Macroeconomic stress testing of a corporate credit portfolio

By

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DECLARATION

I, the undersigned, declare that the dissertation, which I hereby submit for the degree Master of Science at the University of Pretoria is my own work and has not previously been submitted by me for any degree at this or any other tertiary institution.

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Abstract

This dissertation proposes stress testing of a bank's corporate credit portfolio in a Basel Internal Ratings Based (IRB) framework, using publicly available macroeconomic variables. Corporate insolvencies are used to derive a credit cycle index, which is linked to macroeconomic variables through a multiple regression model. Probability of default (PD) and loss given default (LGD) that are conditional on the worst state of the credit cycle are derived from through-the-cycle PDs and LGDs. These are then used as stressed inputs into the Basel regulatory and Economic capital calculation for credit risk. Contrary to the usual expert judgement stress testing approaches, where management apply their subjective view to stress the portfolio, this approach allows macroeconomic variables to guide the severity of selected stress testing scenarios. The result is a robust stress testing framework using Rösch and Scheule (2008) conditional LGD that is correlated to the stressed PD. The downturn LGD used here is an alternative to the widely used Federal Reserve downturn LGD which assumes no correlation between PDs and LGDs.

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Dedicated to my mother, Selebatso Elizabeth Sebolai.

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List of Acronyms

AIRB	Advanced Internal Ratings-based Approach
ASRF	Asymptotic Single Risk Factor
AVC	Asset Value Correlation
BCBS	Basel Committee on Banking Supervision
BIS	Bank for International Settlements
CAR	Capital Adequacy Ratio
CDS	Credit Default Swap
CLGD	Conditional Loss Given Default
CVA	Credit Valuation Adjustment
DF	Degree of Freedom
DLGD	Downturn Loss Given Default
EAD	Exposure-at-Default
EC/ECap	Economic Capital
EL	Expected Loss
EPE	Expected Positive Exposure
FSAP	Financial Sector Assessment Programme
GDP	Gross Domestic Product
ICAAP	Internal Capital Adequacy Assessment Process
IMA	Internal models approach.
IMF	International Monetary Fund
IRB	Internal Ratings-based Approach
LCR	Liquidity Coverage Ratio
LGD	Loss Given Default
MRC	Market Risk Capital
NPLs	Non-Performing Loans
NSFR	Net Stable Funding Ratio
OTC	Over-the-counter
PD	Probability of Default
PIT	Point-in-time
RAF	Risk Appetite Framework
RAPM	Risk-adjusted Performance Measure
REER	Real Effective Exchange Rate
RWA	Risk-weighted Assets
SA	Standardized Approach
SARB	South African Reserve Bank
TTC	Through-the-cycle
UL	Unexpected Loss
VaR	Value-at-Risk

Glossary

Regulatory Capital:	The amount of capital a bank is required to hold as a cushion for unexpected losses. This amount is set to at least 8% of risk-weighted assets, before considering any country buffers.
Economic Capital:	This is the amount of capital calculated internally to absorb unexpected losses, at a predefined confidence level.
Stress Testing:	A range of techniques used to assess the vulnerability of a financial system to macroeconomic shocks.
Risk weighted Assets:	The total amount after assigning regulatory risk weights to the assets and multiplying them together. High risk weighted assets indicate that majority of the assets in the bank's Balance Sheet carry a lot of risk.
Stress Scenarios:	A set of forward looking macroeconomic outcomes used to apply shocks to risk factors in the bank's portfolio.
Credit Cycle:	This is an indication of the expansion or the contraction of access to credit over time.
Downturn LGD:	This refers to an LGD that reflects the lowest levels of the credit cycle.
Insolvencies:	Refer to an individual or partnership which is unable to pay its debt and is placed under final sequestration.
Regression Model:	A statistical method that attempts to determine the strength of the relationship between one dependent variable (usually denoted by Y) and a series of other changing variables (known as independent variables).

Notation

$\Phi()$:	Cumulative standard normal distribution function.
$\Phi^{-1}()$:	The inverse of standard normal distribution.
$\sim N(\mu,\sigma^2)$:	Normally distributed with mean μ and variance σ^2.
σ^2 :	The variance.
$Cov(x, y)$:	Covariance between x and y.
μ :	The mean.
T :	Maturity of an instrument in question.
$\exp()$:	Exponential function.
Δ :	Delta or change.
$E()$:	Expected value.
$\sum_i^n$:	Summation from index i to n.
$\int_0^T$:	Integral from 0 to T.

The following notation is particularly for Chapter 3, Section 3.4.

$f[\mathrm{i}, \mathrm{j}]$:	Forward interest rate function, at node level i and time period j.
$D[\mathrm{i}, \mathrm{j}]$::	Discount function, at node level i and time period j.
$\xi[\mathrm{i}, \mathrm{j}]$:	Default intensity, at node level i and time period j.
$Q[\mathrm{i}, \mathrm{j}]$:	Survival function, at node level i and time period j.
$\phi[\mathrm{i}, \mathrm{j}]$::	Recovery rate, at node level i and time period j.
$S[i, j]$:	Stock price, at node level i and time period j.
$\lambda[i, j]$:	Default probability, at node level i and time period j.
$q[i, j]$:	Risk-neutral probability of reaching node [i, j].
A_N :	Expected present value of the premiums paid on a default swap of maturity N periods
B_N :	Expected present value of loss payments on a default swap of N periods.

Notation for Section 3.3 is defined within the section.

Chapter 1: Dissertation Overview

1.1. Introduction

Financial institutions around the world have learned that severe macroeconomic crisis can occur at least once in every decade or even more frequently. A worldwide stock market crash in September 2008, US debt-ceiling crisis in 2011 and recently, the Eurozone debt crisis in 2012 have put many banks around the world under severe financial pressure. Significant increase in credit losses, market liquidity flight, extremely high trading losses, and a slowdown in lending are typical results of such adverse events, depending on the nature and severity of the crisis. This is mainly due to the systemic nature of these events.

As a result, stress testing of portfolios by banks in an attempt to gauge the impact of adverse macroeconomic events is increasingly becoming an important aspect of risk management worldwide.

Banking regulators have, in turn, became stringent with regard to stress testing and the Internal Capital Adequacy Assessment Process (ICAAP). Pillar I and II of the Basel II framework explicitly state that banks are required to carry out regular stress testing. Beyond regulatory pressure, stress testing is not only a regulatory compliance tool, but also critical from the banks internal risk and capital management perspective. Stress testing can be used internally for capital planning i.e. setting buffers, as well as for risk management purposes where the risk profile is observed under certain stress scenarios.

Banks have adopted different stress testing approaches and practices globally, however, there is a trade-off between complexity and practicality with these models, see Oura and Schumacher (2012). There are Top-Down approaches where expert judgment is applied to risk factors to determine portfolio impacts at a very high level, or Bottom-Up approaches where modelling is based on some quantitative model which uses external factors as inputs to stress the risk parameters of underlying portfolios. The latter is usually the preferred method by different regulators, although a combination of both is usually used in practice.

External data normally used in most stress testing models is macroeconomic data, such as time series of GDP levels, unemployment rate, and many other economic indicators available for the country in question. Internal data could be historical values of non-performing loans (NPLs), credit impairment losses, probabilities of default (PDs) of a bank's portfolio, see Schmieder et al (2011). The most common practice for banks with sufficient historical data is to derive a relationship between their internal variables and macroeconomic variables for stress testing purposes. The approach presented here allows banks to factor the macroeconomic impact of any economic scenario into their stressed losses.

A surprising number of banks in the world do Top-Down guesstimates, expert-driven shocks, or benchmark approaches for stress testing. Majority of banks that have adopted Basel II modelling approaches still ignore the correlation between PDs and loss given default (LGDs), by using a downturn LGDs formula prescribed by the United States' Federal Reserve, see Emery and Cheparev (2007). We show several shortcomings of this approach and derive downturn LGDs which are correlated to the PDs through a credit cycle index.

This dissertation adopts a methodology similar to that of Browne et al (1999), where they investigate the impact of exogenous factors on individual insurers' insolvency rate. In our approach insolvencies are used to derive a systematic credit index which is then used to derive both conditional PDs and LGDs. Orzechowska-Fischer and Taplin (2010), show how insolvencies can be predicted from publicly available macroeconomic variables. Since corporate insolvency rate is directly related to default rate, insolvencies are directly linked to PDs and LGDs in this dissertation.

The approach followed here enables banks to use publicly available macroeconomic variables to stress their PDs and LGDs. This study is done in a South African context, collecting annual macroeconomic data from 1980 to 2012. Although, some of the variables were rejected from the final regression model, initial variables considered are presented in the Appendix Table 26. The data used in our study was collected from multiple sources such as the International Monetary Fund (IMF) website, Statistics South Africa's (Stats SA) website, and World Bank's website.

1.2. Outline of the Dissertation

This dissertation is structured to focus on regulatory and economic capital in the following way. Chapter 2 provides a brief summary of the Basel accords, starting with Basel I, up to the current accord of Basel III. Changes or new proposed measures in each accord are highlighted.

Chapter 3 covers the basics of the Basel II Internal Ratings Based (IRB) approach for regulatory capital. Economic capital under the same assumptions underlying the Basel IRB framework, as shown in Gordy (2002), is discussed. The relationship between capital and the main input parameters PDs and LGDs is outlined, for both regulatory and economic capital. Two reduced form models are recommended for estimating PDs and LGDs internally.

Chapter 4 is devoted to the step-by-step building of the macroeconomic regression model, the data used in the model, results, and how the model can be used practically to predict the number of corporate insolvencies in a given year. The model is backtested using observed historical values of corporate insolvencies.

Chapter 5 covers the fundamental part of this dissertation. The cycle index is derived from corporate insolvencies and validated with the RMB/BER business confidence index, which is commonly used in South Africa. Stressed PDs and LGDs conditional on this cycle index are calculated. A comparison of the USA Federal Reserve's downturn LGD and our downturn LGD based on Rösch and Scheule's (2008) formula, is presented. Our hypothetical credit portfolio used for stress testing illustration is also described in detail.

Chapter 6 summarises the final stress testing results in a tabular form. Important measures to be considered are defined in this chapter. The two chosen historical scenarios to be used for stress testing are presented. A brief comparison between the two scenarios is provided in a commentary format. Lastly, other risk types that can be incorporated into a bank-wide stress testing exercise are defined.

1.3. Dissertation Objectives

The main objective of this dissertation is to propose a less subjective Bottom-Up stress testing approach which relies on the credit cycle, for South African banks that are still using expert judgement to stress their portfolios systematically. The second objective is to recommend a systematic method that stresses both the demand and the supply side of capital.

These objectives are achieved by:

- Using the credit cycle index to stress the main inputs into the Basel Regulatory and Economic capital calculation, i.e. PDs and LGDs.
- Replacing the commonly used downturn LGD recommended by the USA's Federal Reserve Bank with a Rösch and Scheule (2008) downturn LGD that is conditional on the credit cycle index.
- Stressing both the available Tier 1 and Tier 2 capital (capital supply side), and the required capital through an increase in RWAs as a result of stressed PDs and LGDs (capital demand side).

This approach ensures that the downturn LGD are correlated to default rates, through the credit cycle index, which is aligned to recent research by the likes of Frye (2000), and Pykhtin (2003), who prove that recoveries are correlated to default rates to a certain extent. Giese (2005) quantifies the impact of this correlation on credit risk capital.

1.4. Literature Review

This sections covers the literature of previous stress testing models that have been, and continue to be used globally. Previous stress testing research that is specific to South Africa is also reviewed, i.e. Barnhill et al (2000), Havrylchyk (2010), and Fourie et al (2011). According to Foglia (2009), there are three broad stress testing models in general, i.e. structural models, vector autoregressive models (VAR), and statistical models. Melecky and Podpiera (2012) add 'expert judgement' as an additional approach that can be used in cases where aforementioned models cannot produce feasible stress testing results. A brief summary of each is presented here, followed by a motivation of our model choice in this dissertation.

Definition of Stress testing

Basel Committee on Banking Supervision (2009), defines stress testing as the evaluation of a bank's financial position under a severe but plausible scenario to assist in decision making within the bank.

Sorge (2004), defines stress testing as a range of techniques used to assess the vulnerability of a financial system to "exceptional but plausible" macroeconomic shocks.

Stress testing methodologies

As presented in Hoggarth et al (2003) there are two broad approaches for stress testing, each with its own advantages and disadvantages. The choice of a specific approach is driven by many things within the bank in question, mainly the availability of data or the required risk management and stress testing expertise.

i. **Top-down approaches**: these are usually expert-driven shocks, or benchmark approaches applied at an aggregated bank or financial sector level. This method is normally used by regulators or national authorities to assess solvency levels of the entire financial sector.
ii. **Bottom-up approaches**: these are usually statistical analyses such as econometrics models that are used to identify the potential shocks from historical macroeconomic data, and applying them at a granular level of the bank's data.

Table 1 by Oura and Schumacher (2012) summarises the weaknesses and strengths of each method.

Feature	**Bottom-Up (BU)**	**Top-Down (TD)**
Strengths	▪ Reflects granular data and covers exposures and risk-mitigating tools more comprehensively, including those that are hard to cover in TD tests (such as risks from complex structured products, hedging strategies, and counterparty risks). ▪ Utilises advanced internal models of financial institutions, which could potentially yield better results. ▪ May reveal risks that could otherwise be missed. ▪ Provides insights in the risk management capacity and culture of a particular institution. ▪ Application of severe common shocks may encourage individual institutions to prepare for tail events that they might otherwise not be prepared to contemplate.	▪ Ensures uniformity in methodology and consistency of assumptions across institutions. ▪ Ensures full understanding of the details and limitations of the model used. ▪ Provides an effective tool for the supervisory authority or FSAP team to validate BU tests. ▪ Once a core framework is in place, implementation is relatively resource-effective. ▪ Can be implemented in systems where institutions have limited risk management capacities.
Weaknesses	▪ Implementation tends to be resource intensive and depends on the cooperation of individual institutions. ▪ Results may be influenced by institution-specific assumptions, data, and models that hamper comparisons across institutions.	▪ Estimates might not be precise due to data limitations. ▪ Standardisation may come at the cost of not reflecting each institution's strategic and managerial decisions.

Table 1: Comparison of Bottom-Up and Top-Down Stress Tests. (International Monetary Fund, 2012)

Stress testing steps

Stress testing is an iterative process which involves the following common steps irrespective of the method chosen to carry out the actual stress testing. These steps have been defined in a flow diagram in Figure 1.

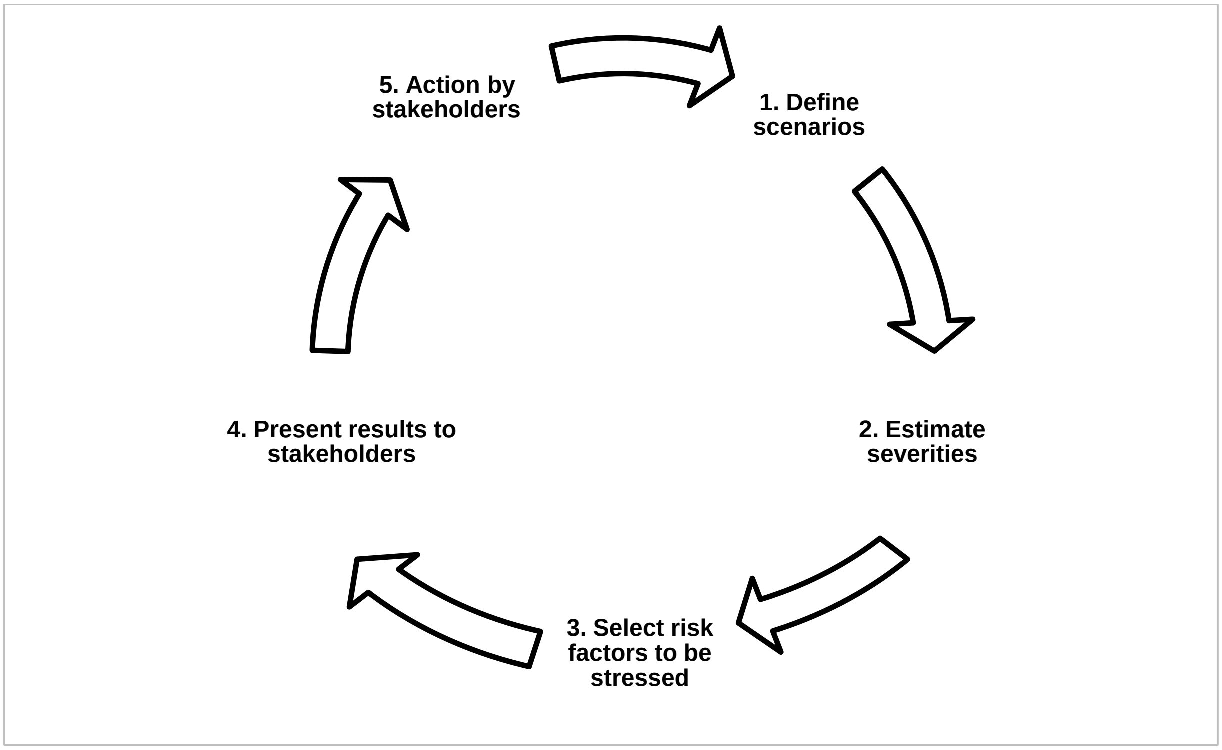

Figure 1: Stress testing flow diagram

1. **Define scenarios**: this is an essential step where a set of macroeconomic outcomes of different severities are formulated with a view to testing their impact on a portfolio.
2. **Estimate severities**: regulators require banks to apply severe but plausible shocks to the risk factors. These could be based on historical crisis events or a future macroeconomic outlook.
3. **Select risk factors**: risk factors should be identified within each risk type and stressed using severe but plausible shocks. For example within credit risk, PDs, LGDs, and EADs are the most commonly stressed risk factors.

4. **Present results to stakeholders**: This is the critical stage of analysing and presenting the risks identified through stress testing to different stakeholders within the bank.
5. **Action by stakeholders**: management might be required to take mitigating actions in case the stress testing results highlight potential breaches of regulatory or risk appetite limits.

Structural models

Melecky and Podpiera (2012) describe these models as econometric stress testing models that use macroeconomic forecasts that are in line with monetary policy analysis. These models mainly rely on market prices to stress the internal risk factors of any bank, or the banking sector as a whole in case of a macroprudential stress testing exercise by the regulatory authority.

Chan-Lau J (2013) illustrates how structural market based top-down stress tests can be constructed using market price data such as bond yields, credit default swaps, equity prices, and many other market variables. He derives a framework that stresses the probabilities of default (PD) based on the Black-Scholes option pricing theory by mapping the capital structure of the bank to the macroeconomic variables and market risk factors. Moody's Ferry D et al (2012) developed a structural econometric framework for stressed expected default frequencies (EDF). They constructed a panel dataset consisting of firm-level EDFs and macroeconomic variables over time. Moody's EDF model is also a structural or asset value model based on the Black-Scholes theory. Because distance-to-default (DD) is easier to work with than the EDF, their framework focuses on stressing the DD, because it is monotonically mapped to EDF.

Next, the two models are considered in turn, first the Chan-Lau J (2013) model, followed by Moody's EDF model by Ferry D et al (2012).

Chan-Lau J (2013), links the probability of default p with the capital structure of the bank, over time horizon T, using the Black-Scholes option pricing formula as follows:

$$p = \Phi\left(-\frac{\ln(\mathrm{V/D}) + (\mu - \sigma^2 T/2)}{\sigma\sqrt{T}}\right), \tag{1.1}$$

where $\Phi()$ is a cumulative normal distribution, μ and σ are the growth rate, and the volatility of the bank's assets V, respectively. D represents the bank's total debt. Capital-to-assets ratio can be calculated from V/D as follows:

$$K/V = 1 - D/V \tag{1.2}$$

From equations (1.1) and (1.2), it follows that the probability of default is a monotonic decreasing function, G, of the capital-to-asset ratio if other model parameters are held constant:

$$p = G(K/V), \quad \frac{\partial G}{\partial_{K/V}} < 0. \tag{1.3}$$

$\frac{\partial G}{\partial_{K/V}} < 0$, is a necessary condition for function G to be a monotonic decreasing function, for all K and V. Furthermore p can be linked to macroeconomic variables, X, and market factors, M, through a function, F, as follows:

$$p_t = F(X_t, M_t) \tag{1.4}$$

If the relationship between probabilities of default and macroeconomic variable and market factors suffices, then the capital structure of the bank can be modelled from (1.3) and (1.4) as:

$$K/V = G^{-1}(F(X_t, M_t)), \tag{1.5}$$

where $G^{-1}()$ is an inverse of the function G in (1.3).

Macroeconomic variables, X, and market factors, M, can be used as stress inputs for stressing the capital structure (K/V) or default probabilities (p).

Moody's EDF stress testing approach is based on the same Black-Scholes option pricing framework as Chan-Lau J (2013). The approach uses distance-to-default (DD) as a stress parameter, instead of the actual EDF. The relationship between DD and EDF can be shown as follows:

Moody's Zhao et al (2012) demonstrate a relationship between DD and EDF. Keeping the same notation as in Chan-Lau J (2013) above, the asset value of the bank is left as

V, and its debt is represented by D. The basic structural model assumes the asset value V follows a stochastic process,

$$dV = \mu V dt + \sigma V dw ,$$

where w is a standard Brownian motion, μ a drift term, and σ the volatility of the asset value V. If the drift coefficient function μ is bounded, then this SDE has a unique solution with initial condition V_0 , and it is given by,

$$V_T = V_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma_V W_T\right) ,$$

where W_T is the Wiener process. Using the Brownian motion assumption that the log of asset values are normally distributed,

$$\ln V_T \sim N(\ln V_0 + \left(u - \frac{\sigma_V{}^2}{2}\right), \sigma_V{}^2) ,$$

the probability of V_T ending below debt D, is given by,

$$\Phi\left(-\frac{\ln\left(\frac{V_0}{D}\right) + (\mu - \frac{\sigma_V{}^2}{2})T}{\sigma_V \sqrt{T}}\right) ,$$

which is the EDF, given the asset V and debt D. If a default occurs when the asset value V_T falls below the debt D, the default point is equal to the debt value D. Distance-to default (DD), can therefore be defined as the distance between the expected value of assets at time T, minus the default point, divided by the volatility of the assets at time T,

$$DD = \frac{E(V_T) - D}{\sigma_V \sqrt{T}} = \frac{\ln(\mathrm{V}_0 / \mathrm{D}) + (\mu - \frac{\sigma_V{}^2}{2})T}{\sigma_V \sqrt{T}} .$$

It follows that DD and EDF have the following relationship,

$$EDF = \Phi(-DD) .$$

Since DD is a function of V and D, Zhao et al (2012) show that a simplifying assumption of letting the second term of the numerator equal to zero; which leads to,

$$DD = \frac{\ln(V_0) - \ln(D)}{\sigma_V \sqrt{T}}.$$

Moody's Ferry D et al (2012), show that default risk or expected default frequency (EDF) can be stressed by regressing change in distance-to-default (DD) with macroeconomic variables using the following firm-level model,

$$\Delta DD_{it} = \alpha + D\rho + M\beta + IND\gamma + IG\delta + e_{it},$$

where i and t are firm and time subscript, respectively.

D: a vector containing the one- and 12-month lags in the dependent variable, i.e. each firm's DD history, for at least 2 recent years.

IND: captures the industry fixed effects.

IG: is a dummy variable classifying firms as investment grade/non-investment grade according to their Through-the-Cycle EDF-implied ratings.

The models presented here are two examples of structural stress testing models which rely mostly on the firm's capital structure, combined with macroeconomic variables. The main benefit of these models is the fact that they can easily be adopted by central banks for macroprudential stress tests since they are consistent with the policy analysis. Their biggest shortfall is the linearity assumptions in their forecasts as well as the linear properties of the Black-Scholes framework. Melecky and Podpiera (2012) state the existence of non-linear relationship among macroeconomic variables and financial variable during the stress period.

Vector Autoregressive (VAR) models

Foglia (2009), and Melecky and Podpiera (2012) describe Vector Autoregressive (VAR) approaches as models that are used either because structural models are not available or they are employed for their greater flexibility and easiness to generate a consistent set of predicted variables. In these models, a set of macroeconomic variables are jointly affected by the initial shock, and the vector process is used to project the stress scenario's combined impact on this set of variables.

Hoggarth et al (2003) estimated the macroeconomic impact on provisions for large commercial banks in the United Kingdom, as part of International Monetary Fund's (IMF's) Financial Sector Assessment Program (FSAP). The top-down VAR model was fitted using quarterly panel data for the period 1987-2001. The model included sector default rates, banks' lending rates, and some macroeconomic variables such as GDP, house prices, and the sterling exchange rate. Overall simulations suggested that the likely increases in credit losses arising under all scenarios are quite small – all scenarios would result in an increase in banks' new provisions charges, both in the first year and cumulatively after three years, of less than 10% of annual profits.

Wong et al (2008), followed a traditional Wilson (1997) autoregressive stress testing model which looks at credit exposures of Hong Kong's retail portfolios. Their stress testing methodology divides their respective portfolios by economic sectors and stress parameters within each sector. Jokivuolle et al (2008) extend the Sorge and Virolainen (2006) autoregressive model to stress Finnish corporate defaults, using quarterly key macroeconomic factors from 1986:Q1 to 2003:Q2. Their model is also classified by default rates into different industry sectors.

Assume a credit portfolio with average default probabilities $p_{j,t}$ for sector j at time t. An industry-specific index $y_{j,t}$ can be derived as follows:

$$y_{j,t} = \left(\frac{1 - p_{j,t}}{p_{j,t}} \right).$$

From this index it is clear that smaller values of $p_{j,t}$ are associated with higher values of the index $y_{j,t}$, and vice versa. The index is assumed to be driven by a set of macroeconomic variables $x_{i,t}, (\mathrm{i} = 1,\ldots,\mathrm{n})$ in the following way:

$$y_{i,t} = \beta_{j,0} + \beta_{j,1} x_{1,t} + \beta_{j,2} x_{2,t} + \ldots + \beta_{\mathrm{j,n}} x_{\mathrm{n},t} + \varepsilon_{j,t},$$

where,

$\varepsilon_{j,t}$ is an independent and identically distributed normal error term. The autoregressive AR(2) come in with the macroeconomic variables that can be modelled as:

$$x_{i,t} = k_{i,0} + k_{i,1} x_{i,t-1} + k_{i,2} x_{i,t-2} + v_{i,t},$$

where:

k_{it} represents coefficients for the i-th macroeconomic variable, at time t.

$v_{i,t}$ is an independent and identically distributed normal error term.

This system of equations is governed by the correlation structure between the error terms for the industry index, and the autoregressive macroeconomic variables error terms i.e. $\varepsilon_{j,t}$ and $v_{i,t}$, respectively. Schechtman and Gaglianone (2011), also investigated the macroeconomic impact on system-wide credit risk of Brazil using the traditional Wilson (1997a) model and extend it to the quantile regression (QR) method based on Koenker and Xiao (2001).

Most VAR models used in stress testing follow a Wilson (1997a) framework which is formulated by the following set of equations:

$$\begin{cases} CRI_t = \dfrac{1}{(1+\exp(-y_t))} & (1.6) \\ y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \gamma_0 z_t + \sum_{j=1}^{q} \gamma_j z_{t-j} + u_t & (1.7) \\ z_t = u + \sum_{k=1}^{m} A_k z_{t-k} + \varepsilon_t, \qquad m > q & (1.8) \\ (u_t, \varepsilon_t) \sim N(0, \text{cov}), \quad \text{cov} = \begin{pmatrix} \text{cov}(u,u) & \text{cov}(u,\varepsilon) \\ \text{cov}(\varepsilon,u) & \text{cov}(\varepsilon,\varepsilon) \end{pmatrix}, & (1.9) \end{cases}$$

where:

y_t is the logit transformation of an observable credit risk indicator CRI ∈ [0,1],

z_t is a vector of macroeconomic variables at time t,

u_t is a normal error, homoscedastic and independent with regard to past information and ε_t is a normal white noise,

Cov is a covariance matrix.

