

**THEORETICAL VALIDATION OF TEST RESULTS FOR THE PRESSURE DROP  
VALUES OF CIRCULAR PINS WITH A MAXIMUM LENGTH TO DIAMETER RATIO  
OF 3.0 USING EXISTING EQUATIONS AND TEST DATA FOR HEAT EXCHANGER  
APPLICATION**

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### ABSTRACT

Pins are a very common type of extended surface used in the field of heat transfer; their main use being in the electronics field. In this report, the use of pins as an extended surface is considered for a Heat Exchanger application in the aerospace field. The Heat Exchanger uses forced convective heat transfer mechanism for the dissipation of heat and the implicated fluid is air. For this application the pin layout and design is completely unique in that the pin's maximum length to diameter ratio is 3.0 and the layout of the pins produces an  $X_T$  value of 7, which has not been explored in any previous work. The Length: Diameter ratio of these new pins is very small when compared to the Length: Diameter ratios of tubes currently used in heat exchangers to enhance heat transfer. Moreover, the distance between the pins in this arrangement is much greater than those for the tubes. Testing has been performed on this pin design and the theoretical validation of those test results is one of the main aspects discussed in this report. Due to the innovative nature of the pin designs, there is insufficient existing test data or established equations that can be used. Assumptions are made in order to be able to apply the current equations for pressure drop calculations with valid justifications. The theoretical results for the total pressure drop show an average deviation of 6% from the test results for mass flow rates between 0.14 kg/s and 0.36 kg/s. The maximum pressure drop was found to be caused by the pins and it was in the range of 89%-91% of the total. In this article, the limitations of existing equations are discussed and the gap in the theoretical knowledge regarding novel pin designs is highlighted.

### INTRODUCTION

The design of a heat exchanger is mainly governed by the pressure drop allowed through the system [1]. Throughout the

years, there have been two popular types of extended surfaces, namely, pins and fins, which have been widely used in various shapes and sizes for different heat exchangers [2-13]. Pins and fins promote heat transfer but at the same time they increase pressure drop. Extensive work has been done to investigate the optimum pin and fin design for maximum heat transfer with minimum pressure drop. Short pins as extended heat transfer surfaces have been most exploited in the heat sink application [8-13].

### NOMENCLATURE

$C_d$		drag coefficient
$d$	[m]	diameter
$D_h$	[m]	hydraulic diameter
$G$	[m/s]	Maximum mass velocity
$S$	[m <sup>2</sup> ]	frontal area
$u$	[m/s]	velocity of fluid
$w$	[m]	width of layer
$F_d$	[N]	Drag force
$A_d$	[m <sup>2</sup> ]	Cross sectional area of duct

Special characters		
$\mu$	[Pa s]	Dynamic viscosity
$\rho$	[kg/m <sup>3</sup> ]	Density
$\nu$	[m <sup>2</sup> /s]	Kinematic viscosity

Subscript	
Eq	Equation
f	friction factor
HE	Heat Exchanger
Re	Reynolds Number
Z	correction factor for tube bundle arrangement.

In this research, short pins are being used in heat exchangers specifically designed for aircrafts. The pins being investigated in this study are circular in cross section, have a maximum Length: Diameter ratio of 3.0 and follow a staggered arrangement. The pins are also equipped with fins on their surface to further enhance the heat transfer.

Historically, tubes have been used in heat exchangers for increased heat transfer. The main difference between the pins and the tubes is that the diameter of the tube is negligible in comparison to the length of the tube, whereas this is not necessarily the case for pins.

The heat exchanger block of 230.5mm by 88.4mm by 72.6mm is tested at room temperature, with air as the working fluid, to obtain pressure drop values for this particular design. The test results are then validated through a theoretical check.

This article illustrates the calculation method for pressure drop across pins, fins, the plate surface and the duct. The main contributor to the total pressure drop is the pins. It constitutes on average 90% of the total pressure drop and, consequently, the pins are studied in greater detail.

