Backtesting Historical Simulation Value-at-Risk for a selected portfolio of South African bonds

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Abstract

As financial asset portfolios have become more complex, it has become more difficult for the management of financial institutions to obtain a useful, yet practical measure of market risk. Since modern portfolios contain more derivative instruments, simple linear measures such as standard deviation and duration are inappropriate. Due to this need a market risk measurement technique called Value-at-Risk (VaR) was developed. VaR can be defined as the predicted maximum potential adverse loss of a single financial asset, or portfolio of assets, over a target horizon, within a given confidence interval.

A backtesting procedure was designed to compare realized trading results of a selection of representative bonds with model generated risk measures in order to evaluate the accuracy of the VaR model. The backtesting procedure used in this study involves the comparison between the number of times the VaR model under-predicted the subsequent day's loss, versus the number of times such an under-prediction is expected.

The empirical results from this study illustrates that VaR underestimated risk during periods of high volatility and overestimated VaR during periods of low volatility, thus rendering it useless as a measure of extreme market movement. The purpose of this study is not to test the validity of VaR, but to illustrate the shortcoming of VaR, in that it measures only market risk. Practitioners should always bear in mind that VaR is a market risk measurement technique and does not warn of extreme market movements.

Key words

Risk management *Value-at-Risk (VaR)*
Financial risk *Historical Simulation Value-at-Risk*
Market risk *Bond Exchange of South Africa*
Bond market risk *Volatility*
Backtesting

[Page 183]

1 Introduction

Recent failures of risk management in investment institutions, international government agencies, and corporations have led to a widespread call for improved quantification of financial risk. Examples illustrating this point include the $1.64 billion loss by California's Orange County in 1994 and the $1.3 billion loss by England's 233-year-old Barings Bank (Jorion 1997:20). As asset portfolios have become more complex, it has become more difficult for the management of financial institutions to obtain a useful, yet practical measure of market risk. Since modern portfolios contain more derivative instruments, simple linear measures such as standard deviation and duration are inappropriate. These measures are unable to accurately measure the risk associated with large moves in the underlying prices of the portfolio's assets (Smithson 1996:25). Derivative-based risk measures are useful at the trading desk for determining the portfolio's sensitivity to individual risk factors but, since they cannot be aggregated across asset classes or instruments to summarise total portfolio market risk, they are not very useful for management reporting purposes. Due to this need a market risk measurement technique called Value-at-Risk (VaR) was developed.

During the last decade the concept of VaR has emerged as the centrepiece of the trend toward the more complex measurement and management of market risk in financial transactions. Its attraction lies in its apparent simplicity, whereby it offers a "snapshot" of how much a participant on a financial market could lose from changes in the price of the instruments he holds. By applying the VaR methodology, an institution can, for the first time, view multiple risks in a single value. This means that equity, bonds and currency exposures, as well as associated hedges in derivative instruments, can be combined to determine an institution's resultant enterprise-wide risk.

Despite the numerous advantages of VaR as a risk management tool it is, as most financial models, subject to shortcomings. The **objective** of this paper is to illustrate one of the shortcomings practitioners experience in using VaR, in that the VaR model only measures market risk. The objective of the research is to illustrate that VaR underestimates risk during periods of high volatility and overestimates risk during periods of low volatility. Backtesting was performed on a selection of South African bonds in order to illustrate this concept.

[Page 185]

2 Market risk and measurement of market risk exposure

Market risk can be defined as the exposure to an adverse variation in future earnings (returns) resulting from unexpected changes in market conditions, (e.g., fluctuation in prices of securities or interest rates) (JP Morgan 2000:15). A distinctive feature of this risk category is that it can be solely attributed to the primary market in which that specific financial instrument is active (Eales 1995:1). Market risk has always been the most significant concern of participants in the financial markets. Research by Golub and Tilman (1997:75) indicates that the greatest risk faced by investors in bonds can be traced to their exposure to changes in interest rates.

