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A NUMERICAL STUDY OF LAMINAR AND TURBULENT NATURAL CONVECTIVE FLOW THROUGH A VERTICAL SYMMETRICALLY HEATED CHANNEL WITH WAVY WALLS

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ABSTRACT

Natural convective flow through a vertical plane channel has been considered. Both of the heated walls are kept at the same temperature. These heated walls have sharp-edged wavy surfaces. Conditions under which both laminar and turbulent flow exists in the channel have been considered. Now using a wavy heated wall can increase the heat transfer rate in external natural convective flows. Using wavy heated walls in vertical channel flows could therefore potentially also increase the heat transfer rate. However the added flow resistance resulting from the wall waviness will normally decrease the flow through the channel which would tend to decrease the heat transfer rate from the heated walls. Therefore a need existed to examine what effect the wall waviness does have on the heat transfer rate in natural convective flow through a vertical channel and it was for this reason that the present study was undertaken. The flow has been assumed to be steady and the Boussinesq approximation has been adopted. The basic *k*-epsilon turbulence model with the effects of the buoyancy forces fully accounted for has been used. The solution has the Rayleigh number, the Prandtl number, the ratio of channel width to the heated channel height, the ratio of the amplitude of the wall waviness to the heated channel height, and the ratio of the pitch of the wall waviness to the heated channel height as parameters. Results have only been obtained for a Prandtl number of 0.74 (the value for air at temperatures near ambient) and for a single value of the dimensionless pitch of the wall waviness. This leaves the Rayleigh number, the width to heated height ratio of the channel, and the amplitude of the wall waviness to the heated channel height ratio as parameters. Results have been obtained for a range of values of these parameters and the effect of these parameters on the mean Nusselt number has been studied.

INTRODUCTION

Heat transfer from the walls of a vertical channel through which there is a natural convective flow effectively occurs in a number of practical situations. Now using a wavy heated wall can increase the heat transfer rate in external natural convective flows. Using wavy heated walls in vertical channel flows could potentially also increase the heat transfer rate. However, the added flow resistance resulting from the wall waviness will normally decrease the flow through the channel which would result in a decrease in the heat transfer rate from the heated walls. Therefore, a need existed to examine what effect the wall waviness does have on the heat transfer rate in natural convective flow through a vertical channel and it was for this reason that the present study was undertaken.

Natural convective flow through a symmetrically heated vertical plane channel has been considered. Both of the heated walls are kept at the same temperature. These isothermal heated walls have sharp-edged wavy surfaces, i.e. have surfaces which periodically rise and fall. The surface waves, which have a saw-tooth (or triangular) cross-sectional shape, run normal to the direction of flow over the surface and have a relatively small amplitude. The flow situation considered is thus as shown in Figure 1. The pitch and amplitude that define the characteristics of the surface waves are shown in Figure 2. Conditions under which both laminar and turbulent flow exists in the channel have been considered. It will be noted from Figure 1 that there are adiabatic wall sections above and below the heated wall sections, i.e., the total channel height was greater the heated channel height, *H*. In obtaining the results presented here these adiabatic wall sections had a height of 0.4*H* so the total channel height was 1.8*H*.

Laminar natural convection in vertical parallel-plate channels with smooth walls has been quite extensively studied, experimental, analytical and numerical studies having been

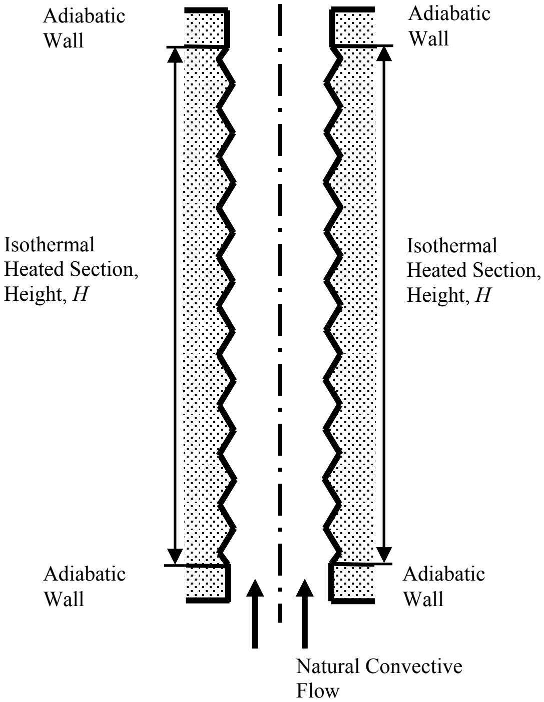

Figure 1 Flow situation considered

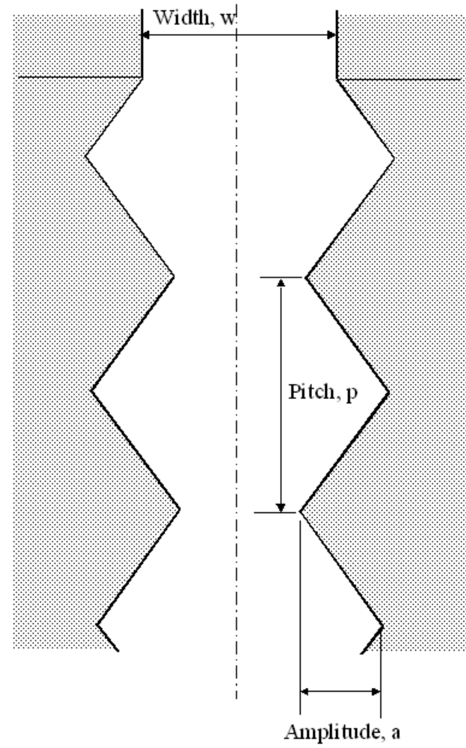

Figure 2 Definitions of amplitude and pitch of surface waviness

undertaken [1–6]. Quite a wide range of geometrical and thermal aspects of the problem such as the edge effects, interactive convection and radiation, channel aspect ratio, effect of channel wall conductivity, effect of a vent on the channel wall, variable fluid property, and flow reversals have been considered [7–36].

