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TURBOSHAFT ENGINES PERFORMANCE OPTIMIZATION USING MULTI-OBJECTIVE GENETIC ALGORITHM

Khorasani Nejad E.[1] * and Javadpour S.M.[2]
*Author for correspondence
Department of Mechanical Engineering,
Azad Islamic University Behbahan branch[1],
University of Sistan & Baluchestan[2],
Iran,
E-mail: Ehsan.kh1002@gmail.com
Javadpour_m@yahoo.com

ABSTRACT

In this paper multi-objective genetic algorithms are employed for Pareto approach optimization of ideal Turboshaft engines. In the multi-objective optimization a number of conflicting objective functions are to be optimized simultaneously. The important objective functions that have been considered for optimization are specific fuel consumption (S_P), output shaft power ($\dot{W}_{shaft}/\dot{m}_0$) and thermal efficiency (η_T). These objectives are usually conflicting with each other.

The design variables consist of thermodynamic parameters, Compressor pressure ratio (π_C), Turbine temperature ratio (x) and Mach number (M_0). In this paper, at the first stage, single objective optimization has been investigated and results have been used to compare of multi-objective optimization results.

In order to investigate the optimal thermodynamic behaviour of two objectives, different set, each including two objectives of output parameters, are considered individually. For each set Pareto front are depicted. The sets of selected decision variables based on this Pareto front, will cause the best possible combination of corresponding objective functions. There is no superiority for the points on the Pareto front figure, but they are superior to any other point.

INTRODUCTION

In most real-world problems, several goals must be satisfied simultaneously in order to obtain an optimal solution. The multiple objectives are typically conflicting and non-commensurable, and must be satisfied simultaneously. For example, we might want to be able to maximize the output shaft power of a Turboshaft engine while minimizing the fuel consumption. Actually, multi-objective optimization is very different than the single-objective optimization.

In single objective optimization, one attempts to obtain the best design or decision, which usually the global minimum or the global maximum depending on the optimization problem is that of minimization or maximization. In multiple objective optimization, there may not exist one solution which is best (global minimum or maximum) with respect to all objectives. In multi-objective optimization problem, there exist a set of solutions which are superior to the rest of solution in the search space when all objectives are considered but are inferior to other solution in the space in one or more objectives. These solutions are known as Pareto-optimal solutions or nondominated solutions. Since none of the solution in the nondominated set is absolutely better than any other, any one of them is an acceptable solution [1-5].

There are many methods to solve multi-objective problems. In this paper we use the Non-dominated Sorting Genetic Algorithm (NSGA-II). NSGA-II proposed in Srinivas and Deb [6].

In this paper, an optimal set of design variables in Turboshaft engines, namely, the input flight Mach number M_0, the pressure ratio of the compressor π_c, and the Turbine temperature ratio x are used by Pareto approach to multi-objective optimization. First, different pairs of conflicting objectives in an ideal Turboshaft engine are selected for optimization. Then, a new diversity preserving algorithm called ε-elimination diversity algorithm is used for enhancing the performance of NSGA-II in terms of diversity of population and Pareto fronts. The modified algorithm has been used for multi-objective optimization with more than two objectives by Atashkari et.al [7].

NOMENCLATURE

M_0	[-]	flight Mach number

T_0	[K]	Inlet temperature
γ	[-]	Ratio of specific heats
C_p	[kJ.kg^{-1}. K^{-1}]	Thermal conductivity
h_{PR}	[kJ.kg^{-1}]	Heating value
T_{t4}	[K]	Burner exit total temperature
π_C	[-]	Compressor pressure ratio
x	[-]	Turbine temperature ratio
$\dot{W}_{shaft}/\dot{m}_0$	[kW. kg^{-1}.sec]	Output shaft power
f	[-]	Fuel/air ratio
S_P	[mg. kW^{-1}.sec^{-1}]	Specific fuel consumption
η_T	[-]	Thermal efficiency
C_{shaft}	[-]	Work output coefficient
$F(X)$	[-]	Vector of objective functions
X^*	[-]	Vector of optimal design variables
P^*	[-]	Pareto set (set of decision variables)
Pf^*	[-]	Pareto front (set of objective function)