An accurate market risk measure estimates a security's potential loss arising from specific market factors and the probability of that loss occurring. Such a technique should be able to help a risk manager accurately asses what could happen and help him avoid surprises associated with changes in these market conditions. In addition, the measure should act as an impetus for the manager to manage market risk exposures more effectively.

Since the exposure to market risk implies the probability of an adverse movement in the price of a security, the foundation of all market risk measurement techniques lies in the volatility or dispersion of the underlying security's price (Duffie & Pan 1997:36). Volatility, which is the essence behind trading in financial securities, is the most basic and commonly accepted statistical risk measure of a single security or portfolio of securities. Volatility is a measure of how stable or unstable the price of a security is (the degree of random variability) and in short, measures the magnitude of a security's price changes over a particular time period, thus the extent of market risk. Since the market risk exposure of a financial security can be derived from volatility of that specific security, the challenge facing risk managers and investors is to estimate volatility and thus try to accurately measure their exposure to market risk. One such a technique that does this is Value-at-Risk.

[Page 186]

3 Development of Value-at-Risk

The VaR methodology was formally developed during the early 1990's by the financial services firm, JP Morgan, to evaluate portfolio risk. VaR originated when the chairman of JP Morgan, Dennis Weatherstone, requested a one-page report delivered to him every day summarising the company's market exposure and providing an estimate of the potential loss over the next trading day (Le Roux 1997:1). Although VaR had been used by JP Morgan and the subsequent investment dealer community for several years, its acceptance in investment management circles came only in the middle of the 1990's. VaR made its first public appearance beyond Wall Street during October 1994 when JP Morgan began to circulate, via the Internet, a version of the daily VaR data it collects in-house (Carey 1996:1). The JP Morgan methodology, called RiskMetrics™ formed a leading standard for the international measurement and description of VaR.

4 Value-at-Risk methodology

VaR is a statistical risk measurement technique that can be used to measure the market risk of a single financial asset or a portfolio of financial assets. The methodology behind VaR is similar to traditional market risk measurement methods used for many years. VaR is based, as are more traditional methods, on measurement of the dispersion of an asset's return during a span of time around the asset's average return during that time (volatility). The major difference between VaR and other methods is that VaR calculates and expresses the downside of that dispersion in a monetary value, while traditional methods apply conventional statistical analysis to determine, for example the standard deviation of those returns.

The key to calculating the VaR value for an asset or portfolio of assets is to estimate the probability distribution of its possible gains and losses over a given holding period. Once this is done, the VaR value can be determined by isolating that portion of the downside, loss-making distribution that corresponds to a chosen probability. Simply stated, VaR is an estimate of the largest loss that a financial asset or portfolio of assets is likely to suffer during all but truly exceptional periods (Haas 2000:1). VaR can formally be defined as the predicted maximum potential adverse change (loss) of a single financial asset, or portfolio of assets, over a target horizon, within a given confidence interval.

[Page 187]

VaR is expressed as a single, monetary value indicating an investor's maximum monetary loss exposure. For example, if an asset's one day, 95% VaR is R5 000, then the asset would be expected to lose less than R5 000 over 95 days out of 100 days. Minnich (1998:39) states that it is important to remember that VaR is not the all-time maximum loss that will occur, but only a predicted loss level threshold that will be pierced some percentage of the time. The actual loss that occurs could be much higher than the predicted VaR value.

5. Value-at-Risk inputs

In order to calculate VaR the model requires the input of the following arbitrary variables:

5.1 Holding period

The advantage of VaR above most other market risk measurement techniques is that it is time-specific in its risk forecast. Whereas standard deviation, for example, indicates the level of anticipated risk exposure, VaR can forecast the level of risk exposure, as well as indicate the time horizon for which that risk exposure is applicable. VaR can measure both short-term risk, such as a financial market trader's risk carried on a book overnight, as well as a longer term risk, such as risk carried by a portfolio over a one month period. This projected risk period is referred to as the VaR holding period.

