THE APPLICATION AND INTERPRETATION OF LINEAR FINITE ELEMENT ANALYSIS RESULTS IN THE DESIGN AND DETAILING OF HOGGING MOMENT REGIONS IN REINFORCED CONCRETE FLAT PLATES

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SUMMARY

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Degree: Masters of Engineering (Civil Engineering)

Structural engineers have used finite element methods for the design of reinforced concrete plate type structures for decades. The theory behind this method is well researched, however, there is still a lack of direction on how to use the information obtained from this type of analysis to practically design reinforced concrete structures for strength and serviceability criteria.

The literature study reviews the analysis of concrete plate type structures using traditional and finite element methods and highlights the difference between linear and non-linear finite element analysis. It is apparent that when designing and detailing using a FE analysis, a great deal is left up to engineering judgement, especially in areas of the structures where peak load effects (singularities) are experienced. In this thesis these peak areas are investigated, in an effort to provide insight into the actual behaviour of the structure as opposed to the theoretical results obtained from a FE analysis.

The research consists of both numerical, (linear and non-linear FE analyses) and practical experimental work performed on different types of concrete plate type structures, including concrete pad foundations and simply supported flat slabs. The response to loading, i.e: cracking characteristics, softening of the concrete, moment redistribution, variation of the strain in reinforcement across the section, and deflection is observed and discussed.

The results show that the traditional simplified methods are adequate with respect to overall strength. Finite element peaks or singularities may be averaged or smoothed without compromising durability and serviceability. Suggestions on how the reinforcement obtained from linear finite element methods be detailed are given.

DECLARATION

I, the undersigned hereby declare that:

I understand what plagiarism is and I am aware of the University's policy in this regard;

The work contained in this thesis is my own original work;

I did not refer to work of current or previous students, lecture notes, handbooks or any other study material without proper referencing;

Where other people's work has been used this has been properly acknowledged and referenced;

I have not allowed anyone to copy any part of my thesis;

I have not previously in its entirety or in part submitted this thesis at any university for a degree.

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Sarah Anne Skorpen

98096355

July 2013

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A_s	area of reinforcement in tension
A_s‘	area of reinforcement in compression
B	width or shorter plan dimension of a pad foundation
b_{col}	width of support column
D	length or longer plan dimension of a pad foundation
d	effective depth of reinforcement in a slab or pad foundation
E_c	concrete modulus of elasticity
$E_{c,eff}$	effective modulus of elasticity of the cracked transformed section
E_s	reinforcement modulus of elasticity
F_s	support reaction
FE M_{peak}	peak moment in finite element analysis
FE M_{total}	total finite element analysis moment
f_r	Modulus of rupture of concrete
f_{cu}	Characteristic cube strength of concrete
f_y	Yield strength of the steel reinforcement
h	depth of concrete in a slab or pad foundation
I_{co}	second moment of area of the un-cracked transformed section
I_{cr}	second moment of area of the cracked transformed section
I_{eff}	effective secant stiffness
M	applied moment
M_{col}	traditional method column strip moment
M_{cr}	moment at first cracking
M_p	peak moment at the centreline of the support
M_s	smoothed support bending moment
M_x	bending moment in the x direction
M_y	bending moment in the x direction
M_{xy}	twisting moment
N	column load
l_x	short span length
l	cantilever length
W_{col}	column strip width
w	uniform pressure under footing

x	lever arm of the resultant force
y_t	distance from centroidal axis to the extreme tension fibre
Ø	diameter of the reinforcement

ABBREVIATIONS

FE	Finite element
SD	Simplified design
RC	Reinforced concrete
3D	Three dimensional

1 INTRODUCTION

Concrete design codes currently in use, i.e. SANS 10100 and Eurocode 2, contain calibrated strength models enabling the user to calculate a safe resistance of a structural member. In many cases, such models are simplifications of quite complex failure modes. Load effects obtained using appropriate methods of analysis provide values of bending moments, shear forces and axial forces. Local peak effects (singularities) cannot be calculated using the traditional methods of analysis. The simplifications are justified, by and large, by the ductile behaviour of the members. The reliability of the models has been proven by the lengthy process of calibration involved and the amount of structures that have safely resisted the applied loads. A specific application can be found in a plate with column supports. Significant differentiation in curvature over the supports is regulated by the traditional methods by simple stepping requirements (see Figure 1-1).

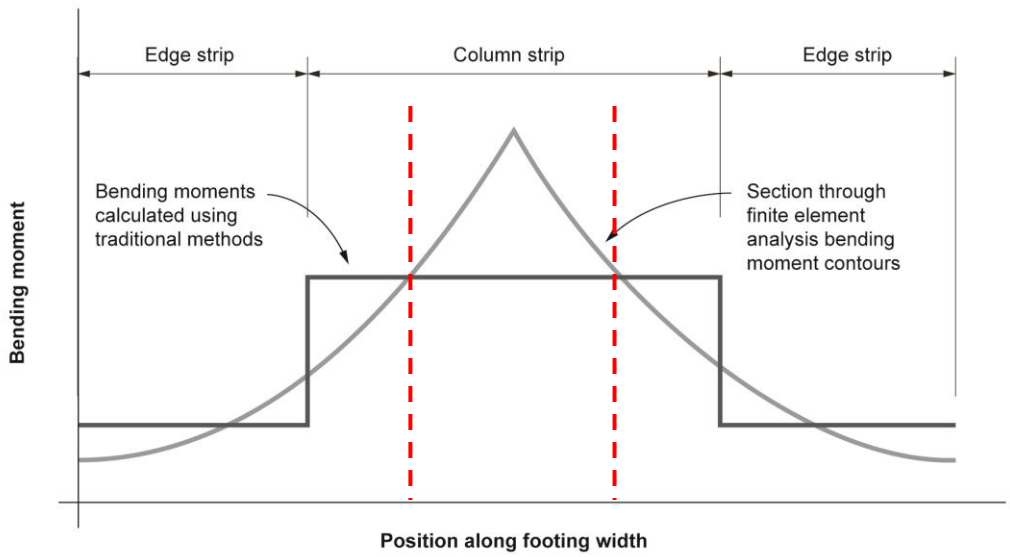


Figure 1-1: Typical transverse distribution of bending moments in a pad foundation

The advent of finite element methods of analysis provides absolute rather than average values of load effects and stresses. Practical detailing of structural elements does not fully consider the peak values obtained from more sophisticated methods of analysis. Given the amount of structures designed using simple methods, and the reliability attached to the proven methods, the use of more sophisticated methods of analysis should be applied in such a manner as to provide consistent results.

Finite Element (FE) methods have been used by civil and structural engineers since the 1960's (Carlton 1993), and the theory behind such methods is well researched. However, there is still a lack of information on how to use the information obtained from this type of analysis to practically design and detail a structure for strength and serviceability criteria. Design codes assume that the designer has engineering judgment and a "feel" for the behaviour of concrete when using FE analysis (Brooker 2006), and leave a lot unstated when dealing with complex force situations (Carlton 1993).

In order to apply and interpret the results of FE analysis consistently to flat plate design problems the following aspects need to be addressed:

- The total resistance achieved with a FE analysis compared with that of traditional methods
- To what extent the peak moment in a FE analysis can be ignored.
- The serviceability performance of a slab detailed to FE analysis compared with simplified methods
- FE analysis detailing of the reinforcing that is practical and acceptable to construction companies.

In this thesis these questions are considered by studying three types of flat plate structures. Numerical and practical experimental work was performed and their response to loading (cracking characteristics; softening of the concrete; moment redistribution; variation of the strain in reinforcement across the section; and deflection) is discussed. The flat plate structures studied are as follows:

Square single column footing – 1.2m x 1.2m x 0.15m thick:

- Several linear FE analyses of this footing were done with different support (constraint) models. The effect of the constraint model on the total FE design moment and peak FE moment was assessed, and compared to the SD moments.

- A nonlinear FE analysis of this footing was done and the effect of cracking on the moment distribution and peak stresses was considered.
- Two footings were constructed and tested, the one reinforced according to the SD method and the other following the FE linear moment contours. Strain gauges were placed on the reinforcement bars at the critical section. Total load carried, strain distribution in the reinforcement, cracking and deflection were observed.

Rectangular combined column footing – 9m x 5m:

- Linear FE analyses of this footing, varying the footing thickness from 400mm to 1300mm were carried out. The effect on varying geometry on the total FE design moment and peak FE moment were assessed, and compared to the SD moments.
- Theoretical crack widths were calculated for FE and SD reinforcement spacing.

Simply supported flat slab – 2m x 2m x 0.15m thick:

- Two, simply supported, two way spanning, flat slabs were tested, one reinforced according the SD method and the other with reinforcement spacing according to a FE linear analysis. Total load carried, deflection and cracking was observed.

The objective of this study is to investigate the use of the linear finite element analysis methods for RC plate type structures, particularly in areas with high peak load effects, and to consider how to use the information obtained from this type of analysis to practically design reinforced concrete structures for strength and serviceability criteria. Typical FE plate element shapes and sizes, as prescribed in various FE design guides, were used in the analyses, in order to assess the analysis results that a practising design engineer would obtain.

The thesis firstly reviews and discusses the analysis of flat plate type structures using traditional methods and finite element methods in Chapter 2, and questions are posed regarding ultimate and serviceability behaviour of a flat plate structure designed in accordance with a FE analysis. The author then considers these questions by studying the behaviour of four different flat plate type structures. Chapters 3 to 6 document numerical and practical experimental work done by the author on these four different flat plate structures. Chapter 7 presents the suggested answers to the questions posed by the literature review as a result of the modelling and experimental work done, as well as suggestions for future work.

The intention of this thesis is not to do a theoretical assessment of the finite element method, but rather to carry out a practical investigation of how this method can be used in the building industry. Guidelines are proposed on the required amount and placement of reinforcement to satisfy ultimate and serviceability limit states.

2 LITERATURE STUDY

This literature study summarizes classic plate theory and flat plate design; the basics of the traditional flat plate design methods are discussed; and finite element analysis is introduced, highlighting the differences between linear and nonlinear analysis.

2.1 CLASSIC PLATE THEORY

FE analysis of plate structures are analysed using classic plate theory which has been formulated by considering equilibrium and strain compatibility in plates which are thin enough for shear deformations to not have a significant effect on the behaviour of the slab, and thick enough that in-plane and membrane forces are not important. Park and Gamble (2005) refer to these plates as “medium thick” but they are generally referred to as thin plates.

Thin plate theory is used for flat structures where transverse shear effects are not important and is based on Kirchhoff’s theory. Thick plate theory is used for flat structures where the effects of transverse shear must be included. This is based on Reissener’s / Mindlin’s theory which takes into account the effect of shear strain.

The basic assumptions of thin plate or shell theory are summarized by Rombach (2004):

- Plane sections remain plane before and after loading
- Linear strain distribution through the slab depth
- Zero strain at the middle of the plane
- Stresses in the normal direction can be ignored
- A thin slab (span/depth > 10)
- Constant slab depth
- Small vertical displacements, w << h.

The simplified analysis of flat plate type structures, such as slabs and pad foundations, described in most codes (TMH7, SANS 0100 and BS8110), ignore transverse shear effects and assume that plane sections remain plane. Where shear effects become important (i.e. deep beams where span/effective depth < 2) the member can be modelled using equivalent truss analogy.

2.2 FLAT PLATE DESIGN

Modern design codes, including Eurocode 2, allow the use of the following methods for flat plate / slab analysis:

- Empirical method – used for small, regular frames. Total bending moments and shear forces are calculated using tabled moment and shear coefficients based on yield line analysis.
- Simplified method– this is commonly known as the equivalent frame method, however this name is misleading as it implies that the full frame including the columns is included in the analysis. It is possible to simplify the equivalent frame method conservatively by considering a continuous beam with simple supports. This method is used for more irregular frames. Total moments are obtained and then split between column and middle strips.
- Linear finite element (FE) method – used where the floor has irregular supports or geometry. A non-linear cracked section FE analysis is helpful in predicting the deflections and crack widths.
- Yield line analysis – this method is based on fully plastic conditions consistent with the most economic and uniform distribution of reinforcement. A separate analysis of cracking and deflection is required.
- Grillage analysis – this method involves the slab being modelled as a series of discreet longitudinal and transverse elements which are interconnected at nodes. This method is quite time consuming.

The simplified and FE methods are the most popular flat slab analysis methods today, with the FE method looking set to become the preferred analysis method in the near future. These two methods of analysis will be discussed in further detail in this literature review.

2.3 TRADITIONAL (SIMPLIFIED) FLAT PLATE ANALYSIS METHODS

Vertical loads are resisted by an equivalent beam with a strip width considering the distribution of total load using the same depth as the slab. Bending moments are not constant across the width of the strip, and it has been experimentally shown (Regan 1981) that they are highest on a line connecting the columns, and then reduce transversely as seen in Figure 2-1 and Figure 2-2. For this reason most codes prescribe the design of flat slabs by considering a “column” and a “middle” strip, with the column strip resisting approximately two thirds of the load effect and the middle strip one third. This apportionment varies between codes, and Table 2-1 summarizes the requirements of various codes. This approach is aimed at satisfying serviceability requirements by placing more reinforcement in regions of higher bending moment, thereby reducing cracking. The column strip reinforcement can be further refined by placing two thirds of the column strip reinforcement in the central half of the column strip in accordance with SANS 10100-1 Cl 4.6.5.4

When considering the hogging moment regions of flat plates a pad foundation is one of the most severe cases to study as the curvature gradient is the largest.

Code	Column strip width	Column strip	Middle strip
TMH 7. 1989 Code of Practice for the Design of Highway Bridges and Culverts in South Africa, Part 3.	b_{col} + 3h if B > 1.5(b_{col} + 3h)	66.67%	33.33%
SANS 10100. 2000 The structural use of concrete, Part 1	$l_x/2$	75%	25%
Eurocode 2: Design of concrete structures EN 1992-1-1:2003 (E)	$l_x/2$	60 – 80%	40 – 20%
BS 8110.1997 Structural use of concrete. Part 1	$l_x/2$	75%	25%

l_x= Short span length; b_{col} = column width; h = depth of the slab; B = width of the pad foundation

Table 2-1: Apportionment between column and middle strip moment expressed as a percentage of the total negative (hogging) moment

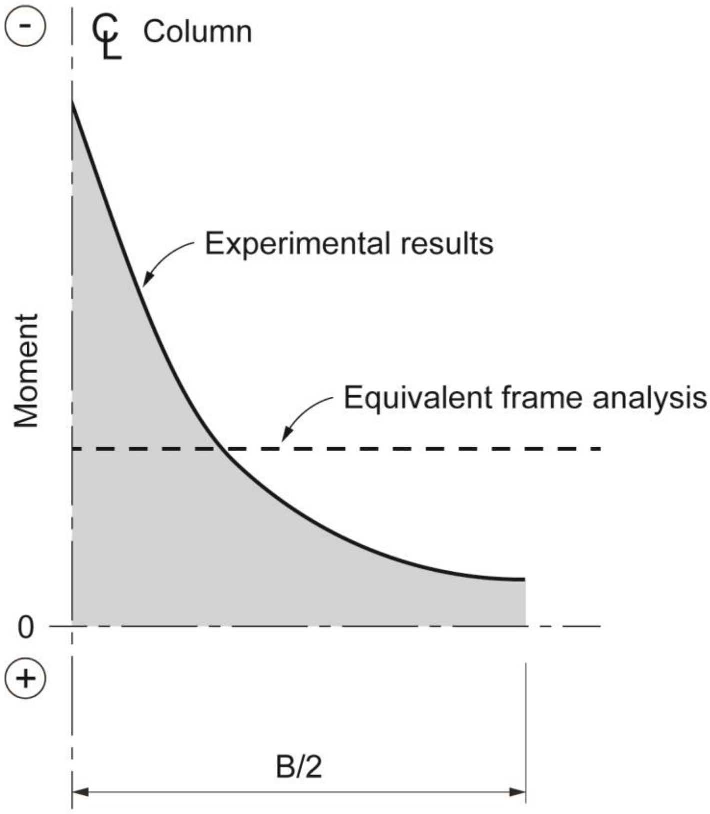


Figure 2-1: Variation of bending moments through the width of a slab (Robberts and Marshall 2007)

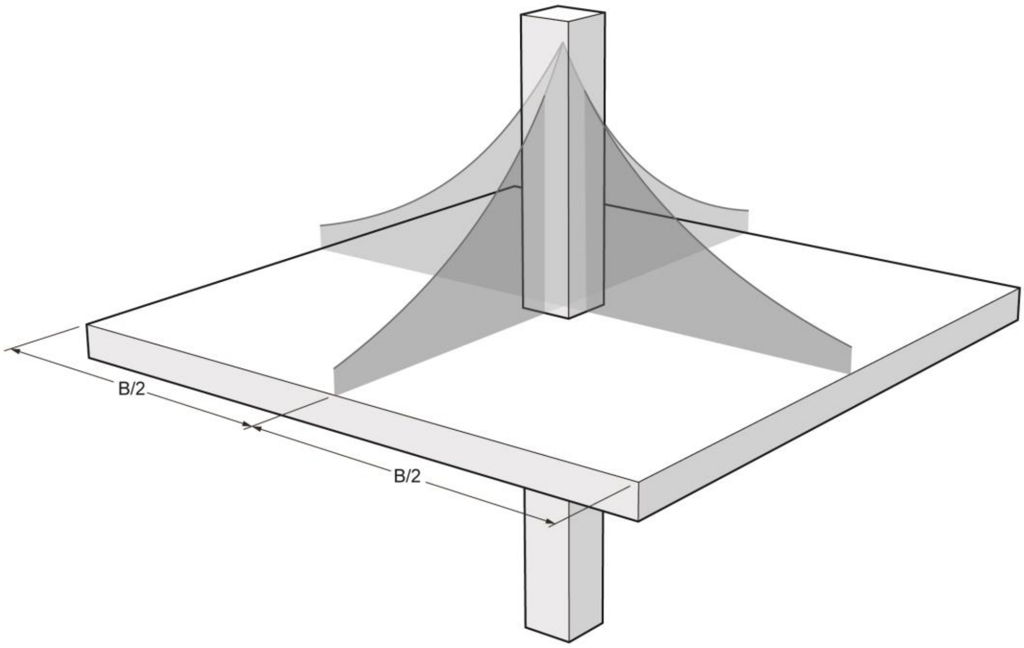


Figure 2-2: Three dimensional variation of moments across the width of the slab (Hossell 2012)

2.4 FINITE ELEMENT ANALYSIS METHODS

The significant advance in computer hardware technology in recent years has resulted in an increase in the use of finite element software to analyse the load effects in structures, and in particular flat plate type structures. Finite element computer packages are becoming more readily available and can run on most personal computers. Large cumbersome computers, which could only be operated by the experienced few are no longer required. Today most graduate Engineers are very familiar with computers and tend towards designing with computer software as opposed to “old fashioned” hand calculations.

Finite element analyses are particularly helpful when analysing a structure that has complex geometry or loading situations, and can be used to determine the forces and moments around large openings in floor slabs. FE analyses can also be used to estimate deflections in complex structures. One must be careful to take cracking of the concrete into account.

With the ever-increasing demand for structures that push the boundaries of both geometry and time constraints, FE analysis has seemed to be the answer for designing complex structures faster. The disadvantages have tended to be overlooked as Engineers strive to move along with the progress of technology. One of the main gaps in FE analysis is the lack of information on how to use the information obtained from this type of analysis to practically design a structure for strength and serviceability criteria. It is not practical to place reinforcement in accordance with the FE output. Design codes assume that the designer has engineering judgment and a “feel” for the behaviour of concrete when using FE analysis, however, this is not the case for most young Engineers who are more often than not the people doing the FE analysis.

A common misconception is that a FE analysis will result in lower bending moments and deflections than obtained using traditional methods. This is incorrect and it has been shown in a study carried out by Jones and Morrison (2005) that FE methods for a rectangular grid give similar results to more traditional methods including yield line and equivalent frame analysis.

The output from a FE analysis is generally not in the most useful form and often additional post-processing programs are required to deal with large amounts of results. Design standards also require careful interpretation in order to ensure that the complex force field information is taken into account to ensure compliance with the standard (Carlton 1993).

2.4.1 SUMMARY OF THE FINITE ELEMENT METHOD OF ANALYSIS

The finite element method is an approximation in which a continuum is replaced by a number of discreet elements (Zienkiewicz et al 1976). Each component representing the system as a whole is known as a finite element. Parameters and analytical functions describe the behaviour of each element and are then used to generate a set of algebraic equations describing the displacements at each node, which can then be solved. The elements have a finite size and therefore the solution to these equations is approximate; the smaller the element, the closer the approximation is to the true solution (Brooker 2006).

When creating a FE model for a flat slab the following aspects highlighted by Brooker (2006) are important to consider:

Element type – Plate elements are generally used for flat slab design; these provide results for flexure, shear and displacement. When a slab is deep (i.e. span to depth < 10) shear deformations should be considered and 3D elements should be used instead of plate elements. Shell elements are used when membrane action is modelled.

Element shape – Plate elements are typically triangular or quadrilateral with a node at each corner, however elements can also include additional nodes on each side (Figure 2-3). Force and displacement are calculated accurately at the nodes and interpolated at other positions, therefore the number of nodes directly affects the accuracy of the model. It is important to have more nodes in the model where the forces change rapidly because it is only at node locations that results are obtained directly. Elements should be “well conditioned” (Figure 2-4) and in general the ratio of maximum to minimum length of the sides should not exceed 2 to 1.

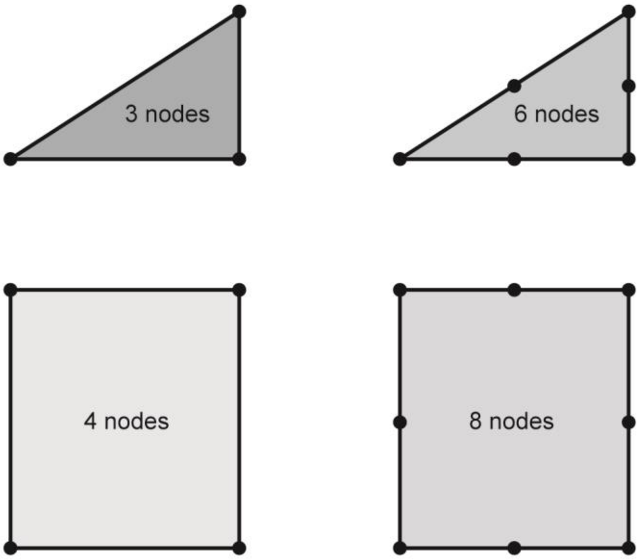


Figure 2-3: Element types (Brooker, 2006)

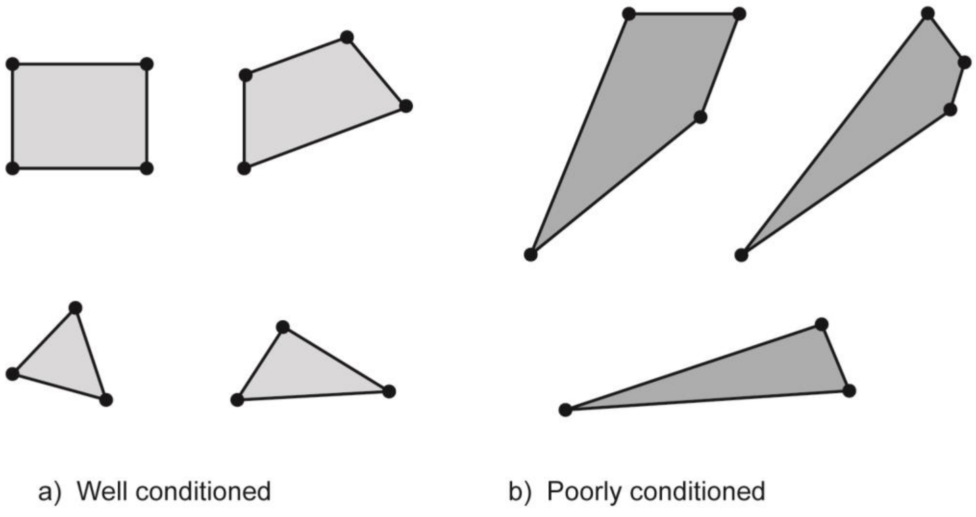


Figure 2-4: Well and poorly conditioned element shape (Brooker, 2006)

Meshing – Meshing describes the sub-division of surface members into elements (Figure 2-5). The finer the mesh the more accurate the results, and the longer the computational time. In general elements should not be greater than span / 10 or 1000mm, whichever is the smallest (Brooker 2006).

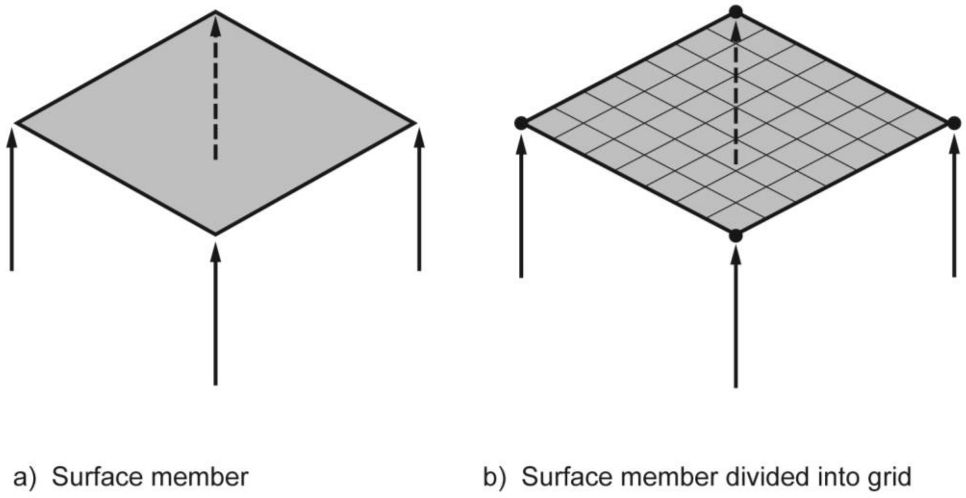


Figure 2-5: Discretization of a surface member (Brooker, 2006)

Supports / Constraints – It is important to correctly model support conditions to ensure realistic bending moments at the supports and at midspan. This will be discussed in more detail further on in this chapter.

2.4.2 FINITE ELEMENT ANALYSIS PROCESS

The analysis process by which an FE program is used to provide the design engineer with valid information to design a structure is described by Carlton (1993):

- Converting the designed structure into an idealised structure
- Converting the idealised structure into a geometrical model
- Interpreting the geometric model as a meshed model
- Converting the meshed model into a solution model
- Running the solution model to create a numerical solution
- Using the numerical solution to perform a results interpretation
- With a satisfactory results set, performing the post-processing
- With an adequate set of post-processed results, obtaining a set of structural characteristics that describe the actual behaviour of the real structure subject to the actual design load.

After processing a structural analysis problem, the finite element system will generate nodal deflections, stress and strain information either at element nodes or, at the element integration points. The results are then viewed by post-processing software, which may be part of the finite element system, or by other software such as the modelling software or by user-written software. The total analysis process should result in a FE output which is interpretable in terms of design decisions.

2.4.3 LINEAR AND NON-LINEAR FINITE ELEMENT ANALYSIS

The materials that structural engineers use have non-linear stress-strain relationships. Concrete changes in behaviour with time because of strength gain and creep, which are both linked to the chemical reaction between cement and water. Reinforced concrete cracks once the tensile strength of the concrete is exceeded, changing the stiffness of the section. A non-linear analysis is therefore desirable, however this type of analysis can be time consuming and not practical when different loading patterns and combinations are considered.

Linear FE analyses are commonly used for reinforced concrete design but are limited in their capabilities. They do not take cracking of the concrete and yielding of the reinforcement causing local loss of stiffness and corresponding transfer of load to the un-cracked concrete into account (refer to Figure 2-6). This type of analysis is suitable for an ultimate limit state design check, but cannot be used to check serviceability deflection and cracking. Non-linear FE analyses model the cracked behaviour of the concrete by means of an iterative process, but are complicated and time consuming to set up and the software cost is significantly more than a linear FE program. In practice, flat plate type structures are generally designed using a linear FE analysis, and serviceability compliance done with 'rule of thumb' span to effective depth ratio checks.

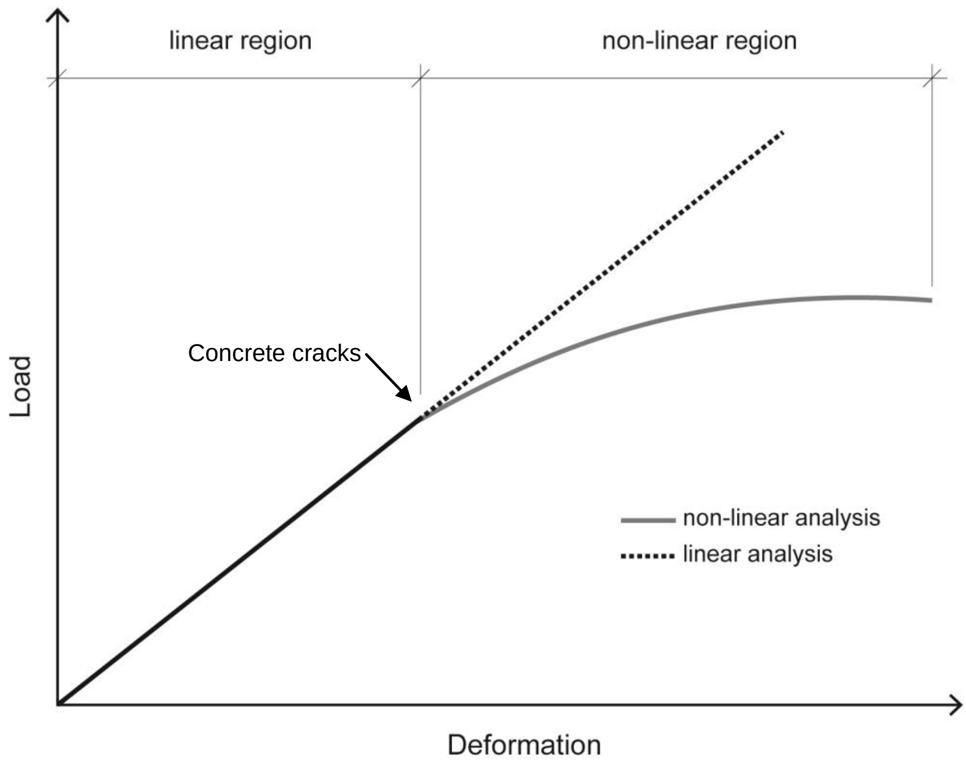


Figure 2-6: Typical load deformation behaviour for a linear and non-linear analysis (Hossell 2012)

The main criticisms of linear FE analyses are that the use of elastic material properties result in overestimated support moments and underestimated deflections (Jones & Morrison 2005) and that the reinforcement contour output that is impractical.

