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CONDENSATION MODELING OF THE UNSTEADY PROCESSES IN TWO PHASE GAS-WATER DROPLETS SYSTEMS

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ABSTRACT

A thermal state variation of water droplets carried by a gas flow during unsteady phase transformations at their surface, with a purpose to explore optimum conditions for utilization of phase transformation heat accumulated in humid flue gas has been modeled. The importance of the case of phase transformations of a “single droplet”, during conductive heating, for a comparative evaluation of warming droplet heating in different heat and mass transfer condition conditions, has been shown. It has been confirmed that the thermal state variation of droplets presented in the time scale expressed in Fourier number is not dependant on water spray dispersity if droplets are heated conductively. Intensity and duration of the process of water vapor condensation on droplet surface is significantly influenced by water temperature and spray dispersity as well as flue gas temperature and its humidity. The radiation flow absorbed by droplets depends on water spray dispersity. It determines formation of a negative gradient temperature field and can shorten duration of the condensation mode.

INTRODUCTION

Humid flue gas is generated in the fuel combustion process. Water vapor is generated as the moisture content of fuel and air evaporates and hydrogen contained in fuel burns. It is a natural phenomenon related to the fuel combustion process. Beside it, humidity can be accumulated in flue gas artificially, when water vapor is used in liquid fuel spray technologies, and when water is sprayed into a furnace due to ecological considerations aiming to reduce nitrogen oxide emissions into the atmosphere using a regulation of a combustion torch temperature. The more humid flue gas is, the more significant part of its enthalpy is constituted by the component related to water phase transformations that can be exerted from flue gas in the water vapor condensation process only. The difficulty is caused by the fact that the condensation process is possible only in a medium with temperature lower than the dew point temperature that is also favorable for corrosion processes to occur. Also, the problem of the use of utilized low potential heat occurs. Recently, despite of the mentioned problems, an increasing attention is given for the utilization of heat accumulated in flue gas. This is related to the continually rising costs of heat energy and ecological considerations. Traditionally, recuperative condensation-type heat exchangers are installed in heat generation systems, where feed water is heated using flue gas. Due to the expensiveness of the condensation unit, its installation is reasonable only in high capacity heat generation systems, and it is practically unaffordable for a small boiler plant network. Extensive opportunities can be opened by contact heat exchangers based on direct spraying of water into flue gas. A number of their advantages can be identified, such as the intensity of heat and mass transfer in the two-phase flow of water spray and flue gas flow, relatively low costs of a heat generation system reconstruction, and possibility for pollutant removing from flue gas. Some emerging problems must also be mentioned, such as possible water contamination during a direct contact with flue gas and the necessity of proper recognition of the complex heat and mass transfer in a droplet system required for optimal management of the water spray heating process during steam condensation on droplet surface.

NOMENCLATURE

a	[m^2/s]	Thermal diffusivity
B_T	[–]	Spalding heat transfer number
c_p	[J/kg·K	Mass specific heat
D	[m^2/s]	Mass diffusivity
Fo	[–]	Fourier number
H	[–]	Optical thickness
I_ω	[W/m·ster	Spectral intensity of radiation
$I_{0\omega}$	[W/m·ster	Spectral intensity of blackbody radiation
L	[J/kg]	Latent heat of evaporation
m	[kg/(m^{2}s]	Mass flux density
M	[kg]	Mass
n	[–]	Number of the term in infinite sum
q	[W/m^2]	Heat flux density
p	[Pa]	Pressure
r	[m]	Radius of a droplet
R_μ	[J/(mol·K]	Universal gas constant
r	[m]	Coordinate
Nu	[–]	Nusselt number
T	[K]	Temperature

Special characters

γ	[rad]	Angle, estimating the peculiarities of spherical geometry, when calculating radiation
χ_ω	m^2	Spectral radiation absorption
$\eta = r / R$		Non-dimensional coordinate
λ	[W/m·K]	Thermal conductivity
$\eta = r / R$	[kg/mol·K]	Molecular mass
ρ	[kg/m^3]	Density
τ	[s]	Time
ω	[m^{-1}]	Wave number

