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FIN SHAPE OPTIMIZATION TO MINIMIZE AERODYNAMIC HEATING USING GENETIC ALGORITHM

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ABSTRACT

The process of developing an optimization tool for the fins of hypersonic vehicles to minimize the aerodynamic heating is described. A code was developed to calculate aerodynamic coefficients and aerodynamic heating of swept isolated fins using shock-expansion theory while taking into account the equilibrium gas effects, and semi-empirical methods. Subsequently, an optimizing program was developed based on the continuous genetic algorithm for finding the global minima. By coupling these two programs, several isolated blunt fin geometries were optimized such that their leading edge aerodynamic heating was minimized while their lift coefficients remained constant. The results show that the leading edge aerodynamic heating and the drag coefficients of the fins were reduced significantly, which indicates the powerful capability of the optimization method.

INTRODUCTION

Aerodynamic heating produced when flying at hypersonic speeds causes the temperatures of the payload interior parts to increase, and may damage them or even change the aerodynamic shape of the flight vehicle. To use thermal protection systems (TPS), hence decreasing the aerodynamic heating, design based on the optimal flight paths or optimal shapes are some of the ways used by the designers of these particular vehicles. By shape optimization the amount of aerodynamic heating which is produced on the vehicle surface can be minimized.

In fact most of the engineering problems finally lead to one or a number of optimization problems. Optimization generally finds one or more points or conditions in which an object function has a limited value (minimized or maximized).

Genetic algorithms are global optimization methods that are actually inspired from the natural evolution models. Genetic algorithms define and execute the genetic processes that occur in the nature and lead to appearance and survival of the fittest of object. These algorithms define some mathematical operators that their successive operations to a set of initial random answers lead to the evolution and optimization of the initial answers.

Genetic algorithms that were first introduced in 70's now are the most widely used optimization methods. One of the reasons that these methods are widely used is their high robustness which makes them capable of solving complex problems. Robustness here means the capability of finding the global optimum point without being trapped in the local optimum points.

Nowadays vehicle shape optimization is one of the new interests of the researchers. So far several shape optimizations of flight vehicles have been attempted. For example Lesieutre et. al. [1] optimized missile fin geometry for a minimum

NOMENCLATURE

C_L	[-]	Lift coefficient
C_{L0}	[-]	Initial design lift coefficient
C_D	[-]	Drag coefficient
h_{aw}	[m^2/s^2]	Adiabatic wall enthalpy
h_w	[m^2/s^2]	Wall enthalpy
$IP_{Mutation}$	[-]	Initial mutation probability
M_∞	[-]	Free-stream Mach number
M_{e2D}	[-]	Free-stream Mach number component normal to the leading edge
N_l	[-]	Laminar Mangler transformation factor
$N_{\mathrm{Re}\,duction}$	[-]	Number of domain reductions
N_t	[-]	Turbulent Mangler transformation factor
$P_{Mutation}$	[-]	Mutation probability
Pr	[-]	Prandtle number
$\dot{q}_w$	[W/m^2]	Aerodynamic heating rate
R_{Ne}	[-]	Local Reynolds number
$\frac{du_e}{dx}$	[1/s]	Local velocity gradient at stagnation point
Special characters		
α	[deg.]	Angle of attack
ΔP	[Pa]	Pressure difference on 3-D parts with free-stream pressures
ΔP_{2D}	[Pa]	Pressure difference on 2-D parts with free-stream pressures
Λ_{LE}	[deg.]	Leading edge sweep angle
μ_0	[-]	Stagnation viscosity
μ_∞	[kg/m.s]	Mach cone angle
μ'	[deg.]	Angle that a conical ray makes with tip
ρ_0	[kg/m^3]	Stagnation density
ρ^*	[kg/m^3]	Density calculated at Eckert's reference enthalpy condition

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hinge moment. Andeson et. al. [2] used genetic algorithm to minimize missile geometry for optimal performance and stability. Obayashi and Sasaki [3] optimized supersonic wing surfaces to minimize the aerodynamic force and moment parameters.