Equation (1.7) is the macro-credit risk link that relates the (transformed) credit risk indicator y_t contemporaneously to the macro vector z_t. Equation (1.8) is an autoregressive system (VAR) of macroeconomic variables. Equation (1.9) states the correlation of error terms between risk indicators and macroeconomic variables.

Schechtman and Gaglianone, (2011), state that the main weakness of VAR models is the correlation specification from Equation (1.9), because it is frequently difficult to prove the existence of the correlation relationship specified. It is also common to find non-normality, and heteroskedasticity, which violates the main assumptions of the ordinary least-squares (OLS) regression.

Statistical models

These approaches involve using statistical distributions of credit losses to stress the risk parameters within a portfolio, see Foglia (2009). Alessandri et al (2007) apply bimodal distributions to the system-wide banking assets in the United Kingdom. The first peak is associated with a healthy banking sector and a considerably smaller second peak in the extreme tail associated with outbreaks of systemic default. Assets distributions conditional on adverse events are simulated using four main elements, i.e. equity prices, interbank spreads, PDs, and market liquidity. The resultant distribution shows that the UK banking sector is adversely affected under stress.

Van den End et al (2006) implement an extended version of the Sorge and Virolainen (2006) VAR macroeconomic model, where a set of macroeconomic shocks are generated through a Monte Carlo simulation. Their approach differs from that of Sorge and Virolainen (2006) since multi-factor simulations are applied taking into account simultaneous changes in macroeconomic variables, and their interactions. The model is driven by the same variance-covariance structure of error terms as in Wilson (1997a), see Equation (1.9). This model can be summarized as a Value-at-Risk (VaR) model where all parameters are a function of macroeconomic variables as follows,

$$VaR_t(\tilde{y}_{t+1} \mid \tilde{x}_{t+1} > \overline{x}) = f\{\mathrm{E}_t(x_t); \mathrm{PD}_t(x_t); \mathrm{LGD}_t(x_t); \mathrm{Cov}_t(x_t)\},$$

where $\tilde{y}_{t+1} \mid \tilde{x}_{t+1} > \overline{x}$ represents the uncertain future realisation of the aggregate credit loss $(\tilde{y}_{t+1})$ for the financial system in the event of a simulated macroeconomic stress scenario. Loss distribution is simulated by a repetitive generation of future macroeconomic variables $(\tilde{x}_{t+1})$, and a VaR value can be read from the tail of this distribution. The VaR is determined by exposures $\mathrm{E}_t(x_t)$, probabilities of default $\mathrm{PD}_t(x_t)$ and loss given default $\mathrm{LGD}_t(x_t)$, which are all driven by macroeconomic variables $\tilde{x}_{t+1}$.

The interactions among macroeconomic variables is taken into account by the covariance matrix $\mathrm{Cov}_t(x_t)$.

Previous stress testing models in a South African context

Barnhill et al (2000) use a structural approach that models the combined effect of both market and credit risk in a South African banking system through a forward looking simulation of a set of market variables. The study uses the characteristics of South African aggregate banking sector to simulate a hypothetical portfolio consisting of 30 banks. These variables include interest rates, equity market indices, foreign exchange rates, gold price, house price index, and inflation rate. All the variables are modelled in a correlated fashion and the banks' assets and liabilities are revalued using the newly simulated variables. The simulation also incorporates credit ratings transition matrix to reflect a deterioration in credit quality, high volatilities, and correlations in a stress event.

Interest rates are simulated using a Hull and White (1990) model. Equity prices, real estate prices, exchange rates, and commodity prices are simulated in correlation with simulated spot interest rates, using a stochastic Brownian motion process.

The results show that market risk alone will not cause South African banks to fail, however, high concentration coupled with poor credit quality can substantially cripple banks in stress periods. This is mainly due to the fact that during stress periods the volatility and correlations of important market variables increase in absolute terms, reducing the diversification benefit and exacerbating the risk of holding a highly concentrated portfolio.

Havrylchyk (2010) investigates the effects of South African macroeconomic variables on credit losses through a multivariate regression model that regresses historical loan loss provisions to macroeconomic variables. The model employed is of a VAR type since it includes lags of each variable in the model. The study used the data submitted by the five largest South African banks to the regulator, namely the South African Reserve Bank (SARB) for the period 1994 - 2007. The five banks constitute approximately 92% of banking assets in South Africa, and so the study is representative of the country, see

Havrylchyk (2010) and South African Reserve Bank (2012). Statistically significant variables at 1%, 5%, and 99% include GDP growth, inflation rate, real interest rate, nominal property prices growth, real effective exchange rate (REER), gold price, and oil price.

Three stress scenarios were chosen, 1) worst historical scenario, 2) unfavourable change of two standard deviation from the current state, and 3) Expert Opinion scenario. The results show that loan loss provisions increase significantly under all three scenarios, and the most severe one is scenario 2. Despite the high credit losses the South African banks can absorb losses because of their high level of capital adequacy.

This dissertation presents an improved approach that can complement both Barnhill et al (2000), and Havrylchyk (2010), by incorporating the credit cycle into the stress testing framework. IMF's Financial Stability Report (2008), points out that South African banks are more exposed to credit risk, which could be exacerbated by adverse macroeconomic changes. This necessitates a more forward looking capital management process that can incorporate macroeconomic cyclicality by increasing capital buffers in a downturn, and reducing them in an upswing. Barnhill et al (2000) and Havrylchyk (2010) models have a common setback of not considering where the economy is in the cycle when stressing the banks' book. Our approach is built on findings by Fourie et al (2011), which is based on the VAR methodology to test the relationship between credit and the business cycle. Their analysis supports an existence of a two-way relationship between credit and insolvencies. The methodology used in this dissertation is a Vector Autoregressive (VAR) model that makes use of insolvencies and other macroeconomic variables to stress the credit book of a hypothetical bank.

Chapter 2 - Summary of the Basel Capital Accords: Basel I – III

This chapter gives a summary of how the Basel accord has evolved over time, starting with the first accord in the year 1988, up to the current Basel III accord. Constituents of capital, risk weights, and the final calculation of the minimum required regulatory capital are outlined in detail. The treatment of market risk and operational risk are also covered to give a complete view of how different risk types are accounted for in the Basel regulatory capital framework. Where possible, reference will be made to paragraphs in the original Basel document, Basel Committee on Banking Supervision (2006).

2.1. Basel I

The Basel Committee (Committee on Banking Regulations and Supervisory Practices) was established by the Central Bank Governors of the G-10 countries in 1974 as a result of international currency and banking crisis, mostly remembered by the failure of Bankhaus Herstatt in West Germany, Styger and Vosloo (2005). The committee serves as a forum where discussions regarding banking regulatory matters take place, among member countries. In December 1987, a consultative process started in all G-10 countries and a proposal for the Basel I accord was tabled.

This sections draws extensively from the Basel I accord document, International Convergence of Capital Measurement and Capital Standards (1988), which was approved and published to supervisory authorities worldwide. The two main aims of this accord were, firstly, to strengthen the soundness and stability of the international banking system; and, secondly, that the framework should be fair and have a high degree of consistency in its application to banks in different countries with a view to diminishing an existing source of competitive inequality among international banks.

The constituents of capital, risk weighting of assets, and the final calculation of the minimum required capital are summarized below as outlined in the document, Basel Committee on Banking Supervision (2006).

The constituents of capital

1) Core capital - The Committee considers that the key element of capital on which the main emphasis should be placed is equity capital and disclosed reserves. This key element of capital is the only element common to all countries' banking systems; it is wholly visible in the published accounts and is the basis on which most market judgements of capital adequacy are made; and it has a crucial bearing on profit margins and a bank's ability to compete. See Basel Committee on Banking Supervision (2006), Para. 49i.

2) Supplementary capital – This consists of undisclosed reserves and revaluation reserves.

    i. Undisclosed reserves: Under this heading are included only reserves which, though unpublished, have been passed through the profit and loss account and which are accepted by the bank's supervisory authorities.

    ii. Revaluation reserves:

    These can arise in two ways:

        i. From a formal revaluation, carried through to the balance sheets of the banks' own premises; or

        ii. From a notional addition to capital of hidden values which arise from the practice of holding securities in the balance sheet valued at historic costs.

        iii. General provisions/general loan-loss reserves: General provisions or general loan-loss reserves are created against the possibility of losses not yet identified. Where they do not reflect a known deterioration in the valuation of particular assets, these reserves qualify for inclusion in tier 2 capital.

    iii. Hybrid debt capital instruments: It has been agreed that, where these instruments have close similarities to equity, in particular when they are able to support losses on an on-going basis without triggering liquidation, they may be included in supplementary capital.

    iv. Subordinated term debt: subordinated term debt instruments with a minimum original term to maturity of over five years may be included within the supplementary elements of capital, but only to a maximum of 50% of the core capital element and subject to adequate amortisation arrangements. See Basel Committee on Banking Supervision (2006), Para. 49v.

3) Deductions from capital - These can be in the following ways:
   i. Goodwill, as a deduction from tier 1 capital elements;
   ii. Investments in subsidiaries engaged in banking and financial activities which are not consolidated in national systems.

   Basel Committee on Banking Supervision (2006), Para. 49xv.

Credit risk - risk weights

Minimum capital is calculated using a capital adequacy ratio, where a bank's capital divided by the risk-weighted assets should be at least 8%. Risk weighted assets is derived by multiplying the face value of assets by prescribed risk weighting percentages, depending on the asset type. Table 2 below summarises risk weights applied to different categories.

Risk-Weight Category	**Types of Assets Included in the Risk Category**
0%	Cash assets involving sovereigns
20%	Assets involving banks
50%	Loans secured by mortgages on residential property
100%	Assets involving businesses; personal consumer loans; assets involving non-governments (unless the transaction is denominated and funded in the same currency)

Table 2: Basel I on-balance sheet risk weighting percentages. BCBS (2001)

The Basel I accord also catered for off-balance sheet items. These refer to those items held by the bank, but do not appear in the bank's balance sheet. A risk weight of 100% was applied to the commercial letter of credit, and 20% to a letter of credit issued by banks.

Market risk treatment

In 1993, the Basel Committee on Banking Supervision proposed two alternative approaches to measure capital charges related to market risk activities in banks, i.e. the Standardised approach (SA), and Internal models approach (IMA). Amendment to the Capital Accord to Incorporate Market Risks (1996) was published by the Basel Committee on Banking Supervision, and both the SA and IMA approaches were incorporated into the Basel accord to address the gap regarding the treatment of market risk related activities in banks.

The two approaches are summarised by Lobanov (2011) as follows:

1. The standardised approach (SA) consists of the following components:
    - Interest risk rate in trading.
    - Equity risk in the trading book. (Equity risk in the banking book is covered either through deductions from total capital, for non-consolidated equity holdings in subsidiaries, or by credit risk capital charge - 100% risk weight for other equity investments).
    - Currency risk across the bank.
    - Commodity risk across the bank.
2. Internal models approach (IMA):
    - Under this approach, the bank's market risk capital charge (MRC) is based on the internal Value-at-Risk estimates:

      $$MRC = \max\left(k.\frac{1}{60}\sum_{i=1}^{60} VaR_{t-i}, VaR_{t-i}\right)$$

      This is subject to the following quantitative requirements:
        - Both VaR and MRC computed daily
        - 99% one-tail confidence level
        - 10-day holding period (scaling from shorter holding periods using $\sqrt{T/t}$ where possible)
        - 250 days minimum historical observation period
        - Multiplier k is set based on backtesting results and equals 3 for adequate ('green-zone') models, 3.4 to 3.85

for 'yellow-zone' models and 4 for inadequate ('red-zone') models

- Backtesting of VaR model conducted quarterly based on sample of past 250 trading days.

- Specific risk interest rate and equity risk should be captured by VaR model, otherwise capital surcharge applies.
- No model type is prescribed; model must not be used solely for capital calculations.
- Stress-testing scenarios and results must be regularly reported to the regulator.

After computing the total risk-weighted assets using relevant risk weights for on-balance sheet and off-balance sheet assets, the bank's capital can now be calculated as: Capital = (Tier 1 + Tier 2). This should equal to at least 8% of risk-weighted assets, with tier 1 equalling at least 4% of risk-weighted assets. This risk weighted assets number includes the market risk component as incorporated in 1996.

2.1. Basel II

In June 1999, the Basel Committee on Banking Supervision (BCBS) released a proposal to replace the Basel I accord with a revised framework, i.e. Basel II. The rationale for the new accord was to:

i. Introduce a more flexible and risk sensitive framework.
ii. Allow more advanced banks to estimate their risk parameters through their own internal methodologies.
iii. Encourage supervisory review, and market discipline.

The fundamental objective of the Committee's work to revise the 1988 Accord had been to develop a framework that would further strengthen the soundness and stability of the international banking system while maintaining sufficient consistency that capital adequacy regulation will not be a significant source of competitive inequality among internationally active banks. More generally, they have expressed support for improving capital regulation to take into account changes in banking and risk management practices while at the same time preserving the benefits of a framework that can be applied as uniformly as possible at a national level.

The first version of the new accord was published in International Convergence of Capital Measurement and Capital Standards (2004). A revised version then followed in November 2005, which did not have any new elements. The New Basel Capital Accord: an explanatory note (2001) gives a good summary of the transition from the Basel I to the Basel II accord. This section draws extensively from that paper:

Basel II is divided into 3 separate pillars:

1. Minimum capital requirements,
2. Supervisory review,
3. Market discipline.

Pillar 1: Minimum capital requirements

The Committee retained key elements of the 1988 capital adequacy framework, including the general requirement for banks to hold total capital equivalent to at least 8% of their risk-weighted assets; the basic structure of the 1996 Market Risk Amendment regarding the treatment of market risk; and the definition of eligible capital.

The main amendment in the calculation of capital adequacy ratio, from Basel I to II, is the denominator, which now incorporates Operational risk. The market risk component remained unchanged from the previous accord.

$$\frac{\text{Total Capital (Same as Basel I)}}{\text{Credit risk + Market risk (Same as Basel I) + Operational risk}} = \begin{array}{l}\text{Bank's Capital ratio} \\ \text{(minimum 8\%)}\end{array}$$

Unlike the previous accord, Basel II offers different types of approaches for capital charge calculation, for each risk type. Document, Basel Committee on Banking Supervision (2001), summarizes a list of approaches available for each risk type, depending on the bank's level of sophistication:

Credit risk

- Standardised Approach (a modified version of the Basel I approach): The standardised approach is conceptually the same as the previous Accord, but is more risk sensitive. The bank allocates a risk-weight to each of its assets and off-balance-sheet positions and produces a sum of risk-weighted asset values.

  Individual risk weights previously depended on the broad category of borrower (i.e. sovereigns, banks or corporates). Under the Basel II Accord, the risk weights are refined by reference to a rating provided by an external credit assessment institution (such as a rating agency) that meets strict standards. Basel Committee on Banking Supervision (2001).

- Foundation and Advanced Internal Rating Based Approach: Under the IRB approach, banks are allowed to use their internal estimates of borrower creditworthiness to assess the credit risk in their portfolios, subject to strict methodological and disclosure standards. Distinct analytical frameworks are provided for different types of loan exposures, for example corporate and retail lending, whose loss characteristics are different.

  Under the IRB approach, a bank estimates each borrower's creditworthiness, and the results are translated into estimates of a potential future loss amount, which form the basis of minimum capital requirements. The framework allows for both a foundation method and more advanced methodologies for corporate, sovereign and bank exposures.

  In the foundation methodology, banks estimate the probability of default associated with each borrower, and the supervisors supply the other inputs.

  In the advanced methodology, a bank with a sufficiently developed internal capital allocation process is permitted to supply other necessary inputs as well. Under both the foundation and advanced IRB approaches, the range of risk weights are far more diverse than those in the standardised approach, resulting in greater risk sensitivity.

Credit risk mitigation and securitisation: The new framework introduces more risk sensitive approaches to the treatment of collateral, guarantees, credit derivatives, netting and securitisation, under both the standardised approach and the IRB approach. Basel Committee on Banking Supervision (2001).

Market risk

The basic structure of the 1996 Market Risk Amendment regarding the treatment of market risk remains the same. In 2004, both the Standardised and Internal models approaches were incorporated into the Basel II accord.

Operational risk

Three different approaches of increasing sophistication are available for operational risk charge calculation:

- The basic indicator approach utilises one indicator of operational risk for a bank's total activity.
- The standardised approach specifies different indicators for different business lines.
- The internal measurement approach requires banks to utilise their internal loss data in the estimation of required capital.

The second pillar: supervisory review process

The supervisory review process requires supervisors to ensure that each bank has sound internal processes in place to assess the adequacy of its capital based on a thorough evaluation of its risks. The new framework stresses the importance of bank management developing an internal capital assessment process and setting targets for capital that are commensurate with the bank's particular risk profile and control environment. Supervisors would be responsible for evaluating how well banks are assessing their capital adequacy needs relative to their risks. This internal process would then be subject to supervisory review and intervention, where appropriate.

The third pillar: market discipline

The third pillar of the new framework aims to bolster market discipline through enhanced disclosure by banks. Effective disclosure is essential to ensure that market participants can better understand banks' risk profiles and the adequacy of their capital positions.

2.3. Basel III

In the wake of recent financial crises, the Basel Committee on Banking Supervision realised the need for a more stringent and robust regulatory framework. During the 2007 crisis, many banks experienced problems, despite being well capitalised because they did not manage liquidity well. One such bank was Northern Rock, a British bank known for becoming the first bank in 150 years to suffer a bank run after having had to approach the Bank of England for a loan facility. The crisis again drove home the importance of liquidity to the proper functioning of financial markets and the banking sector.

The introduction of Basel III is not a replacement to the existing Basel II accord, but rather an enhancement that focuses on quantity and quality of capital held by banks. The framework is expected to be phased in from 2013, with parallel runs up to 2019. The BCBS promulgated Basel III in September of 2010 with a document titled: A Global Regulatory Framework for More Resilient Banks and Banking Systems (2010). Among the most important parts of Basel III is its new definition of regulatory capital, which is more restrictive and emphasises greater quality.

Key aspects of Basel III

Basel III retains the tier 1 and tier 2 distinction, but limits their composition to higher-quality capital that is better able to absorb losses. Under Basel III, tier 1 capital must be mostly of 'core capital', which consists of equity stock and retained earnings. In addition, many items that were formerly included in a bank's capital calculation under Basel II, including some forms of subordinated debt, will be excluded under Basel III. Below is a summary of items revised from the previous accord, including the newly introduced components.

Capital

- Overall minimum capital remains unchanged at 8% of risk-weighted assets (RWA).

- Increase in common equity requirement from 2% to 4.5% of risk-weighted assets.
- Increase in tier 1 capital from 4% to 6% of risk-weighted assets.
- This leaves tier 2 to no more than 2% of RWA.
- Tier 1 capital can no longer include hybrid instruments.
- Tier 3 previously used for market risk has been eliminated altogether.

Capital buffers

- Introduction of Capital Conservation Buffer: designed to ensure that banks build up capital buffers outside periods of stress which can be drawn down as losses are incurred. This is set at 2.5% of the common equity tier 1.
- Introduction of Counter Cyclical buffer: The countercyclical buffer aims to ensure that banking sector capital requirements take into account the cyclical nature of the macro-financial environment in which banks operate. This ranges from 0 to 2.5% of RWA.

Risk management

- Credit Valuation Adjustment (CVA): Banks will be subject to a capital charge for potential mark-to-market losses associated with deterioration in the credit worthiness of the counterparty. This was not covered in the Basel II accord, which only considered counterparty default risk.
- Stressed parameters must be used to calculate the Counterparty Credit Risk.
- Effective Expected Positive Exposure (EPE) with stressed parameters to be used to address general wrong-way risk.
- Banks expected to keep track of trades at counterparty level and perform regular stress testing.
- Asset value correlation (AVC) multiplier of 1.25 will be used for exposures associated with large financial institutions.
- For netting sets containing one or more trades involving either illiquid collateral, or an OTC derivative that cannot be easily replaced, a supervisory floor of 20 business days is imposed for the margin period of risk.

Liquidity measures

- To promote short-term resilience of a bank's liquidity risk profile by ensuring that it has sufficient high quality liquid resources to survive an acute stress scenario lasting for one month, the Committee developed the Liquidity Coverage Ratio (LCR), which will only be implemented in 2015.
- Another newly introduced liquidity measure is the Net Stable Funding Ratio (NSFR). The NSFR requires a minimum amount of stable sources of funding at a bank relative to the liquidity profiles of the assets, as well as the potential for contingent liquidity needs arising from off-balance sheet commitments, over a one-year horizon. The NSFR aims to limit over-reliance on short-term wholesale funding during times of buoyant market liquidity and encourage better assessment of liquidity risk across all on- and off-balance sheet items.

Leverage

- The leverage ratio is intended to constrain the build-up of leverage in the banking sector, helping to avoid destabilising and deleveraging processes which can damage the broader financial system and the economy. A minimum tier 1 leverage ratio of 3% is being tested as of January 2013.

In additional to all of the items mentioned above, the BCBS have set higher reporting standards. For example, Banks need to monitor their maturity mismatches between assets and liabilities. Banks are also expected to perform regular stress testing on their portfolios. Basel III emphasises lower reliance on external rating agencies.

Transitional plans are in place to help active banks comply by 2019. National authorities have discretion to impose a shorter timeline where they see fit.

2.4. Chapter summary

In this chapter we presented, in chronological order, different changes to the Basel accord by the Basel Committee on Banking Supervision in order to strengthen the soundness and stability of the international banking system. For example Basel II introduced a more flexible and risk sensitive capital framework through the Advanced, Foundation, and Standardised approaches. Three approaches for Operational risk capital were also introduced in Basel II. While Basel III did not change the way capital is being calculated entirely, new liquidity measures and capital buffers were introduced to ensure that Banks maintain a high level of liquidity and reduce procyclicality in their capital.

Chapter 3 - Basel IRB capital and economic capital

This section demonstrates why internally calibrated PDs and LGDs, are such a critical part of capital calculation. Capital referred to here is both economic and regulatory capital. Although the main focus is on regulatory capital adequacy ratios, we show how economic capital calculated under the same IRB approach assumptions changes with stressed PDs and LGDs.

The last two sections of this chapter cover the modelling of both PDs and LGDs using market implied approaches. Hull and White's (2000) CDS pricing model is used for modelling PDs from CDS spreads. For LGDs, we use a jump-to-default Das and Hanouna (2009) model on a Cox, Ross and Rubinstein (1979) binomial tree.

3.1. Basel Advanced Internal Ratings Based (AIRB) approach

Basel Committee on Banking Supervision (2006), International Convergence of Capital Measurement and Capital Standards, has a section dedicated to credit risk's IRB approach with quantitative requirements that should be followed by banks that are approved to take this approach. The Basel Committee on Banking Supervision (2004) explains the technical details, as well as the underlying model.

According to the AIRB approach, banks rely on their own internal estimation of PDs and LGDs as main inputs into the regulatory capital calculation. This paper shows why these inputs are such a critical part of regulatory capital calculation in the IRB approach. In addition, we show how a small change in any of these inputs can impact the required capital. The stress testing approach presented here assumes the bank has already estimated their own PDs and LGDs internally.

Basel capital calculations are based on the assumptions that expected losses (EL) are provisioned for or priced into the products. Any further capital charges are thus only dependent on unexpected losses (UL) i.e. the difference between Value-at-Risk (VaR) at a predetermined confidence level based on a target credit rating's default probability, and EL. An expected loss formula is given by:

$$EL = EAD \times PD \times LGD\,.$$

- Exposure-at-default (EAD): is the outstanding amount of the obligor at the time of default.
- Probability of default (PD): is the obligor's 1-year default probability, depending on the obligor's credit rating.
- Loss-given-default (LGD): is the portion of the outstanding amount (EAD) that will not be recovered when the obligor defaults.

Figure 2 demonstrates the EL concept better.

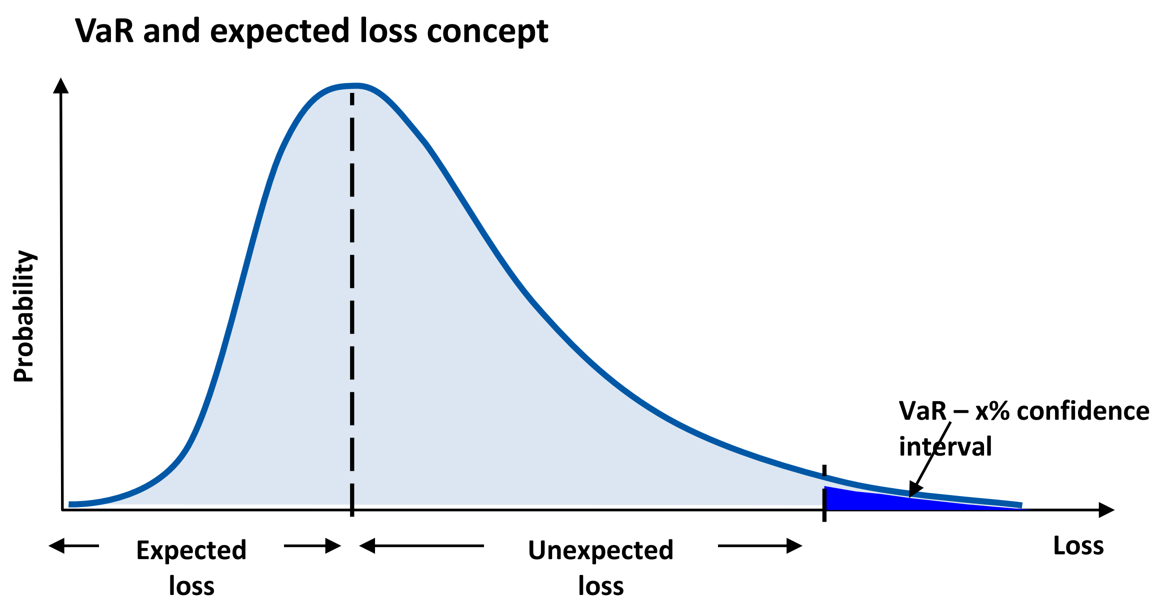

Figure 2: Illustration of expected loss and Value-at-Risk concepts

The ASRF Basel II model is based on the two assumptions from Gordy (2002), i.e. capital charges are portfolio invariant only if (a) there is only a single systematic risk factor driving correlations across obligors, and (b) no exposure in a portfolio accounts for more than an arbitrarily small share of total exposure. Consider a portfolio with N obligors and a single-step model. Without loss of generality, assume that each obligor j has (unconditional) default probability PD_j and a single loan with loss given default and exposure at default given by LGD_j and EAD_j respectively. Each obligor's (j) credit losses at the end of the horizon are driven by a single systemic risk factor. Obligor j defaults when its credit index falls below a given threshold $\Phi^{-1}(PD_j)$. Vasicek (2002), uses the property of jointly standard normal with equal pairwise correlation R, and shows that this creditworthiness index Y can be written as follows,

$$Y_j = \sqrt{R_j} Z + \sqrt{1 - R_j} \varepsilon_j , \tag{3.1}$$

where,

Z: is a standard Normal variable representing the single systematic, economy-wide factor.

R_j: is the correlation factor of obligor j to the systematic factor Z.

ε_j: is an independent standard Normal variable representing the idiosyncratic movement of the obligor's creditworthiness.

