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A NUMERICAL STUDY OF NATURAL CONVECTIVE HEAT TRANSFER FROM A HORIZONTAL ISOTHERMAL SQUARE ELEMENT IMBEDDED IN AN ADIABATIC SURFACE WITH A PARALLEL ADIABATIC COVERING SURFACE

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ABSTRACT

Natural convective heat transfer from a square horizontal flat isothermal heated element imbedded in a larger flat square adiabatic surface with a square flat horizontal adiabatic surface mounted parallel to the heated surface at a relatively short distance from it has been numerically studied. The surface of the heated element is in the same plane as the surface of the surrounding adiabatic material. The square heated element considered in this study is at a higher temperature than that of the surrounding fluid. Both the case where the square heated element is facing upwards and the surrounding adiabatic covering surface is above the heated element (upward facing case) and the case where the square heated element is facing downwards and the surrounding adiabatic covering surface is below the heated element (downward facing case) have been considered. The situation considered is a simplified model of some situations that arise in engineering practice examples occurring in some electrical and electronic component cooling problems. In this study, the range of conditions considered is such that laminar, transitional, and turbulent flows occur. The purpose of this study was to numerically determine how the heat transfer rate from the square heated element varies with the distance of the adiabatic covering surface from the heated element. The solution was obtained by numerically solving the governing equations subject to the boundary conditions using the commercial CFD solver ANSYS FLUENT© using the *k*-epsilon turbulence model with full account being taken of buoyancy force effects. Because of the applications that motivated this study, results have been obtained for a Prandtl number of 0.74, i.e., effectively the value for air. The effect of the dimensionless distance between the heated element and the cover on the variation of the Nusselt number with Rayleigh number has been studied in detail for both the upward and the downward facing cases.

INTRODUCTION

The natural convective heat transfer rate from a horizontal square isothermal element imbedded in a larger square flat adiabatic surface has been studied numerically. The surface of the heated element is in the same plane as the surface of the surrounding adiabatic surface and there is a square flat horizontal adiabatic surface mounted parallel to the heated surface at a relatively short distance from it, i.e., there is a covering surface over the heated element. This square adiabatic covering surface has the same size as the square adiabatic surface in which the heated element is imbedded. The flow situation considered is therefore as shown schematically in Figure 1.