CALCULATIONS OF THE AERODYNAMIC FORCES AND AERODYNAMIC HEATING

In every optimization problem there is a need for a solver and an optimizer that are coupled together. The solver calculates the objective function and the optimizer finds the best conditions related to the best objective function.

In this work, in order to optimize the fin shape in hypersonic speeds a computer program was developed. The basis of this program is described. In order to calculate the aerodynamic properties of a fin, this program first calculates the thermodynamic properties of the flow at the outer edge of the boundary layer on the fin based on shock-expansion theory, but for an for equilibrium gas. Then using these properties the aerodynamic forces of the fin are calculated taking into account both the leading edge sweep angle and the tip effects. Consequently the aerodynamic heating on the fin surface based on the semi-empirical methods are calculated. In these calculations it is assumed that the boundary layer on the whole surface of the fin is attached. The main aspects of the solver code are described in the following paragraphs.

High Temperature Effects

At hypersonic speeds the flow temperature increases drastically and as a result air begins to chemically dissociate. The dissociated air will no longer follow the perfect gas flow properties. Here it is assumed that when the flow temperature is high enough, the flow condition is at the state of chemical equilibrium. In 1986, Srinivasan et. al. [4] obtained some useful curve-fittings for equilibrium dissociative air. In the present program these curve-fittings are widely used for the calculation of the thermodynamic properties of the flow.

Shock-Expansion Method

This method is based on the conservation laws and is used for the calculations of the flow properties across the normal and oblique shocks, as well as the expansion waves on the body surfaces. These methods in addition to the modified Newtonian method are implemented. Solution of the flow equations through the normal and oblique shocks, and expansion waves with the perfect gas assumption will result in some simple equations that the flow conditions after them can easily be calculated knowing the upstream conditions. But if we want to calculate the flow conditions taking into account the dissociation effects some nonlinear equations will be obtained instead. These equations should be iteratively and or numerically integrated and solved. It should be mentioned that applying these equilibrium gas effects in the program involves delicate considerations. And finally in order to calculate the flow properties at the outer edges of the boundary layer on each part of the fin section, the program combines these methods together.

These flow equations are described in detail in ref. [5]. The related subroutines are written and for each shape they are combined together. The results are validated with the available data.

Leading Edge Sweep Angle Effects

Leading edge sweep effects are considered based on the method mentioned in ref. [6]. In this reference an analytical method is presented for infinite wings where 2-D aerodynamic coefficients of a selected section are first calculated, then using some geometrical conversions, 3-D aerodynamic coefficients are obtained. We have used this method for parts of the finite fins that are in the 2-D flow regions. Further, the tip effects are included using the following method.

Tip Effects

Perturbations in supersonic flows are propagated into cones whose centres are source of the perturbation. So some parts of the wings behaves in a manner as they are in a 2-D flows and some regions positioned in these imaginary cones are affected by the effects of the tips. Therefore the flow on an isolated fin can be divided into two groups of 2-D and 3-D flows. In ref. [7] an approximate method is presented for the pressure distribution in this region:

$$\frac{\Delta P}{\Delta P_{2D}} = \frac{2}{\pi} \sin^{-1} \sqrt{\frac{\tan \mu'}{\tan \mu_\infty}} \qquad (1)$$

Although this relation is presented for rectangular wings, it can be used as an engineering approximation and we choose this method for the types of fin geometries we are dealing with. So the 3-D effects of the fin tips on the pressure distribution are modelled using the above approximation method, equation 1. It is further assumed that these conical regions never interfere because their angle at hypersonic speeds is very low.

Calculation of fin aerodynamic forces

It is well known that in hypersonic speeds the pressure forces are much higher than the other forces values; hence we considered the pressure forces as the whole aerodynamic forces. At first the middle section of the fin is analyzed using the shock-expansion method then using the method of ref. [6] the 2-D forces of this section are extended to the 3-D values for the entire fin and the tip effects are then taken into account using equation 1.