Less attention has been given to turbulent natural convection in vertical parallel-plate channels. Early work in this area is reported by Miyamoto et al. [37] who undertook an experimental study for the case where there is a uniform wall heat flux on the walls. Fraser et al. [38] measured velocity profiles in turbulent natural convection in an asymmetrically heated vertical channel while La Pica et al. [39] experimentally studied flow in an asymmetrically heated vertical parallel-plate channel with horizontal inlet and outlet with a uniform wall heat flux. Cheng and Mueller [40] studied turbulent natural convection coupled with radiation in large vertical channels experimentally and numerically. Habib et al. [41] undertook an experimental study of turbulent natural convection in a vertical flat plate channel. Fedorov and Viskanta [42] numerically studied turbulent natural convection in a vertical parallel-plate channel, considering both the case where there is a uniform wall heat flux at the walls and where there is a uniform wall temperature. Other studies of turbulent natural convection in channels are given in [43–48].

Natural convection flows in vertical channels with obstructions occurs in a number of engineering applications and a number of experimental and numerical studies have been undertaken in this area [49–60]. However none of these deal with the type of wavy wall situation considered here.

There have been a number of previous studies of natural convective heat transfer from vertical plates with various types of wavy surface, most of these studies being based on the assumption that the flow is laminar. Yao [61] studied natural convection from a semi-infinite, vertical, sinusoidal, isothermal surface while Moulic et al. [62] studied heat transfer from the same type of surface for the case where there is a uniform heat flux at the surface. In both cases, the local Nusselt number was found to vary periodically with half the wavelength of the plate. Kumari et al. [63] undertook a numerical study of natural convection to a non-Newtonian power-law fluid from a semi-infinite, vertical plate with an isothermal surface having sinusoidal waves, a similar study also being undertaken by Kim [64]. These authors also found that the local Nusselt number varied periodically with half the wavelength of the plate and Kim [64] found that the local Nusselt number decreased with increasing surface wave amplitude. Rees and Pop [65,66] studied natural convection from wavy surfaces in porous media. A numerical study of natural convective flow from a surface in which the "waves" are normal to the direction of flow is described by Oosthuizen and Garrett [67]. Oosthuizen [68] considered laminar, transitional and turbulent flow over an isothermal plate with sharp triangular waves.

NOMENCLATURE

A	[-]	Dimensionless amplitude of surface waves.
A_F	[m^2]	Frontal area of heated surface
a	[m]	Amplitude of surface waves
g	[m^2/s]	Gravitational acceleration
H	[m]	Height of heated surfaces
k	[W/mK]	Thermal conductivity

Nu	[-]	Nusselt number based on frontal area of heated surface and on channel height
Nu_l	[-]	Local Nusselt number based on channel height
Nu_w	[-]	Nusselt number based on frontal area of heated surface and on channel width
P	[-]	Dimensionless pitch of surface waves
Pr	[-]	Prandtl number
p	[m]	Pitch of surface waves
Q'	[W]	Mean heat transfer rate from heated surface
q'	[W/m^2]	Local heat transfer rate per unit area
Ra	[-]	Rayleigh number based on channel height
Ra_w	[-]	Rayleigh number based on channel width
T_w	[K]	Wall temperature of heated surfaces
T_a	[K]	Fluid temperature at channel inlet
W	[m]	Dimensionless channel width
w	[m]	Channel width

Special characters

α	[m^2/s]	Thermal diffusivity
β	[K^{-1}]	Bulk expansion coefficient
ν	[m^2/s]	Kinematic viscosity

SOLUTION PROCEDURE

The flow has been assumed to be steady and fluid properties have been assumed constant except for the density change with temperature that gives rise to the buoyancy forces, this being treated by means of the Boussinesq type approximation. A standard *k*-epsilon turbulence model with the effects of the buoyancy forces being fully accounted for has been used in obtaining the solution. The channel has been assumed to be wide in the transverse direction so edge-effects are not accounted for, i.e. the flow has basically been assumed to be two-dimensional. However, because the possibility exists that a three-dimensional flow pattern could develop when transition from laminar to turbulent flow is occurring, a three-dimensional solution covering part of the channel width has been adopted. However, in no case was such three-dimensional flow found to occur. The governing equations were solved using the commercial finite-volume based cfd software package Fluent. Extensive grid independence and convergence-criteria independence testing was undertaken and this indicated that the heat transfer results given here are grid and convergence criteria independent to within about one per cent. The mean heat transfer rate per unit surface area can be expressed relative to the actual surface area or relative to the projected normal surface area, the Nusselt numbers based on these two mean heat transfer rates being denoted by Nu_T and *Nu* respectively.

