

## COMPARISON OF THERMAL CHARACTERISTICS BETWEEN THE PLATE-FIN AND PIN-FIN HEAT SINKS IN NATURAL CONVECTION

Younghwan Joo and Sung Jin Kim\*

\*Author for correspondence

School of Mechanical, Aerospace & Systems Engineering,  
 Korea Advanced Institute of Science and Technology,  
 Daejeon 305-701,  
 Republic of Korea,  
 E-mail: [sungjinkim@kaist.ac.kr](mailto:sungjinkim@kaist.ac.kr)

### ABSTRACT

Thermal performances of optimized plate-fin and pin-fin heat sinks with vertical base plate were compared in natural convection. Comparison is performed under the same base plate dimensions and fin height condition. A Nusselt number correlation for plate-fin array was adopted to optimize plate-fin heat sinks. For pin-fin heat sinks, a new correlation of heat transfer coefficient was developed. With the new correlation, diameter, horizontal spacing, and vertical spacing of pin-fin array are optimized. Optimized plate-fin and pin-fin heat sinks are compared analytically. They were compared for various base plate dimensions, fin heights, temperature differences. In most regions, plate-fin heat sinks show better performance than pin-fin heat sinks.

### INTRODUCTION

With the development of technologies, electronic devices have been integrated. Therefore, power dissipation rate per unit area has been increasing. Increased temperature of components caused by high heat dissipation rate leads to performance reduction and failure in systems. Therefore, cooling problem became an important issue and development of cooling devices is necessary to advance electronic devices. Heat sink is one of these cooling devices. It increases the area of heat transfer using materials with high thermal conductivity.

Heat sinks are typically divided into forced convection and natural convection heat sinks based on the operating conditions. Forced convection heat sinks dissipate large amount of heat with additional devices such as fan but the reliability is lower than natural convection heat sinks because of additional devices. Therefore, natural convection heat sinks are widely used for the applications in which high reliability is required and additional devices are not easy to be implemented.

Natural convection heat sinks have various configurations to enhance the thermal performance but typically plate-fin and pin-fin array are widely used because they are cost-effective.

Plate-fin array usually has larger surface area than pin-fin array because of the area loss caused by cross cut in pin-fin array. Pin-fin array usually has higher heat transfer coefficient because of the depression of growth of the thermal boundary layers. Then, which array type is better?

### NOMENCLATURE

$A$	[m <sup>2</sup> ]	Surface area
$c_p$	[kJ/kg-K]	Specific heat
$d$	[m]	Fin diameter
$g$	[m/s <sup>2</sup> ]	Standard acceleration of gravity
$H$	[m]	Fin height
$h$	[W/m <sup>2</sup> K]	Convective heat transfer coefficient
$K$	[m <sup>2</sup> ]	Permeability
$k$	[W/m-K]	Thermal conductivity
$L$	[m]	Heat sink length
$n$	[-]	Number of fins
$Nu$	[-]	Nusselt number
$Pr$	[-]	Prantl number
$R_{th}$	[K/W]	Thermal resistance
$S$	[m]	Spacing of pin-fin array
$T$	[K]	Temperature
$W$	[m]	Heat sink width
$w_c$	[m]	Fin thickness of plate-fin
$w_w$	[m]	Channel spacing of plate-fin
Special characters		
$\alpha$	[m <sup>2</sup> /s]	Diffusion coefficient
$\beta$	[1/K]	Volumetric thermal expansion coefficient
$\varepsilon$	[-]	Emissivity
$\eta$	[-]	Fin efficiency
$\mu$	[N-s/m <sup>2</sup> ]	Dynamic viscosity
$\nu$	[m <sup>2</sup> /s]	Kinematic viscosity
$\rho$	[kg/m <sup>3</sup> ]	Density
$\sigma$	[W/m <sup>2</sup> K <sup>4</sup> ]	Stefan-Boltzmann constant
$\varphi$	[-]	Porosity
Subscripts		
$array$		Array
$base$		Base
$f$		Fluid
$fin$		Fin

$h$	Horizontal
$pin$	Pin-fin heat sink
$plate$	Plate-fin heat sink
$ratio$	Ratio
$v$	Vertical

About the question, there are some previous researches. Sparrow and Vemuri [1] studied the optimal number of pin-fin with fixed base plate dimensions and fin diameter. With the optimized result, they compared pin-fin and plate-fin heat sinks. However, their comparison was performed based on the same surface area. Therefore, their results didn't consider the optimal combination of surface area and heat transfer coefficient which is different for each array type. Iyengar and Bar-Cohen [2] tried to compare optimized plate-fin and pin-fin array with least-material method. In many cases, however, thermal performance itself is usually more important than the amount of material. Therefore, there may be other practical constraints to compare plate-fin and pin-fin heat sinks.