Subscripts

c	Convective
e	Equilibrium evaporation
f	Phase transformation
g	Gas
k	Conductive
ko	Condensation
m	Mass average
r	Radiation
R	Droplet surface
l	Liquid
v	Vapor
vg	Vapor-gas mixture
ω	Spectral
Σ	Total
0	Initial state
∞	Far from droplet

Superscripts

+	External side of a droplet surface
–	Internal side of a droplet surface
‘	Variable

A condensation process on the surface of water droplets sprayed into a humid flue gas flow is investigated in this work.

MODELING METHODOLOGY OF PHASE TRANSFORMATIONS AT THE SURFACE OF HEATING DROPLETS

Theoretical investigation methods are preferred in investigations of unsteady heat and mass transfer in water spray two-phase systems [1-8], and others. They are designed on the basis of the "droplet" problem. The classical "droplet" problem consists of calculation of heat and mass transfer intensity in a two-phase gas-droplet flow. It integrates an internal and an external problem. Solutions of the internal problem are: the unsteady temperature field, $T_l(r,\tau)$, in the droplet, and density of the heat flux $\vec{q}_\Sigma(R^-,\tau) = \vec{q}_c(R^-,\tau) + \vec{q}_r(R^-,\tau)$ heating the liquid. Solutions of the external problem are: density of the heat flux supplied to the droplet $\vec{q}_\Sigma(R^+,\tau) = \vec{q}_c(R^+,\tau) + \vec{q}_r(R^+,\tau)$, density of the liquid vapor flux $m_v^+(\tau)$ at the droplet surface, and gas flow parameters, among which $T_g(\tau)$ and $p_{v,\infty}(\tau)$ are the most important. When liquid is sprayed into gas, droplets warm and evaporate, simultaneously, the gas mixture is cooling and its composition is changing. Consequently, variation of the state of a two-phase flow is determined by a sum of interrelated factors. In order to explore their individual impact, effects of each of them have to be investigated in boundary cases, when other factors can be neglected. This work argues that such volume of water is sprayed into a humid flue gas flow, heating and evaporation of which has no influence on gas parameters. This corresponds to the "single droplet" task conditions.

When the intensity of droplet convective heating is calculated using similarity theory methods, and when heat transfer in a droplet is described using a conduction and radiation model, the "single droplet" heat and mass transfer problem can be described by a system of nonlinear equations [1, 6]:

$$\frac{\partial M_l(\tau)}{\partial \tau} = -4\pi \cdot R^2(\tau) \cdot m_v^+(\tau) \tag{1}$$

$$\begin{gathered} m_v^+(\tau) = \frac{D_{vg}(\tau)\mu_v}{T_R(\tau)R(\tau)R_\mu}\left\{p_{v,R}(\tau) - p_{v,\infty}(\tau) + \frac{\mu_v}{\mu_{vg}} \times \right. \\ \left. \times\left[p\ln\frac{p - p_{v,\infty}(\tau)}{p - p_{v,R}(\tau)} - p_{v,R}(\tau) + p_{v,\infty}(\tau)\right]\right\}, \end{gathered} \tag{2}$$

$$q_f^+(\tau) = m_v^+(\tau) \cdot L(\tau) \tag{3}$$

$$q_c^+(\tau) = \frac{\left[2 + F_T^{-1} \cdot (Nu - 2)\right] \cdot B_T^{-1} \cdot \ln(1 + B_T) \cdot \lambda_{vg}(\tau)}{2R(\tau)} \cdot \left[T_g - T_R(\tau)\right] \cdot f(B_T) \tag{4}$$