Non-linear analyses require specialized skills from the analyst controlling the process as well as a good knowledge and understanding of the material models and restrictions of the system. Software with good non-linear analysis capabilities is also very costly. The kinds of problems where a non-linear analysis is particularly helpful are:

- Overloading analysis of structures
- Behaviour of mass concrete pours
- Ground stability and settlement

Modern codes allow for non-linear analysis of reinforced concrete structures, but in practice such a complex analysis is seldom justified. Designs are usually based on linear-elastic material behaviour, assuming that the ductile properties of reinforced concrete allow for a limited redistribution of forces. Rombach (2004) states that the accuracy of such a simplified approach is generally sufficient. A conservative design approach is to have two slab models, one where columns are assumed to be pinned supports, to determine the worst case sagging moment; and the second where the column supports are fixed to determine the worst case hogging moments.

2.4.4 OUTPUT FROM A LINEAR FINITE ELEMENT ANALYSIS

The output from a linear finite element flat slab analysis provides contour plots of stresses and moments. Bending moments in the x and y directions, M_x and M_y, and a local twisting moment M_{xy} (see Figure 2-7) are given. This twisting moment is important because of the three dimensional behaviour of a flat slab and must be taken into account in the design of the reinforcement; however it does not act in the direction of the reinforcement. A popular method of considering the twisting moment is known as Wood-Armer moments (Armer (1968); Wood (1968)), and most design software will automatically calculate the Wood-Armer moments for the user. There are four components, top (hogging) moments in the x and y directions, M_{xT} and M_{yT}, and bottom (sagging) moments in each direction, M_{xB} and M_{yB}. This method is conservative and these moments form an upper limit envelope of the worst-case design moments. The four components can be used to calculate the required reinforcement for each of the reinforcement layers in a flat slab type structure. (Brooker, 2006)

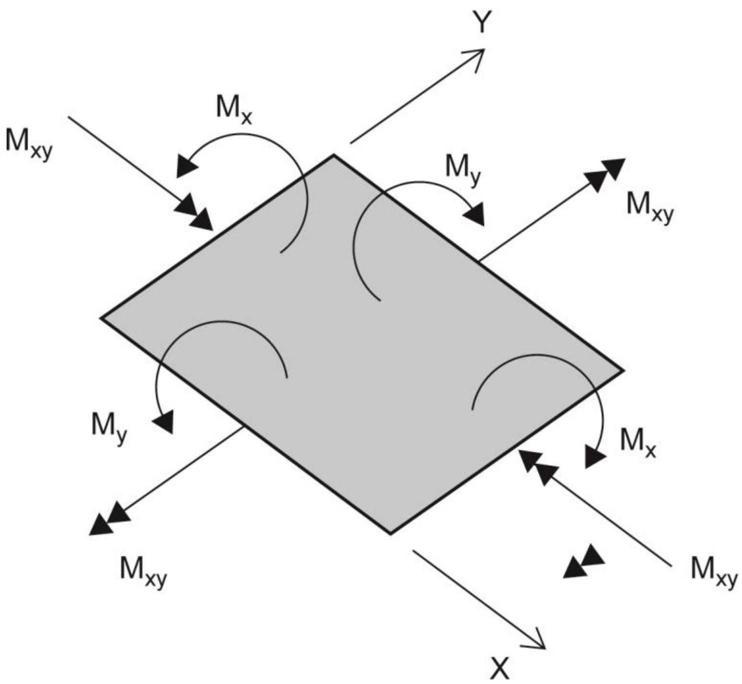


Figure 2-7: Finite element design bending moments (Brooker, 2006)

In order to maintain a practical reinforcement layout Brooker's (2006) recommends using the total bending moment under the FE moment curve and then stepping the reinforcement as for the inner and outer columns strip detailing rules given in SANS 10100-1 (2000).

In 2005 Jones and Morrison performed a comparative study on flat slab design methods including finite element analysis. The comparative designs were carried out by the two separate practices and in certain cases significant discrepancies were found. The discrepancies in design load effects were due to different interpretations of the code, different interpretations of the finite element models or different conceptual ideas for the yield lines.

2.4.5 SUPPORTS IN A LINEAR FINITE ELEMENT ANALYSIS

Contour plots of bending moments show very large peaks at the supports. These peak bending moments can vary considerably depending on how the support conditions are modelled. While the basics of using a linear FEM to analyse flat slabs is commonly understood by most designers the modelling of column to flat plate connections is still open to numerous forms of interpretation and designer preference. The most common support models are shown in Figure 2-8 (Rombach (2004)).

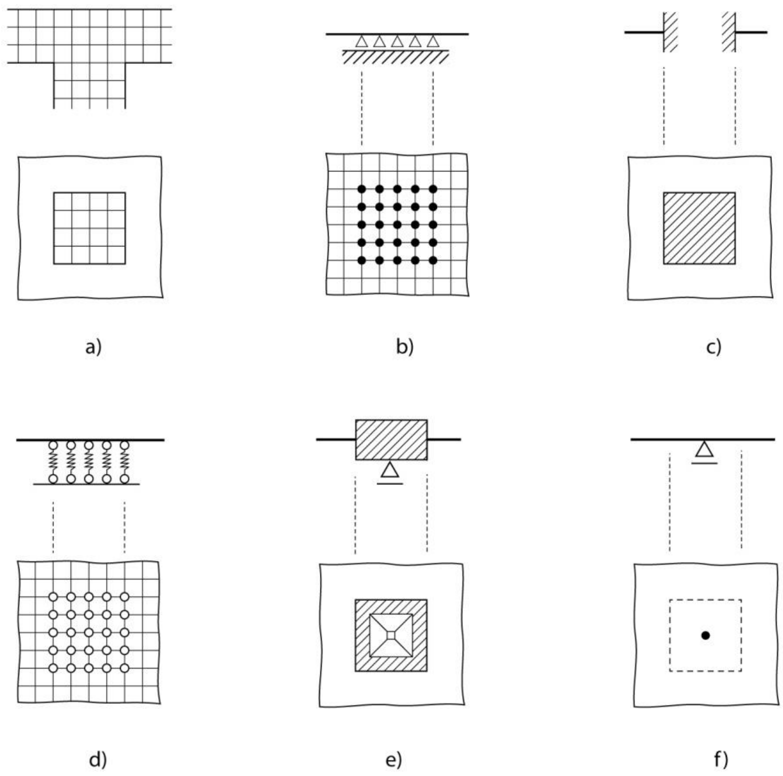


Figure 2-8: Idealisations of column/flat slab connections

The models in Figure 2-8 can be interpreted as follows:

a) Full 3D continuum model – this models accurately, but is very time consuming.
b) Pinned supports over all nodes above the column – this is not suitable where the column is relatively flexible.

c) Encased supports assigned to the edge of the column in the shell model – this is not suitable where the column is relatively flexible.

d) Spring supports assigned to the column area in the shell model.

e) Rigid column head – this allows rotation of the column cross section and is suitable for flexible columns.

f) Point support at one node – this is the least accurate way of modelling a support, but the most commonly used.

2.4.6 SINGULARITIES IN LINEAR FINITE ELEMENT ANALYSES

Peak load effects (singularities) from linear elastic FE models are consistent with high elastic stresses. These peaks are reduced by yielding and cracking, or "softening", of the concrete, and are never actually realized in real structures. In a two dimensional analysis, the bending moment in a one way spanning slab supported on pinned supports is generally smoothed using the following equation given by Rombach (2004):

$$M_s = M_p - \frac{F_s b_{col}}{8} \qquad (1)$$

M_s = smoothed support bending moment

M_p = peak moment at the centreline of the support

F_s = support reaction

b_{col} = width of support column

This method is consistent with considering the moment at the edge of the column support.

Singularities in FE analyses commonly occur where pinned-supports and concentrated loads are modelled. The stress and bending moment contour output from a finite element modal will indicate peaks as seen in Figure 2-9.

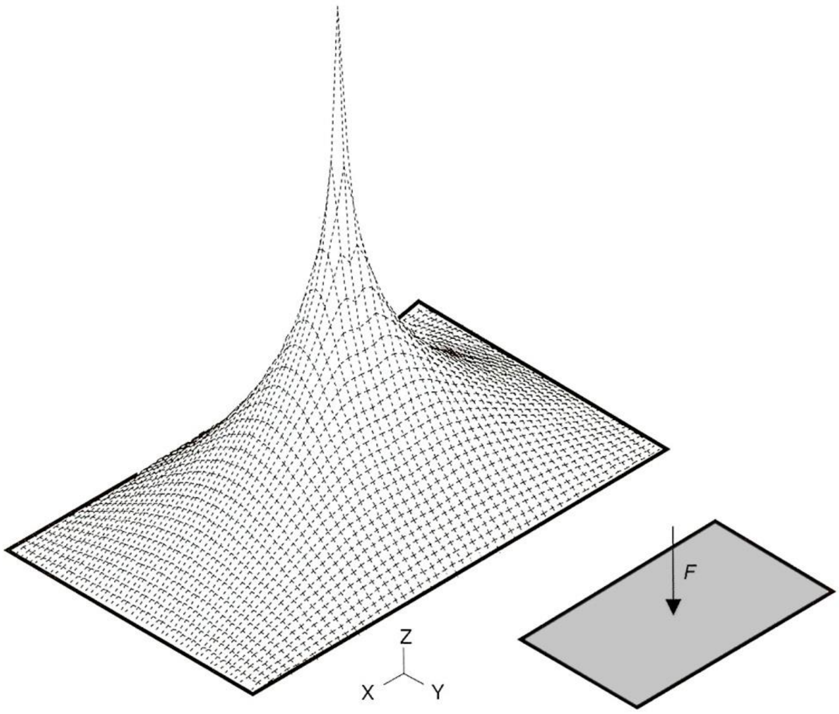


Figure 2-9: Singularity due to a point load (Rombach, 2004)

Flat slab / plate behaviour is 3 dimensional and much more difficult to analyse. It is widely accepted that these singularities in a flat slab analysis do not need to be considered in design. However, if this is assumed then it is not clear to what extent a peak value obtained from a simplified FE model may be smoothed or averaged in a two way spanning slab.

2.4.7 REDUCED STIFFNES DUE TO CRACKING OF CONCRETE

When concrete cracks the stiffness of the section is reduced. Branson (1963 and 1977) proposed an effective secant stiffness I_{eff} from experimental studies on short term deflections:

$$I_{eff} = \left(\frac{M_{cr}}{M}\right)^3 I_{co} + \left(1 - \frac{M_{cr}}{M}\right)^3 I_{cr} \qquad (2)$$

M = applied moment

M_{cr} = moment at first cracking

I_{co} = second moment of area of the un-cracked transformed section

I_{cr} = second moment of area of the cracked transformed section

I_{eff} = effective secant stiffness

The cracking moment is given by:

$$M_{cr} = \frac{f_r I_{co}}{y_t} \qquad (3)$$

f_r = tensile strength of the concrete (modulus of rupture)

y_t = distance from centroidal axis to the extreme tension fibre

2.5 SUMMARY

In conclusion, the FE method looks set to become the preferred analysis method for flat slab design in the industry in the near future, however, a finite element analysis is only as good as:

- The model of the structure
- The assumptions used in the properties for each element
- The representation of the external loads and constraints.
- The interpretation of the results

Eurocode 2 does not prescribe a specific analysis or dictate how to interpret FE method load effects, which are open to a wide range of interpretations depending on how the column supports are modelled. Most commonly used FE packages give no clear directive on how to detail the reinforcement for flat slabs designed using FE. The most useful recommendation on how to use an FE analysis result to detail a concrete flat slab was obtained from Brooker's (2006), all other sources were very vague on this point.

In general it is accepted that the design engineer will use the required reinforcement contour plots to decide how to place the slab reinforcement. It is, however, obvious that if the FE reinforcement contours are followed exactly this would lead to a very impractical reinforcement layout.

From the above it is clear that further research into the practical application and interpretation of linear FEM analysis results for design and detailing of flat plate type structures is needed.

3 SQUARE SINGLE COLUMN FOUNDATION

The design of a single column square pad foundation was undertaken using the traditional design methods (SD), and then compared to the results of a linear elastic FE analysis, and a simplified non-linear FE analysis. The moment variation at the critical design section (i.e. at the face of the column) for the methods of analysis were compared. Different support models were also considered for the linear FE analysis, and the effect on the peak FE and total FE moments noted.

This footing was then constructed with two different reinforcement layouts, according to the SD and FE analysis, and each was loaded to failure.

3.1 ANALYSIS OF PAD FOUNDATION

The analysis of a reinforced concrete pad foundation is a multi-parameter problem. The stiffness of the flat slab is significantly influenced by the non-linear properties of concrete (i.e. cracking, which in turn influences member forces and deflections), and furthermore there is the added complexity of soil-structure interaction. The deformation characteristics of the soil can play a significant role in the distribution of the pressure and hence the load effect. For this study, a conservative uniform bearing pressure under the pad foundation was assumed, with the column acting as the support.

A square spread footing supporting one column, acts as a double cantilever, which is statically determinate with respect to the total moment. Figure 3-1 and Figure 3-2 show the foundation dimensions and the analysis parameters are summarized in Table 3-1.

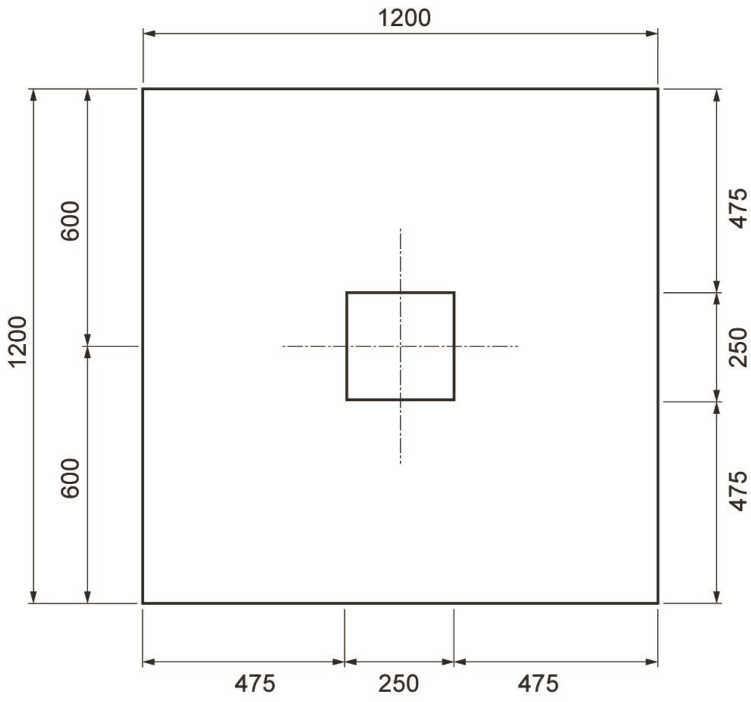


Figure 3-1: Pad foundation plan dimensions

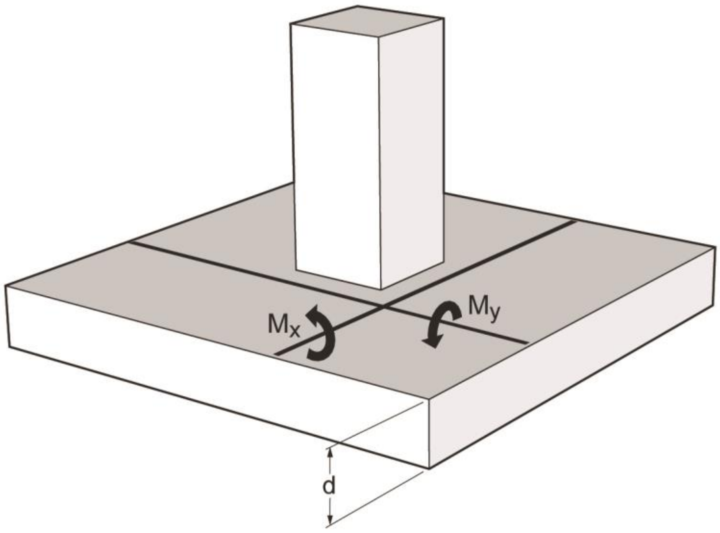


Figure 3-2: Three dimensional pad foundation model showing moment sign convention

Item	Description
Flat plate type	Pad foundation
Plan dimensions	1.2m x 1.2m
Thickness, h	0.15m
Concrete strength, f_{cu}	36.7MPa
Yield stress of reinforcement, f_y	450MPa
Reinforcement (high tensile)	Y8
Concrete Modulus of Elasticity, E_c	33.1GPa
Reinforcing Modulus of Elasticity, E_s	200 GPa
Concrete tensile strength, f_r	4.9MPa
Design uniformly distributed load, w	220kPa
Design point load applied to column	318kN

Table 3-1: Pad foundation analysis parameters

3.2 TRADITIONAL FOOTING DESIGN METHOD

The conventional flat slab / pad foundation design method described in the South African bridge code TMH7 (1989) was used for the simplified method of design. The critical section for the design of the flexural hogging reinforcement in the x and y directions was taken at the face of the column. If the width of the pad foundation is greater than $1.5(b_{col} + 3h)$ the code requires the slab to be split into column and middle strips, and designed and detailed accordingly, where b_{col} is the column width and d the effective depth to the tension reinforcement of the slab. The width of the column strip is thus governed by the width of the column and the depth of the slab using the equation $W_{col} = b_{col} + 3h$. The column strip was then designed to resist two thirds of the total bending moment, and the middle strip was designed to resist one third of the total bending moment.

The total SD design moment is taken at the edge of the column shown in Figure 3-3 is calculated as follows:

$$M = \frac{Bwl^2}{2} \qquad (4)$$

M = design bending moment at the face of the column

B = width of the footing

l = cantilever length

The lever arm (x) for the resultant force of a uniformly distributed load under the critical area is half the distance between the critical section and the edge of the footing.

$$x = \frac{D - b_{col}}{2} \qquad (5)$$

x = lever arm of the resultant force

D = length of footing

b_{col} = width of column support

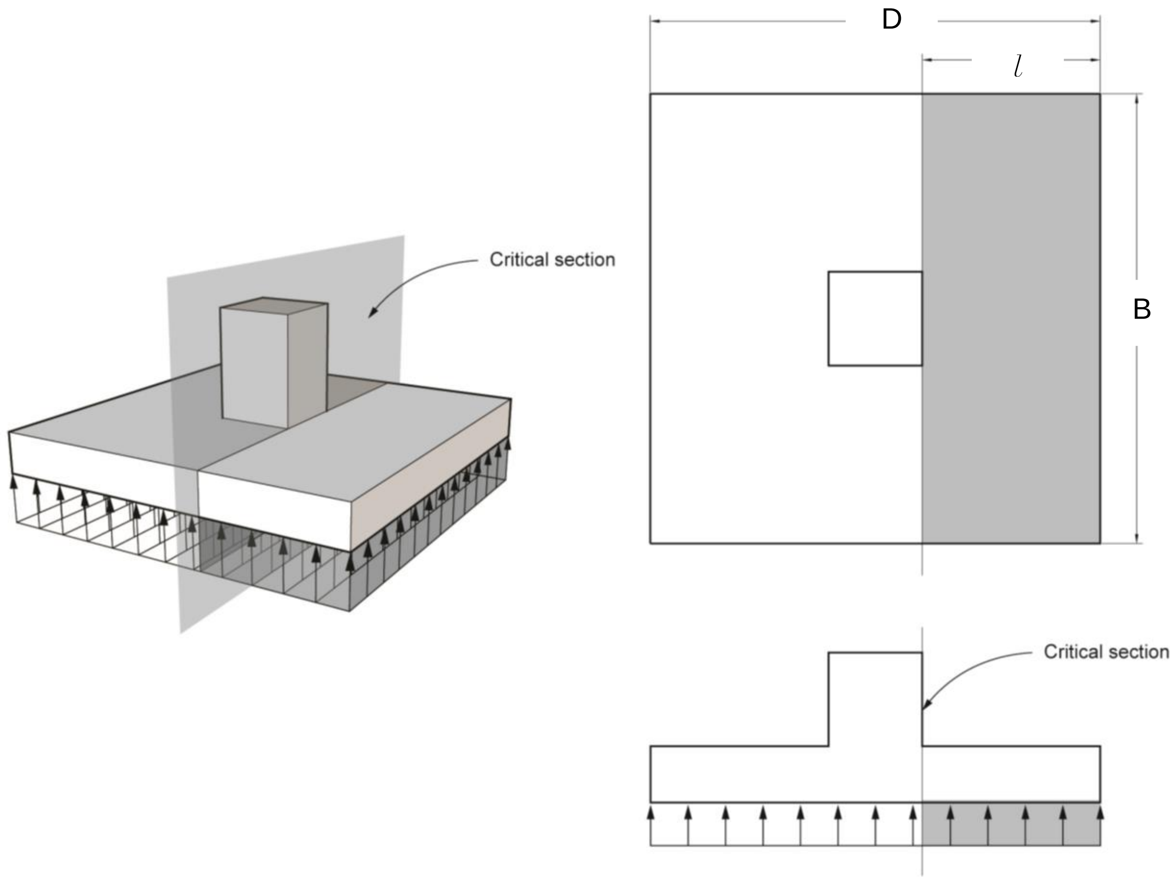


Figure 3-3: Critical section for the determination of the flexural design moment

3.3 FINITE ELEMENT FOOTING DESIGN METHOD

The pad foundation was modelled using the linear elastic FE program Prokon (2011), which is available to the majority of designers in South Africa. The model as shown in Figure 3-4 consisted of square 0.025m x 0.025m plate elements, keeping within the bounds of the standard element size as prescribed by Brooker (2006). The elements used to analyse the pad foundations are discreet Kirchoff-Mindlin quadrilaterals which provides good results for both thick and thin plates and are free from shear locking. Shear deformations were not considered here in order to be consistent with the code requirements. The author notes that shear strain and deformation should be considered in thick pad foundations.

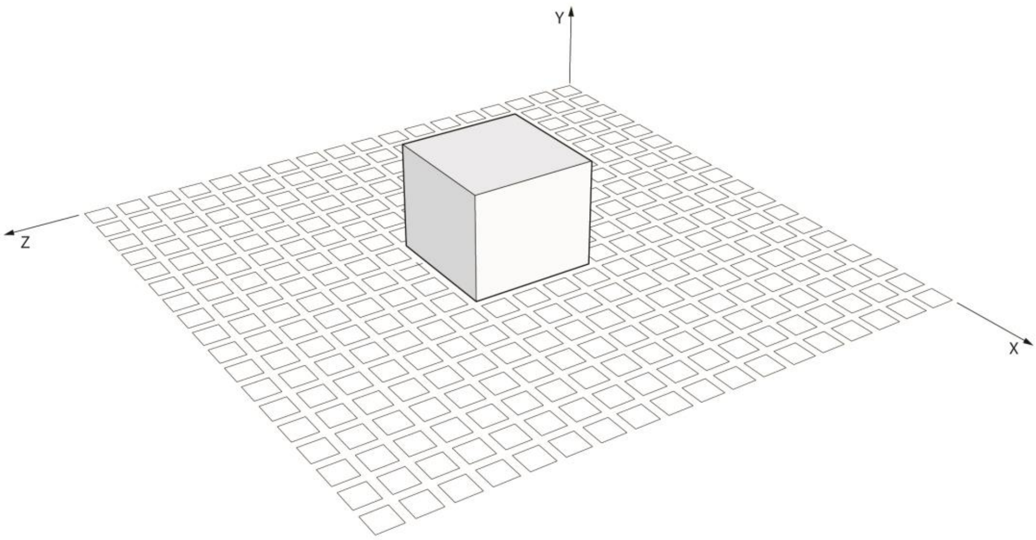
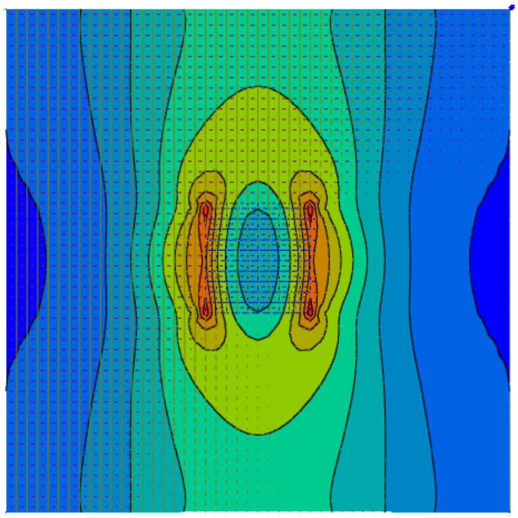


Figure 3-4: Finite element pad foundation model - slab/column modelled with solid elements

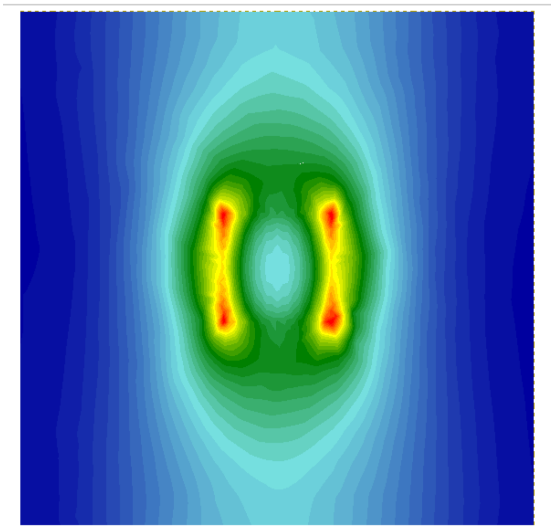
A German FE modelling package, SOFiSTK, was used to check the Prokon results, and similar moment distributions were obtained. It seemed reasonable to do the linear FE analysis on a package that is used in the South African industry today, therefore all the linear FEM analyses where done using Prokon 2012. Figure 3-5 shows the M_y axis moment plot for a footing modelled with rigid supports in the footprint of the column. The placement of the pinned supports was the same for both models. The results show very similar peak moments at the column edges, the vertical displacement at the edges of the footing and moment contours.



Peak moment at column corners: 43.9kNm/m

Max deflection at edges: 0.59mm

Prokon Analysis



Peak moment at column corners: 46.2kNm/m

Max deflection at edges: 0.577mm

Sofistik Analysis

Figure 3-5: Comparison between Prokon and Sofistik Analysis

The square footing was analysed using the different support conditions discussed in Section 2.4.5 and the following noted for each:

- peak M_y axis moment (FE M_{peak})
- peak Wood and Armer M_y moment (FE W&A M_{peak})
- total M_y axis moment - the sum under the moment curve at the face of the column. (FE M_{total})
- total Wood and Armer M_y moment - the sum under the moment curve at the face of the column (FE W&A M_{total})

The peak moment, FE M_{peak} is defined as the maximum FE hogging moment and is affected by the curvature of the pad foundation. The total FE moment is defined as the sum under the moment curve at the face of the column. See Appendix A for calculation details.

These peak and total moments were then compared to the SD column strip (SD M_{col}) and total moments (SD M_{total}) as well as the concentrated SD column moment where two thirds of the column strip moment is concentrated into an inner column strip. The applied load corresponded to a typical pressure under a footing founded on dense sand (SABS 0160, 1980)

A summary of the results is as follows:

- For each different support model the total FE M_x or M_y moments were the same as the total SD moment.
- The total FE Wood and Armer moment were greater than the SD moment (up to 20% more) because Wood and Armer moments are design moments, which include the M_{xy} twisting moment.
- The peak FE Wood and Armer moments were higher than the peak FE M_x or M_y moments because of the M_{xy} twisting moment.
- The M_{xy} twisting moment were significantly affected by how the supports are modelled.
- Different supports (constraint) conditions caused the FE M_x or M_y peak moment to vary by as much as 36%.
- Different supports (constraint) conditions caused the FE Wood and Armer peak moment to vary by as much as 88%.
- The FE peak M_x or M_y moment can be more than double the SD column strip moment, depending on the support model.
- The column strip moment approaches the FE peak moment if the concentrated column strip detailing rules specified in SANS 10100 are used.
- The most realistic moment distribution through the footing were obtained from modelling the columns support as a 3D continuum and modelling the edge of the column with springs. The peak moment is within 5% of the inner column strip stepped SD method moment.
- For both spring models different the sensitivity of the model to support models was checked. In general a more realistic moment distribution was obtained as the stiffness of the spring decreased.

- Ignoring the stiffness of the column, and modelling the support as pinned over the footprint of the column shows reverse curvature in the column area (see Figure 3-6). This reduction in moment over the column could be attributed to the fact that fixing the translational degrees of freedom on the column footprint prevents the movement at the nodes, curvature within the element still occurs and therefore a reduced moment over the footprint of the column is observed. If the rotational degrees of freedom are also fixed in the column footprint the moment over the column reduces to almost zero, which in reality is impossible.