In calculating the forces on the mid-span section of the fin, first the thermodynamic flow properties on the blunt leading edge are calculated using modified Newtonian theory up to a point where the blunt part is joined to the aft wedge parts. Then using the oblique shock wave method for the windward surface and expansion wave method for the leeward surface the flow properties are calculated on the remaining downstream parts. The flow properties are then calculated both in the windward and the leeward parts of this section. In figures 1 and 2 the results of this program is shown for a fin with the geometric properties given in table 1 at a free-stream Mach number of 10 in different angles of attack. The results are compared with the results of a CFD invicsid code and an engineering code VMD. As seen from these figures the present results are in good agreement with the above two methods.

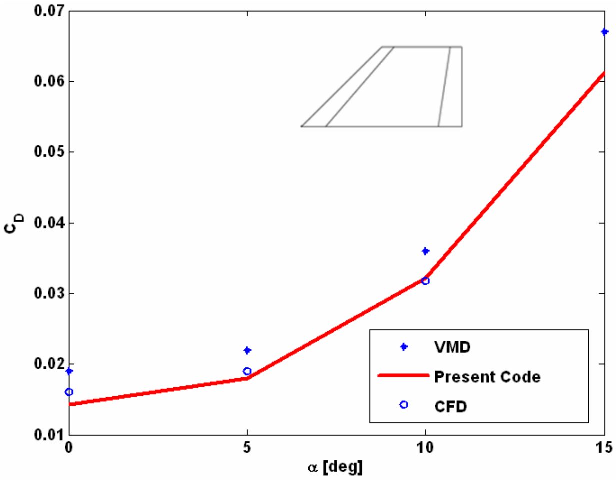

Figure 1 Drag coefficient for a 45° leading edge sweep angle blunt fin F1 at $M_\infty = 10$, altitude=10000 m

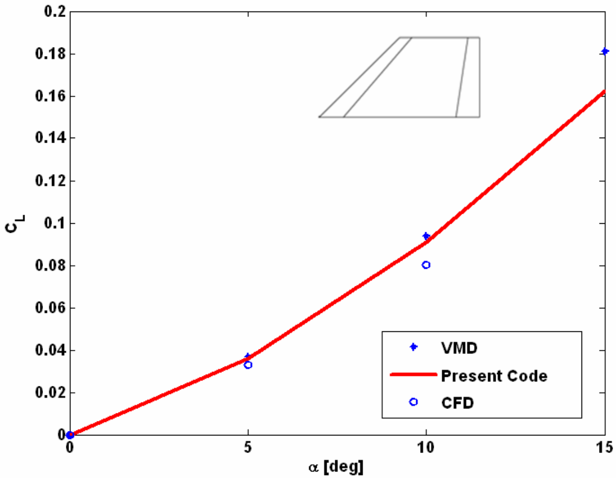

Figure 2 Lift coefficient for a 45° leading edge sweep angle blunt fin F1 at $M_\infty = 10$, altitude=10000 m

Calculation of the fin aerodynamic heating

In the last decades various researchers tried to develop engineering methods to calculate aerodynamic heating on hypersonic vehicles. As a result, several methods along with software have been developed. MINIVER, AP98, AEROHEAT are some of the most famous engineering programs for aerodynamic heating calculations.

In this paper the method of Beckwith and Gallagher [8] is used to calculate aerodynamic heating at the blunt leading edge regions of fins. In this method swept cylinder heating on the wing leading edge is calculated using the following relation:

$$\dot{q}_w = 0.57\,\mathrm{Pr}^{-0.6}\sqrt{\rho_0\mu_0}\sqrt{\frac{du_e}{dx}}\left(h_{aw}-h_w\right) \times \left(\cos\Lambda_{LE}\right)^{1.1} \tag{2}$$