$$F_T = (1 + B_T)^{0,7} B_T^{-1} \cdot \ln(1 + B_T) \tag{5}$$

$$B_T(\tau) = \frac{c_{p,vg}(\tau) \cdot \left[T_g - T_R(\tau)\right]}{L(\tau)} \cdot \left[1 + \frac{q_c^-(\tau)}{q_c^+(\tau)}\right] \tag{6}$$

$$q_c^-(\tau) = -\lambda_l(\tau) \cdot \left.\frac{\partial T(r,\tau)}{\partial r}\right|_{r \equiv R^-} \tag{7}$$

$$\begin{gathered} \left.\frac{\partial T(r,\tau)}{\partial r}\right|_{r \equiv R^-} = \sum_{n=1}^{\infty}\left\{(-1)^n \cdot \frac{2\pi n}{R} \cdot \int_0^\tau (-1)^n \frac{R}{n\pi}\frac{dT_R}{d\tau'} + \right. \\ + \frac{1}{R\rho_l c_{p.l}}\int_0^R q_r(r',\tau')\left[\sin(n\pi\eta) - n\pi\eta \cdot \cos(n\pi\eta)\right]dr' \times \\ \times \exp\left[-a_l(\tau')\left(\frac{n\pi}{R}\right)^2(\tau - \tau')\right]d\tau', \end{gathered} \tag{8}$$

$$\begin{gathered} q_r(r) = 2\pi\int_0^\infty\int_0^{\pi/2}\cos\gamma\sin\gamma\left[I_\omega(R,\gamma)\exp\left(-H_r^R\right) + \right. \\ + \int_r^R n_\omega^2 I_{0\omega}\exp\left(-H_r^{r'}\right)dH_r^{r'} - I_\omega(R,\gamma)\exp\left(-H_{r\sin\gamma}^R - H_r^r\right) - \\ - \int_{r\sin\gamma}^r n_\omega^2 I_{0\omega}\exp\left(-H_{r'}^r\right)dH_{r'}^r - \\ \left. - \int_{r\sin\gamma}^r n_\omega^2 I_{0\omega}\exp\left(-H_{r\sin\gamma}^r - H_{r\sin\gamma}^{r'}\right)dH_r^{r'}\right]d\gamma d\omega, \end{gathered} \tag{9}$$

$$H_{r_1}^{r_2} = \int_{r_1}^{r_2} dH_{r_1}^{r_2} = \int_{r_1}^{r_2}\frac{\chi_\omega dr'}{\sqrt{1 - (r/r')^2\sin^2\gamma}} \tag{10}$$

A requirement of the equality of the energy fluxes taken to the surface and taken away from it is posed for droplet surface temperature variation:

$$\vec{q}_{\Sigma}^{\,+}(\tau)+\vec{q}_{\Sigma}^{\,-}(\tau)-\vec{q}_{f}^{\,+}(\tau)=0 \qquad (11)$$

Equation (1) describes droplet mass variation due to phase transformations on its surface. Equation (2) describes vapor flow density at the droplet surface, considering the influence Stefan hydrodynamic flow according to the model of Kuzikovskij [8]. Equation (3) describes the intensity of heat flux at the droplet surface related to phase transformations. Expressions (4, 5) describe the convective heating intensity of the droplet according to the Abramzon-Sirignano model [7]. The modified expression of Spalding heat transfer number (6) enables consideration of the influence of Stefan hydrodynamic flow on the convective heating intensity of an evaporating droplet in the unsteady phase transformation mode [6]. For droplets carried by the gas flow without slip, Nu=2. Equation (7) is an expression of Fourier's law of heat conduction, in which the unsteady temperature field gradient is described by the integral equation (8), when heat is transferred in a semitransparent droplet by conduction and radiation [1]. Local density of the radiation flux is described by equation (9), which has been constituted considering light-ray propagation peculiarities in a spherical volume, and using symbolic notations of optical thickness (10) [9], when symbols r_1 and r_2 correspond to optical density limits in the expression (9).