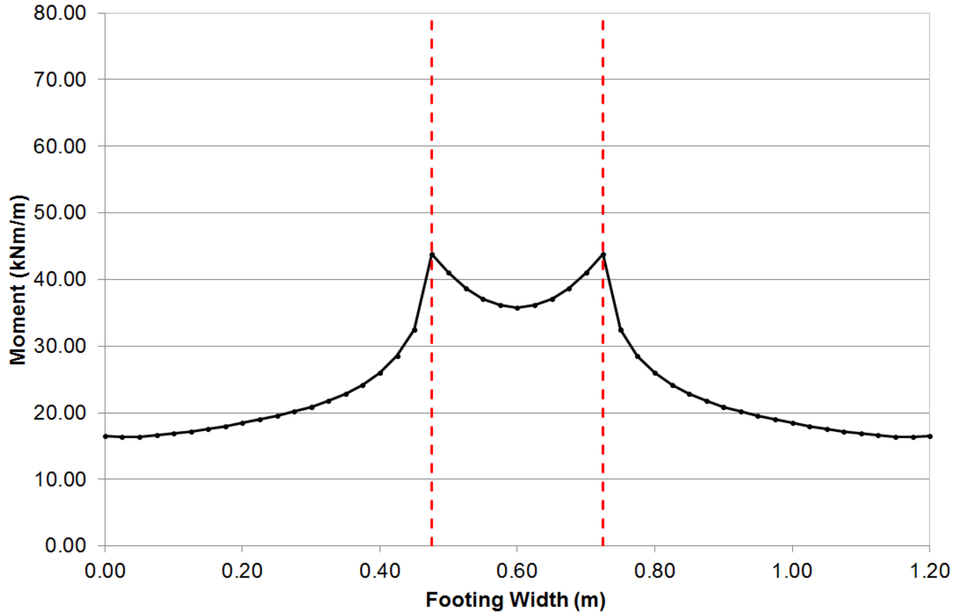
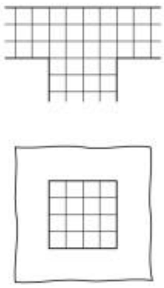


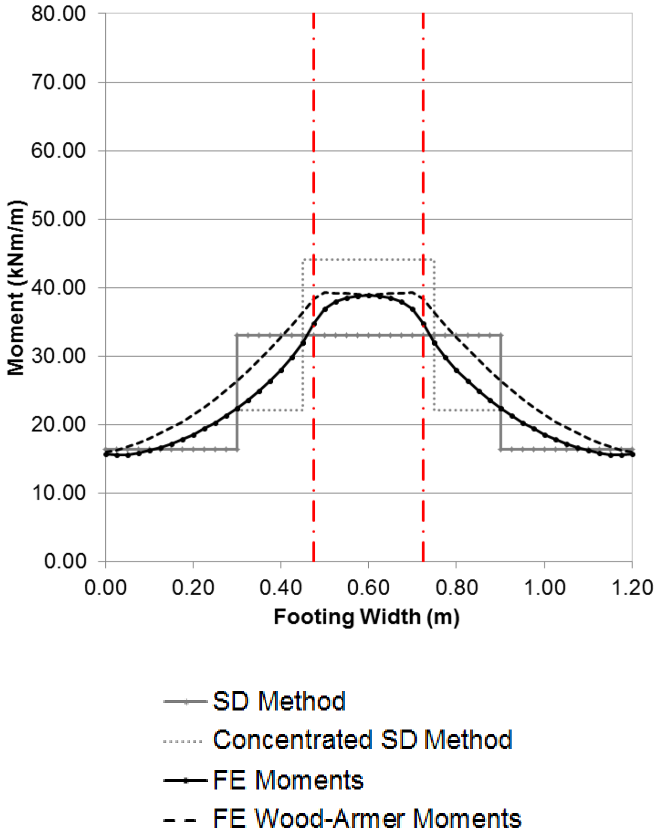
Figure 3-6: FE Bending moment distribution showing reverse curvature over the column width when the translation degrees of freedom were fixed over the footprint of the column

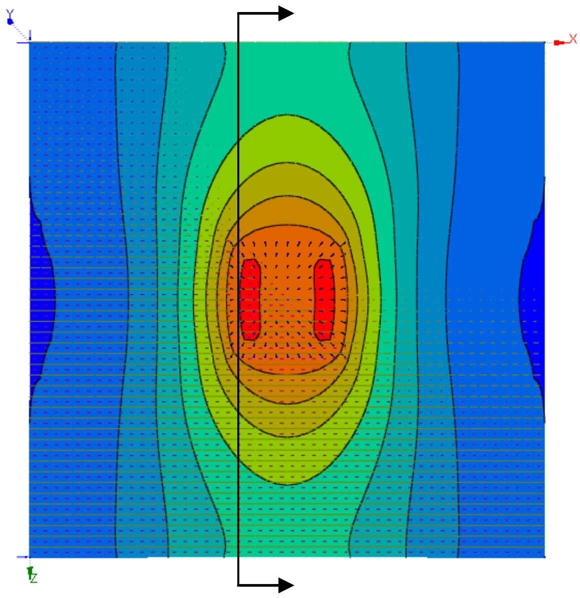
Support type: Full 3D continuum model

Footing analysis	Moment
FE M_{peak} (M_x or M_y)	38.86 kNm/m
FE W&A M_{peak}	39.24 kNm/m
SD M_{col}	33.1 kNm/m
Concentrated SD M_{col}	44.12 kNm/m
FE M_{peak} / SD M_{col}	1.17
FE M_{peak} / Concentrated SD M_{col}	0.88
SD M_{total}	29.69 kNm
FE W&A M_{total}	33.03 kNm
FE M_{total}	29.78 kNm
FE W&A M_{total} / SD M_{total}	1.11
FE M_{total} / SD M_{total}	1.003



Support model



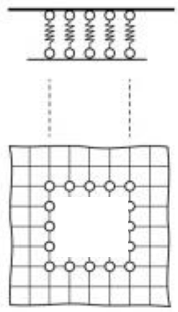


FE M_x or M_y Moments

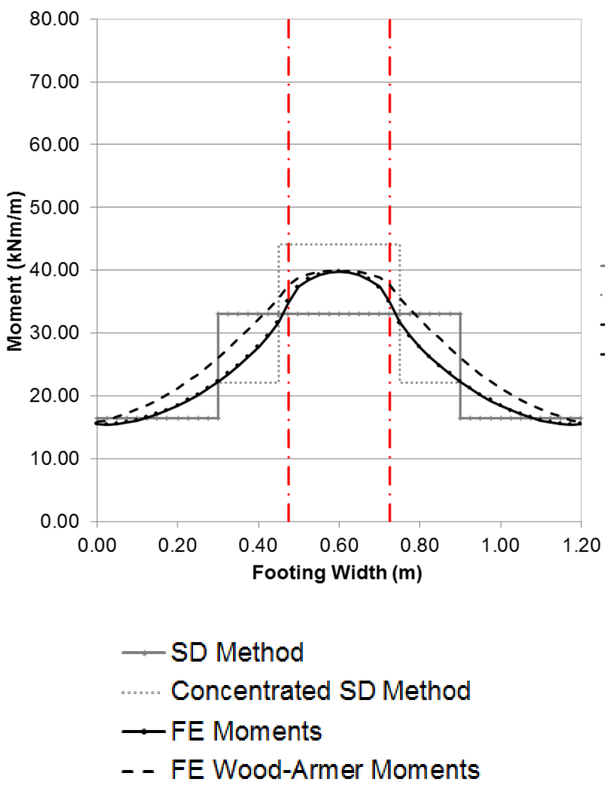
- The total FE M_x or M_y moments were the same as the total SD moment.
- The total FE W&A moment was 11% greater than the total SD moment.
- The peak FE Wood and Armer moment was higher than the peak FE M_x or M_y moment.
- The peak FE M_x or M_y moment was 17% greater than the SD column strip moment.
- The peak FE M_x or M_y moment was 12% less than the concentrated SD column strip moment.

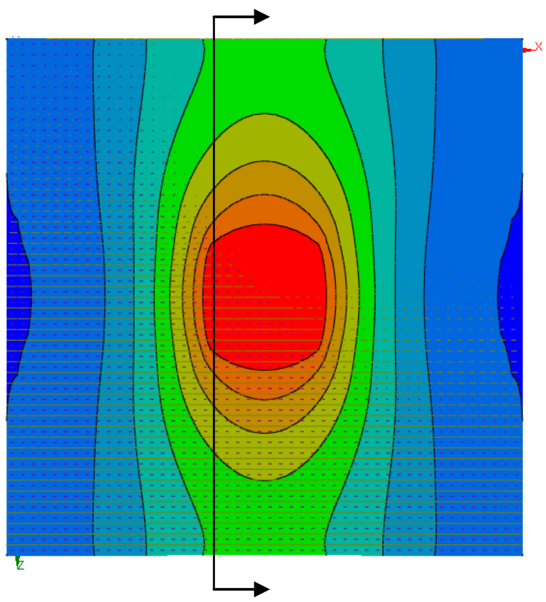
Support type: Spring support to column edges

Footing analysis	Moment
FE M_{peak} (M_x or M_y)	39.74 kNm/m
FE W&A M_{peak}	39.75 kNm/m
SD M_{col}	33.1 kNm/m
Concentrated SD M_{col}	44.12 kNm/m
FE M_{peak} / SD M_{col}	1.2
FE M_{peak} / Concentrated SD M_{col}	0.9
SD M_{total}	29.69 kNm
FE W&A M_{total}	32.79 kNm
FE M_{total}	29.78 kNm
FE W&A M_{total} / SD M_{total}	1.10
FE M_{total} / SD M_{total}	1.003



Support model; k = 50000kN/m



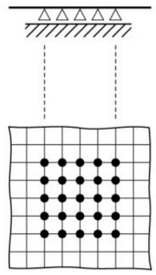


FE M_x or M_y Moments

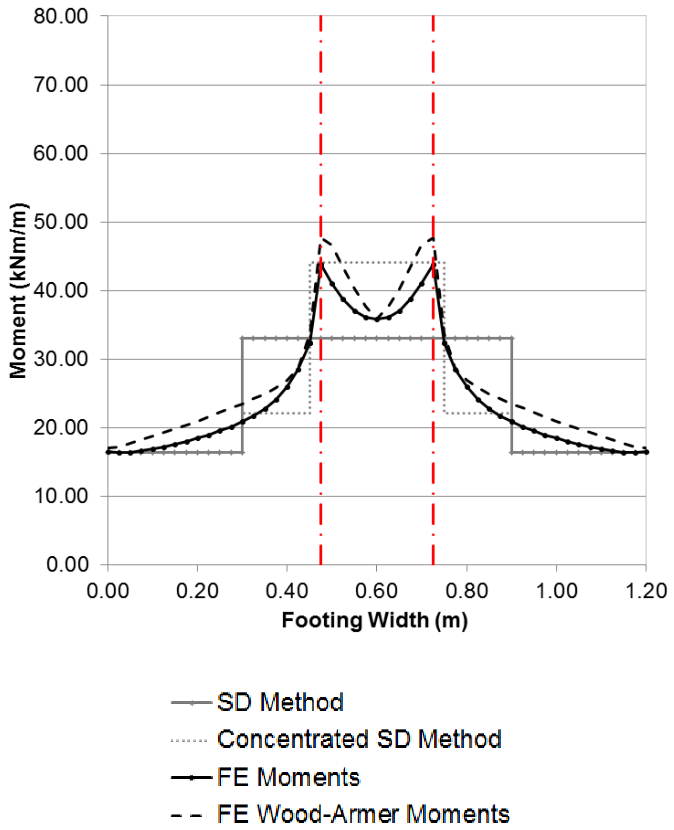
- The total FE M_x or M_y moments were the same as the total SD moment.
- The total FE W&A moment was 10% greater than the total SD moment.
- The peak FE Wood and Armer moment was higher than the peak FE M_x or M_y moment.
- The peak FE M_x or M_y moment was 20% greater than the SD column strip moment.
- The peak FE M_x or M_y moment was 10% less than the concentrated SD column strip moment.

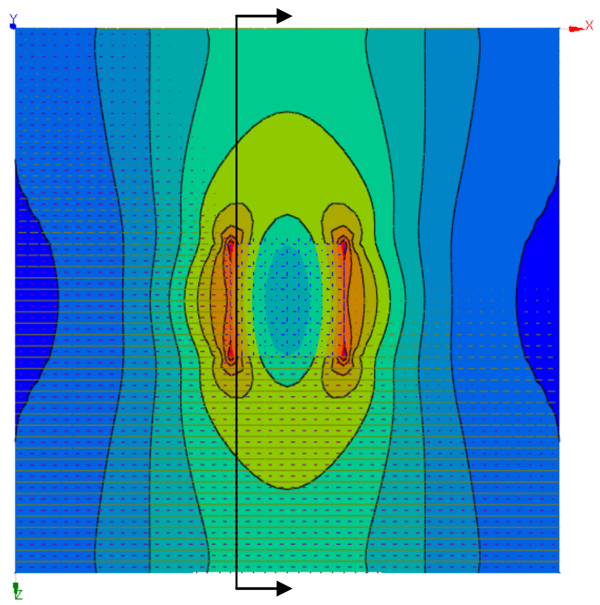
Support type: Rigid supports

Footing analysis	Moment
FE M_{peak} (M_x or M_y)	43.87 kNm/m
FE W&A M_{peak}	47.74 kNm/m
SD M_{col}	33.1 kNm/m
Concentrated SD M_{col}	44.12 kNm/m
FE M_{peak} / SD M_{col}	1.325
FE M_{peak} / Concentrated SD M_{col}	0.99
SD M_{total}	29.69 kNm
FE W&A M_{total}	32.44 kNm
FE M_{total}	29.78 kNm
FE W&A M_{total} / SD M_{total}	1.11
FE M_{total} / SD M_{total}	1.003



Support model



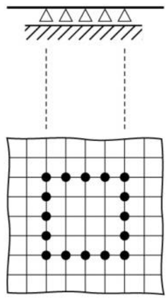


FE M_x or M_y Moments

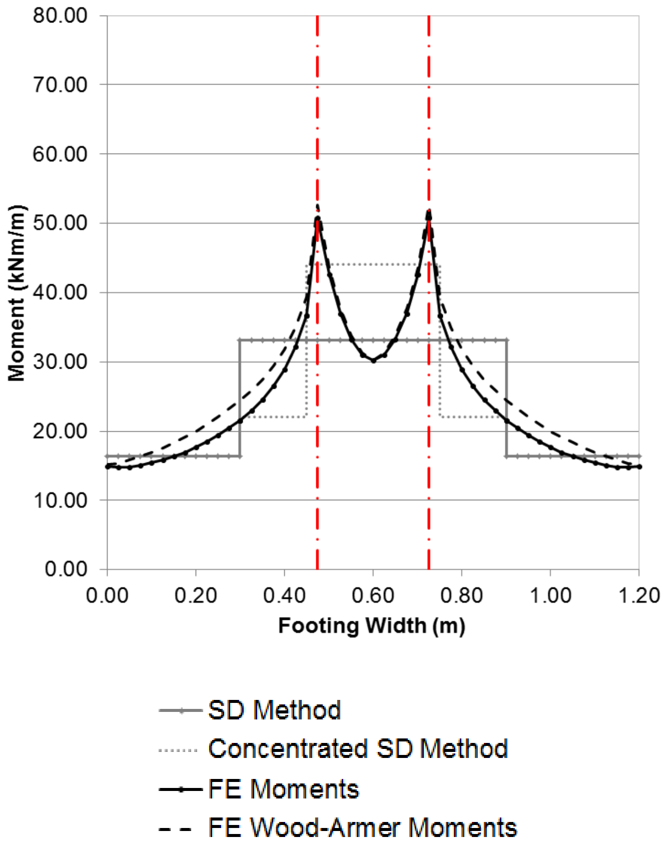
- The total FE M_x or M_y moments were the same as the total SD moment.
- The total FE W&A moment was 11% greater than the total SD moment.
- The peak FE Wood and Armer moment was higher than the peak FE M_x or M_y moment.
- The peak FE M_x or M_y moment was 32.5% greater than the SD column strip moment.
- The peak FE M_x or M_y moment was 1% less than the concentrated SD column strip moment.

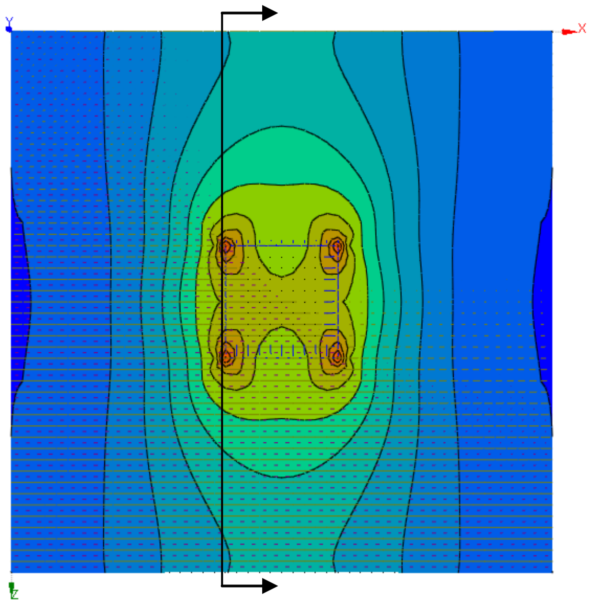
Support type: Rigid supports (edges)

Footing analysis	Moment
FE M_{peak} (M_x or M_y)	50.8 kNm/m
FE W&A M_{peak}	52.6 kNm/m
SD M_{col}	33.1 kNm/m
Concentrated SD M_{col}	44.12 kNm/m
FE M_{peak} / SD M_{col}	1.53
FE M_{peak} / Concentrated SD M_{col}	1.15
SD M_{total}	29.69 kNm
FE W&A M_{total}	32.03 kNm
FE M_{total}	29.78 kNm
FE W&A M_{total} / SD M_{total}	1.08
FE M_{total} / SD M_{total}	1.003



Support model



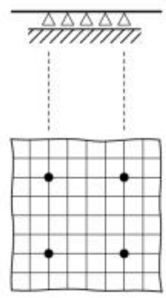


FE M_x or M_y Moments

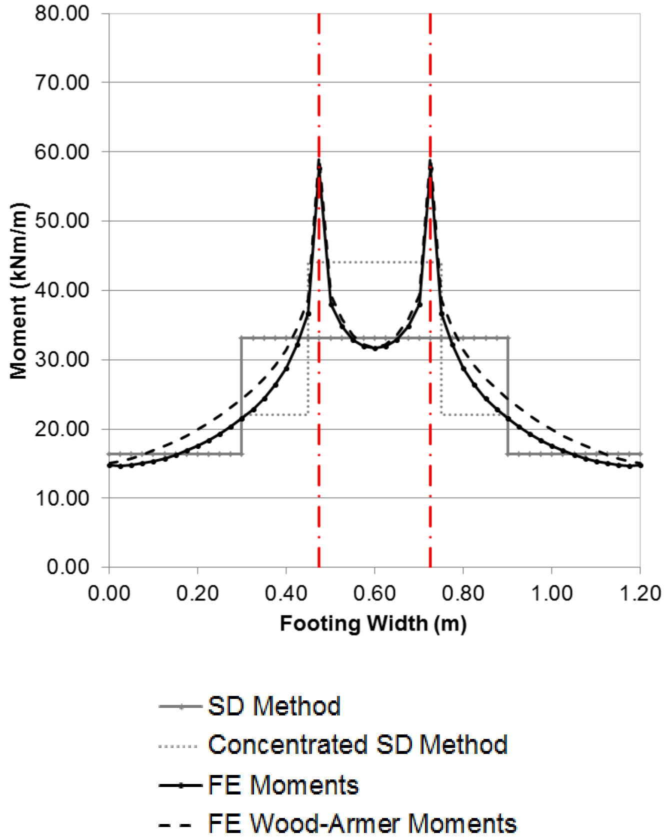
- The total FE M_x or M_y moments were the same as the total SD moment.
- The total FE W&A moment were 8% greater than the total SD moment.
- The peak FE Wood and Armer moment were higher than the peak FE M_x or M_y moment.
- The peak FE M_x or M_y moment was 53% greater than the SD column strip moment.
- The peak FE M_x or M_y moment was 15% greater than the concentrated SD column strip moment.

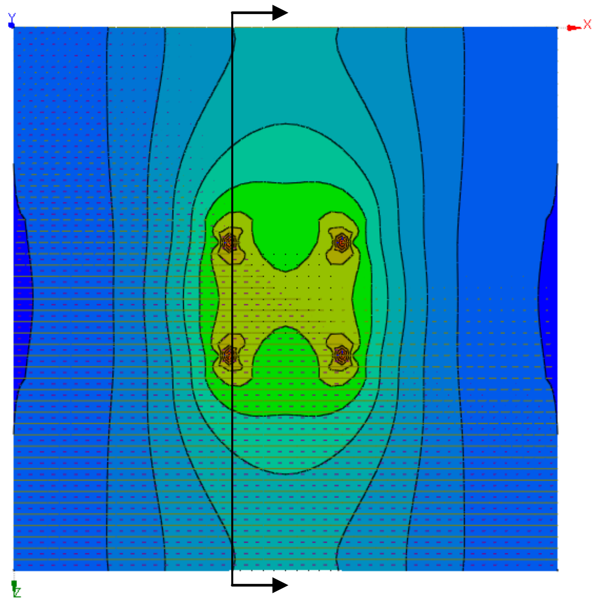
Support type: Rigid supports (corners)

Footing analysis	Moment
FE M_{peak} (M_x or M_y)	57.6 kNm/m
FE W&A M_{peak}	59.4 kNm/m
SD M_{col}	33.1 kNm/m
Concentrated SD M_{col}	44.12 kNm/m
FE M_{peak} / SD M_{col}	1.74
FE M_{peak} / Concentrated SD M_{col}	1.31
SD M_{total}	29.68 kNm
FE W&A M_{total}	31.98 kNm
FE M_{total}	29.78 kNm
FE W&A M_{total} / SD M_{total}	1.08
FE M_{total} / SD M_{total}	1.003



Support model



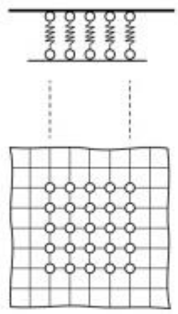


FE M_x or M_y Moments

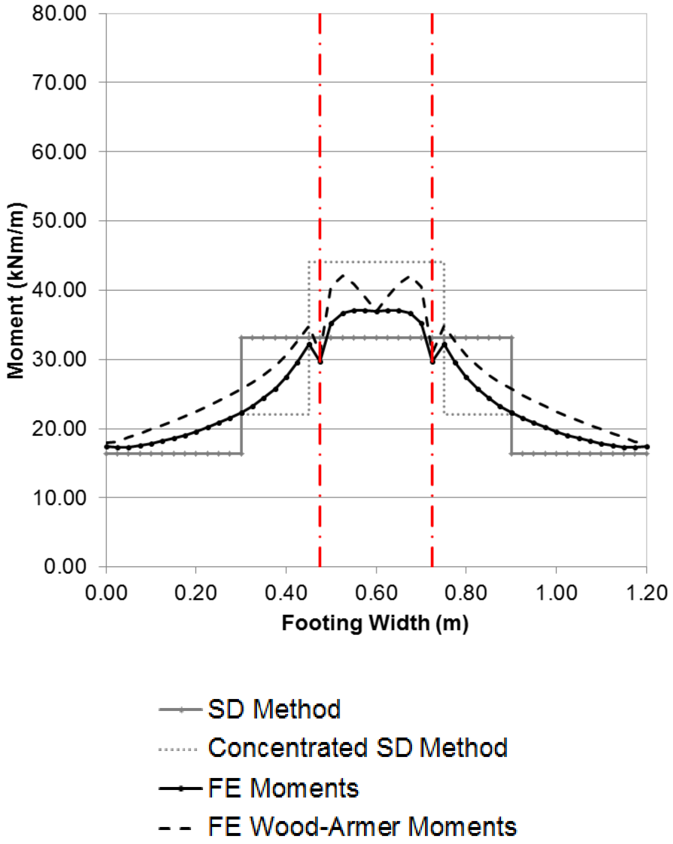
- The total FE M_x or M_y moments were the same as the total SD moment.
- The total FE W&A moment were 8% greater than the total SD moment.
- The peak FE Wood and Armer moment were higher than the peak FE M_x or M_y moment.
- The peak FE M_x or M_y moment was 74% greater than the SD column strip moment.
- The peak FE M_x or M_y moment was 31% greater than the concentrated SD column strip moment.

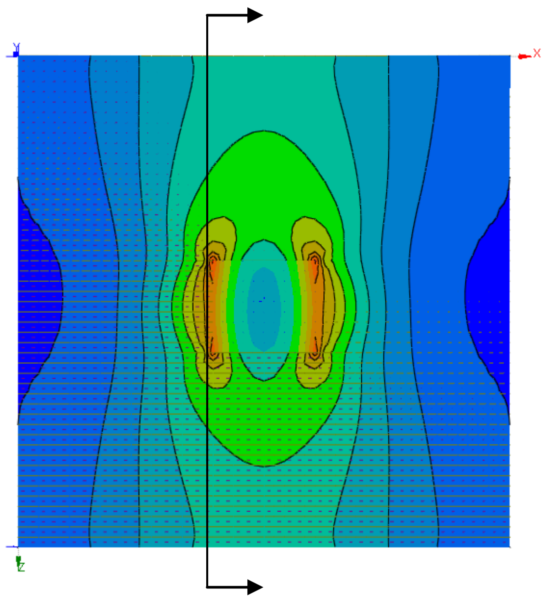
Support type: Springs

Footing analysis	Moment
FE M_{peak} (M_x or M_y)	37.05kNm/m
FE W&A M_{peak}	42.08 kNm/m
SD M_{col}	33.1 kNm/m
Concentrated SD M_{col}	44.12 kNm/m
FE M_{peak} / SD M_{col}	1.12
FE M_{peak} / Concentrated SD M_{col}	0.84
SD M_{total}	29.69 kNm
FE W&A M_{total}	33.10 kNm
FE M_{total}	29.78 kNm
FE W&A M_{total} / SD M_{total}	1.12
FE M_{total} / SD M_{total}	1.003



Support model; k = 5000kN/m



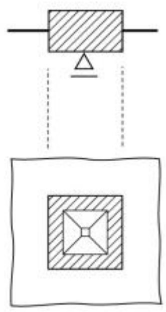


FE M_x or M_y Moments

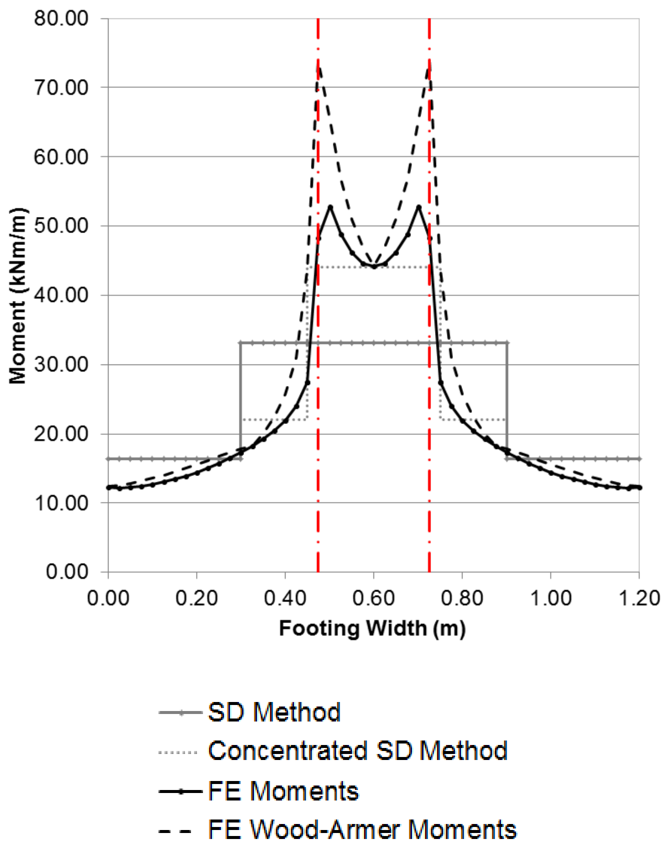
- The total FE M_x or M_y moments were the same as the total SD moment.
- The total FE W&A moment was 12% greater than the total SD moment.
- The peak FE Wood and Armer moment was higher than the peak FE M_x or M_y moment.
- The peak FE M_x or M_y moment was 12% greater than the SD column strip moment.
- The peak FE M_x or M_y moment was 16% less than the concentrated SD column strip moment.