Since Y_j: is a linear combination of two normally distributed variables, it is also normally distributed. Given the above, Vasicek, (2002), shows that probability of default $p_j(Z)$, for obligor j, that is conditional on the systematic factor Z can be derived as,

$$p_j(z) = p[\mathrm{Y}_j < \Phi^{-1}(PD_j) \mid Z]$$

$$p_j(z) = p[\sqrt{R_j} Z + \sqrt{1 - R_j} \varepsilon_j < \Phi^{-1}(PD_j) \mid Z]$$

$$p_j(z) = p\left[\varepsilon_j < \frac{\Phi^{-1}(PD_j) - \sqrt{R_j} Z}{\sqrt{1 - R_j}} \mid \mathrm{Z} \right]$$

$$p_j(z) = \Phi\left(\frac{\Phi^{-1}(PD_j) - \sqrt{R_j} Z}{\sqrt{1 - R_j}} \right) , \tag{3.2}$$

where,

$\Phi()$: is the cumulative standard normal distribution function.

$\Phi^{-1}()$: is the inverse of the cumulative standard normal distribution function.

Because AIRB Basel capital covers the difference between expected losses (EL) and unexpected losses (UL), the capital formula is actually the difference between losses calculated with through-the-cycle PDs at 99.9%, and expected losses. This implies that from Equation (3.2) above, Z would be a standard normal variable from the 99.9th percentile of a normal distribution. LGDs in this case would be downturn or conservative LGDs. Capital, as a percentage of exposure-at-default (EAD), is calculated as follows:

$$K = \sum_j [LGD_j \times \Phi\left(\Phi^{-1}(PD_j) + \frac{\sqrt{R_j} \times \Phi^{-1}(0.999)}{\sqrt{1-R_j}} \right) - LGD_j \times PD_j] \times (1-1.5 \times b(PD_j))^{(-1)} \times (1+(M_j - 2.5) \times b(PD_j)) . \quad (3.3)$$

k : capital as a percentage of EAD,

LGD: downturn LGDs derived from expected LGDs,

$\Phi()$: standard normal distribution applied to a conservative value of a systematic factor,

R_j : corporate asset class correlation representing the systematic risk factor for obligor j, calculated as follows:

$$R_j = 0.12 \times \frac{1 + e^{(-50 \times \mathrm{PD}_j)}}{1 - e^{(-50)}} + 0.24 \times \left(1 - \frac{1 - e^{(-50 \times \mathrm{PD}_j)}}{1 - e^{(-50)}} \right) \quad (3.4)$$

$\Phi^{-1}()$: inverse of standard normal distribution,

$b(PD)$: smoothed maturity adjustment, given as $(0.11852 - 0.05478 \times log(PD))^2$,

M : an instrument's time to maturity.

Minimum required capital can then be calculated as: Capital = K x EAD. We will look at capital after stressing the PDs and converting the long term average or expected LGDs to downturn LGDs. Consequently the correlation factor R automatically depends on PDs and LGDs.

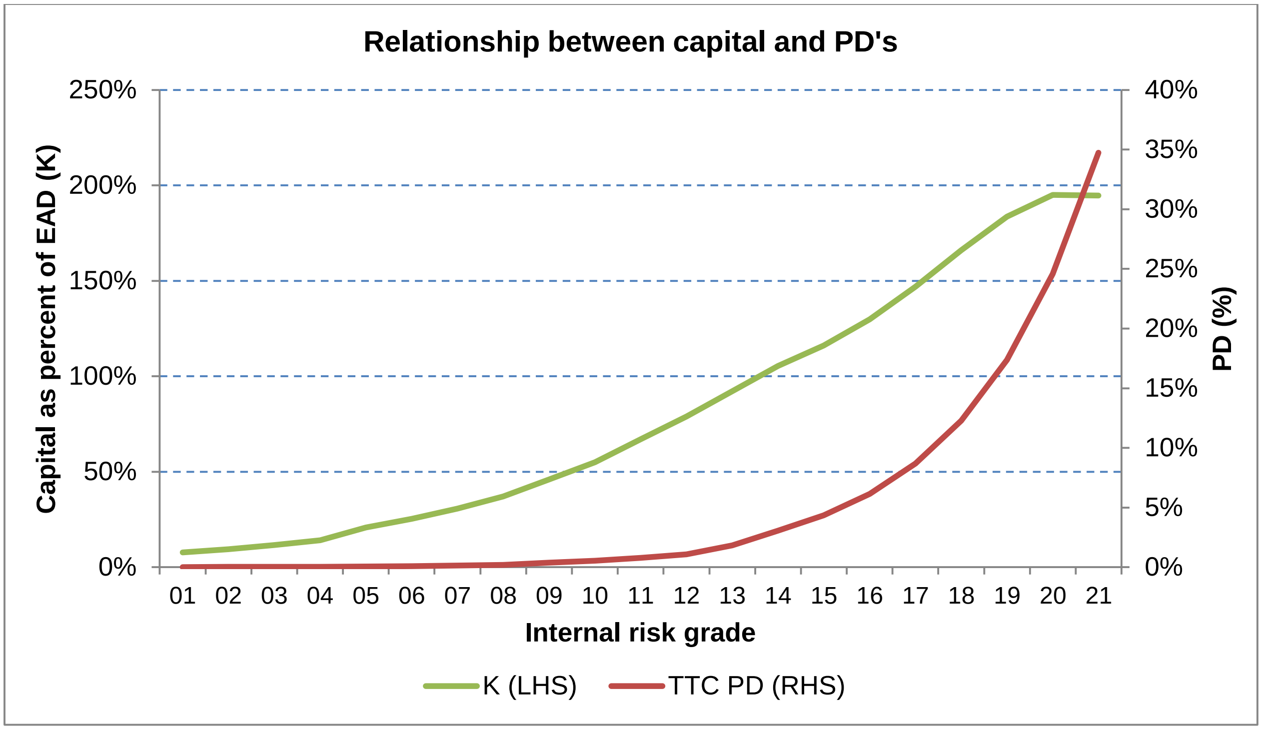

Figure 3: Relationship between PDs and Basel required capital as a percentage of exposure

In Figure 3 it appears that the required capital increases exponentially as the risk grade deteriorates, at any fixed LGD point. These are internal risk grades mapped to Moody's ratings (See Appendix, Table 21).

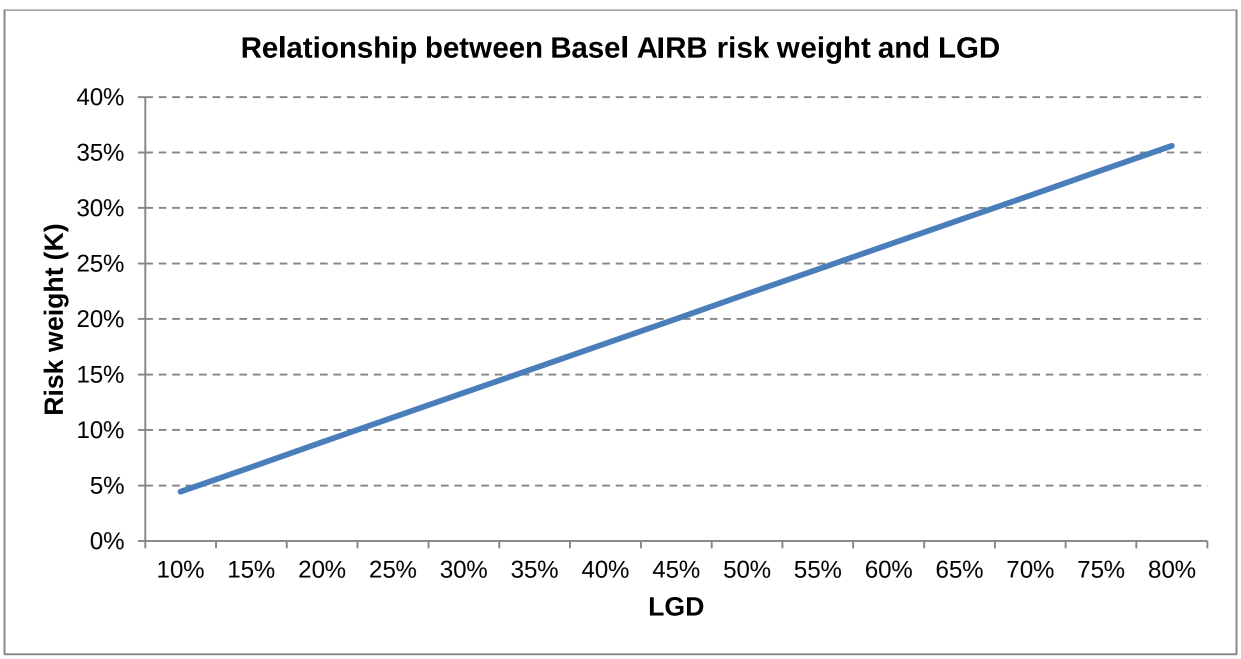

Figure 4: Relationship between LGDs and required capital

Unlike the PDs, Figure 4 shows that LGDs exhibit a linear relationship with the required capital when keeping the PD constant.

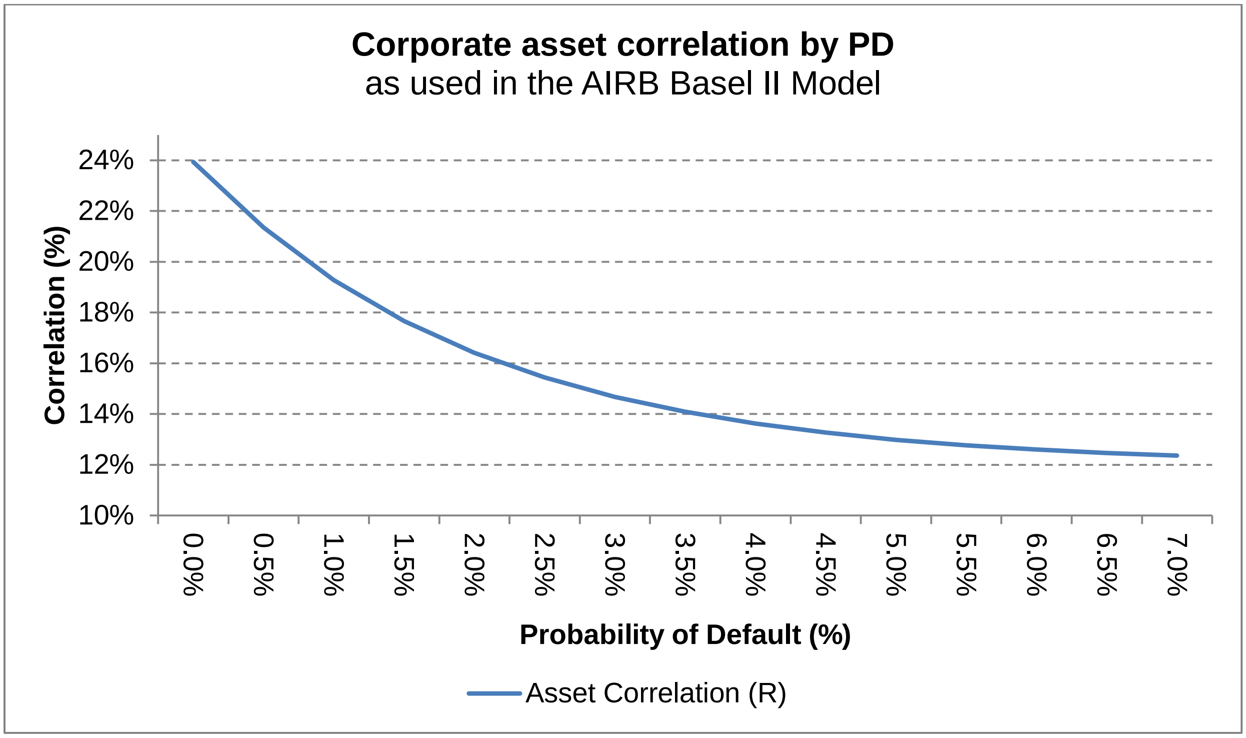

Figure 5: Relationship between Basel corporate asset correlation and PDs

Figure 5 presents the inverse relationship between PDs and the asset correlation factor (R), where (R) decreases as PDs increase. This also explains why the required capital (K) in Figure 3 above remains flat in the last two risk grades. This is because the incremental effects of PDs, for higher risk grades, are neutralised by the reducing (R).

3.2. Economic capital under IRB Basel assumptions

Economic capital (ECap) is mainly used by banks to assess the potential size of unexpected losses that could arise within a predefined horizon, usually 1 year. Since ECap is not imposed by the regulator, it is mostly used by more advanced banks as an internal measure of risk. Most common benefits of ECap include risk aggregation, risk-adjusted performance measures (RAPMs), pricing, and active portfolio risk management. The calculation of economic capital is mainly driven by the risk parameters in the bank's portfolio. The main input parameters include PDs, LGDs, and maturities.

This section draws extensively from Mausser and Rosen (2008) wherein economic credit capital allocation and risk contributions, are discussed in detail. Gordy (2002) demonstrates that ratings based capital rules can be reconciled with the general class of credit VaR models. Since economic capital is defined as the difference between VaR at a specified percentile and expected losses, the incremental effects of stressed PDs and LGDs should have the same impact on both IRB regulatory capital and economic capital, under the same IRB assumptions.

Economic capital can be loosely defined as the amount of capital calculated to absorb large unexpected losses, at a confidence level associated with an institution's target credit rating, over a certain time horizon. Because expected losses are normally priced into products or provisioned for, Economic capital is typically calculated as Value-at-Risk (VaR) minus the expected loss (EL), at a confidence level α:

$$EC_{\alpha} = VaR_{\alpha} - EL \tag{3.5}$$

Using the same creditworthiness index analogy as derived in section 2.1, and redefining each obligor's creditworthiness index in Equation (3.1) as:

$$Y_j = R_j Z + \sqrt{1 - R_j^{\,2}}\,\varepsilon_j \,.$$

Portfolio VaR can be defined as:

$$VaR_{\alpha} = \sum_j LGD_j \times EAD_j \times \Phi\left(\frac{\Phi^{-1}(PD_j) - R_j Z(\alpha)}{\sqrt{1 - R_j^2}}\right), \tag{3.6}$$

where,

PD is defined as Equation (3.2) above, i.e. conditional on a standard normal variable Z (α), predefined at a specific confidence level α. From this it follows economic capital can be calculated as:

$$EC_{\alpha} = \sum_j LGD_j \times EAD_j \times \left[\Phi\left(\frac{\Phi^{-1}(PD_j) - R_j Z(\alpha)}{\sqrt{1 - R_j^2}}\right) - PD_j\right] \tag{3.7}$$

This is simply unexpected loss using Equation (3.6), minus expected loss using expected PDs, instead of conditional PDs. Notice that Equations (3.3) and (3.7) are the same in principle, with the exception of maturity adjustment applied to Equation (3.3).

It should be noted that the incremental effect of stressed PDs and LGDs on regulatory and economic capital are only comparable because of the same assumptions under which both are being calculated. In actual portfolios where concentrations and multiple risk factors affect the bank's portfolio, economic capital will come out different to the IRB regulatory capital. Economic capital is usually calculated through Monte Carlo simulations, where a portfolio is simulated forward with a predefined time horizon. A correlation matrix is incorporated into the simulation to account for multiple risk factors, i.e. counterparty, country, or industry correlations, see Mausser and Rosen (2008).

3.3. Internal calibration of probabilities of default (PD)

The South African Reserve Bank, Accord Implementation Forum (2006), stipulates Basel II requirements for those banks that have been approved to use the IRB approach to calculate their regulatory capital. This section addresses internal modelling of PDs which could be either point-in-time (PIT) or TTC by design. The study suggests that Basel acknowledges different modelling methodologies used in different banks around the world, and concludes that most South African banks follow an approach which is some hybrid between TTC and PIT.

The South African Reserve Bank further recommended that banks estimate their own internal PDs for their corporate customers by using an internal rating scale that is mapped to the rating scale used by the three international rating agencies i.e. Fitch, S&P, and Moody's. Once the mapping is completed, empirical default probabilities based on each rating bucket can be derived using different statistical and mathematical techniques. Packer and Taravesh (2007), look into how these three rating agencies model default probabilities. A table showing the bank's internal risk grades mapped to Moody's and S&P ratings is presented in the Appendix, Table 21.

There are two main approaches for credit risk modelling, which can eventually be used to derive default probabilities associated with specific counterparties, i.e. reduced form and structural approaches. Jarrow and Protter (2004), compare these models as follows:

1. Structural models:

These models assume complete knowledge of a very detailed information set, similar to that held by the firm's managers. In most cases, this informational assumption implies that a firm's default time is predictable. These models originated with the risky debt model of Black and Scholes (1973), and Merton (1974).

Structural models are generally used to price corporate bonds based on the internal structure of the company. They therefore require information about the balance sheet of the firm and can be used to establish a link between pricing in the equity and debt markets. Their biggest setback is therefore the fact that company information, which includes the financials, are only published several times in a year, in most cases bi-annually. This means the model can be outdated when there is new information about the company that has not been published.

2. Reduced form models:

Contrary to structural models that assume knowledge of detailed information about the firm, these models rely on publicly available market prices to model the probability of the credit event itself. The most widely used reduced-form approach is based on the work of Jarrow and Turnbull (1995), who characterise a credit event using a Poisson process. Hull and White (2000) derived a valuation formula for a plain vanilla CDS contract which is widely used today.

Reduced form models also generally have the flexibility to refit the prices of a variety of credit instruments of different maturities. They can also be extended to price more exotic credit derivatives. Because market prices are readily observable and attainable, this approach is usually preferred to structural models. Jarrow and Protter (2004), argue that structural models can be transformed into reduced form models as the information set changes and become less refined, from that observable by the firm's management to that which is observed by the market.

As argued by Jarrow and Protter (2004), the choice of a model will depend on the available information. Since market prices are always available and continually updated, we recommend a reduced form approach using CDS prices available in the market. This section demonstrates the derivation of a PIT PD from CDS spreads, and eventually show how these PIT PDs can be converted to a TTC PDs, or vice versa. CDS prices are chosen over bond prices because the CDS market is the most responsive indicator of corporate credit risk available. This follows mainly from the fact that the structure of a CDS contract separates the credit risk component of the reference obligation(s) from other risks such as interest rate and currency risk.

Hull and White (2000) derive a valuation formula for a plain vanilla CDS contract, under the following assumptions: default events, treasury interest rates, and recovery rates are mutually independent, and that the claim in the event of default is the face value plus accrued interest. The following parameters are defined for notation:

T: Life of credit default swap or maturity,

$q(t)$: Risk-neutral default probability density at time t,

R: Expected recovery rate on the reference obligation in a risk-neutral world,

$u(t)$: Present value of payments at the rate of \$1 per year on payment dates between time zero and time t,

$e(t)$: Present value of an accrual payment at time t equal to $t-t^*$ where t^* is the payment date immediately preceding time t,

$v(t)$: Present value of \$1 received at time, t,

w: Total payments made by credit default swap buyer,

s: Value of w that causes the credit default swap to have a value of zero,

π: Risk-neutral probability of no credit event during the life of the swap,

$A(t)$: Accrued interest on the reference obligation at time t as a percent of face value.

The value of π is one minus the probability that a credit event will occur by time T. It can be calculated from $q(t)$: as follows:

$$\pi = 1 - \int_0^T q(t)dt \tag{3.8}$$

The payments last until a credit event or maturity T, whichever comes first. If a default occurs at time t (t < T), the present value of the payments is $w[u(t)+e(t)]$. If there is no default before maturity, the present value of the payments is given as $wu(t)$.The expected present value of the payments is shown to be:

$$w\int_0^T q(t)[u(t)+e(t)]dt + w\pi u(T) \tag{3.9}$$

Given the assumption that the claim amount is equal to face value, plus accrued interest, Hull and White (2000) shows that the risk-neutral expected payoff from the CDS is:

$$1-[1+A(t)]R = 1-R-A(t)R \tag{3.10}$$

The present value of the expected payoff from the CDS is:

$$\int_0^T [1-R-A(t)R]q(t)v(t)dt \tag{3.11}$$

And the value of the credit default swap to the buyer is the present value of the expected payoffs from the CDS minus the expected present value of the payments made by the buyer or

$$\int_0^T [1-R-A(t)R]q(t)v(t)dt - w\int_0^T q(t)[u(t)+e(t)]dt + w\pi u(T) \tag{3.12}$$

Equating this equation to zero, and solving for w yields s that makes the CDS zero as:

$$s = \frac{\int_0^T [1-R-A(t)R]q(t)v(t)dt}{\int_0^T q(t)[u(t)+e(t)]dt + \pi u(T)} \tag{3.13}$$

The variable s is referred to as the credit default swap spread or CDS spread. It is the total of the payments per year, as a percent of the notional principal, for a newly issued credit default swap. Through an exercise of bootstrapping spread curves for different tenors, default probabilities can be derived by a relationship between spreads and default probabilities.

Figure 6 shows what a typical spread curve looks like, with the horizontal axis representing an external rating, and CDS spread is presented on the vertical axis. Spread curves can be extracted using Equation (3.13) above, for different tenors. The resultant spreads can then be used to derive point-in-time default probabilities, which can be adjusted accordingly to become through-the-cycle.

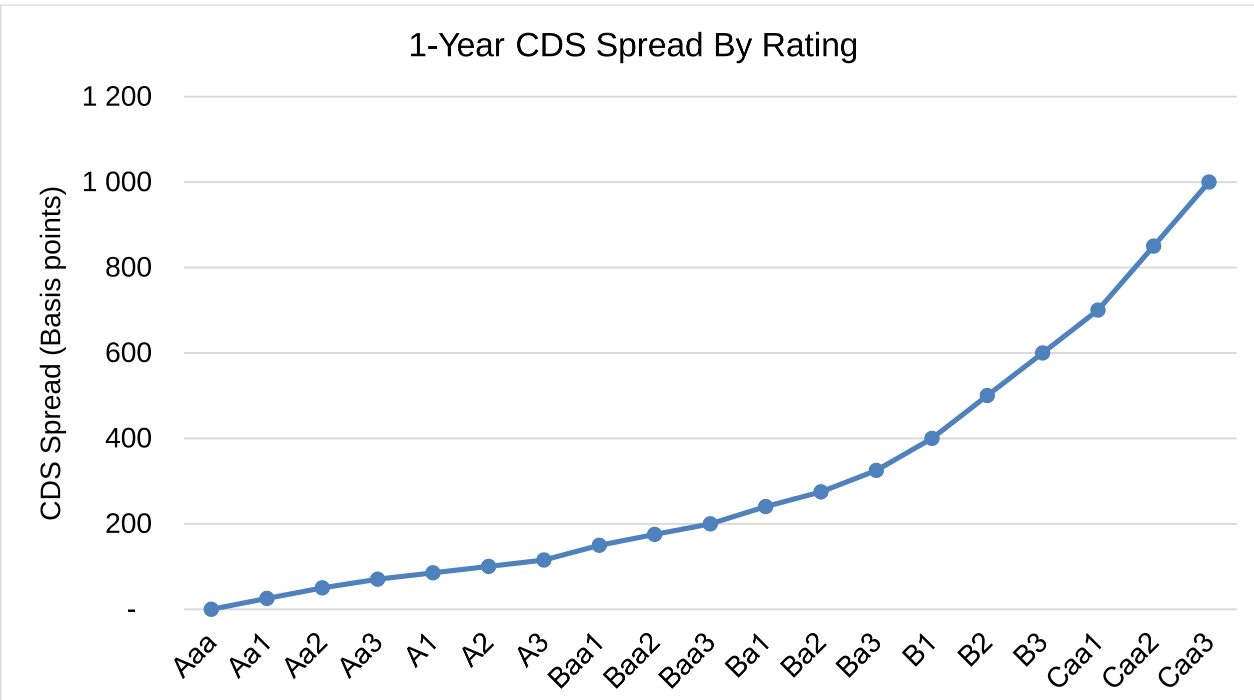

Figure 6: Typical relationship between CDS spread and external ratings. (Damodaran, 2012)

Hull et al (2005) show that the best approximation of spread implied default intensity is given by the relationship:

$$\lambda = \frac{s}{1-R},$$

where, λ represents a default intensity for the given spread. Probability of default curves can be derived using the following formula, for different tenors:

$$PD = 1 - e^{-\lambda T},$$

where T represents maturity, see Chan-Lau J (2006).

Since spread based PDs are point-in-time, they will have to be converted to through-the-cycle PDs for capital calculation. Carlehed and Petrov (2012) show that TTC PDs can be derived from PIT PDs. As shown in Section 3.1, a point-in-time probability of default that is conditional on the credit cycle can be derived from the through-the-cycle PD as follows:

$$p_j(z) = \Phi\left(\frac{\Phi^{-1}(PD_{TTC\,j}) - \sqrt{R_j}Z}{\sqrt{1-R_j}}\right).$$

This is assuming the creditworthiness of each obligor event is driven by an index Y made of two correlated normally distributed variables as follows:

$$Y_j = \sqrt{R_j}Z + \sqrt{1-R_j}\varepsilon_j .$$

At a portfolio level, we therefore drop the obligor index j, and solving for PD_{TTC} yields:

$$PD_{TTC} = \Phi\left(\Phi^{-1}\left(PD_{PIT}\right)\sqrt{1-R} + \sqrt{R}Z\right),$$

where, $\Phi()$, $\Phi^{-1}()$, Z, R, and ε are still defined as per Section 3.1. Therefore given a PIT PD can be converted to TTC PD, and vice versa using this relationship. Figure 7 shows typical cumulative default rates by rating class.

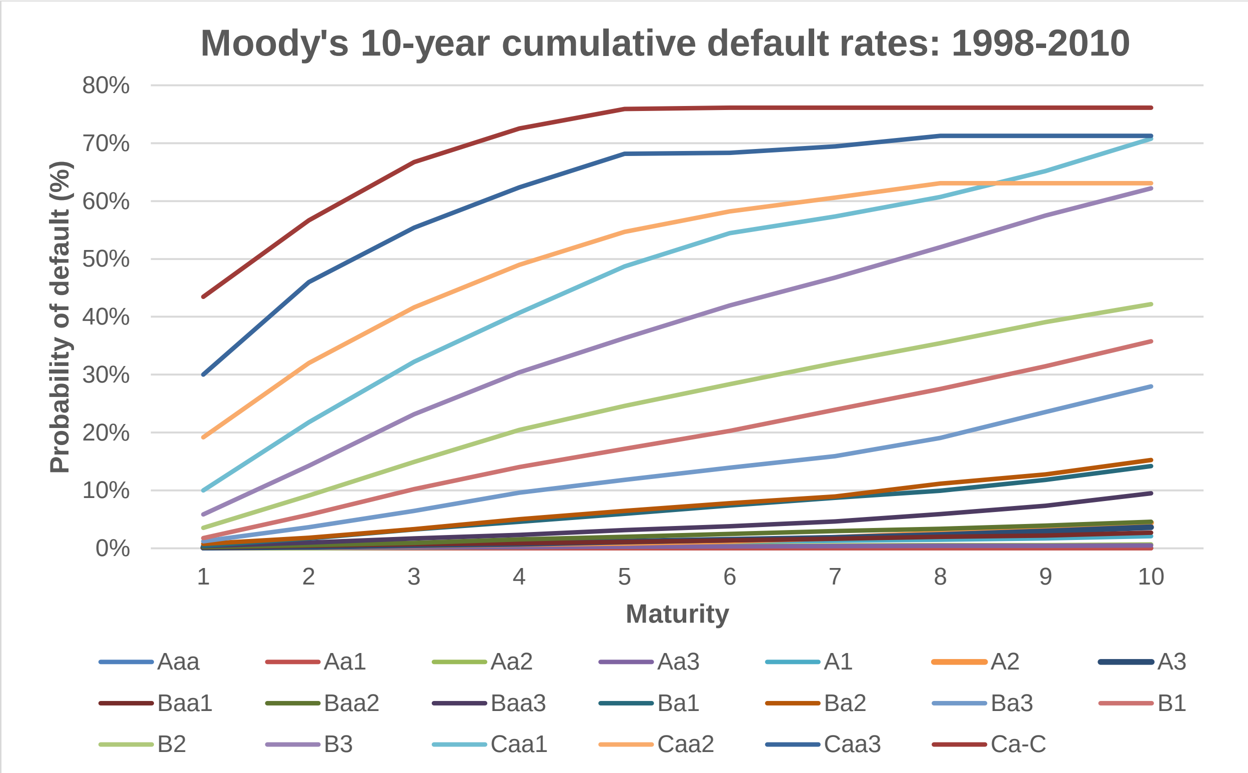

Figure 7: Corporate 10-year cumulative default rates by rating. (Moody's, 2011)

A table showing the bank's internal risk grades mapped to Moody's ratings is presented in the Appendix section in Table 21.