RESULTS

The solution has the following parameters:

- the Rayleigh number, *Ra*, based on the height of the heated channel wall section, *H*, and the overall temperature difference $T_w - T_a$, i.e.:

$$Ra = \frac{\beta g\left(T_w - T_a\right)H^3}{\nu\alpha} \tag{1}$$

- the Prandtl number, *Pr*
- the dimensionless amplitude, *A*, the amplitude being expressed relative to the height of the heated channel wall section, *H,* i.e.:

$$A = \frac{a}{H} \tag{2}$$

- the dimensionless pitch, *P,* the pitch also being expressed relative to the height of the heated channel wall section, *H,* i.e.:

$$P = \frac{p}{H} \tag{3}$$

- the dimensionless width of the channel, *W*, the width also being expressed relative to the height of the heated channel wall section, *H*, i.e.:

$$W = \frac{w}{H} \tag{4}$$

Results have only been obtained here for *Pr* = 0.74 and for a dimensionless pitch of 0.095. Results for other dimensionless pitch values showed the same basic characteristics as those for a dimensionless pitch value of 0.095. This leaves *Ra, A,* and *W* as parameters. Results are presented here for *Ra* values between approximately 10^5 and 10^{16} and for *A* values between 0 and approximately 0.025.

The mean heat transfer rate from the heated walls has been expressed in terms of two Nusselt numbers:

$$Nu = \frac{Q'H}{k\,A_F\,(T_w - T_a)} \quad \text{and} \quad Nu_T = \frac{Q'H}{k\,A_T\,(T_w - T_a)} \tag{5}$$

where *Q'* is the mean heat transfer rate from the heated wavy walls and A_F and A_T are the frontal area and the total surface area of the heated wall surfaces respectively.

Attention will first be given to the case where the heated walls are smooth, i.e., where *A*=0. At low values of *Ra* and *W* the flow will be fully developed while at the larger values of *Ra* and *W* the flow essentially consists of separate non-interacting boundary layer flows over the two heated walls. If the flow remains laminar, the Nusselt numbers for the two limiting cases are given by:

(a) Fully-Developed Flow

$$Nu_w = \frac{WRa_w}{12} \tag{6}$$

(b) Boundary Layer Flow

$$Nu_w = 0.619\left(WRa_w\right)^{0.25} \tag{7}$$

Based on their experimental and numerical results, Azevedo and Sparrow [15] derived the following correlation equation that applies for all values of *Ra* and *W* provided that the flow remained laminar:

$$Nu_w = \left\{\left(\frac{WRa_w}{12}\right)^{-2} + \left[0.619\left(WRa_w\right)^{0.25}\right]^{-2}\right\}^{-0.5} \tag{8}$$

The variation of Nu_w with WRa_w given by this equation is compared with the results obtained here for *A* = 0 in Figure 3. It will be seen that at the smaller values of WRa_w while the present numerical results are such that Nu_w is a function of WRa_w, the form of the function differs from that given by equation (8). This is because in the present study the overall height of the channel is greater than *H* because of the presence of the unheated sections below and above the heated wall

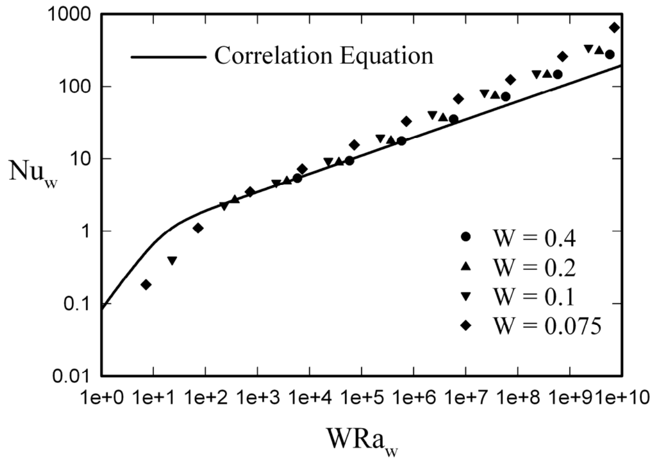

Figure 3 Comparison of the variation of Nusselt number based on channel width with WRa_w for a channel with a smooth surface, i.e. A = 0, as given by the Sparrow and Azevedo [12] correlation equation with the present results for A = 0 and various values of W

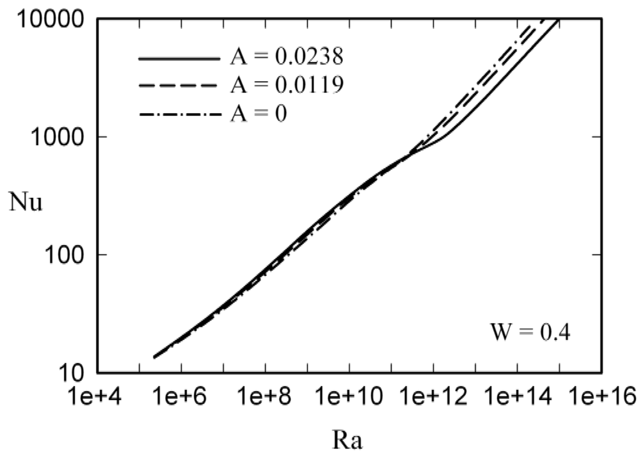

Figure 4 Variation of Nusselt number with Ra for W = 0.4 and for three values of the dimensionless amplitude of the surface waviness

section. At larger values of WRa_w in the boundary layer type flow regime the present results are in good agreement with equation (9) as long as the flow remains laminar. When transition occurs Nu_w is no longer a function of WRa_w alone and the Nusselt numbers are higher than those given by equation (9).