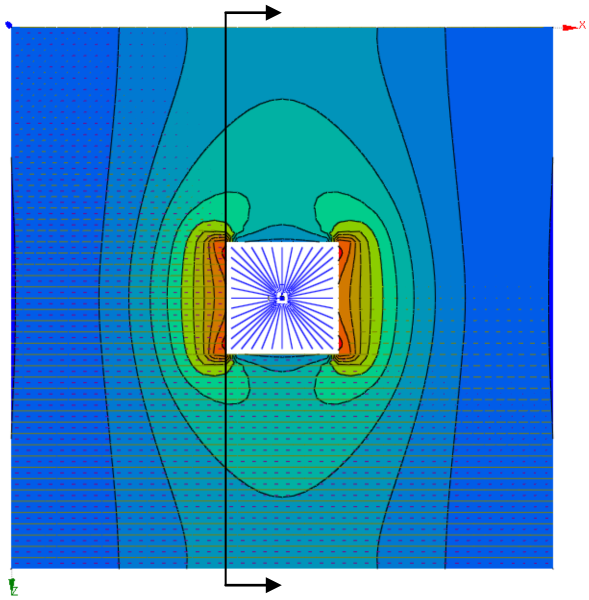
Support type: Rigid links

Footing analysis	Moment
FE M_{peak} (M_x or M_y)	52.85 kNm/m
FE W&A M_{peak}	73.84 kNm/m
SD M_{col}	33.1 kNm/m
Concentrated SD M_{col}	44.12 kNm/m
FE M_{peak} / SD M_{col}	1.60
FE M_{peak} / Concentrated SD M_{col}	1.20
SD M_{total}	29.69 kNm
FE W&A M_{total}	33.12 kNm
FE M_{total}	28.47 kNm
FE W&A M_{total} / SD M_{total}	1.12
FE M_{total} / SD M_{total}	0.96



Support model



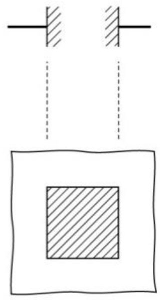


FE M_x or M_y Moments

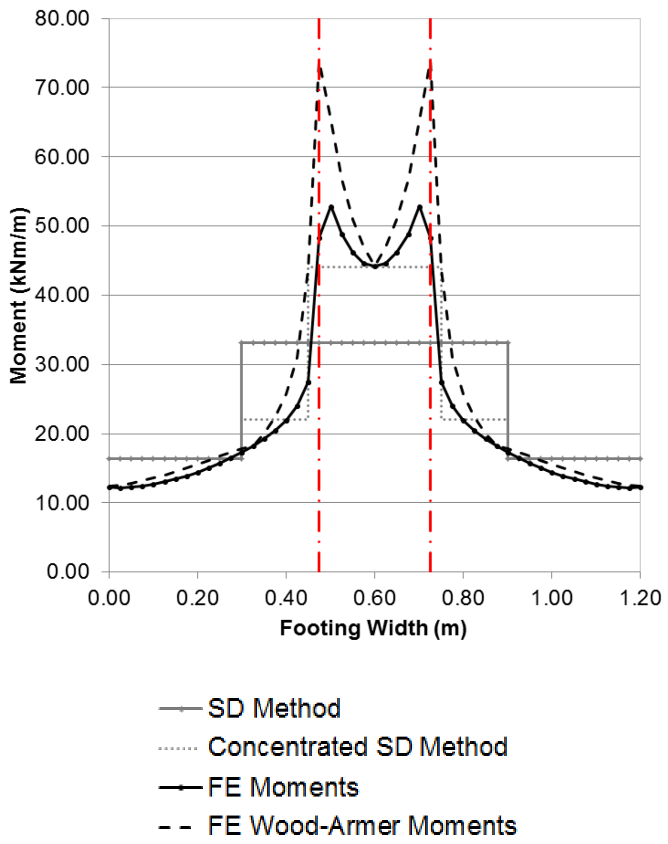
- The total FE M_x or M_y moments were the same as the total SD moment.
- The total FE W&A moment were 12% greater than the total SD moment.
- The peak FE Wood and Armer moment were higher than the peak FE M_x or M_y moment.
- The peak FE M_x or M_y moment was 60% greater than the SD column strip moment.
- The peak FE M_x or M_y moment was 20% greater than the concentrated SD column strip moment.

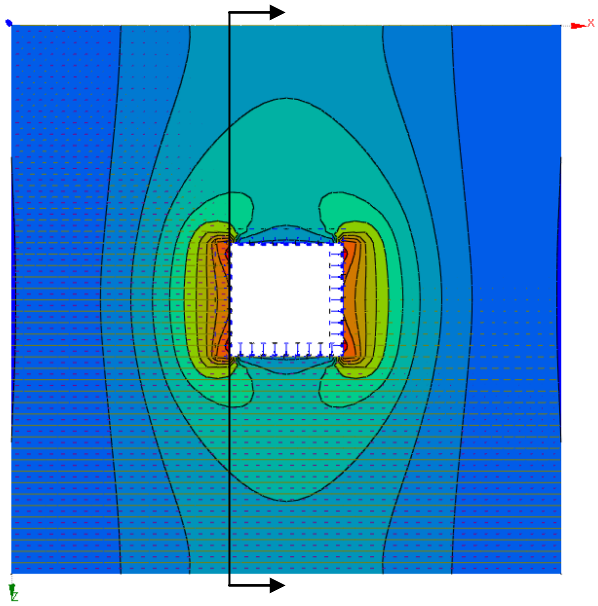
Support type: Encased support

Footing analysis	Moment
FE M_{peak} (M_x or M_y)	52.85 kNm/m
FE W&A M_{peak}	73.84 kNm/m
SD M_{col}	33.1 kNm/m
Concentrated SD M_{col}	44.12 kNm/m
FE M_{peak} / SD M_{col}	1.60
FE M_{peak} / Concentrated SD M_{col}	1.20
SD M_{total}	29.69 kNm
FE W&A M_{total}	33.12 kNm
FE M_{total}	28.47 kNm
FE W&A M_{total} / SD M_{total}	1.12
FE M_{total} / SD M_{total}	0.96



Support model



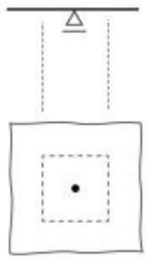


FE M_x or M_y Moments

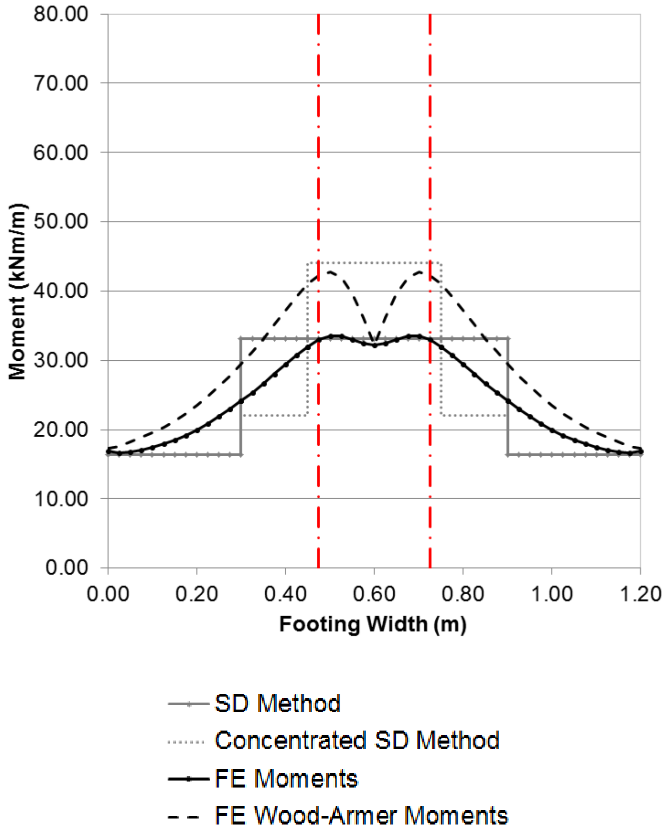
- The total FE M_x or M_y moments were the same as the total SD moment.
- The total FE W&A moment were 12% greater than the total SD moment.
- The peak FE Wood and Armer moment were higher than the peak FE M_x or M_y moment.
- The peak FE M_x or M_y moment was 60% greater than the SD column strip moment.
- The peak FE M_x or M_y moment was 20% greater than the concentrated SD column strip moment.

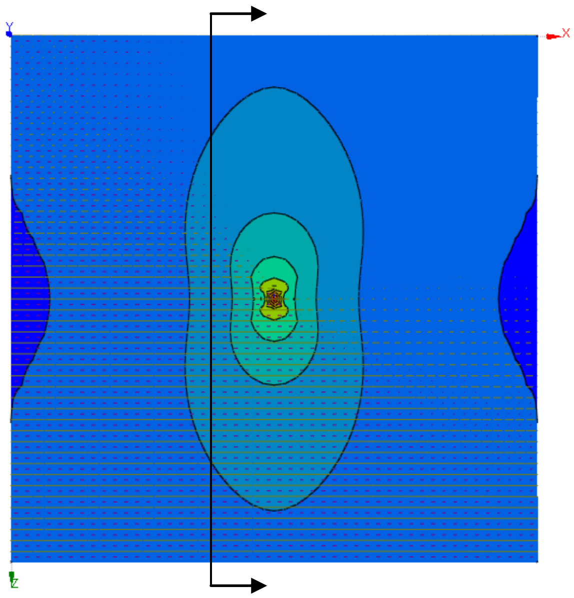
Support type: Point support

Footing analysis	Moment
FE M_{peak} (M_x or M_y)	33.48 kNm/m
FE W&A M_{peak}	42.74 kNm/m
SD M_{col}	33.1 kNm/m
Concentrated SD M_{col}	44.12 kNm/m
FE M_{peak} / SD M_{col}	1.01
FE M_{peak} / Concentrated SD M_{col}	0.76
SD M_{total}	29.69 kNm
FE W&A M_{total}	35.72 kNm
FE M_{total}	29.78 kNm
FE W&A M_{total} / SD M_{total}	1.20
FE M_{total} / SD M_{total}	1.003



Support model





FE M_x or M_y Moments

- The total FE M_x or M_y moments were the same as the total SD moment.
- The total FE W&A moment was 20% greater than the total SD moment.
- The peak FE Wood and Armer moment was higher than the peak FE M_x or M_y moment.
- The peak FE M_x or M_y moment was 1% greater than the SD column strip moment.
- The peak FE M_x or M_y moment was 24% less than the concentrated SD column strip moment.

3.4 DESIGN MOMENTS

The pad foundation requires bottom reinforcement in the transverse (x) and longitudinal (y) directions to resist the M_y and M_x hogging moments respectively.

The Simplified Design, SD, method of analysis results in a constant moment, which is then split into a column strip moment and an edge strip moment for the pad foundation. The column strip can be stepped again by concentrating two thirds of the column strip moment into half of the column strip width to form an inner and outer column strip, according to the detailing rules of SANS 10100. See Figure 3-7.

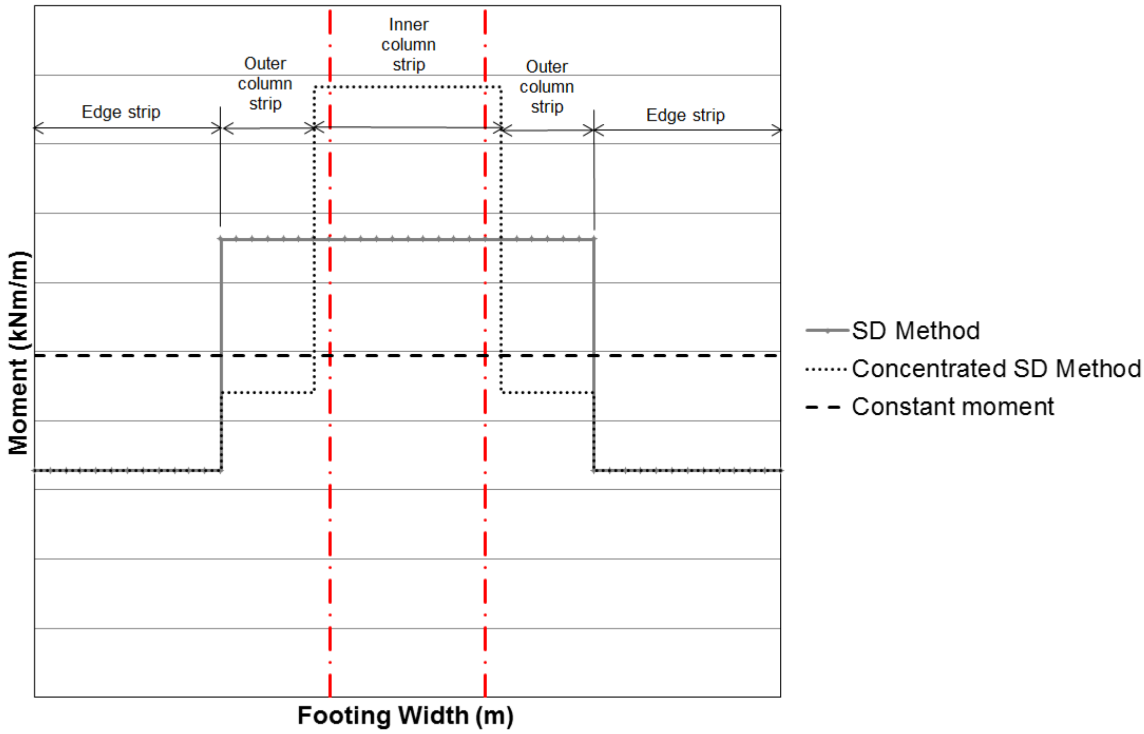


Figure 3-7: Simplified design moment

Considering the results of the FE analyses with different support constraints the supports modelled with full 3D continuum are the most realistic and will be used for comparison with the SD analysis. The linear FE moment outputs are the M_x and M_y axis moments and the commonly used Wood-Armer moments which include the twisting moment M_{xy}. Figure 3-8 shows the FE M_x moment contours for the hogging moments in the pad foundation. Figure 3-9 shows the FE Wood-Armer (W&A) moment contours for the M_{xT} and M_{yT} hogging moments in the pad foundation.

A section taken through the SD and FE bending moment diagrams at the face of the column in the x and y directions is shown in Figure 3-10. Both the peak FE M_y and M_x hogging moments occur at the face of the column as there is a cantilever on both sides.

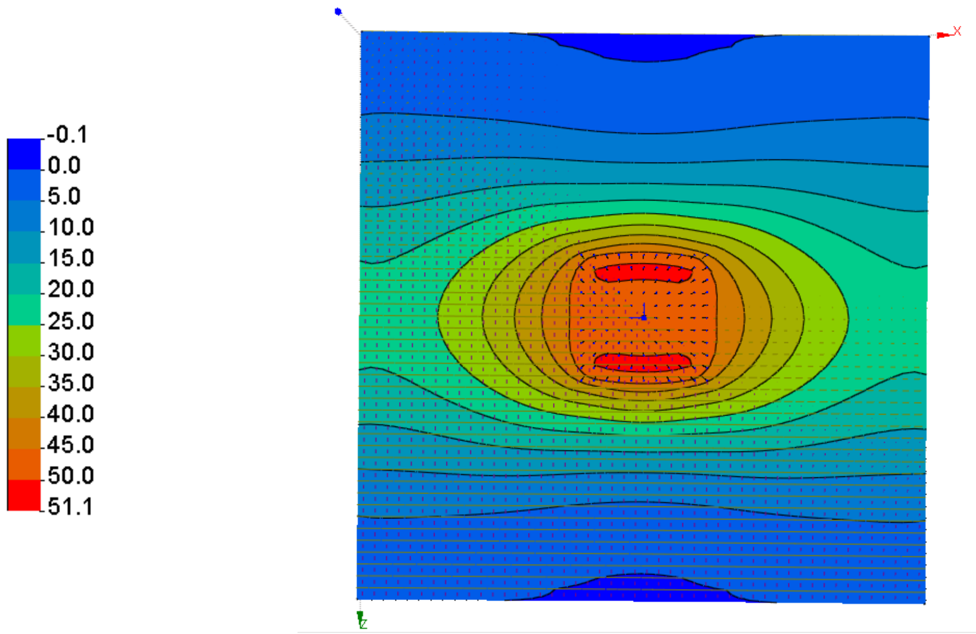


Figure 3-8: M_x moment contours (kNm/m)

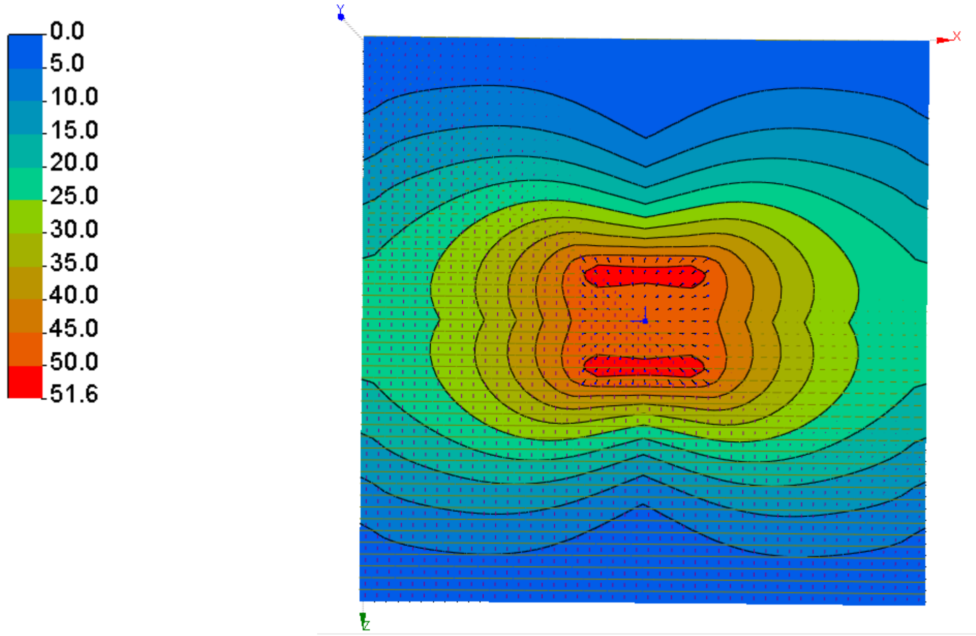


Figure 3-9: Wood and Armer M_x moment contours (kNm/m)

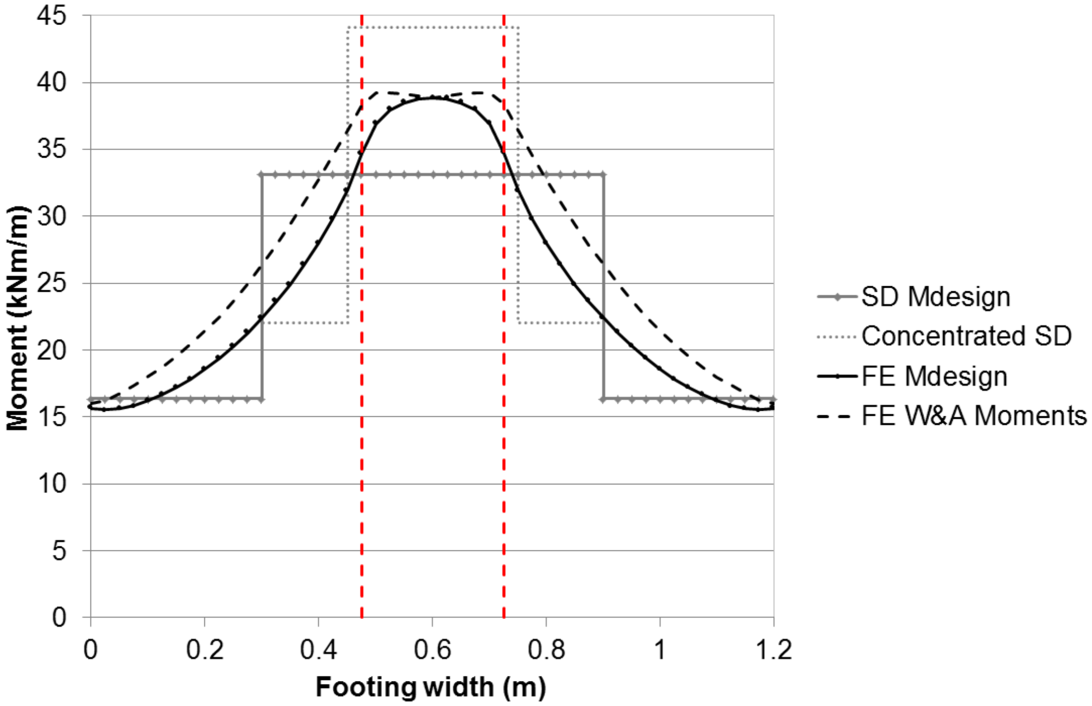


Figure 3-10: Simplified design method moment compared to FE design moments

A section through the FE moment contours shows a realistic moment distribution, increasing to a maximum value at the support. To simplify this for design of flat plates the column strip rules were introduced. The column strip requirement of the simplified design method ensures an increase in the design moment over the column strip as the peak moment in the FE analysis increases. As the FE peak moment increases, the SD column strip reduces, resulting in an increased SD moment.

The integration of the area under the moment diagram gives the total SD and FE load effect. The total SD design moment, and total FE M_x moment are the same. This does not change with geometry or constraint model as the principal of equilibrium has to apply. The total FEM Wood and Armer design moments are, however, greater than the SD design moments. These moments were intended for use in design, and because of the M_{xy} twisting moment and of the unique solution and optimisation requirement the capacity is always greater than the applied moment (Denton & Burgoyne 1996).

3.5 NON LINEAR ANALYSIS

A three-dimensional non-linear analysis on both the SD and FE reinforced concrete footings analyses including the designed reinforcement bars (see Figure 3-15) was attempted. The conversion of the output into bending moments became very tedious, as the output from all non-linear FE programs is in the form of stresses in the x, y and z directions. To design a reinforced concrete structure, moments are required and stresses would need to be integrated over the width of the elements to calculate M_x, M_y and M_z and the twisting moment M_{xy}. This is relatively simple to do for concrete brick elements prior to cracking, but becomes very complex when the concrete cracks (i.e. non-linear behaviour starts) and the strain in the reinforcement starts to increase, as the nodes of the concrete brick elements and rebar truss elements do not generally coincide. This is beyond the scope of this research project, and therefore a simpler non-linear analysis was investigated.

The linear elastic FE model was compared with the results of a linear elastic FE model with reduced stiffness in regions where the moments exceeded the cracking moment (M_{cr}). This was a simplified non-linear analysis used to assess the effects of cracking in regions of high curvature. The variation in moment at the critical design section (i.e. at the face of the column) for both methods of analysis were compared.

The cracking moment was calculated using the method described in Chapter 2.4.7

$$M_{cr} = \frac{f_r I_{co}}{y_t} = 17.45\ kNm/m$$

I_{co} = second moment of area of the un-cracked transformed section = 284,79 x 10^6 mm^2

f_r = tensile strength of the concrete = 4.57MPa (concrete tested in Section 3.6)

y_t = distance from centroidal axis to the extreme tension fibre = 75.4mm

The pad foundation was again modelled with plate (shell) elements using a linear elastic FE analysis. The advantage of using plate elements as opposed to three dimensional brick elements was that the moments were calculated directly from the stiffness matrix and defection at the nodes.

The column load associated with the cracking moment was 225kN, and the corresponding uniformly distributed load 156.25kPa. The linear FE moment distribution at this load was observed and all areas where the cracking moment was exceeded were noted (Figure 3-11a and Figure 3-11b).

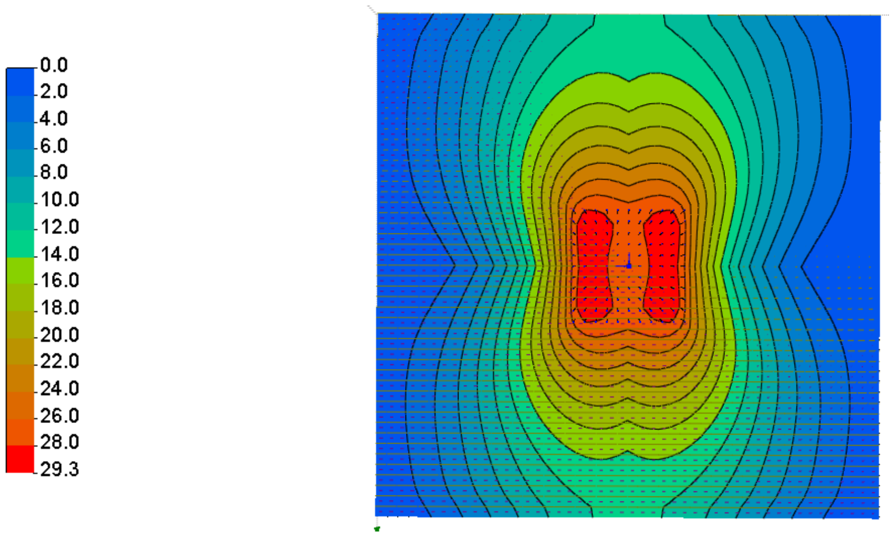


Figure 3-11a: Wood and Armer M_x moment contours (kNm/m) showing where FE moment exceed M_{cr} in one direction

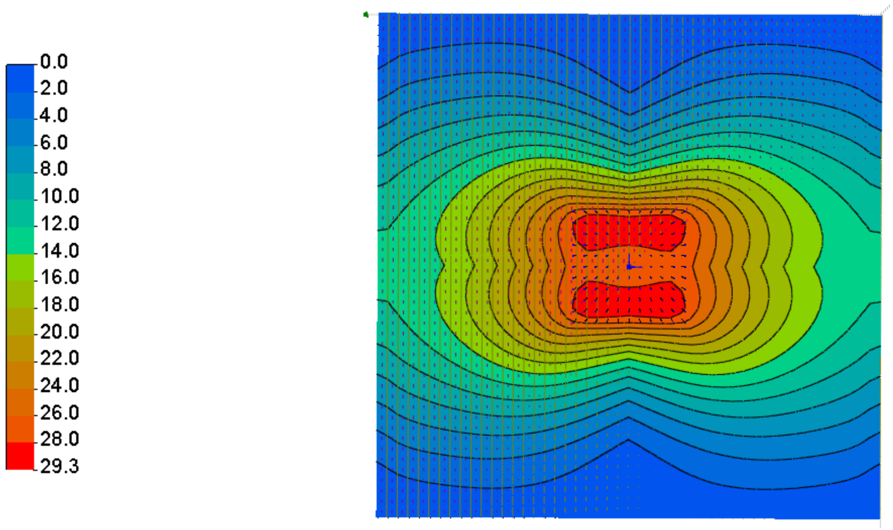


Figure 3-11a: Wood and Armer M_y moment contours (kNm/m) showing where FE moment exceed M_{cr} in one direction

A second linear FE analysis was then run on a model where an effective modulus of elasticity ($E_{c,eff}$) value was used in all elements where the linear FE moment had exceeded the cracking moment. Figure 3-11c shows the concrete E value decreasing in increments of 3GPa from 33GPa (uncracked) to 11GPa at the column. Table 3-2 shows the effective modulus of elasticity value for various linear FE moments, calculated using a modification of the method described in section 2.4.7, shown below.

$$E_{c,eff} = \frac{I_{eff}}{I_{co}} E_c$$

$E_{c,eff}$ = effective modulus of elasticity of the cracked transformed section

I_{co} = second moment of area of the un-cracked transformed section

I_{eff} = effective secant stiffness

E_c = modulus of concrete

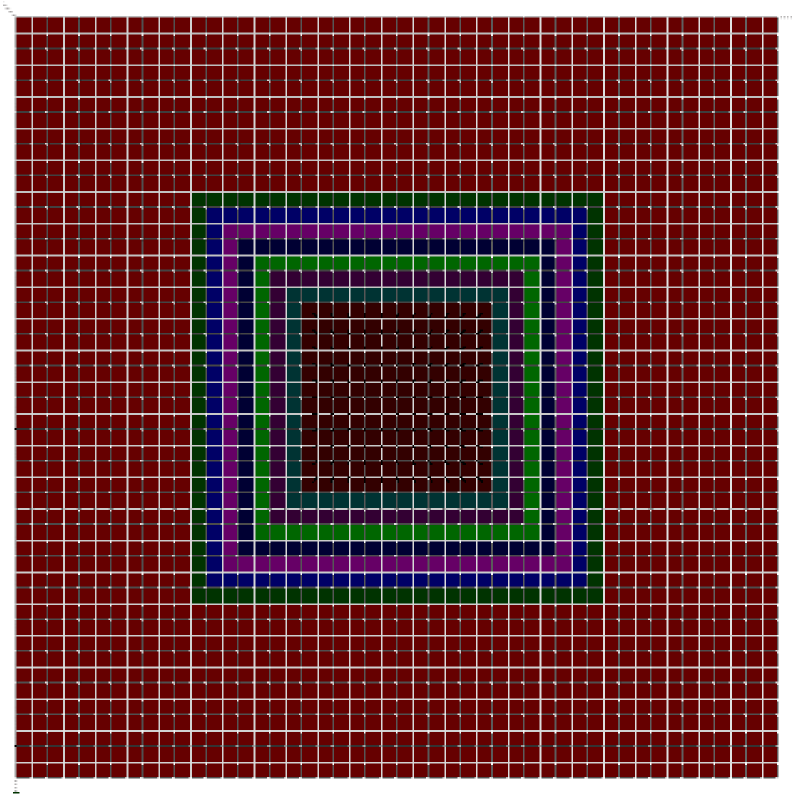


Figure 3-11c: Non-linear analysis showing change in concrete E value where concrete has cracked.

h mm	dia mm	d mm	E_s MPa	E_c MPa	f_{cr} MPa	I_{co} / I_{eff}	E_{eff} MPa	A_s mm^2/m	I_{eff} mm^4/m	FE M kNm/m	M_{cr} kNm/m	I_{co} mm^4/m
150	8	121	200000	33100	4.57	1.1	**30177**	335	259,63 x 10^6	**18**	17.5	284,79 x 10^6
150	8	121	200000	33100	4.57	1.5	**22004**	335	189,32 x 10^6	**20**	17.5	284,79 x 10^6
150	8	121	200000	33100	4.57	2.0	**16552**	335	142,41 x 10^6	**22**	17.5	284,79 x 10^6
150	8	121	200000	33100	4.57	2.6	**12786**	335	110,01 x 10^6	**24**	17.5	284,79 x 10^6
150	8	121	200000	33100	4.57	3.3	**10109**	335	86,98 x 10^6	**26**	17.5	284,79 x 10^6
150	8	121	200000	33100	4.57	4.1	**8162**	335	70,22 x 10^6	**28**	17.5	284,79 x 10^6
150	8	121	200000	33100	4.57	4.9	**6716**	335	57,7 x 10^6	**30**	17.5	284,79 x 10^6

Table 3-2: Effective stiffness of concrete

A comparison in the moment distribution at the face of the column shows the decrease in the peak moment and subsequent redistribution of moments (Figure 3-11d). The influence of concrete cracking on the transverse distribution of bending moments thus observed. Only one iteration of this process was done, as this is an approximate method, and the purpose was to show the trend of a decrease in the peak moment with cracking.