The VaR holding period can be defined as the time horizon for which possible losses will be projected. The possible loss in the portfolio's value (thus the VaR value) depends on the amount of time given for that loss. Longer holding periods are generally associated with greater risk, hence a ten-day VaR value will be larger than a one-day VaR value (Stambaugh 1996:614). This is due to the fact that absolute volatility (market risk) increases over time (Iacono 1996:8). Ideally the VaR holding period should correspond to the longest period needed for an orderly selling of the asset or liquidation of the portfolio (Minnich 1998:42).

5.2 Confidence level

The VaR confidence level can be defined as the tolerance level, stated as a percentage, for which the loss estimated by a portfolio's VaR value can and will be exceeded. Due to the "confidence level" terminology, some risk managers make the mistake of equating their VaR expectation to a [Page 188] certainty that a portfolio will not lose more than the stated VaR value. VaR does not, however, provide certainty of or confidence in outcomes, but rather an expectation of outcomes based on a specific set of assumptions and a specific time horizon. For example, actual losses using a 90% confidence level should exceed the VaR value 10% of the time while a 95% VaR value should be exceeded 5% of the time.

In practice, VaR estimates are calculated from the 90^{th} to the $99{,}9^{th}$ percentiles, but the most commonly used range is the 95^{th} to the 99^{th} percentile range (Hendricks 1996:40). The choice of confidence level, however, depends on its use in the organisation. If VaR values are used in the calculation of capital allocations, then the choice of the confidence level is crucial, as it should reflect the degree of risk aversion of the company and the cost of a loss of exceeding the VaR value (Jorion 1996:48). Higher risk aversion implies that a greater amount of capital should be allocated to cover potential losses, thus leading to a higher confidence level. In contrast, if VaR values are used to provide a company-wide yardstick to compare risk across different markets, then the choice of the confidence interval is not too important (Minnich 1998:42).

The Bank for International Settlement recommends a confidence level of 99%, while most VaR practitioners recommend the calculation of VaR values over a 95% confidence level, such as used in the JP Morgan RiskMetrics™ methodology (Duffie & Pan 1997:9). Empirical research shows that a 95%

confidence interval performs best under backtesting due to the presence of "fat-tails" (Minnich 1998:42). "Fat-tails" refers to the fact that large market moves occur more frequently than what would occur if market returns were normally distributed. This implies that extreme movements up or down tend to be more likely than the normal distribution suggests it would (thus, there are more occurrences away from the mean that predicted by the normal distribution).

5.3 Data window

VaR calculations are based on the historic price movements (volatility) of the underlying asset or portfolio of assets. VaR is thus, by most accounts, fairly data intensive. The problem facing risk managers is to decide on how much historical data to include in the calculation of VaR values. Minnich (1998:43) contends that longer periods of data have a richer return distribution, while shorter periods allow the VaR values to react more quickly to changing market events (i.e., capture short-term movements in the underlying risk of a portfolio).

[Page 189]

A problem of long data periods (for example one year) is that if there is even just one unusual return during the past year, it will continue to keep volatility estimates high for exactly one year following that day, even though the underlying volatility may have long since returned to normal levels (Alexander & Leigh 1997:53). Generally speaking, there may be a number of extreme market movements during the course of the past year, keeping volatility estimates artificially high in periods of tranquillity.

A study by Beder (1995:12) found that using 100 days compared to 250 days of trading data as input appears to be inadequate. The discrepancies between results appeared to be large since the small sample size made the 5% left tail of a distribution (on a 95% confidence interval) difficult to measure. The use of longer data windows (e.g., 250 days) is also supported by the empirical research of Hendricks (1996), where the longest samples produced the best performance.