MULTI-OBJECTIVE OPTIMIZATION

Multi-objective optimization, which is also called multicriteria optimization or vector optimization, has been defined as finding a vector of decision variables satisfying constraints to give acceptable values to all objective functions [7, 8]. In general, it can be mathematically defined as: find the vector $X^* = [x_1^*, x_2^*, \ldots, x_n^*]^T$ to optimize

$$F(X) = [f_1(x), f_2(x), \ldots, f_k(x)]^T \quad (1)$$

Subject to m inequality constraints

$$g_i(X) \le 0, \quad i = 1, \ldots, m \quad (2)$$

and p equality constraints

$$h_j(X) = 0, \quad i = 1, \ldots, p \quad (3)$$

Where $X^* \in \Re^n$ is the vector of decision or design variables, and $F(X) \in \Re^k$ is the vector of objective functions, which must each be either minimized or maximized. However, without loss of generality, it is assumed that all objective functions are to be minimized. Such multi-objective minimization based on Pareto approach can be conducted using some definitions:

- Definition of Pareto dominance

A vector $U = [u_1, u_2, \ldots, u_k] \in \Re^k$ is dominant to vector $V = [v_1, v_2, \ldots, v_k] \in \Re^k$ (denoted by $U < V$) if and only if $\forall i \in \{1, 2, \ldots, k\}$, $u_i \le v_i \wedge \exists j \in \{1, 2, \ldots, k\} : u_j \le v_j$. In other words, there is at least one u_j which is smaller than v_j whilst the remaining u's is either smaller or equal to corresponding v's.

- Definition of Pareto optimality

A point $X^* \in \Omega$ (Ω is a feasible region in $\Re^n$ satisfying Equations (2) and (3)) is said to be Pareto optimal (minimal) with respect to all $X \in \Omega$ if and only if $F(X^*) < F(X)$. Alternatively, it can be readily restated as $\forall i \in \{1, 2, \ldots, k\}$, $\forall X \in \Omega - \{X^*\}$ $f_i(X^*) \le f_i(X) \wedge \exists j \in \{1, 2, \ldots, k\} : f_i(X^*) < f_i(X)$. In other words, the solution X^* is said to be Pareto optimal (minimal) if no other solution can be found to dominate X^* using the definition of Pareto dominance.

-Definition of a Pareto set

For a given MOP, a Pareto set P^* is a set in the decision variable space consisting of all the Pareto optimal vectors $P^* = \{X \in \Omega \mid \nexists X' \in \Omega : F(X') < F(X)\}$. In other words, there is no other X' as a vector of decision variables in Ω that dominates any $X \in P^*$.

- Definition of a Pareto front

For a given MOP, the Pareto front Pf^* is a set of vector of objective functions which are obtained using the vectors of decision variables in the Pareto set P^*, that is $Pf^* = \{(f_1(X), f_2(X), \ldots, f_k(X)) : X \in P^*\}$. In other words, the Pareto front Pf^* is a set of the vectors of objective functions mapped from P^*.

Different algorithms have been widely used for multiobjevtive optimization because of their natural properties suited for these types of problems. The NSGA-II is one of these algorithms. In order to show this algorithm more clearly, some basics of NSGA-II are represented. In Fig. 1 demonstrated now selects individuals from the entire population R_t to construct the next parent population R_{t-1}. The entire population R_t is simply the current parent population P_t plus its offspring population Q_t which is created from the parent population P_t by using usual genetic operators. The selection is based on non-dominated sorting procedure which is used to classify the entire population R_t according to increasing order of dominance [7].

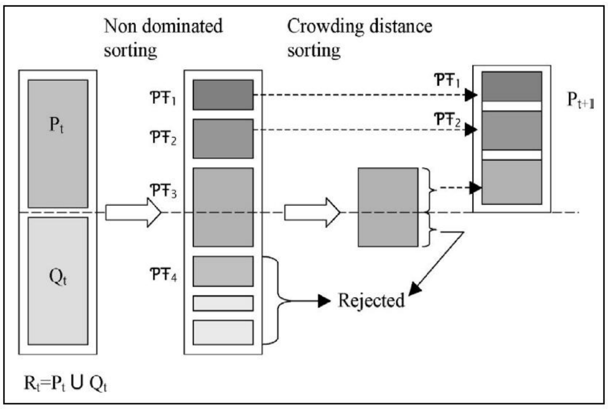

Figure 1 Basics of NSGA-II procedure [7]