Turning next to a consideration of the effect of the wall waviness on the heat transfer rate, Figure 4 and Figure 5 show typical variations of mean Nusselt number, Nu, with Rayleigh number for A values of 0, 0.0119, and 0.0238. Figure 4 shows results for W = 0.4 while Fig. 5 shows results for W = 0.2. It will be seen from these two figures that, for the conditions considered, the wall waviness has a relatively weak effect on the Nusselt number variation, the wall waviness effect being more pronounced in the turbulent flow region than in the laminar flow region. In the laminar boundary flow region the

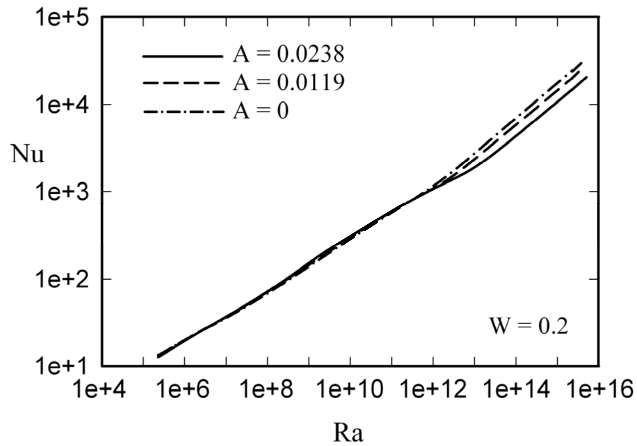

Figure 5 Variation of Nusselt number with Ra for W = 0.2 and for three values of the dimensionless amplitude of the surface waviness

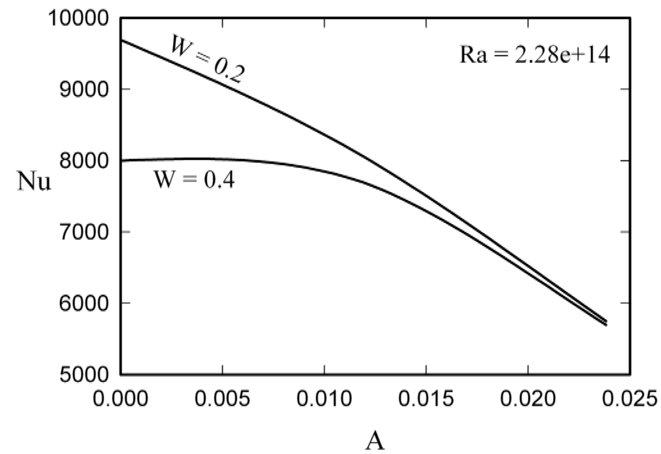

Figure 6 Variation of Nusselt number with dimensionless amplitude of the surface waviness, A, for two values of W and for $Ra = 2x10^{14}$

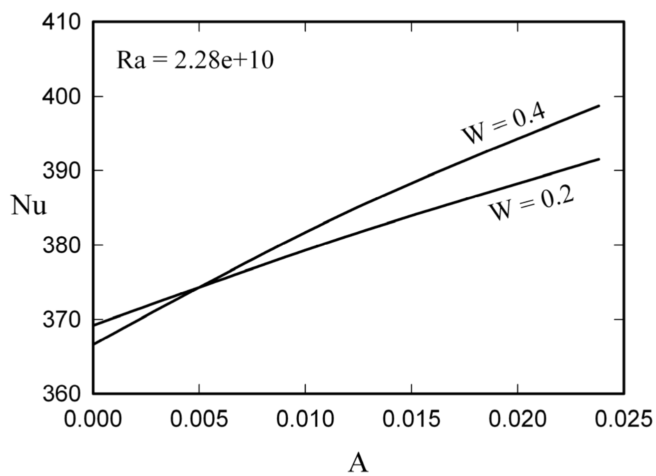

Figure 7 Variation of Nusselt number with dimensionless amplitude of the surface waviness, A, for two values of W and for $Ra = 2x10^{10}$.

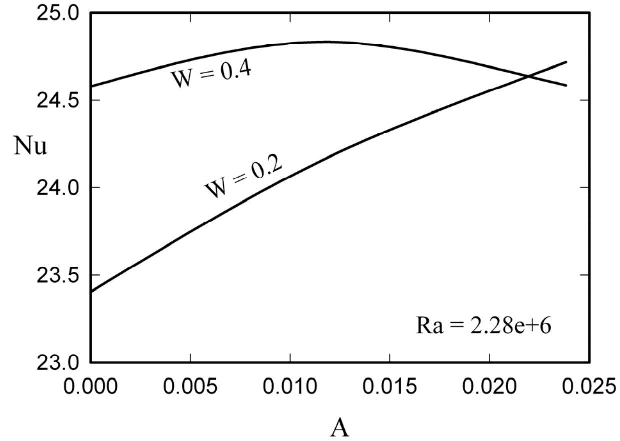

Figure 8 Variation of Nusselt number with dimensionless amplitude of the surface waviness, *A*, for two values of *W* and for $Ra = 2\text{x}10^6$