3.4. Internal calibration of loss given default (LGD)

Basel requires downturn LGDs to be used for IRB capital calculation, these are derived from expected LGDs which is the long run average calculated as a long term average through time, see Basel Committee on Banking Supervision (2006), International Convergence of Capital Measurement and Capital Standards, a Revised Framework, Comprehensive Version. Similar to PDs, banks that are approved for the IRB approach are allowed to estimate their own LGDs internally using the bank's historical data. The resultant LGD should then be adjusted to the downturn LGD, which means they should be stressed to reflect unfavourable economic conditions.

LGD models are used to estimate the proportion of the outstanding amount that will not be recovered when the counterparty defaults. According to the Bank for International Settlements (BIS), default is a situation when an obligor is 90 days past due on any credit obligation. Schuermann (2004), summarises the three main methods of measuring an LGD:

- ***Market LGD***: observed from market prices of defaulted bonds or marketable loans soon after the actual default event.
- ***Workout LGD***: The set of estimated cash flows resulting from the workout and/or collections process, properly discounted, and the estimated exposure.
- ***Implied Market LGD***: LGDs derived from risky (but not defaulted) bond prices using a theoretical asset pricing model.

Dwyer and Korablev (2009), describe the model used by Moody's to estimate LGDs across geographies using historical recovery data. They show that while debt type and seniority are important dimensions in LGD classification, there are many other combinations of dimensions that can be used. A typical corporate LGD have dimensions that can influence the LGD level, such as industry sector, company size, and credit rating. These factors are summarised in Table 3.

Factors External to the Issuer	**Factors Specific to the Issuer**	**Factors Specific to the Issue**
Geography	Distance-to-default (public firms)	Debt type
Industry	Probability of default or leverage (private firms)	Relative standing in capital structure
Credit cycle stage	---	Collateral

Table 3: Drivers of recovery. (Moody's KMV LossCalc V3.0, 2009)

Homogeneous LGD pools can be created using the dimensions above, and more dimensions can be used if homogeneity can be proved to exist within pools. The next section recommends a model that can be used to estimate LGDs.

Implied Market LGD modelling

For internal LGD modelling, we choose the implied market approach which is consistent with the one used to derive PDs in Section 3.4. This is a reduced form jump-to-default (JTD) model by Das and Hanouna (2009), which relies on observed CDS spreads to extract term structures of both recoveries, and probabilities of default. A theoretical LGD formula is given by:

$$LGD_t = 1 - \frac{(R_t - c_t)}{EAD_t},$$

EAD_t : refers to exposure at time of default t,

R_i : the recovered amount discounted back to the time of default,

c_i : the costs associated with the recovery process discounted back to the time of default.

The costs component is more relevant when using the workout LGD approach. In our model the variable of interest is the recovery rate, which is the recovered amount divided by the exposure-at-default (EAD). The LGD formula, disregarding costs then becomes:

$$LGD = 1 - R \tag{3.14}$$

Das and Hanouna (2009), model is based on the following principles of a CDS contract. They assume an N period model, indexed by j = 1,..., N. With each period consisting of length h, of units of years; and it is also assumed that h is the coupon frequency in the model. Time intervals in the can therefore be presented as $\{(0,h),(h,2h),...,((N-1)h,Nh)\}$. The corresponding end of period maturities are $T_j = jh$.

All $f((j-1)h, jh) \equiv f(T_{j-1}, T_j)$, represents Risk free forward interest rates, i.e. the rate over the jth period in the model. And this can be shortened to f_j , the forward rate applicable to the jth time interval. The discount functions are presented as functions of the forward rates as follows,

$$D(T_j) = \exp\left\{-\sum_{k=1}^{j} f_k h\right\},$$

which is the value of \$1 received at time T_j .

For a given firm, Das and Hanouna (2009) show that default is likely with an intensity denoted as $\xi_j \equiv \xi(T_{j-1}, T_j)$, constant over forward period j. Given these intensities, they define the survival function of the firm as

$$Q(T_j) = \exp\left\{-\sum_{k=1}^{j} \xi_k h\right\}.$$

They also assume that at time zero, a firm is solvent, which implies that, $Q(T_0) = Q(0) = 1$.

Their model is based on the usual credit default swap, with a bond or loan as an underlying. The periodic premium payments by the buyer are denoted as a "spread" CN, which represent an annualised percentage of the nominal value of the contract. For simplicity nominal value is assumed to be \$1. They further assume that defaults occur at the end of the period, and that premiums will be paid until the end of the period. Since premium payments are made as long as the reference instrument survives, the expected present value of the premiums paid on a default swap of maturity N periods is shown to be:

$$A_N = C_N h \sum_{j=1}^{N} Q(T_{j-1}, T_j) .$$

This accounts for the expected present value of payments made from the buyer to the seller.

The other possible payment on the default swap arises in the event of default, and goes from the seller to the buyer. The expected present value of this payment is shown to depend on the recovery rate denoted as $\phi_j \equiv \phi(T_{j-1}, T_j)$, which is the recovery rate in the event of a default in period j. The loss payment on default is then becomes $(1-\phi_j)$, for every \$1 of the notional principal. Das and Hanouna (2009), assume the "recovery of par" (RP) convention.

The expected loss payment in period j is based on the probability of default in period j, assuming no default occurred before. This probability is given by the probability of surviving till period (j -1) and then defaulting in period j, as follows:

$$Q(T_{j-1})(1-e^{-\xi_j h}).$$

Therefore, applying the discount function and loss payment on default, the expected present value of loss payments on a default swap of N periods equals the following:

$$B_N = \sum_{j=1}^{N} Q(T_{j-1})(1-e^{-\xi_j h})D(T_j)(1-\phi_j).$$

The fair pricing of a default swap must be such that the expected present value of payments made by buyer and seller are equal, i.e. $A_N = B_N$.

Jump-to-default model

Das and Hanouna (2009) define the inputs to their model as:

1. The term structure of CDS swap rates, C_j: j = 1,…,N;
2. Forward risk free interest rates f_j: j = 1,…,N;
3. The stock price S and its volatility σ.

The outputs from the model are:

1. Implied functions for default intensities and recovery rates, and
2. The term structures of forward default probabilities λ_j and forward recovery rates ϕ_j.

The single driving state variable in the model is the stock price S. Its stochastic behaviour is modelled on a Cox-Ross-Rubinstein binomial tree with an additional feature: the stock can jump to default with probability λ, where λ is state-dependent. Hence, from each node, the stock will proceed to one of three values in the ensuing period,

$$S \rightarrow \begin{cases} Su = Se^{\sigma\sqrt{h}} & w/\,prob \quad q(1-\lambda) \\ Sd = Se^{-\sigma\sqrt{h}} & w/\,prob \quad (1-q)(1-\lambda) \\ 0 & w/\,prob \quad \lambda \end{cases}$$

The stock can go up by factor $u = e^{\sigma\sqrt{h}}$ or fall by factor $d = e^{-\sigma\sqrt{h}}$, conditional on no default. The third branch assumes the recovery on equity is zero in the event of default.

This creates the jump-to-default (JTD) feature of the model. $\{q, 1-q\}$ are the branching probabilities when the default does not occur. Das and Hanouna (2009) argue that if f is the risk free rate of interest for the period under consideration, then under risk-neutrality, the discounted stock price must be a martingale, which allows them to imply the following jump-compensated risk-neutral probability:

$$q = \frac{R/(1-\lambda) - d}{u - d}, \quad R = e^{fh}.$$

Each node on the tree is denoted by the index [i; j], where j indexes time and i indexes the level of the node at time j. The initial node is therefore the [0; 0] node. At the end of the first period, we have 2 nodes [0; 1] and [1; 1]; there are three nodes at the end of the second period: [0; 2], [1; 2] and [2; 2], and so on. There are different default probability $\lambda[i, j]$ at each node. Hence, the default intensity is assumed to be dynamic with time and state. Further, for any reference instrument, a recovery rate at each node, denoted as $\boldsymbol{\phi[i, j]}$ is applied, which again, is dynamic over [i; j]. Das and Hanouna (2009) define functions for the probability of default and the recovery rate are defined as follows:

$$\lambda[i, j] = 1 - e^{-\xi[i,j]h}, \quad \xi[i, j] = \frac{1}{S[\mathrm{i, j}]^{b}},$$

$$\phi[i, j] = N(a_0 + a_1 \lambda[i, j]),$$

$$S[i, j] = S[0,0]u^{j-i}d^{i} = S[0,0]\exp[\sigma\sqrt{h}(\mathrm{j} - 2\mathrm{i})],$$

where N(.) is a cumulative normal distribution. Thus, the default probabilities and recovery rates at each node are specified as functions of the state variable S[i; j], and are parsimoniously parameterised by three variables: $\{a_0, a_1, \mathrm{b}\}$. Functional specifications for λ and ϕ result in values that remain within the range (0,1). The intermediate variable ξ is the hazard rate of default. When the stock price goes to zero, the hazard rate of default ξ becomes infinite, i.e. immediate default occurs. And as the stock price gets very high, ξ tends towards zero.

Given the values of $\{a_0, a_1, \mathrm{b}\}$, the jump-to-default tree may be used to price CDS contracts. This is done as follows. The fair spread C_N on a N-period CDS contract is that which makes the present value of expected premiums on the CDS, denoted $A_N[0,0]$, equal to the present value of expected loss on the reference security underlying the CDS, $B_N[0,0]$. These values may be computed by recursion on the tree using calculations shown above. Values of $\lambda[i,j]$ and $\phi[\mathrm{i},\mathrm{j}]$ are used on the tree to compute the fair CDS spreads by backward recursion.

$$A[i,j] = C_N / R + \frac{1}{R}\{q[i,j](1-\lambda[i,j])A[i,j+1] + (1-q[i,j])(1-\lambda[i,j])A[i+1,j+1]\},$$

$$\mathrm{B}[i,j] = \lambda[i,j](1-\phi[i,j]) + \frac{1}{R}\{q[i,j](1-\lambda[i,j])\,\mathrm{B}[i,j+1] + (1-q[i,j])(1-\lambda[i,j])\,\mathrm{B}[i+1,j+1]\},$$

for all N, i.

The recursion ends in finding the values of A[0,0] and B[0,0]. The fair spread CN is the one that makes the initial present value of expected premiums A[0,0] equal to the present value of expected losses B[0,0]. The term structure of fair CDS spreads may be written as $C_j(a_0, a_1, b) \equiv C_j(S, \sigma, f, a_0, a_1, b), \mathrm{j} = 1, \ldots, N$. The model parameters are fitted by solving the following least-squares program:

$$\frac{min}{a_0, a_{1,} b}\ \frac{1}{N}\sum_{j=1}^{N}[\mathrm{C}_j(\mathrm{a}_0, \mathrm{a}_1, \mathrm{b}) - C_j^0]^2,$$

where $\{C_j\}, \mathrm{j} = 1 \ldots \mathrm{N}$, are observable market CDS spreads, and $C_j(a_0, a_1, b)$ are model fitted spreads. This provides the root mean-squared fit of the model to market spreads by optimally selecting three model parameters $\{a_0, a_1, \mathrm{b}\}$. Once parameters have been calibrated, values of $\lambda[i,j]$ and $\phi[\mathrm{i},\mathrm{j}]$ at each node of the tree, can be computed. The forward curve of default probabilities λ_j and recovery rates ϕ_j are defined as the set of expected forward values:

$$\phi_j = \sum_{j=1}^{N} p[\mathrm{i},\mathrm{j}]\phi[\mathrm{i},\mathrm{j}], \forall\, \mathrm{j},$$

$$\lambda_j = \sum_{j=1}^{N} p[i,j]\lambda[i,j], \forall j \quad ,$$

where $p[i,j]$ represent the total probability of reaching node $[i,j]$ on the tree, via all possible paths.

Having used Das and Hanouna's (2009) model to derive recovery rates, we can go back to our initial LGD formula, Equation 3.14, and substitute the recovery term structure to calculate LGD as follows:

$$LGD_t = 1 - \phi_t \, .$$

This can be calculated for all homogenous pools of obligors grouped into the dimensions shown in Table 3.

3.5. Chapter Summary

This chapter emphasised the importance of both PDs and LGDs as main inputs into the Basel IRB capital, and Economic capital calculation. We have also demonstrated that Economic capital can be reconciled to the Basel IRB Capital under the same assumptions. A Hull-White (2000) CDS pricing model is recommended for internal estimation of PDs. For LGDs, a jump-to-default model by Das and Hanouna (2009), is used for calibration of market implied LGDs from market observable CDS prices. Both these models are reduced form models with the advantage of using market information which is constantly available and up to date.

Chapter 4 - Macroeconomic Regression Model

Now that the modelling of main inputs that go into stress testing, i.e. PDs and LGDs have been covered, this section demonstrates how those parameters can be stressed to produce stress regulatory, and economic capital. Macroeconomic data, the fitting of a regression model, creation of a credit index, and finally the stressing of PDs and LGDs using the credit index, are covered in this chapter.

Schechtman and Gaglianone (2011), probe the effects of macroeconomic variables such as GDP, inflation, and unemployment rate on credit risk of the Brazilian financial sector. Credit risk in this instance is measured by the ratio of non-performing loans (NPL's) to the performing loans. Wong et al (2008), regressed macroeconomic variables against a historical average default rate to stress the Hong Kong banking sector. Both these studies suggest that macroeconomic variables have a significant relationship with credit risk. Schmieder et al (2011) summarise lessons learned from different stress testing approaches as part of IMF surveillance work on the financial sector. Hoggarth et al (2003), describe the results of a range of macroeconomic stress tests carried out in the year 2002 on large United Kingdom based banks as part of the International Monetary Fund's (IMF's) Financial Sector Assessment Programme (FSAP) on the United Kingdom.

In our approach, insolvencies are used to derive a systematic credit cycle index which is then used to derive both PDs and LGDs, conditional on the index level. Annual corporate insolvencies were chosen to be the response variable in the regression model, and the main variable in deriving the credit cycle index because they are directly related to corporate default rates. An analysis by Fourie et al (2011) confirmed that there is a two way relationship between bank-extended credit and insolvencies. Their Vector Autoregression model also substantiates a positive relationship between credit and the business cycle. By implication, insolvencies could be used to get an indication of the business cycle status.

4.1. Data used: Macroeconomic variables

The data used in this study is from International Monetary Fund's (IMF) world economic outlook database, and World Bank's website. Insolvencies were obtained from the Statistics South Africa website. The data goes as far back as 1980 to 2012, with an annual forecast from 2013 up to 2017. A summary of variables used, including those variables that did not make it into the final model, is presented in the Appendix Table 26.

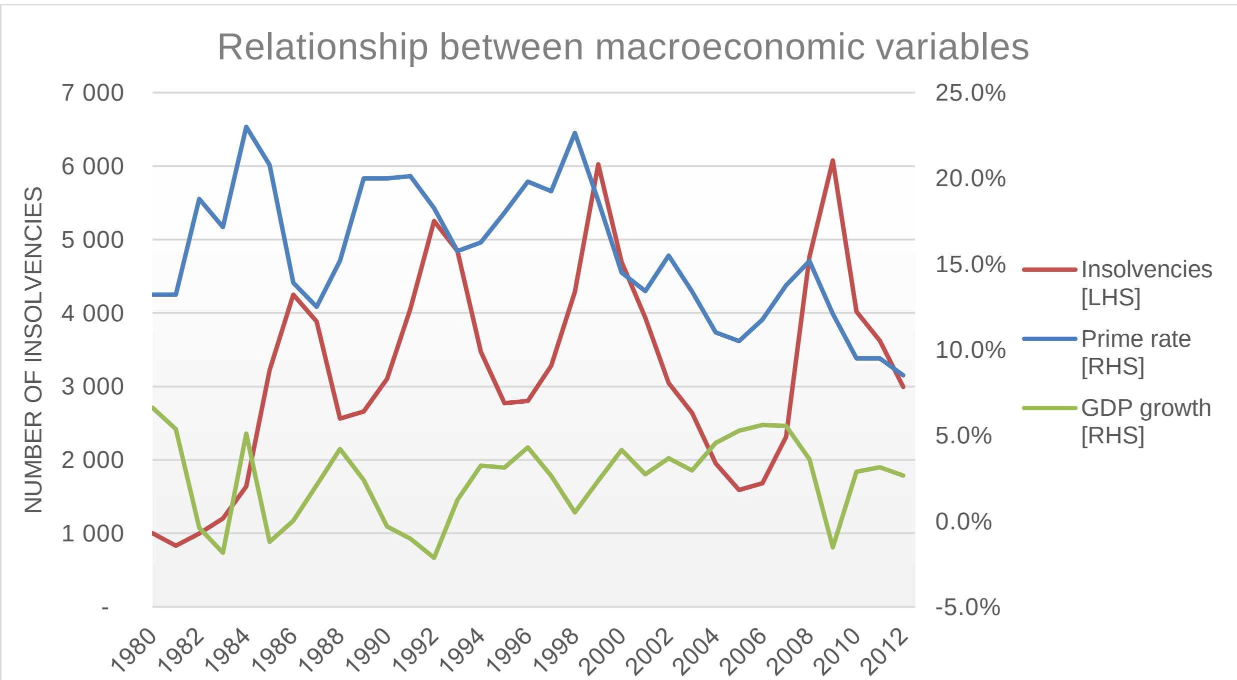

Figure 8: South African historical macroeconomic variables

In Figure 8, for visibility, we have chosen two variables, GDP and prime rate, and observed their relationship with Insolvencies. What stands out the most is the clear relationship between both these variables and insolvencies. The trend here makes intuitive sense. Notice that periods characterised by high insolvencies appear to have a lower GDP growth. On the other hand Prime rate appears to be a leading indicator of insolvencies, exhibiting the same cyclical trends, with one or two years lag.

Even more interesting is the ability of this graph to reveal historical financial crisis experienced by South Africa. The spike in insolvencies in the year 1985 can be related to the debt crisis SA suffered at the time, see Hirsch, (1989). The 1997-1998 periods is known for the severe crisis that hit Asia, and subsequently spread all over the world. US financial crisis of 2008-2009 that sent shocks across the global financial market is well represented on the graph as well.

The main purpose of this study is to show how exogenous factors can affect financial institutions in any country, with focus on South African banks. So far there seem to be a relationship between insolvencies, GDP growth, and prime rate, at least graphically. The regression model in the next section will confirm the existence and the extent of this relationship, and whether there are more variables related to insolvencies, other than GDP growth and prime rate.

4.2. Multiple regression model

Since we are using Insolvencies as the main economic cycle indicator, the purpose of a regression model is to quantify how much each of the variables listed in Appendix Table 26 relate to insolvencies. We will therefore build a regression model with number of insolvencies as a response variable, and the remainder of variables as explanatory variables.

The model has a general form:

$$y_t = \alpha + \sum_{j}^{n} \beta_j x_{tj} + \varepsilon_t \ ,$$

where,

y_t : is a response variable at time t,

α : is a constant intercept,

x_{tj} : predictor variables,

β_j : coefficient of x_{tj},

ε_t : is a white noise residual term $\sim N(0,\sigma^2)$.

The next table, Table 4, shows correlations among all variables available to build the regression model. This is a first step that helps identify variables that have a clear relationship with the response variable, i.e. insolvencies. This also reveals variables that are highly correlated, and such variables can be excluded in the model to avoid multicollinearity. Only variables that show a significant correlation with the response variable will be considered for inclusion in the final model.

To avoid including variables that are highly correlated in the model, Gujarati (2005) recommends that predictor variables with a correlation in excess of 0.9 be excluded from the model. This is directly related to the variance inflation factor (VIF). If highly correlated predictor variables are included in the model, standard errors for parameter estimates become inflated. The problem is that highly correlated explanatory variables make it difficult to identify variables that actually impact the response variable. This problem is defined as multicollinearity. From the table above none of the variables can be removed at this stage.

Pearson Correlation Coefficients, N = 33															
Prob > |r| under H0: Rho=0															
	Insolvencies	Prime_rate	inflation_index	unemployment_rate	gross_national_saving_gd	imports_volumes	inflation_prices	export_growth	exports_gdp	gdp_growth	import_gdp	producer_price_index	real_effective_exchange_ra	real_int_rates	risk_premium
Insolvencies	**1**	0.061	0.325	0.455	-0.445	-0.334	-0.107	-0.137	-0.157	-0.459	-0.204	-0.258	-0.529	0.213	0.320
		0.736	0.065	*** 0.0078	*** 0.0095	0.057	0.555	0.449	0.382	*** 0.0072	0.254	0.147	*** 0.0015	0.233	0.069
Prime_rate		**1**	-0.590	-0.335	0.134	-0.224	0.411	0.114	-0.392	-0.362	-0.515	0.244	0.301	0.432	0.122
			0.000	0.057	0.459	0.210	0.017	0.526	0.024	0.038	0.002	0.171	0.089	0.012	0.500
inflation_index			**1**	0.771	-0.731	0.069	-0.782	-0.024	0.194	0.240	0.624	-0.601	-0.751	0.264	-0.023
				<.0001	<.0001	0.705	<.0001	0.896	0.280	0.179	0.000	0.000	<.0001	0.137	0.901
unemployment_rate				**1**	-0.829	0.077	-0.752	0.127	0.025	0.137	0.254	-0.615	-0.868	0.423	0.211
					<.0001	0.670	<.0001	0.481	0.890	0.446	0.154	0.000	<.0001	0.014	0.239
gross_national_saving_gdp					**1**	-0.024	0.726	-0.180	0.279	-0.025	-0.203	0.651	0.728	-0.687	-0.145
						0.895	<.0001	0.317	0.116	0.892	0.257	<.0001	<.0001	<.0001	0.420
imports_volumes						**1**	-0.265	0.502	0.036	0.786	0.137	-0.013	-0.017	-0.035	0.068
							0.135	0.003	0.841	<.0001	0.447	0.942	0.924	0.845	0.705
inflation_prices							**1**	-0.197	0.072	-0.413	-0.406	0.815	0.565	-0.436	-0.088
								0.271	0.689	0.017	0.019	<.0001	0.001	0.011	0.627
export_growth								**1**	-0.055	0.362	-0.101	0.108	-0.103	0.154	0.058
									0.763	0.039	0.577	0.550	0.568	0.393	0.749
exports_gdp									**1**	0.439	0.693	0.386	-0.289	-0.397	-0.042
										0.011	<.0001	0.026	0.103	0.022	0.817
gdp_growth										**1**	0.561	-0.011	-0.164	-0.020	0.028
											0.001	0.954	0.362	0.912	0.879
import_gdp											**1**	-0.105	-0.368	0.047	-0.092
												0.561	0.035	0.794	0.610
producer_price_index												**1**	0.333	-0.476	-0.137
													0.059	0.005	0.447
real_effective_exchange_rate													**1**	-0.334	-0.131
														0.058	0.468
real_int_rates														**1**	0.125
															0.487
risk_premium															**1**

Table 4: Correlations among macro-economic variables

*** Significant at 1% confidence level

Lagged Variable	Pearson Correlation Coefficients with Insolvencies	Prob > |r| under H0: Rho=0
prime_1	0.41882	** 0.017
prime_2	0.4942	*** 0.0047
prime_3	0.32793	0.0769
CPI_1	0.26136	0.1485
CPI_2	0.17488	0.3467
CPI_3	0.08812	0.6433
unemploy_1	0.30631	0.0882
unemploy_2	0.12354	0.5079
unemploy_3	-0.01477	0.9383
gross_saving_1	-0.43402	** 0.0131
gross_saving_2	-0.38595	** 0.032
gross_saving_3	-0.30053	0.1066
immport_vol_1	-0.25496	0.1591
immport_vol_2	-0.10701	0.5667
immport_vol_3	0.07797	0.6821
inflation_1	0.01728	0.9252
inflation_2	-0.04959	0.7911
inflation_3	-0.13342	0.4821
export_grw_1	0.04839	0.7926
export_grw_2	0.17724	0.3402
export_grw_3	0.30055	0.1066
export_gdp_1	-0.09485	0.6056
export_gdp_2	-0.21482	0.2458
export_gdp_3	-0.29025	0.1197
gdp_1	-0.3491	0.0502
gdp_2	-0.15126	0.4166
gdp_3	-0.3491	0.0502
import_gdp_1	-0.05669	0.7579
import_gdp_2	-0.06909	0.7119
import_gdp_3	-0.06546	0.7311
ppi_1	-0.05571	0.7620
ppi_2	-0.0341	0.8555
ppi_3	-0.02043	0.9147
refx_1	-0.44893	*** 0.01
refx_2	-0.23339	0.2064
refx_3	-0.04152	0.8275
real_int_1	0.42617	** 0.015
real_int_2	0.44305	** 0.0126
real_int_3	0.32176	0.0829
risk_prem_1	0.16662	0.3621
risk_prem_2	-0.10828	0.5620
risk_prem_3	-0.37984	0.0384

Table 5: Lagged variables correlation to Insolvencies

*** Significant 1% confidence level

** Significant at 5% confidence level

Table 5 shows the correlation between insolvencies and all variables lagged up to three years. This is mainly because some variables have a lagged effect on the economy. For example, if the GDP growth has reduced this year in comparison to the previous year, the effect of this reduction could only be seen in the following year. The correlation between these lagged variables and the response variables will then be examined, and only the ones that show some sort of a relationship with response variables will be considered for inclusion in the model. In this case only those variables that show a significant correlation with insolvencies at both 1% and 5% confidence level (marked with stars).

Including all selected variables marked by stars in Table 4 and Table 5 above, the first functional model before eliminating all statistically insignificant variables is as follows:

$$insolvencies = \alpha + \beta_1 unemploy + \beta_2 savings + \beta_3 \Delta GDP + \beta_4 REER + \beta_5 \,\mathrm{prime}(2) + \beta_6 savings(1) + \beta_7 refx(1) + \beta_8 realrate(2) + \varepsilon,$$

where the variables are as defined in Table 26. The number in brackets after a variable represents the lag applied to it. For example the variable prime (2) refers to annual Prime rates with a lag of two years.

To arrive at the final model, we follow an iterative process where variables with insignificant parameter estimates are removed, one at a time from the model, starting with those that have the highest p-value. The model is refitted until all variables have statistically significant parameters, at least at 95% confidence level.

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	12192	3248.94	3.75	0.0011
unemployment_rate	1	-11766	6803.41	-1.73	0.0977
gross_national_saving_gdp	1	-6239.87816	9540.10	-0.65	0.5198
gdp_growth	1	-31935	7412.66	-4.31	0.0003
real_effective_exchange_rate	1	-38.90792	14.94	-2.6	0.0162
prime_2	1	18170	5948.69	3.05	0.0058
gross_saving_1	1	-12851	10486	-1.23	0.2334
refx_1	1	-7.73866	14.16	-0.55	0.5903
real_int_2	1	-685.87638	5475.25	-0.13	0.9014

Table 6: Fitted model 1

Table 6 above contains parameter estimates from the first model fit. The variable real_int_2, which is the two year lag of real interest rates is removed and the model is refitted. Table 7 below is the resultant parameter estimates of the model.