Since the pin design is novel, externally, there is limited supporting test data available to validate the test results gathered during this investigation. Even the existing equations have their limitations, in that, they are not directly applicable to this novel pin design. Consequently, a few assumptions have to be made during the calculations in order to use the existing equations.

This undoubtedly gives rise to differences between the test and theoretical values for the pressure drop. Both sets of results are compared, analysed and justification statements provided for the deviations.

### CALCULATION OF PRESSURE DROP VALUES

There are many equations that have been developed over the years to calculate the pressure drop experienced by fluids when they flow over a series of pins. Each equation is best suited to specific types of flows and conditions.

Historically, the “pin surface technology” used in traditional heat exchanger has been classified as a tube. This is a classification mainly driven by the geometry of the pins. Pins that are included in the design of heat exchangers to increase their thermal performance normally have diameters that are considered negligible in comparison with their heights and therefore are termed as tubes. As a result of this, most equations or theories currently developed tend to take that assumption into consideration.

For this investigation, a heat exchanger with a pin array arrangement is investigated. To calculate the pressure drop for the pins, the equation (1) is usually recommended. This equation is normally used to calculate the pressure drop for fluids flowing across tube bundles. It takes into account the arrangement of the tubes, that is, staggered or in line. Since the pins, being studied, have a staggered arrangement too, this equation was the first choice as it calculates the pressure drop with the acknowledgement of the pins’ arrangement. The main assumption when using this equation is that the pins act as tubes. This is not very accurate given that the pin’s diameter is not negligible when compared to its length. However this is the

only equation that provides correction factors, which account for the type of pin arrangement.

$$\delta P = \frac{\left( f * \left( N * G^2 \right) * Z \right)}{(2 * \rho)} \quad (1) \quad [14]$$

where

f – friction factor

N – number of tube rows in direction of flow

G -  $\rho * u$

-Maximum mass velocity

$\rho$  - Density of fluid

Z – correction factor for effects of tube bundle arrangement.

The main variables that determine if this equation can be applied are the values for f and Z. These values are based on historical data and graphs. The other variables such as velocity and number of rows can be easily calculated. As for the value of fluid density, it can be obtained from the data sheets available which lists the properties of the fluid at different temperatures.

For square or equilateral triangle tube arrangement, Z =1. However, the particular pin arrangement being studied does not have the square or equilateral arrangement; consequently, the values for Z are obtained from the graphs derived from existing data. [19]

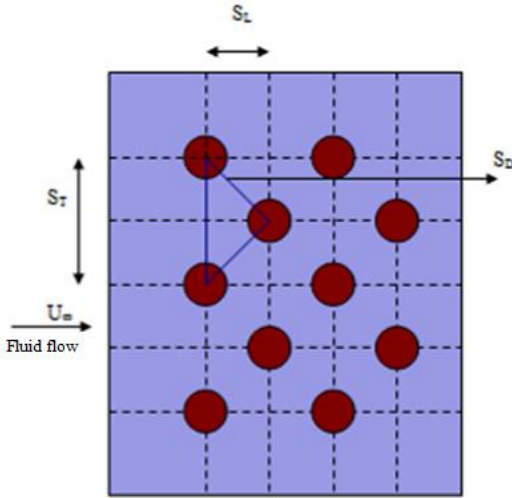
The values for  $x_T$ ,  $x_L$  and  $x_D$  are derived from the pin arrangement.

$$x_T = \frac{S_T}{d} \quad (2)$$

$$x_L = \frac{S_L}{d} \quad (3)$$

$$x_D = \frac{S_D}{d} \quad (4)$$

In the above equations, d is the diameter of the pin. Figure 1 illustrates what  $S_T$ ,  $S_L$  and  $S_D$  represent.