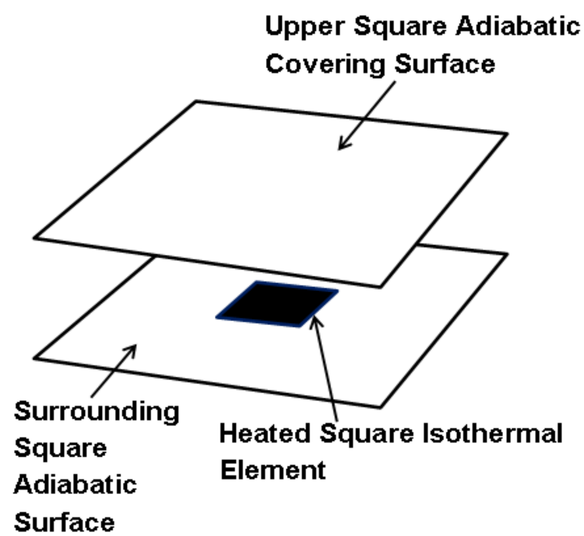

Figure 1 Flow situation considered

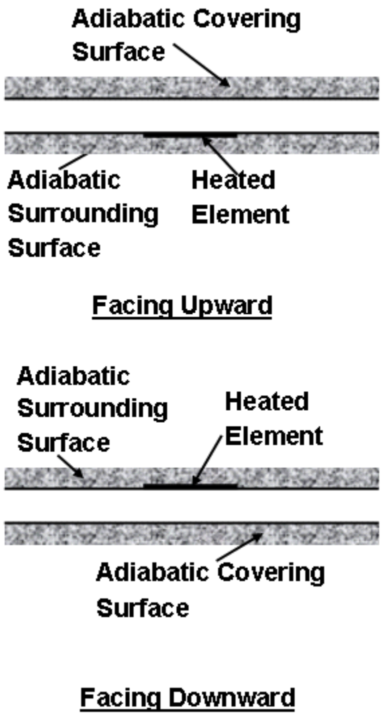

Figure 2 Upward and downward facing heated element situations

The square heated element considered in this study is at a higher temperature than that of the surrounding fluid. Both the case where the square heated element is facing upwards and the surrounding adiabatic covering surface is above the heated element (upward facing case) and the case where the square heated element is facing downwards and the surrounding adiabatic covering surface is below the heated element (downward facing case) have been considered. These cases are as shown in Figure 2. This study considers a simplified model of some situations that arise in engineering practice. Situations of this general type occur, for example, in some electrical and electronic component cooling problems. Here, the range of conditions considered is such that laminar, transitional, and turbulent flows can occur over the heated element. The aim of the present numerical study was to numerically investigate the variation of the heat transfer rate from the square heated element with the distance of the adiabatic covering surface from the heated element for both the upward and downward facing element cases.

NOMENCLATURE

A	[m^2]	Surface area of heated element
g	[m/s^2]	Gravitational acceleration
h	[m]	Vertical distance of covering surface from heated element
H	[-]	Dimensionless vertical distance of covering surface from heated element, h/w
k	[W/mK]	Thermal conductivity
Pr	[-]	Prandtl number
Nu	[-]	Nusselt number
$\overline{Q'}$	[W]	Mean heat transfer rate from element
Ra	[-]	Rayleigh number
T	[K]	Temperature
T_f	[K]	Undisturbed fluid temperature
T_w	[K]	Heated element wall temperature
w	[m]	Side length of square heated element
w_{out}	[m]	Side length of surrounding and covering adiabatic surfaces
W_{out}	[-]	Dimensionless side length of surrounding and covering adiabatic surfaces, w_{out} / w

Greek symbols

α	[m^2/s]	Thermal diffusivity
β	[1/K]	Coefficient of thermal expansion
ν	[m^2/s]	Kinematic viscosity
ρ	[kg/m^3]	Density

There have been many studies of natural convective heat transfer from upward facing heated horizontal surfaces without a covering surface, e.g., see [1] to [16]. Most of these, however, have considered situations that involve only laminar flow. There have also been many studies of natural convective heat transfer from downward facing heated horizontal surfaces without a covering surface, e.g., see [17] to [23]. The present study differs from these available studies by considering the effect of the covering surface and by obtaining results for a wide range of Rayleigh numbers that cover conditions in which laminar, transitional, and turbulent flow occur. Some investigations of the effect of a covering surface on natural convective heat transfer from heated horizontal surfaces have been undertaken, e.g., see [24] and [25]. However, these studies cover narrower ranges of the Rayleigh number than considered in the present work.

SOLUTION PROCEDURE

Steady flow has been assumed. Fluid properties have been assumed constant except for the density change with temperature that gives rise to the buoyancy forces. This was treated by means of the Boussinesq type approximation. Effects of radiant heat transfer have been neglected. Allowance has been made for the possibility that turbulent flow can occur in the system. In order to deal with this, the basic *k*-epsilon turbulence model has been used with standard wall functions and with full account being taken of buoyancy force effects. The numerical approach adopted to allow for the development of turbulence, i.e., solving the Reynolds averaged governing equations together with a turbulence model for all conditions considered and then assessing the solutions obtained with increasing Reynolds number in order to determine when turbulence effects begin to develop has been used quite extensively. For example, this approach was used in the early studies [26-28]. Some work on the use of this approach in natural convective flows has also been undertaken [29, 30]. The commercial CFD solver ANSYS FLUENT© has been used to numerically solve the three-dimensional governing equations subject to the boundary conditions. It has been assumed that the flow is symmetric about the longitudinal centre-lines shown in Figure 3.

Extensive grid independence and convergence-criteria independence testing was undertaken indicating that with the grids used in obtaining the results presented here the heat transfer results are grid- and convergence criteria independent to within about one per cent.