6. Calculating Value-at-Risk

6.1 Value-at-Risk models

Although there is only one definition of VaR, there are in essence three different methods for estimating VaR values. The three methods are:

- Historical Simulation Value-at-Risk
- Variance-covariance Value-at-Risk
- Monte Carlo Simulation Value-at-Risk

One of the most difficult aspects of calculating VaR values is selecting from among the different VaR methodologies and their associated assumptions. Each method has its own particular strengths and weaknesses, and should be properly viewed not as competing methodologies, but as alternatives which might be appropriate in certain circumstances.

The three methods share the same basic premise: that the behaviour of the financial market over the recent past is a good and unbiased indicator of the way it will behave in the near future. In mathematical terms, it is said that the probability distributions of the market price movements exhibit stationary. The task of the VaR practitioner is then to take the recent past and use it to develop the probability distribution of future portfolio gains and losses (Stambaugh 1996:614).

[Page 190]

The major difference in the methods lies in the assumption of normality of asset returns. The Variance-covariance method assumes normality, while the two simulation methods (Historical and Monte Carlo) can be calculated for non-normal distributions. Given this, VaR values for portfolios containing derivatives can be more accurately derived by the latter two methods (Smithson 1996:27). Jordan and Mackay (1996) in Smithson (1996:27) calculated VaR for portfolios made up of equities and equity option positions. Their findings indicated that the VaR values generated by the Variance-covariance method differed dramatically from that generated by the two simulation methods. Furthermore, a study commissioned by the Bank of England found that the Variance-Covariance method systematically underestimated the risk of a sample portfolio of multi-currency fixed-income portfolios (Stambaugh 1996:616). The Historical Simulation model was used in this study. This is due to the fact that this VaR model requires a simple, atheoretical approach that requires relatively few assumptions about the statistical distributions of the underlying prices of the portfolio's individual instruments.

6.2 Variance-covariance: Model description

The Variance-covariance method is so named because the VaR can be derived from a variance-covariance matrix of the relevant underlying market prices of a portfolio. This variance-covariance matrix contains information on the volatility and correlation of all market prices relevant to the portfolio. It is a symmetric matrix which has the variances of every asset down the diagonal axis and the covariances between the various assets on the off-diagonal axis. Variances are calculated by means of the standard deviations (or variances) of the market prices, and the covariances by means of the correlation coefficients of the market prices (Simons 1996:7).

This method is based on the assumption that the risk factors that have an influence on the market price of a portfolio exhibit a normal distribution around a zero mean. Such risk factors might include various interest rates, share prices or exchange rates. Using this assumption, it is possible to determine the distribution of portfolio profits and losses, which is also normal. Once the distribution of possible profit and losses has been obtained, the standard mathematical properties of a normal distribution are used to determine the VaR - the loss that will be exceeded only a certain percentage (e.g., 5%) of the time (Linsmeier & Pearson 1996:10).

[Page 191]

6.3 Monte Carlo Simulation: Model description

This method is very similar to the Historical Simulation VaR with the major difference that the price changes against which the portfolio is re-valued are simulated rather than historical (Linsmeier & Pearson 1996:8). Instead of carrying out the simulation using the observed changes in the market prices over the last e.g. 250 days to create 250 hypothetical portfolio profits or losses, one chooses a statistical distribution that is similar to possible changes in the market prices.

The ability to select a distribution is the feature that distinguishes Monte Carlo simulation from the other VaR approaches, for in the other methods the distribution of changes in market prices is specified as part of the method. Linsmeier & Pearson (1996:15) contends that the distribution selected need not be the normal, though the natural interpretations of its parameters (means, standard deviations and correlations) and the ease with which these parameters can be estimated weigh in its favour. Users of Monte Carlo simulation are free to select any distribution that they think reasonably describes possible future changes in the market prices.

A random number generator is then used to generate an x-amount of hypothetical changes in the market prices. These are then used to construct an x-amount of hypothetical portfolio profits and losses on the current portfolio, and the distribution of possible profit and loss. This distribution is then interpreted to reveal the VaR - the loss that will be exceeded only a certain percentage (e.g., 5%) of the time (Stambaugh 1996:618).

