HEFAT2007
5th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics
Sun City, South Africa
Paper number: AA4

TRANSIENT CONJUGATED HEAT TRANSFER IN THICK WALLED PIPES WITH UNIFORM HEAT FLUX BOUNDARY CONDITION

Ateş A., Darıcı S. and Bilir Ş.*
*Author for correspondence
Department of Mechanical Engineering,
Selçuk University, 42070 Konya, Turkey
E-mail: sbilir@selcuk.edu.tr

ABSTRACT

Transient conjugated heat transfer in thick walled pipes for thermally developing laminar flow is investigated involving two dimensional wall and axial fluid conduction. The problem is solved numerically by a finite difference method for hydrodynamically developed flow in a two regional pipe, initially isothermal in which the upstream region is insulated and the downstream region is subjected to a suddenly applied uniform heat flux. A parametric study is done to analyze the effects of four defining parameters namely, wall thickness ratio, wall-to-fluid thermal conductivity ratio, wall-to-fluid thermal diffusivity ratio and the Peclet number. The results are given by non-dimensional interfacial heat flux values, and it is observed that, heat transfer characteristics are strongly dependent on the parameter values.

INTRODUCTION

Analysis of conjugated heat transfer in transient regime is important during start up, shutoff or any change in the operating conditions. This problem may be faced in regenerative and recuperative heat exchangers, in cooling of gas turbine blades, in nuclear reactors, aircraft engines and spacecrafts, and is more likely to be analyzed in pipes or in flow sections which can be modeled as a pipe or channel.

Transient heat transfer for laminar pipe or channel flow was analyzed by many investigators and in some of them the pipe wall is considered extremely thin. In this case the wall conduction may be ignored and the condition at the outer wall surface can be assumed to prevail along the inner surface. However, in thick walled pipes the conditions at the wall-fluid interface are not known a priory and the energy equations must be solved simultaneously by assuming continuity in temperatures and in heat fluxes at the interface. When Peclet number of the flow is low, the axial fluid conduction may be comparable to convection and can not be ignored. Diffusion of heat backward through the upstream region, results preheating of the fluid before the beginning of the heating section. Therefore such problems are usually analyzed in two-regional pipes. A brief literature survey on steady conjugated problems and on the effect of axial fluid conduction is given in [1] and [2].

Unsteady conjugated problems for laminar flow considering one or two-dimensional wall conduction and fluid axial conduction were also studied by many investigators under various boundary conditions. Schneider [3] solved the problem for parallel plates and Vick, Özışık and Ullrich [4] for pipes with uniform flow and convection from the outer surface by analytical methods. Campo and August [5] worked on a problem with parabolic velocity profile and both with convective and radiative boundary conditions. Numerical methods are used for solving the problem in pipes, heated in finite length, with a step change in heat flux, by Lin and Kuo [6] and in temperature, by Yan, Tsay and Lin [7]. With variable inlet fluid conditions in parallel plates, the problem is investigated by Travelho and Santos [8], with uniform flow, and by Olek, Elias and Wacholder [9], with parabolic flow. Yapıcı and Albayrak [10] solved a problem with non-uniform heat fluxes and Yin and Bau [11] with and without axial fluid conduction.

Recently numerical methods were used in investigations considering two-dimensional wall and axial fluid conduction. Schutte, Rahman and Faghri [12] solved the problem, for combined development region, Lee and Yan [13] and Bilir [14], with step change in wall temperature, Yan [15] and Bilir and Ateş [16], with convective boundary conditions and Li and Kakaç [17], with step and sinusoidal change in wall heat flux.

NOMANCLATURE

c_p	specific heat at constant pressure
d	thickness of the pipe wall
Fo	Fourier number
Gz	Graetz number
k	thermal conductivity
Nu	Nusselt number
Pe	Peclet number
q	heat flux

r radial coordinate
t time
T temperature
T_0 initial temperature of the system
u axial velocity
x axial coordinate

Greek symbols
α thermal diffusivity
ρ density

Subscripts
b bulk
f fluid
i inner wall
m mean
o outer wall
w wall
wf ratio of wall to fluid

Superscripts
' dimensionless

PROBLEM FORMULATION

The schematics of the problem and the coordinate system are shown in Fig.1. The flow pipe is two-regional and infinite in both sides. At the far upstream, the fluid temperature is T_0 and uniform. The upstream region of the pipe wall is externally insulated; the flow is laminar and hydrodynamically developed at the beginning of the downstream region. Initially the whole system is isothermal at temperature T_0 , and at time $t=0$ a constant and uniform heat flux q_{wo} is suddenly applied on the external surface of the downstream side of the pipe. Physical properties of the fluid are assumed to be constant and the viscous dissipation is neglected.