The presented system of equations (1-11) is determined unambiguously if the temperature $T_R(\tau)$ function meeting the condition (11) is known. As it can be observed that this function must be known in order to solve the external (1-6) as well as the internal (7-10) "droplet" problem, the necessity of an iterative scheme for the solution of the equation system (1-11) becomes evident. The iterative scheme for the equation system (1-11) is designed projecting J control sections in a droplet $\eta_J \equiv \sum_{j=2}^{J}\left(\eta_j-\eta_{j-1}\right)=1$ and distinguishing I control time moments $\tau_I \equiv \sum_{i=2}^{I}\left(\tau_i-\tau_{i-1}\right)$.

When the function $T_R(\tau)$ is calculated in the *it* iteration, corresponding values calculated in the previous iteration are attributed to droplet heat and mass transfer parameters that are required but unknown, and these values are corrected in subsequent iterations. Surface temperature of the heating droplet is selected using the method of the fastest descent, requiring for the confidence of condition (11) not lower than one tenth of one percent.

When local density of the radiation flux in the droplet is calculated, the radiation spectrum is separated equally according to the wave number $1{,}25 \cdot 10^6 \div \omega_2$ and an integration is performed using the rectangular method. The limit of the radiation spectrum interval ω_2 corresponds to the droplet diameter. Integration according to the angle γ is performed using the Gauss method. Reflection of the light ray on the water surface and Brewster angle are considered, and the influence of the warming droplet temperature on the complex refraction index for water is evaluated [10, 11].

RESULTS

Unsteady phase transformations at the droplet surface, when they warm without slipping carried by a gas flow with temperature T_g, have been modeled. It is argued that warming of sprayed water and phase transformations don't have a significant impact on the thermal state of flue gas, therefore $T_g(\tau)\approx const$ and $p_{v,\infty}(\tau)\approx const$. The imbalance of energy fluxes at the surface of evaporating liquids is controlled during the numerical experiment (Figure 1). In the selection of an instant value of the droplet surface temperature using the fastest descent method, the temperature variation in the range of one-thousandth of a degree was considered. Then, the balance of energy fluxes is secured at a confidence not less than 0.05% during the numerical investigation. Imbalance fluctuation is related to the change of the phase transformation mode, when the vapor flux density module reaches its zero value and the direction of its vector changes as vapor condensation at the droplet surface is replaced by liquid evaporation. The control of the energy flux imbalance at the droplet surface was performed in all cases of phase transformations.

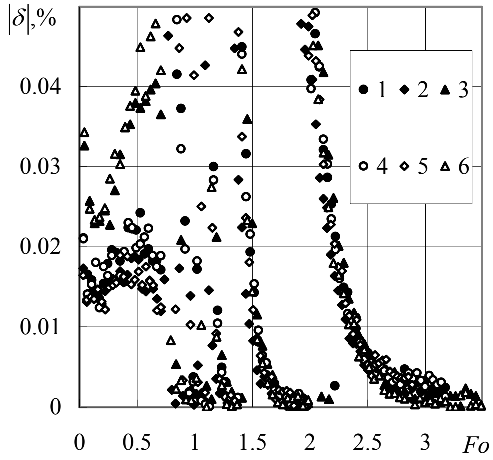

Figure 1 Imbalance control of energy fluxes at the droplet surface during the numerical investigation. Droplet heating mode: (1-3) conduction, (4-6) conduction and radiation. Droplet radius $R_0 \cdot 10^6$, m: (1, 4) 25, (2, 5) 50, (3, 6) 100; T_g =500K; $p_{v,\infty}/p$ =.1; $T_{l,0}$ =278K.