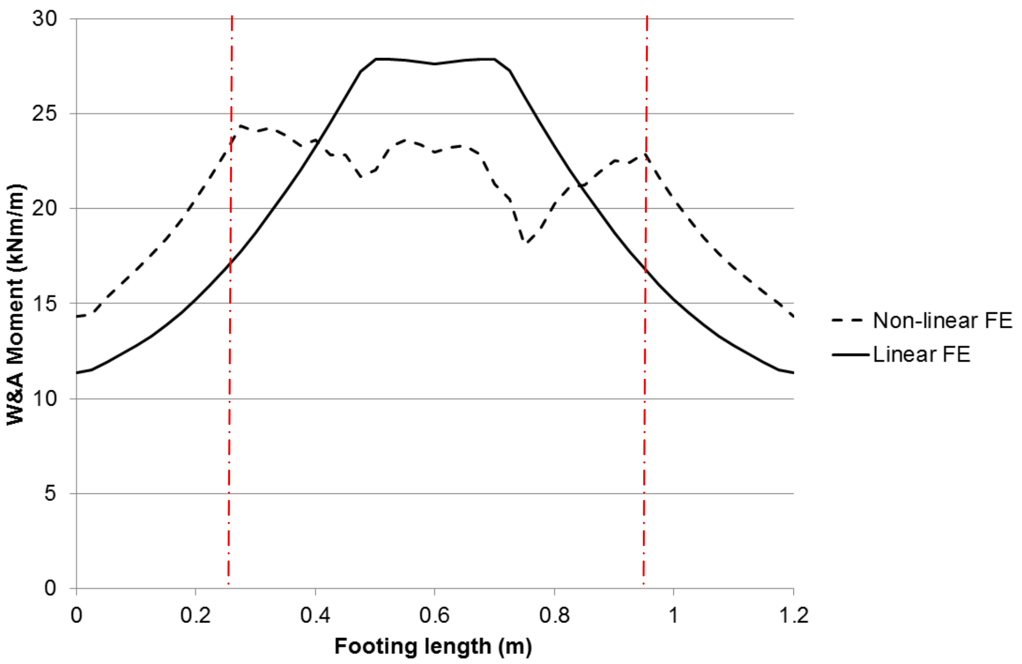


Figure 3-11d: Non-linear analysis compared to linear analysis at the face of the column

3.6 TESTS ON PAD FOUNDATIONS

Experimental work carried out at the Department of Civil Engineering at the University of Pretoria on the footing support the above numerical analysis. Hossell (2012), under the supervision of the author, undertook tests on a reinforced concrete pad foundation supported on springs. Two specimens were designed and reinforced, one according to the SD method and the other according to the linear FE method (See Appendix B). The influence of the reinforcement layout on the slab's response to ultimate limit state and serviceability limit state characteristics was observed.

The spring supported pad foundation test setup is shown in Photo 3-1, Photo 3-2 and Figure 3-13, with the springs simulating the founding support conditions. The test parameters are shown in Table 3-3 and the reinforcement layouts are shown in Figure 3-15. Pad foundation (a) was reinforced according to a SD analysis, and pad foundation (b) according to an FE analysis. Strain gauges were placed on the flexural reinforcement bars at the critical design section along the face of the column (See Figure 3-14), and linear variable differential transformers (LVDT) at the centre of each support spring were used to measure the displacement of the foundation. The strain in the reinforcement across the foundation was logged at a rate of 1 Hz. The change or variation in strain is shown in Figure 3-16, Figure 3-17 and Figure 3-18; the pad foundation displacement is shown in Figure 3-19; and a summary of the pad foundations response to the load is included in Table 3-4. It should be noted that as a result of using Wood and Armer moments (see Appendix A) to calculate the FE reinforcement the FE pad foundation required slightly more reinforcement than the SM pad foundation.

Photo 3-1: LVDT placement on the pad foundation supported on springs (Hossell 2012)

Item	Description
Flat slab type	Foundation
Support	Springs (k = 2500 kN/m)
Plan dimensions	1.2m x 1.2m
Thickness, h	0.15m
Effective depth, d	0.131m
Concrete strength, f_{cu}	36.7MPa
Concrete Young's Modulus, E_c	33.1GPa
Concrete tensile strength, f_r	4.57MPa
Reinforcement (high tensile) SD	8No. Y8
Reinforcement (high tensile) FE	9No. Y8
Yield stress of reinforcement, f_y	545MPa
Design point load applied to column	318kN

Table 3-3: Pad foundation test parameters

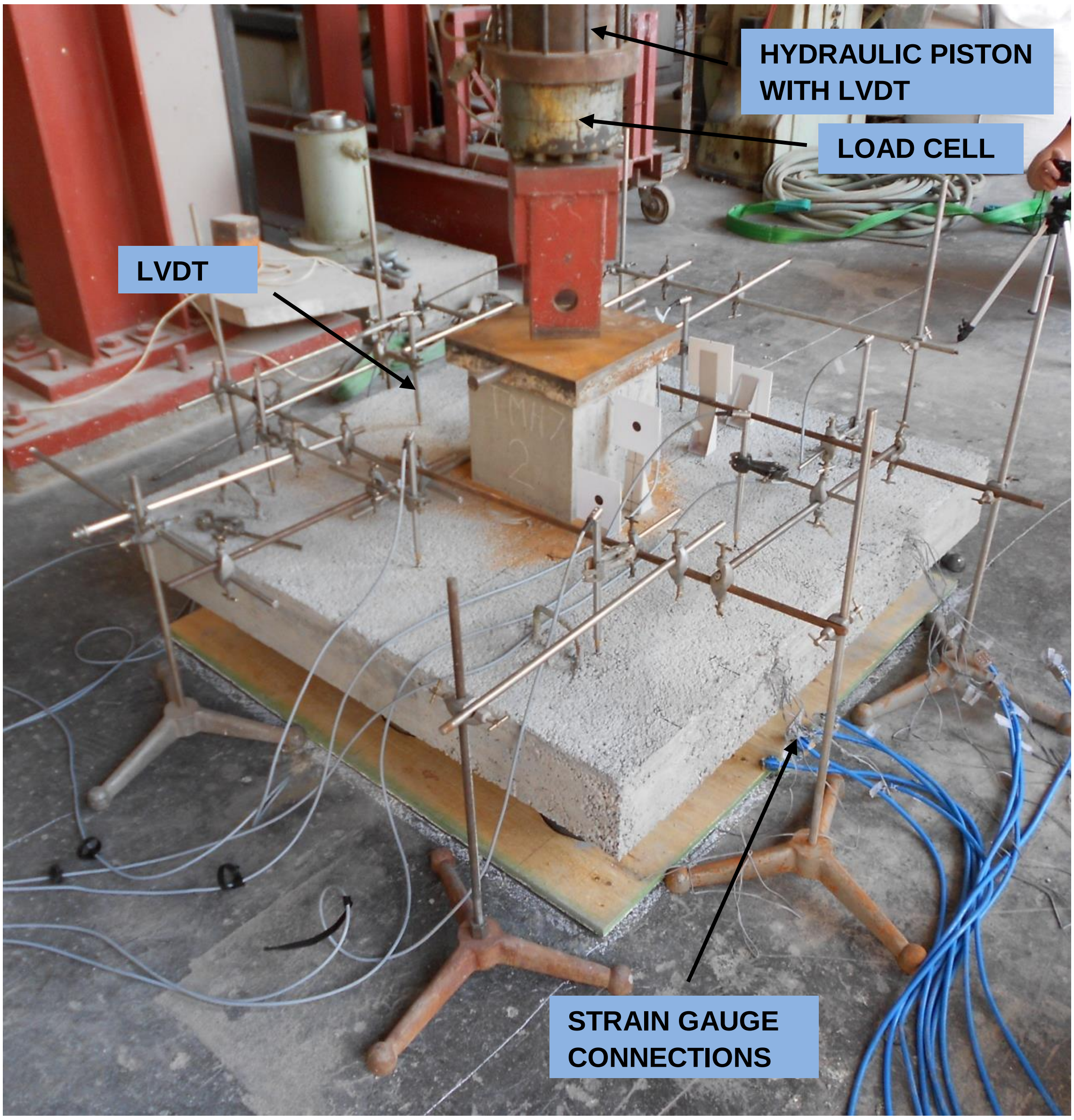


Photo 3-2: Test setup on testing floor (Hossel 2012)

Springs were used to simulate the soil accordance with Winkler's hypothesis. The springs used were 190mm in diameter and height with a 38mm thread. Figure 3-12 shows the results of the tests done on the springs in order to determine the spring stiffness. Using the linear region in this graph the spring stiffness calculated was 2500 kN/m.

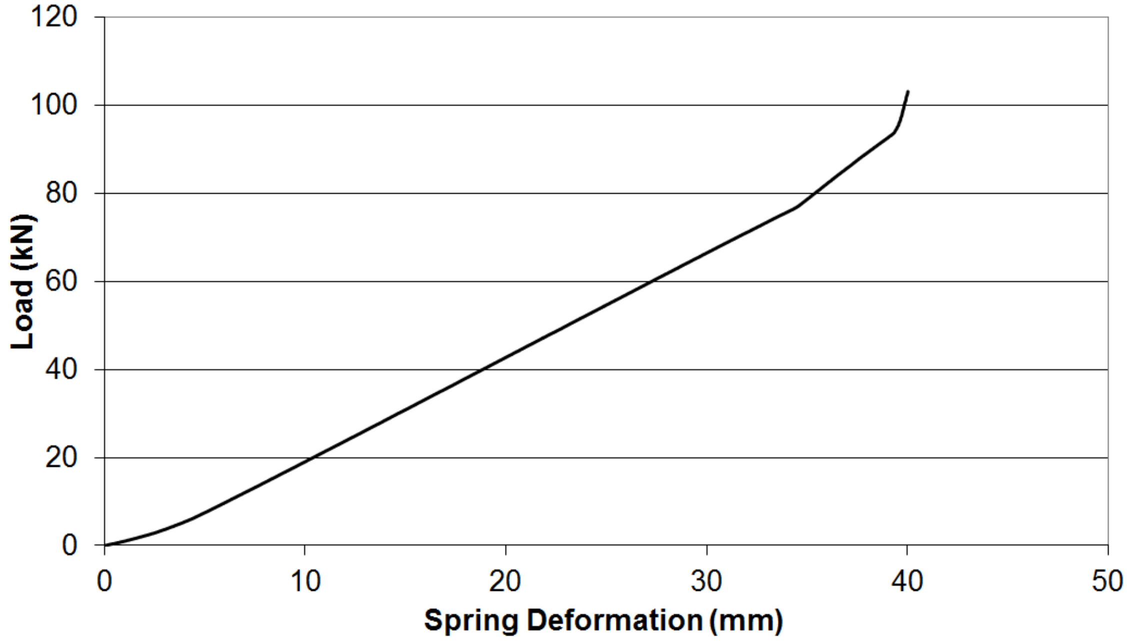


Figure 3-12: 190x190x38mm Spring stiffness test results

With the spring stiffness of 2500 kN/m (per spring) and a tributary area of 0.4 × 0.4 m, the modulus of subgrade reaction calculated was typical of medium dense sand and therefore the configuration of the springs shown in Figure 3-13 was chosen.

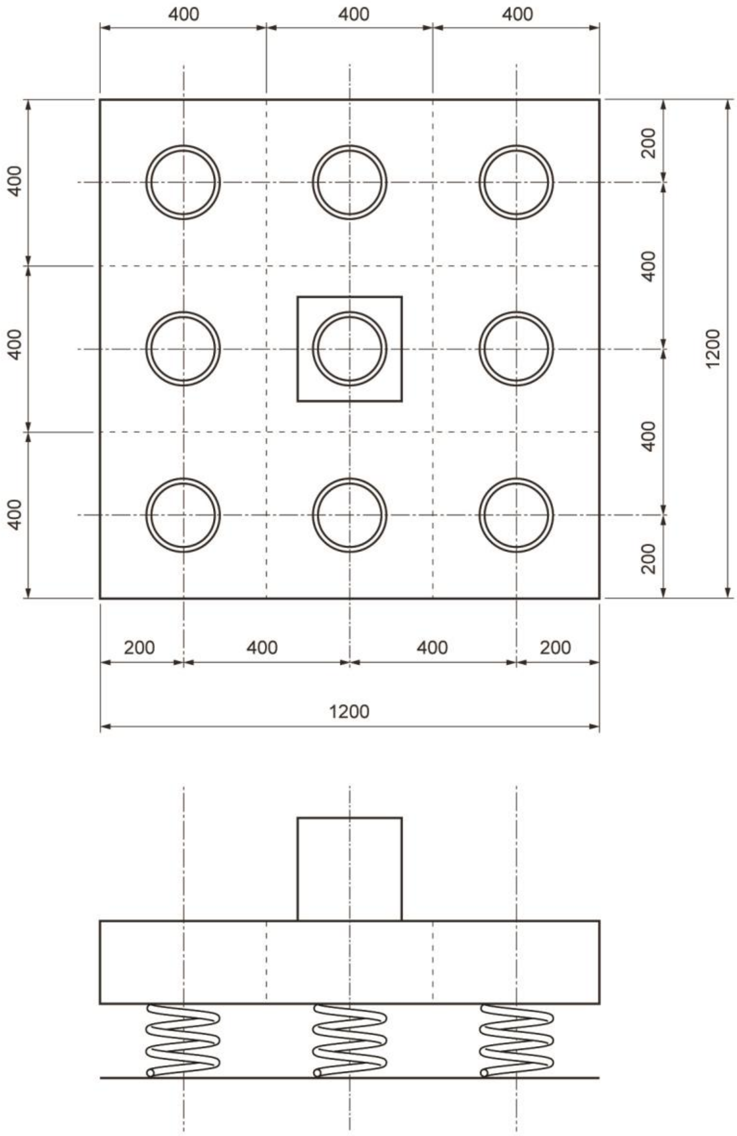


Figure 3-13: Spring layout

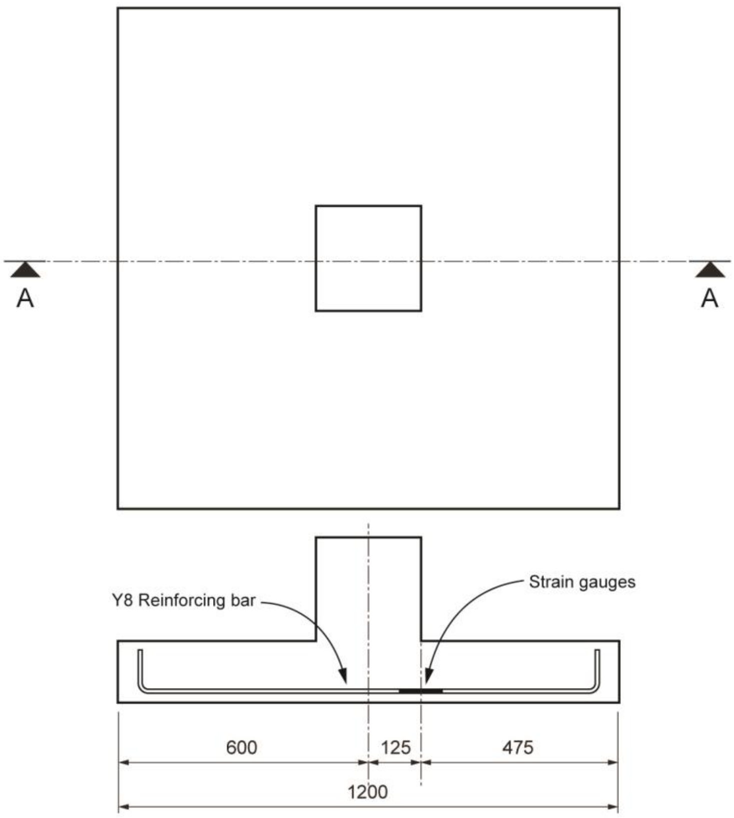


Figure 3-14: Position of strain gauges on reinforcing bars

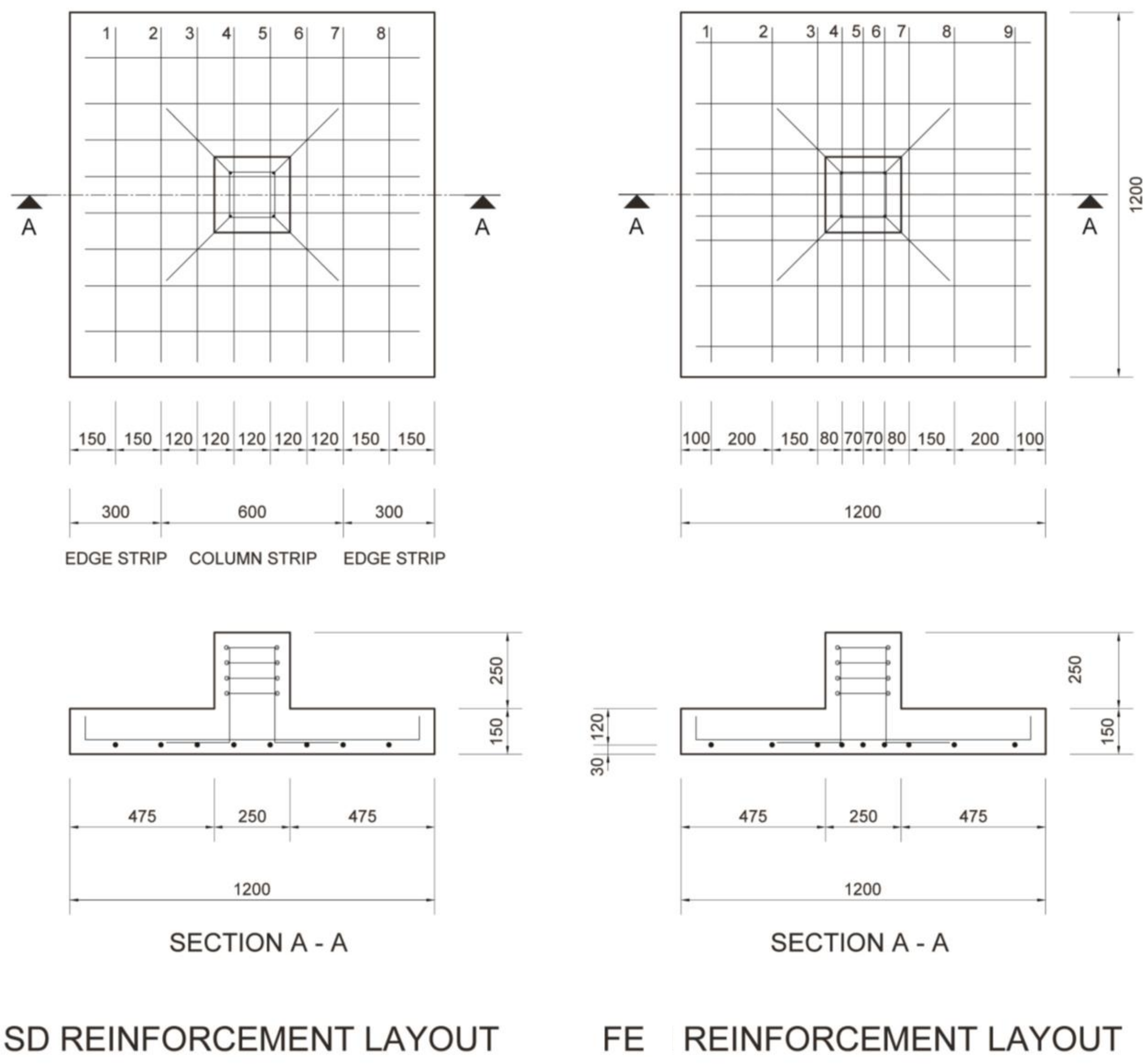


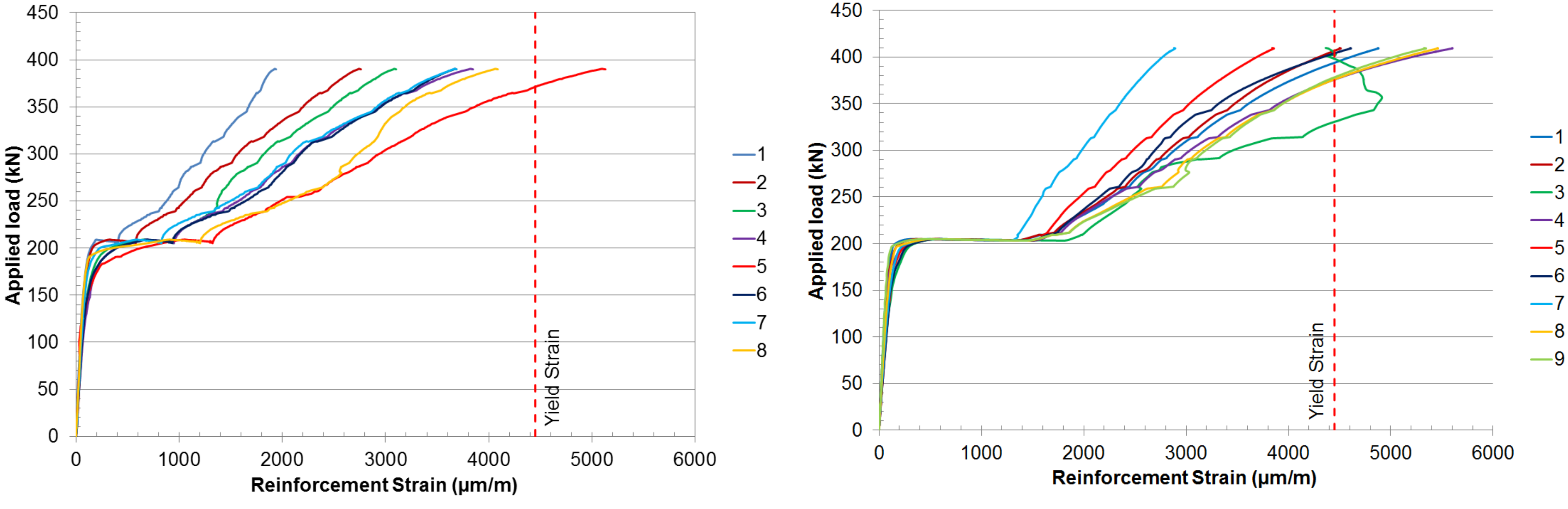
Figure 3-15: Reinforcement numbering for SD and FE Pad foundations

3.6.1 STRAIN IN REINFORCEMENT WITH LOAD

Strain levels for each reinforcement bar in both the SD and FE footing are shown in Figure 3-16. The first crack (sudden increase in strain at a load of approximately 205kN) in both pad foundations occurred at very similar loads, as this is primarily dependent on the tensile strength of the concrete. These two graphs show that the strain distribution in the FE designed footing is more uniform than in the SD footing.

The SD pad foundation test had to be stopped at a load of 412kN, before failure, as the testing machine piston moved out of alignment. The FE pad foundation failed in punching at 480kN. At the design load of 318kN the reinforcement strain in both pad foundations was well below the yield strain. The strain in the FE footing showed a much larger increase in strain when the concrete first cracked than the SD footing. At first, it was thought that there was a problem with the strain gauge readings, but on closer inspection of the data this strain increase did actually occur. During the test an audible noise occurred at the same load. It is assumed that this increase in strain can be attributed to the cracking of the concrete and subsequent load transfer to the reinforcement.

The yield strain was calculated using the 0.2% proof stress method described in TMH7 (1989).



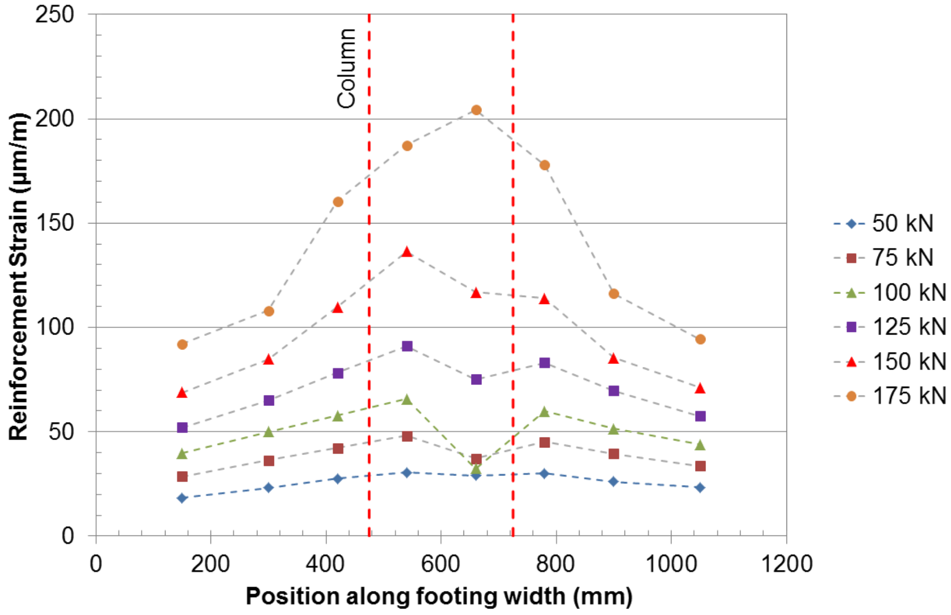
(a) SD **(b) FE**

Figure 3-16: Strain in reinforcement with applied load for (a) SD and (b) FE Pad foundations

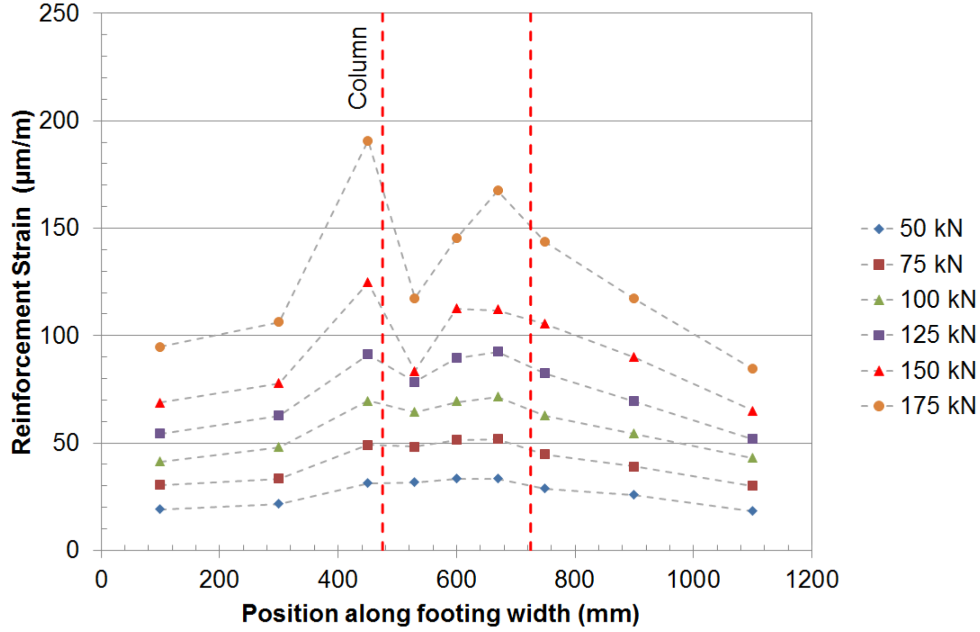
3.6.2 VARIATION IN REINFORCEMENT STRAIN PRIOR TO CRACKING

Change in strain levels prior to the concrete cracking (under 205kN) for each reinforcement bar in both the SD and FE footing are shown in Figure 3-17. This figure shows that prior to the concrete cracking the reinforcement at the face of the pad foundations (i.e. design section) is experiencing the highest strain. The SD pad foundation showed a greater variation in strain between the reinforcement under column and the reinforcement at the edge, than the reinforcement in the FE pad foundation.

The reinforcement strain in the FE pad foundation appears to be more uniform across the pad foundation width than when compared with the SM pad foundation.



(a) SD



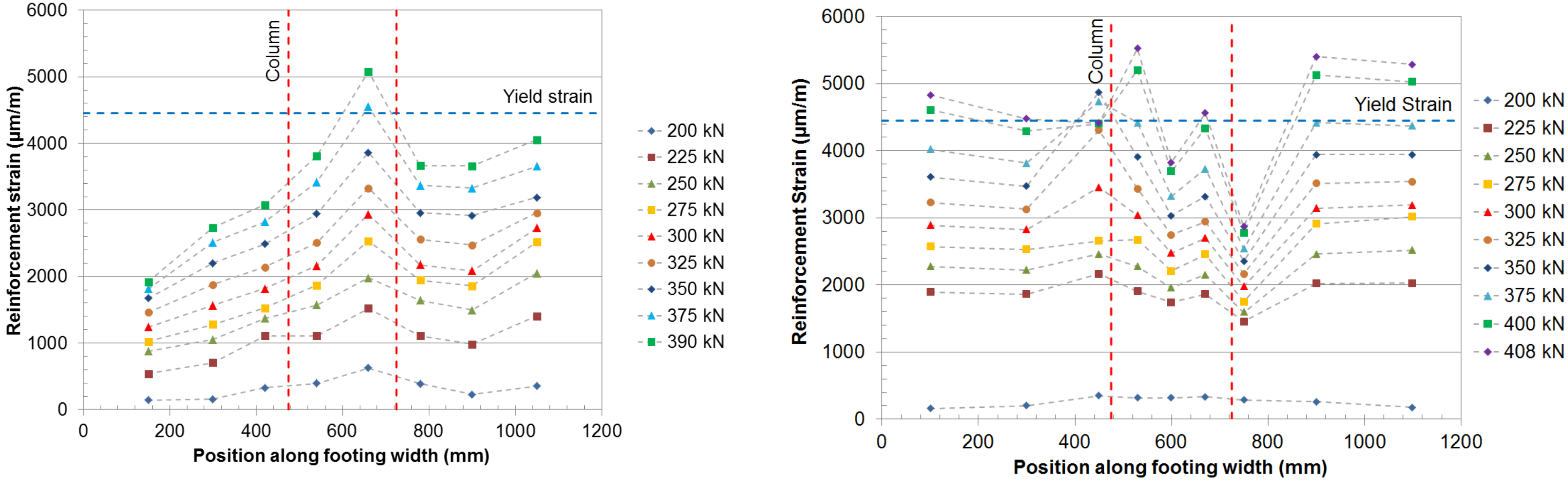
(b) FE

Figure 3-17: Strain in reinforcement prior to cracking for (a) sample SD and (b) sample FE along section AA

3.6.3 VARIATION IN REINFORCEMENT STRAIN AFTER CRACKING

Flexural cracking occurred at an applied load of approximately 205kN in both test, shown by the sudden increase in strain in the reinforcement shown in Figure 3-16. The increase in strain in the central reinforcement shown in Figure 3-18 indicates the formation of cracks at the face of the column, and shows the transfer of load from the concrete to the reinforcement.