The above-described problem may be formulated in non-dimensional form as follows. In the wall side, the differential equation is

$$\frac{1}{\alpha_{wf}}\frac{\partial T'_w}{\partial t'} = \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial T'_w}{\partial r'}\right) + \frac{1}{Pe^2}\frac{\partial^2 T'_w}{\partial x'^2}. \quad (1a)$$

The initial and boundary conditions are

at $t'=0 \quad T'_w = 0$; (1b)

at $x'=-\infty \quad T'_w = 0$; (1c)

at $x'=+\infty \quad \dfrac{\partial T'_w}{\partial x'} = 4(1+d')$; (1d)

at $r'=1+d'$ for $x'<0 \quad \dfrac{\partial T'_w}{\partial r'} = 0$; (1e)

at $r'=1+d'$ for $x'\geq 0 \quad \dfrac{\partial T'_w}{\partial r'} = \dfrac{1}{k_{wf}}$; (1f)

at $r'=1 \quad T'_w = T'_f$ and $\dfrac{\partial T'_w}{\partial r'} = \dfrac{1}{k_{wf}}\dfrac{\partial T'_f}{\partial r'}$. (1g,h)

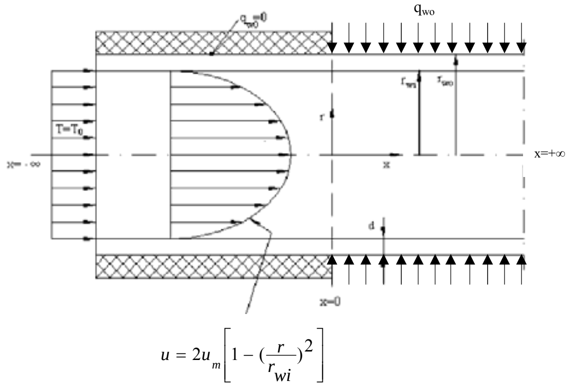

Figure 1 Schematics of the problem and the coordinate system

In the fluid side, the differential equation is

$$\frac{\partial T'_f}{\partial t'} + (1-r'^2)\frac{\partial T'_f}{\partial x'} = \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial T'_f}{\partial r'}\right) + \frac{1}{Pe^2}\frac{\partial^2 T'_f}{\partial x'^2} \quad (2a)$$

The initial and boundary conditions are

at $t'=0 \quad T'_f = 0$; (2b)

at $x'=-\infty \quad T'_f = 0$; (2c)

at $x'=+\infty \quad \dfrac{\partial T'_f}{\partial x'} = 4(1+d')$; (2d)

at $r'=1 \quad T'_f = T'_w$ and $\dfrac{\partial T'_f}{\partial r'} = k_{wf}\dfrac{\partial T'_w}{\partial r'}$; (2e,f)

at $r'=0 \quad \dfrac{\partial T'_f}{\partial r'} = 0$. (2g)

Non-dimensional parameters of the problem are defined as:

$$x' = \frac{x}{r_{wi}Pe} \equiv \frac{2}{Gz}, \quad r' = \frac{r}{r_{wi}}, \quad q'_{wi} = \frac{q_{wi}}{q_{wo}}$$

$$T' = \frac{T-T_0}{q_{wo}r_{wi}/k_f}, \quad d' = \frac{d}{r_{wi}}, \quad k_{wf} = \frac{k_w}{k_f}$$

$$\alpha_{wf} = \frac{\alpha_w}{\alpha_f}, \quad t' = \frac{t\alpha_f}{r_{wi}^{\ 2}} = Fo, \quad Pe = \frac{2r_{wi}u_m\rho_f c_{p_f}}{k_f}.$$

