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AN UNCONSTRAINED MELTING ANALYSIS OF A SOLID PCM IN A SPHERICAL CAPSULE

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ABSTRACT

A melting model of a solid phase change material (PCM) in a spherical capsule using the stream function in which a solid sphere is moving in a stagnant viscid fluid was proposed. The enthalpy method based on this model clarifies the sedimentation of a solid PCM and the appearance of close contact melting which is very significant. Furthermore, the analyzed result elucidates qualitatively the melting process of a solid PCM which was obtained by the experiment.

INTRODUCTION

Phase change problems are encountered extensively in nature and in a wide variety of technological important processes. Such processes include thermal energy storage, melting of ice, crystal growth, and thermal control of electronic equipments using PCMs. In particular, thermal energy storage (TES) systems play an important role in providing enormous potential for facilitating energy saving and reducing environmental impact. Indeed, TES appears to provide one of the most advantageous solutions for correcting the gap that often occurs between the supply and demand of energy. TES is a term widely used to describe the storage used for both the heating and cooling of energy. It deals with the storing of energy by cooling, heating, melting, and solidifying a substance, and the energy becomes available as heat when the process is reversed. In the TES system, a spherical capsule is most commonly used for storing PCM. This is mainly due to its low volume to heat transfer surface area ratio, and because it can be easy packed into the storage system [1]-[4]. Recently, Regin et al. [1] reviewed the development of available latent heat thermal energy storage technologies. The different aspects of storage such as material, encapsulation, heat transfer, applications and new PCM technology innovation have been carried out.

Assis et al. [5] explored numerically and experimentally the process of melting of a PCM in spherical geometry. Transient numerical simulations were performed using the commercial software. These simulations show the melting process from beginning to the end, and incorporate such phenomena as convection in the liquid phase, volume expansion due to melting, sedimentation of the solid in the liquid, and close contact melting. The results of the experimental investigation, which included visualization, compare favorably with the numerical results and thus serve to validate the numerical approach. Fomin and Saito [6] analyzed close-contact melting within spherical capsule. A complete mathematical model is solved numerically by utilizing the boundary fixing method. Koizumi [7] communicated the limited results of an experimental study of constrained melting of PCM (three thin copper plates inserted at right angles in a spherical capsule) and unconstrained melting of PCM within a spherical capsule which was placed in an upwardly directed heated flow. More recently, Koizumi et al. [8] revealed the temporal and spatial heat transfer performance and the transition process from a steady axisymmetric separated flow to a chaotic flow of an isothermally heated sphere placed in a uniform, downwardly directed flow by experimentation and transient 3D numerical simulation from the standpoint of mixed convective heat transfer science. This flow situation is the approximate adverse flow phenomena around an encapsulated PCM capsule which was placed in an upwardly directed heated flow.

Regin et al. [2] analyzed the behavior of a packed bed latent heat thermal energy storage system. The packed bed is composed of spherical capsules filled with paraffin wax for a solar water heating system. The effects of the inlet heat transfer fluid temperature (Stefan number), mass flow rate and phase change temperature range on the thermal performance of the capsules of various radii have been investigated numerically. The results indicate that for the proper modeling of performance of the system, the phase change temperature range of the PCM must be accurately known and should be taken into account. The melting process is chiefly governed by the magnitude of the Stefan number, phase change temperature range and the capsule radius.

The objective of this study is to elucidate the unconstrained melting process of a solid PCM in a spherical capsule using the stream function in which a solid sphere is moving in a stagnant viscid fluid. The melting process is analyzed by the Stefan number (*Ste*) and the capsule diameter (*D*) using the enthalpy method based on this model.

