature increases inside the cavity (the rate of heat transfer increases) as result of adding nanoparticles into pure water inside the cavity.
2. The aspect ratio has great influence on the rate of heat transfer inside the cavity. In this work, it is found that for $Ra=10^4$ and AR=2, the rate of heat transfer enhancement is much more than the cavity with $Ra=10^5$ and AR=1. The high concentration of solid nanoparticles leads to accelerate the flow inside the cavity. Rising nanoparticles volume fraction in the cavity enhances thermal conductivity and the existing temperature in the cavity.
3. The variation of local Nusselt number along the heated surface using water- Al_2O_3 nanofluid at $Ra=10^5$ for AR=1 and $Ra=10^4$ for AR=2, indicate that in general when nanoparticles volume fraction is high, the local Nusselt number along the heated surface decreases and vice versa. It is also found that an increase in AR leads with increase in local Nusselt number. However, there is an optimum value of local Nusselt number where whereby begins to decrease.

Andre Maripia was one of the PhD students of the Department of Mechanical and Aeronautical Engineering at University of Pretoria.

He unfortunately passed away on 28th March 2014 from kidney disease. May his sole rest in peace; Andre will live on in our memories forever. This paper was his last work.

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