Dimensionless fluid bulk temperatures, T'_b, interfacial heat flux values, q'_{wi}, and local Nusselt numbers, *Nu*, may be of engineering interest and can be calculated as follows:

$$T'_b = 4\int_0^1 r'(1-r'^2)T'_f\,dr', \quad (3)$$

$$q'_{wi} = -\left(\frac{\partial T'_f}{\partial r'}\right)_{r'=1}, \quad (4)$$

$$Nu = \frac{-2\left(\frac{\partial T'_f}{\partial r'}\right)_{r'=1}}{T'_{wi} - T'_b}. \quad (5)$$

SOLUTION METHODOLOGY

The systems of Eqs. (1a)-(1h) and (2a)-(2g) are solved simultaneously by a numerical finite-difference method. The conductive terms are discretized by central-difference schemes and the convective terms in the energy differential equation for the fluid side by an exact method which is given in [2]. This method of discretization is a two-dimensional version of the "exact or exponential scheme" defined by Patankar [18]. For the transient terms a fully implicit formulation is used to assure stability in the solutions.

The grids are laid both in the wall and the fluid side and due to axial symmetry bounded between the outer surface of the wall and the pipe axis. The boundaries of the computational region in the axial direction are guessed by the results of some trial runs with coarse grid systems as to satisfy the conditions at these boundaries. Axial grids are contracted in the vicinity of the beginning of the heating section while radial grids are uniform in size. The first axial step size is taken 0.001 for both upstream and downstream region and linearly stretched in both directions by increasing the axial step sizes by the one third of the previous grid. The optimum number for the grid system is found to be 50 X 28 in order both to minimize the solution time and maximize the sensitivity.

Time steps are also chosen non-uniform. Since, heat transfer characteristics change rapidly at the beginning of the transient and rather slowly when approaching the steady state, the first time step is taken 0.0001 and increased 10% for the subsequent steps. Temperature distributions are obtained by Gauss-Siedel iteration technique. In each time step iterations are made by the line-by-line method [18], by traversing from outer surface of the wall to the pipe axis and by sweeping from upstream to downstream region. At the interface, the harmonic mean formulation [18] is used for the discretization of the boundary conditions and a consecutive procedure is used in the solutions. By so, previously calculated temperatures are used to transfer information from wall to fluid side and interfacial heat flux values from fluid to wall side in the iterations.

Convergence limit is taken to be 10^{-5} and at a time step solutions were obtained in 7400 iterations at the average. When the number of iterations at a time step decreases below two, the system was assumed to reach steady state and solutions were obtained approximately in 10^{6} total iterations. Some accuracy tests were done by increasing the number and changing the positions of the grids, by decreasing the time steps and the convergence limit and by changing the traversing and sweeping directions for the solutions and no considerable difference in computed values were obtained.

RESULTS AND DISCUSSION

Inspection shows that the results of the problem depend on four parameters, namely wall thickness ratio, wall-to-fluid thermal conductivity ratio, wall-to-fluid thermal diffusivity ratio and the Peclet number. Solutions are then made for different combinations of these parameters: d'=0.02, 0.1 and 0.3; k_{wf} =0.1, 1, 10, 100 and 1000; α_{wf} =0.1, 1, 10, 100 and 1000 and Pe =1, 5, 20 and 50. These values are chosen as appropriate for problems of engineering interest and from the range that all presumed effects of the problem are in a significant level. The results are given in interfacial heat flux values.

In Fig. 2, axial distribution of interfacial heat flux values at different instants of time is shown, obtained by a run with a typical combination of average parameter values. The characteristics of the curves are almost similar obtained with other parameter value combinations. As shown in the figure, due to axial conduction in both wall and fluid sides, there is a substantial amount of heat transfer in the upstream region. Backward heat penetration in axial direction increases with time and therefore the magnitude of heat transfer and the length of preheating increase as the time elapses. In the downstream region, heat flux values increase first and after rising to a maximum decrease and attains a constant value. At the beginning of the heating section inner wall temperatures are somewhat higher due to rapid radial wall conduction. Due to axial convection, bulk temperatures of the fluid also increase in the flow direction and this fact results the decrease in heat flux values after a peak value.