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	12197	3178.43	3.84	0.0008
unemployment_rate	1	-11841	6630.61	-1.79	0.0873
gross_national_saving_gdp	1	-6641.56	8790.68	-0.76	0.4576
gdp_growth	1	-32219	6905.57	-4.67	0.0001
real_effective_exchange_rate	1	-39.12498	14.52	-2.69	0.013
prime_2	1	17624	3957.44	4.45	0.0002
gross_saving_1	1	-12136	8605.58	-1.41	0.1719
refx_1	1	-7.37	13.56	-0.54	0.5919

Table 7: Fitted model 2

From Table 7, it is clear that with every statistically insignificant variable removal, the p-values for the remaining variables' parameter estimates improve. The variable refx_1 is next to be removed as it has the highest p-value of 0.5919.

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	11 827	3 059	3.87	0.0007
unemployment_rate	1	-10 739	6 220	-1.73	0.0971
gross_national_saving_gdp	1	-7 071	8 626	-0.82	0.4204
gdp_growth	1	-33 031	6 642	-4.97	<.0001
real_effective_exchange_rate	1	-44	11	-4.05	0.0005
prime_2	1	17 225	3 831	4.5	0.0001
gross_saving_1	1	-11 977	8 474	-1.41	0.1703

Table 8: Fitted model 3

Following the same variable elimination procedure as above, the variable 'gross_national_saving_gdp' leaves the model.

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	10 987	2 863	3.84	0.0008
unemployment_rate	1	-9 578	6 017	-1.59	0.1240
gdp_growth	1	-31 917	6 459	-4.94	<.0001
real_effective_exchange_rate	1	-43	11	-4.02	0.0005
prime_2	1	17 252	3 806	4.53	0.0001
gross_saving_1	1	-16 226	6 660	-2.44	0.0223

Table 9: Fitted model 4

At this stage all variables, except for 'unemployment_rate' are statistically significant at 95% confidence level. The variable is again eliminated and the model is refitted with the remaining variables.

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	6 761	1 104	6.12	<.0001
gdp_growth	1	-32 473	6 637	-4.89	<.0001
real_effective_exchange_rate	1	-33	9	-3.73	0.0009
prime_2	1	16 266	3 865	4.21	0.0003
gross_saving_1	1	-9 515	5 306	-1.79	0.0846

Table 10: Fitted model 5

After removing 'unemployment_rate' on the 5th trial of model fitting (see Table 10), the first lag of gross national savings 'gross_saving_1' is no longer significant at 95% confidence level. As a result the variable is removed and the model is refitted.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	36 388 502	12 129 501	20.79	<.0001
Error	27	15 749 396	583 311		
Corrected Total	30	52 137 898			
Root MSE	763.75	R-Square	0.70		
Dependent Mean	3343.45	Adj R-Sq	0.66		
Coeff Var	22.84				

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	6 194.98	1 100.52	5.63	<.0001
gdp_growth	1	-31 038.00	6 853.77	-4.53	0.0001
real_effective_exchange_rate	1	-42.92	7.19	-5.97	<.0001
prime_2	1	15 593.00	4 001.05	3.9	0.0006

Table 11: Final model Analysis of Variance and Parameter Estimates

Table 11 shows that all remaining variables, the intercept, GDP growth, real effective exchange rate, and the 2nd lag of Prime rate, are statistically significant at 99% confidence level. Analysis of variance also shows that the global F-test is significant at the same confidence level. The model also have a decent adjusted R-Squared of about 0.66. Gujarati (2005), explains that the R-Squared or multiple coefficient of determination measures the amount of variation in the response variable that is explained by the predictor variables. The adjusted R-Squared, takes into account the degrees of freedom, which is based on how many variables were included in the regression model.

Finally to ensure that no multicollinearity exists among the final variables included in the model, we will check pair-wise correlation of all predictor variables in the model. This is important since the correlation checks in Table 4, and Table 5 were done in isolation, i.e. the lagged variables correlations were performed separately from original variables without lags.

	Insolvencies	gdp_growth	real_effective_ exchange_rate	prime_2
Insolvencies	1			
gdp_growth	-0.46	1		
real_effective_exchange_rate	-0.53	-0.16	1	
prime_2	0.49	-0.29	0.13	1

Table 12: Pearson correlation of final variables included in the regression model

From the correlation Table 12 it appears there is no significant correlation among the predictor variables to suspect the existence of multicollinearity. Figure 9 also confirms that our model assumptions have been satisfied, i.e. normally distributed residuals with a zero mean.

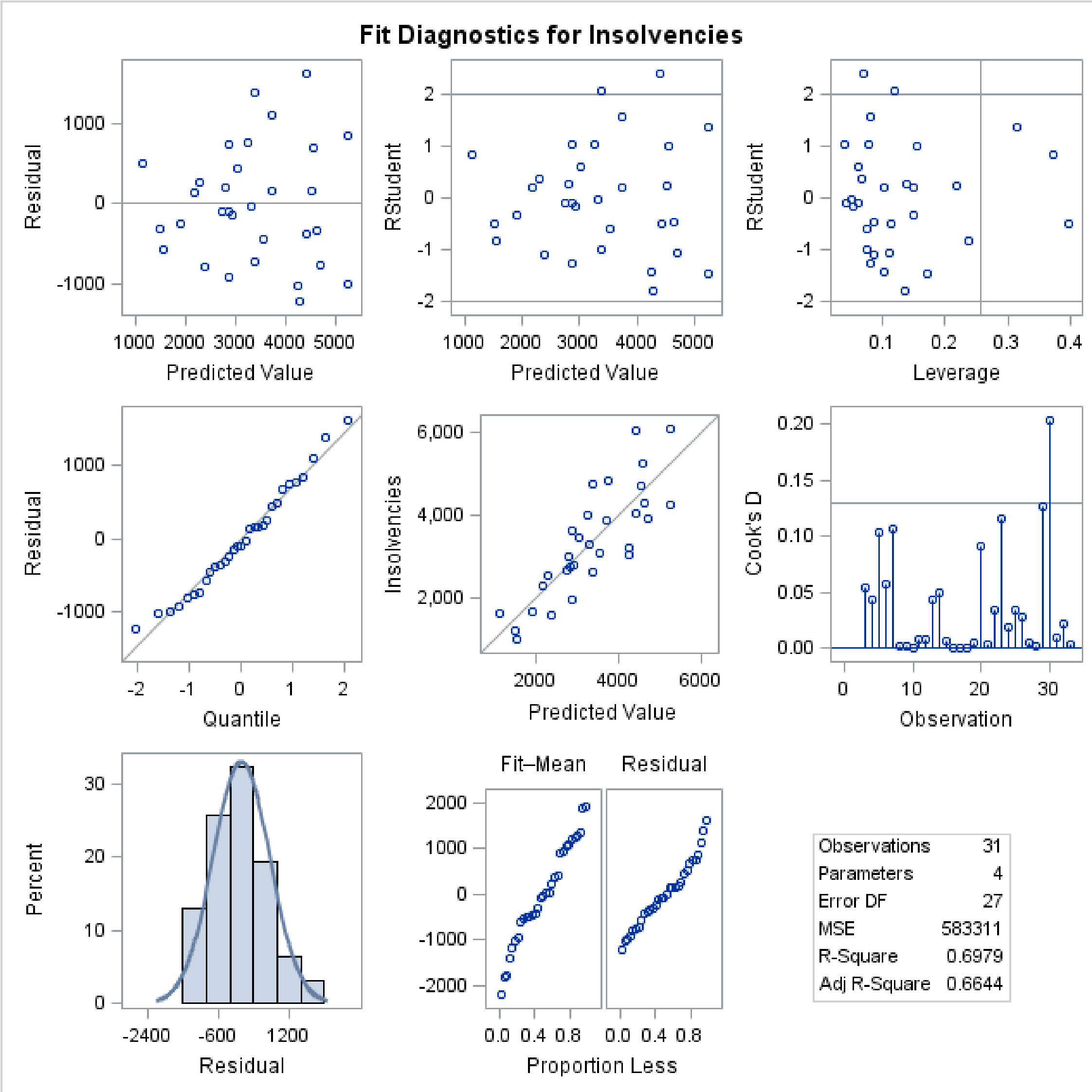

Figure 9: Model fit diagnostics

With satisfactory parameter estimates, and goodness of fit, the final functional form of the regression model therefore reduces to:

$$insolvencies = \alpha + \beta_1 \Delta GDP + \beta_2 REER + \beta_3 \,\mathrm{prime}(2) + \varepsilon.$$

Substituting the parameter with their estimates, this becomes:

$$insolvencies = 6194.98 - 31038 \Delta GDP - 42.92 REER + 15593 \,\mathrm{prime}(2).$$

4.3. Regression model prediction error

The purpose of the regression model built in the previous section is to help forecast insolvencies for the stress testing forecast period, using the variables selected in the model. In this case GDP, real effective exchange rates, and a two year lagged Prime rate will be used as main drivers for insolvencies. An economic cycle index will be built using the predicted insolvencies from this model.

Stress testing is normally done on a three year forecast of the balance sheet and income statement. As a result the chosen macroeconomic scenario should have a three year view for every macroeconomic variable in the model. In our case, based on a chosen scenario, a three year forecast for GDP growth, real effective exchange rates, and lagged prime rates will be required to predict the number of insolvencies over the same period.

To validate if the model is doing a great job in terms of forecasting, it has to be tested against actual historical data. We will look at the historical values of our model variables to predict insolvencies using our model. These predicted values can then be compared to actual observed insolvencies at the time. Table 13 shows values produced by the regression model against actual historical values observed.

Year	gdp_growth	real_effective_exchange_rate	prime_2	Observed Insolvencies	Model Insolvencies
1980	7.0%	156	.	1 003	.
1981	5.0%	164	.	831	.
1982	0.0%	155	13.0%	998	1 557
1983	-2.0%	171	13.0%	1 201	1 507
1984	5.0%	151	19.0%	1 637	1 135
1985	-1.0%	114	17.0%	3 221	4 242
1986	0.0%	106	23.0%	4 248	5 245
1987	2.0%	119	21.0%	3 883	3 729
1988	4.0%	113	14.0%	2 563	2 301
1989	2.0%	113	13.0%	2 658	2 746
1990	0.0%	116	15.0%	3 104	3 542
1991	-1.0%	121	20.0%	4 057	4 428
1992	-2.0%	125	20.0%	5 254	4 567
1993	1.0%	123	20.0%	4 843	3 735
1994	3.0%	117	18.0%	3 473	3 028
1995	3.0%	114	16.0%	2 770	2 857
1996	4.0%	105	16.0%	2 803	2 936
1997	3.0%	111	18.0%	3 283	3 311
1998	1.0%	102	20.0%	4 289	4 633
1999	2.0%	96	19.0%	6 025	4 405
2000	4.0%	93	23.0%	4 694	4 538
2001	3.0%	82	19.0%	3 936	4 695
2002	4.0%	70	15.0%	3 048	4 267
2003	3.0%	91	13.0%	2 643	3 371
2004	5.0%	100	16.0%	1 954	2 861
2005	5.0%	100	13.0%	1 589	2 378
2006	6.0%	96	11.0%	1 680	1 922
2007	6.0%	90	11.0%	2 314	2 169
2008	4.0%	80	12.0%	4 763	3 373
2009	-2.0%	88	14.0%	6 078	5 236
2010	3.0%	101	15.0%	4 020	3 258
2011	3.0%	99	12.0%	3 624	2 880
2012	3.0%	94	10.0%	2 994	2 801

Table 13: Backtesting on historical insolvencies

Table 13 above shows that the model is accurate in predicting insolvencies to a large extent, given the historical predictor variables. This is clearer in Figure 10.

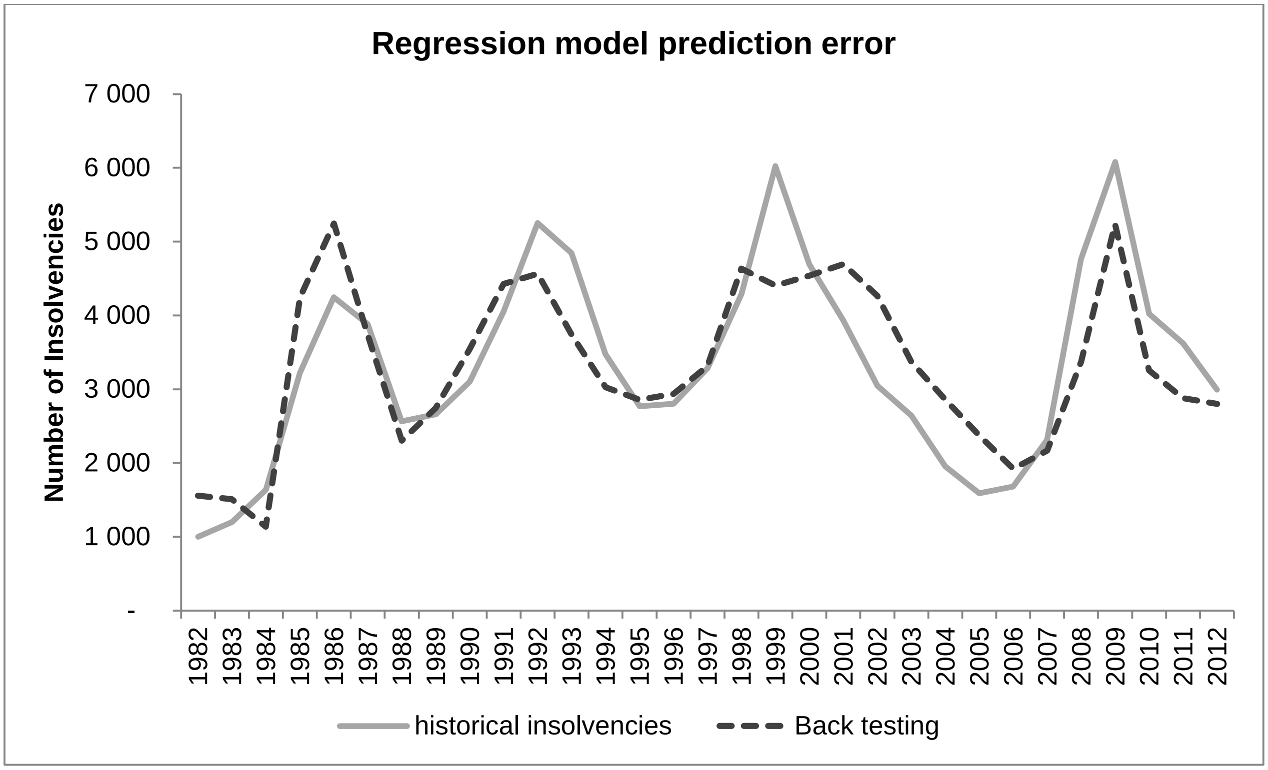

Figure 10: Model prediction error on historical insolvencies

Figure 10 shows there is a tolerable amount of prediction error in the model. The model will be used to predict insolvencies for the three forecast years of stress testing.

4.4. Chapter Summary

In this chapter we used South African macroeconomic variables to build a multiple regression model to identify variables that are more predictive to insolvencies, which were then used to build an economic cycle indicator. In our case corporate insolvencies were chosen to be the cycle indicator, because they are a good indication of the economic pressure from number of insolvent companies. GDP growth, real effective exchange rate, and prime rate (lagged by 2 years), appeared to be the most influential variables to the cycle indicator.

The model's prediction error was tested by comparing the model output with historically observed insolvencies, and the model performs well with minimum prediction error.

Chapter 5 - Stressed PDs and downturn LGDs using a credit cycle index

At this point, we have demonstrated how the key stress testing inputs, i.e. PDs and LGDs can be modelled internally, and we have also built a regression model that can forecast insolvencies for the stressed testing horizon of three years. This section takes us through the process of deriving the credit cycle index or the systematic risk factor from insolvencies and other macroeconomic variables. The resultant cycle reflects the status of the economy depending on the chosen stress testing scenario, and it is then used to stress both PDs and LGDs.

5.1. Credit cycle index

The credit cycle index is built from macroeconomic factors, and it is normally used to gauge the status of the cycle. It has been used widely in credit modelling where variables and parameters can be scaled up or down to reflect the cycle. Blümke (2010) explores probability of default in the context of credit cycle, and shows that the credit cycle can be used to adjust and validate PDs in the same context as that of Basel IRB capital charge Equation (3.3) in Section 3.1.

This section draws extensively from Carlehed and Petrov (2012). They propose a way of adjusting PDs from a hybrid model to be either TTC or PIT using a credit cycle index. Assuming our PDs are TTC, we follow the same analogy of deriving stressed PDs that are conditional on credit cycle index. They derive the credit cycle index Z as follows:

Using the property of equicorrelated normal distribution, the creditworthiness of obligor j can be presented as, see Vasicek (2002):

$$Y_j = \sqrt{\rho}Z + \sqrt{1-\rho}\varepsilon_j, \tag{5.1}$$

where

Z: is a standard Normal variable representing the single systematic, economy-wide factor.

ρ: is the correlation factor of obligor j to the systematic factor Z.

ε_j : is independent standard Normal variable representing the idiosyncratic movement of the obligor j's creditworthiness.

Obligor j's conditional $p_j(z)$ becomes:

$$p_j(z) = \Phi\left(\frac{\Phi^{-1}(PD_j) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right). \tag{5.2}$$

As we cannot observe default frequency for an individual obligor, Carlehed and Petrov (2012) use the Merton model for the whole portfolio with a global $\Phi^{-1}(PD) = \mathrm{B}$, which can be interpreted in Merton model terms as the B of the average obligor. We therefore have a portfolio conditional PD:

$$p_P(z) = \Phi\left(\frac{B - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right). \tag{5.3}$$

Given a historical data series d_t of default frequencies, inverting Equation (5.3) above, and solving for Z yields the following:

$$Z_t = \frac{B - \Phi^{-1}(d_t)\sqrt{1-\rho}}{\sqrt{\rho}}. \tag{5.4}$$

The biggest challenge with this formula is that we need to know the correlation parameter ρ in order to calculate the index value Z_t. Moments approach can be used to find an estimation to Equation (5.4). From Equation (5.3) we see that

$$E[\Phi^{-1}(p_P)] = \frac{B}{\sqrt{1-\rho}},$$

and

$$\mathrm{Var}[\Phi^{-1}(p_P)] = \frac{\rho}{1-\rho}.$$

Applying the transformation $d \rightarrow \Phi^{-1}(d)$, we define m as the mean, and σ as the standard deviation of the transformed data series. Identifying moments and solving for m and ρ, we get $B \approx 1/\sqrt{1+\sigma^2}$, and $\rho \approx \sigma^2/(1+\sigma^2)$. Substituting these into Equation (4), we get:

$$Z_t = \frac{m - \Phi^{-1}(d_t)}{\sigma} \quad . \tag{5.5}$$

Z_t can now be calculated from the inverted time series d_t, its mean m, and the standard deviation σ. In this dissertation we have replaced d_t by the frequency of historical insolvencies under the observed period, (1980 – 2012). Table 14 illustrates how the credit cycle index Z is derived from Equation (5.5) using historical insolvencies.

Year	Observed Insolvencies	Frequency (dt)	Normsinv (dt)	Cycle Index (Zt)
1980	1 003	0.95%	-2.345	1.93
1981	831	0.79%	-2.415	2.24
1982	998	0.95%	-2.347	1.93
1983	1 201	1.14%	-2.277	1.62
1984	1 637	1.55%	-2.157	1.08
1985	3 221	3.05%	-1.873	-0.20
1986	4 248	4.03%	-1.748	-0.77
1987	3 883	3.68%	-1.789	-0.58
1988	2 563	2.43%	-1.972	0.24
1989	2 658	2.52%	-1.957	0.17
1990	3 104	2.94%	-1.889	-0.13
1991	4 057	3.85%	-1.769	-0.67
1992	5 254	4.98%	-1.647	-1.22
1993	4 843	4.59%	-1.686	-1.05
1994	3 473	3.29%	-1.839	-0.35
1995	2 770	2.63%	-1.939	0.09
1996	2 803	2.66%	-1.934	0.07
1997	3 283	3.11%	-1.865	-0.24
1998	4 289	4.07%	-1.743	-0.79
1999	6 025	5.71%	-1.579	-1.53
2000	4 694	4.45%	-1.701	-0.98
2001	3 936	3.73%	-1.783	-0.61
2002	3 048	2.89%	-1.897	-0.09
2003	2 643	2.51%	-1.959	0.19
2004	1 954	1.85%	-2.085	0.75
2005	1 589	1.51%	-2.168	1.13
2006	1 680	1.59%	-2.146	1.03
2007	2 314	2.19%	-2.015	0.44
2008	4 763	4.52%	-1.694	-1.01
2009	6 078	5.76%	-1.575	-1.55
2010	4 020	3.81%	-1.773	-0.65
2011	3 624	3.44%	-1.820	-0.44
2012	2 994	2.84%	-1.905	-0.06
		Average (m)	**-1.92**	
		Std Dev.	**0.22**	

Table 14: Derivation of the credit cycle index

A positive index reflects favourable levels of the cycle, and therefore when insolvencies are high (unfavourable levels), the index will be low reflecting the downturn. The next section illustrates how Z can be used to stress both PDs and LGDs.

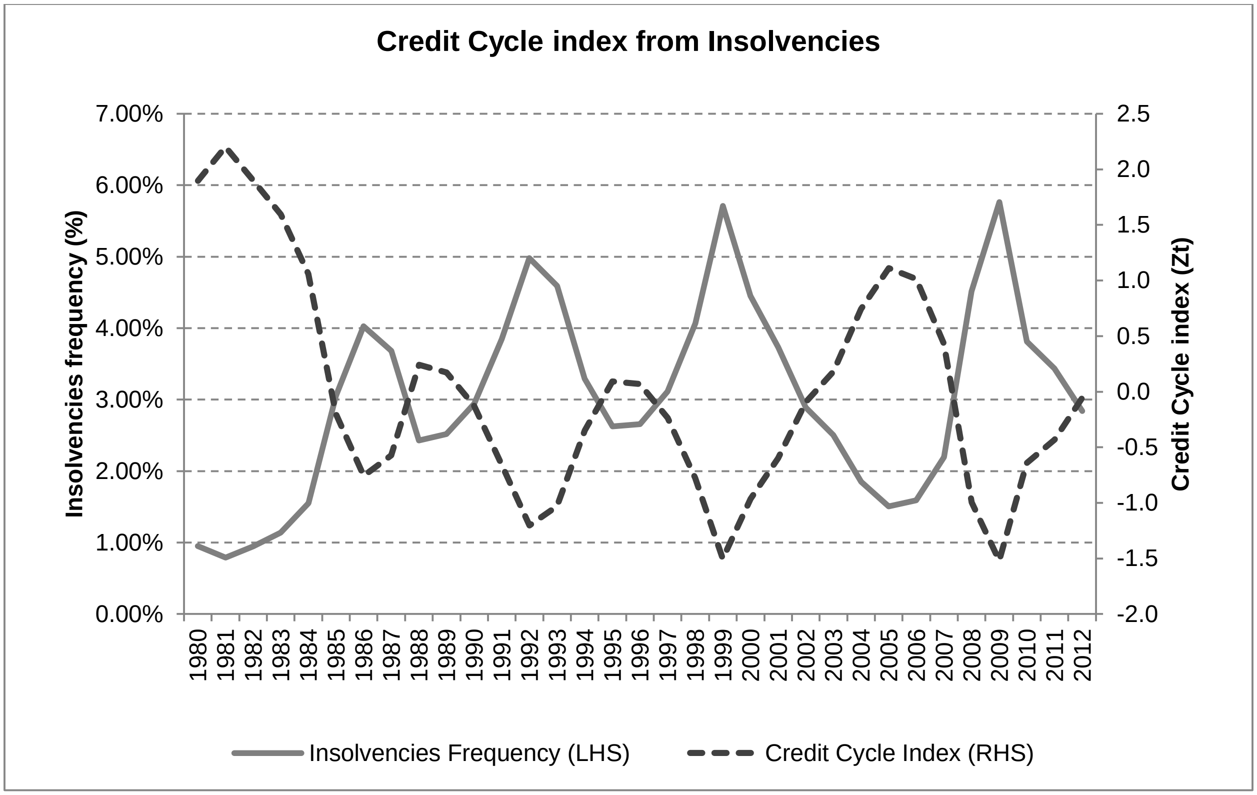

Figure 11: Credit cycle index from insolvencies

Figure 11 shows the frequency of insolvencies on the left-hand side axis, as a solid line. On the right-hand side axis, the credit cycle index is shown. Notice the inverse relationship between the insolvencies and the credit cycle. Because a high insolvency rate represents unfavourable credit cycle, when insolvencies increase the credit cycle index declines.

To validate our credit cycle index derived from corporate insolvencies, we compare it to the RMB/BER business confidence index (see Figure 12). This is the most commonly used cycle indicator in South Africa, and it is published quarterly by the Bureau for Economic Research (BER), a division of Stellenbosch University.

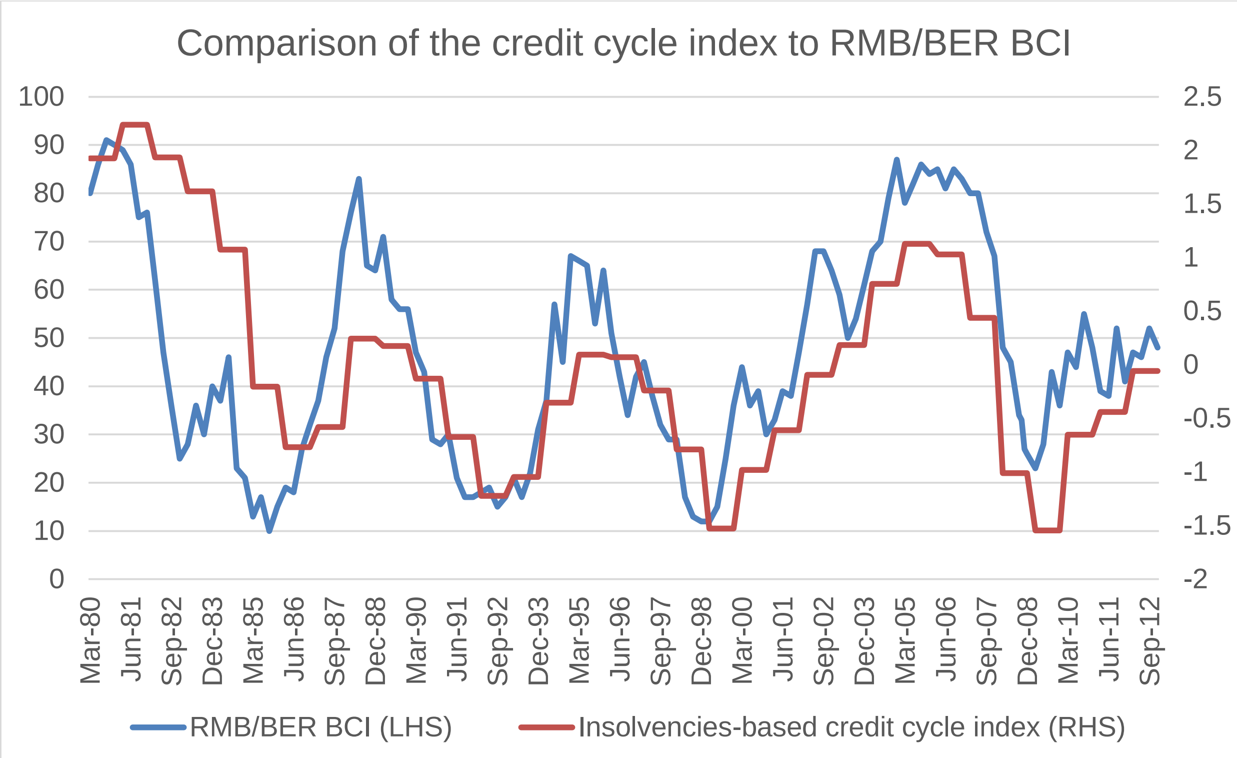

Figure 12: Comparison of the insolvencies-based credit cycle index to the RMB/BER Business Cycle Index

The red line in Figure 12 represents the credit cycle index derived from insolvencies in this section, and the blue line is the RMB/BER business confidence index. Since our index is annual, values for all four quarters were kept constant for comparison with the RMB/BER index, which is quarterly. Our insolvencies-based credit cycle index seems to be tracking the RMB/BER business confidence index very closely, and this confirms a positive relationship between insolvencies and the business cycle.