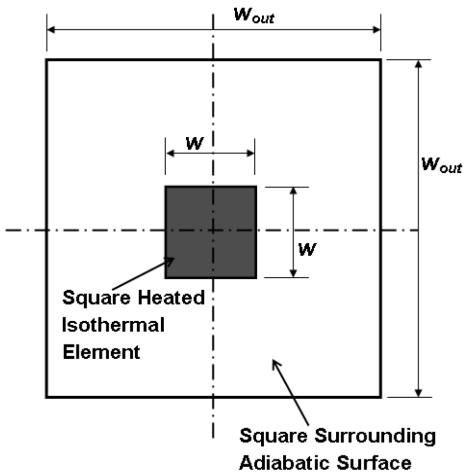

Figure 3 Heated element and surrounding adiabatic surface

RESULTS

The solution has the following parameters:

1. The Rayleigh number, *Ra*, based on the side length, *w*, of the square heated element (see Figure 3) and the difference between the temperature of the surface of the heated element, T_w, and the temperature of the undisturbed fluid well away from the system, T_f, i.e.:

$$Ra = \frac{\beta g w^3 (T_w - T_f)}{\nu\alpha} \tag{1}$$

2. The dimensionless size of the square surrounding adiabatic surface and the square adiabatic covering surface, $W_{out} = w_{out} / w$ (see Figure 3).
3. The dimensionless vertical distance of the adiabatic covering surface from the heated element, $H = h / w$ (see Figure 3).
4. The Prandtl number, *Pr*.
5. The orientation of the heated element, i.e., facing upward or facing downward (see Figure 2).

Results have been obtained only for a Prandtl number of 0.74, i.e., effectively the value for air because of the applications that motivated this study. Results will be presented here only for $W_{out} = 4$. Results obtained for other values of W_{out} showed the same basic characteristics as those presented here. Dimensionless heated element to adiabatic covering surface distances, *H*, of between 0.125 and 1 and Rayleigh numbers between approximately 10^4 and 10^{12} have been considered. Results have been obtained for both upward and downward facing heated element orientations.

The mean heat transfer rate from the heated surface, $\overline{Q}'$, has been expressed in terms of a mean Nusselt number based on the side length, *w*, of the square heated element and on the difference between the temperature of the surface of the heated element, T_w, and the temperature of the undisturbed fluid well away from the system, T_f, i.e.:

$$Nu = \frac{\overline{Q}' w}{kA(T_w - T_f)} \tag{2}$$

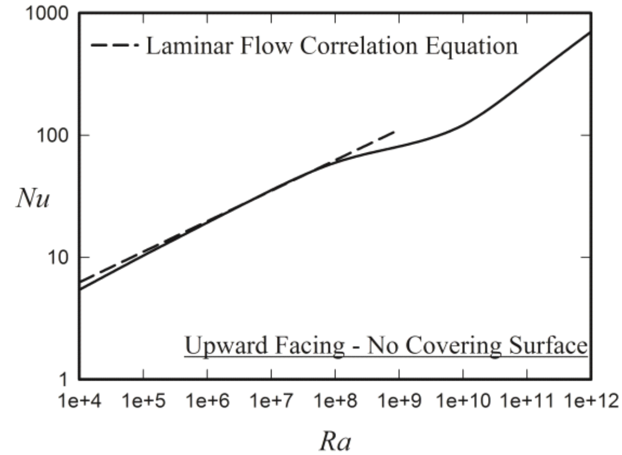

Figure 4 Variation of Nusselt number with Rayleigh number for upward facing heated element with no covering surface

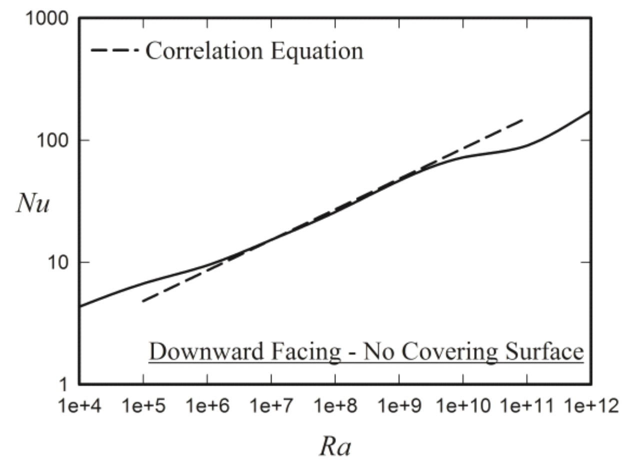

Figure 5 Variation of Nusselt number with Rayleigh number for downward facing heated element with no covering surface

where *A* is the surface area of the heated surface, i.e., w^2. Because values of *Pr* and W_{out} are fixed in this study, *Nu* is a function of *Ra*, *H*, and the element orientation, i.e., upward or downward facing.