Water droplets intensively warm during unsteady phase transformations. When their surface temperature finally reaches dew point temperature, water starts to evaporate, and flue gas starts to humidify. Therefore, water heating in a contact heat exchanger must be organized in such way that wouldn't allow droplet warming to dew point temperature. With this purpose duration of droplet and flue gas contact must be controlled. The problem at this point is the fact that the duration is different for droplets of different sizes and it is determined by heat and mass process conditions. In order to evaluate the influence of the mentioned factors on variation of the thermal state of droplets in the condensation mode of phase transformations, a consistent systemically comparative analysis of transfer process interaction is required. A completed comprehensive investigation of the simplest heat and mass transfer of the droplet is needed. Then, a comparative analysis can be performed, consistently changing boundary heat and mass transfer conditions in the direction of their complexity, considering new factors, simultaneously evaluating their individual influence thoroughly. The case of the simplest heat and mass transfer is when droplets are heated conductively.

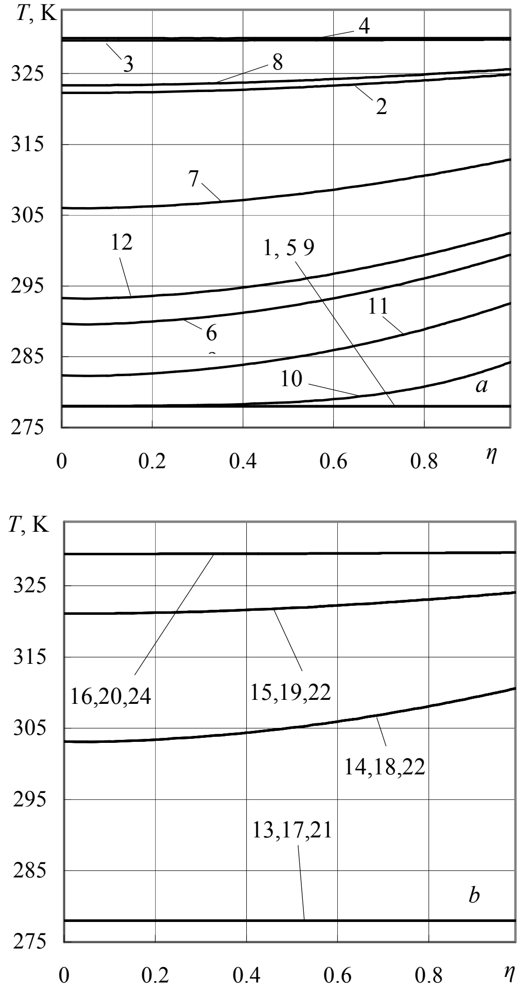

Figure 2 Water droplet warming in real time (a) and Fourier number (b) scales. $R_0 \cdot 10^{-6}$, m: (1-4, 13-16) 25 (5-8, 17-20) 50, (9-12, 21-24) 100; Warming time τ, s: (1, 5, 9) 0, (2, 6, 10) .005, (3, 7, 11) .01, (4, 8, 12) .02; Fourier number $Fo = \left(a_0 / R_0^2\right) \cdot \tau$: (13, 17, 21) 0, (14, 18, 22) .5, (15, 19, 23) 1, (16, 20, 24) 2.

Droplets heated by conduction in flue gas, irrespective of their size, warm to the steady vaporization temperature during phase transformations (Figure 2, a), which is a function of gas temperature and partial pressure of vapor in gas [1]. The phase transformation mode, when heat supplied to the droplet vaporizes water, is called steady vaporization. The peculiarity of the liquid heating by conduction is the fact that the change of their thermal state in the time scale expressed in Fourier number is identical (Figure 2, b). An unsteady temperature field in droplets of different sizes in Figure 2, a is presented for an equal conductive heating duration for droplets of different sizes, and an unsteady temperature field in Figure 2, b corresponds to equal values of Fourier number. When smaller droplets have already reached steady evaporation conditions, larger droplets are still intensively warming at that time. However, the warming dynamics of droplets of different sizes appears identical in the time scale expressed in Fourier number. This is verified in all modeled cases of droplet thermal state variations, when they are heated by conduction. Dynamics of droplet surface temperature and its bulk average temperature are very important in the unsteady temperature field of the warming droplet, which can be also presented by characteristic curves.