5.2. Stressed PDs

Stressing PDs simply means deriving PDs that are conditional on the credit index Z from long term average through-the-cycle PDs. This is because Z from insolvencies on the basis that insolvency rate is the best indicator of the cycle status at any point in time. These conditional PDs are similar to the ones used in the Basel IRB capital charge formula, except Z in this case not derived from a normal distribution curve at a predefined confidence level of 99.9%. Instead a standard normal credit cycle index derived from insolvencies, and other macroeconomic variables is used to stress PDs.

From Chapter 3, Section 3.1, the conditional PDs are given as,

$$p_j(z) = p[\mathrm{Y}_j < \Phi^{-1}(PD_j) \mid Z_t] = \Phi\left(\frac{\Phi^{-1}(PD_j) - \sqrt{R_j}Z_t}{\sqrt{1-R_j}}\right),$$

where PD_j refers to through-the-cycle probabilities of default estimated internally by the bank. Z is the credit cycle index as calculated in Section 5.1. Conditional PDs in Table 15 below are based on a fixed minimum corporate correlation of R = 0.12, for illustration purpose.

Year	Cycle Index	TTC PD	Cycle based PD
1980	1.93	5.89%	3.66%
1981	2.24	5.89%	3.36%
1982	1.93	5.89%	3.65%
1983	1.62	5.89%	3.96%
1984	1.08	5.89%	4.55%
1985	-0.20	5.89%	6.22%
1986	-0.77	5.89%	7.10%
1987	-0.58	5.89%	6.80%
1988	0.24	5.89%	5.59%
1989	0.17	5.89%	5.69%
1990	-0.13	5.89%	6.12%
1991	-0.67	5.89%	6.94%
1992	-1.22	5.89%	7.87%
1993	-1.05	5.89%	7.56%
1994	-0.35	5.89%	6.45%
1995	0.09	5.89%	5.80%
1996	0.07	5.89%	5.83%
1997	-0.24	5.89%	6.28%
1998	-0.79	5.89%	7.13%
1999	-1.53	5.89%	8.42%
2000	-0.98	5.89%	7.45%
2001	-0.61	5.89%	6.84%
2002	-0.09	5.89%	6.06%
2003	0.19	5.89%	5.67%
2004	0.75	5.89%	4.93%
2005	1.13	5.89%	4.49%
2006	1.03	5.89%	4.61%
2007	0.44	5.89%	5.33%
2008	-1.01	5.89%	7.50%
2009	-1.55	5.89%	8.46%
2010	-0.65	5.89%	6.91%
2011	-0.44	5.89%	6.58%
2012	-0.06	5.89%	6.01%

Table 15: Cycle-conditional PDs

The relationship between the cycle Z, and the cycle based PDs is shown in Figure 13 below. Using hypothetical through-the-cycle PDs, it is clear the PDs worsen as the index declines and improve when it improves, as expected. Figure 13 clearly highlights this fact.

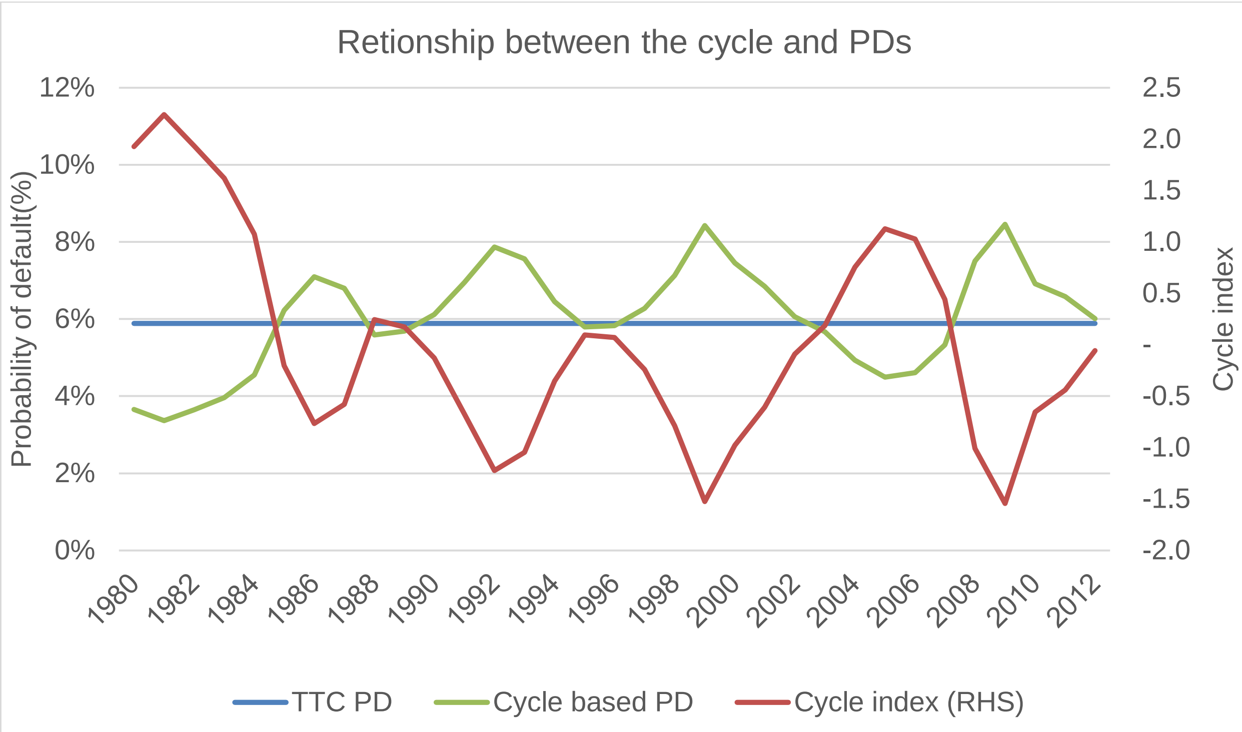

Figure 13: Credit cycle conditional PD compared to through-the-cycle PD

5.3. Downturn LGDs

Besides PDs, Downturn LGDs are the main input into the IRB Basel capital calculation. The framework requires banks to calculate their own LGDs that are reflective of the stress economic conditions, i.e. when the cycle is on a downturn. This section is devoted to deriving the downturn LGD based on the credit cycle index that was built from a regression model in Chapter 4, Section 4.3.

There has been a significant amount of research done on finding the right methodology to derive downturn LGDs from the internally derived expected or TTC LGDs. Because banks are not equally advanced when it comes to modelling expertise, and collection of historical data required for such an exercise is difficult, the Board of Governors of the Federal Reserve System (USA) proposed a generic formula for deriving the downturn LGD from the long-term-average LGD estimate, see Cheparev (2007). The formula is given as:

$$\mathrm{LGD}_{\mathrm{Downturn}} = 0.08 + 0.92 \times \mathrm{LGD}_{TTC}$$

While proposing a single formula across banks is good for consistency, this specific formula has its own shortcomings. For example, it can be shown that the downturn adjustment is higher for banks with historically lower LGDs and lower for those with higher LGDs. In other words, the formula penalises banks that had good historical recoveries with a higher downturn adjustment, and incentivises those that had bad recoveries with a smaller adjustment. Figure 14, confirms this by ranking LGDs from 25% to 75%, and calculating the incremental effect of downturn LGD formula.

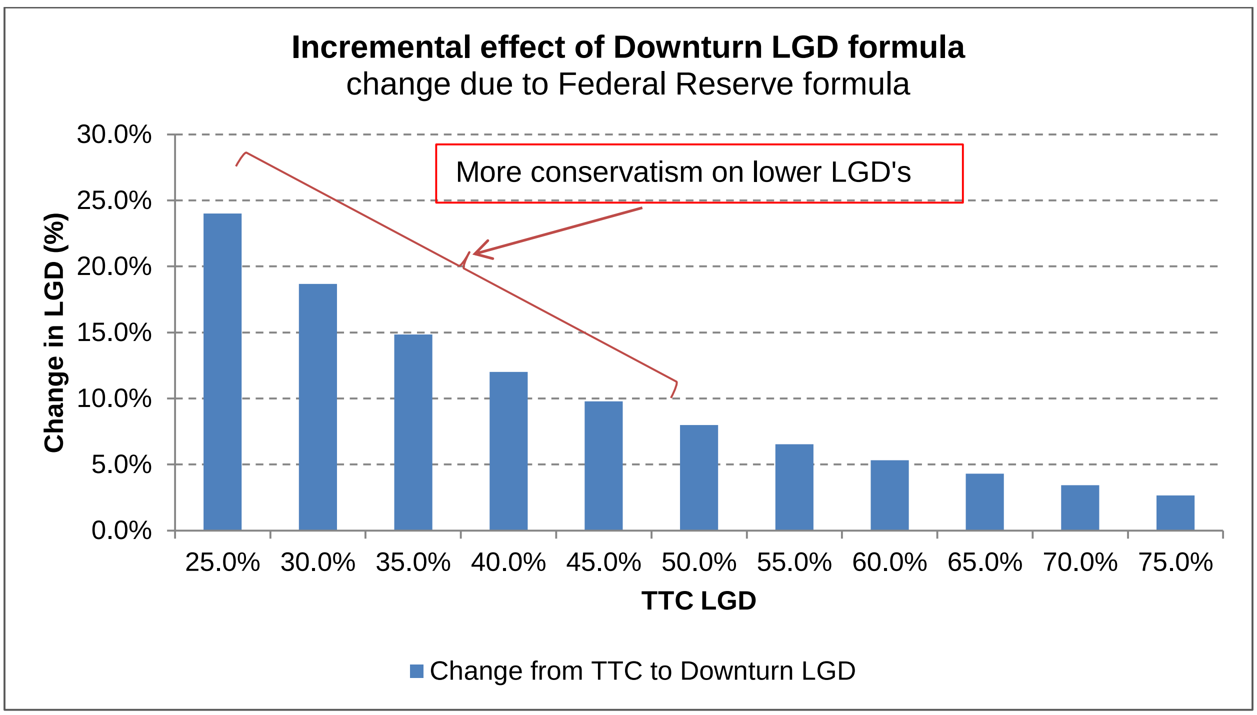

Figure 14: Incremental Effects of Federal Reserve downturn LGD formula

One other setback with the Federal Reserve downturn LGD is the fact that it does not take into account the PD-LGD correlation. Miu and Ozdemir (2005) explain why Basel requires downturn LGDs and show how conservative point-in-time LGDs can be derived to account for lack of PD-LGD correlation in the Basel downturn formula. They show that the mean LGD needs to be increased by about 35% to 41% for a corporate portfolio and about 16% for a mid-market portfolio in order to compensate for the lack of correlations.

Frye (2000), and Pykhtin (2003), explore deriving LGDs or recoveries from traditional loss models, resulting in LGDs that are correlated to default rates. Merton and Vasicek loss models are common underlying models and the core of these studies. All these studies prove the correlation of LGDs or recoveries with default rates and show how systematic LGDs can be modelled.

Rösch and Scheule (2008), follow a similar PD-LGD correlation approach which is relatively practical and simple to implement. They propose an LGD that is correlated to PDs through two bivariately normally distributed systematic factors. The only difference is the extent to which these factors affect both PDs and LGDs. Their approach allows a bank to choose the extent to which the LGD systematic factor is correlated to the same systematic factor used to derive conditional PDs. A conservative assumption of a correlation of one (1) is recommended where there is no data to estimate this parameter.

$$LGD_{downturn} = \Phi\left(\left(\Phi^{-1}(LGD_{TTC}).\sqrt{1+b^2} + b\rho\Phi^{-1}(0.999)\right).\sqrt{\frac{1}{1+b^2(1-\rho^2)}}\right),$$

LGD_{TTC}:	is a through-the-cycle LGDs as derived in section (3.3).
$\Phi^{-1}()$:	inverse of standard normal distribution,
b:	represents the sensitivity of LGDs to the unknown systematic risk drivers.
ρ:	the correlation of LGDs to the systematic risk factor Z that drives the conditional PDs.

Next is the step-by-step derivation of this formula. Using the Basel II conditional PD formula assuming the worst realisation of the systematic random variable $Z = \Phi^{-1}(0.999)$

$$p_j(z) = p[X_j < \Phi^{-1}(PD_j) \mid Z_t] = \Phi\left(\frac{\Phi^{-1}(PD_j) - R_j Z_t}{\sqrt{1-R_j^2}}\right).$$

Similarly, the conditional recovery rate can be defined as:

$$R_t(X_t) = \Phi(\beta_0 + \beta Y_{t-1}^R + bX_t),$$

where β_0 is an intercept, Y_{t-1}^R is systematic risk drivers, and X_t is a standard normally distributed unobservable systematic factor, such as the cycle index Z, derived in Section 5.1. Conditional loss given default rate can therefore be defined as:

$$CLGD_t(X_t) = 1 - R_t(X_t) = 1 - \Phi(\beta_0 + \beta Y_{t-1}^R + bX_t).$$

The unconditional loss rate given default (LGD) is then given by:

$$LGD_t = 1 - \int_{-\infty}^{\infty} \Phi(\beta_0 + \beta Y_{t-1}^R + bx_t) d\Phi(x_t) = 1 - \Phi\left((\beta_0 + \beta Y_{t-1}^R).\sqrt{\frac{1}{1+b^2}}\right).$$

The link between the recovery rate and default process is introduced by modelling the dependence of the two systematic risk factors Z and X. Their dependence is modelled by assuming they are bivariately normally distributed with correlation parameter ρ. According to law of conditional expectation, the downturn expected loss rate given default (DLGD) conditional on worst case realisation of Z is:

$$DLGD(z_t) = 1 - \Phi\left(\mu(z_t).\sqrt{\frac{1}{1+\sigma(z_t)^2}}\right).$$

With the conditional mean of the average transformed recovery rate:

$$\mu(z_t) = \beta_0 + \beta Y_{t-1}^{R} - b\rho z_t .$$

A conditional standard deviation of the average transformed recovery rate:

$$\sigma(z_t) = b\sqrt{1-\rho^2} .$$

Unconditional LGD can then be written as:

$$LGD_{downturn} = \Phi\left(\left(\Phi^{-1}(LGD_{TTC}).\sqrt{1+b^2} + b\rho\Phi^{-1}(0.999)\right).\sqrt{\frac{1}{1+b^2(1-\rho^2)}}\right).$$

Since this dissertation is exploring stressing both the PDs and LGDs using the credit cycle index derived from macroeconomic variables, an obligor j LGD that is conditional on the credit cycle Z, can be derived from the following changes:

1. Substituting $\Phi^{-1}(0.999)$ with the cycle index Z as calculated in Section 5.1,
2. A conservative perfect correlation assumption between the LGD systematic factor X, and the PD systematic factor Z ($\rho = 1$),
3. Replacing the sensitivity of the unknown systematic risk drivers b with the Basel II corporate correlation factor R,

$$R_j = 0.12 \times \frac{1 + e^{(-50 \times \mathrm{PD}_j)}}{1 - e^{(-50)}} + 0.24 \times \left(1 - \frac{1 - e^{(-50 \times \mathrm{PD}_j)}}{1 - e^{(-50)}}\right).$$

Substituting all of these in the downturn LGD formula above, the final conditional or stressed LGD becomes:

$$CLGD_j = \Phi\left(\left(\Phi^{-1}(LGD_{jTTC}).\sqrt{1+R_j^{\,2}} - R_j Z_t\right)\right).$$

Note that the sign before the systematic factor Z changed to negative due to the inverse relationship of the cycle and the LGDs. Favourable (high) levels of the index exhibit lower LGDs than the lower levels of the index which are usually implied by low recovery rates. Table 16 demonstrates LGDs that are calculated from the systematic factor Z, using the Federal Reserve LGDs as a benchmark. The cycle index values from Table 16 for the last 12 years (2001 to 2012) are used to calculate cycle based conditional LGDs (CLGD) from a hypothetical TTC LGD of 55%. Federal Reserve's conservative downturn LGD is calculated on the last column.

Table 16: LGD comparison

Year	Cycle Index	TTC LGD	CLGD	Fed downturn LGD
2001	-0.61	55%	58%	58%
2002	-0.09	55%	55%	58%
2003	0.19	55%	54%	58%
2004	0.75	55%	51%	58%
2005	1.13	55%	49%	58%
2006	1.03	55%	50%	58%
2007	0.44	55%	53%	58%
2008	-1.01	55%	59%	58%
2009	-1.55	55%	62%	58%
2010	-0.65	55%	58%	58%
2011	-0.44	55%	57%	58%
2012	-0.06	55%	55%	58%

**** Corporate correlation factor: R = 0.12**

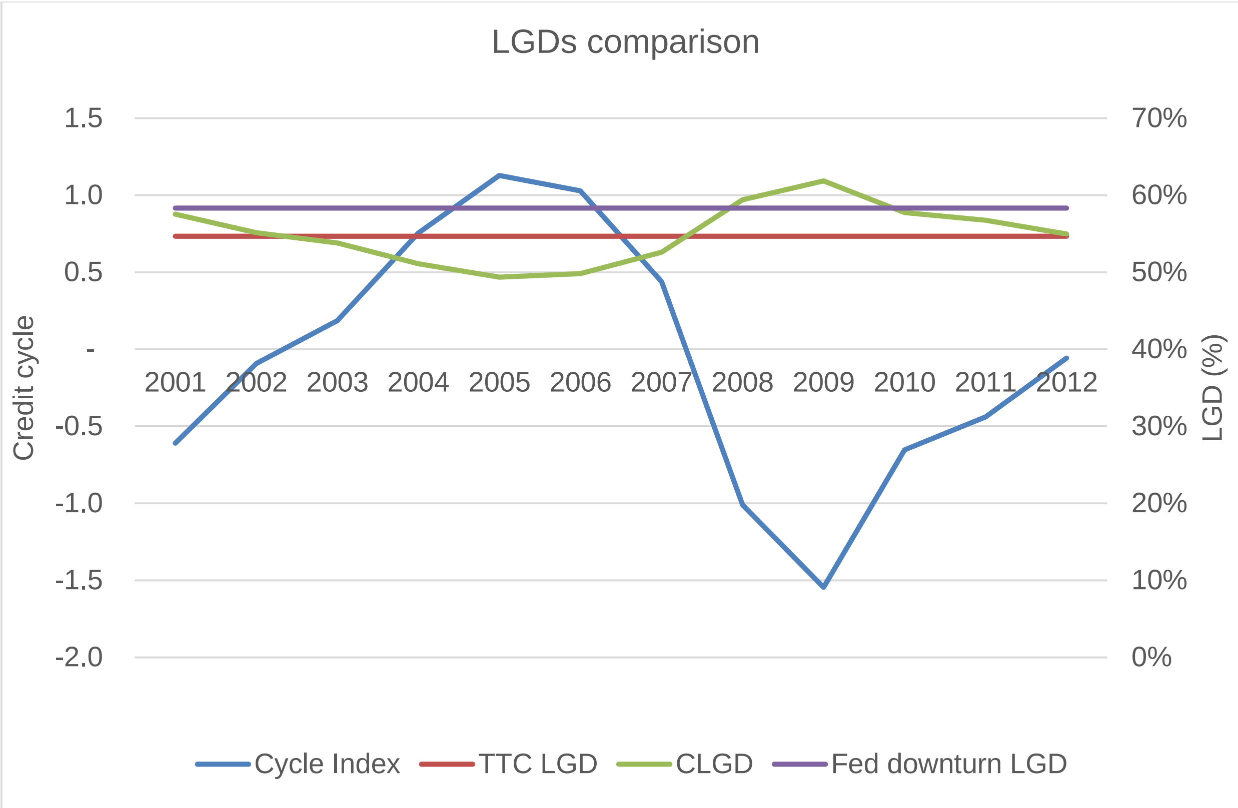

Figure 15: Conditional LGD comparison to Federal Reserve downturn LGD

The left-hand side of Figure 15 shows different level of the cycle, and the right-hand side shows LGD values in percentages. TTC LGD is a constant hypothetical long term average LGD for across years, starting from year 2001. The Federal Reserve downturn LGD is calculated using the formula shown in the beginning of Section 5.3. Notice that this is just a parallel upward shift from the TTC LGD, and it is independent from the credit cycle index. CLGD on the other hand, is negatively correlated to the index, such that high levels of the index reflect lower LGD, and vice versa. It is interesting to note that the Federal Reserve Downturn LGD formula is not always as conservative as it was meant to be. For example, in Figure 15, in 2009 conditional LGDs are higher than the Federal Reserve Downturn LGDs, reflecting low levels of the credit cycle.

At this stage we have established a framework for stressing both portfolio TTC PD and LGD through an index that reflects the status of the economic or credit cycle, at any point in time. The next section describes properties of the credit portfolio that is to be stressed.

5.4. Hypothetical credit portfolio

Now that we have established a stress testing framework, we now illustrate the practical application of the stressed PD and downturn LGD on a hypothetical portfolio. In this section, a basic description of our hypothetical credit portfolio is provided. For illustration and simplicity, we have created a credit portfolio aggregated at a counterparty level. To align the underlying Basel II assumptions as explained in Gordy (2002), the portfolio is designed in such a way that all counterparties have equal exposures, and no single obligor accounts for more than an arbitrarily small share of total exposure.

A hypothetical corporate portfolio is created, consisting of 200 obligors with risk parameters attached to each obligor. Every counterparty has an internal risk grade that corresponds to Moody's rating with a corresponding default rate (See Table 21 in the Appendix). PDs, LGDs, exposure-at-default (EAD), and average time-to-maturity across the counterparty's facilities are displayed on a portfolio snapshot in the Appendix Table 24.

The total EAD is R 33,461 million, which is equally distributed as R167 million among obligors to avoid any counterparty concentration. Risk grades were assigned to ensure that the average portfolio rating is realistic, i.e. not too good or bad. Average time-to-maturity was defaulted to a conservative 3 years, since Basel proposes 2.5 maturity where data is missing. Average portfolio PD is 4.8% which suggests an average rating of 15 on the rating scale.

Default rate for each rating was assigned in line with Moody's ratings as presented in the Appendix Table 21. Obligor level expected LGD was assigned based on an internal rating-LGD scale with a minimum LGD of 10% on the best rated counterparties and a maximum of 95% for the worst rated ones. LGDs were assigned to be consistent with Moody's LGD categories as shown in the Appendix Tables 22 and Table 23.

5.5. Base financial ratios

Financials are the important part of stress testing since they are indicative of how much loss the company can tolerate in a stress event. We start off by defining our financial periods, as well as stress forecasts period. The income statement and the balance sheet have actual data as at year end of 2012. In addition, a three year base forecast of both the Income Statement and the Balance Sheet are required for 2013 to 2015 (see Appendix A.5). Base forecast refers to the forecast under normal economic conditions, and this is stressed to reflect the macroeconomic impact. Table 17 below shows key measures that will be observed under the base case and stress scenarios.

Stress testing metrics	**2012 - Actual**	**2013**	**2014**	**2015**
Profit after Tax	2 257	2 765	3 066	3 677
IRB Basel II Capital	8 547	10 634	12 808	15 689
Risk weighted assets	106 839	132 931	160 106	196 115
Tier 1 capital	9 096	10 632	12 440	14 526
Tier 2 capital	2 000	2 347	2 950	2 926
Total qualifying capital	11 096	12 978	15 390	17 452
Tier 1 Ratio	8.51%	8.00%	7.77%	7.41%
Total capital ratio (CAR >8%)	10.39%	9.76%	9.61%	8.90%
Credit Value-at-Risk (99.9%)	1 786	1 964	1 982	2 000
Economic Capital	865	909	954	1 002

Table 17: Key stress testing metrics

A quick glance through Table 17 shows a bank that continues to grow its asset base, judging from the growing risk-weighted assets over the three year projection. Although profits are not that large, this bank is well capitalised under the South African minimum capital adequacy ratio of 8%. Tier 1 capital alone satisfies this minimum requirement, for 2012 and 2013, dropping slightly below 8% in 2014 and 2015. These are the core ratios that will be observed before and after stressing both PDs and LGDs. There are many other ratios that can be considered in a stress testing exercise, such as leverage ratio, credit loss ratio, and many other liquidity ratios that were introduced with the new Basel III accord. However, measures in Table 17 will suffice for this exercise.

5.6. Chapter Summary

In this chapter the credit cycle approach of Carlehed and Petrov (2012) was implemented to derive the credit cycle using South African historical corporate insolvencies. The resultant credit cycle was then used to stress both PDs and LGDs using parameters estimated from the regression model that was fitted in Chapter 4.

Our hypothetical credit portfolio that was used to demonstrate the stress testing was also described in detail. The last part of this chapter presented some of the useful measures that need to be closely watched when performing a stress testing exercise.

Chapter 6 - Stress testing scenarios and results

So far we have established the required building blocks for performing a bank wide macroeconomic stress testing. We established a stress testing framework for main risk parameters into the capital calculation, i.e. PDs and LGDs. Key stress testing metrics from the forward looking Income Statement and Balance Sheet are presented in the Appendix (see Table 27), for the projected stress testing period of three years (2013 - 2015).

This chapter covers the design of stress testing scenarios, presents stressed PDs and LGDs under different scenarios, and then finally presents stress testing results of the same metrics shown in Table 17 of Chapter 5.

6.1. Stress testing scenarios

To illustrate the importance of chosen scenarios in a stress testing exercise, consider the following quote from Basel Committee on Banking Supervision (2009), Principles for Sound Stress Testing Practices and Supervision, *"An effective stress testing programme should comprise scenarios along a spectrum of events and severity levels. Doing so will help deepen management's understanding of vulnerabilities and the effect of non-linear loss profiles."*

There are a number of ways in which scenarios can be chosen, or developed. Usually, this part of stress testing relies on the economics department of the bank to recommend events that are likely to affect the bank in terms of losses or liquidity. Such events are chosen with the portfolio dynamics in mind, i.e. the link between different exogenous factors and portfolio losses in a stress period are well thought through.

Arya P (2008), discusses different stress testing methodologies that are generally used, and go into detail regarding their advantages and disadvantages. The methodology is as follows:

i. Sensitivity analysis: calculates the impact of a large predefined shock in specific risk factors.

ii. Historical scenario analysis: based on actual historical events to identify changes in risk factors.

iii. Hypothetical scenario analysis: extreme yet plausible hypothetical shocks are applied to risk factors.

In this dissertation we consider a historical scenario analysis. This means selecting severe historical events, replicating them with the macroeconomic model, and assessing their impact on our hypothetical portfolio. The current macroeconomic variables are scaled to reflect the relative impact experienced historically. The reason for choosing this methodology is because the likelihood or the severity of the event cannot be challenged because it has occurred previously. On the other hand it can be argued that events that occurred in the past are less likely to repeat themselves.