6.4 Historical simulation: Model description

JP Morgan (2000:272) refers to the Historical Simulation model as: "A non-parametric method of using past data to make inferences about the future. One application of this technique is to take today's portfolio and re-value it using past historical price and rates data."

This method involves the taking of actual price changes that occurred over the last x trading days (the data window), and re-values the asset or portfolio as if those changes were to occur again in the next day (for a one-day holding period). Specifically, a historical VaR value is calculated by using historical changes in market prices to construct a distribution of potential future portfolio profit and losses, and then reading off the VaR as [Page 192] the loss that is exceeded only a certain percentage (e.g., 5%) of the time (Linsmeier & Pearson 1996:7). The distribution of profits and losses is constructed by taking the current portfolio and subjecting it to actual changes in the market prices experienced during each of the last x days (Jones 1996:88). No in depth statistical calculations are required because the methodology uses the actual observed changes to estimate expected future market changes.

Many financial models assume that markets and prices of instruments are continuous in nature and that there are no sharp jumps or discontinuities in prices. The advantage of Historical Simulation VaR is that no assumptions are required regarding the structure of the financial market on which it will be used to measure market risk. Since actual historic returns are used, this method captures true market behaviour and does not rely on the assumption of a log-normal or normal distribution of financial market returns. Changes in market prices are used as input to calculate prospective gains and losses, so any "fat-tails" or other distortions are fully captured in the model (Stambaugh 1996:617). This holds true even for a portfolio containing derivative instruments.

Stambaugh (1996:617) contends that a major advantage of this approach is that it is intuitively obvious. The data window of, for example 250 trading days (derived from actual market movements), can be explained with relative ease to management, traders and regulators. This in turn helps in gaining acceptance of the outcome of the VaR analysis and of the disciplines of putting risk management to work.

7. Backtesting Value-at-Risk

7.1 Introduction

In order to validate the accuracy of the Historical Simulation VaR methodology on the South African Bond market, a backtesting procedure was developed. Backtesting is a statistical procedure designed to compare realised trading results with model generated risk measures in order to evaluate the accuracy of the model (JP Morgan 2000:39). The backtesting procedure used in this study is a simple technique which involves the comparison between the number of times the VaR model under-predicted the subsequent day's loss, versus the number of times such an under-prediction is expected. If, under a 95% confidence interval, losses [Page 193] exceeding the VaR values have a 5% chance of occurring, then we expect to see between 12 and 13 of these in a year (250-days * 5% confidence level = 12½ days).