Results for the case where there is no covering surface will first be considered. The variations of Nusselt number with Rayleigh number for this case for an upward facing heated element and for a downward facing heated element are shown in Figures 4 and 5 respectively. Also shown in these figures are the results given by the following commonly used empirical equations for laminar flow over a horizontal square isothermal surface, e.g. see for example [31] and [32].

Upward facing surface:

$$Nu = 0.622\, Ra^{1/4} \tag{3}$$

Downward facing surface:

$$Nu = 0.27\, Ra^{1/4} \tag{4}$$

Figures 4 and 5 illustrate that these empirical equations produce results that are in quite good agreement with the numerical results for the upward facing element for Rayleigh numbers between 10^5 and 10^8 and for the downward facing element for Rayleigh numbers between 10^6 and 10^9.

The case of a covered heated element facing upward will next be considered. The effect of the dimensionless vertical distance of the adiabatic covering surface from the heated element, *H*, for this case is demonstrated by the variations of Nusselt number with Rayleigh number for various values of *H* shown in Figures 6 to 11. Also shown in Figures 7 to 11 is the variation of Nusselt number with Rayleigh number for the no-cover case. This variation is not shown in Figure 6 because for *H* = 1 the covered element results for an upward facing element are essentially the same as the no-cover results. From Figures 7 to 11 it will be seen that the covered element variations of Nusselt number with Rayleigh number differ from that for the no-cover case, the difference increasing as *H* decreases. It will be seen that, generally, the covered element Nusselt number results are lower than the no-cover values at low Reynolds numbers and are higher than the no-cover values in the transition region. The complex form of the variation of Nusselt number with Rayleigh number for the covered element case is the result of changes in the flow pattern with Rayleigh number. The effect of *H* on the Nusselt number variation for the upward facing covered element case is further illustrated by the results given in Figure 12. This figure shows the variation of Nusselt number with *H* for various Rayleigh number values. For the lowest Rayleigh number for which results are given it will be seen that the presence of the covering surface affects the Nusselt number value when *H* is less than approximately 1. For the intermediate Rayleigh numbers considered the value *H* at which the presence of the covering surface effects the Nusselt number value decreases with increasing Rayleigh number being approximately 0.4 at a Rayleigh number of 10^8. At the higher Rayleigh number values considered the presence of the covering surface effects the conditions under which transition occurs and the value of *H* at which the presence of the covering surface affects the Nusselt number value increases.

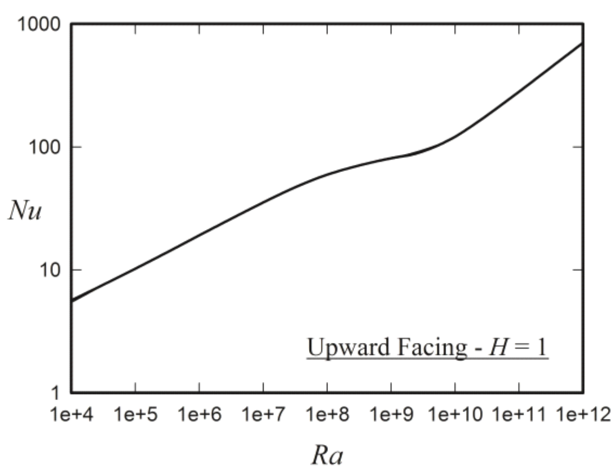

Figure 6 Variation of Nusselt number with Rayleigh number for upward facing heated element with covering surface for a dimensionless element to covering surface distance of 1