Once the concrete cracked, a greater variation in strain was observed in the SD pad foundation, compared to the FE pad foundation, shown in Figure 3-18. With a larger variation in strain the reinforcing bars beneath the column in the SD pad foundation strained more than the bars towards the edge; indicating that fewer, but larger, cracks developed when compared to the FE pad foundation. The FE pad foundation showed a more uniform variation in strain, indicating that more cracks had formed but these cracks were smaller because of the lower strain levels.



(a) SD **(b) FE**

Figure 3-18: Strain variation after cracking for (a) sample SD and (b) sample FE along section AA

3.6.4 LOAD-DEFLECTION CURVES

Figure 3-19 shows the load deflection curves at the centre of the two pad foundations. Cracking and flexural failure can be seen by the change in gradient of the curves. The first crack occurred at very similar loads and deflections for both the FE and SD pad foundation as shown in Table 3-4.

Flat slab response	Simplified Design	Finite Element
Load at first crack	205kN	205kN
Deflection at 200kN (centre of pad foundation)	19mm	18.5mm
Load at failure	412kN (piston out of alignment)	480kN (punched)
Deflection at 390kN (centre of pad foundation)	35.3mm	33.5mm

Table 3-4: Summary of pad foundation test results

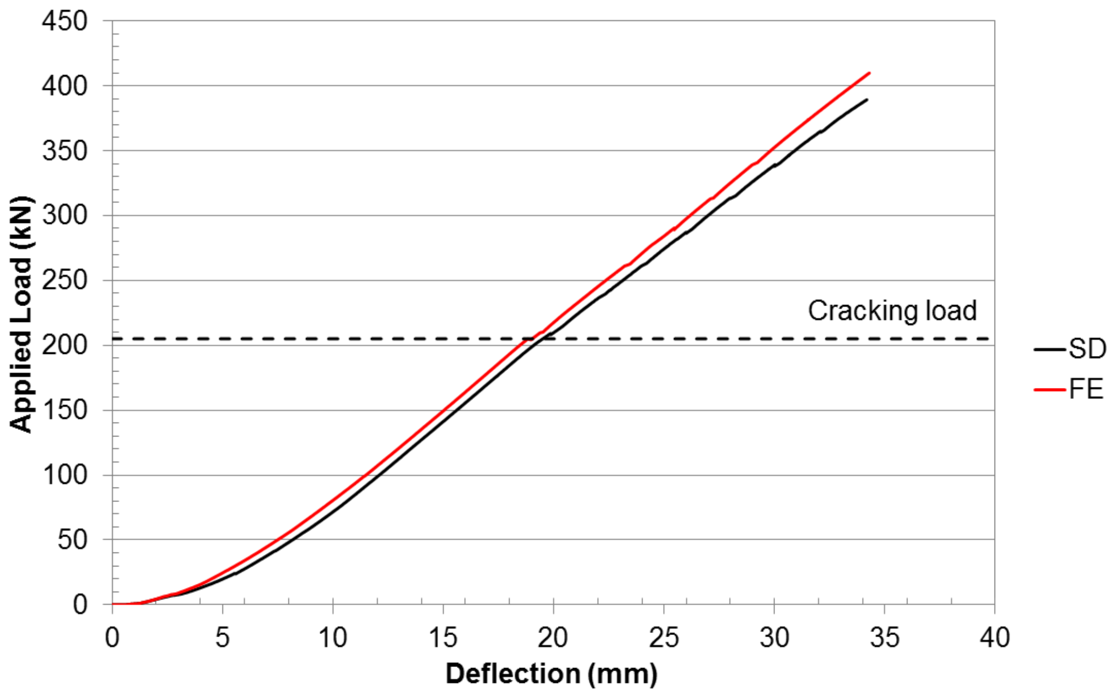


Figure 3-19: Deflection at centre of pad foundation for SD and FE

3.7 OBSERVATIONS FROM NUMERICAL AND EXPERIMENTAL WORK

From the above numerical and experimental work, the following conclusions are drawn regarding SD and FE analysis and design:

- The total FE M_x or M_y moments are the same as the total SD moment.

- Both the simplified method and finite element analysis and reinforcement layouts provided adequate and similar flexural capacity.

- The peak and total Wood and Armer moments obtained from a linear FE analysis are significantly influenced by how the supports are modelled. This is because of the change in the twisting moment with the support / constraint model.

- The FE peak M_x or M_y analysis moment can be almost double the SD column strip moment, depending on how the support constraints are modelled.

- A simplified non-linear analysis shows a decrease in the peak moment and subsequent redistribution of moments after cracking of the concrete occurs. The moment at the edge of the foundation subsequently increases. This redistribution of moments was also observed in the strain distribution of the footings that were tested.

- Detailing reinforcement in accordance with the variation in moments produced from a linear finite element analysis results in a more uniform distribution of strains across the width of the footing before and after cracking occurs.

- Cracking would appear to be better controlled by reinforcing placed in accordance with the distribution shown by the FE analysis.

- There is no apparent benefit in controlling deflection by detailing reinforcing to follow the FE peak moment.

4 RECTANGULAR COMBINED COLUMN FOOTING

The design of a rectangular combined column foundation was undertaken using the traditional design methods (SD), and then compared to the results of a linear elastic FE analysis. The moment variation at the critical design section for both methods of analysis was compared, and the effect of a change in plate geometry (depth) on peak load effects in a linear elastic FE model was investigated. Theoretical crack widths were also considered.

The spread pad foundation considered was assumed to support two columns which in turn support a conventional concrete bridge deck. The column size was calculated to meet the criterion of a $0.4f_{cu}$ MPa maximum concrete stress, South African bridge code, TMH7 (1989), in order to maximize local effects. A range of pad foundation depths (h) was then considered, varying in 100mm increments from 400mm to 1300mm, with a constant load effect. The chosen variation in pad foundation depth covered a range of reinforcing percentages from maximum to nominal values. It also allowed a study of the variation in peak values in FE methods. For the purposes of this study only resistance to hogging (negative) bending moments was considered. Resistance to the sagging (positive) moments, shear and punching forces was not investigated. The author notes that as a pad foundation's depth decreases, punching becomes the governing failure mechanism. Figure 4-1and Figure 4-2 show the overall pad foundation dimensions, and the analysis parameters are summarized in Table 4-1.

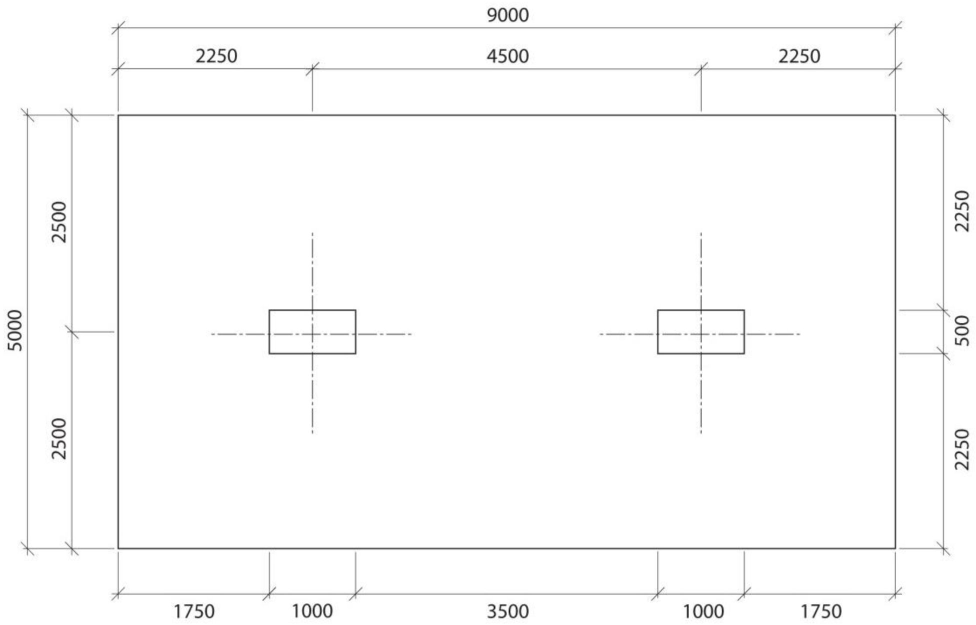


Figure 4-1: Pad foundation plan dimensions

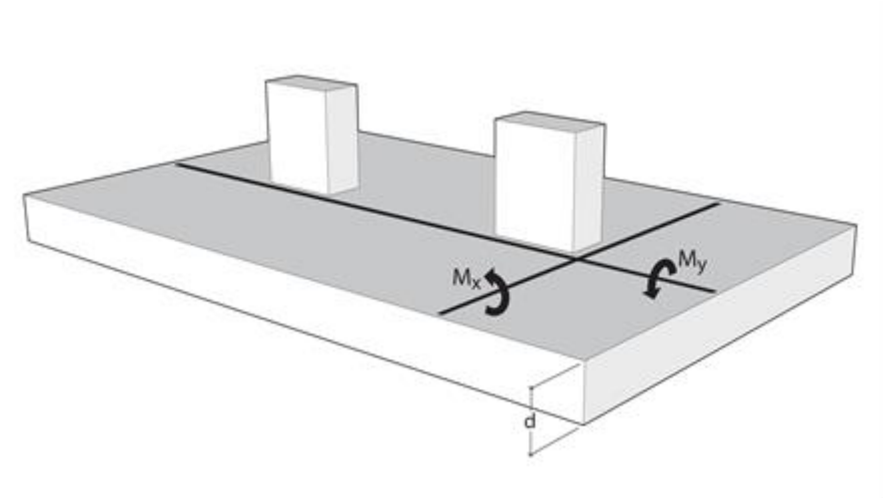


Figure 4-2: Three dimensional pad foundation model showing moment sign convention

Item	Description
Flat plate type	Pad foundation
Support in FE Analysis	Pinned at nodes in the column area
Plan dimensions	9m x 5m
Thickness, h	Varies from 0.4m to 1.3m
Concrete strength, f_{cu}	30MPa
Concrete Young's Modulus, E_c	28GPa
Concrete tensile strength, f_r	2.4MPa
Design uniformly distributed load (factored), w	347.5kPa
Design uniformly distributed load (unfactored), w	250.0kPa

Table 4-1: Pad foundation analysis parameters

4.1 TRADITIONAL FOOTING DESIGN METHOD

The conventional flat slab / pad foundation design method described in the South African bridge code TMH7 (1989) was used for the simplified method of design. If the width of the pad foundation is greater than $1.5(b_{col} + 3d)$ the code requires the slab to be split into column and middle strips, and designed and detailed accordingly, where b_{col} is the column width and d the effective depth to the tension reinforcement of the slab. The width of the column strip (W_{col}) is thus governed by the width of the column and the depth of the slab using the equation $W_{col} = b_{col} + 3d$. The column strip is then designed to resist two thirds of the total bending moment, and the middle strip is designed to resist one third of the total bending moment.

4.2 FINITE ELEMENT METHOD

The pad foundation was modelled using the linear elastic FE program Prokon (2012), which is available to the majority of designers in South Africa. The model as shown in Figure 4-3 consisted of square 0.25mx0.25m plate elements (span/10 is recommended by Brooker (2005)) with column supports modelled as 3D continuum models, as this gives the most realistic moment distribution. The elements used to analyse the pad foundations are discreet Kirchoff-Mindlin quadrilaterals that provide good results for both thick and thin plates and are free from shear locking. Shear deformations are not considered here in order to be consistent with the code requirements. The author notes that shear strain and deformation should be considered in thick pad foundations.

The worst case negative (hogging) bending moment envelope along the face of the support in each direction was then used to design the required finite element model (FE) reinforcement.

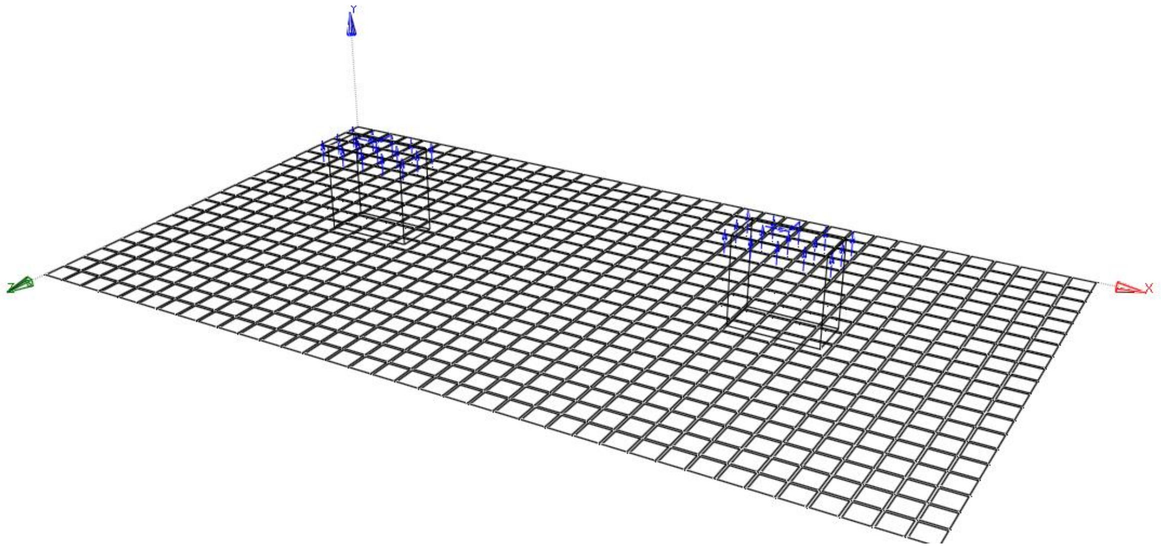


Figure 4-3: Finite element pad foundation model

4.3 DESIGN MOMENTS

The pad foundation requires bottom reinforcement in the transverse (x) and longitudinal (y) directions to resist the M_y and M_x hogging moments respectively, and top steel in the longitudinal (y) direction to resist the M_x sagging moment.

The linear FE moment outputs used were the M_x or M_y moments and the Wood-Armer moments. Figure 4-4 and Figure 4-6 show the FE moment contours in the x and y directions for a pad foundation depth of 700mm. Figure 4-5 and Figure 4-7 show the FE Wood-Armer moment contours for the M_{xT} and M_{yT} hogging moments in the pad foundation with a depth of 700mm. The effect of the twisting M_{xy} moment at the constraints (column) can be seen in the Wood-Armer moment contours.

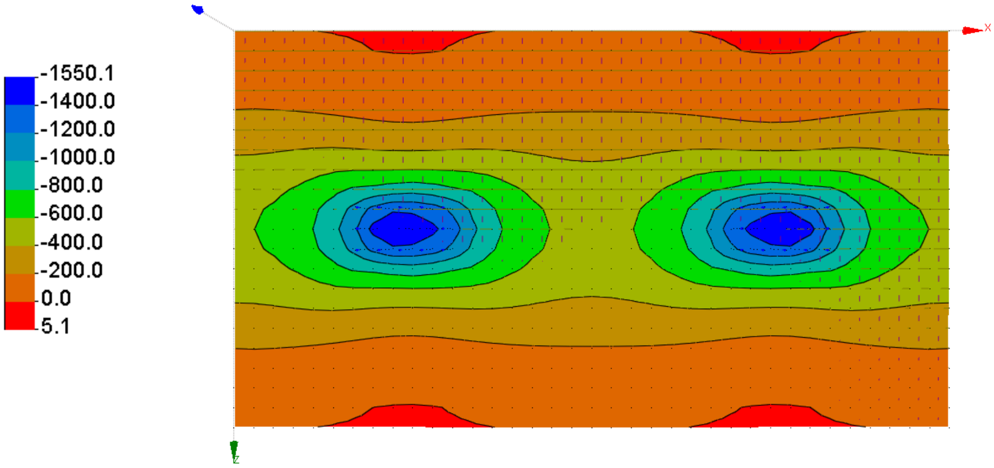


Figure 4-4: M_y moment contours (kNm/m)

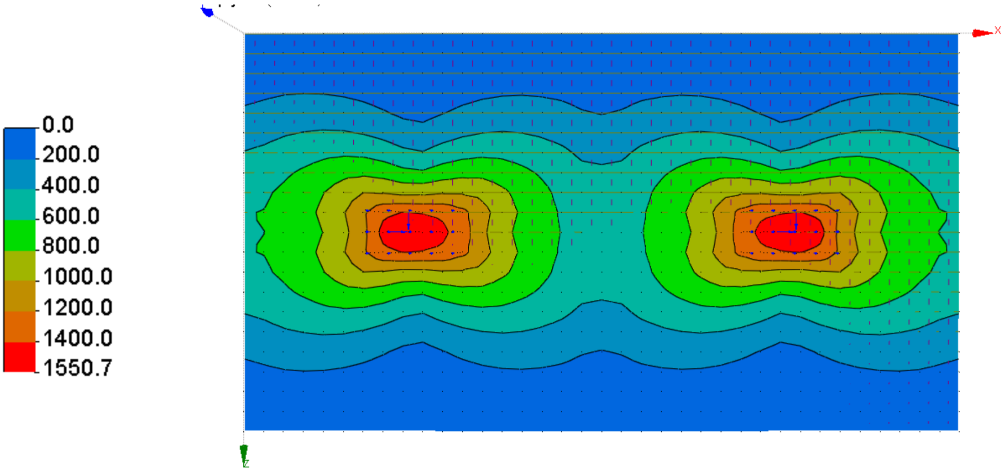


Figure 4-5: Hogging M_y Wood and Armer Moment contours (kNm/m) - bottom rebar

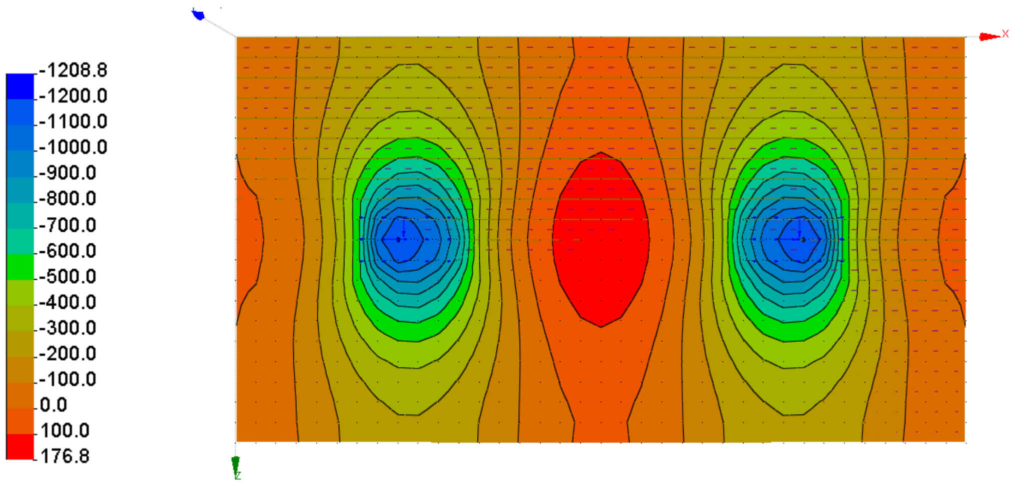


Figure 4-6: M_x moment contours (kNm/m)

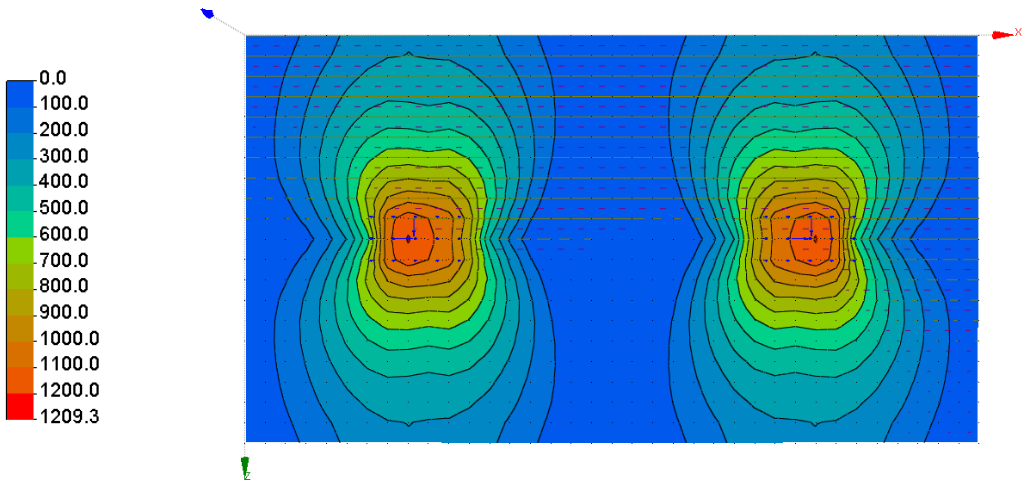


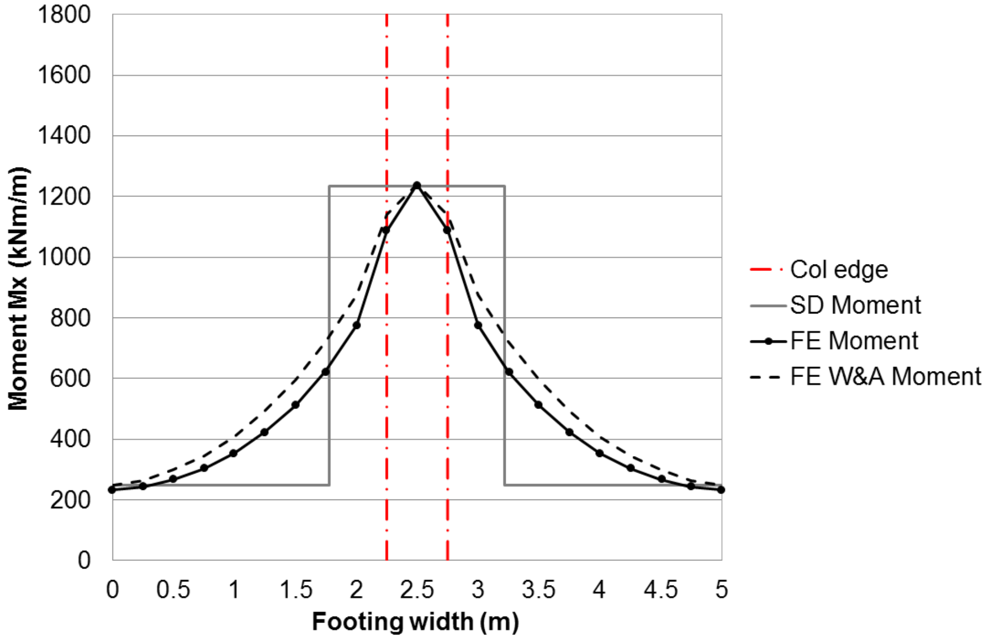
Figure 4-7: Hogging M_x Wood and Armer Moment contours (kNm/m) - bottom rebar

A section taken through the SD and FE bending moment diagrams at the face of the column in the x and y directions for pad foundation depths of 400mm, 700mm and 1200mm are shown in Figure 4-8 to Figure 4-10. Both the FE M_x or M_y moment and the FE Wood and Armer moment were shown in each graph.

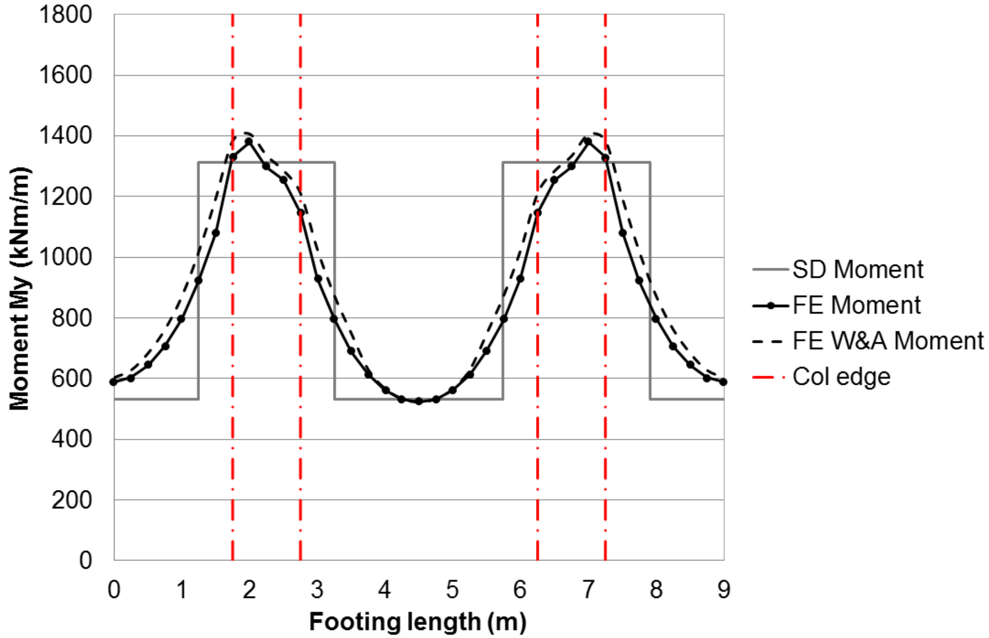
The peak FE M_y hogging moment occurs on the cantilever side of the column whereas the peak FE M_x hogging moment is mirrored about the pad foundation centreline, as there is a cantilever on both sides on the column. The Wood and Armer design moments are greater than the SD design moments as these moments are intended for use in design and include the twisting M_{xy} moment. Because of the unique solution and optimisation requirement the capacity is always greater than the applied moment (Denton & Burgoyne 1996).

The peak FE moment, M_{peak} (maximum FE hogging moment) was affected by the curvature of the pad foundation. As the stiffness of the footing decreased the peak FE moment increased. The SD method of analysis results in a constant moment which is split into a column strip and edge strip moment. The column strip requirement ensures that as the FE peak moment increases with a footing depth decrease, the SD column strip reduces, resulting in an increased SD moment, thus ensuring that the increase in curvature is provided for.

These three different depths of footing show how the traditional SD column strip width increase with the footing depth, until at a depth of 1200mm the transverse distribution of curvature has reduced sufficiently to not warrant differentiation between the column and middle strip.

