HEFAT2007
5th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics
Sun City, South Africa
HO1

The Analysis of Natural Convection in Rectangular Porous Cavities with four Squared Isothermal Bodies

O. B. Hmood and A. A. Salem
Department of Mech. Eng. University of Sebha
Brack, P.O. Box: 53808
Libya
E-mail: osamabasil@hotmail.com
abubakerawidat@hotmail.com

ABSTRACT

The paper deals with porous cavities with impermeable boundaries. Four squared isothermal bodies are located at the corners of the cavity. Adequate numerical methods, finite difference and Gauss-Siedel iterative methods were used to solve the governing equations. The analysis of natural convection heat transfer was meant for Modified Rayleigh Number, Aspect Ratio, and Nusselt Number. In the findings, the Nusselt Number emerged as a strong function of Rayleigh Number and Aspect Ratio.
Keywords: porous cavities, aspect ratio, Nusselt number, Rayleigh number

NOMENCLATURE

A	aspect ratio, H/L
c_p	specific heat at constant pressure, (J/kg.K)
g	acceleration due to gravity, (m/s^2)
H	height of porous cavity, (m)
K	permeability of porous medium, (m^2)
k_e	eff. thermal conductivity of porous medium,(W/m.K)
L	width of porous cavity, (m)
Nu	Nusselt number
p	pressure, Pa
Ra	modified Rayleigh number
T	dimensional temperature, K
ΔT	temperature difference (T_h-T_c), K
u	fluid velocity in x direction, m/s
v	fluid velocity in y direction, m/s
x , y	Cartesian coordinates
α	thermal diffusivity of porous medium, (m^2/s)
β	coefficient of thermal expansion of fluid, (K^{-1})
θ	dimensionless temperature
μ	dynamic viscosity, (kg/m.s)
ρ	density of fluid, (kg/m^3)
ψ	stream function

Subscript

c	cold wall
e	effective
f	fluid
h	hot wall
o	reference condition (for the media)

Superscript

ˆ	dimensionless parameter

INTRODUCTION

Natural convection heat transfer in the porous enclosure between two planes, each of constant temperature has been studied for many years. In the on-going search for techniques to augment natural convective heat transfer, attention has been recently directed at the possibility of embedding isothermal bodies in the enclosures.

Two kinds of problem have been considered in the literature with respect to the type of boundary conditions of saturated porous cavity. The first kind of the problem, vertically difference temperature with horizontal insulated walls [1,2,3]. The second, horizontally difference temperature with vertical insulated walls[4].

The works that include bodies embedded in porous media are explained here. Oothuizen [5] studied two dimensional flow over a horizontal hot plate in a saturated porous medium mounted near an impervious adiabatic horizontal surface and subjected to horizontal forced flow, where the usually Darcy model is adopted. This problem was numerically investigated by using the finite element method. The heat transfer from the plate is influenced by the dimensionless depth of the plate below the surface and the importance of the buoyancy forces.

Lai and Kulacki[6] analyzed another practical problem of natural convection heat transfer from a buried sphere in an infinite porous medium. Also here, the governing equations based on Darcy's law. Different cases had been studied analytically, of temperature, concentration, heat flux, and mass flux to the sphere. Results were presented as streamlines, temperature, and concentrations, where the flow and temperature fields are significantly modified by inclusion of mass transfer effects.

Arnold [7] studied the natural convection in a porous medium between two concentric, horizontal cylinders. Two-dimensional equations had been solved, using a very fine mesh. For radius ratio (R=2), a steady four-cell regimes were seen to occur at a Rayleigh number of about 120, further increase of the Rayleigh number does not result in the appearance of more cells.

The present work deals with four squared isothermal bodies located at the corners of the porous cavity. The porous cavity has an impermeable boundary. Two of the previous isothermal bodies are of high temperature and the others are of low temperature. The study was for natural convection heat transfer in this cavity. The problem geometry and the coordinated are depicted in figure(1).