In order to calculate the laminar and turbulent aerodynamic heating in regions far from stagnation line, Eckert's reference enthalpy method [9] is used. So for laminar flow

$$\dot{q}_w = 0.332\left(\mathrm{Pr}^*\right)^{-0.667}\frac{\rho^* V_b}{\sqrt{\dfrac{R^*_{Ne}}{N_l}}}\left(h_{aw}-h_w\right) \tag{3}$$

and for turbulent flow

$$\dot{q}_w = 0.185\left(\mathrm{Pr}^*\right)^{-0.667}\frac{\rho^* V_b}{\left(\ln\sqrt{\dfrac{R^*_{Ne}}{N_t}}\right)^{2.584}}\left(h_{aw}-h_w\right) \tag{4}$$

are used. In figure 3 the aerodynamic heating calculations on a wedge in Mach number of 8.2 have been compared with the experimental results presented in ref. [10]. As seen from this figure, good agreement for all angles of attack is achieved.

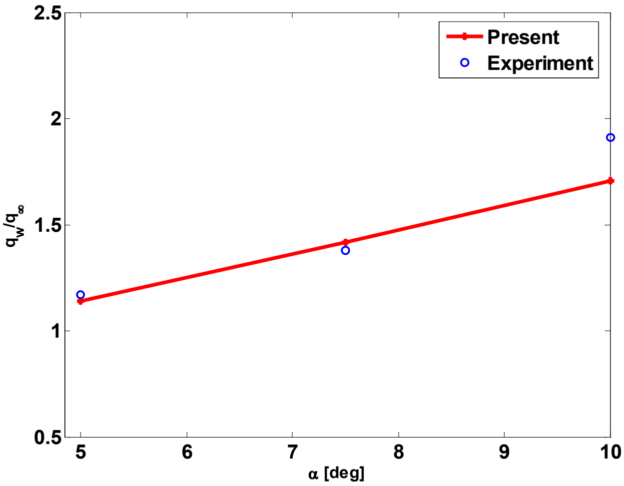

Figure 3 comparison of the present calculated and measured, [10], aerodynamic heating results on a point on a wedge in different angles of attack

OPTIMIZATION ALGORITHM

Genetic algorithms are stochastic search approaches based on randomized operators, such as selection, crossover and mutation, inspired by the natural reproduction and evolution of the living creatures. An algorithm has been implemented and used in order to optimize the fin shape geometries. This algorithm is called Continuous Genetic Algorithm (CGA) and is proposed for the global optimization of multiminima functions in ref. [11]. In order to cover a wide domain of possible solutions, it first takes care of the choice of the initial population. Then it locates the most promising area of the solution space, and continues the search through an "intensification" inside this area. The selection, the crossover and the mutation are performed by using the decimal code.

In this algorithm the size of the population must be initially large enough to achieve a better convergence of the algorithm. To avoid a prohibitive CPU time, it is then necessary to dynamically reduce the size of this population. The reductions in the search space and the population sizes are performed after a given number of consecutive generations without any improvement of the objective function. The variation steps of the crossover and mutation operators directly depend on the search space size. Thus at each reduction of the search space size, these steps are also reduced.

In ref. [11] it is proposed that in each reduction process the population size should be reduced at least 5 units. Furthermore, the mutation probability should be reduced by the following experimental relation:

$$P_{Mutation} = IP_{Mutation} \exp(-N_{Reduction}) \tag{5}$$

where the initial value of $IP_{Mutation}$ is set to be 0.9.

FIN SHAPE OPTIMIZATION PROCESS

Having developed the solver and the optimizer programs, these two programs are then coupled to optimize the given fin geometries. The aerodynamic heating on the fin leading edge is several times larger than the values on other portions of the fin surface, and the normal force on the fin is usually important for the designers, hence it was decided that the fins geometry be optimized in order to minimize the leading edge aerodynamic heating while the lift coefficient remains unchanged. So the objective function was chosen to be the stagnation line (leading edge) aerodynamic heating.

The program is such that it first asks for the geometrical properties of the initial design and the domain of the allowable change for each property. It then produces an initial population of the chromosomes and calls the aerodynamic solver to calculate the aerodynamic force coefficients and aerodynamic heating at the stagnation line for each chromosome. Then evaluating the objective function values, the best chromosomes are chosen and again another initial population is produced and in the same way this process is repeated until an optimized value for the objective function is found. The corresponding chromosome defines the optimized shape parameters.