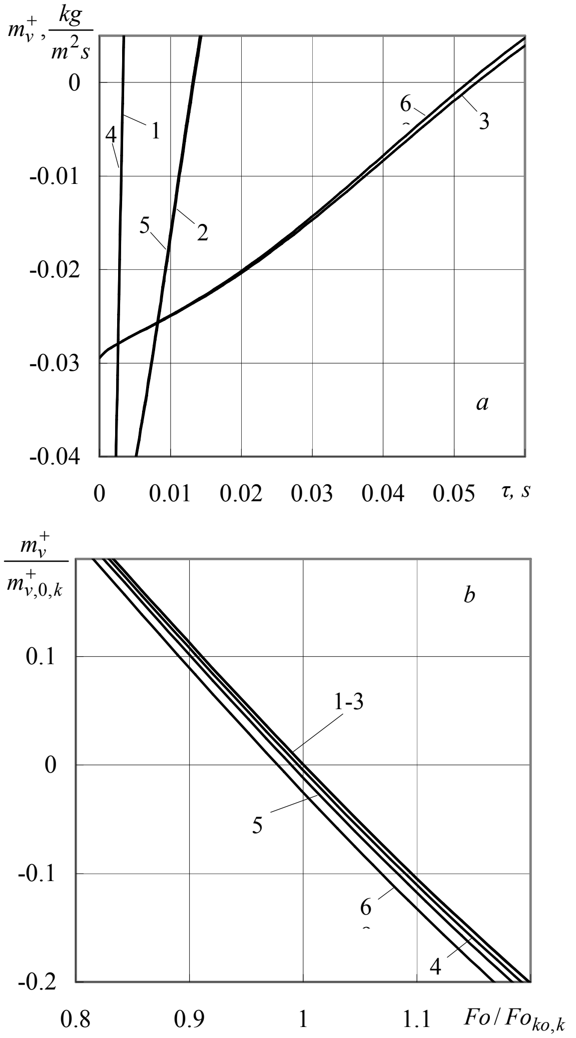

Figure 3 Condensation intensity in real time (a) and Fourier number (b) scales. $R_0 \cdot 10^{-6}$, m: (1, 4) 25 (2, 5) 50, (3, 6) 100; Droplet heating: (a) by conduction, (b) by conduction and radiation.

The growth of droplet volume in the initial stage of unsteady phase transformations is determined by water expansion due to warming and water vapor condensation on the droplet surface. The more humid flue gas is, the more intensive condensation occurs (Figure 3), and droplet expansion is more significant (Figure 4). When condensation is replaced by liquid evaporation, an extreme point appears in the volume variation curve.

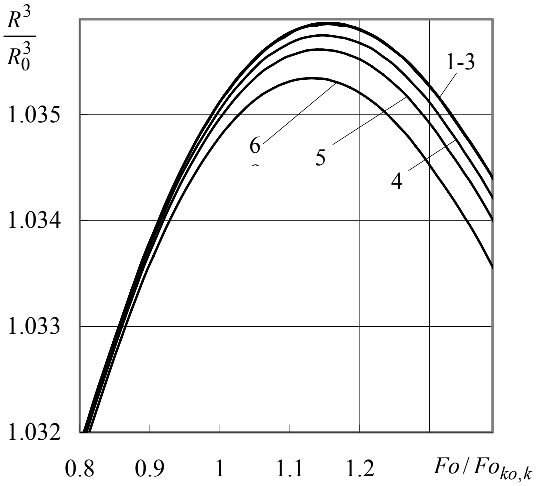

Figure 4 Variation of droplet volume in Fourier number scale. Boundary conditions the same as in Fig. 3.