Our historical scenarios will be based on the lowest levels of the credit cycle index presented in Table 14 for the periods 1999 and 2009, where the index is (-1.53) and (-1.55), respectively.

The lowest level of the credit cycle index in year 1999 can be attributed to the recession that hit Asia in 1998. Hussain et al (2002) explain the impact of 1998 Asian crisis to the South African economy. They state that in 1996, Africa's exports to Asia accounted for about 13% of Africa's exports to the rest of the world. As a result, the Asian crisis adversely affected South Africa when the demand for manufactured goods and commodities suddenly decreased significantly.

Similarly the downturn in the index for the period 2009 is attributed to the global financial crisis that broke out in 2007 and 2008. Padayachee (2011) states that as a result of the Global financial crisis, South Africa fell into a technical crisis when the GDP growth rate dropped to 1.8% in the last quarter of 2008, and further plunged to -6.4% in the first quarter of 2009.

We have established two sets of historical scenarios for the periods 1999 and 2009 which will be named Asian crisis, and the Global financial crisis, respectively. Because we have a three year stress testing horizon, for each scenario, we will replicate the lowest historical level, with two subsequent years. Table 18 below shows how the cycle index associated with each recession periods will be replicated over the stress testing period 2013 – 2015.

Year	Asian crisis Cycle Index (Z)	Global financial crisis Cycle Index (Z)
2013	-1.53	-1.55
2014	-0.98	-0.65
2015	-0.61	-0.44

Table 18: Worst historical cycle index values to be replicated

The alternative to this historical scenario approach is to forecast annual values for each of the variables in our model over the stress testing period, and then recalculate the index based on the projected macroeconomic variables. The challenge with this approach is justifying the projected levels for each macroeconomic variable. The next section presents stressed results based on the worst levels of the cycle presented in Table 18.

6.2. Stressed results per stress testing scenario

This section presents the stress testing using stressed PDs and LGDs derived in Chapter 5. The metrics of interest here are the main Basel minimum capital ratios, i.e. tier 1 ratio, total qualifying capital ratio, and other measures displayed in Table 17. A comparison of the two chosen historical scenarios, i.e. the Asian crisis and the Global financial crisis, is presented in tabular format below.

		Actual – 2012	2013	2014	2015
Asian crisis					
	cycle index (Z)	**-0.06**	**-1.53**	**-0.98**	**-0.61**
	PD (%)	6.04%	7.9%	7.1%	6.7%
	%Δ from actual		30.8%	18.1%	10.2%
	LGD (%)	55.7%	65.2%	61.8%	59.5%
	%Δ from actual		17.0%	10.9%	6.8%
Global Financial crisis					
	cycle index (Z)	**-0.06**	**-1.55**	**-0.65**	**-0.44**
	PD (%)	4.78%	7.9%	6.7%	6.4%
	%Δ from actual		66.1%	40.5%	35.1%
	LGD (%)	55.7%	65.3%	59.7%	58.4%
	%Δ from actual		17.2%	7.3%	4.9%

Table 19: Stress testing impact on risk parameters

Table 19 compares the severities of the two scenarios, i.e. the Asian crisis and the Global financial crisis with the actuals of the base year 2012. The stress testing period for the next three years therefore spans from 2013 to 2015. Comparing the cycle indices over the stress testing horizon for both scenarios shows that in 2013 the severity of both scenarios is almost the same. In both 2014 and 2015, the Asian crisis appear to be more severe than the Global financial crisis. Both PDs and LGDs are compared to the 2012 actuals. Both scenarios have a u-shaped impact, starting off in the benign levels, deteriorating to the worst point in 2013, and slowly recovering in 2014 and 2015.

Table 20 shows the full stress testing results, per scenario, including all metrics presented in Table 17. Looking at the measures in this table from the top to the bottom, we start with credit impairment charges. Some of the key measures are defined below:

- Credit impairment charges: are based on expected loss where the incremental expected loss based on stressed PDs and LGDs is added to the initially forecasted impairments.
- Profit after tax: are equal to the forecast revenue, less costs, less tax, and less additional impairments losses under the stress scenario in question.
- Required capital: this is calculated using Equation (3) in Chapter 3, Section 3.1. The only difference is that the PDs and LGDs are replaced by the stressed, and downturn LGDs, respectively.
- Risk-weighted assets: This is a reciprocal of required capital, and it is given as 12.5 times required capital, since the required minimum capital is 8% of RWA.
- Tier 1 capital: this is equity plus the profit and loss for the year.
- Total qualifying capital: this is tier 1 capital including the subordinated debt.
- Expected loss: this is EAD multiplied by PD, multiplied by LGD. For the stressed periods, PDs and LGDs are replaced by stressed values.
- Credit Value-at-Risk: is calculated using Equation (3.6) in Chapter 3.
- Economic capital: is calculated from Equation (3.7) in Chapter 3.

	Measure - R'mn	2012 - Actual	Stressed output 2013	Stressed output 2014	Stressed output 2015
Base	Credit impairment charges	920	1 142	1 379	1 493
	Profit after Tax	2 257	2 765	3 066	3 677
	Required Capital (IRB)	8 547	10 634	12 808	15 689
	Risk Weighted Assets	106 839	132 931	160 106	196 115
	Tier 1 capital	9 096	10 632	12 440	14 526
	Total qualifying capital	11 096	12 978	15 390	17 452
	Tier 1 Ratio	8.51%	8.00%	7.77%	7.41%
	Total capital ratio (CAR >8%)	10.39%	9.76%	9.61%	8.90%
	Expected Loss (EL)	1 629	1 104	1 248	1 398
	Credit Value-at-Risk (99.9%)	2 709	1 964	1 982	2 000
	Economic Capital (99.9%)	1 080	952	960	969
Asian financial crisis	Credit impairment charges		1 734	2 112	1 928
	Profit after Tax		2 338	2 538	3 364
	Required Capital (IRB)		13 691	15 063	17 328
	Risk Weighted Assets		171 134	188 293	216 595
	Tier 1 capital		10 205	11 913	14 213
	Total qualifying capital		12 552	14 862	17 139
	Tier 1 Ratio		5.96%	6.33%	6.56%
	Total capital ratio (CAR >8%)		**7.33%**	**7.89%**	**7.91%**
	Expected Loss (EL)		2 221	1 981	1 833
	Credit Value-at-Risk (99.9%)		2 910	2 843	2 795
	Economic Capital (99.9%)		1 281	1 213	1 166
Global financial crisis	Credit impairment charges		1 743	1 981	1 864
	Profit after Tax		2 332	2 632	3 410
	Required Capital (IRB)		13 735	14 252	16 830
	Risk Weighted Assets		171 683	178 155	210 370
	Tier 1 capital		10 199	12 007	14 260
	Total qualifying capital		12 546	14 957	17 185
	Tier 1 Ratio		5.94%	6.74%	6.78%
	Total capital ratio (CAR >8%)		**7.31%**	**8.40%**	**8.17%**
	Expected Loss (EL)		2 231	1 850	1 769
	Credit Value-at-Risk (99.9%)		2 913	2 801	2 773
	Economic Capital (99.9%)		1 283	1 172	1 144

Table 20: Stress testing results by scenario

Table 20 consists of key capital adequacy measures that enable us to track capital levels as the macroeconomic environment changes. Forward looking stress testing scenarios are applied to the bank's credit portfolio for the projection period 2013 – 2015. Note that for both scenarios, 2013 total capital ratio breaches the minimum capital ratio of 8%, recovering in the following 2 years only in the Global financial crisis.

The 2 important lines to keep track of under all scenarios for the three year stress testing horizon are the Total qualifying capital and Risk weighted assets. The capital adequacy formula is the ratio of these two metrics, with total qualifying capital as a numerator and the risk weighted assets as a denominator. For both stress scenarios, these measures need to be compared to the ones in the base scenario (forecast under current macroeconomic environment). The impact on Total qualifying capital, which is a function of tier 1 equity and net profit after tax, is an average reduction of 3% from the base case in both scenarios. On the other hand, Risk weighted assets increase by an average of 19%, and 16% from the base scenario, in the Asian crisis scenario, and Global financial crisis scenario, respectively, due to stressed PDs and LGDs. Since the numerator (Total qualifying capital) is reducing, and the denominator (Risk weighted assets) is increasing significantly, the resultants Total capital ratio becomes substantially small.

There are a few things worth noting in this table. Firstly, the stress impact on the bottom line, profit after tax, is quite minimal for both scenario. This is mainly because for both scenarios, additional losses in the income statement are derived as the difference between budget and stressed EL. Budget usually anticipates an increase in EL in a growing credit book, so the increase in expected loss was initially provided for in the initial budget.

Secondly, even with a small impact on capital supply side, i.e. the total qualifying capital and the capital adequacy ratios seem to be haemorrhaged by the significant increase in risk-weighted assets (RWA) on the demand side of capital, which is a combination of stressed risk parameters, and a the portfolio risk grade mix for new assets on book. Figure 16 shows that minimum capital adequacy ratio breaches in 2013, for both stress scenarios, and further breaches in 2014 and 2015 on the Asian crisis scenario. This is a clear indication of that our capital ratios are impacted more by the demand side of capital (RWA).

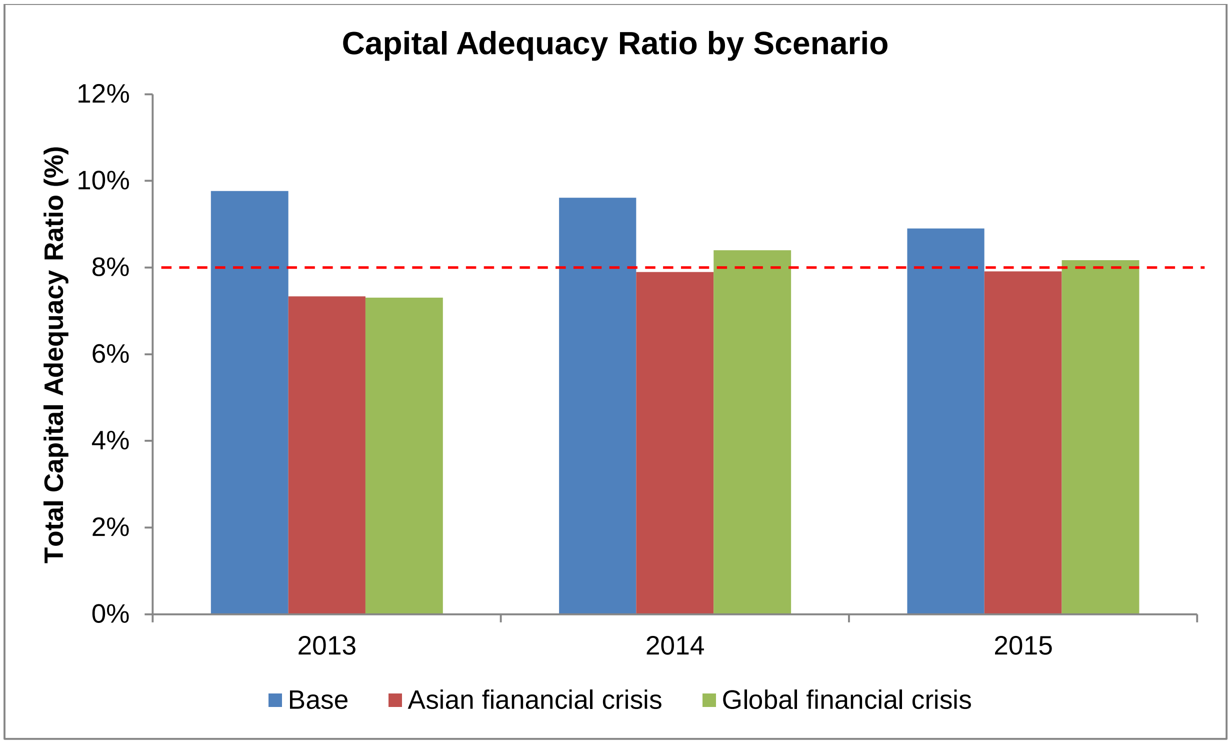

Figure 16: Capital adequacy ratios under by scenarios

Figure 17 shows the difference between total qualifying capital (available capital), and the required capital calculated from the Basel IRB approach. Similar to capital adequacy ratios in Figure 16 above, under the base scenario there's enough excess capital up the year 2015. In the first stress testing year (2013), required capital exceeds the available capital for in both scenarios, which can be clearly seen in Figure 17. In 2014 and 2015, only the Asian crisis scenario continues to show a shortfall in available capital.

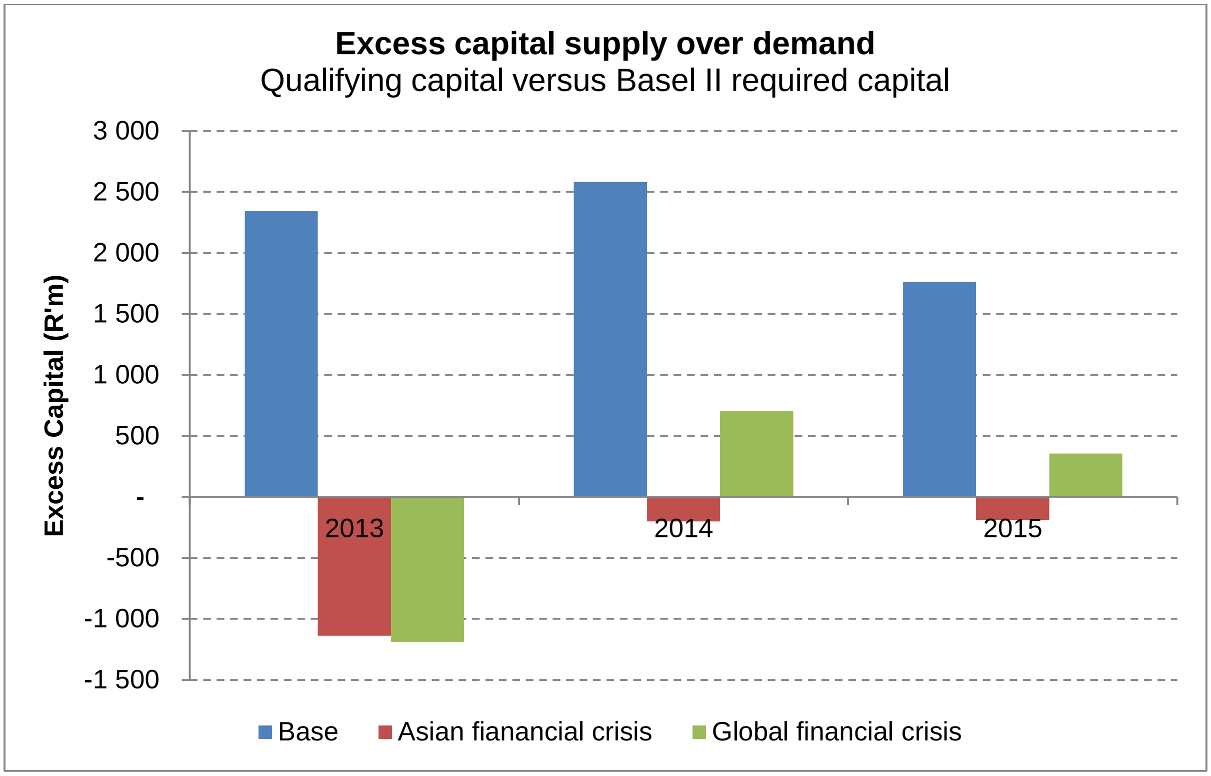

Figure 17: Excess capital by scenario

Finally, it is crucial to put all these results into perspective, given that our portfolio is hypothetical, and that historical stress scenarios were applied. Basically we need to know what these results tell us, given the structure of both our hypothetical Balance sheet and Income statement. This is very important because a well-capitalised bank could have survived the stress scenarios applied here, while our hypothetical bank did not. In South Africa, since the Basel II implementation in 2008, the minimum capital adequacy ratio has been set to 9.75%, see IMF (2008). It is therefore important to note that this dissertation uses the international minimum capital requirement of 8% Total capital to Risk weighted assets, and therefore only the relative change in capital ratios is important instead of the absolute change. The relative change in both capital demand and supply should give a bank an idea of the size of capital buffers, should they have these in place, in anticipation of adverse economic conditions.

6.3. Incorporating Other Risk Types

The scope of this dissertation is mainly the credit risk component of stress testing. Credit risk typically accounts for a significantly large portion of the bank's total regulatory and Economic capital. This ranges from anything between 60-80%, depending on the portfolio mix and the operations of the bank in question.

When performing a bank wide stress testing exercise, other risk types should be incorporated into the stress testing framework, to have a complete view of the macroeconomic impact on the bank's total book. These risk types include:

- Market risk: this is the risk of a loss in asset value due to changes in market variables.
- Business risk: the risk that the bank will generate profits lower than expected.
- Operational risk: the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events.
- Liquidity risk: the risk that an asset cannot be sold at an anticipated price because there no demand in the market at the time.

Risk types are usually added together using a correlation matrix that takes into account an overlap in different risk types. Aggregating different risk types by adding them up without a correlation matrix can overstate the total risk reported. Our complete stress testing process is illustrated in the Appendix, Figure 19.

6.4. Chapter Summary

This chapter presented results from the stress testing exercise using all other inputs that were derived in the preceding chapters. Firstly, different stress testing methods were covered and briefly outlined, and historical scenario analysis is recommended. The last important part of this chapter explained how different risk types such as Operational risk, Market risk, Business risk, and Liquidity risk are usually incorporated into the stress testing framework.

Chapter 7 – Conclusions

Banks are very important to the economic growth of a country, however, an unregulated financial system can cripple the whole country's economy in case of a recession. Their core business is to accept deposits and extend loans to households and companies stimulating the economy. In the South African context four main banks dominate the market, with a market share so significant that one bank's failure could have a systemic effect on the entire economy.

This dissertation is a step-by-step manual on how a bank that has been approved to be on the Basel AIRB approach can perform a systematic macroeconomic stress testing on their capital, using publicly available information. Historical South African macroeconomic data dating as far back as 1980 was collected and used to stress PDs and LGDs, which are main inputs in both regulatory and economic capital. Our model stressed both the demand side (required capital), as well as the supply side (available capital, i.e. Tier 1 and Tier 2).

In Chapter 3, we have provided detailed mechanics of the Advanced Internal Rating-Based (AIRB) approach which is useful in helping the banks to calculate their minimum required capital (capital demand). We have also recommended two models for internal estimation of PDs, and LGDs, respectively. These models could be very useful to banks that are approved to be on the AIRB approach, since they are allowed to estimate their own risk parameters internally, such as PDs, LGDs, and exposure-at-default (EAD).

The credit cycle index is used widely in credit modelling where variables and parameters can be scaled up or down to reflect the status of the credit cycle. Blümke (2010); Miu and Ozdemir (2005), demonstrated how through-the-cycle PDs and LGDs can be scaled up or down to reflect the status of the economic cycle. We have adopted Carlehed, and Petrov (2012) methodology to convert our long-run average PDs to conditional ones using the credit cycle approach.

The multiple regression model in Chapter 4 is a key link between macroeconomic scenarios and stressed parameters. Using corporate insolvencies as the economic cycle indicator, capital is stressed assuming the worst possible levels of the cycle. The rationale for choosing to use insolvencies as our credit cycle indicator is the fact that they are directly related to the corporate default rate. An analysis by Fourie et al (2011), shows that there is a two way relationship between bank-extended credit and insolvencies. Their Vector Autoregression model also substantiates a positive relationship between credit and the business cycle. By implication, insolvencies could be used to get an indication of the business cycle status.

We addressed the problem of subjective expert judgement Top-down stress testing approaches, where management come up with risk factors stress testing guesstimates. This is achieved by using the credit cycle index to determine the stress scenario severity. One important feature of our model, is the ability to derive downturn LGDs which are correlated to PDs through the same credit cycle index that was used to stress PDs. This downturn LGD approach can be adopted by South African banks which are still using a benchmark downturn LGD recommended by the United States' Federal Reserve. Shortfalls of the Federal Reserve LGD are presented in Chapter 5. Our stress testing model is less subjective and more reflective of the state of the economy in the projected stress testing period.

The impact of the two memorable historical stress events, namely the 1998 Asian crisis and the 2008 Global financial crisis, are replicated on our mock Balance Sheet and credit portfolio. The results are quite useful in revealing strengths and weaknesses of a bank's balance sheet. In our case we have shown that a growth in assets can lead to significant growth in risk weighted assets (capital demand), if not controlled properly. Where the minimum capital ratios have been breached in the stress testing exercise, our model clearly indicates how much more capital supply is required to be within the required level of capital adequacy.

The importance of stress testing goes beyond a regulatory compliance exercise of proving that a bank has enough capital to survive a crisis. Stress testing forms an integral part of a Risk Appetite Framework (RAF) as a tool that validates limits and triggers set by the internal risk management committees. A Risk Appetite Framework ensures enterprise risk management practice which is essential when monitoring the amount of risk the bank is taking. In addition stress testing also informs the budgeting process and capital planning process, by indicating how much buffers should be held in anticipation of a stress event.

Although this dissertation focuses on the credit component of risk, it is emphasised that other risk types should also be stressed in order to gain the full benefit of the stress testing exercise. As presented in Chapter 2 (Section 2.3), new measures introduced in Basel III attempt to ensure that all potential risks are covered, with emphasis on liquidity risk. The model presented in this dissertation can be very useful when setting procyclicality buffers that are in line with the projected macroeconomic outlook. This approach relies more on the status of credit cycle index, therefore it is less subjective and reflective of the economic environment in the country.

References

Alan Hirsch, (1989), The Origins and Implications of South Africa's continuing Financial Crisis. Transformation 9. 32-42.

Alessandri, Gai, Kapadia, Mora and Puhr (2007), A framework for quantifying systemic stability, December, preliminary paper presented at the workshop "Stress Testing of Credit Risk Portfolios: The Link between Macro and Micro", hosted by the BCBS and the DNB, Amsterdam, 7 March 2008.

Altman E, (2006), Default Recovery Rates and LGD in Credit Risk Modelling and Practice: An Updated Review of the Literature and Empirical Evidence. New York, USA.

Arya O, (2008), Effective Techniques for Stress Testing and Scenario Analysis, Federal Reserve Bank of New York.

Barnhill T, Papapanagiotou P, and Schumacher L, (2000), Measuring Integrated Market and Credit Risks in Bank Portfolios: An Application to a Set of Hypothetical Banks Operating in South Africa. IMF Working Paper No. 00/212.

Basel Committee on Banking Supervision, (1995), An Internal Model-based Approach to Market Risk Capital Requirements. Bank for International Settlements.

Basel Committee on Banking Supervision, (1996), Amendment to the Capital Accord to Incorporate Market Risks. Bank for International Settlements.

Basel Committee on Banking Supervision, (2004), An Explanatory Note on the Basel II IRB Risk Weight Functions. Bank for International Settlements.

Basel Committee on Banking Supervision, (2006), International Convergence of Capital Measurement and Capital Standards, a Revised Framework, Comprehensive Version. Bank for International Settlements.

Basel Committee on Banking Supervision, (2009), Principles for Sound Stress Testing Practices and Supervision. Bank for International Settlements.

Basel Committee on Banking Supervision, (2010a), A Global Regulatory Framework for more Resilient Banks and Banking Systems. Bank for International Settlements.

Basel Committee on Banking Supervision, (2010b), Basel III: International framework for liquidity risk measurement, standards and monitoring. Bank for International Settlements.

Basel Committee on Banking Supervision, (2011), The Transmission Channels between the Financial and Real Sectors: a Critical Survey of the Literature. Bank for International Settlements.

Blümke O, (2010), Probability of Default Estimation and Validation within the Context of the Credit Cycle. The Journal of Risk Model Validation (27–45) Volume 4/Number 2, Summer 2010.

Browne M, Carson J, and Hoyt R, (1999), Economic and Market Predictors of Insolvencies in the Life-Health Insurance Industry. The Journal of Risk and Insurance, 1999, Vol. 66, No. 4, 643-659.

Buncic D and Melecky M, (2012), Macroprudential Stress Testing of Credit Risk, A Practical Approach for Policy Makers. The World Bank. Policy Research Working Paper 5936. 2-39.

Carlehed M and Petrov A, (2012), A Methodology for Point-in-Time–Through-the-Cycle Probability of Default Decomposition in Risk Classification Systems. Journal of Risk Model Validation Volume 6/Number 3, Fall 2012. 3–25.

Carling K, Jacobson T, Lindé J, and Roszbach K, (2002), Capital Charges under Basel II: Corporate Credit Risk Modelling and the Macro Economy. Sveriges Riksbank Working Paper Series No. 142.

Chan-Lau J, (2006), Market-Based Estimation of Default Probabilities and Its Application to Financial Market Surveillance. IMF Working Paper WP/06/104.

Damodaran A, (2012), Equity Risk Premiums (ERP): Determinants, Estimation and Implications – The 2012 Edition. Stern School of Business.

Das S and Hanouna P, (2009), Implied Recovery, Santa Clara University, Leavey School of Business. USA.

Dwyer D and Korablev I, (2009), Modeling Methodology: Moody's KMV LossCalc V3.0. Moody's Analytics.

Emery K and Cheparev T, (2007), Adjusting Moody's LGD Assessments to Meet Basel II Downturn Requirements. Moody's.

Ferry, Danielle H., Tony Hughes, and Ming Ding, (2012b), Stressed EDF Credit Measures for Western Europe. Modeling Methodology (Moody's Analytics).

Financial Services Authority, (2009), Variable Scalar Approaches to Estimating Through the cycle PDs. Wholesale & Prudential Policy Division.

Foglia, A. (2009), Stress Testing Credit Risk: A survey of Authorities' Approaches. International Journal of Central Banking, vol.6, no. 2.

Fourie L, Botha I and Mears R, (2011), Credit extension in South Africa: A business cycle perspective for the period 1985 to 2009. African Journal of Business Management Vol. 5(34), pp. 13074-13083.

Frye J and Jacobs M Jr., (2012), Credit Loss and Systematic Loss Given Default. The Journal of Credit Risk (109–140) Volume 8/Number 1, Spring 2012.

Frye J. (2010), Modest means. Risk 23(1), pp. 94–98.

Giese G, (2005), The Impact of Pd-LGD Correlations on Credit Risk Capital. Risk April 2005, available at http://www.risk.com

Gordy M and Lütkebohmert E, (2006), Granularity Adjustment for Basel II. Board of Governors of the Federal Reserve System, Washington DC.

Gordy M, (2002), A Risk-Factor Model Foundation for Ratings-Based Bank Capital Rules. Board of Governors of the Federal Reserve System.

Gujarati N, (2005), Basic Econometrics. 4th Ed. New York: The McGraw-Hill Companies. 341-353.

Havrylchyk O, (2010), A macroeconomic credit risk model for stress testing the South African banking sector. South African Reserve Bank Working Paper WP/10/03.

Hoggarth G and Whitley J (2003), Assessing the strength of UK banks through macroeconomic stress tests. Financial Stability Review, Bank of England, June.

Howard S, (2009), Stress testing with incomplete data: a practical guide. Economist at the Central Bank of Barbados. IFC Bulletin No 31.

Hull J and White A, (2000), Valuing Credit Default Swaps I. University of Toronto. Canada. 13-21.

Hull J, Predescu M, and White A, (2005), Bond Prices, Default Probabilities and Risk Premiums. Journal of Credit Risk Volume 1/Number 2, Spring 2005. Pp. 54-55.

Institute of International Finance, (2011), Implementing Robust Risk Appetite Frameworks to Strengthen Financial Institutions.

International Monetary Fund, (2008), South Africa: Financial System Stability Assessment, Including Report on the Observance of Standards and Codes on the following topic: Securities Regulation. IMF Country Report No. 08/349.

International Monetary Fund, Accessed 1 April 2013, http://www.imf.org/external/data.htm.