In the present study, thermal performances of plate-fin and pin-fin heat sinks with vertical base plate would be compared based on the same base plate dimensions and fin height condition. To achieve maximum thermal performance for the constraints, each array type would be optimized using correlations of convective heat transfer coefficient. There have been many researches about plate-fin heat sinks [3, 4, 5, 6, and 7], so plate-fin heat sinks would be optimized using correlations suggested by previous researches. For pin-fin heat sinks, there have been many researches and correlations [1, 8, 9, and 10]. Aihara et al. [9] studied highly populated pin-fin heat sinks and suggested a Nusselt number correlation. However, their correlation cannot be used for optimization of pin-fin array in above constraints. For given horizontal spacing and diameter, their correlation predicts same thermal performance along the vertical spacing of pin-fin array, so optimal vertical spacing cannot be achieved. Therefore, we need a new correlation of convective heat transfer coefficient to optimize pin-fin heat sinks.

## ANALYTIC EQUATION FOR PIN-FIN HEAT SINK

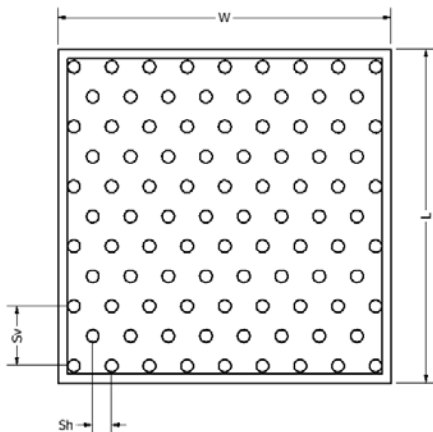


Figure 1 Configuration of pin-fin heat sink

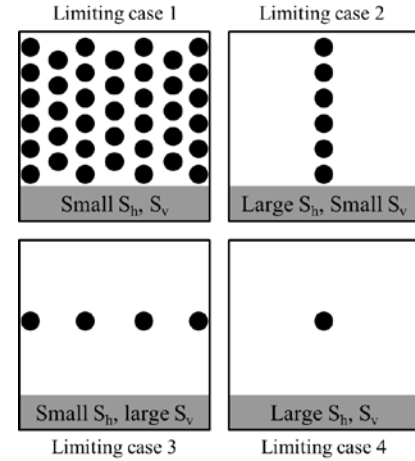


Figure 2 Limiting cases for pin-fin array

Figure 1 shows pin-fin heat sink. For given  $W$ ,  $L$ , and  $H$ , configuration of array is determined with  $d$ ,  $S_h$ ,  $S_v$ . Therefore, correlation for convective heat transfer coefficient consists of these three parameters. With variations of  $S_h$ ,  $S_v$ , pin-fin array configurations can be divided into 4 limiting cases as Figure 2. Limiting case 1 means densely positioned pin-fins with small  $S_h$  and  $S_v$ . Limiting case 2 means vertical single array of pin-fins with large  $S_h$  and relatively small  $S_v$ . Limiting case 3 means horizontal single array of pin-fins with large  $S_v$  and relatively small  $S_h$ . Limiting case 4 means isolated horizontal cylinder with large  $S_h$  and  $S_v$ . With these limiting cases, we are going to apply asymptotic method to get a new correlation for convective heat transfer coefficient of pin-fin heat sink. As in the plate-fin heat sink, we need to get equations of heat transfer coefficient in each limiting case.