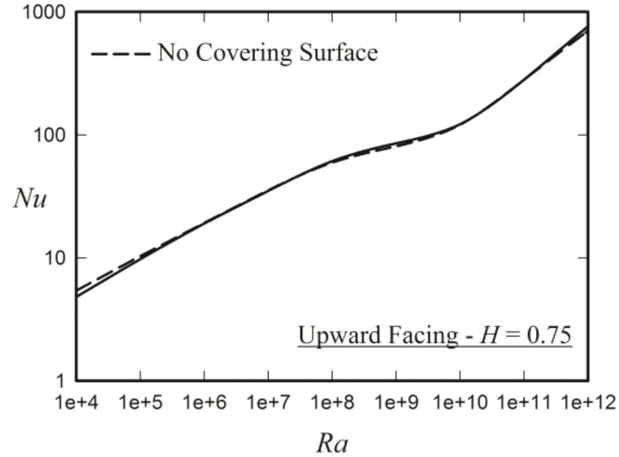

Figure 7 Variation of Nusselt number with Rayleigh number for upward facing heated element with covering surface for a dimensionless element to covering surface distance of 0.75

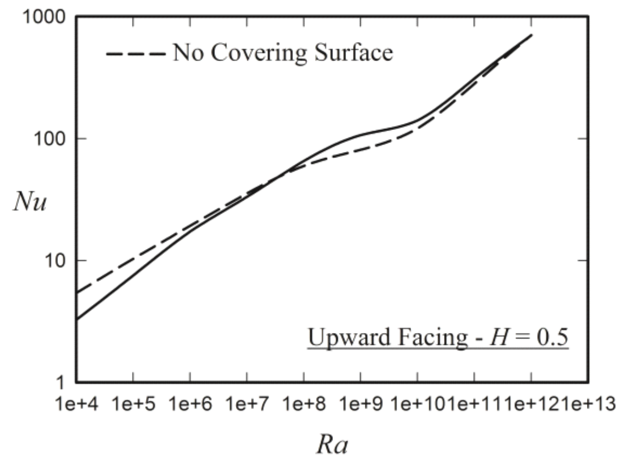

Figure 8 Variation of Nusselt number with Rayleigh number for upward facing heated element with covering surface for a dimensionless element to covering surface distance of 0.5

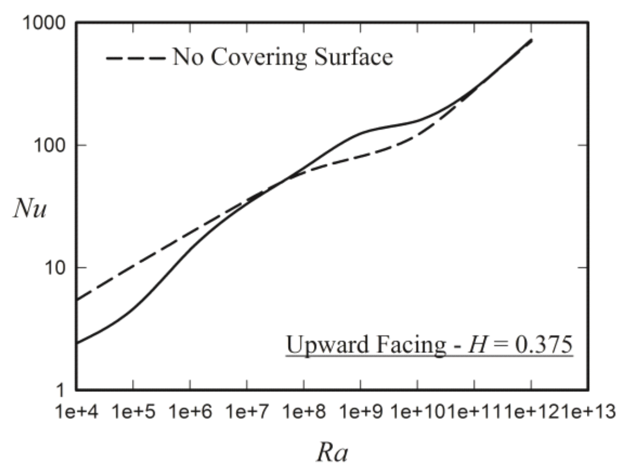

Figure 9 Variation of Nusselt number with Rayleigh number for upward facing heated element with covering surface for a dimensionless element to covering surface distance of 0.375

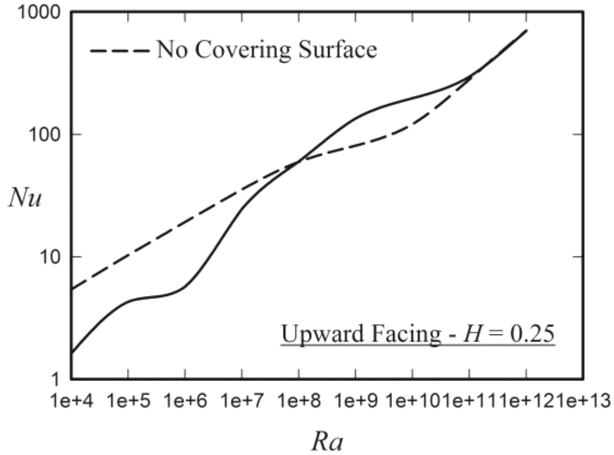

Figure 10 Variation of Nusselt number with Rayleigh number for upward facing heated element with covering surface for a dimensionless element to covering surface distance of 0.25