NOMENCLATURE

c_p	[J/(kg· K)]	Specific heat at constant pressure
D	[mm]	Outer diameter of the spherical capsule
d	[mm]	Inner diameter of the spherical capsule
F	[N]	Buoyancy force
Gr	[-]	Grashof number, $g\beta\Delta TD^3/\nu^2$
h	[J/m^3]	Enthalpy per unit volume
k	[W/(m· K)]	Thermal conductivity of PCM
L	[J/kg]	Latent heat of melting
MF	[%]	Fraction of the melt volume, liquid PCM volume/total PCM volume
P	[Pa]	Pressure
R	[mm]	Radius of curvature at the melting phase front as shown in Fig. 3
r	[m]	Radial direction, as shown in Fig. 1
S	[m]	Position of melting front
Ste	[-]	Stefan number, $c_p\Delta T/L$
Th	[mm]	Thickness of the capsule wall, $(D-d)/2$
t	[s]	Time
T	[K]	Temperature
T_m	[K]	Melting temperature of PCM
u	[mm/s]	Velocity in the melt layer in the θ-direction as shown in Fig. 2
u_{ave}	[mm/s]	Mean velocity in the melt layer
Greek symbols		
α	[m^2/s]	Thermal diffusivity
β	[1/K]	Coefficient of thermal expansion at constant pressure
θ	[rad]	Tangential direction as shown in Fig. 1
ΔT	[K]	Temperature difference, T_w - T_m
δ	[mm]	Thickness of the melt layer
ρ	[kg/m^3]	Density
ν	[m^2/s]	Kinetic viscosity of the heat transfer fluid
Ψ	[m^2/s]	Stream function
Subscripts		
i		Solid-liquid interface
l		Liquid PCM
s		Solid PCM
w		Wall

Melting analysis of a solid PCM

Enthalpy method

The model analysis shows the melting process from the beginning to the end and incorporates the sedimentation of the solid PCM in the liquid PCM and close contact melting. Among various methods, the enthalpy method is the most suitable method to solve the phase change problems in which the phase change takes place in a range of temperature. The major reason is that this method does not require explicit treatment of conditions on the phase change boundary [9]. A melting model of a solid PCM in a spherical capsule using the stream function in which a solid sphere is moving in a stagnant viscid fluid was proposed. **Figure 1** shows the spherical coordinate system used in this model analysis.

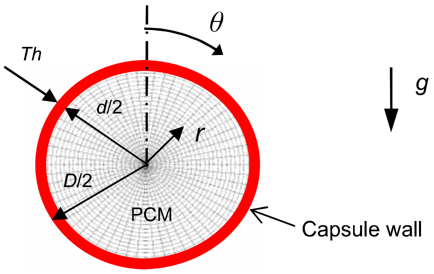


Figure 1 The spherical coordinate system.

Figure 2 shows a schematic sketch of close contact melting in a spherical capsule. The following assumptions are introduced in this melting analysis:

(i) The PCM is homogeneous and isotropic.
(ii) The capsule wall-temperature is uniform and constant.
(iii) The phase change process in the PCM is isothermal.
(iv) The heat transfer process in the PCM is dominated by conduction.
(v) The volumetric expansion due to melting and convection in the liquid phase is completely neglected.
(vi) The melting process is axisymmetrical, that is the derivative of all variables in the azimuthal direction is zero ($\frac{\partial}{\partial \Phi} = 0$).

Incorporating the foregoing assumptions, the conservation of energy in the spherical coordinate for the conduction dominated phase change in the PCM can be expressed in terms of enthalpy and temperature as:

$$\frac{\partial h}{\partial t} = k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)\right] \qquad (1)$$

In the above equation, the total volumetric enthalpy h is the sum of the sensible and latent heats of the PCM and is related to the temperature of the PCM as follows:

$$h = \int \rho c_p * dT = \int \rho c_p\, dT + \rho L \qquad (2)$$

The energy balance for the solid-liquid interface in the melting process is given in the following form:

$$k\left.\frac{\partial T}{\partial n}\right|_{solid} - k\left.\frac{\partial T}{\partial n}\right|_{liquid} = \rho L \frac{d S_n}{dt} \quad (3)$$

where S is the solid-liquid phase change interface, n the normal of the solid-liquid interface and L the latent heat.

Sedimentation model of a solid PCM

For unconstrained melting, the solid PCM exists near the bottom wall across the thin melt layer along the inner capsule wall due to gravity throughout the melting process. Sedimentation of the solid PCM in the liquid phase is presented by the following model.