In limiting case 1, pin-fins are densely positioned, so it can be considered as porous media. By assuming 1-D flow, we can start from following governing equation.

$$\frac{\Delta P}{\Delta x} = \frac{\mu_f}{K} u_D + \frac{0.55}{K^{1/2}} \rho_f u_D^2 \quad (1)$$

Brinkmann term which considers the effects of wall friction can be ignored because there is no wall for pin-fin array. On the other hand, we can assume that exit temperature of air becomes almost same as the temperature of fins when intake flow is thermally fully developed immediately. Then, following equation of energy balance can be used.

$$q = \dot{m} c_p (T_{fin} - T_\infty) = h_1 A_{array} (T_{fin} - T_\infty) \quad (2)$$

By combining equation (1) and (2), we can suggest following equation for limiting case 1.

$$h_1 = \frac{S_h S_v c_p}{1.1 \pi d L} \left[ -\frac{\mu_f}{\sqrt{K}} + \sqrt{\frac{\mu_f^2}{K} + 2.2 \sqrt{K} \rho_f^2 g \beta \Delta T} \right] \quad (3)$$

$$K = \frac{4 S_h S_v - \pi d^2}{48} \quad (4)$$

In limiting case 2, horizontal cylinders are arranged in a vertical direction.  $S_h$  is very large, so it can be considered as an isolated vertical single array of horizontal cylinders. There are previous researches about the situation. Aihara *et al.* [9]

performed experiments about the heat transfer phenomena from uniformly heated horizontal cylinders and suggested a Nusselt number correlation as follow.

$$h_2 = \frac{k_f}{L} [0.311 + 0.454 \ln(S_v/d)] Gr_L^{1/4} \quad (5)$$

$$Gr_L = \frac{g\beta\eta(T_b - T_\infty)L^3}{\nu_f^2} \quad (6)$$

This correlation is applicable to  $S_v/d=1-4$  and  $Gr_L=10^6-10^8$ . Above equation would be used for limiting case 2.

In limiting case 3, horizontal cylinders are arranged in a horizontal direction.  $S_v$  is large enough to be considered as a vertically isolated horizontal array. To analyse this case, correlation form of immersed body was employed because it approaches the situation of an isolated horizontal cylinder as  $S_h$  becomes large. Then, we can postulate following form of equation for limiting case 3.

$$h_3 = k_f \left[ f\left(\frac{S_h}{d}\right) \right] \left( \frac{g\beta\eta(T_b - T_\infty)1}{\alpha_f \nu_f} \frac{1}{d} \right)^{1/4} \quad (7)$$

To find the form of the function in the square bracket, we made use of numerical simulation which replaces experiments. For different fin diameters, heat transfer coefficients were evaluated as dimensionless horizontal spacing ( $S_h/d$ ) varies. Then, the results were fitted into a single equation as follow.

$$h_3 = 0.59k_f \left[ 1 - 2.29 \exp\left(-\frac{1}{0.61211} \frac{S_h}{d}\right) \right] \left( \frac{g\beta\eta(T_b - T_\infty)1}{\alpha_f \nu_f} \frac{1}{d} \right)^{1/4} \quad (8)$$

Above equation would be used for limiting case 3.

Limiting case 4 means an isolated horizontal cylinder. There have been many researches about this case. Churchill and Chu [11] suggested a correlation applicable for a wide range of Rayleigh number, so following equation would be used for limiting case 4 by modifying the correlation.

$$h_4 = \frac{k_f}{d} \left\{ 0.60 + \frac{0.387 Ra_d^{1/6}}{\left[ 1 + (0.559 / Pr)^{9/16} \right]^{8/27}} \right\}^2 \quad (9)$$

In asymptotic method, equations for each limiting case should be integrated with proper exponents. The exponents were decided by comparison with experimental results including previous researches. Then, final form of average convective heat transfer coefficient is obtained.

$$h_m = \left[ h_1^{-8} + (h_2^{-3.5} + h_3^{-3.5} + h_4^{-3.5})^{-\frac{8}{3.5}} \right]^{\frac{1}{8}} \quad (10)$$

By using above equation, thermal performance of pin-fin heat sink would be predicted.

## OPTIMIZATION OF HEAT SINK

To optimize heat sink for given volume, analytic equations predicting the convective heat transfer coefficient and equations to calculate surface area are needed. For plate-fin heat sinks, we are going to use following equation suggested by Kim *et al.* [7].