The porous medium is considered to be homogeneous, isotropic, and in local thermal equilibrium with saturated fluid. The governing equations (momentum and energy) solved numerically by using finite difference method for two dimension with Gauss-Siedel iterative method[8].

GOVERNING EQUATIONS

The equations that govern the fluid and heat flow in a saturated porous medium are the continuity, momentum, and energy equations.

Continuity Equation

The continuity equation for steady two-dimensional incompressible flow in an isotropic porous medium is:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.0 \qquad (1)$$

Momentum Equation

In porous medium the momentum equation is derived from Darcy law, which is based on measurement alone. Where this law stated on: the area averaged fluid velocity through a column of porous material is directly proportional to the pressure gradient established along the column, in addition, the velocity is inversely proportional to the viscosity (μ) of the fluid seeping through the porous medium [4]. Then for two dimensions with the presence of gravitational acceleration the Darcy model is:

$$u=-\frac{K}{\mu}\frac{\partial p}{\partial x} \qquad (2)$$

$$v=-\frac{K}{\mu}\left(\frac{\partial p}{\partial y}+\rho g\right) \qquad (3)$$

Where K is an empirical constant called permeability.

To admit the buoyancy effect the Boussinesq approximation will be used. This approximation can be applied to both liquids and gases:

$$\rho\cong\rho_{O}\left[1-\beta(T-T_{O})\right] \qquad (4)$$

Using equations (3) and (4)

$$v=\frac{-K}{\mu}\left[\frac{\partial p}{\partial y}+\rho_{O}g(1-\beta(T-T_{O}))\right] \qquad (5)$$

It is convenient to eliminate the pressure term between equation (2) and (5) by differentiating the first w.r.t. (y) and the other w.r.t. (x) thus the momentum equation becomes:

$$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=\frac{-\rho_{O}g\beta K}{\mu}\frac{\partial T}{\partial x} \qquad (6)$$

Energy Equation

The derivation of energy equation is based on the following assumptions: the medium is homogeneous, isotropic, and the solid matrix is in thermal equilibrium with the fluid filling the pores. These assumptions are adequate for small-pores media such as geothermal reservoir and fibrous insulation. Then for steady-state flow in two-dimensions the energy equation is:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right] \qquad (7)$$

$$\alpha=\frac{k_e}{\rho_f Cp_f}$$

DIMENSIONLESS FORMULATION

To more conveniently elucidate the mathematical manipulations, the study of the governing equations will be carried out in a non-dimensional form. All the spatial dimensions are non-dimensionalized with respect to L and H, then

$$\hat{x}=\frac{x}{L} \quad , \quad \hat{y}=\frac{y}{H} \quad , \quad A=\frac{H}{L}$$

Where A is the aspect ratio (height, H, to width, L).

$$\theta=\frac{T-T_c}{T_h-T_c}$$

The stream function concept is commonly used for convective heat transfer problem, and is defined in terms of the velocity components, which satisfies the continuity equation:

$$u=\frac{-\partial\psi}{\partial y} \quad , \quad v=\frac{\partial\psi}{\partial x}$$

The stream function is non-dimensionalized as follow:

$$\hat{\psi}=\frac{L}{H\alpha}\psi$$

Then the governing equations will take the following non-dimensional forms:

Momentum Equation

$$A^2 \frac{\partial^2 \hat{\psi}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{\psi}}{\partial \hat{y}^2} = RaA\frac{\partial \theta}{\partial \hat{x}} \qquad (8)$$

Where (Ra) is a modified Rayleigh number which is the ratio between buoyancy force to drag force.

$$Ra = \frac{\rho g \beta K L \Delta T}{\mu \alpha}$$

Energy Equation

$$\frac{\partial \hat{\psi}}{\partial \hat{x}}\frac{\partial \theta}{\partial \hat{y}} - \frac{\partial \hat{\psi}}{\partial \hat{y}}\frac{\partial \theta}{\partial \hat{x}} = \frac{\partial^2 \theta}{\partial \hat{x}^2} + \frac{1}{A^2}\frac{\partial^2 \theta}{\partial \hat{y}^2} \qquad (9)$$