(i) Calculate the buoyancy force, F, exerted on the solid PCM.

$$F = \int_V (\rho_{solid} - \rho_{liquid})gdV \quad (4)$$

(ii) Stokes approximation is used for obtaining the pressure distribution around the lower half of the sphere.

$$\mu \nabla^2 u = \nabla P \quad (5)$$

(iii) Stream function, in which a solid sphere is falling in a stagnant viscid fluid, is presented by the combination of the Stokeslet and the doublet as

$$\Psi = (A/r - Br)\sin^2\theta \quad (6)$$

where A and B are constants. A and B are determined by the boundary condition of the velocity in the tangential θ–direction at the inner capsule wall which is zero, and the mean velocity of u_{ave} in the thin melt layer of δ (δ << $D/2$) between the solid PCM and the capsule inner wall. δ is exaggeratedly depicted in comparison with $D/2$ as shown in Fig. 2.

(iv) The drag force in the gravitational direction is obtained by integrating the local normal stress (τ_{rr}) and the tangential stress ($\tau_{r\theta}$) around the sphere. Both stresses are obtained by adding the shear stresses which can be calculated using the stream function of equation (6) to the static pressure.

(v) u_{ave} in the melt layer is obtained by the force balance between the drag force exerted on the solid PCM and the buoyancy force during the sedimentation. Then the new melt layer of thickness δ' is obtained by calculating the exit volume flow rate Q $(= \pi (d^2/4 - r_i^2)\cdot u_{ave})$.

(vi) Solid PCM is forced to sink downwardly up to $r_i = d/2 - \delta'$. Then, the processes from (i) to (vi) are repeated until the complete melting of a solid PCM.

Furthermore, it is difficult to obtain the deformation of the melting phase front only by the analysis, and then we used the constant radius of curvature, R=40 mm, throughout the melting process at the melting front as shown in **Figure 3** which was formerly obtained by the experiment [7]. Therefore, we used the same upper melting configurations which kept the same curvature during melting as shown in the figure. That is, the solid PCM is sinking with the same radius of curvature at the upper melting phase front.

Numerical analysis

The unsteady term was discredited using second order accuracy of the Crank-Nicholson method. A uniform 250 (radial r-direction)×30 (tangential θ-direction) grid was used. The program was made by C code. The initial condition was set to the constant and uniform temperature at 25℃ with a sub-cooling of 3℃, and the boundary condition was set to the isothermal condition at 58℃ (Ste=0.27). The initial thickness of the melt layer along the bottom inner capsule wall of δ_0 was used from the result of Fomin and Saito [6]. The specific heat at constant pressure of c_p* in equation (2) was used by the differential scanning calorimeter (DSC) curve for melting the n-Octadecane [10]. The spherical capsule was made of plastic 2 mm thick with an outer diameter (D) of 100 mm. The thermal conductivity of the plastic was 0.15 W/(m· K). The analysis performed included the capsule wall thickness of 2 mm.

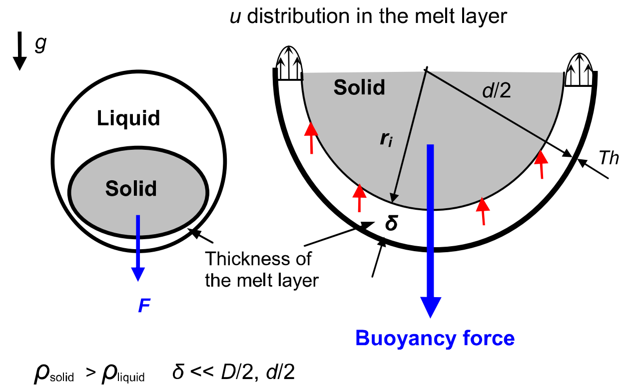


Figure 2 A schematic sketch of close-contact melting in a spherical capsule.