**Figure 1** Diagram showing the variables.  $S_T$ ,  $S_L$  &  $S_D$  in a staggered pin arrangement

The friction factor values for the corresponding Reynolds numbers are obtained from the existing graphs [19]. Unfortunately, the amount of data available is only for a limited number of tube arrangements. The graph provides friction factors only for staggered tube arrangements with the following values of  $x_T$ :  $x_T = 1.25, 1.5, 2.0$  &  $2.5$ . Small values of  $x_T$  imply that the tubes are very close to each other.

For the pin arrangement being studied, the value of  $x_T$  is 7. Therefore, the friction factor data was not readily available to read from the graph [19]

The  $x_T$  values for the pin arrangement under study were much greater than the ones studied previously. This meant that the pins were quite far from each other and therefore it was deemed that a fair assumption would be to treat the pins as separate pins instead of a pin arrangement. Consequently, the approach adopted to calculate the pressure drop resulting from the pins was the one that is utilised to calculate the pressure drop across individual pins.

First of all, the drag force is calculated for each pin and then the value is multiplied by the number of pins in the path of the fluid flow. The total drag force,  $F_D$ , obtained is subsequently divided by the cross sectional area of the duct ( $A_d$ ) to yield the pressure drop due to the pins,  $Pins\delta P$ . This is shown in equation (14).

$$F_D = \frac{1}{2} * \rho * S * C_d * u^2 * n \quad (5)$$

where,

- $\rho$  - density of fluid
- $S$  - frontal area of pin
- $C_d$  - drag coefficient
- $u$  - velocity of fluid
- $n$  - number of pins in the flow path

The frontal area of the pin,  $S$  is given by

$$S = \pi * d * l \quad (6)$$

where  $d$  is the diameter of pin and  $l$  is the length of pin

The drag coefficient is dependent on the type of flow. The drag coefficients are obtained from existing graphs [20], based on the Reynolds numbers for the pins under different conditions.

First for each mass flow rate being investigated, the Reynolds numbers for the pins are calculated. The equation used to calculate the Reynolds number for the pins is as shown below

$$Re = \frac{(\rho * u * d)}{\mu} \quad (7)$$

Where:-

- $\mu$  - Absolute viscosity
- $u$  - velocity of fluid
- $\rho$  - Density of fluid
- $d$  - diameter of pin

Normally, to calculate the maximum flow velocity across staggered pin arrangements, Equation (8) is applied as shown below.

$$u = \frac{U_\infty * \left(\frac{S_T}{2}\right)}{\left[\left(\frac{S_T}{2}\right)^2 + S_L^2\right]^{\frac{1}{2}} - d} \quad (8)$$

However, this equation is valid only if

$$2 * \left\langle \left[\left(\frac{S_T}{2}\right)^2 + S_L^2\right]^{\frac{1}{2}} - d \right\rangle \leq S_T - d \quad (9)$$

[11]

In the current arrangement,

$$2 * \left\langle \left[\left(\frac{S_T}{2}\right)^2 + S_L^2\right]^{\frac{1}{2}} - d \right\rangle > S_T - d$$

Therefore, the above equations for the calculation of maximum velocity could not be utilized. Consequently, the maximum velocity of the fluid is calculated by dividing the mass flow rate of the fluid flowing across the pins by the minimum cross sectional area of the duct.

In order to calculate the total pressure drop experienced by the heat exchanger, the pressure drop resulting from the following aspects are also calculated and then the values are added to the pressure drop caused by the pins.

- Flow in a Duct
- Flow over the Fins
- Flow over the surface plate

When fluid flows in a duct, it experiences frictional forces that result in the reduction of the fluid's initial pressure. The main factors that influence the magnitude of the pressure loss are fluid viscosity, duct diameter, duct surface texture and fluid velocity. The equation adopted for the pressure drop calculation is as shown:

$$\text{Pressure drop, } \delta P = f * \left( \frac{L}{D_h} \right) * \left[ \frac{(\rho * u^2)}{2} \right] \quad (10)$$

where:-

- f = friction factor
- L – length of the duct
- D<sub>h</sub> – hydraulic diameter
- u – velocity of fluid
- ρ - Density of fluid

Each variable listed in equation (10) is calculated separately and then substituted into the equation to yield the pressure drop value. The friction factor, f, is a constant that depends on the type of flow of the fluid, that is, it depends on whether the flow is laminar or turbulent.