RESULTS AND DISCUSSION

Several fin geometries with different airfoil sections (Hexagonal and tetragonal) at Mach Numbers of 10 and 15, altitude of 1000 m, angle of attack of 5 deg., and wall temperature of 300 K were optimized. In figure 4 a schematic configuration of a hexagonal fin with its parameters is shown.

Tables 1 and 2 show the constraints considered in the optimization of fins with the hexagonal and tetragonal sections, respectively. As is shown, the Mach number normal to the leading edge is not allowed to become less than 1.2 (supersonic leading edge). The lift coefficient also is forced to be constant, equal to the value of the initial design. Other constraints are set such that the general geometric configuration does not change and the thickness ratio does not exceed the specified values. Also in table 3 the allowable changing domain of the geometric parameters for fins F1 and F2 are shown. The changing domain for fin F3 is similar.

Figures 5 and 6 show configurations of fins F1 and F2 with hexagonal section shapes, respectively, and figure 7 shows the configurations of fin F3 with tetragonal section shape before and after the optimization. The changes in geometric parameters of fin F1, F2 and F3 before and after the optimization are also shown in tables 4, 5 and 6 respectively, at Mach numbers of 10 and 15. Furthermore, tables 7, 8 and 9 show the changes in the aerodynamic parameters of these fins before and after the optimization process.

Optimizations of these fins took an average CPU time of 10 hours on a P4 personal computer. After the optimization of fin F1 at a free-stream Mach number 15, the leading edge aerodynamic heating was reduced 7 times and its drag was automatically reduced 40 percent in comparison with the original values. For fin F2 at a free-stream Mach number of 10 the leading edge aerodynamic heating for the optimized one, figure 6b, was reduced 18 times and its drag again was automatically reduced 26 percent in comparison with the original values. In addition after optimization of fin F3 at a free-stream Mach number of 10 its leading edge aerodynamic heating was reduced 21 times and its drag again was automatically reduced 64 percent in comparison with the original values.

Although the optimizations were performed just for the leading edge heating values, in all cases it was observed that the aerodynamic heating on the entire surface of the fins has been reduced with respect to the initial conditions (i.e., Figure 8).

In addition, to ensure that the resulting corresponding minima are for each case the global ones, the process was repeated several times. The calculated global minima remained unchanged within the program accuracy. Thus we were sure that the optimizing algorithm, CGA, is able to find the global minima correctly, which can be used for shape optimizations of different geometries.

CONCLUSION

The process of developing an optimization tool to minimize the aerodynamic heating of hypersonic vehicles fins is described. The performed optimizations showed that the leading edge aerodynamic heating is reduced significantly and moreover the drag coefficient is reduced automatically. The solver results are in good agreement with the experimental and numerical results and also it is much faster than the numerical codes. Therefore, a good optimization tool is provided for the designers of hypersonic vehicles to optimize shape of the fins in order to reduce the application and/or the amount of TPS used on them.

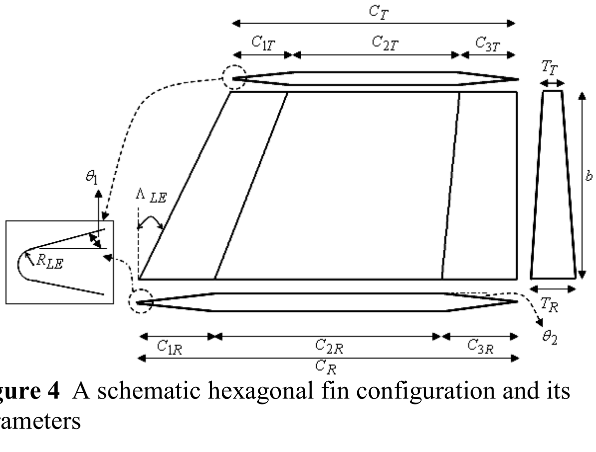

Figure 4 A schematic hexagonal fin configuration and its parameters

Table 1 Constraint considered in fins F1 and F2 optimizations

$M_{e\,2D} > 1.2$	$T_R > T_T$
$C_R > b\tan(\Lambda_{LE})$	$C_{1R} + C_{2R} < C_R$
$T_T / C_T < 0.1$	$C_{1T} + C_{2T} < C_T$
$T_R / C_R < 0.1$	$C_L \ge C_{L0}$