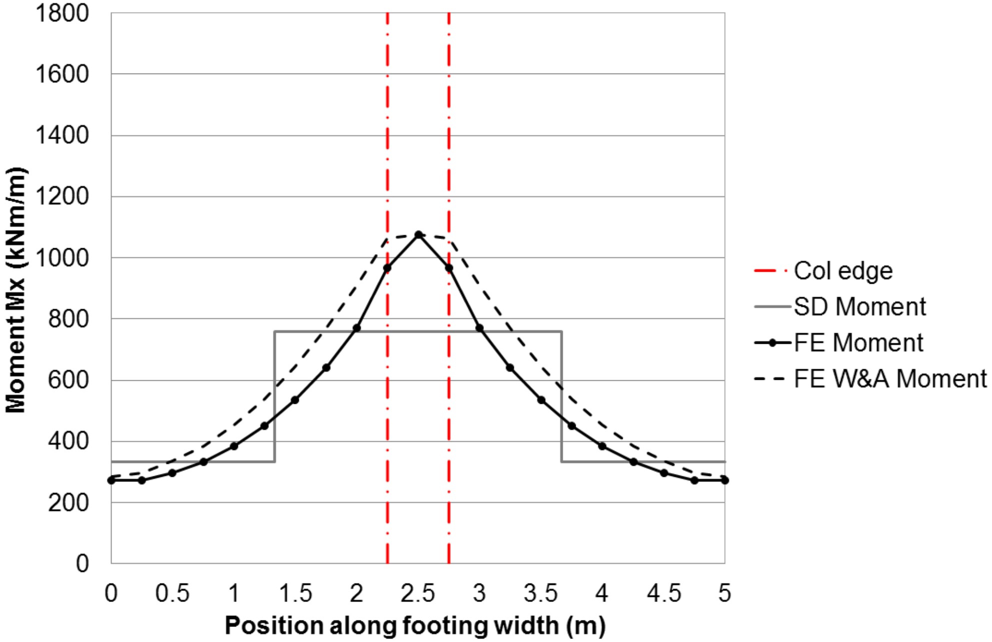


M_x

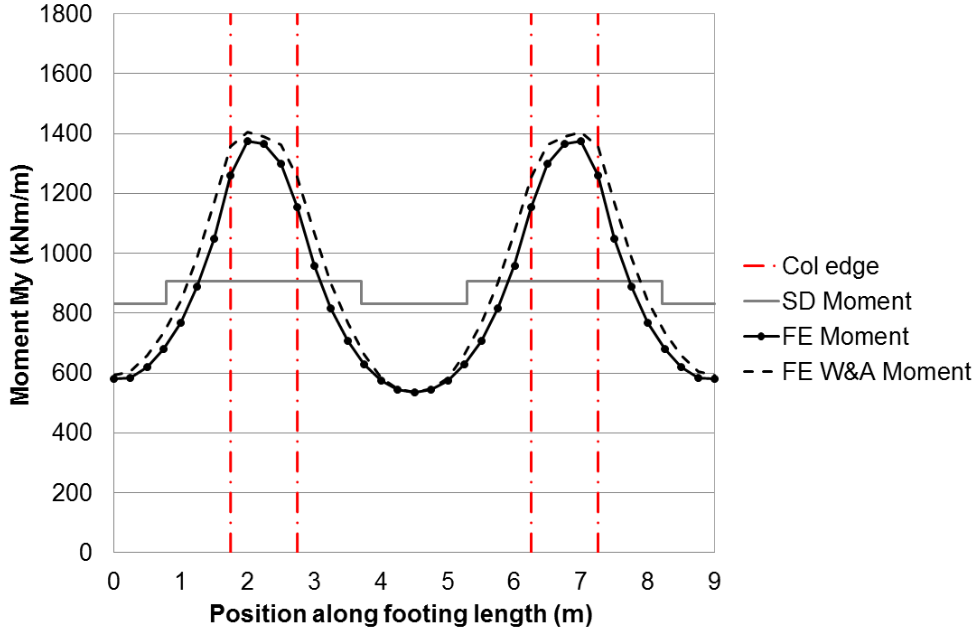


M_y

Figure 4-8: Simplified design method moment compared to FE design moment (d=400mm)



M_x



M_y

Figure 4-9: Simplified design method moment compared to FE design moment (d=700mm)

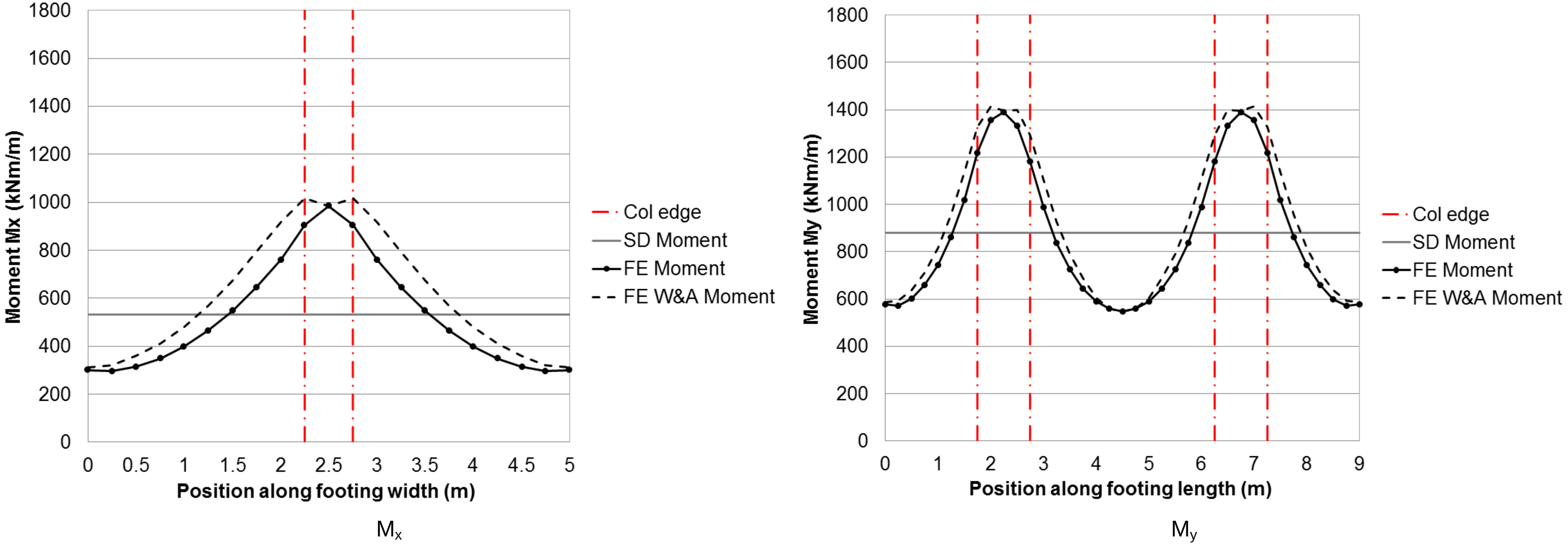


Figure 4-10: Simplified design method moment compared to FE design moment (d=1200mm)

The integration of the area under the moment diagram gives the total SD and FE load effect. The total SD design moment does not vary with the change in pad foundation depth as the self-weight of the pad foundation does not have an effect on the applied load effect. The total FE M_x or M_y axis moments are also constant with respect to change in footing depth, and are the same as the SD design moments. The FE Wood and Armer moments are affected by the twisting moment, which in turn is affected by how the constraints are modelled, and slab geometry. An increase in slab stiffness leads to an increase in the twisting M_{xy} moment. A comparison of the total FE M_x and M_y axis moment (equal to total SD moment) to the total FE Wood and Armer moment is shown in Figure 4-11 and Figure 4-12.

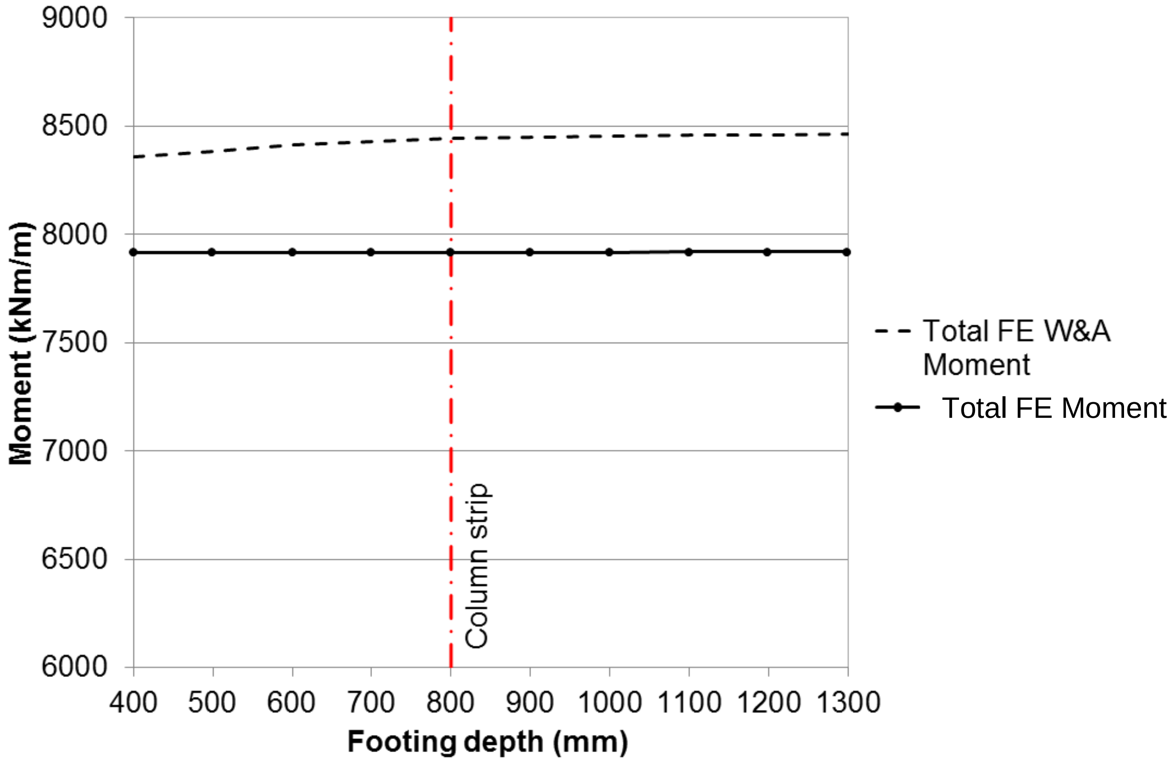


Figure 4-11: Total FE W&A moment compared to Total FE M_y moment

The total FE Wood and Armer moment M_y (cantilever) was approximately 5.6% greater than the total SD moment at a depth of 400mm, increasing to approximately 6.9% at a depth of 1300mm, a 23% increase. The M_y moment was the same as the SD moment.

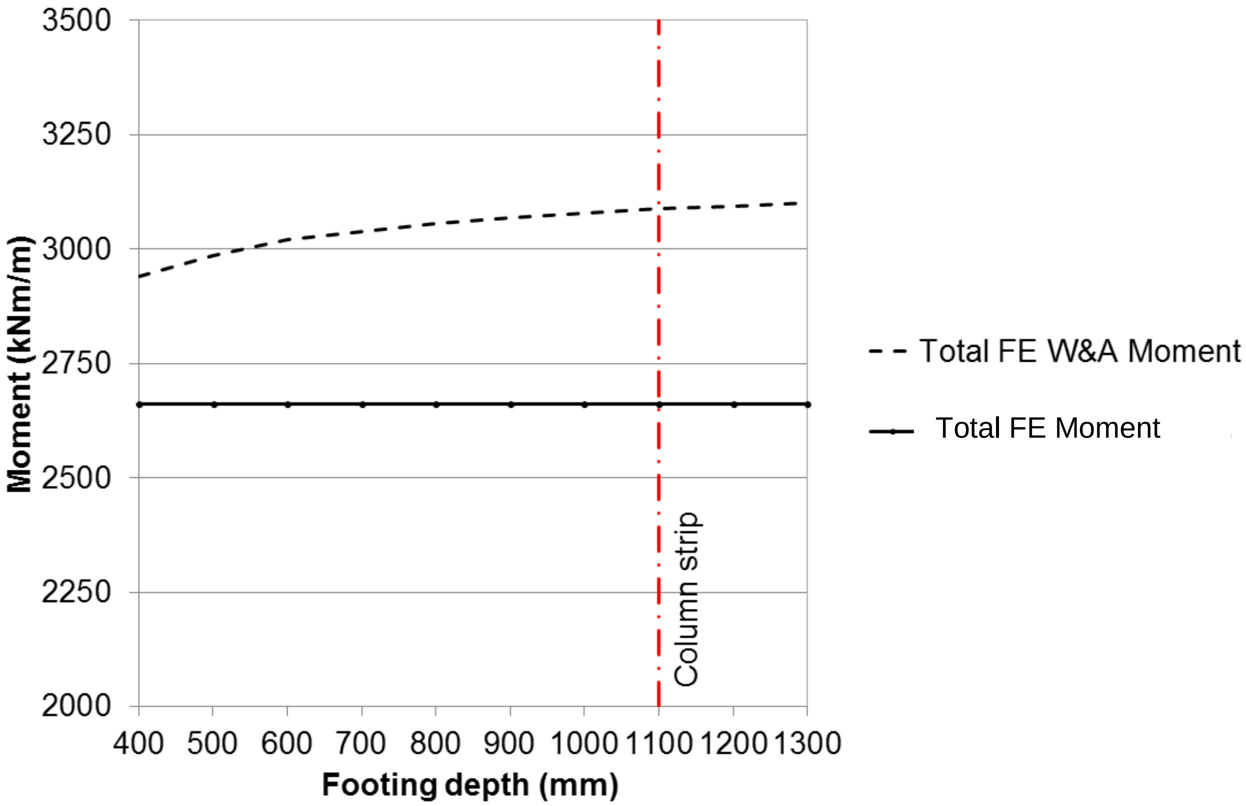


Figure 4-12: Total FE W&A moment compared to Total FE M_x moment

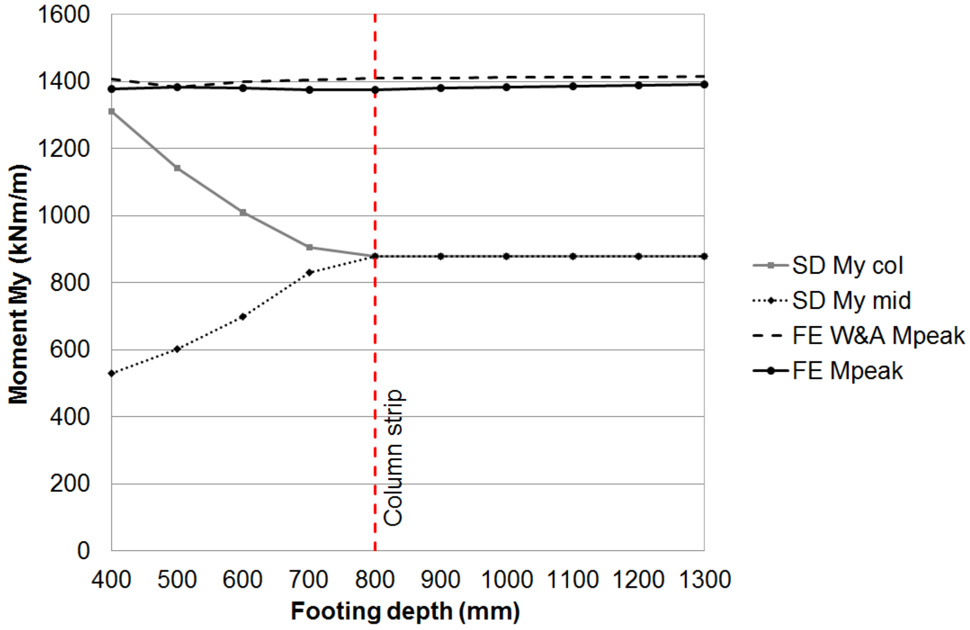
The total FE Wood and Armer moment M_x (beam) was approximately 10.5% greater than the total SD moment at a depth of 400mm, increasing to approximately 16.5% at a depth of 1300mm. The M_x moments were the same as the SD moment.

Figure 4-13 shows the change in the SD moment, column strip and middle strip, compared to the FE peak moment (M_{peak}), as the depth of the pad foundation (h) varies. Both the M_x or M_y moment and Wood and Armer moments were plotted, and the SD requirement of differentiating between the column and middle strip is shown. For a pad foundation width of greater than 1.5 x (b_{col} + 3h), the transverse distribution of curvature has reduced sufficiently so as to not warrant the differentiation between the column and middle strip.

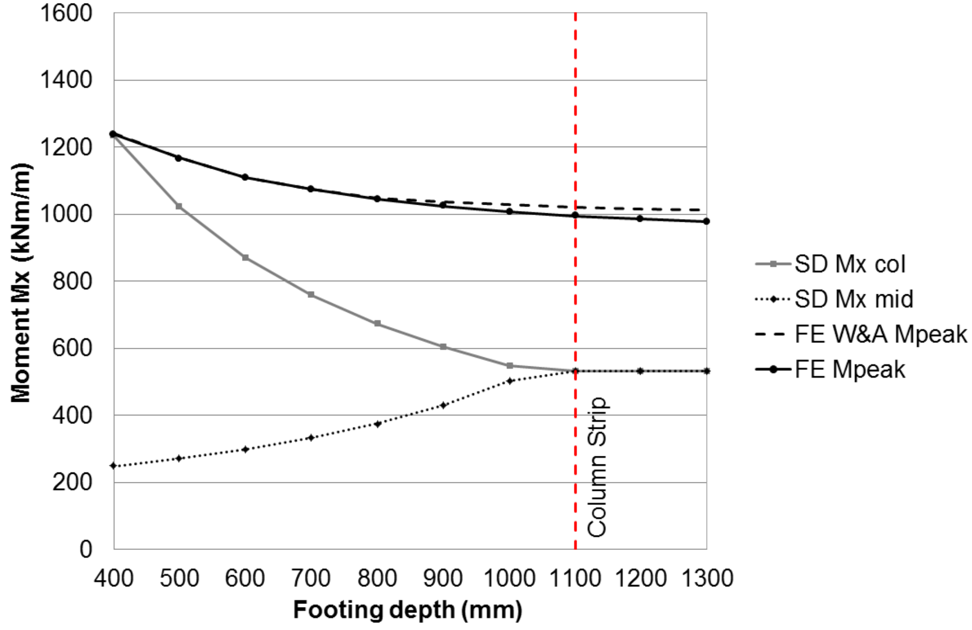
The M_X FE M_{peak} moment remained constant as the footing depth increased. The M_y FE M_{peak} moment decrease with the increase a footing depth, until a footing depth of 1100mm, and then levelled out.

Figure 4-14 shows a comparison of the ratio of the FE M_{peak} to the SD column strip moment and the SD concentrated column strip moment

Both the M_x and M_y FE M_{peak} / SD M_{column} ratios approach one as the stiffness of the slab decreases, and levelled out to a constant value at a depth consistent with the limit for the column strip of the code. Again showing that the SD column strip requirement ensures that as the FE peak moment increases with a footing depth decrease, the SD column strip reduces, resulting in an increased SD moment, thus ensuring that the increase in curvature is provided for.

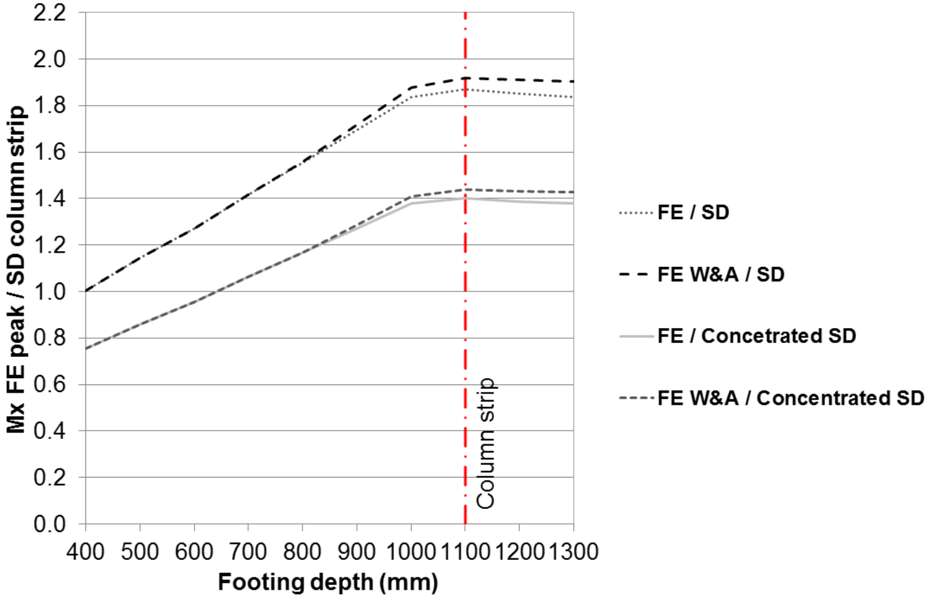


M_x

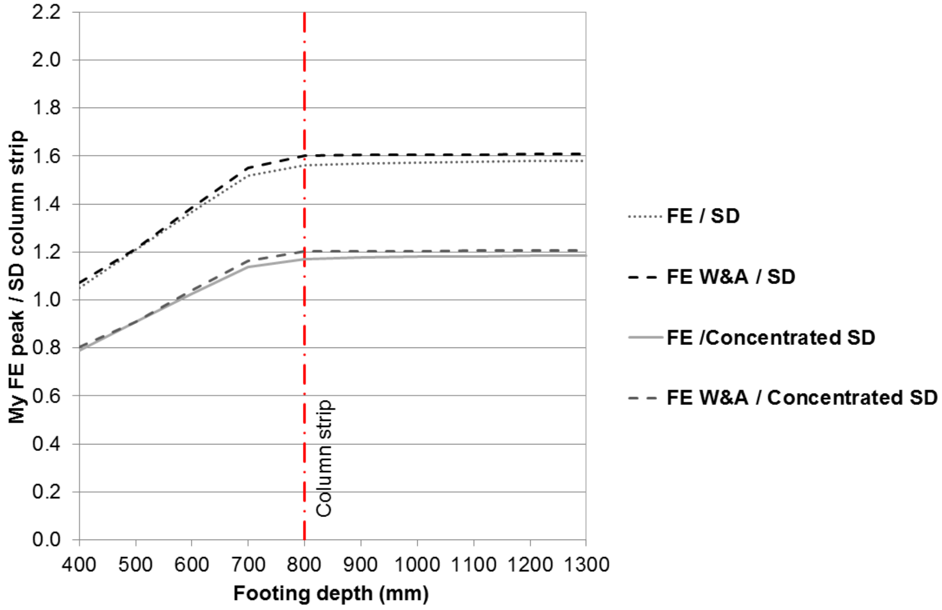


M_y

Figure 4-13: Simplified design method moment compared to FE peak moment



M_x



M_y

Figure 4-14: Ratio of FE M_{peak} to SD method load effect

4.4 CRACK WIDTH CHECKS FOR LINEAR FE MODEL

The South African bridge code requires cracking in pad foundations to be restricted to 0.2mm in order to meet the serviceability limit criterion of TMH7 (1989).

Linear FE model output gives reinforcement contours for providing resistance for the peak load effect; however, these singularities are never actually realised in the structure, as cracking and softening of the concrete result in a flattening out of the bending moment curve, and a reduction in the peak moment. Pad foundations are generally reinforced according to the SD requirements for column and middle strip. If the reinforcement detailing follows the FE contours, less softening and cracking is expected with more differentiation in curvature, when compared to the simplified method.

The predicted crack width, using the method described in SANS 10100-1 (2000), for the theoretical linear FE peak moment was compared using the FE and SD required reinforcement (SD on Figure 4-16). Certain parameters which influence crack widths were specified in the design, i.e. cover was kept constant (50mm) and the reinforcement spacing was made a function of slab depth (s = 0.25d). See Appendix D.

Figure 4-16 shows that by concentrating the reinforcement in the column strip the simplified design method reduces crack widths significantly as the slab becomes thinner, with a maximum crack width occurring when the column strip is no longer required. The nominal reinforcement requirement also results in a reduction the crack width.

Detailing according to the required FEM reinforcement results in a fairly constant estimated crack width as the slab depth varies, averaging approximately 0.3mm for both M_y and M_x. This is consistent with what is expected as the FEM reinforcement follows the peak moment.

SANS 10100-1 (2000) detailing guidelines for flat slabs suggests that the column strip can be further split into an inner strip with two outer edge strips. The inner strip is half of the column strip width, and the edge strips each a quarter of the column strip width. Two thirds of the column strip reinforcement is then placed in the inner strip, and the remaining one third equally distributed between the two edge strips. If this detailing method is used for the simplified method (Concentrated SD on Figure 4-16), the SD design crack width is reduced and tends to approach the FE crack width as the stiffness of the slab is reduced. This stepped method has been proposed for use in flat slabs where the column strip width is independent of the thickness of the slab. In pad foundations where the column strip width is a function of the pad foundation depth the percentage of reinforcement apportioned to the column and edge strips would need to be adjusted in order to ensure that the outer column strip has sufficient reinforcement.

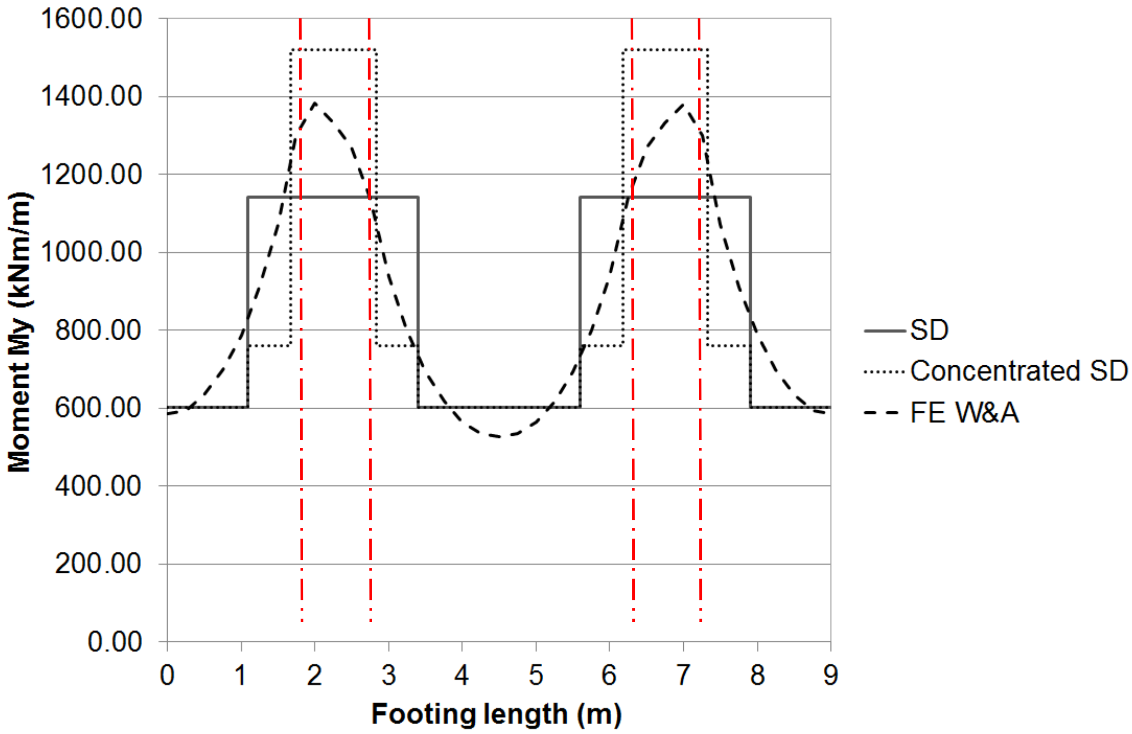
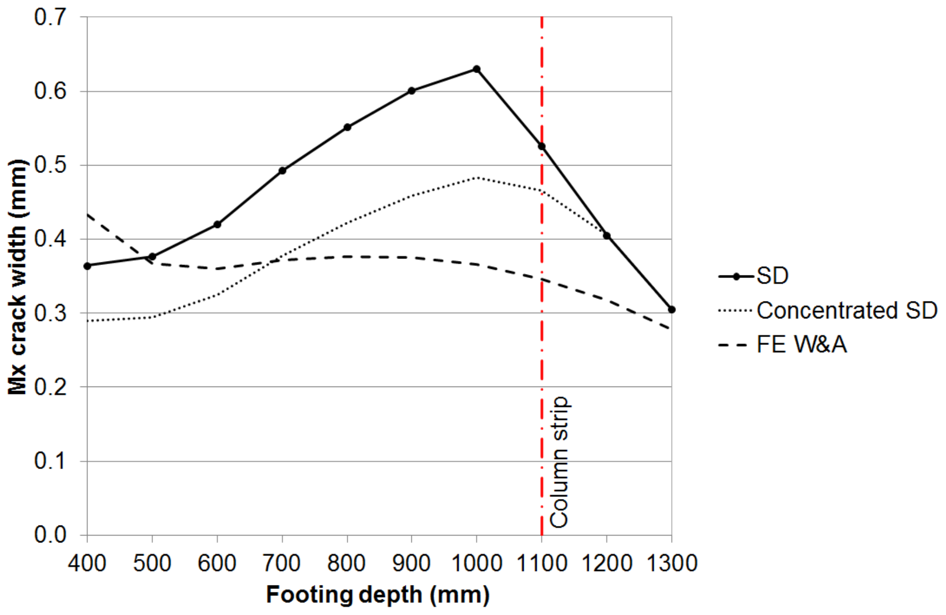
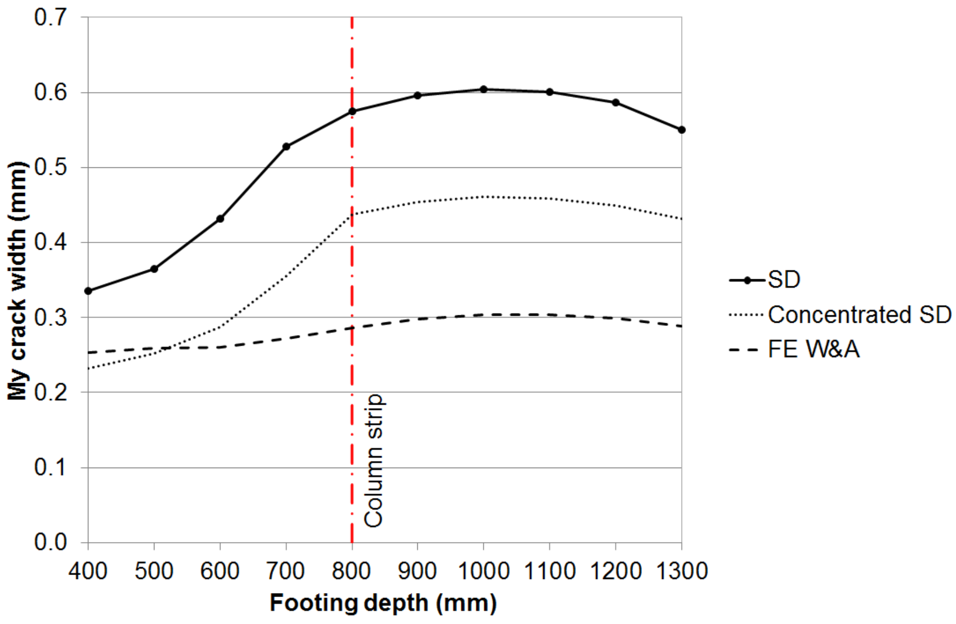


Figure 4-15: Concentrated reinforcement moment resistance distribution - footing depth of 500mm



M_x



M_y

Figure 4-16: Theoretical crack widths

4.5 OBSERVATIONS FROM NUMERICAL ANALYSIS

From the above numerical analyses the following can be concluded about the simplified method overall strength, finite element peak values and detailing according to linear FE methods:

- The total FEM M_x or M_y moments are the same as the total SD moment.
- Both the simplified method and finite element analysis and reinforcement layouts provided adequate and similar flexural capacity.
- The peak and total Wood and Armer moments obtained from a linear FE analysis is affected by the change in plate thickness - this is because of the change in the twisting moment.
- The peak moment from the FE model can exceed the column strip moment by a significant amount, it is however commonly assumed that this peak is reduced by cracking of the concrete and yielding of the reinforcement. The non-linear analysis confirms this. This may be compensated for by considering a column strip with a reduced width.
- Concentrating two thirds of the column strip reinforcement into an inner column strip reduces the theoretical peak FE moment crack width by approximately 20%, compared to using SD reinforcement.