RESULTS AND DISCUSSIONS

Solution of the flow field and temperature distributions within the enclosure may be found by standard numerical methods (finite difference and Gauss-Siedel methods). The governing equations for the stream function, eq.(8), and temperature, eq.(9), are first discredited according to the well-known central difference scheme for a regular mesh size. The discredited equations for ψ, θ are then solved at each point, until the convergence to a steady or a stationary oscillating state is achieved. The iterative procedure was repeated until the following condition was satisfied: $\sum_i \sum_j \left| \theta_{i,j}^{n+1} - \theta_{i,j}^{n} \right| < 10^{-5}$, where the superscripts n and $(n+1)$ indicate the value of the n^{th} *and* $(n+1)^{th}$ iterations respectively, i *and j* indices denote grid location in the (x,y) plane.

The most effective parameters in this study are Rayleigh number (Ra) and aspect ratio (A). The range of Ra was (0-600) and aspect ratios were 0.5, 1, 1.5, and 2. As mentioned before the isothermal bodies are square and of dimension (0.25H×0.25H) for A=0.5, and (0.25L×0.25L) for other aspect ratios.

Typical numerical results are presented in figures (2-5) for isotherms and streamlines and figure(6) for heat transfer result. Firstly, for pure conduction, Ra=0, there is no flow and the isotherms will be vertical lines across the cavity.

As Ra increased, figure(2), the isotherms deflect toward the right (Ra=300). This deflection increased more at Ra=600 because of buoyancy effects. It has seen that the flow consists of one cell rotating in the clockwise direction (Ra=300). As Ra increased to 600 the flow comprises to multi cellular flow of two weak secondary cells.

Figure(3), A=1, the core region looks greater and the main streams appear between vertically spaced isothermal bodies. Also here, most of heat released from the hot boundaries and that gained by cold boundaries are due to horizontal temperature gradient $\frac{\partial T}{\partial x}$ and that is clear at the lower hot body and the upper cold body. The flow remains unicellular because of the narrow space between different temperature bodies.

Figures(4,5) for A>1. In figure(4,a) , A=1.5, there is no great change in isotherm shape as Ra increased. Also, the flow remains unicellular, the core region enlarged and the flow proceeds to the boundary layer regime. The change in the flow and direction of heat be clear at A=2, figure(5). The flow comprises to three main cells, Ra=600. It can be seen vertical temperature gradient at the place of the middle cell. Most of the heat released from the lower hot boundary gained by the lower cold boundary. We can say all the heat released from the upper hot boundary gain by the upper cold boundary.

Figure(6) illustrates the effect of Raleigh number on the convection heat transfer presented by Nusselt number at different aspect ratios. Generally, Nu increased as Ra increased. For A=0.5, the onset of convection was at high Ra and the heat is transferred mainly by conduction. The behavior of the curve, at A=2, differs on the other curves in the range of Ra from 400 to 500 because the nature of the flow changes. This change effects on the convective heat transfer which reflect on the value of Nu.

CONCLUSIONS

In this paper a two-dimensional numerical investigation of the natural convection in a porous cavity contained in four isothermal bodies at its corners. The governing equations were solved in the ranges of Ra and A. We have found that for aspect ratio less than one the enhancement to convection is low. Whereas for A>1,(i.e. A=2), the flow structure effect greatly on heat transfer as Rayleigh number increased.