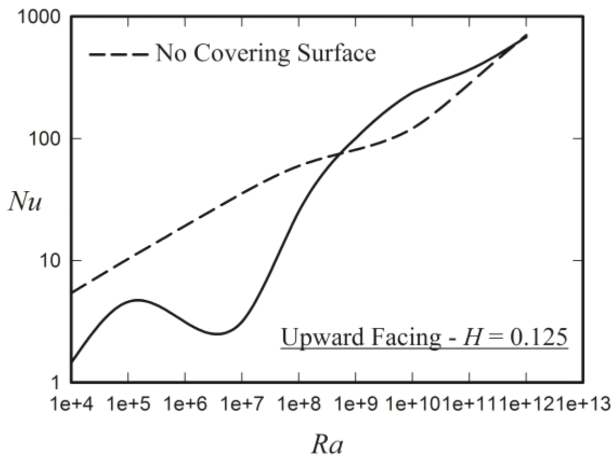

Figure 11 Variation of Nusselt number with Rayleigh number for upward facing heated element with covering surface for a dimensionless element to covering surface distance of 0.125.

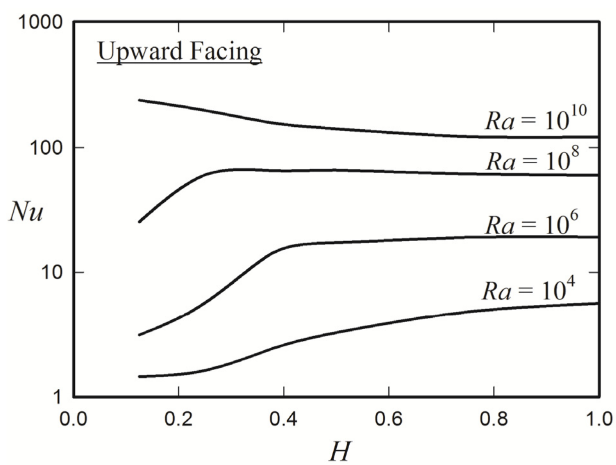

Figure 12 Typical variations of Nusselt number with dimensionless element to covering surface distance for various Rayleigh numbers for upward facing heated element

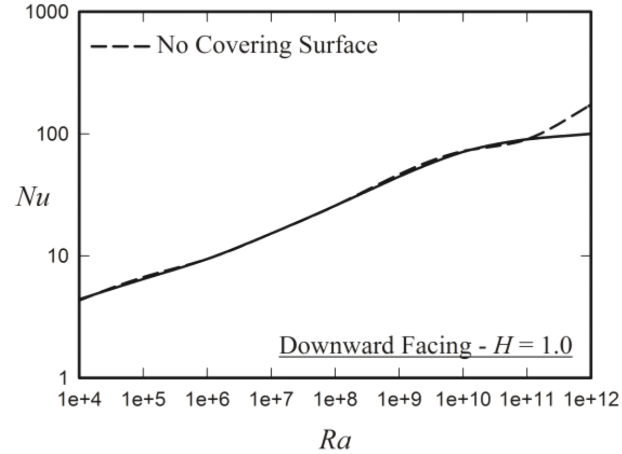

Figure 13 Variation of Nusselt number with Rayleigh number for downward facing heated element with covering surface for a dimensionless element to covering surface distance of 1