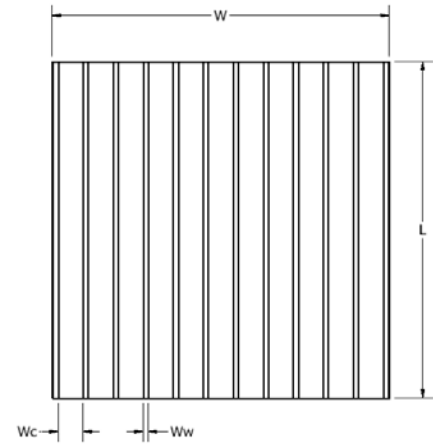


Figure 3 Configuration of plate-fin heat sink

$$\overline{Nu}_{fin} = \frac{h_m w_c}{k_f} = \left[ \left( 0.09112 El^{0.6822} \right)^{-3.5} + \left( 0.5170 El^{0.2813} \right)^{-3.5} \right]^{-1/3.5} \quad (10)$$

$$El = \frac{g\beta(T_b - T_\infty)Pr w_c^4}{\nu_f^2 L} \quad (11)$$

This equation considers the effect of intake flow from the front side of plate-fin heat sinks, so it can predict thermal performance of plate-fin heat sink more precisely.

Surface area of plate-fin heat sink can be expressed with design parameters shown in Figure 3.

$$A_{array} = n_{fin} \times (2LH + 2w_w H + w_w L) \quad (12)$$

$$A_{base} = WL - n_{fin} w_w L \quad (13)$$

$$n_{fin} = \frac{W - w_w}{w_c + w_w} \quad (14)$$

For pin-fin heat sinks, equation (10) is used to predict convective heat transfer coefficient. Surface area of pin-fin array can be calculated using equations as follows.

$$A_{array} = n_{fin} \times \pi d (H + d / 4) \quad (15)$$

$$A_{base} = WL - n_{fin} \frac{\pi d^2}{4} \quad (16)$$

$$n_{fin} = n_v n_h + (n_v - 1)(n_h - 1) \quad (17)$$

$$n_v = (L - d) / S_v + 1 \quad (18)$$

$$n_h = (W / 2 - d) / S_h + 1 \quad (19)$$

For un-finned base plate, following correlation for natural convection from vertical wall is used.

$$\overline{Nu}_L = \frac{h_{base} L}{k_f} = 0.59 Ra_L^{1/4} \quad (20)$$

$$Ra_L = \frac{g\beta(T_b - T_\infty)L^3}{\nu_f \alpha_f} \quad (21)$$

With above equations, we can express  $Q_{total}$  which means total heat dissipation from heat sink.

$$Q_{total} = (h_{base} A_{base} + h_{fin} A_{array} \eta) (T_b - T_\infty) \quad (22)$$

Then, thermal resistance ( $R_{th}$ ) which is target dependent variable for thermal performance comparison is expressed as follow.

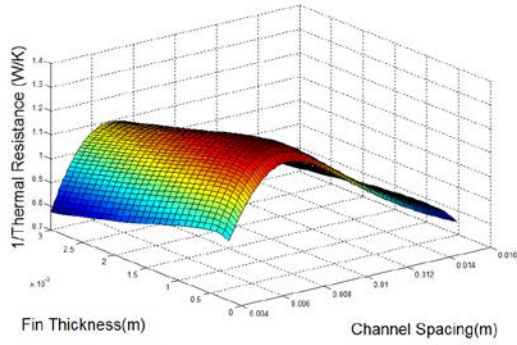
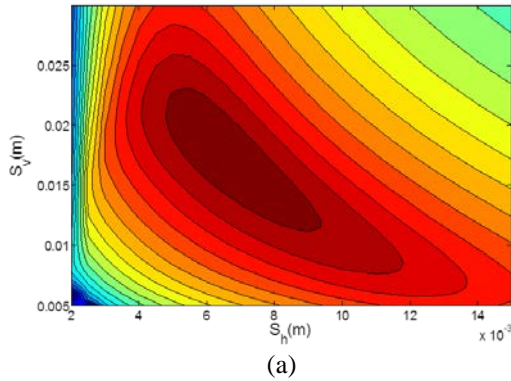
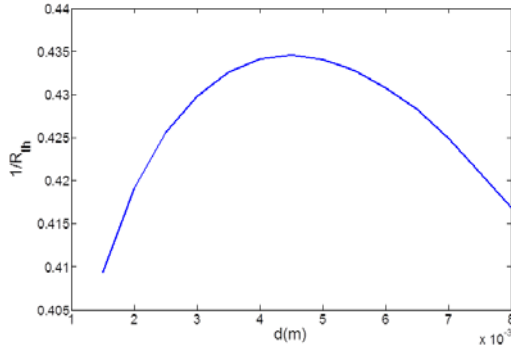