Thus, the best Pareto fronts from the top of the sorted list is chosen to create the new parent population P_{t+1} which is half the size of the entire population R_t. So, it should be noted that all the individuals of a certain front cannot be modified in the new parent population because of space, as shown in Fig. 1. To choose an exact number of individuals of that particular front, a crowded comparison operator is used in NSGA-II to find the best solutions to complete the new parent population. The crowded comparison procedure is based on density estimation of solutions surrounding a particular solution in a population or front. So, the solutions of a Pareto front are first sorted in each objective direction in the ascending order of that objective value. The crowding distance is then assigned equal to the half of the perimeter of the enclosing hyper box. Other objectives are sorted too and the overall crowding distance is calculated as the sum of the crowding distances from all objectives. The less crowded non-dominated individuals of that particular Pareto front are then selected to fill the new parent population. It is important to know that in a two-objective Pareto optimization, if the solutions of a Pareto front are sorted in a decreasing order of importance to one objective, these solutions are then automatically ordered in an increasing order of importance to the second objective. In other words, the hyper-boxes surrounding an individual solution remain unchanged in the objective-wise sorting procedure of the crowding distance of NSGA-II in the two-objective Pareto optimization problem. However, in multi-objective Pareto optimization problem with more than two objectives, such sorting procedure of individuals based on each objective in this algorithm will cause different enclosing hyper boxes. Therefore, the overall crowding distance of an individual computed in this way may not exactly reflect the true measure of diversity or crowding property for the multi-objective Pareto optimization problems with more than two objectives.

In reference [7], a new method is presented which modifies NSGA-II so that it can be safely used for any number of objective functions (particularly for more than two objectives). The modified method is then used for a two objective thermodynamic optimization of Turboshaft engines and the results are compared with those of the original NSGA-II.

THE ε - ELIMINATION DIVERSITY ALGORITHM

In the ε-elimination diversity approach that is used to main loop in NSGA-II, all the clones and/or ε-similar individuals based on Euclidean norm of two vectors are recognized and simply eliminated from the current population. Therefore, based on a pre-defined value of ε as the elimination threshold ($\varepsilon = 0.001$ has been used in this paper) all the individuals in a front within this limit of a particular individual are eliminated. It should be noted that such ε-similarity must exist both in the space of objectives and in the space of the associated design variables. This will ensure that very different individuals in the space of design variables having ε-similarity in the space of objectives will not be eliminated\ from the population. The pseudo-code of the ε-elimination approach is depicted in Figure 2 Evidently, the clones or ε- similar individuals are replaced from the population with the same number of new randomly generated individuals[7].

```
Pseudo-code of e-elimination
ε-elim=ε-elimination (pop)          //pop includes design variables and
                                      objective functions.
define ε                            //Define elimination threshold.
get k (k = 1 for the first front)   //Front No.
i = l
until i + 1 < pop_size
        j = i + 1
        until j < pop_size
        IF {‖F(X(i)), F(X(j))‖ < ε ∧ ‖X(i), X(j)‖ < ε}
            F(X(i)), F(X(j)) ∈ PF*_k   X(i), X(j) ∈ P*_k
        THEN pop = pop\pop(j)   //Remove the ε-similar individual.
r_new_ind = make_new_random_individual
                                    //Generate new random individual.
pop = pop ∪ r_new_ind               //Add the newly generated individual.
        end
end
```

Figure 2 Pseudo-code of ε-elimination for preserving genetic diversity [7]

MULTI-OBJECTIVE THERMODYNAMIC OPTIMIZATION OF TURBOSHAFT

The Turboshaft engine is similar to the Turboprop except that power is supplied to a shaft rather than a propeller. The Turboshaft engine is used quite extensively for supplying power for helicopters [9]. We saw that the addition of regenerator to the Brayton engine cycle increased the cycle`s thermal efficiency when the compressor exit temperature (station 3) was below the turbine exit temperature (station 5). For analysis, we consider an ideal Turboshaft engine with regeneration, as show in Figures 3 and 4. The high temperature/low-pressure gas enters the regeneration at station 5 and deports as station 6.