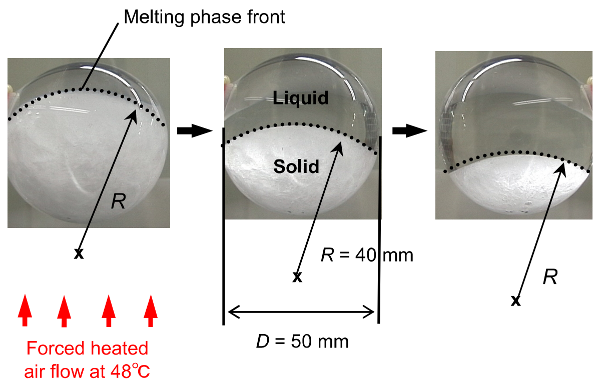


Figure 3 Visualization photos in the unconstrained melting process of a spherical capsule obtained by Koizumi [7].

Experiment

Experimental apparatus and procedure

The experiment was performed in a transparent plastic tank, filled with water at 58℃ (Gr=4×10^8). In order to maintain the water temperature at a certain level, an electric heater is used, and its power was adjusted. The spherical capsule was made of plastic 2 mm thick with an outer diameter (D) of 100 mm. At first the spherical capsule was filled with liquid PCM, and then the capsule was maintained in the constant temperature container in order to set the constant and uniform temperature of the PCM at about 25 ℃, which is the sub-cooling temperature of 3℃ below the melting temperature of PCM at 28.1℃. That is, the solid PCM occupied initially 85% of the enclosed space, having a flat top. The heat transfer fluid was place in stagnant water at about 58℃ of which is 30℃ higher than the PCM melting temperature of 28.1℃ (Ste=0.27). The inner diameter of the sphere was 96 mm and the inner volume of the capsule was 463 cm^3. The melting process was monitored throughout the melting process by video-camera. The phase change material mainly used in the experiment is n-Octadecane, and the material properties are presented in **Table 1**.

Table 1 Material properties of n-Octadecane.

Melting point	T_m = 28.1℃	
Latent heat of melting	L = 244 kJ/kg	
Density	ρ_s = 890 kg/m^3	ρ_l = 773 kg/m^3
Thermal conductivity	k_s = 0.3 W/(m· K)	k_l = 0.15 W/(m· K)
Specific heat	c_{ps} = 1.80 kJ/(kg· K)	c_{pl} = 2.18 kJ/(kg· K)

Convection velocity in the liquid PCM

There is little to clarify about the convection effects of the melting process experimentally, therefore we attempted to measure the local velocities in the liquid PCM following the flow path of the same tracer for a short period of time. The results of the experimental investigation, which included the visualization of the melting process, compare favourably with the analyzed results and thus serve to validate the analyzing approach. The calcium chloride hexahydrate $CaCl_2 \cdot 6H_2O$ [11] was used as a PCM for only the flow visualization experiment. (Melting temperature: 29℃, latent heat: 190.8 kJ/kg and density: 1562 kg/m^3 in the liquid phase at 32℃, 1802 kg/m^3 in the solid phase at 24℃.) The spherical capsule was made of 2 mm thick plastic with an outer diameter of 100 mm. The local velocity was obtained by tracking the path lines of the same tracer. Fine nylon fine thread of 1 mm long with an outer diameter of 0.5 mm was used as a tracer, and its density was almost the same as that of the liquid $CaCl_2 \cdot 6H_2O$. The heat transfer fluid was uniform upwardly directed heated air of which velocity was 0.3 m/s and temperature 58℃.

Results and discussion

PCM melting process

At first we checked the analytical accuracy using this model for comparison with the transient numerical results using Fluent 6.0 software [5]. **Figure 4** shows the relationship between the melt volume fraction, MF, and melting time for the capsule with an outer diameter of 80 mm. The solid line shows the result of this model analysis, and the dotted line the shows the numerical simulation from Assis et al. [5] which is an attempt to solve complete transient conservation equations simultaneously for solid PCM, liquid PCM, and air, while allowing for PCM expanding, convection in the fluid media (melted PCM and air), and solid phase motion in the liquid. The simulation conditions are as follows: The wall-temperature is uniform, the PCM is RT27 paraffin wax (a melting interval of 28-30℃), the latent heat is 179 kJ/kg, the solid density is 870 kg/m^3, the liquid is 760 kg/m^3, the spherical capsule has an outer diameter of 80 mm, and the wall thickness is 2 mm (thermal conductivity of glass is 0.81 W/(m·℃)), respectively.