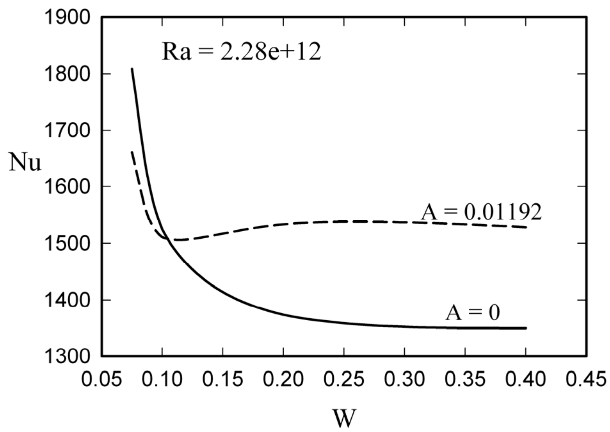

Figure 9 Variation of Nusselt number with dimensionless channel width, *W*, for two values of the dimensionless amplitude of the surface waviness, *A*, for $Ra = 2\text{x}10^{12}$

waviness can produce a small increase in the Nusselt number while in the turbulent flow region a decrease in the Nusselt number is produced. These effects are shown more clearly by the results given in Figure 6 to Figure 8 which illustrate the form of the variation of Nusselt number with dimensionless surface waviness amplitude, *A*, for two values of *W* and for three values of *Ra,* each of the figures giving results for a different value of *Ra* .

The effect of the dimensionless channel width, *W*, on the wall heat transfer rate, i.e., on the Nusselt number, is illustrated by the results shown in Figure 9 to Figure 11. These figures show the variation of Nusselt number with *W* for two values of the dimensionless surface waviness amplitude, *A*, for three values of *Ra*, each of the figures giving results for a different value of *Ra*. It will be seen from these results that at the larger values of *Ra* and *W* the dimensionless channel width has a comparatively small effect on the Nusselt number, the flow then being in the boundary layer flow regime. It will also be seen that the form of the variation of *Nu* with *W* is dependent on the values of *Ra* and *A*.

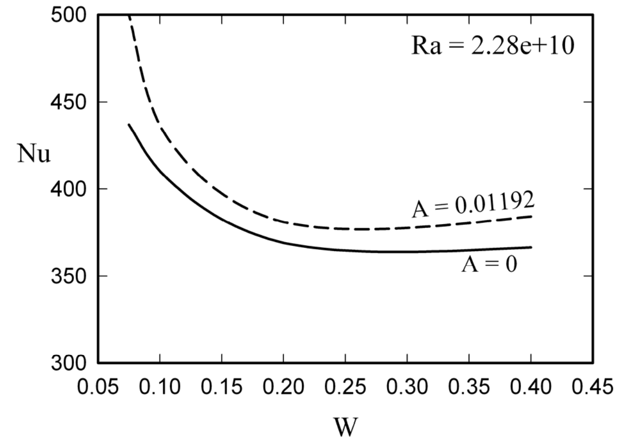

Figure 10 Variation of Nusselt number with dimensionless channel width, *W*, for two values of the dimensionless amplitude of the surface waviness, *A*, for $Ra = 2\text{x}10^{10}$

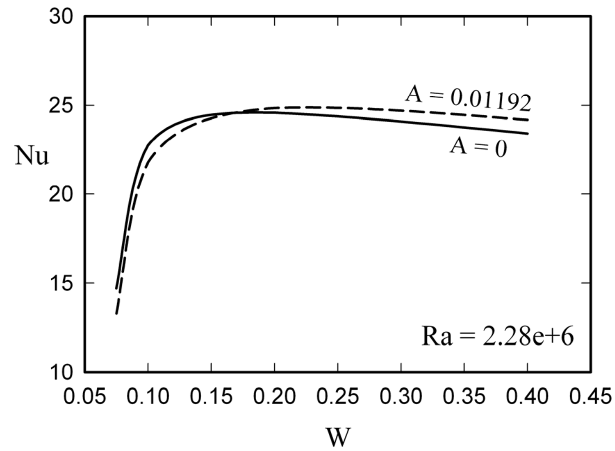

Figure 11 Variation of Nusselt number with dimensionless channel width, *W*, for two values of the dimensionless amplitude of the surface waviness, *A*, for $Ra = 2\text{x}10^6$

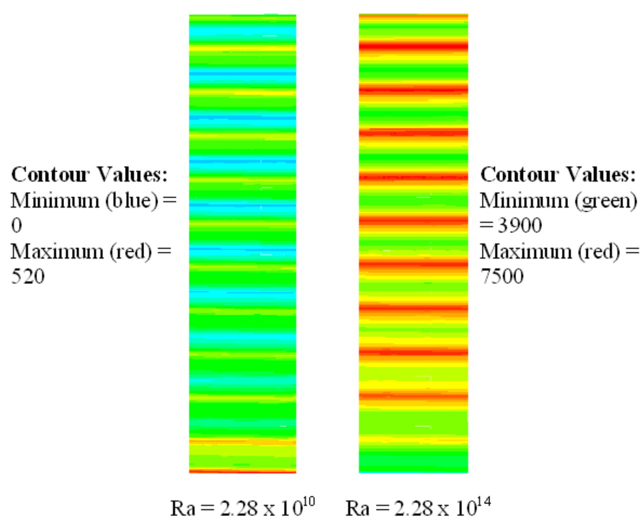

Figure 12 Typical local Nusselt number, Nu_l, contours. These results are for $A = 0.02382$ and $W = 0.4$ for the Rayleigh numbers indicated.

Two typical local Nusselt number distributions over the surface of the heated plate are shown in Figure 12. It will be seen from these results that the local heat transfer rate is much higher at the top of the wall waves that at the valleys between these waves.