In this study, the flow in the duct is found to be turbulent through the calculations of the Reynolds numbers and therefore equations (11) & (12) are used to calculate the friction factors.

For  $Re < 2 * 10^4$

$$f = 0.316 Re^{-0.25} \quad (11)$$

For  $2 * 10^4 < Re < 3 * 10^5$

$$f = 0.184 Re^{-0.2} \quad (12)$$

[14]

The Reynolds numbers are calculated using the same equation as the one used for the pins.

$$Re = \frac{(\rho * u * d)}{\mu} \quad (7)$$

However, in this case, d is the hydraulic diameter. In contrast, for the fins and surface plate, d is the length of the fin/plate in the direction of flow.

Fins have extensively been used as 'extended surfaces' in heat exchangers for a very long time. They vary in shapes and sizes. The fins used in this study have a rectangular cross section and are treated as flat plates in the calculations since the thickness of the fin is negligible in comparison to its length. Surface plates are also considered as flat plates as their thicknesses are negligible to their lengths too. The approach taken to calculate the pressure drops for both the surface plates and the fins is similar to the one adopted for the pins. First, the drag force is calculated, which is then used to calculate the pressure drop. The equation used to calculate the drag force for the fins or surface plate is identical to the one used for the pins.

$$F_D = \frac{1}{2} * \rho * S * C_d * u^2 * n \quad (5)$$

where,

- ρ - density of fluid
- S – surface area of fin/surface plate in contact with flow
- C<sub>d</sub> – drag coefficient
- u – velocity of fluid
- n – number of fins in the flow path (not applicable for surface plate)

The mean drag coefficient is calculated using the following equation, which is best suited for Reynolds numbers  $< 5 * 10^5$ .

$$C_m = \frac{1.328}{\left( Re^{1/2} \right)} \quad (13)$$

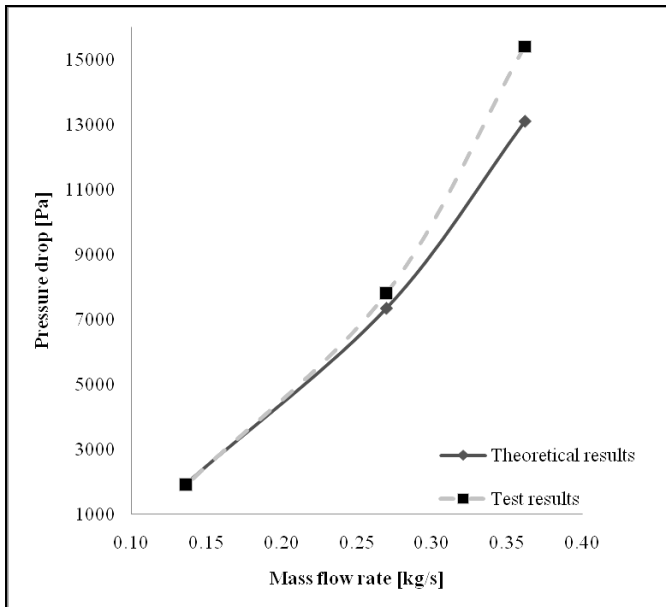
[14]

As previously mentioned, the pressure drop, δP is then calculated by dividing the total drag force, F<sub>D</sub> by the cross sectional area of the duct (A<sub>d</sub>).