Figure 4 Optimization of plate-fin heat sink



(a)



(b)

Figure 5 Optimization of pin-fin heat sink

$$R_{th} = \frac{(T_b - T_\infty)}{Q_{total}} = \frac{1}{h_{base}A_{base} + h_{fin}A_{array}\eta} \quad (23)$$

Figure 4 shows optimization of plate-fin heat sink. Optimal configuration seems to have small fin thickness and optimal channel spacing is about 7-8 mm. Figure 5 shows optimization of pin-fin heat sink. Two graphs are shown because there are three main parameters ( $S_b$ ,  $S_y$ ,  $d$ ) for pin-fin heat sinks. Figure 5 (a) shows a contour of  $R_{th}^{-1}$  for a fixed diameter ( $d=4$  mm). Figure 5 (b) means collection of optimized results for different diameters. Therefore, the optimal performance in Figure 5 (a) is expressed as a point on Figure 5 (b). Above results are gained from  $L=0.1$  m,  $W=0.1$  m,  $H=0.03$  m,  $T_b-T_\infty=50$ K.

## COMPARISON OF THERMAL PERFORMANCE

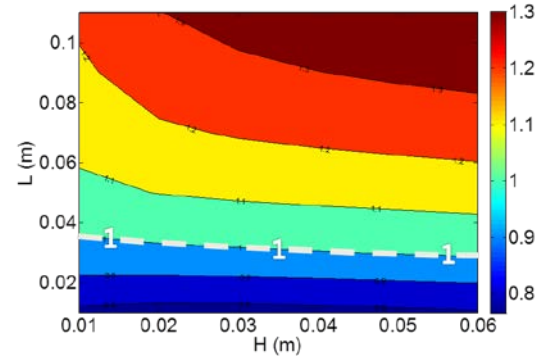


Figure 6 Region map of  $R_{th,ratio}$  at  $W=0.1$  m,  $T_b-T_\infty=50$ K

In the present study, thermal performances of plate-fin and pin-fin heat sinks are compared analytically. The constraints for the comparison are 1) same base plate dimensions, 2) same fin height. Based on the optimization process, region maps of thermal resistance ratio are obtained as Figure 6. The definition of thermal resistance ratio is as follow.

$$R_{th,ratio} = \frac{R_{th,pin}}{R_{th,plate}} \quad (24)$$

In above equation, each thermal resistance is minimum thermal resistance for a given volume of heat sink. According to the definition of  $R_{th,ratio}$ , plate-fin heat sinks have better performance when  $R_{th,ratio}$  is larger than 1. From the region map, we can see that thermal performance of plate-fin heat sinks is superior to that of pin-fin heat sinks when  $L$  is larger than 0.03 m. When  $L$  is larger than 0.03 m, plate-fin heat sinks become superior with large  $H$ . However, the effect of  $L$  is more dominant than that of  $H$  on the thermal performance.

## CONCLUSION

In the present study, thermal performances of plate-fin and pin-fin heat sinks were compared for the fixed base plate dimensions and fin height. To compare the optimized heat sinks, each heat sink type was optimized based on the correlations for convective heat transfer coefficient of fin array. For the pin-fin heat sink with staggered circular fin, however, we couldn't find appropriate correlation which is available for optimization in our constraints. Therefore, a new correlation for convective heat transfer coefficient of pin-fin heat sink was suggested. Using the correlations for both types of heat sinks, thermal performances were compared and a region map was suggested. According to the region map, we can conclude that plate-fin heat sinks show better thermal performance in most practical regions.

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