CONCLUSIONS

The results of the present study indicate that for the flow situation considered:

- The presence of the surface waviness has only a relatively small effect on the mean Nusselt number values under all conditions covered in the present study.
- At the lower values of Ra considered the waviness produces almost no change in the Nusselt number, at the intermediate values of *Ra* considered the waviness produces a small increase in the Nusselt number while at the higher values *Ra* considered the waviness produces a decrease in the Nusselt number, the heat transfer rate being based on the projected frontal area of the heated surfaces.

Turbulence starts to have a significant effect on the mean heat transfer rate at approximately the same Rayleigh numbers for all condition considered in the present study

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REFERENCES

[1] Elenbaas, W., Heat dissipation of parallel plates by free convection, *Physica,* Vol. 9, No. 1, 1942, pp. 1–28.

[2] Ostrach, S., Laminar natural convection flow heat transfer of fluids with and without heat sources in channels with constant wall temperatures, *NACA Technical Note*, *2863*, 1952.

[3] Bodoia, J.R., and Osterle, J.F., The development of free convection between heated vertical plates, *Journal of Heat Transfer*, Vol. 84, 1962, pp. 40–44.

[4] Engel, R.K., and Mueller, W.K., An analytical investigation of natural convection in vertical channels, *ASME HT 16*, Paper No. 67, 1967.

[5] Aung, W., Fletcher, L.S., and Sernas, V., Developing laminar free convection between vertical flat plates with asymmetric heating, *International Journal of Heat and Mass Transfer*, Vol. 15, 1972, pp. 2293–2308.

[6] Naylor, D., Floryan, J.M., and Tarasuk, J.D., A numerical study of developing free convection between isothermal vertical plates, *Journal of Heat Transfer*, Vol. 113, 1991, pp. 620–626.

[7] Sparrow, E.M., and Bahrami, P.A., Experiments on natural convection from vertical parallel plates with either open or closed edges, *Journal of Heat Transfer,* Vol. 102, 1980, pp. 221–227.

[8] Li, Z., Gui, Y. and Guo, Z., Experimental investigation on the edge effect of natural convection between two vertical plates, in: R.K. Shah, E.N. Ganic, K.T. Yang (Eds.), *Experimental Heat Transfer, Fluid Mechanics, and Thermodynamics*, Elsevier, New York, 1988, pp. 419–424.

[9] Carpenter, J.R., Briggs, D.G. and Sernas, V., Combined radiation and developing laminar free convection between vertical flat plates with asymmetric heating, *Journal of Heat Transfer,* Vol. 98, 1976, pp. 95–100.

[10] Sparrow, E.M., Shah, S. and Sernas, V., Natural convection in a vertical channel: I. Interacting convection and radiation, II. The vertical plate with and without shrouding, *Numerical Heat Transfer*, Vol. 3, 1980, pp. 297–314.

[11] Moutsoglou, A., Rhee, J.H. and Won, J.K., Natural convection–radiation cooling of a vented channel, *International Journal of Heat and Mass Transfer*, Vol. 35, 1992, pp. 2855–2863.

[12] Sparrow, E.M. and Azevedo, L.F.A., Vertical channel natural convection spanning between the fully developed limit and single-plate boundary layer limit, *International Journal of Heat and Mass Transfer*, Vol. 28, 1985, pp. 1847–1857.

[13] Burch, T., Rhodes, T. and Acharya, S., Laminar natural convection between finitely conducting vertical plates, *International Journal of Heat and Mass Transfer*, Vol. 28, 1985, pp. 1173–1186.

[14] Kim, S., Anand, N.K. and Aung, W., Effect of wall conduction on free convection between asymmetrically heated vertical plates: uniform wall heat flux, *International Journal of Heat and Mass Transfer*, Vol. 33, 1990, 1013–1023.

[15] Azevedo, L.F.A. and Sparrow, E.M., Natural convection in a vertical channel vented to the ambient through an aperture in the channel wall, *International Journal of Heat and Mass Transfer*, Vol. 29, 1986, pp. 819–830.

[16] Aihara, T. and Maruyama, S., Laminar free convective heat transfer in vertical uniform heat flux ducts (numerical solution with constant/temperature-dependent fluid properties), *Heat Transfer-Japanese Research,* Vol. 15, 1986, pp. 69–86.

[17] Sparrow, E.M., Chrysler, G.M. and Azevedo, L.F., Observed low reversals and measured-predicted Nusselt numbers for natural convection in a one-sided heated vertical channel, *Journal of Heat Transfer,* Vol. 106, 1984, pp. 325–332.

[18] Aung, W. and Worku, G., Developing flow and flow reversal in a vertical channel with asymmetric wall temperatures, *Journal of Heat Transfer,* Vol. 108, 1986, pp. 299–304.

[19] Auletta, A.A. and Manca, O., Entropy generation in natural convection in a symmetrically and uniformly heated vertical channel, *International Journal of Heat and Mass Transfer*, Vol. 49, 2006, pp. 3221–3228.

[20] Andreozzi, A., Campo, A. and Manca, O., Compounded natural convection enhancement in a vertical parallel-plate channel, *International Journal of Thermal Sciences,* Vol. 47, 2008, pp. 742–748.

[21] Andreozzi, A, Buonomo, B. and Manca, O., Transient Natural Convection in Vertical Channels Symmetrically Heated at Uniform Heat Flux, *Numerical Heat Transfer, Part A*, Vol. 55, 2009, pp. 409–431.

[22] Ayinde, T.F., Said, S.A. and Habib, M.A., Turbulent natural convection flow in a vertical channel with anti-symmetric heating, *Heat and Mass Transfer,* Vol. 44, 2008, pp.1207–1216. DOI 10.1007/s00231-007-0359-z .