As the time goes on, the position of the peak shifts slowly to downstream and both the peak and average values of heat flux increase and the curves reach the expected asymptotic value at steady state. An interesting feature can be seen from the figure, that the peak heat flux value is greater than the final asymptotic value. This means that the heat flux at the inner wall is somehow greater than the heat flux

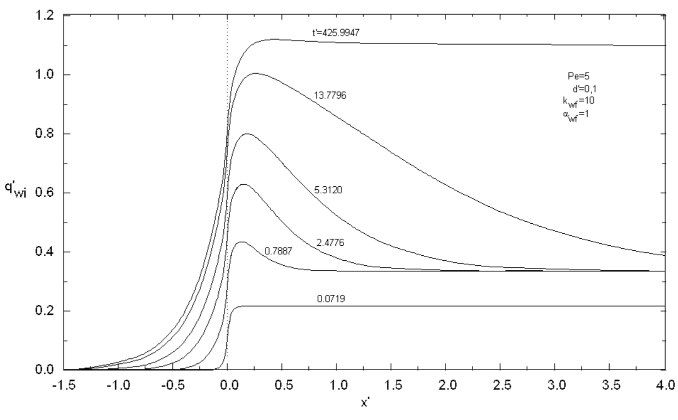

Figure 2 Transient axial distribution of interfacial heat flux

entering from the outer wall of the pipe on that region. This may be explained by the excess heat transferred by axial conduction from the downstream region in the wall side.

In Fig.3, the effect of wall thickness ratio on interfacial heat flux is shown. The curves are drawn for three different instants of time and are parametized by three different values of thickness ratio. More heat is shown penetrated backward through the upstream region by axial conduction in thick walled pipes. In thin walled pipes, since the radial conduction in the wall side is easy and rapid, at early and intermediate transient periods heat flux values are high.

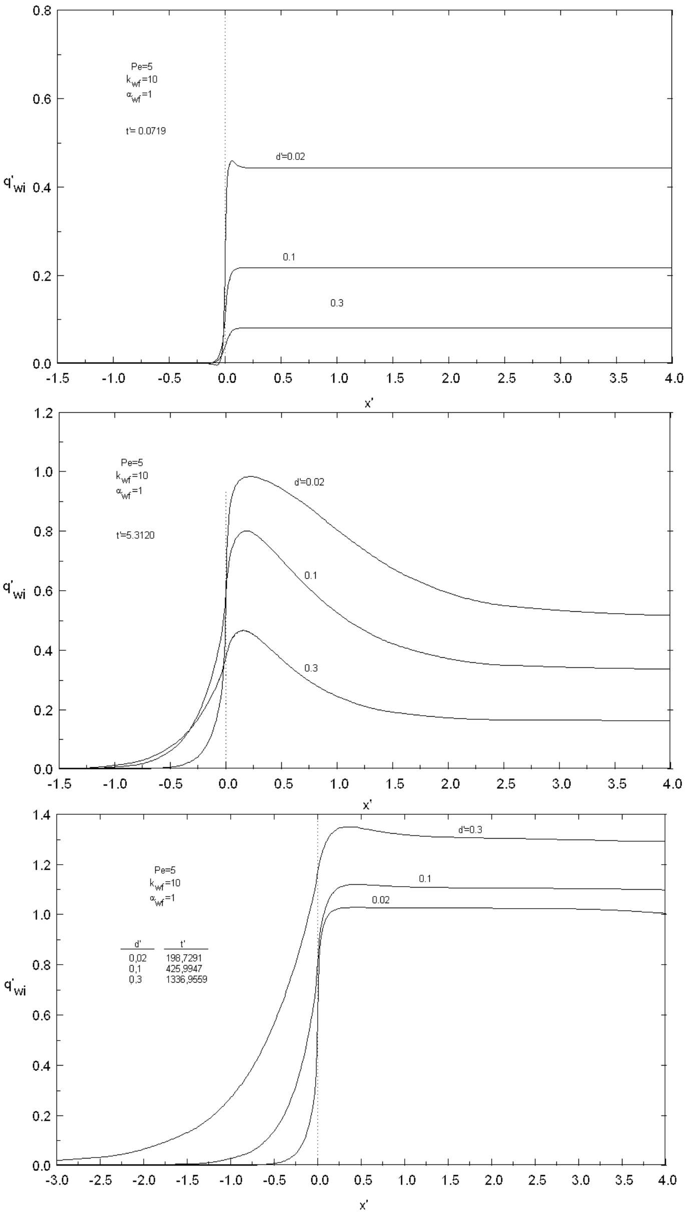

Figure 3 Effect of wall thickness ratio on interfacial heat flux

In the early transient, at the end of the upstream region due to backward axial fluid conduction, fluid bulk temperatures are higher than the interface temperatures in thin walled pipes. This is the reason of negative heat flux, i.e. from fluid to the wall side, shown in the figure. The negative heat fluxes disappear by the time due to convection of heat by the flow through the downstream region. At steady state, in the downstream region the peak is more evident and therefore the amount of excess heat diffused backward by wall axial conduction at the beginning of the heating section is more in thick walled pipes. The time to reach the steady state is increasing with increasing wall thickness due to the increased thermal inertia.