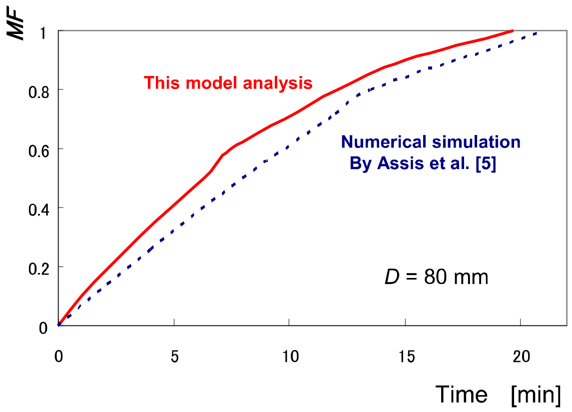


Figure 4 Relationship between MF and melting time.

Good agreement between the analysis and the numerical simulation was obtained. Considering the various effects such as the convection in the liquid phase and allowing for PCM expanding in the numerical simulation [5], it is indirectly confirmed that this simplified model analysis without convection effects in the liquid PCM could elucidate qualitatively the melting process from the beginning to the end.

Figure 5 shows the normal and tangential stresses around the lower half part of the sphere which was calculated from the Stokes approximation of equation (5). Drag force, which is balanced by the downwardly directed buoyancy force (F), was obtained by integrating the forces in the gravitational direction from the local normal stress τ_{rr} and the tangential stress $\tau_{r\theta}$ around the lower half of the sphere.

$$\tau_{rr} = \mu A \left[\frac{12}{(\frac{d}{2})^4} + \frac{6}{(\frac{d}{2})^2 (\frac{d}{2} - \delta)^2} \right] \cos\theta \qquad \tau_{r\theta} = -\mu A \frac{6}{(\frac{d}{2})^4} \sin\theta$$

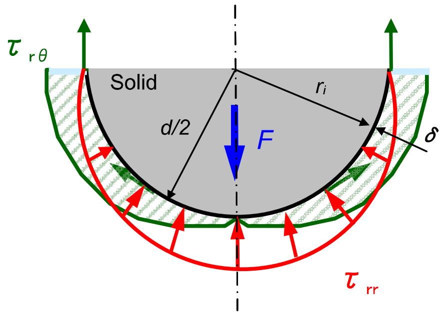
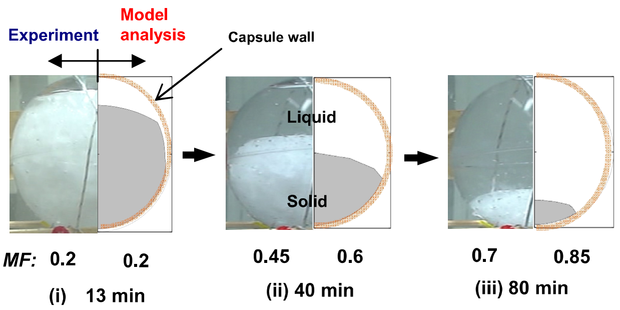


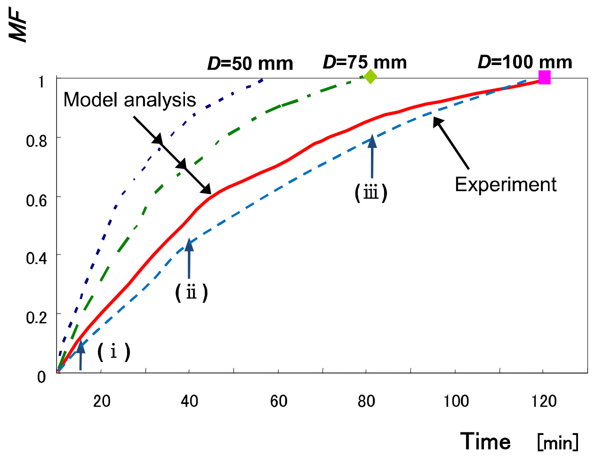
Figure 5 Normal and tangential stresses around the lower half of the sphere.



(b) Instantaneous visualization photos for *D*=100 mm capsule.