[23] Bianco, N., Langellotto, L., Mancab, O., and Nardin, S., Thermal design and optimization of vertical convergent channels in natural convection, *Applied Thermal Engineering,* Vol. 26, 2006, pp. 170–177.

[24] Chappidi, P.R., and Eno, B.E., Natural convection in a heated parallel plate vertical channel; influence of inlet conditions, *Proceedings of the 1988 ASME National Heat Transfer Conference*, *HTD*, Vol. 96, 1988, pp. 135-143.

[25] Cheng, C-Y., Fully developed natural convection heat and mass transfer of a micropolar fluid in a vertical channel with asymmetric wall temperatures and concentrations, *International Communications in Heat and Mass Transfer,* Vol. 33, 2006, pp. 627–635.

[26] Higuera, F.J., and Ryazantsev, Y.S., Natural convection flow due to a heat source in a vertical channel, *International Journal of Heat and Mass Transfer*, Vol. 45, 2002, pp. 2207–2212.

[27] Leith, J.R., Thermal design considerations in vertical-channel natural convection, *Journal of Heat Transfer,* Vol. 109, No. 1, 1987, pp. 249-251.

[28] Lin, S.C., Chang, K.P., and Hung, Y.H., Natural convection within a vertical finite-length channel in free space**,** *Journal of Thermophysics and Heat Transfer*, Vol. 8, No. 2, 1994, pp. 366-368.

[29] Manca, O., Musto, M., and Naso, V., Experimental investigation of natural convection in an asymmetrically heated vertical channel with an asymmetric chimney, *Journal of Heat Transfer,* Vol. 127, 2005, pp, 888-896.

[30] Morrone, B., Campo, A., and Manco, O., Optimum plate separation in vertical parallel-plate channels for natural convective flows: incorporation of large spaces at the channel extremes, *International Journal of Heat and Mass Transfer*, Vol. 40, No. 5, 1997, pp. 993-1000.

[31] Naylor, D., and Tarasuk, J.D., Natural convective heat transfer in a divided vertical channel. Part I. Numerical study, *Journal of Heat Transfer,* Vol. 115, No. 2, 1993, pp. 377-387.

[32] Naylor, D. and Tarasuk, J.D., Natural convective heat transfer in a divided vertical channel. Part II. Experimental study, *Journal of Heat Transfer,* Vol. 115, No. 2, 1993, pp 388-394.

[33] Rodrigues, A.M., Canha da Piedade, A., Lahellecb, A., and Grandpeix, J.Y., Modelling natural convection in a heated vertical channel for room ventilation, *Building and Environment*, Vol. 35, 2000, pp. 455-469.

[34] Wanga, X., and Pepper, D.W., Numerical simulation for natural convection in vertical channels, *International Journal of Heat and Mass Transfer*, Vol. 52, 2009, pp. 4095–4102.

[35] Webb, B.W., and Hill, D.P., 1989, High Rayleigh number laminar natural convection in an asymmetrically heated vertical channel, *Journal of Heat Transfer,* Vol., 111, No. 3, 1989, pp. 649-656.

[36] Yan, W.M., and Lin, T.F., 1987, Natural Convection Heat Transfer in Vertical Open Channel Flows with Discrete Heating, *International Communications in Heat and Mass Transfer*, Vol. 14, pp. 187–200.

[37] Miyamoto, M., Katoh, Y., Kurima, J. and Sasaki, H., Turbulent free convection heat transfer from vertical parallel plates, in: C.I. Tien, V.P. Carey, J.K. Ferrel (Eds.), *Heat Transfer*, Vol. 4, Hemisphere, Washington, DC, 1986, pp. 1593–1598.

[38] S.M. Fraser, A. Gilchrist, T. Yilmaz, Natural convection LDA measurements, in: A. Dybbs, B. Ghorashi (Eds.), *Laser Anemometry Advances and Applications,* ASME, New York, 1991, pp. 547–554.

[39] La Pica, A., Rodono, G., and Volpes, R., An experimental investigation on natural convection of air in a vertical channel, *International Journal of Heat and Mass Transfer*, Vol. 36, 1993, pp. 611–616.

[40] Cheng, X., and Mueller, U., Turbulent natural convection coupled with thermal radiation in large vertical channels with asymmetric heating, *International Journal of Heat and Mass Transfer*, Vol. 41, 1998, pp. 1681–1691.

[41] Habib, M.A., Said, S.A.M., Ahmed, S.A., and Asghar, A., Velocity characteristics of turbulent natural convection in symmetrically and asymmetrically heated vertical channels, *Experimental Thermal and Fluid Sciences*, Vol. 36, 2002, pp. 77–87.

[42] Fedorov, A.G., and Viskanta, R., Turbulent natural convection heat transfer in an asymmetrically heated, vertical parallel-plate channel, *International Journal of Heat and Mass Transfer*, Vol. 40, 1997, pp. 3849–3860.

[43] Habib M.A., Said S.A.M., and Ahmed S.A., Velocity characteristics of turbulent natural convection in convergent-plates vertical channels. *International Journal of Fluid Mechanics Research*, Vol. 29, No. 2, 2002, pp. 158–178.

[44] Yilmaz T., and Gilchrist A., Temperature and velocity field characteristics of turbulent natural convection in a vertical parallel-plate channel with asymmetric heating, *Heat and Mass Transfer*, Vol. 43, 2007, pp. 707–719.