5 SIMPLY SUPPORTED, TWO WAY SPANNING, FLAT SLABS

Els (2012), under the supervision of the author, undertook tests on two way spanning, corner supported flat slabs with an applied patch load (300mm x 300mm) in the centre of the slab, as shown in Photo 5-1 and Photo 5-2. Slab (a) was reinforced according to a SD analysis, and Slab (b) according to an FE analysis. The test parameters are shown in Table 5-1 and the reinforcement layouts in Photo 5-3 and Figure 5-1. LVDT's were used to measure the displacement of the slab. The load-deflection curves for the centre of both flat slabs are shown in Figure 5-2 and Table 5-2 shows a summary of the response of the flat slab to the load.

Photo 5-1: Test set-up for simply supported flat slab (Els, 2012)

Photo 5-2: Slab supported on ball bearings to ensure rotation is permitted (Els, 2012)

Item	Description
Flat slab type	Flat slab
Support	Corner supports
Plan dimensions	2.0m x 2.0m
Loaded area	0.3m x 0.3m
Thickness, h	0.15m
Effective depth, d	0.115m
Cover, c	0.03m
Concrete strength, f_{cu}	37MPa
Concrete Young's Modulus, E_c	3.5MPa
Concrete tensile strength, f_r	39GPa
Reinforcement (high tensile) SD	19 No. Y10
Reinforcement (high tensile) FE	21 No. Y10
Yield stress of reinforcement, f_y	450MPa
Design load, N	150kN

Table 5-1: Flat slab test parameters

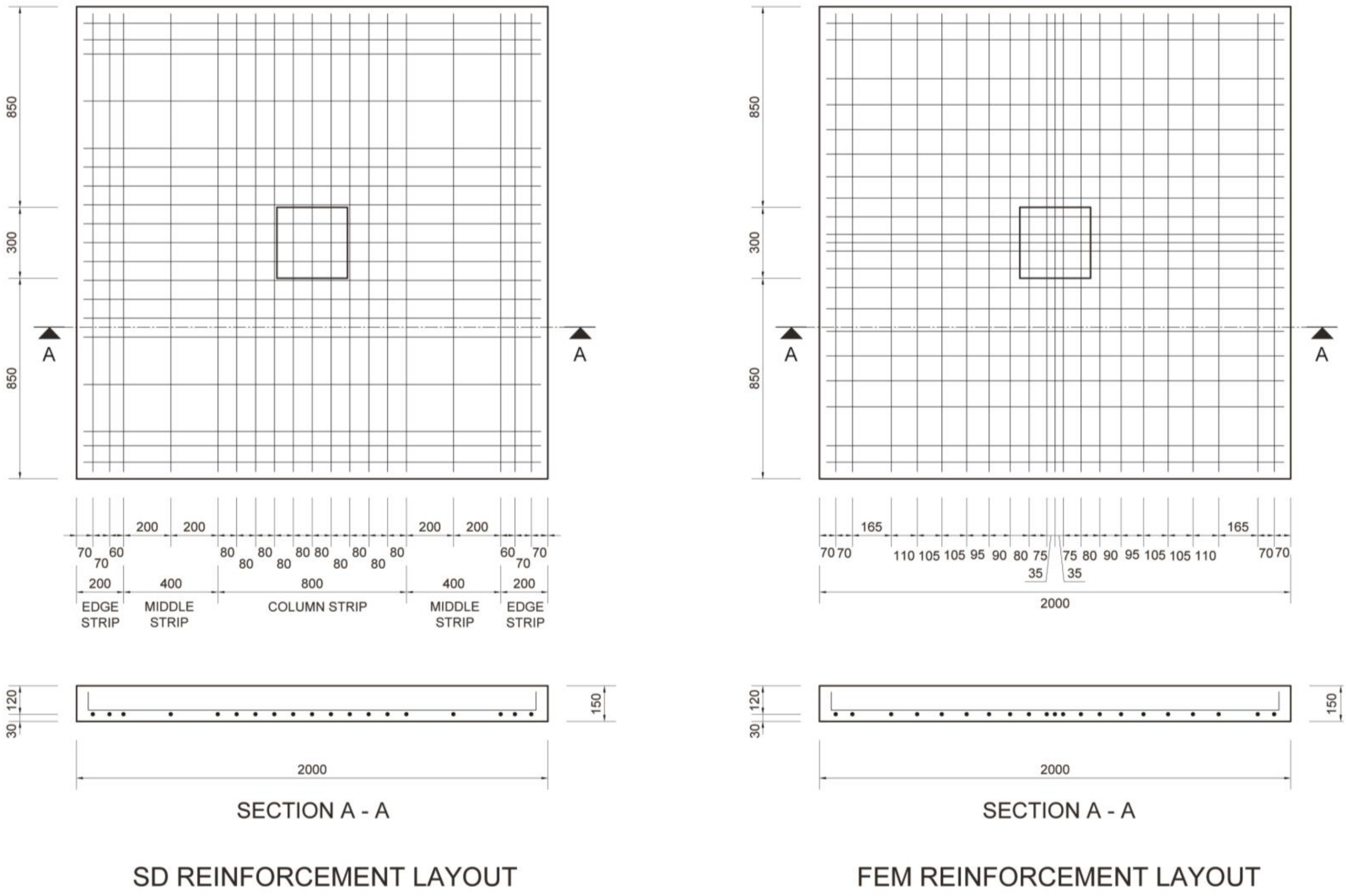
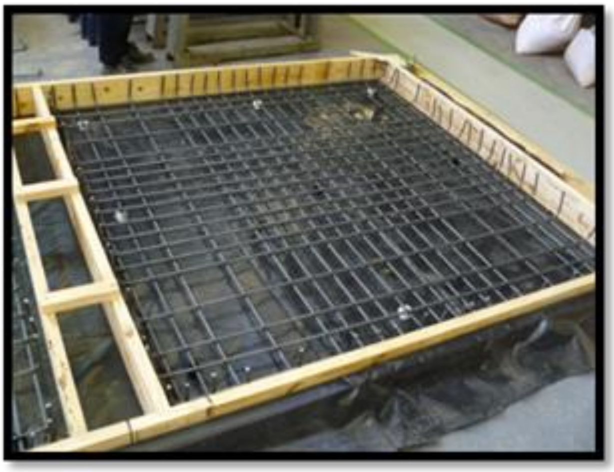


Figure 5-1: Reinforcement spacing for (a) SM and (b) FE flat slab samples



(a) (b)

Photo 5-3: Reinforcement for (a) SM and (b) FE flat slab samples (Els, 2012)

5.1 LOAD-DEFLECTION CURVES

Figure 5-2 shows the load deflection curves at the centre of the two slabs that were designed using the SM and FE methods. Cracking and flexural failure can be clearly seen by the change in gradient of the curves. First crack and failure occurred at very similar loads and deflections for both slabs, indicating that the placement of the reinforcement has very little effect on deflection control.

Flat slab response	Simplified Method	Finite Element
Load at first crack	60kN	63kN
Deflection at centre of slab at 60kN	3.6mm	4.5mm
Load at failure	207kN	203kN
Deflection at failure	20.7mm	20.7mm

Table 5-2: Summary of slab test results

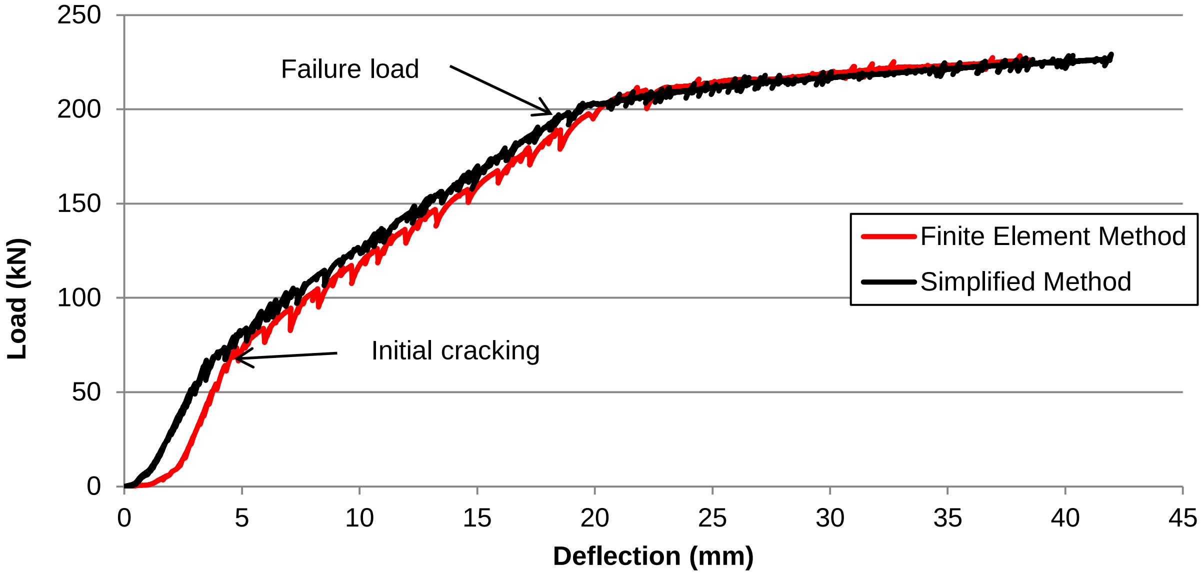


Figure 5-2: Load-deflection curve at the centre of the slab for SM and (b) FE slabs

5.2 CRACKING OF SLAB

The crack pattern of each slab was photographed at failure (Photo 5-4). The SM slab shows fewer but larger cracks (visual inspection) when compared to the FE slab, an indication that cracking is better controlled when the slab reinforcement follows the linear FE moment distribution.

(a) (b)

Photo 5-4: Cracking at failure for (a) SM and (b) FE Flat slabs (Els, 2012)

5.3 OBSERVATIONS FROM EXPERIMENTAL WORK ON TWO WAY SPANNING FLAT SLABS

From the above experimental work, the following can be concluded regarding SD and FE analysis and design:

- Both the simplified method and finite element reinforcement layouts provided adequate and similar flexural capacity.
- Cracking would appear to better controlled by reinforcing placed in accordance with the distribution shown by the FE analysis.
- There is no apparent benefit in controlling deflection by detailing reinforcing to follow the FE peak moment

6 FINAL CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

The questions posed at the start of this research regarding how to apply and interpret FE analysis results were answered as follows:

Total resistance achieved with FE design compared to that of traditional methods

Both the numerical analysis and experimental work support the conclusion that at the ultimate limit state there is very little difference, if any, between a flat plate analysed and reinforced using the SD method and one analysed with a FE model.

For each different flat plate structure modelled, irrespective of the support model, the total FE M_x and M_y moment was the same as the total SD moment. The total FE Wood and Armer moment was always greater than the SD moment, it is design moments which includes the M_{xy} twisting moment.

To what extent the peak moment in a FE analysis can be ignored.

The support (constraint) model has a significant effect on peak moments calculated in a FE analysis. If the stiffness of the column / support was taken into account peak moments are not observed in the FEM analysis and a very realistic moment distribution obtained (Figure 6-1). Pinned supports are not advised as the stiffness of the support must be taken into account.

Support modelled as full 3D continuum — Springs at edge of column

Figure 6-1: Support models which resulted in the most realist moment distribution

The peak moment from the FE model may exceed the SD column strip moment, however this peak is reduced by cracking of the concrete and yielding of the reinforcement. This may be compensated for by considering a column strip with a reduced width.

Serviceability performance of a FE design

Detailing reinforcement to follow the FE moments at the serviceability limit state results in a more uniform distribution of strain across the width of the slab, and therefore more, but smaller cracks.

The reinforcement distribution according to the FE method does not have a significant effect on the overall stiffness of the slab, and therefore does not appear to influence the deflection of the slab. This was show in both the experimental testing performed on the signal column footing and on the flat slab.

Detailing of reinforcement for an FE design that is practical and acceptable to construction companies

The principal advantage in detailing reinforcement using a linear FEM is related to crack control. In order to maintain a practical reinforcement layout Brooker's (2006) recommendations for using the total bending moment under the FE moment curve and then concentrating the reinforcement as for an the inner and outer columns strip detailing rules given in SANS 10100-1 (2000) is supported. In pad foundations where the column strip width is a function of the pad foundation depth, the percentage of reinforcement apportioned to the column and edge strips would need to be adjusted in order to ensure that the outer column strip has sufficient reinforcement.

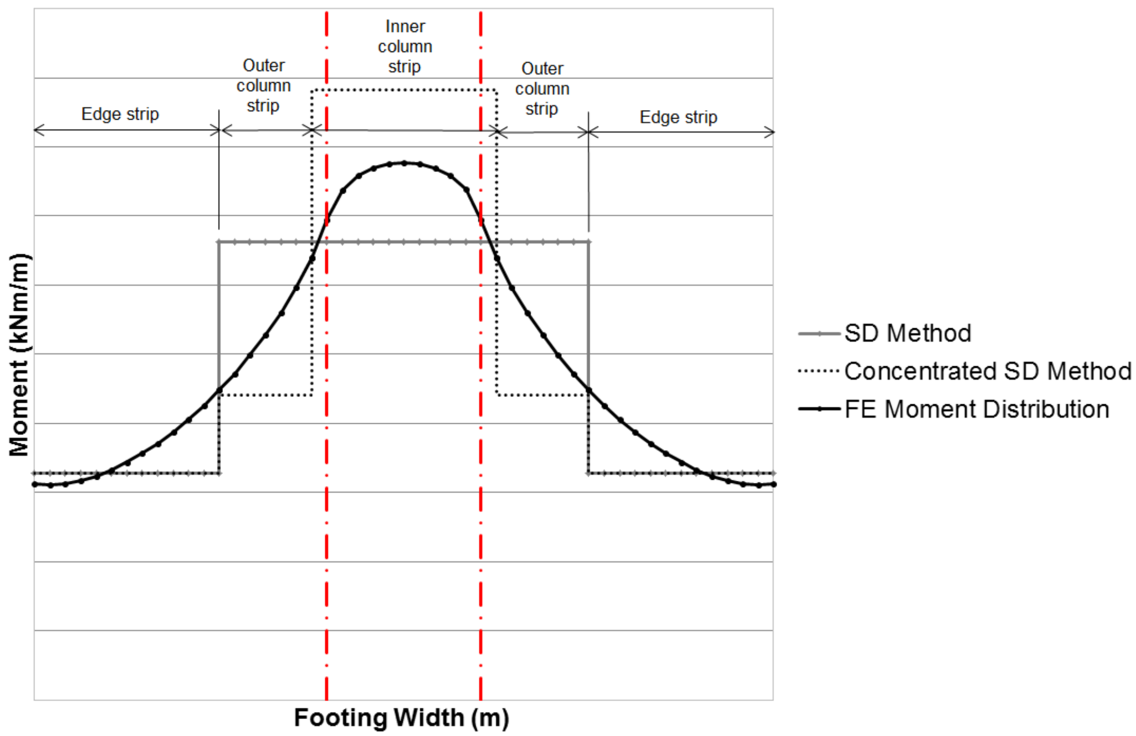


Figure 6-2: Moment distribution through hogging moment region of a flat plate

6.2 RECOMENDATIONS

Based on the findings from this study the following recommendations for future study were made:

- Further non-linear analysis to determine the effect of cracking and softening of the concrete on M_{peak}.

- Determining the influence that soil structure interaction has on the design of the footing and how the pressure under the footing redistributes as a result of cracking and softening of the concrete;

- Tests should be carried out with a concentrated column strip reinforcement layout to determine if the response characteristics are similar to those of the FED design.

- Investigation into the shear and punching capabilities of flat slabs with reinforcement layouts from different methods should be considered.

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APPENDIX A

FINITE ELEMENT MOMENT CALCULATIONS

This appendix gives the method used to determine the total design moment in the finite element analysis design.

The total moment was calculated by integrating beneath the unfactored variation in moment shown in Figure A-1. The moment per metre was calculated at each of the nodes in the finite element model along the critical section, and the sum of the moment was calculated by using Equation A.1.

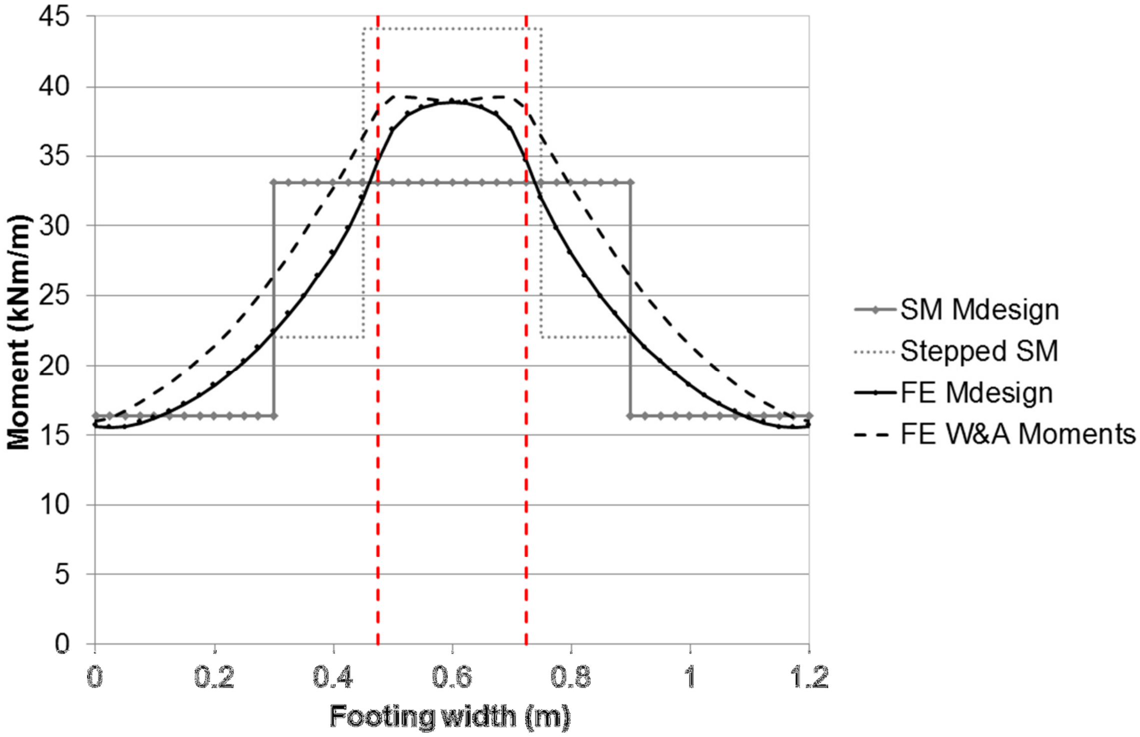

Figure E.1: Variation in moment at the critical section for design

$$M_t = \sum_{i=1}^{n} \frac{(M_{i-1} + M_i)}{2} . (d_i - d_{i-1})$$ A.1

M_t:	Total moment (kNm)
M_i:	The moment per metre at node i (mm^2/m)
d_i:	Distance from origin to node i (m)

APPENDIX B

DESIGN OF SINGLE COLUMN PAD FOUNDATION

This appendix gives the design of the test single column pad foundation described in Chapter 3.

Variables

Characteristic concrete strength: f_{cu} = 36.7MPa

Characteristic reinforcement strengths: f_y = 450MPa

Unit weight of concrete: γ_c = 25kN/m^3

Footing dimensions

Footing length: D = 1200mm

Footing width: B = 1200mm

Column width: b_{col} = 250mm

Cantilever length: ℓ = 475mm

Slab thickness: h = 150mm

Cover: c = 30mm

Loads

Column design load: N = 200kN

Uniform pressure under footing: w = 138.89kN/m^2

No partial material safety factors used

Flexural Reinforcement

Maximum bending moment

Maximum bending moment at the face of the column

$M_{total} = \frac{bwl^2}{2} = 18.8kNm$

Column and Edge Strips

Width of column strip: $b_{col} + 3h = 0.6m$

Width of middle strip: $D - (b_{col} + 3h) = 0.6m$

Division of moments in strips:

Column strip: 67%

Edge strip: 33%

Moments resisted by column strip = $M_{col} = 0.67\ M_t = 12.6\ kN.m$

Moments resisted by edge strip = $M_{edge} = 0.33\ M_t = 6.2\ kN.m$

Reinforcement in column strip

$Column\ strip\ width = 0.6m$

$Bar\ diameter = \emptyset_t = 8mm$

$\mathrm{d = h - cover - 0.5(\emptyset_t) = 150 - 30 - 0.5(8) = 116mm}$

$M_{col} = \mathrm{f_y.As.z}$

$\mathrm{z} = \left[1 - \frac{\mathrm{1.1.fy.As}}{\mathrm{fcu.b.d}}\right].\mathrm{d}$

$12.6 \times 10^6 = 450 \times \mathrm{As} \times \left[1 - \frac{1.1 \times 450 \times \mathrm{As}}{30 \times 1200 \times 116}\right] \times 116$

$\mathrm{As = 253mm^2}$

Therefore provide 5Y8 (251mm^2)

Reinforcement in edge strip

Column strip width: 0.6m

Bar diameter: 8mm

d = h – c – ϕ/2 = 116mm

$M_{col} = f_y.As.z$

$z = \left[1 - \frac{1.1.fy.As}{fcu.b.d}\right].d$

$6.2 \times 10^6 = 450 \times As \times \left[1 - \frac{1.1 \times 450 \times As}{30 \times 1200 \times 116}\right] \times 116$

$As = 125mm^2$

Therefore provide 3Y8 (151mm^2) in two edge strips either side of column strip

Punching Shear Check for Design Load

Maximum shear

Check maximum shear at column

$u = 2(bcol_x + bcol_y) = 2(500) = 1000mm$

$d_{avg} = 112mm$

$v_{max} = \frac{N}{u.d_{avg}} = 1.78\ MPa$

$v_{max} = 1.78\ MPa \quad < min \begin{cases} 4.75\ MPa \\ 0.75\sqrt{f_{cu}} \end{cases} = 0.75\sqrt{f_{cu}} = 4.5\ MPa$

First critical parameter at 1.5d from edge of column

$1.5d = 1.5(112) = 168mm$

Length of one side of punching perimeter: $x = c_x + 2(1.5d) = 250 + 2(168) = 586mm$

Total length of shear perimeter: $u = 4x = 4(586) = 2344mm$

Shear stress on perimeter: $v = \frac{V_{eff}}{u.d} = \frac{152000}{(2344)(112)} = 0.58\ MPa$

Shear resistance of concrete:

$$v_c = \frac{0.75}{y_{mv}}\left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}}.\left(\frac{100.(A_s)}{b.d}\right)^{\frac{1}{3}}.\left(\frac{400}{d}\right)^{\frac{1}{4}}$$

$$= 0.75\left(\frac{36.7}{25}\right)^{\frac{1}{3}}\left(\frac{100(250)}{586.112}\right)^{\frac{1}{3}}\left(\frac{400}{112}\right)^{\frac{1}{4}}$$

$$= 0.85\ MPa$$

$v_c > v$

$\therefore No\ shear\ reinforcement\ required$

APPENDIX C

DESIGN OF COMBINED COLUMN PAD FOUNDATION

This appendix gives the design of the combined column pad foundation discussed in Chapter 4.

It was assumed that the pad footing supported a typical highway overpass (15m jack span with 30m main span) voided reinforced concrete deck with conventional New Jersey parapets and 100mm asphalt. NA loading according to TMH7 Part 1&2 was assumed for the live load.

The design of the reinforcement was carried out in accordance with TMH7 Part 3

Variables

Characteristic concrete strength: f_{cu} = 30MPa

Concrete modulus of elasticity : E_c = 680 f_{cu}

Modulus of rupture: f_r = 2.4MPa

Characteristic reinforcement strengths: f_y = 450MPa

Unit weight of concrete: γ_c = 25kN/m^3

Footing dimensions

Footing length: D = 9000mm

Footing width: B = 5000mm

Column length: b_{col} = 1000mm

Column width: b_{col} = 500mm

Slab thickness: h varies between 400mm and 1300mm

Cover: c = 50mm

Loads

Column load from bridge deck (un-factored): N = 11237kN

Column load from bridge deck (factored): N = 15640kN

Uniform pressure under footing (un-factored): w = 250kN/m^2

Uniform pressure under footing (factored): w =347.5kN/m^2

Maximum Design M_x bending moment (factored)

Maximum M_x bending moment at the face of the column

$$M_{total} = \frac{bwl^2}{2} = \frac{5*347.5*1.75^2}{2} = 2661kNm$$

Maximum Design M_y bending moment (factored)

Maximum M_y bending moment at the face of the column

$$M_{total} = \frac{bwl^2}{2} = \frac{9*347.5*2.25^2}{2} = 7917kNm$$

Division of strips (TMH7 Part 3 3.7.3.1)

Width of column strip: $b_{col} + 3h$

Width of edge strip: $D - (\,b_{col} + 3h)$

Division of moments in strips:

Column strip: 67%

Edge strip: 33%

Moments resisted by column strip = $M_{col} = 0.67\, M_{total}$

Moments resisted by edge strip = $M_{edge} = 0.33\, M_{total}$

Tension reinforcement (TMH7 Part 3 3.3.2.3)

Where only tension reinforcement is required:

$M_u = \mathrm{f_y}.A_s.\mathrm{z}$

$z = \left[1 - \frac{1.1.fy.As}{fcu.b.d}\right].d$

Compression reinforcement (TMH7 Part 3 3.3.2.3)

Where compression reinforcement is required:

$M_u > 0.15 f_{cu} b d^2$

$M_u = 0.15 f_{cu} b d^2 + 0.72 f_y A'_s (d - d')$

$0.87 f_y A_s = 0.2 f_{cu} b d + 0.72 f_y A'_s$

Crack widths (SANS 10100-1)

Design crack width:

$$w_{max} = \frac{3 a_{cr} \varepsilon_m}{1 + 2\left(\frac{a_{cr} - c_{min}}{h - x}\right)}$$

$$\varepsilon_m = \varepsilon_1 - \frac{b_t (h - x)(a' - x)}{3 E_s A_s (d - x)}$$

ε_1= concrete strain at the level under consideration

a' = distance from te compression edge to the level under consideration

a_{cr} = distance from the point under consideration to the nearest longitudinal bar

b_t = width of the section at the level of the tension reinforcement

c_{min} = minimum cover to tension reinforcement

APPENDIX D

DESIGN OF SIMPLY SUPPORTED FLAT SLAB

This appendix gives the design of the simply supported flat slab tested in Chapter 5 according to SANS 10100-1.

Variables

Characteristic concrete strength: f_{cu} = 30MPa

Concrete modulus of elasticity : E_c = 28GPa

Modulus of rupture: f_r = 2.4MPa

Characteristic reinforcement strengths: f_y = 450MPa

Unit weight of concrete: γ_c = 24kN/m^3

Slab dimensions

Slab length: D = 2000mm

Slab width: B = 2000mm

Slab span (distance to edge of supports) = 1600mm

Column length: b_{col} = 300mm

Column width: b_{col} = 300mm

Slab thickness: h = 150mm

Cover: c = 30mm

Loads

Self weight of slab: 24 x 0.15 x 1.6 x 1.6 = 9.2kN

Column load (un-factored): N = 150kN

Support reactions: R = 150 + 92 = 159.2kN

Load per support = 159.2kN / 4 = 39.8kN

Flexural Reinforcement

Maximum bending moment

Maximum bending moment at the edge of the centre plate $(300mm \times 300mm)$.

$$\therefore M_{max} = 78.2(0.65) - \frac{4.61(0.65)^2}{2}$$

$$\therefore M_{max} = 49.856\ kN.m^{-1}$$

Column and Edge Strips

Long span = Short span = 1.6m

$\therefore\ l_2 = l_x = l_1 = l_y = 1.6m$

Width of column strip: $b_{col} = \frac{l_x}{2} = \frac{1.6}{2} = 0.8m$

Width of middle strip: $b_{mid} = l_2 - \frac{l_x}{2} = 1.6 - 0.8 = 0.8m$

Division of moments in strips:

Column strip: 75%

Edge strip: 25%

Moments resisted by column strip
= $M_{col} = 0.75\ M_{max} = 37.392\ kN.m$

Moments resisted by edge strip
= $M_{mid} = 0.25\ M_{max} = 12.464\ kN.m$

Reinforcement in column strip

$b_{col} = 0.8m$

d = 120mm = h - c – ϕ/2

$$K = \frac{M_{col}}{B_{col}.d^2.f_{cu}} = \frac{37392000}{0.8(1000)(120)^2(30)} = 0.108 < 0.156$$

$No\ compression\ reinforcement$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) = 103.3 < 0.95d$$

$$A_s = \frac{M_{col}}{f_y.z} = \frac{37392000}{450.103.3} = 804.4\ mm^2$$

$$\frac{A_{s.req}}{B} = 1005.49\frac{mm^2}{m}$$

Therefore provide 10 Y10's @ 80mm spacing

Reinforcement in edge strip

$b_{edge} = 0.8m$

d = 120mm = h - cover – ϕ/2

$$K = \frac{M_{mid}}{B_{col}.d^2.f_{cu}} = \frac{1246400}{0.8(1000)(120)^2(30)} = 0.108 < 0.156$$

$No\ compression\ reinforcement$

$$z = d\left(0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right) = 115 > 0.95d$$

z = 114mm

$$A_s = \frac{M_{mid}}{f_y.z} = \frac{12464000}{450.114} = 243\ mm^2$$

$$\frac{A_{s.req}}{B} = 304\frac{mm^2}{m}$$

Therefore provide 4 Y10's @ 200mm spacing

Edge strip

Additional reinforcement was placed in the edge strip to prevent punching at the supports. See Figure 5.1 for reinforcement layouts.