Speed of phase transformations at the droplet surface is characterized by vapor flow density dynamics (Figure 3). Droplet volume variation is determined by peculiarities of phase transformations at its surface and water expansion due to warming (Figure 4). Water vapor condensing on the droplet surface enforces growth of its volume. When the condensation mode ends, the droplet keeps warming, until it reaches temperature characteristic for steady evaporation. While the influence of water expansion exceeds the impact of water evaporation, the droplet volume keeps growing. When the evaporation factor starts to outweigh, an extreme point is observed in the droplet volume evaporation curve.

The impact of gas radiation on the water condensation process can be evaluated based on variations from characteristic curves reflecting variations of thermal state and phase transformation parameters of an infinite number of different size droplets, when droplets are heated by conduction. Gas radiation is more significant for larger water droplets (Figures 3, 4, curves 4-6). The larger droplets are, the more effectively they absorb a radiation flux falling on them. The smaller droplets are optically more transparent; consequently, they transmit more radiation energy without absorption. Specifics of the role of radiation energy propagating from a radiation source in semi-transparent droplet phase transformations must be noted: it is absorbed in the droplet bulk and is not present directly in the process of phase transformations. However, due to the radiation influence an intense interaction of complex heat and mass transfer processes occurs in the droplet, which can exert a significant influence on water droplet warming, raising a negative-gradient temperature field in it, which starts to occur at the moment when the heat conduction vector in the droplet changes its direction [1]. During condensation of liquid vapor on droplet surface, the droplet is heated by external heat convection and phase transformation energy emerging at its surface very intensively (Figure 5).

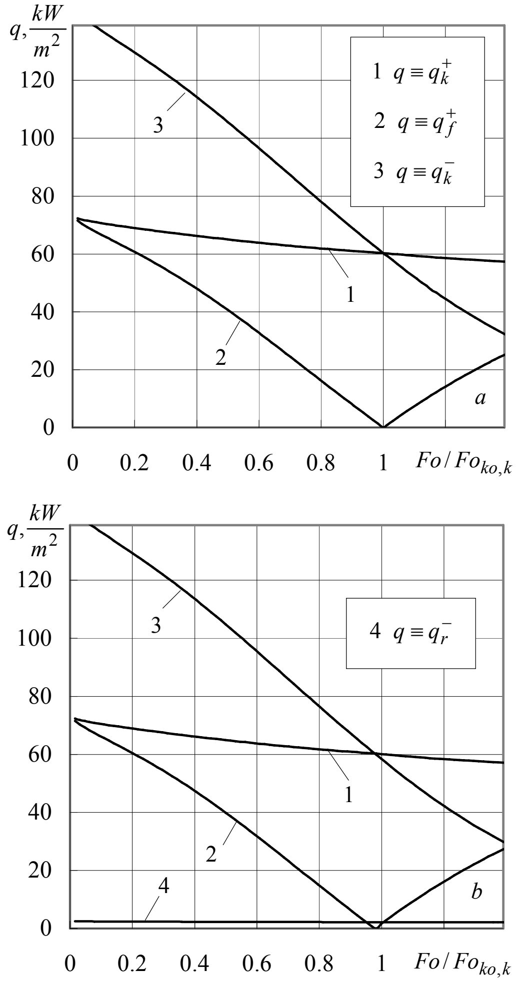

Figure 5 Energy fluxes at the droplet surface, when the droplet is heated by conduction (a); by conduction and radiation (b).

Temperature of flue gas expelled into the atmosphere is not high, and contribution of the radiation flux in the droplet energy balance is not significant. Therefore, a negative-gradient temperature field does not occur in the condensation mode of phase transformations, and the radiation flux cannot act directly in the phase transformations process, but its influence on the droplet warming speed is detectable (Figure 6). The rate of droplet surface temperature variation is slowing consistently (Figure 6, curves 1, 2), and the one of droplet bulk average temperature, that has been accelerating initially, starts slowing later (Figure 6, curves 3, 4).