Table 2 Constraint considered in fin F3 optimizations

$M_{e\,2D} > 1.2$	$T_R > T_T$
$C_R > b\tan(\Lambda_{LE})$	$C_{1R} < C_R$
$T_T / C_T < 0.1$	$C_{1T} < C_T$
$T_R / C_R < 0.1$	$C_L \ge C_{L0}$

Table 3 Allowable changing domain of geometric parameters of fins F1 and F2

Geometric Parameter	Minimum Value	Maximum Value
Λ_{LE}	10°	70°
R_{LE}	$R_{LE\,0}$	$6 \times R_{LE\,0}$
b	$b_0 / 1.5$	$2 \times b_0$
T_R	$T_{R0} / 2$	$1.5 \times T_{R0}$
T_T	$T_{T0} / 2$	$1.5 \times T_{T0}$
w_{1R}	0.1	0.25
w_{2R}	0.5	0.8
w_{1T}	0.1	0.25
w_{2T}	0.5	0.8

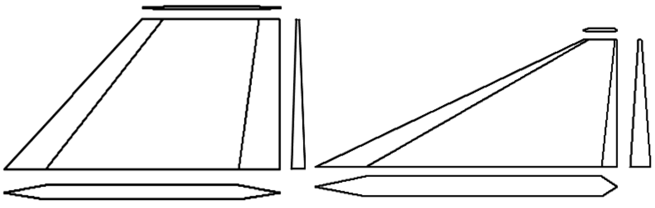
(a) (b)

Figure 5 Shape of fin F1, before (a) and after optimization (b), $M_\infty = 15$

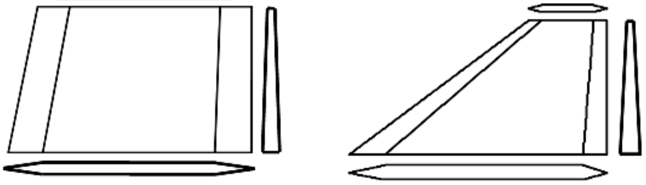
(a) (b)

Figure 6 Shape of fin F2 before (a) and after optimization (b), $M_\infty = 10$

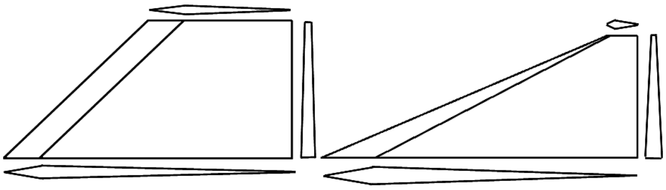
(a) (b)

Figure 7 Shape of fin F3, before (a) and after optimization (b), $M_\infty = 10$

Table 4 Geometric parameters before and after the optimization, for fin **F1** (parameters are introduced in figure 6)

Geometric Parameters	Initial Design	After Optimization, $M_\infty = 10$	After Optimization, $M_\infty = 15$
C_R	1200	1200	1200
C_T	600	87.5	67
Λ_{LE}	45	64.5	61.8
R_{LE}	20	40.5	35.6
b	600	506.2	608.1
T_R	55	81.2	67.3
T_T	10	13.6	6.6
C_{1R}	180	199.4	171.7
C_{2R}	840	939.6	902.5
C_{1T}	90	12.3	9.6
C_{2T}	420	69.4	41.2
θ_{1R}	7.1	11.6	11.1
θ_{2R}	7.1	33.9	14.9
θ_{1T}	3.2	29.6	19
θ_{2T}	3.2	50.4	11.5

Table 5 Geometric parameters before and after the optimization, for fin F2 (parameters are introduced in figure 6)