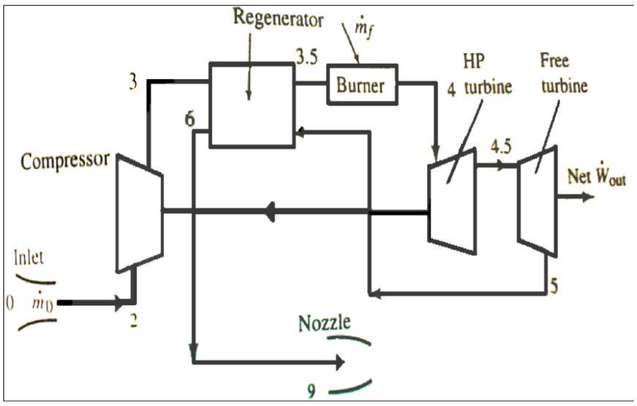

Figure 3 stations numbering of Turboshaft engine with regeneration

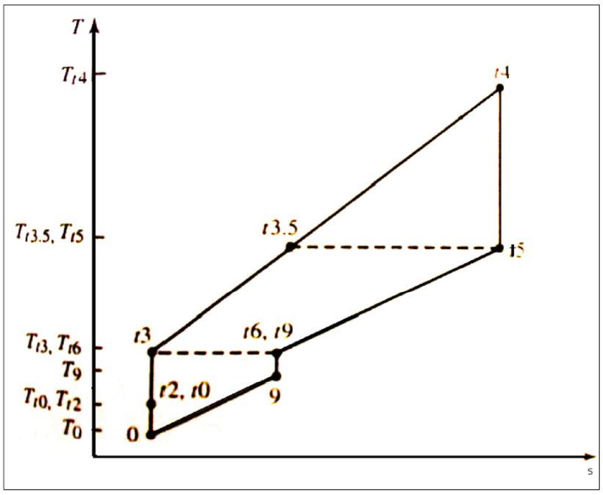

Figure 4 The T-S diagram of ideal Turboshaft engine with regeneration

A detailed description of the thermodynamic analysis and equations [8] of ideal Turboshaft engines is given in Appendix A. This elementary thermodynamic model is sufficient to capture the principles of behavior and interactions among different input and output parameters in a multi-objective optimal sense. The input parameters involved in such thermodynamic analysis in an ideal Turboshaft engine given in Appendix A are Compressor pressure ratio (π_C), Turbine temperature ratio (x) and Mach number (M_0). The output parameters involved in the thermodynamic analysis in the ideal Turboshaft engine given in Appendix A are, output shaft power ($\dot{W}_{shaft} / \dot{m}_0$), specific fuel consumption (S_p) and thermal efficiency (η_T). However, in the multi-objective optimization study, some input parameters are already known or assumed as, $T_0 = 290K$, $\gamma = 1.4$, $C_P = 1.004 kJ.kg^{-1}.K^{-1}$, $h_{PR} = 42800$ kJ·kg^{-1}, and $T_{t4} = 1600K$. The input flight Mach number $0 < M_0 < 0.4$, Turbine temperature ratio $1.01 < X < 1.09$ and the compressor pressure ratio $2 < \pi_C < 30$ are considered as design variables to be optimally found based on multi-objective optimization of 3 output parameters, namely, S_P, η_T and $\dot{W}_{shaft} / \dot{m}_0$. In order to investigate the optimal thermodynamic behavior of Turboshaft engines,2 different sets, each including two objectives of the output parameters, are considered individually. Such pairs of objectives to be optimized separately have been chosen as ($\dot{W}_{shaft} / \dot{m}_0$, η_T) and ($\dot{W}_{shaft} / \dot{m}_0$, S_P), evidently, it can be observed that $\dot{W}_{shaft} / \dot{m}_0$, η_T are maximized whilst S_P is minimized in those sets of objective functions. A population size of 40 has been chosen in runs with crossover probability Pc and mutation probability Pm as 0.75 and 0.7, respectively for single-objective optimization and a population size of 120 has been chosen in runs with crossover probability Pc and mutation probability Pm as 0.94 and 0.1 respectively for two-objective optimization.