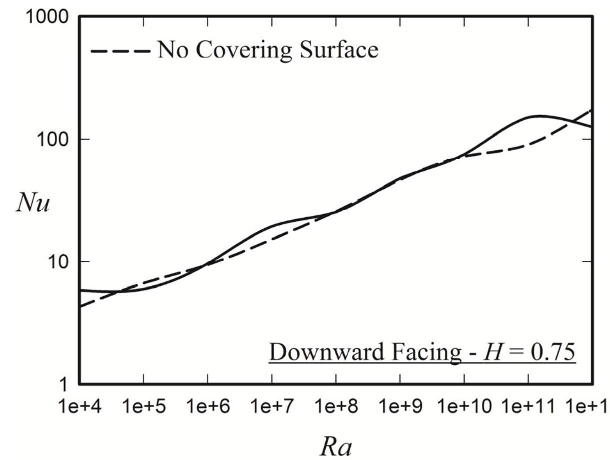

Figure 14 Variation of Nusselt number with Rayleigh number for downward facing heated element with covering surface for a dimensionless element to covering surface distance of 0.75

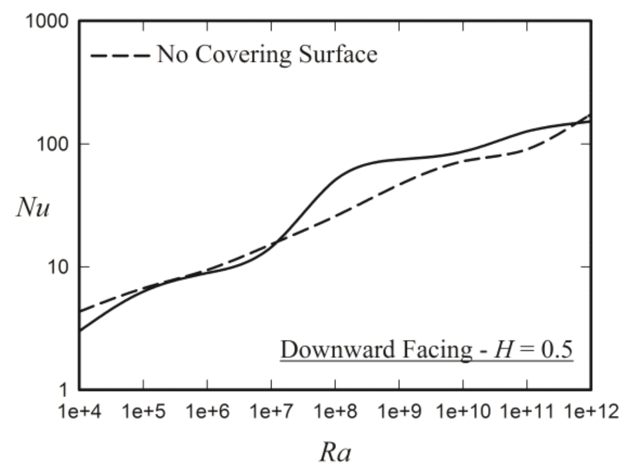

Figure 15 Variation of Nusselt number with Rayleigh number for downward facing heated element with covering surface for a dimensionless element to covering surface distance of 0.5

The results for the case of a covered heated element facing downward will be considered next. The effect of the dimensionless vertical distance of the adiabatic covering surface from the heated element, *H*, for this case is illustrated by the variations of Nusselt number with Rayleigh number for various values of *H* shown in Figures 13 to 18. Also shown in these figures is the variation of Nusselt number with Rayleigh number for the no-cover case. It will be seen from Figures 13 to 18 that the difference between the covered element variations of Nusselt number with Rayleigh number and the variations for the no-cover case are generally smaller than the difference that exists in the upward facing element case. The effect of *H* on the Nusselt number variation for the downward facing covered element case is further illustrated by the results given in Figure 19. This figure shows the variation of Nusselt number with *H* for various Rayleigh number values. For the lowest Rayleigh number for which results are given it will be seen that the presence of the covering surface affects the Nusselt number value when *H* is less than approximately 1. At the higher Rayleigh numbers considered the value *H* at which the presence of the covering surface affects the Nusselt number value is approximately 0.8.

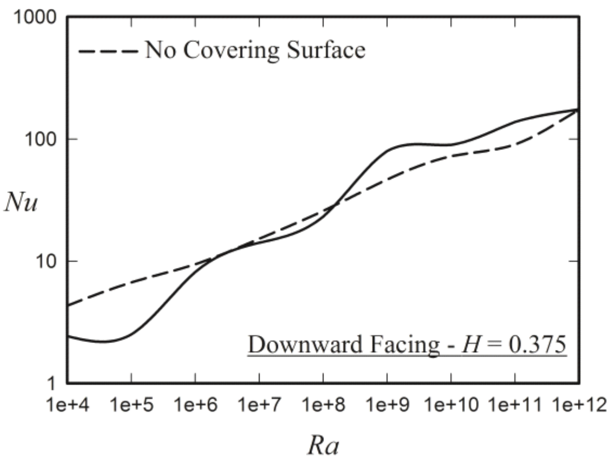

Figure 16 Variation of Nusselt number with Rayleigh number for downward facing heated element with covering surface for a dimensionless element to covering surface distance of 0.375

CONCLUSIONS

The results of the present study indicate that:

1. The effect of the dimensionless distance of the covering surface from the heated element on the mean heated element Nusselt number is significantly different for the upward facing element than it is for the downward facing element.
2. For the upward facing element case the covered element Nusselt number results are generally lower than the no-cover values at low Reynolds numbers and are generally higher than the no-cover values in the transition region.
3. For the upward facing element case, the value of the dimensionless distance of the covering surface from the heated element at which the presence of the covering surface begins to affect the mean element Nusselt number lies between 0.4 and 1, the value depending on the Rayleigh number.
4. For the downward facing element case, the difference between covered element variations of Nusselt number with Rayleigh number and the variations for the no-cover case are generally smaller than in the upward facing element case.
5. For the downward facing element case, the value of the dimensionless distance of the covering surface from the heated element at which the presence of the covering surface starts to effect the mean element Nusselt number lies between approximately 0.8 and 1, this value depending on the Rayleigh number.

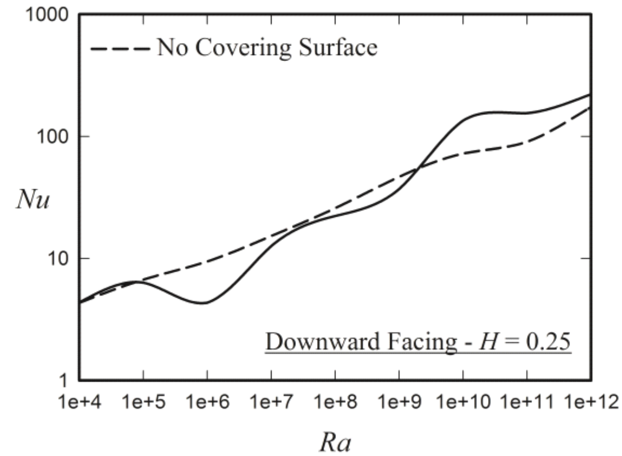

Figure 17 Variation of Nusselt number with Rayleigh number for downward facing heated element with covering surface for a dimensionless element to covering surface distance of 0.25

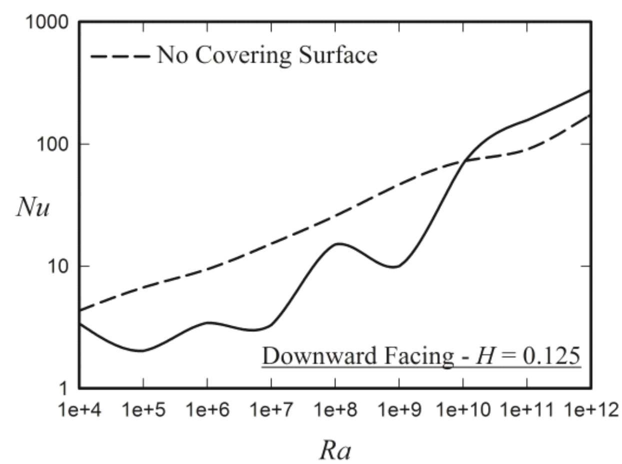

Figure 18 of Nusselt number with Rayleigh number for downward facing heated element with covering surface for a dimensionless element to covering surface distance of 0.125

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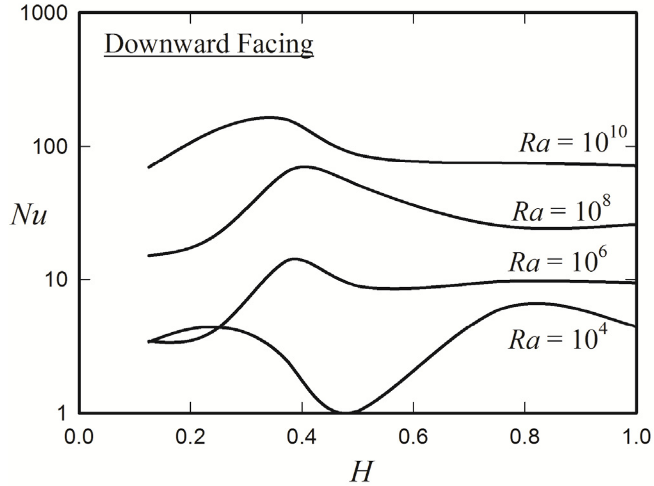

Figure 19 Typical variations of Nusselt number with dimensionless element to covering surface distance for various Rayleigh numbers for downward facing heated element

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