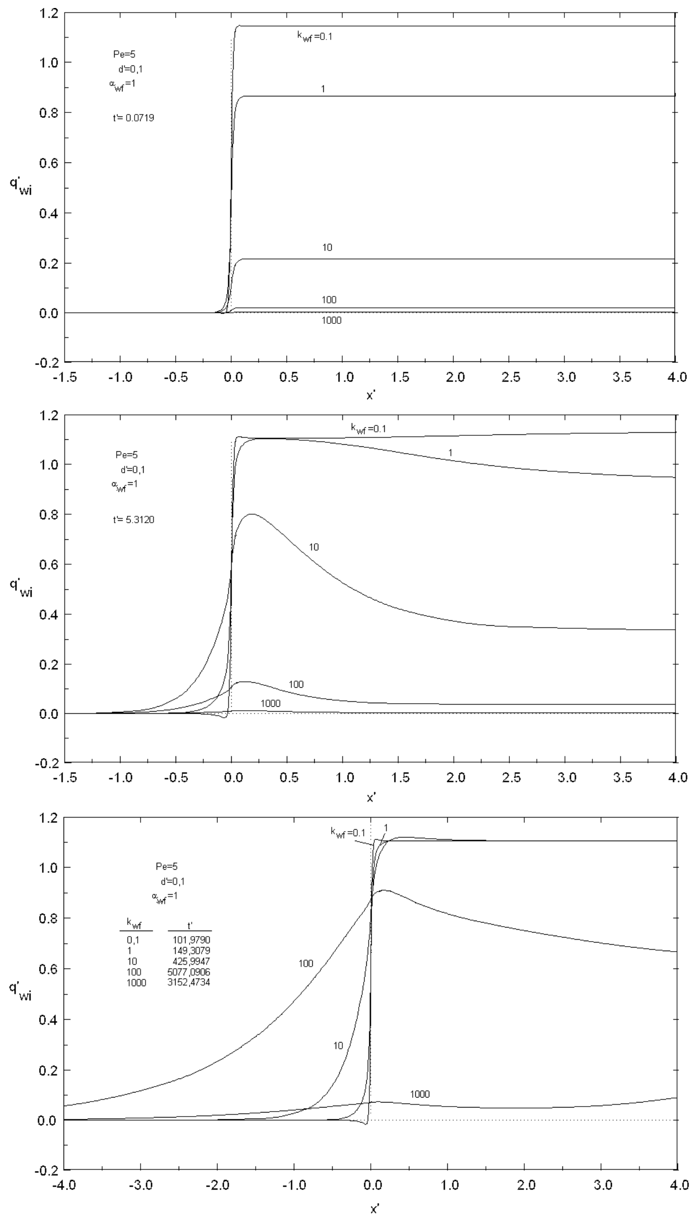

Figure 4 Effect of wall-to-fluid thermal conductivity ratio on interfacial heat flux

Fig. 4 shows the effect of wall-to-fluid thermal conductivity ratio on interfacial heat flux. For large k_{wf} values both preheating and thermal development lengths are increasing due to the increased wall axial conduction. For the whole transient and also in steady state, the magnitudes of the interfacial heat flux values are smaller for large k_{wf}. The time to reach the steady state is also considerably long for large k_{wf} values. These may be explained by the relation between k_{wf} and α_{wf} . If k_{wf} is increased, $k_{wf} = \alpha_{wf}\,(\rho_w c_{p_w} / \rho_f c_{p_f})$, and α_{wf} is kept constant, the heat capacity of the wall, $\rho_w c_{p_w}$, becomes much larger than that of the fluid [6],[12],[15]. Negative heat flux values are also shown in the upstream region for very small k_{wf} values, because of more heat diffusion backward through the upstream region in the fluid side.