The results of the single-objective optimizations are summarized in Table 1.

$2 < \pi_C < 30$, $0 < M_0 < 0.4$, $1.01 < X < 1.09$	
$\frac{\dot{W}_{shaft}}{\dot{m}_0} = 532.2548$ $\pi_C = 18.1008$ $X = 1.01$ $M_0 = 0.4$	
$S_p = 64.9558$ $\pi_C = 16.0864$ $X = 1.01$ $M_0 = 0.4$	$\eta_T = 79.2209$ $\pi_C = 2$ $X = 1.01$ $M_0 = 0.4$

Table 1 Values of decision variables and objective functions

Some Pareto fronts of each pair of two objectives have been shown through Figures 5-6.

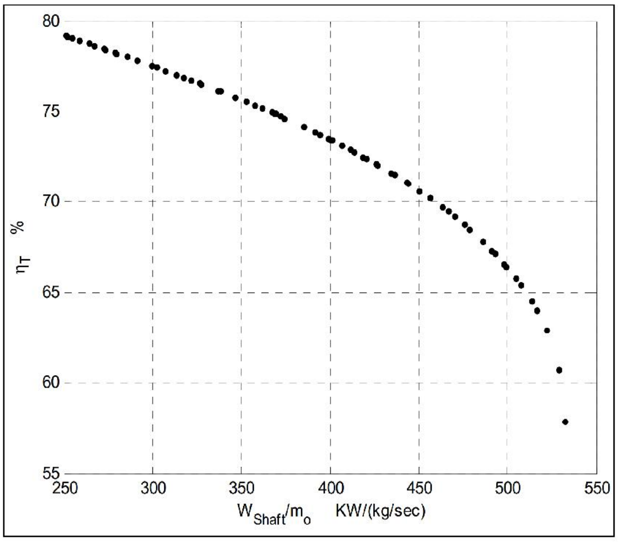

Figure 5 Pareto front of thermal efficiency and output shaft power in 2-objective optimization

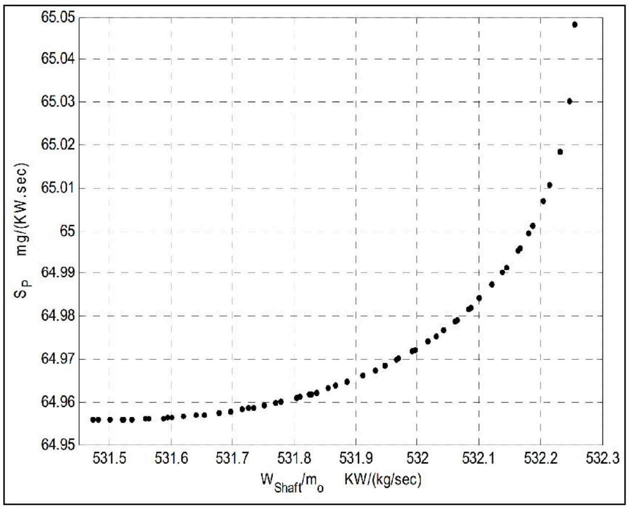

Figure 6 Pareto front of specific fuel consumption and output shaft power in 2-objective optimization

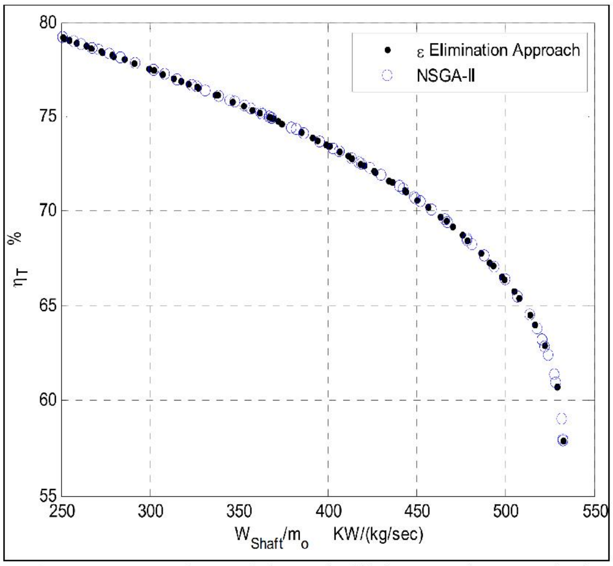

Figure 7 Pareto front of thermal efficiency and output shaft power in 2-objective optimization

These figures and the associated values of the decision variables and the objective functions given in Table 1 simply cover all the 3 objectives studied in the two-objective Pareto optimization. The first and the end points of this diagrams that is explanatory extremum points at single-objective optimization are compared with the results given in Table 1. The result of this comparison indicates the similar conformity.