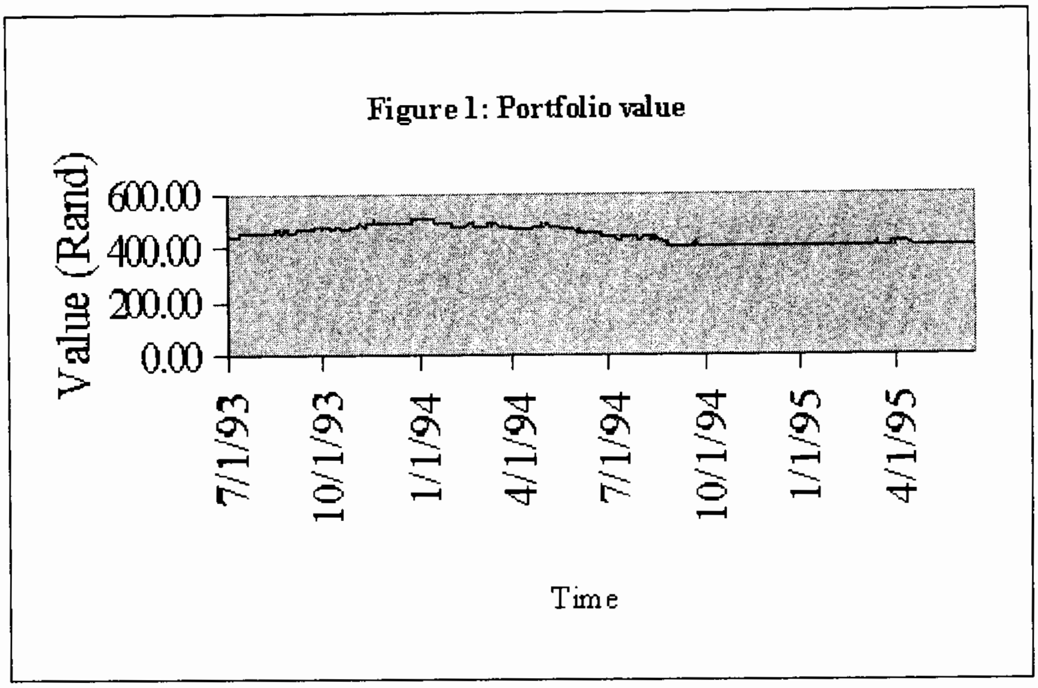
7.2 Sample selection

In order to conduct backtesting on the bond market, a selection of five bonds were made. These bonds were combined into a single portfolio based on the clean price (R100%) of the bonds, combined on an equal investment of one bond each. During the time of the backtesting, the five bonds accounted for 63% of the daily trades on the Bond Exchange of South Africa (Bond Exchange of South Africa 1997:4). The bonds and their respective issuers are:

- R150 & R153 (Government)
- E168 (Eskom)
- T016 (Transnet)
- TK05 (Telkom)

The backtesting was done over a two-year time period between July 1993 and June 1995. This period was specifically chosen for the fact that the country's first democratic elections were held during this time (April 1994). As can be seen from Figure 1, the time period saw a mixture of volatility on the bond markets with bond yield to maturaties fluctuating 0.5% in 12 months before and 3% in six-months after the elections, with 0.10% in six-months after that.

[Page 194]



7.3 Backtesting design

The following is a description of the procedure followed in the calculation and backtesting of the Historical Simulation VaR values for the above bond portfolio:

- The daily mark-to-market clean prices of the bonds were combined into one portfolio for the time period 1 July 1993 to 15 June 1995. Weekends and public holidays were not included, thus leaving a data set of 490 daily prices of the bond portfolio.
- Percentage changes in the value of the portfolio were calculated from one day to the next over the whole 490 days.
- The following VaR parameters were selected: a one-day holding period, a 95% confidence level and a 250 trading-day data window.
- Using the first 250 days (1 June 1993 - 15 June 1994) as data window input, the VaR value for the 251st day was calculated as being between the 12th and 13th biggest losses over the previous 250 days (250-days * 5% confidence level = 12½ days).
- This procedure was repeated for the next 240 days, on each instance dropping the first day and adding a new day of data, thereby ensuring that the data window stays at a constant 250 days.

Following the backtesting procedure, a VaR violation would occur each time that the actual portfolio loss exceeded the loss predicted by the VaR model for that specific day. One would thus expect daily losses to [Page 195] fall inside the 95% confidence level for 95% of the time and only violate the VaR on 5% of occasions.

7.4 Backtesting results

The backtesting procedure is summarised in Figure 2, from which the following observations can be drawn:

- **15 June 1994 – 14 December 1994**

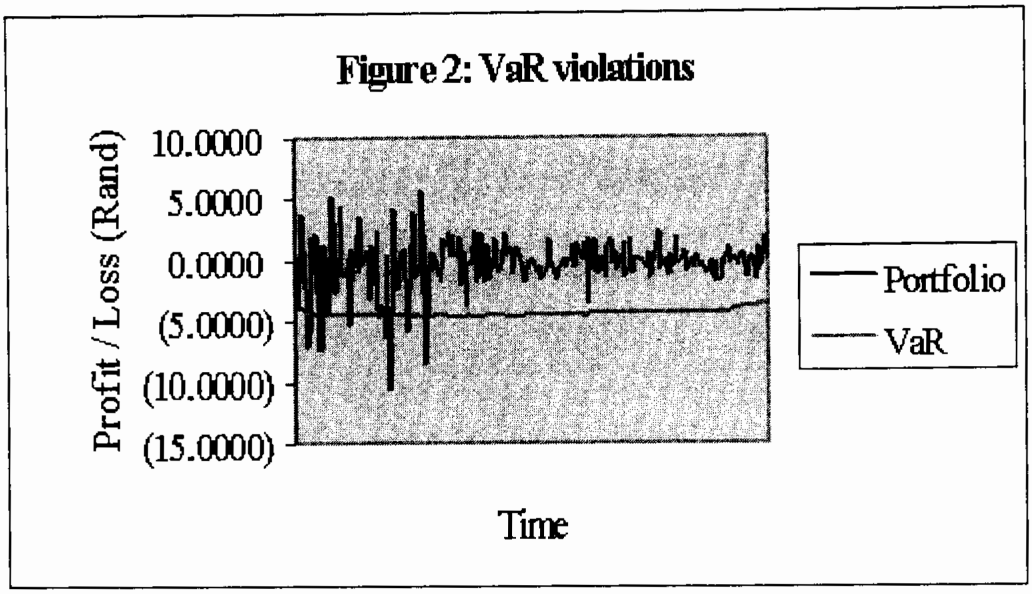
  During this time period one would expect between six and seven VaR violations given the 95% confidence interval. The VaR limit was however broken ten times during this six month time period.

- **15 December 1994 – 15 June 1995**

  Once again, during this time period one would expect between six and seven VaR violations.

However, the VaR limit was not broken even once during this six month time period.

Figure 2



7.5 Backtesting conclusions

Since Historical VaR measures normal market risk based on historical data as input, one of the most important inputs in the model is the 250-day data window. For the first backtesting period (15 June 1994 - 14 December 1994), the data window used spanned from 1 July 1993 to [Page 196] 15 June 1994, a period with relatively normal volatility (normal market risk). It is not surprising then to find that the backtesting period (with higher volatility) would yield so many VaR violations. The opposite is true for the following backtesting period (15 December 1994 - 15 June 1995), were highly volatile inputs were used to calculate VaR during a low volatility period.

The underlying problem thus seems to be the fact that Historical Simulation VaR assumes that historical volatility was constant over the time of the data window. However, volatility can change over time, sometimes quite abruptly, and it makes sense to pay more attention to the most recent observations in the data window when forecasting future volatility (Simons 1996:9). One way to correct this problem is through exponential weighting of observations in the data window. This approach emphasises more recent observations at the expense of the more distant ones because the weights assigned to past observations decline with time.

The backtesting done in this study illustrates the major danger in using VaR. The purpose of this study was not to test the validity of VaR, but to simply illustrate the shortcoming of VaR, in that it measures only market risk. Practitioners should always bear in mind that VaR is a market risk measurement technique and does not warn of extreme market movements. This study illustrated that VaR underestimates risk during periods of high volatility and overestimates risk during periods of low volatility, thus rendering it useless as a measure of extreme market movement.

8. Conclusion

In conclusion it should be noted that the rapidly spreading use of VaR should be seen as a vast improvement over the antiquated or non-existent risk management practices, some of which have caused major financial disasters. The benefits of VaR should not, however, mask its shortcomings. Any VaR value is itself subjected to some form of error or estimation risk. Thus, understanding the statistical methodology is important in order to interpret VaR values. This interpretation would be made easier not only by reporting a single VaR value, but also by reporting the confidence interval and holding period applicable to the VaR value. The most important point is however to remember that VaR does one thing, and one thing only: measure *market* risk.

[Page 197]

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