Jarrow R and Protter P, (2004), Structural versus Reduced Form Models: A New Information Based Perspective. Journal of Investment Management, Vol. 2, No. 2, (2004), 1–10.

Jarrow R and Turnbull S (1995), Pricing Derivatives on Financial Securities Subject to Credit Risk. Journal of Finance 50(1), 53–85.

Jokivuolle E, Virolainen K, and Vähämaa O, (2008), Macro-model-based Stress Testing of Basel II capital Requirements. Bank of Finland Research. Discussion Papers 17.

Jones C, (2008), the Global Financial Crisis of 2007. A Supplement to Macroeconomics (W.W. Norton, 2008).

Koenker R and Xiao Z (2001), Inference on the quantile regression process. Journal of Economic Perspectives, Volume 15, No. 4, 2001.

Le Leslé V and Avramova S, (2012), Revisiting Risk-Weighted Assets: Why Do RWAs Differ Across Countries and What Can Be Done About It? IMF Working Paper WP/12/90.

Lobanov A (2011), Prudential Regulation of Market Risk: From Basel I to Basel III, PowerPoint presentation, 5 February 2011.

Mausser H and Rosen D, (2008), Chapter 16: Economic Credit Capital Allocation and Risk Contributions. Birge J.R. and Linetsky V (Eds.), Handbooks in OR & MS, Vol. 15.

Melecky M and Podpiera A, (2012), Macroprudential Stress-Testing Practices of Central Banks in Central and South Eastern Europe - An Overview and Challenges Ahead. The World Bank Policy Research Working Paper 5434.

Miu P and Ozdemir B, (2005), Basel Requirement of Downturn LGD: Modeling and Estimating PD & LGD Correlations. Journal of Credit Risk (43–68) Volume 2/ Number 2, Summer 2006.

Moody's Investor Services, (2011), Corporate Default and Recovery Rates, 1920-2010. 30-40.

Moody's Investors Services, (2009), Moody's Rating Symbols & Definitions.

Nureldin H, Mlambo K, and Oshikoya T, (2002), Global Financial Crisis: Implications and Lessons for Africa. African Development Bank. Strategic Planning and Research Department. pp. 1-24.

O'Kane D and Turnbull S, (2003), Valuation of Credit Default Swaps. Fixed Income Quantitative Credit Research, Lehman Brothers.

Orzechowska-Fischer E, and Taplin B, (2010), Forecasting Corporate Insolvency Rates in Australia. Australian Conference of Economists, 27-29th September 2010.

Oura H and Schumacher L, (2012), Macrofinancial Stress Testing—Principles and Practices. International Monetary Fund. Monetary and Capital Markets Department.

Packer and Taravesh, (2007), Rating Methodologies for Banks. BIS Quarterly Review, June 2011.

Padayachee V, (2011), Global Economic Recession: Effects and Implications for South Africa at a Time of Political Challenges. Claves de la Economia Mundial.

Pykhtin M, (2003), Unexpected Recovery Risk. Risk 16(8).

Rösch D and Scheule H, (2008), Credit Losses in Economic Downturns - Empirical Evidence for Hong Kong Mortgage Loans. Hong Kong Institute for Monetary Research.

Rosen D, (2004), Practical Application of Economic Credit Capital Allocation Methodologies. Risk Lab. Madrid. Algorithmics Incorporated.

Schechtman R and Gaglianone W, (2011), Macro Stress Testing of Credit Risk Focused on the Tails. Banco Central Do Brasil. Working Series Paper 241.

Schmieder C, Puhr C, and Hasan M, (2011), Next Generation Balance Sheet Stress Testing, IMF Working Paper (WP/11/83). pp. 3-36.

Schuermann T, (2004), What Do We Know About Loss Given Default? Credit Risk Models and Management 2nd Edition, London, UK: Risk Books.

Sorge M, (2004), Stress-testing Financial Systems: An Overview of Current Methodologies, BIS Working Paper No. 165. Monetary and Economic Department.

Sorge M and Virolainen K (2006), A comparative analysis of macro stress testing methodologies with application to Finland. Journal of financial stability, 2, pp. 113–151.

South African Reserve Bank, (2006), Accord Implementation Forum, point-in-time vs. Through-the-cycle credit Rating Systems. Accord Implementation Forum Steering Committee, Position Paper 149 v.2.

South African Reserve Bank, (2012), Bank Supervision Department Annual Report. Chapter 4: Banking Sector Overview. pp. 28.

Statistics South Africa, accessed 1 April 2013, http://www.statssa.gov.za/Publications

Styger P and Vosloo P, (2005), The Banker's Guide to the Basel II Framework. The Banking Association South Africa.

The World Bank, accessed 1 April 2013, http://www.worldbank.org/

Van Den End, Hoeberichts, and Tabbae (2006), Modelling Scenario Analysis and Macro Stress-testing, WP No. 119, De Nederlandsche Bank.

Vasicek O, (2002), Loan portfolio value. Risk 15(12), pp. 160–162.

Wilson, T C (1997a), Portfolio credit risk I. Risk 10(9), pp. 111–117.

Wong J, Choi K, and Fong P, (2008), A Framework for Stress-testing Banks' Credit Risk. The Journal of Risk Model Validation (3–23) Volume 2/Number 1, Spring 2008.

Appendix

Risk grade	Moody's Rating	1-year Default rate
1	Aaa	0.00%
2	Aa1	0.00%
3	Aa2	0.00%
4	Aa3	0.05%
5	A1	0.12%
6	A2	0.11%
7	A3	0.08%
8	Baa1	0.19%
9	Baa2	0.23%
10	Baa3	0.36%
11	Ba1	0.46%
12	Ba2	0.78%
13	Ba3	1.20%
14	B1	1.74%
15	B2	3.54%
16	B3	5.86%
17	Caa1	9.99%
18	Caa2	19.19%
19	Caa3	30.04%
20-21	Ca-C	43.45%

Table 21: Internal ratings mapping to Moody's ratings and Default rate. (Moody's, 2011)

Table 21 shows Moody's ratings with their corresponding historical default rate as measured by Moody's on a large sample of firms globally.

Assessments	Loss range
LGD1	≥ 0 and < 0.1
LGD2	≥ 0.1 and < 0.3
LGD3	≥ 0.3 and < 0.5
LGD4	≥ 0.5 and < 0.7
LGD5	≥ 0.7 and < 0.9
LGD6	≥ 0.9 and ≤ 1

Table 22: Moody's Loss Given Default (LGD) assessments. (Moody's, 2009)

Table 22 shows Moody's LGD groupings based on counterparty or instrument rating, as measured by Moody's on a large sample of firms globally.

Risk grade	Moody's Rating	LGD
1	Aaa	10%
2	Aa1	15%
3	Aa2	20%
4	Aa3	20%
5	A1	25%
6	A2	25%
7	A3	25%
8	Baa1	30%
9	Baa2	35%
10	Baa3	40%
11	Ba1	45%
12	Ba2	50%
13	Ba3	55%
14	B1	60%
15	B2	65%
16	B3	70%
17	Caa1	75%
18	Caa2	80%
19	Caa3	85%
20	Ca	90%
21	C	95%

Table 23: Internal risk grades mapped to Moody's LGDs

Table 23 is the bank's internal ratings mapped to Moody's ratings and expected LGDs presented in Table 22.

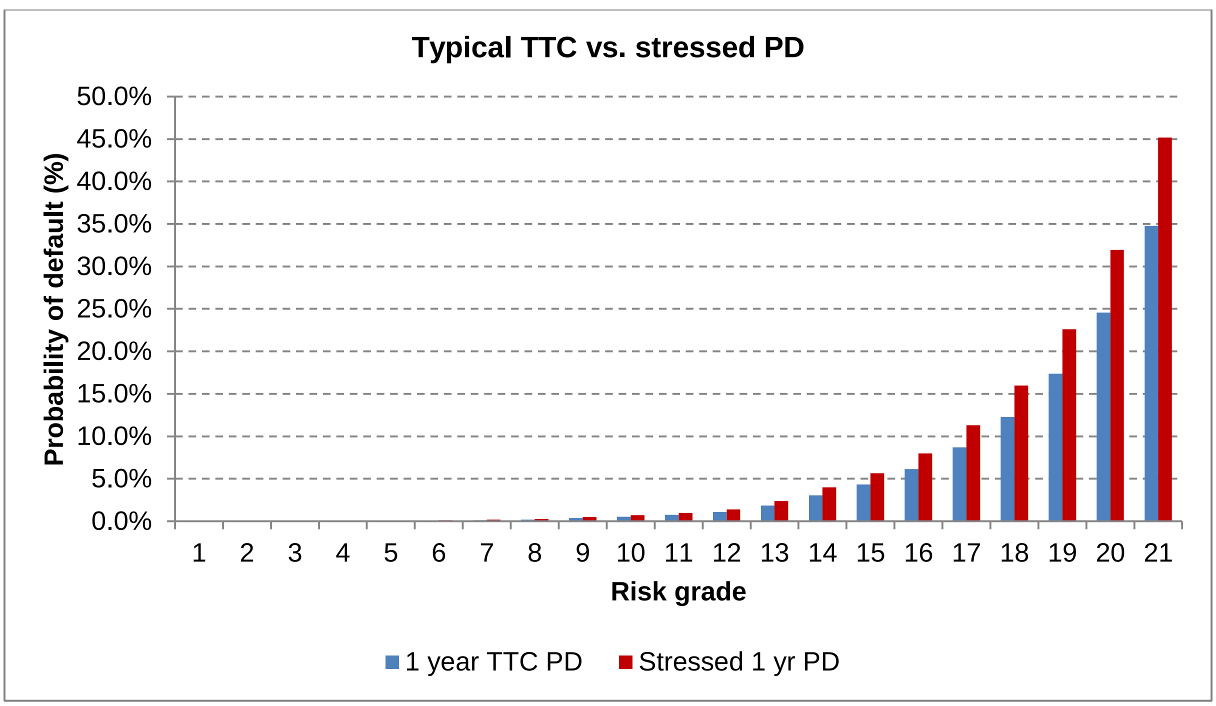

Figure 18: Typical 1-year PDs versus Stressed PDs

Figure 18 shows typical probabilities of default by rating category, before stressing and after stressing. The impact of stress testing is typically more on lower rated instruments or counterparties.

Internal risk grade	Exposure R'mn (EAD)	Maturity (yrs.)	1-year PD	LGD
Grade 17	167	3	9.99%	75%
Grade 11	167	3	0.46%	45%
Grade 06	167	3	0.11%	25%
Grade 18	167	3	19.19%	80%
Grade 10	167	3	0.36%	40%
Grade 13	167	3	1.20%	55%
Grade 09	167	3	0.23%	35%
Grade 15	167	3	3.54%	65%
Grade 14	167	3	1.74%	60%
Grade 13	167	3	1.20%	55%
Grade 12	167	3	0.78%	50%
Grade 16	167	3	5.86%	70%

Table 24: Credit portfolio snapshot with mapped PDs and LGDs

Table 24 is a snippet of the counterparty level hypothetical credit portfolio, showing counterparties' ratings, exposure, average time-to-maturity of all their instruments, a rating-based probability of default (PD) and loss given default (LGD).

year	GDP_growth	gross_national _saving_GDP	Inflation_index	inflation_prices	imports_ volumes	Unemployment rate	Insolvencies	Prime rate	PPI	real_int_rates	risk_premium	real effective exchange rate	Exports (% GDP)	Export growth	Import (%GDP)
1980	0.07	33.61	10.80	14.24	21.62	9.24	1 003	13.2%	15%	-12.3%	4.86%	155.77	35%	0%	27%
1981	0.05	26.39	12.50	15.74	14.22	9.82	831	13.2%	13%	3.7%	4.20%	163.88	28%	-5%	30%
1982	-0.00	20.46	14.30	14.40	-17.85	10.76	998	18.8%	13%	4.7%	3.74%	155.28	26%	-3%	27%
1983	-0.02	24.79	16.10	12.59	-18.68	12.54	1 201	17.2%	10%	0.1%	3.22%	170.91	25%	-1%	21%
1984	0.05	21.67	17.90	11.18	21.47	13.72	1 637	23.0%	9%	9.7%	3.00%	150.77	25%	3%	24%
1985	-0.01	24.24	20.80	16.20	-14.70	15.45	3 221	20.8%	15%	4.0%	3.94%	114.49	31%	10%	23%
1986	0.00	23.17	24.70	18.75	-5.32	16.05	4 248	13.9%	19%	-2.3%	3.90%	105.69	31%	-4%	22%
1987	0.02	21.94	28.70	16.19	3.96	16.59	3 883	12.5%	14%	-1.7%	3.79%	119.28	30%	5%	20%
1988	0.04	22.52	32.40	12.89	23.94	17.24	2 563	15.2%	14%	0.1%	3.30%	112.66	29%	8%	23%
1989	0.02	22.21	37.10	14.51	-1.54	17.83	2 658	20.0%	16%	2.2%	3.00%	113.12	27%	2%	21%
1990	-0.00	18.93	42.40	14.29	-4.27	18.78	3 104	20.0%	12%	4.7%	3.20%	116.30	24%	0%	19%
1991	-0.01	18.30	49.00	15.57	0.38	20.16	4 057	20.1%	11%	4.0%	3.63%	121.05	22%	-2%	17%
1992	-0.02	16.44	55.70	13.67	5.54	21.21	5 254	18.3%	8%	3.8%	5.14%	125.06	21%	5%	17%
1993	0.01	16.14	61.20	9.87	6.70	22.16	4 843	15.8%	7%	2.7%	4.85%	122.74	22%	10%	18%
1994	0.03	16.83	66.60	8.82	18.92	22.89	3 473	16.3%	9%	5.5%	4.65%	117.49	22%	2%	20%
1995	0.03	16.51	72.40	8.71	19.68	16.71	2 770	18.0%	11%	6.9%	4.37%	114.20	23%	11%	22%
1996	0.04	15.99	77.70	7.32	11.85	19.32	2 803	19.8%	8%	10.6%	4.48%	105.13	25%	7%	23%
1997	0.03	15.18	84.40	8.62	6.73	20.95	3 283	19.3%	8%	11.0%	4.74%	110.89	25%	5%	23%
1998	0.01	15.24	90.20	6.87	1.22	25.20	4 289	22.7%	4%	13.1%	5.26%	101.82	26%	3%	25%
1999	0.02	15.71	94.90	5.21	-9.29	23.35	6 025	18.7%	5%	10.2%	5.15%	96.28	25%	1%	23%
2000	0.04	15.57	100.00	5.37	7.73	25.61	4 694	14.5%	7%	5.2%	4.39%	93.25	28%	8%	25%
2001	0.03	15.33	105.70	5.70	0.16	29.40	3 936	13.4%	8%	5.7%	4.09%	82.29	30%	2%	26%
2002	0.04	16.70	115.40	9.18	5.61	30.41	3 048	15.5%	13%	4.5%	4.59%	70.48	33%	1%	29%
2003	0.03	15.66	122.10	5.81	4.65	27.96	2 643	13.4%	2%	8.9%	4.29%	91.32	28%	0%	26%
2004	0.05	15.04	123.80	1.39	17.90	26.21	1 954	11.0%	2%	4.6%	3.76%	99.65	26%	3%	27%
2005	0.05	14.49	128.00	3.39	11.78	26.73	1 589	10.5%	4%	4.9%	3.72%	100.00	27%	9%	28%
2006	0.06	14.38	134.00	4.69	19.89	25.54	1 680	11.8%	8%	4.4%	3.83%	96.13	30%	7%	32%
2007	0.06	14.27	143.50	7.09	9.43	22.23	2 314	13.8%	11%	4.7%	4.05%	90.38	31%	7%	34%
2008	0.04	15.53	160.06	11.54	2.94	22.91	4 763	15.2%	14%	6.6%	4.32%	80.42	36%	2%	39%
2009	-0.02	15.59	171.46	7.13	-20.46	23.94	6 078	12.1%	0%	3.2%	3.86%	87.66	27%	-20%	28%
2010	0.03	16.61	178.78	4.27	8.85	24.91	4 020	9.5%	6%	2.5%	3.41%	101.24	27%	5%	28%
2011	0.03	16.47	187.72	5.00	9.71	24.51	3 624	9.5%	8%	2.8%	3.51%	99.13	29%	6%	30%
2012	0.03	14.79	198.50	5.75	4.30	23.81	2 994	8.5%	6%	3.1%	3.46%	93.72	28%	0%	31%

Table 25: Historical macro-economic variables

Variable	Unit description	Std Dev.	Mean	Median
prime_rate	Annual average	3.9%	15.7%	15.0%
consumer_price_index	Consumer price Index	56.92	84.21	77.70
Insolvencies	Number of corporate insolvencies in a given year	1 405.51	3 196.39	3 104.00
unemployment_rate	Percent of total labour force	5.6%	20.7%	22.2%
gross_national_saving_gdp	Percent of GDP	4.4%	18.4%	16.5%
imports_volumes	Percent change	12.0%	5.1%	5.6%
inflation_prices	Annual percentages of average consumer prices	4.6%	9.8%	8.8%
export_growth	Exports of goods and services (annual % growth)	5.8%	2.6%	3.0%
exports_gdp	Exports as a percentage of GDP	3.6%	27.3%	27.0%
gdp_growth	Annual GDP growth	2.5%	2.5%	3.0%
import_gdp	Imports as a percentage of GDP	5.0%	25.1%	25.0%
producer_price_index	Wholesale price index	4.5%	9.4%	9.0%
real_effective_exchange_rate	Nominal effective exchange rate	24.35	111.47	105.69
real_int_rates	Lending rates adjusted for inflation	4.6%	4.4%	5.0%
risk_premium	Rate on loans in excess of risk-free rate	0.7%	4.0%	4.0%

Table 26: Data summary statistics: years 1980 – 2012

Table 26 shows some basic statistics on all macroeconomic variables that were used in the study, irrespective of whether they were included in the final model or not.

Income statement – R'mn	2012 - Actual	2013	2014	2015
Net interest income	**2 611**	**3 547**	**4 152**	**4 999**
Interest income	4 821	4 747	5 558	6 690
Interest expense	2 210	1 200	1 406	1 691
Non-interest revenue	**4 586**	**4 874**	**5 441**	**6 150**
Total income	**7 197**	**8 421**	**9 593**	**11 149**
Credit impairment charges	-920	-1 142	-1 379	-1 493
Income after credit impairment charges	6 276	7 279	8 214	9 656
Operating expenses	**3 142**	**3 440**	**3 956**	**4 549**
Net income before taxation	**3 134**	**3 840**	**4 258**	**5 107**
Tax Rate	28.0%	28.0%	28.0%	28.0%
Taxation	878	1 075	1 192	1 430
Profit after Tax	**2 257**	**2 765**	**3 066**	**3 677**
Dividends paid	300	400	600	700

Balance sheet – R'mn				
Cash and balances with the central bank	6 368	5 490	5 782	6 739
Pledged assets	1 249	1 600	1 760	2 112
Derivative assets	785	864	950	1 045
Trading securities	5 578	6 780	4 800	5 760
Financial investments	2 403	5 354	6 240	7 488
Loans and advances	**33 461**	**41 828**	**54 979**	**68 211**
Other assets	1 060	1 331	1 610	1 932
Intangible assets	204	256	282	339
Property and equipment	906	1 214	1 544	1 852
Total Assets	**52 014**	**64 717**	**77 947**	**95 478**
Trading & derivatives liabilities	1 390	2 624	2 650	4 690
Deposit and current accounts	39 934	49 655	60 561	75 076
Other liabilities	1 852	2 224	2 412	1 937
Subordinated bond/debt	2 000	2 347	2 950	2 926
Equity	6 839	7 867	9 375	10 850
Total Equity and Liabilities	**52 014**	**64 717**	**77 947**	**95 478**

Table 27: Actual and base forecast Income Statement and Balance Sheet

Table 27 shows an Income Statement and a Balance Sheet of the hypothetical bank being studied. The 2012 values are actuals as at the year, followed by a 3 year forecast for the stress testing period. Net loans make up to 70% of total assets on average, with deposits and current accounts making up to 78% of total liabilities.

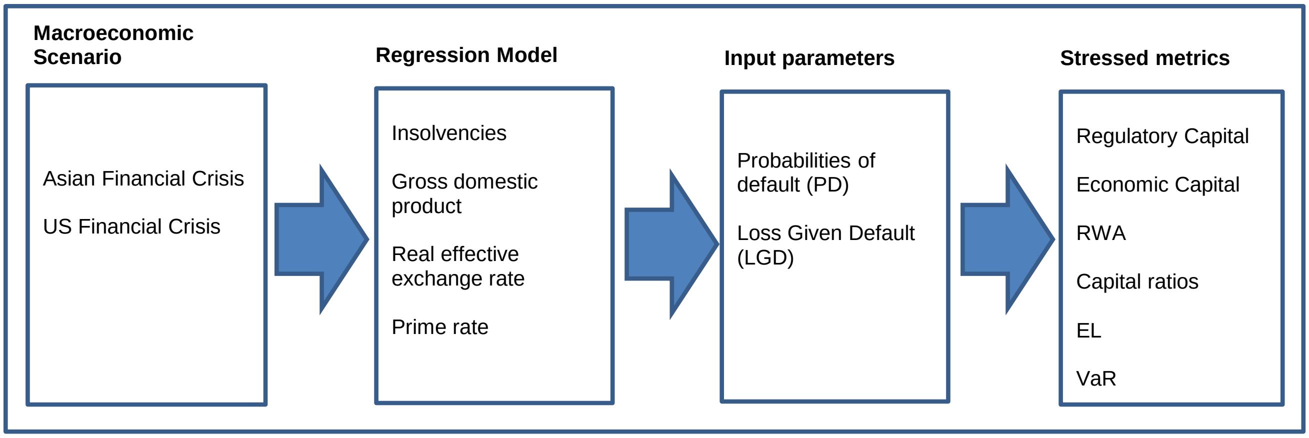

Figure 19: Stress testing flow diagram for selected scenarios

Figure 19 shows the overview of the stress testing flow, starting with the two scenarios, i.e. Asian crisis and US Financial crisis, the regression model, stressing of risk parameters PDs and LGDs, and the output metrics that show the impact of stress testing.

A.1. VBA code for IRB Basel capital calculations

```
Function calcKCorp (pd As Double, lgd As Double, sig As Double, m As Double)

Dim N1, N2, N3, N4 as Double
Dim N as Double
Dim K1, K2, K3 as Double
Dim b As Double 'maturity adjustment

Call calcRCorp(pd)
R = calcRCorp(pd)
N1 = (1 - R) ^ (-0.5)
N2 = WorksheetFunction.NormSInv(pd)
N3 = (R / (1 - R)) ^ (0.5)
N4 = WorksheetFunction.NormSInv(sig)
N = (N1 * N2) + (N3 * N4)
K1 = ((lgd * WorksheetFunction.NormSDist(N) - lgd * pd))
b = (0.11852 - 0.05478 * WorksheetFunction.Ln(pd)) ^ 2
K2 = (1 - 1.5 * b) ^ (-1)
K3 = 1 + (m - 2.5) * b
calcKCorp = K1 * K2 * K3
End Function

Function calcRCorp(pd As Double)
Dim R1 As Double
Dim R2 As Double
Dim R3 As Double
R1 = (1 - Exp(-50 * pd))
R2 = (1 - Exp(-50))
R3 = (1 - (1 - Exp(-50 * pd)) / (1 - Exp(-50)))
calcRCorp = 0.12 * R1 / R2 + 0.24 * R3
End Function

Function calcRsme(pd As Double, turnover As Double)

Call calcRCorp(pd)
calcRsme = calcRCorp(pd) - 0.04 * (1 - (turnover - 40) / 360)

End Function

Function calcDrawnRWACorp(pd As Double, lgd As Double, sig As Double, m As Double, EAD As Double)

Call calcKCorp(pd, lgd, sig, m)
K = calcKCorp(pd, lgd, sig, m)

calcDrawnRWACorp = K * 12.5 * EAD

End Function

Function calcULCorp(pd As Double, lgd As Double, sig As Double, m As Double)

Call calcRCorp(pd)
R = calcRCorp(pd)
N1 = (1 - R) ^ (-0.5)
N2 = WorksheetFunction.NormSInv(pd)
N3 = (R / (1 - R)) ^ (0.5)
N4 = WorksheetFunction.NormSInv(sig)
N = (N1 * N2) + (N3 * N4)
calcULCorp = lgd * WorksheetFunction.NormSDist(N)

End Function
```

A.2. SAS code for macroeconomic regression model

```
proc  contents data = master.econ out=ttt;run;

/*get some basic statistical information on the variables*/
Proc means data=master.econ std mean median;
var
prime_rate
Inflation_index
Insolvencies
Unemployment_rate
gross_national_saving_gdp
imports_volumes
inflation_prices
export_growth
exports_gdp
gdp_growth
import_gdp
producer_price_index
real_effective_exchange_rate
real_int_rates
risk_premium

;
run;

/*Check for collinearity of variables: corr > 0.9*/
Proc corr data=master.econ;
var Insolvencies prime_rate Inflation_index
Unemployment_rate gross_national_saving_gdp
imports_volumes inflation_prices
export_growth exports_gdp
gdp_growth import_gdp
producer_price_index
real_effective_exchange_rate
real_int_rates
risk_premium
;
run;

/*Step 1: First model attempt with lagged variables included*/
Proc reg data=master.lagecon;
model Insolvencies = Unemployment_rate gross_national_saving_gdp  gdp_growth
                               real_effective_exchange_rate prime_2
gross_saving_1 refx_1 real_int_2 /  p r influence pcorr1 pcorr2;
Ods output outputstatistics = diag;
quit;

/*Step 2: Remove the 2nd lag of real interest rate*/
Proc reg data=master.lagecon;
model Insolvencies = Unemployment_rate gross_national_saving_gdp  gdp_growth
                               real_effective_exchange_rate prime_2
gross_saving_1 refx_1  /  p r influence pcorr1 pcorr2;
Ods output outputstatistics = diag;
quit;
```

```
/*Step 3: remove the first lag of real effective exchange rates*/
Proc reg data=master.lagecon;
model Insolvencies = Unemployment_rate gross_national_saving_gdp  gdp_growth
                               real_effective_exchange_rate prime_2
gross_saving_1  /  p r influence pcorr1 pcorr2;
Ods output outputstatistics = diag;
quit;

/*Step 4*: remove gross national saving*/
Proc reg data=master.lagecon;
model Insolvencies = Unemployment_rate  gdp_growth
real_effective_exchange_rate prime_2 gross_saving_1  /  p r influence pcorr1
pcorr2;
Ods output outputstatistics = diag;
quit;

/*Step 5: remove unemplyment rate*/
Proc reg data=master.lagecon;
model Insolvencies = gdp_growth real_effective_exchange_rate prime_2
gross_saving_1  /  p r influence pcorr1 pcorr2;
Ods output outputstatistics = diag;
quit;


/*Step 6: remove gross_saving_1 lag*/
Proc reg data=master.lagecon;
model Insolvencies = gdp_growth real_effective_exchange_rate prime_2 /  p r
influence pcorr1 pcorr2;
Ods output outputstatistics = diag;
quit;


proc univariate data=diag;
histogram StudentResidual /normal;
run;


proc univariate data=diag plots;
var Residual;
run;

/*final check for multicollinearity*/
Proc corr data=master.lagecon;
var Insolvencies  gdp_growth real_effective_exchange_rate prime_2;
run;

/*buid the final model using parameter estimates*/

data model (keep= Insolvencies  gdp_growth real_effective_exchange_rate
prime_2 calc_insolv );
 set master.lagecon;
 calc_insolv = 6194.975 - 31038*gdp_growth - 42.92*
real_effective_exchange_rate + 15593 * prime_2;
run;
```