Figure 6 Melting performance for *Ste*=0.27.

Figure 6 shows the melting performances for *Ste*=0.27 (ΔT=30℃). Figure (a) shows the relationship between *MF* and the melting time for three different capsule sizes. The dotted line shows that for *D*=50 mm, the dotted and dashed line shows that for *D*=75 mm, and the solid line shows that for *D*=100 mm, respectively, which were obtained by the model analysis. In addition, the dashed line shows the result of *D*=100 mm which was obtained by the experiment. *MF* is defined as liquid PCM volume/total PCM volume, and *MF*=1 corresponds to the complete melting. Figure (b) shows the instantaneous visualizations for the *D*=100 mm capsule which corresponds to the three different representative melting instances as shown by the marks ↑ in Fig. (a). Figure (b-i) shows these visualizations at 13 minutes after the initiation of melting, Fig. (b-ii) shows them at 40 minutes, and Fig. (b-iii) shows them at 80 minutes, respectively. The photos on the left hand side of the sphere were obtained by the experiment, and those on the right hand side were obtained by the model analysis.



(a) Relationship between *MF* and melting time.

The complete melting times by the model analysis for three different capsule sizes are 53 minutes, 77 minutes and 124 minutes, respectively. These complete melting times for three the capsule sizes are in good agreement with those of the experimental result. That is, the melting time increases rapidly as the capsule diameter becomes larger. It is found that the characteristic during the melting process is almost captured by this conduction dominated model analysis, but *MF* by this model analysis is slightly larger than that of the experimental result. Also, the photos by the model analysis show the same earlier melting throughout the melting process.

Convection velocity in the liquid PCM

Figure 7 shows the flow visualization photos in the case of the *D*=100 mm capsule. Figure (a) shows the result at *MF*=0.25, Fig. (b) at *MF*=0.50, and Fig. (c) at *MF*=0.75, respectively.

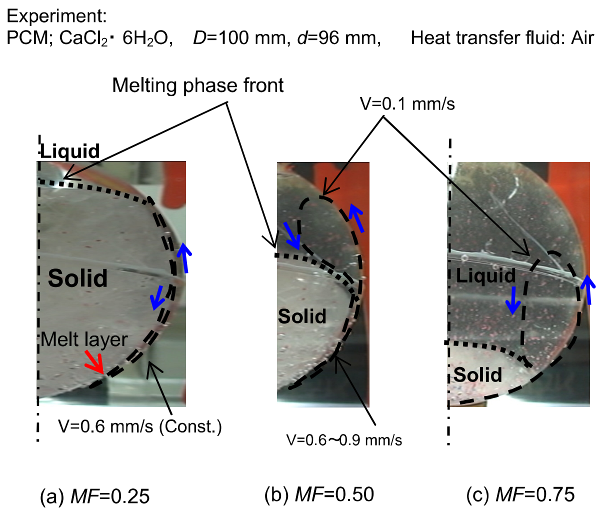


Figure 7 Path lines for representative three melting instances.

$CaCl_2 \cdot 6H_2O$ was used as a PCM only in this flow visualization experiment. It was found that the local velocity was very low from v=0.6 mm/s to 0.9 mm/s even in the thin melt layer as shown in Figs. (a) and (b). In particular, the velocity in the liquid PCM above the solid PCM was extremely low, i.e., v=0.1 mm, due to its stable liquid layer. Therefore, velocities in the liquid PCM including the melt layer was extremely for unconstrained melting in the spherical capsule. That is, the close contact melting is extremely significant for unconstrained melting compared to the convection effect in the liquid PCM. This result proves indirectly that the conduction dominated equation could thoroughly elucidate the melting process from the beginning to the end, and it incorporates the sedimentation of the solid PCM in the liquid PCM, and close contact melting.

CONCLUSION

A melting model of a solid PCM in a spherical capsule using the stream function in which a solid sphere is moving in a stagnant viscid fluid was proposed. The enthalpy method based on this model demonstrates the effect of solid PCM sedimentation throughout the melting process, and the appearance of close contact melting is crucially significant. Furthermore, good agreement for the melting process of a solid PCM was obtained between the analysis and the experiment.

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