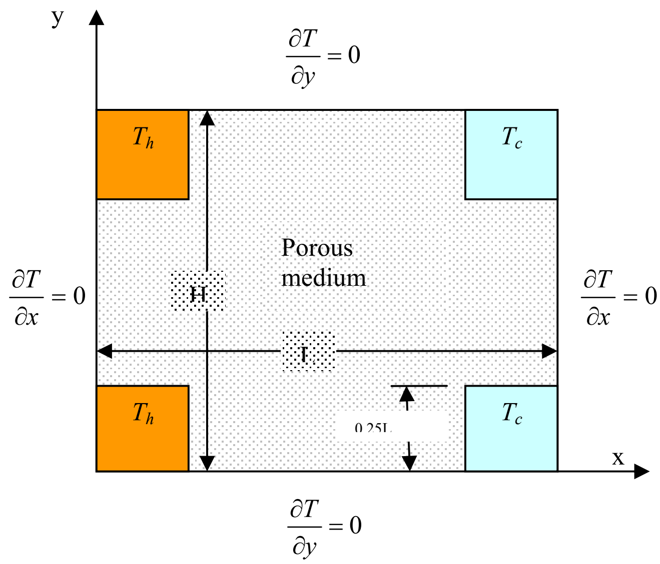

Figure 1: *Geometry of the problem*

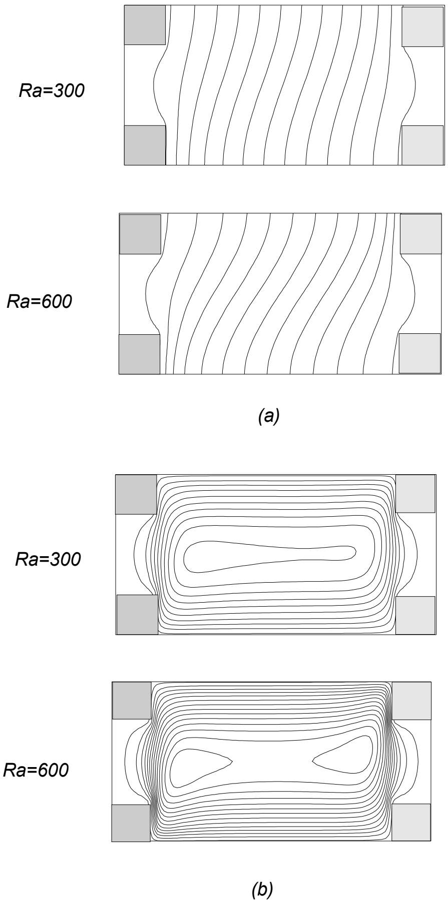

Figure2: (a)Isotherms and, (b)Streamlines for A=0.5 and different Ra

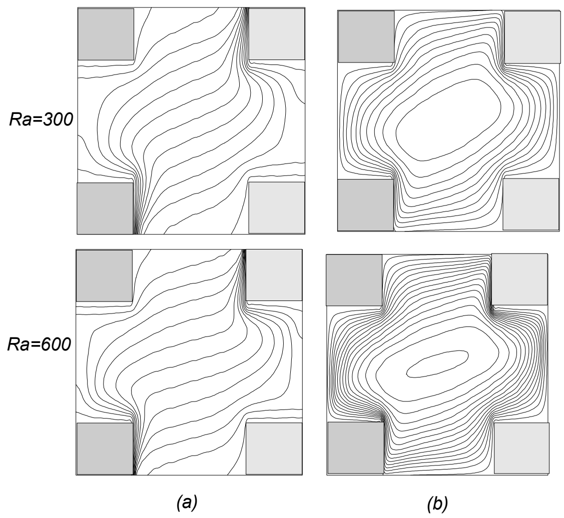

Figure3: (a)Isotherms and, (b)Streamlines for A=1 and different Ra

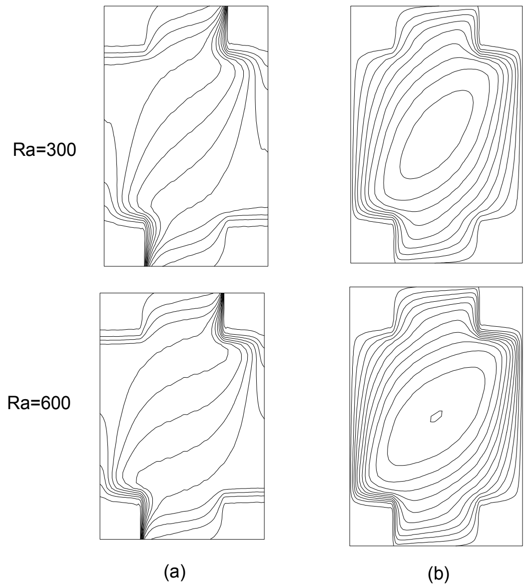

Figure4: (a)Isotherms and, (b)Streamlines for A=1.5 and different Ra

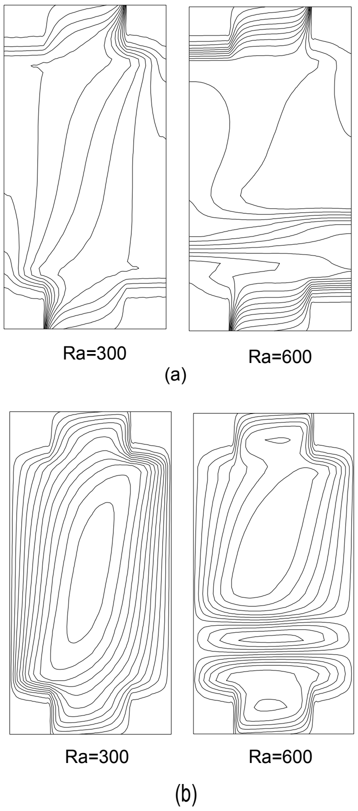

Figure5: (a)Isotherms and, (b)Streamlines for A=2 and different Ra

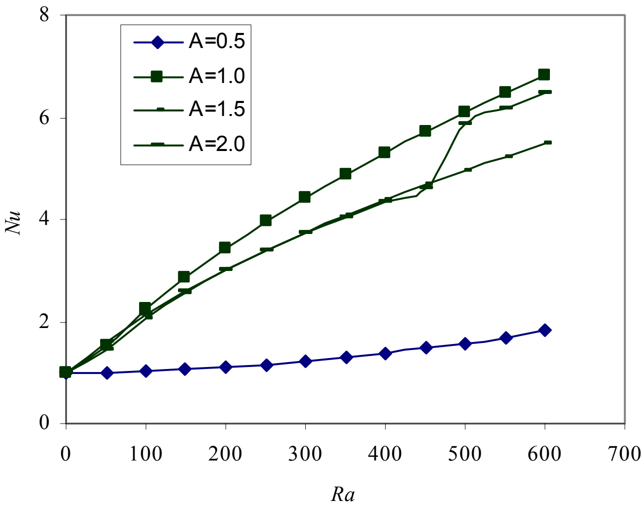

Figure6: *Relation between Raleigh no. and Nusselt no. for different Aspect ratio A*

REFERENCES

[1] Bejan, A and Tien, C.L, 1978, Natural Convection in Horizontal Porous Medium Subjected to an End-to-End Temperature Difference, *J. Heat Transfer* 100, 191-198.