The effect of wall-to-fluid thermal diffusivity ratio on axial distribution of interfacial heat flux values is shown in Fig. 5. This parameter is effective especially in the early and intermediate periods of the transient as can be seen from the figure. The final shapes of the curves are identical, irrespective of the value of α_{wf} , in the steady state as expected. Due to small thermal capacity, during the transient for small α_{wf} values, interfacial heat flux values are also high.

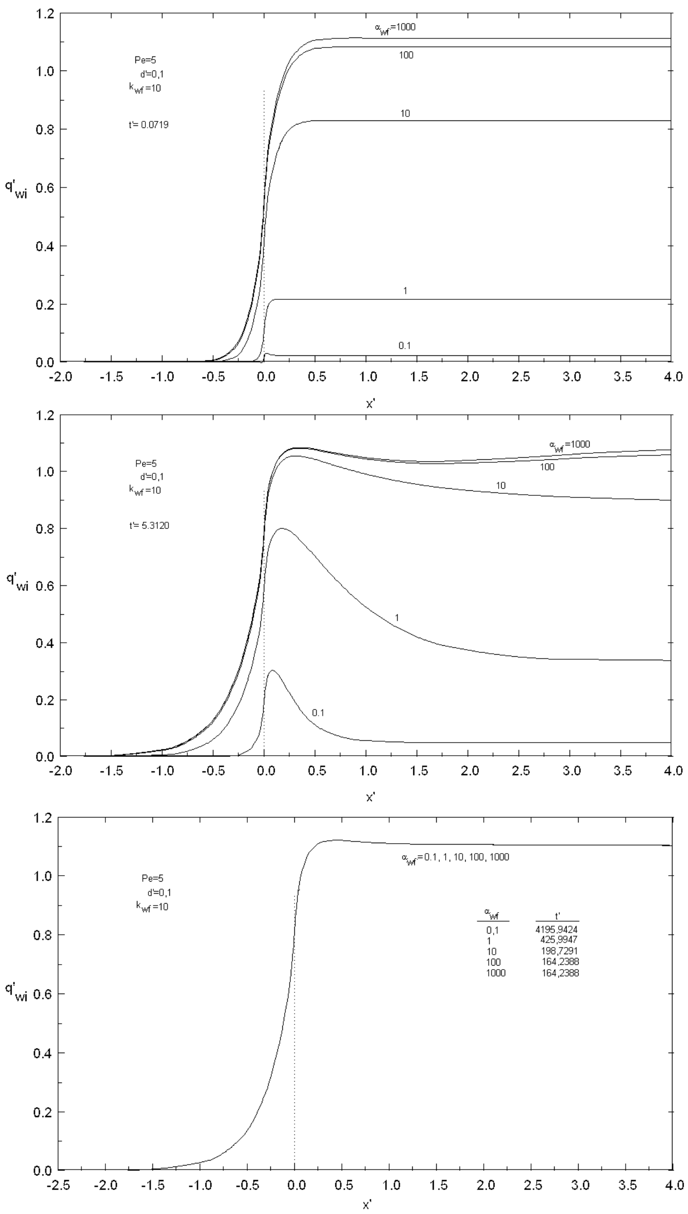

Figure 5 Effect of wall-to-fluid thermal diffusivity ratio on interfacial heat flux