Geometric Parameters	Initial Design	After Optimization, $M_\infty = 10$	After Optimization, $M_\infty = 15$
C_R	1000	1000	1000
C_T	876.5	241.9	302.8
Λ_{LE}	10	54.2	50
R_{LE}	10	37.8	27.7
b	700	547.3	588.2
T_R	45	66.8	64.5
T_T	22.5	24	30.2
C_{1R}	150	107.4	140.7
C_{2R}	700	768.7	765.9
C_{1T}	131.5	28.1	48.7
C_{2T}	613.5	148.5	200.5
θ_{1R}	8.5	17.3	12.9
θ_{2R}	8.5	15.1	19
θ_{1T}	4.9	23.1	17.2
θ_{2T}	4.9	10.4	15.7

Table 6 Geometric parameters before and after the optimization, for fin F3 (parameters are similar to those introduced in figure 6)

Geometric Parameters	Initial Design	After Optimization, $M_\infty = 10$
C_R	1200	1200
C_T	600	121.8
Λ_{LE}	45	68.3
R_{LE}	20	39.2
b	600	430
T_R	55	64
T_T	10	10.2
C_{1R}	180	204
C_{2R}	1020	996
C_{1T}	90	14.9
C_{2T}	510	106.9
θ_{1R}	7.1	8.9
θ_{2R}	1.3	1.8
θ_{1T}	3.2	18.9
θ_{2T}	0.6	2.7

Table 7 Aerodynamic coefficients and fin leading edge aerodynamic heating before and after the optimization, for fin F1, $M_\infty = 15$

Aerodynamic Parameter	Initial Design	After Optimization
C_L	0.0314	0.0314
C_D	0.0173	0.0102
C_L/C_D	1.82	3.05
$\dot{q}_w$	3.36×10^7	4.7×10^6

Table 8 Aerodynamic coefficients and fin leading edge aerodynamic heating before and after the optimization, for fin F2, $M_\infty = 10$

Aerodynamic Parameter	Initial Design	After Optimization
C_L	0.049	0.049
C_D	0.031	0.0228
C_L/C_D	1.58	2.15
$\dot{q}_w$	5.36×10^7	3.02×10^6

Table 9 Aerodynamic coefficients and fin leading edge aerodynamic heating before and after the optimization, for fin F3, $M_\infty = 10$

Aerodynamic Parameter	Initial Design	After Optimization
C_L	0.039	0.039
C_D	0.0124	0.0045
C_L/C_D	3.15	8.38
$\dot{q}_w$	9.35×10^6	4.35×10^5

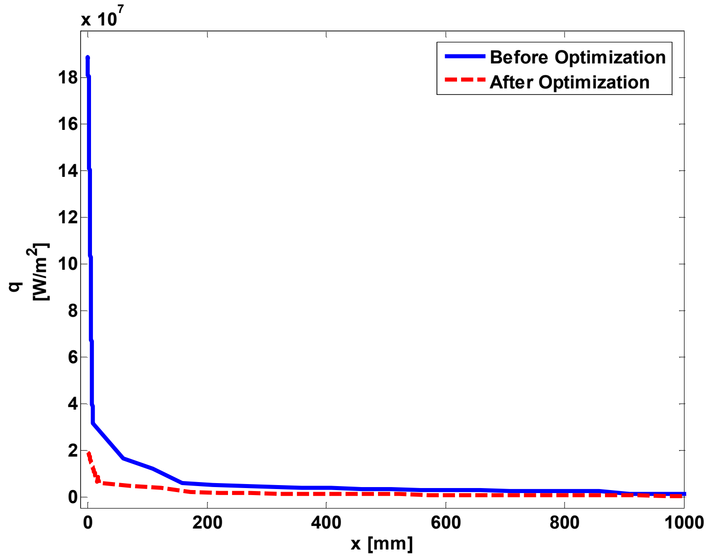

Figure 8 Windward aerodynamic heating distribution on the fin mid-span section of fin F2, before and after the optimization, $M_\infty = 15$

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