Figure 5 shows variation of output shaft power and thermal efficiency. Interval variations are (250.8, 532.25) and (57.91, 79.21) for output shaft power and thermal efficiency, respectively. The initial and the end of values of this diagram are very similar to the optimal values of single-objective condition. If the designer wants to have more output shaft power for the engine, he should use the higher points of this diagram. And he wants to have more thermal efficiency he should use the lower points of this diagram. Also as it is shown the thermal efficiency decreased drastically, if the output shaft power goes higher than 450 kW/(kg/Sec). Therefore choosing these points in a very good design can be avoid.

Figure 6 shows variation of output shaft power and specific fuel consumption. Interval variations are (531.45, 532.25) and (64.955, 65.048) for output shaft power and specific fuel consumption, respectively. At this diagram by attention to characteristic problem designer can be determined optimal point. At single-objective condition (Table 1) minimum point of specific fuel consumption and maximum point of output shaft power are 64.9558 and 532.2548, respectively, that this points is closer to the initial and the end points of this diagram.

Figure 7 depicts comparison of approach NSGA-II with elimination approach. As seen elimination approach is smoother than other one.

CONCLUSION

In the single objective optimization an objective function was investigated by changing several design variables, simultaneously. The correlation between the optimal point and the objective function and design variable are obtained. In the two-objective optimization, the comparison of the first and the end points of Pareto curvature with the result of single-objective show the compatibility with these diagrams.

APPENDIX A. THERMODYNAMIC MODEL OF IDEAL TURBOSHAFT ENGINE

Assumptions: Inlet diffuser, compressor, turbine and exit nozzle, all operate isentropically.

No pressure loss in the burner. f = (fuel/air)<<1, P_e (Turboshaft exit pressure) = P_o (ambient pressure), C_P = 1.004 $(kJ.kg^{-1}.K^{-1})$, $T_0 = 290\,K$, γ = 1.4, $h_{PR} = 42800\ kJ.kg^{-1}$, $T_{t4} = 1600\,K$.

Input parameters: M_0, $T_0\,(K)$, γ, $C_p(kJ.kg^{-1}.K^{-1})$, X, $h_{PR}\,(kJ.kg^{-1})$, $T_{t4}(K)$, π_C

Output parameters: $\dot{W}_{shaft}/\dot{m}_0(kW.kg^{-1}.\sec)$, $S_P(mg.kW^{-1}.\sec^{-1})$, f, η_T, C_{shaft}.

Equation:

$$\tau_r = 1 + \frac{\gamma - 1}{2} M_0^2 \qquad (A.1)$$

$$\tau_\lambda = \frac{T_{t4}}{T_0} \qquad (A.2)$$

$$\tau_C = \pi_C^{(\gamma-1)/\gamma} \qquad (A.3)$$

$$C_{shaft} = \tau_\lambda \left(1 - \frac{X}{\tau_r \tau_C}\right) - \tau_r (\tau_C - 1) \quad (A.4)$$

$$\frac{\dot{W}_{shaft}}{\dot{m}_0} = C_P T_0 C_{shaft} \quad (A.5)$$

$$f = \frac{C_P T_0 \tau_\lambda}{h_{PR}} \left(1 - \frac{X}{\tau_r \tau_C}\right) \quad (A.6)$$

$$S_P = \frac{f}{\dot{W}_{shaft} / \dot{m}_0} \quad (A.7)$$

$$\eta_T = 1 - \frac{\tau_r (\tau_C - 1)}{\tau_\lambda \left[1 - X / (\tau_r \tau_C)\right]} \quad (A.8)$$

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