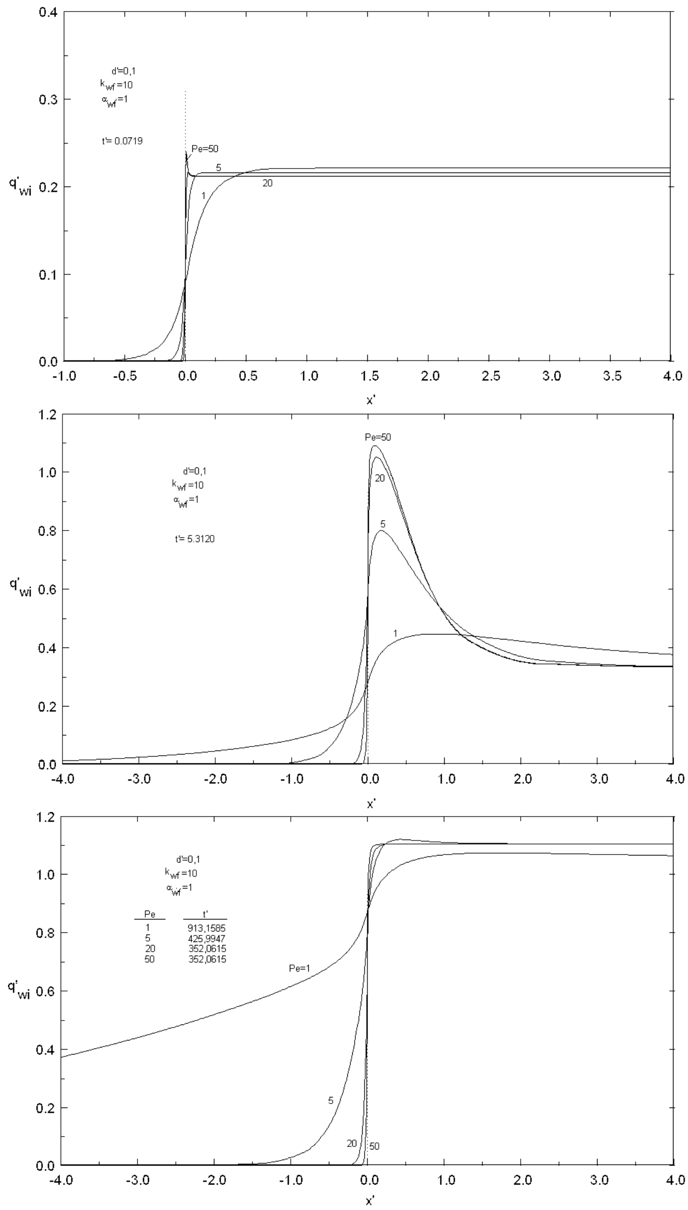

Figure 6 Effect of Peclet number on interfacial heat flux

By the same reason, both the extent and the magnitude of preheating in the upstream region are high for large α_{wf} values. At the very early transient, a small amount of negative heat flux is shown in the upstream region for very small α_{wf} values, because of the large thermal capacity in the pipe wall and therefore low interfacial temperatures. The time to reach the steady state is increasing with increasing thermal inertia of the wall, (i.e., decreasing α_{wf} values).

Fig. 6 is drawn in order to see the effect of Peclet number. Since the fluid axial conduction is increasing with decreasing Peclet number, heat is more penetrated backward through the upstream region. In the intermediate periods of the transient, convection predominates over both wall and fluid axial conduction and therefore the effect of Peclet number on the interfacial heat flux is more felt, than in the early and late transient periods especially in the downstream region. The peak values for heat flux are smaller and curves change more gradually for small Peclet number flows. The reason for this is the decreased convection effect and therefore increased bulk fluid temperatures. The thermal development length is also increased and the time to reach the steady state is longer for low Peclet numbers, because of decreased convection effect.

CONCLUDING REMARKS

In the present work, transient conjugated heat transfer in thick walled pipes for thermally developing laminar flow with uniform heat flux boundary condition is analyzed. A numerical finite difference method is used and a parametric investigation is done in order to understand the effects of four defining parameters of the problem. The results obtained may be outlined as follows.

Due to axial conduction both in the wall and in the fluid sides, heat penetrates backward through the upstream region and this results preheating of the fluid before the beginning of the heating section. The preheating effect is felt more as the time elapses, and the extent and the amount of preheating are strongly dependent on the parameter values. Reverse heat flux, from fluid to wall side is shown, in the early transient at the upstream region, when backward heat penetration is more rapid in the fluid side than that in the wall side. In the downstream region, at the beginning of the heating section peak values are shown in the interfacial heat flux distribution, due to the surpassing radial wall conduction on convection. At steady state, these peak values of interfacial heat flux are somewhat greater than the entering heat flux through the outer surface due to the excess heat transferred by axial conduction from the downstream region in the wall side. The thermal development length and the time to reach the steady state are also changing with the parameter values. The effects of wall conjugation and fluid axial conduction are more pronounced on heat transfer characteristics with increasing wall thickness ratio and thermal diffusivity ratio and with decreasing thermal conductivity ratio and the Peclet number. The effects of change of the parameter values on interfacial heat flux are generally seen for the whole period of the transient and also at the steady state.