[2] Prasad, V and Kulacki, F.A, 1984, Convective Heat Transfer in a Rectangular Porous Cavity-Effect of Aspect Ratio on Flow Structure and Heat Transfer, *J. Heat Transfer* 106, 158-165.

[3] Vafia, K, and Kim, S.J, 1990, Analysis of Surface Enhancement by a Porous Substrate, *ASME J. Heat Transfer* 11, 700-705.

[4] Nield, D. A, and Bejan, A, 1999, *Convection in porous Media*, Springer Verlag.

[5] Oothuizen, P.H, 1988, Mixed Convective Heat Transfer from a Heated Horizontal Plate in a Porous Medium Near an Impermeable Surface, *J. Heat Transfer* 110, 390-394.

[6] Lai, F.C, and Kulacki, F.A, 1990, Coupled Heat and Mass Transfer from a Sphere Buried in an Infinite Porous Medium, *Int. J. Heat Mass Transfer* 33, 209-215.

[7] Arnold, F, 1991, Natural Convection in a Porous Medium Between Concentric Horizontal Cylinders, *Numerical Methods in Thermal Problems* 7, Part2, 1081-1091.

[8] Hoffman, J. D., 1992, *Numerical Methods for Engineers and Scientist*, Mc Graw-Hill, Inc.