$$\delta P = \frac{F_D}{A_d} \quad (14)$$

## RESULTS AND DISCUSSION

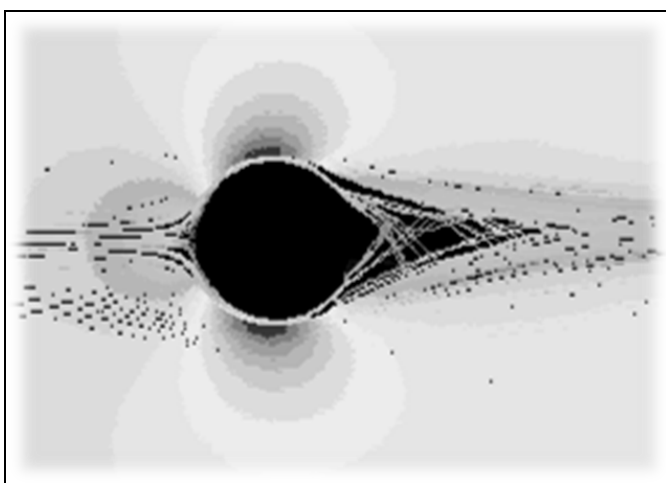
Using the approach described in the previous section, the results for the pressure drop of the heat exchanger are obtained for a mass flow rate of 0.27 kg/s. The same calculations are then repeated for mass flow rates of 0.14 kg/s and 0.36 kg/s. The theoretical results and test results are subsequently plotted and compared.



**Figure 2** Graph showing the trends of the theoretical results and test results

As seen from figure 2, the theoretical results correlate very well with the test results at low mass flow rates. As the mass flow rate increases, the deviation between the theoretical results and test results increases. At the lowest mass flow rate of 0.14 kg/s, the theoretical values are within 1% of the test results. As the mass flow rate increases from 0.14 kg/s to 0.36 kg/s, the difference between the two sets of results increases from 1% to 15%. The theoretical results also show that, on average, for the three different mass flow rates, 90% of the total pressure drop experienced by the fluid is due to the pins.

When calculating the pressure drop resulting from the pins, it is assumed that the pins behave as individual cylinders in order to be able to use the existing equations. However, as the mass flow rate increases, this assumption becomes increasingly less applicable.



**Figure 3** Picture showing the fluid flow around a circular pin.

As the mass flow rate increases, the flow around the pins changes, especially at the rear of the pin. This affects the flow and consequently the pressure drop for the next pin in line. Therefore the pin behaves less as individual cylinders and more as an arrangement.

This phenomenon is more apparent at higher mass flow rates, thus, at higher mass flow rates the theoretical test results show a greater deviation from the test results.

## CONCLUSION

Theoretical validation of Pressure Drop test results of Circular Pins is very challenging. This is due mainly to the fact that existing equations do not fully represent the test conditions, therefore quite a few assumptions have to be made and this results in the theoretical results deviating from the test results mainly at higher mass flow rates.

The test results for the pressure drop of a Circular Pin Heat Exchanger are studied and validated through a theoretical check in this article. The pressure drops that resulted due to the different features in the construction of the heat exchanger are calculated. The results show that, on average, 90% of the total pressure drop that is experienced by the fluid is caused by the pins. Therefore the main aspect of heat exchanger design that is responsible for causing pressure drop is identified as the Pin.

Historically, pins have been used in the electronics field. Traditionally, for heat exchangers, tubes are used to enhance heat transfer. In this study, an innovative pin design and arrangement has been applied to a heat exchanger design. There is currently insufficient theoretical knowledge and test data available in support of the application of this novel pin design in Heat Exchangers and this makes the calculations of the various pressure drops, experienced in the Heat Exchanger, difficult. However, with reasonable assumptions, the theoretical results obtained were on average 6% lower than the test results.

This work highlights the gaps in the theoretical knowledge and test data currently available for the application of new pin designs in heat exchangers.

Hopefully, in future, with the development in computational fluid dynamics techniques, validation of test results will not have to depend solely on theoretical knowledge.

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