REFERENCES

[1] Bilir, Ş., Laminar Flow Heat Transfer in Pipes Including Two-Dimensional Wall and Fluid Axial Conduction, *Int. J. Heat Mass Tr.*, Vol. 38, No. 9, 1995, pp. 1619-1625.

[2] Bilir, Ş., Numerical Solution of Graetz Problem with Axial Conduction, *Numerical Heat Tr.*, Vol. 21, 1992, pp. 493-500.

[3] Schneider, P.J., Effects of Axial Fluid Conduction on Heat Transfer in the Entrance Region of Parallel Plates and Tubes, *Trans. ASME*, Vol. 79, 1957, pp. 765-773.

[4] Vick, B., Özışık, M.N., and Ullrich, D.F., Effects of Axial Conduction in Laminar Tube Flow with Convective Boundaries, *J. of Franklin Inst.,* Vol. 316, 1983, pp. 159-173.

[5] Campo, A. and Auguste, J.C., Axial Conduction in Laminar Pipe Flows with Nonlinear Wall Heat Fluxes, *Int. J. Heat Mass Tr.*, Vol. 30, No. 21, 1978, pp. 1597-1607.

[6] Lin, T.F. and Kuo, J.C., Transient Conjugated Heat Transfer in Fully Developed Laminar Pipe Flows, *Int. J. Heat Mass Transfer*, Vol. 31, No. 5, 1988, pp. 1093-1102.

[7] Yan, W.M., Tsay, Y.L., and Lin, T.F., Transient Conjugated Heat Transfer in Laminar Pipe Flows, *Int. J. Heat Mass Tr.,* Vol. 32, No. 4, 1989, pp. 775-777.

[8] Travelho, J.S. and Santos, W.F.N., Solution for Transient Conjugated Forced Convection in the Thermal Entrance Region of a Duct with Periodically Varying Inlet Temperature, *Trans. ASME J. of Heat Tr.*, Vol. 113, 1991, pp. 558-562.

[9] Olek, S, Elias, E., Wacholder, E., and Kaizerman, S., Unsteady Conjugated Heat Transfer in Laminar Pipe Flow, *Int. J. Heat Mass Transfer*, Vol. 34, No. 6, 1991, pp. 1443-1450.

[10] Yapıcı, H. and Albayrak, B., Numerical Solutions of Conjugate Heat Transfer and Thermal Stresses in a Circular Pipe Externally Heated with Non-uniform Heat Flux, *Energy Conversion and Management,* Vol. 45, No. 6, 2004, pp. 927-937.

[11] Yin, X. and Bau, H.H., The Conjugate Graetz Problem with Axial Conduction, *Journal of Heat Transfer,* Vol. 118, 1996, pp. 482-485.

[12] Schutte, D.J., Rahman, M.M., and Faghri, A., Transient Conjugate Heat Transfer in a Thick-Walled Pipe with Developing Laminar Flow, *Numerical Heat Transfer, Part A*, Vol. 21, 1992, pp. 163-186.

[13] Lee, K.T. and Yan, W.M., Transient Conjugated Forced Convection Heat Transfer with Fully Developed Laminar Flow in Pipes", *Numerical Heat Tr.,* Part A, Vol. 23, 1993, pp. 341-359.

[14] Bilir, Ş., Transient Conjugated Heat Transfer in Pipes Involving Two-Dimensional Wall and Axial Fluid Conduction, *Int. J. Heat Mass Tr.*, Vol. 45, 2002, pp. 1781-1788.

[15] Yan, W.M., Transient Conjugated Heat Transfer in Channel Flows with Convection from the Ambient", *Int. J. Heat Mass Tr.,* Vol. 36, No. 5, 1993, pp. 1295-1301.

[16] Bilir, Ş. and Ateş, A. Transient Conjugated Heat Transfer in Thick Walled Pipes with Convective Boundary Conditions", *Int. J. Heat Mass Tr.*, Vol. 46, No. 14, 2003, pp. 2701-2709.

[17] Li, W and Kakaç, S., Unsteady Thermal Entrance Heat Transfer in Laminar Flow with a Periodic Variation of Inlet Temperature", *Int. J. Heat Mass Transfer*, Vol. 34, No. 10, 1991, pp. 2581-2592.

[18] Patankar, S.V., Numerical Heat Transfer and Fluid Flow, *Hemisphere*, Washington, DC, 1980.