The process of vapor condensation in a contact heat exchanger will not depend on the water spray dispersity only, but also, it will largely depend on water temperature and flue gas moisture content (Figure 7). Therefore, practical use of a contact heat exchanger for the utilization of phase transformation heat accumulated in flue gas requires consideration of boundary conditions of heat and mass transfer processes in each particular case.

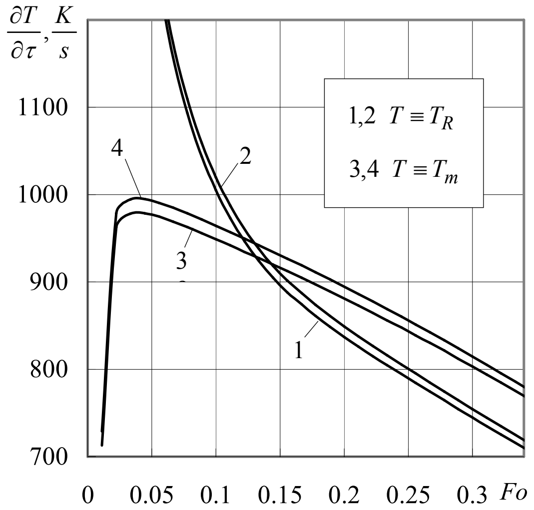

Figure 6 Influence of flue gas radiation on the droplet warming intensity during condensational phase transformation mode. T_g =500K; $p_{v,\infty}/p$=.1; $T_{l,0}$=278K; R_0=0001 m.

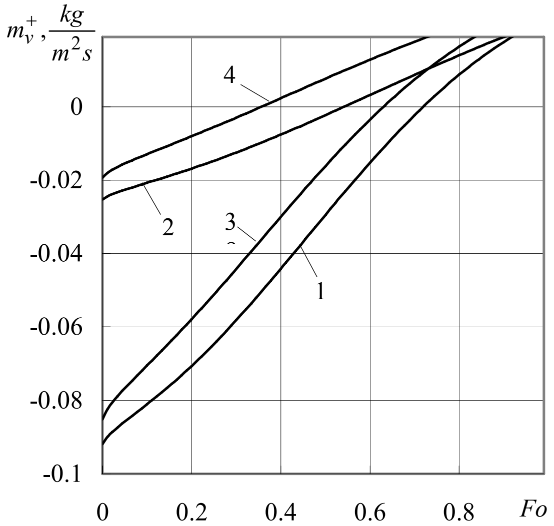

Figure 7 Influence of flue gas moisture content and injected water temperature on the vapor condensation process: $T_{l,0}$, K (1, 2) 280 (3, 4) 290; $p_{v,\infty}/p$: (1, 3) .15 (2, 4) .05; T_g =500K; R_0=.00005 m.

Only thorough consideration of these conditions enables a strict control of condensation process on the droplet surface and adequate regulation of their thermal state variation intensity by proper adjusting of injected water temperature and droplet dispersity.

CONCLUSIONS

Heat and mass transfer processes in a two-phase gas-water droplet flow can be conveniently expressed through a universal chain of time variation $Fo \equiv 0 \div Fo_{ko} \div Fo_e \div \infty$, restricting it to characteristic duration of phase transformations Fo_f. When phase transformation heat accumulated in flue gas is utilized, the condition $Fo_f < Fo_{ko}$ must be met. Then, water vapor contained in flue gas air will condensate on the surface of warming droplets, and the flue gas moisture content will lower. A universal duration of gas utilization $0 \div 1$, which is constituted using a time scale $\overline{Fo} \equiv Fo / Fo_{ko}$, expressed in a Fourier number relation, is convenient in practical investigations.

Gas radiation accelerates droplet warming, and it can reduce duration of condensation phase transformations significantly. Phase transformation intensity at the droplet surface is largely influenced by water spray dispersity. This must be considered in the utilization of phase transformation heat accumulated in flue gas using water sprays.

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