[45] Yilmaz T., and Fraser S.M., Turbulent natural convection in a vertical parallel-plate channel with asymmetric heating, *International Journal of Heat and Mass Transfer*, Vol. 50, 2007, pp. 2612–2623.

[46] Pallares, J., Vernet, A., Ferre, J.A., and Grau, F.X., Turbulent large-scale structures in natural convection vertical channel flow, *International Journal of Heat and Mass Transfer*, Vol. 53, 2010, pp. 4168–4175.

[47] Shiri, A., and George, W.K., Turbulent natural convection in a differentially heated vertical channel , *Proceedings of the 2008 ASME Summer Heat Transfer Conference,* , Vol. 1, 2008, pp. 286-291.

[48] Versteegh, T.A.M., and Nieuwstadt, F.T.M., Turbulent budgets of natural convection in an infinite, differentially heated, vertical channel, *International Journal of Heat and Fluid Flow*, Vol. 19, 1998, pp. 135-149.

[49] Cruchaga, M., and Celentano, D., Modeling natural and mixed convection in obstructed channel, *International*

Journal of Numerical Methods in Heat and Fluid Flow, Vol. 13, No.1, 2003, pp. 57–84.
[50] Viswatmula, P., and Amin, M.R., Effects of multiple obstructions on natural convection heat transfer in vertical channels, *International Journal of Heat and Mass Transfer*, Vol. 38, No. 39, 1995, pp. 2039–2046.
[51] Wirtz, R.A., and Stutzman, R.J., Experiments on natural convection between vertical plates with symmetric heating, *Journal of Heat Transfer,* Vol. 104, 1982, pp. 501–507.
[52] [52] Said, S.A.M., and Krane, R.J., An analytical and experimental investigation of natural convection heat transfer in vertical channels with a single obstruction, *International Journal of Heat and Mass Transfer*, Vol. 33, No. 6, 1990, pp. 1121–1134.
[53] Burch, T., Rhodes, T., and Acharya, S., Laminar natural convection between finitely conducting vertical plates, *International Journal of Heat and Mass Transfer*, Vol. 28, 1985, pp. 1173–1186.
[54] Desrayaud, G., and Fichera, A., Laminar natural convection in a vertical isothermal channel with symmetric surface-mounted rectangular ribs, *International Journal of Heat Transfer and Fluid Flow,* Vol. 23, 2002, pp. 519–529.
[55] Hung, Y.H., and Shiau, W.M., Local steady-state natural convection heat transfer in vertical parallel plates with a two-dimensional rectangular rib, *International Journal of Heat and Mass Transfer*, Vol. 31, No. 6, 1988, pp. 1279–1288.
[56] Andreozzi, A., and Manca, O., Natural convection in vertical channels with an auxiliary plate," *International Journal of Numerical Methods for Heat and Fluid Flow,* Vol. 12 No. 6, 2002, pp. 716-734.
DOI 10.1108/09615530210438364
[57] Bessaih, R., and Kadja, M., Turbulent natural convection cooling of electronic components mounted on a vertical channel, *Applied Thermal Engineering*, Vol. 20, 2000, pp. 141-154.
[58] Daloglu, A. and Ayhan, T., Natural convection in a periodically finned vertical channel, *International Communications in Heat and Mass Transfer*, Vol. 26, No. 8, 1999, pp. 1175-1182.
[59] Kelkar, K.M., and Choudhury, D., Numerical prediction of periodically fully developed natural convection in a vertical channel with surface mounted heat generating blocks, *International Journal of Heat and Mass Transfer*, Vol. 5, 1993, pp. 1133-1145.
[60] Mehrotra, A., and Acharya, S., Experimental study of natural convection in smooth and ribbed vertical channels, *Proceedings of the ASME 1990 Winter Annual Meeting, HTD,* 1990, Vol. 144, pp. 23-31.
[61] Yao, L.S., Natural convection along a vertical wavy surface, *Journal of Heat Transfer,* Vol. 105, 1983, pp. 465-468.
[62] Moulic, S., Ghosh, P., and Yao, L.S., Natural convection along a vertical wavy surface with uniform heat flux, *Journal of Heat Transfer,* Vol. 111, 1989, pp. 1106-1108.
[63] Kumari, M., Pop, I., and Takhar, H.S., Free-convection boundary-layer flow of a non-Newtonian fluid along a vertical wavy surface, *International Journal of Heat and Fluid Flow*, Vol. 18, 1997, pp. 625-631.
[64] Kim, E., Natural convection along a wavy vertical plate to non-Newtonian fluids, *International Journal of Heat and Mass Transfer*, Vol. 40, No. 13, 1997, pp. 3069-3078.
[65] Rees, D.A.S., and Pop, I., A note on free convection along a vertical wavy surface in a porous medium, *Journal of Heat Transfer,* Vol. 116, 1994, pp. 505-508.
[66] Rees, D.A.S., and Pop, I., Free convection induced by a vertical wavy surface with uniform heat flux in a porous medium, *Journal of Heat Transfer*, Vol. 117, 1995, pp. 547-550.
[67] Oosthuizen, P.H., and Garrett, M., A numerical study of natural convective heat transfer from an inclined plate with a "wavy" surface, *Proceedings of the ASME 2001 National Heat Transfer Conference*, *2001. Paper NHTC2001-20205.*
[68] Oosthuizen, P.H., A numerical study of laminar and turbulent natural convective heat transfer from an isothermal vertical plate with a wavy surface, *Proceedings of the ASME 2010 International Mechanical Engineering Congress & Exposition IMECE2010*, 2010, Paper IMECE2010-38167.