

FEASIBLE GENERALISED LEAST SQUARES ESTIMATORS IN SERIALLY CORRELATED ERROR MODELS FROM AN ASYMMETRY VIEWPOINT

S. Nakale

Department of Statistics, Faculty of Natural and Agricultural Sciences, University of Pretoria, Pretoria, South Africa.

e-mail: *sammynakale@gmail.com*

J. Coetsee

Department of Statistics, Faculty of Natural and Agricultural Sciences, University of Pretoria, Pretoria, South Africa.

M. Arashi

Department of Statistics, School of Mathematical Sciences, Shahrood University, Shahrood, Iran.
Department of Statistics, Faculty of Natural and Agricultural Sciences, University of Pretoria, Pretoria, South Africa.

A. Bekker

Department of Statistics, Faculty of Natural and Agricultural Sciences, University of Pretoria, Pretoria, South Africa.

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Summary: In this paper we investigate the finite sample risk performance of feasible generalised least squares estimators applied in models with serially correlated error terms. The risk functions of the ordinary least squares, generalised least squares and feasible generalised least squares estimators are derived under the asymmetric Linear-Exponential loss function. A numerical evaluation using simulation is used to compare the risk functions. Our numerical results show that the relative risk gains of the feasible generalised least squares estimators over the ordinary least squares estimator increases with higher loss asymmetry, particularly for larger serial correlation coefficients.

1. Introduction

There are a number of assumptions underlying the classical linear regression model (CLRM). In practice, however, when using finite sample data to estimate the unknown regression coefficients there is usually some doubt about the validity of some of these assumptions. Serially correlated error terms, for example, is commonly encountered in models involving time series data. In particular, Cochrane and Orcutt (1949) found that error terms in most economic models are highly positively serially correlated. It is well known that when the error terms in a model are serially correlated, the Ordinary Least Squares (OLS) estimator is inefficient, though it remains unbiased. Therefore, there is a strong motivation to consider alternative estimators that account for serial correlation when

estimating models where the assumption of no serial correlation is violated. The generalised least squares (GLS) estimator is the best linear unbiased estimator, however, it requires the serial correlation coefficient to be known, which is rarely the case in practice. Various feasible Generalised Least Squares (FGLS) estimators have been proposed in the literature. Rao and Griliches (1969) investigated the finite sample performance of these FGLS estimators based on the mean square error criterion. Their Monte Carlo results suggest that when compared to the OLS estimator, the FGLS estimators are more efficient, particularly for moderate and high levels of serial correlation in the error terms. We consider the risk performance of the FGLS estimators under the asymmetric Linear-Exponential (LINEX) loss function and evaluate the effects of the asymmetry by considering various combinations of the loss parameters by making use of Monte Carlo experiments.

Section 2 discusses the model and the estimators under consideration. The risk functions of the OLS, GLS and FGLS estimators are derived in section 3 and the numerical evaluations of the risk functions follow in section 4.

2. The model and estimators

We consider the classical linear regression model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u} \quad (1)$$

where \mathbf{y} is a $T \times 1$ vector of observations on a dependent variable, \mathbf{X} is a $T \times k$ nonstochastic design matrix of full column rank k , $\boldsymbol{\beta}$ is a $k \times 1$ vector of unknown regression coefficients and \mathbf{u} is a $T \times 1$ random vector of the error terms. In most econometric models it is commonly assumed that the error terms are generated by a stationary first order autoregressive process given by

$$u_t = \rho u_{t-1} + e_t, \quad \mathbf{e} \sim N(\mathbf{0}, \sigma_e^2 \mathbf{I})$$

where $\mathbf{e} = (e_1, \dots, e_T)'$ and ρ is the serial correlation coefficient (for positive stationary processes $0 \leq \rho < 1$).

Under this assumption, it follows that

$$E(\mathbf{u}) = \mathbf{0}$$

and

$$E(\mathbf{u}\mathbf{u}') = \frac{\sigma_e^2}{(1-\rho^2)} \begin{bmatrix} 1 & \rho & \rho^2 & \dots & \rho^{T-1} \\ \rho & 1 & \rho & \dots & \rho^{T-2} \\ \rho^2 & \rho & 1 & \dots & \rho^{T-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & \dots & 1 \end{bmatrix}_{(T \times T)} = \sigma_e^2 \boldsymbol{\phi}$$

therefore,

$$\mathbf{u} \sim N(\mathbf{0}, \sigma_e^2 \boldsymbol{\phi})$$

If the serial correlation coefficient, ρ , is equal to zero and assuming that all other assumptions of the CLRM holds, it follows from the Gauss-Markov theorem that the OLS estimator, (eq. 2), is the best linear unbiased estimator of the unknown regression coefficient vector $\boldsymbol{\beta}$.

$$\hat{\boldsymbol{\beta}}_{OLS} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \quad \hat{\boldsymbol{\beta}}_{OLS} \sim N(\boldsymbol{\beta}, \sigma_e^2 (\mathbf{X}'\mathbf{X})^{-1}) \quad (2)$$

However, as discussed by Judge, Griffiths, Hill, Lutkepohl and Lee (1985), even though the OLS estimator remains unbiased in the presence of serial correlation, it is no longer efficient relative to estimators obtained by taking into account knowledge of serial correlation in the error terms. They further point out that the least squares estimator for the variances of the regression coefficients will be biased and inconsistent, leading to invalid inferences about the regression coefficients. Therefore, estimators that adjust for serial correlation by making use of ϕ through some transformation techniques, have more desirable statistical properties and should be considered in models with serially correlated error terms.

If the value of ρ is known, then the problem of serial correlation can be remedied by using ϕ in estimating β . The GLS estimator is then the minimum variance linear unbiased estimator and is given by:

$$\tilde{\beta}_{GLS} = (\mathbf{X}'\phi^{-1}\mathbf{X})^{-1}\mathbf{X}'\phi^{-1}\mathbf{y} \quad \tilde{\beta}_{GLS} \sim N(\beta, \sigma_e^2(\mathbf{X}'\phi^{-1}\mathbf{X})^{-1}) \quad (3)$$

However, the value of ρ is rarely known in practice, therefore it is not possible to obtain the GLS estimator directly. Some variants of the GLS estimator based on consistent estimates of ρ have been suggested in the literature and the general FGLS estimator is given by:

$$\tilde{\beta}_{FGLS} = (\mathbf{X}'\hat{\phi}^{-1}\mathbf{X})^{-1}\mathbf{X}'\hat{\phi}^{-1}\mathbf{y} \quad \tilde{\beta}_{FGLS} \sim N(\beta, \sigma_e^2(\mathbf{X}'\hat{\phi}^{-1}\mathbf{X})^{-1}) \quad (4)$$

The various FGLS estimators differ in the method used in estimating the value of $\hat{\rho}$ and in how they deal with the first observation in the transformation matrix. Cochrane and Orcutt (1949) suggested an estimation procedure that estimates both ρ and β iteratively. The model (eq. 1) is estimated by the usual OLS procedure and the first sample autocorrelation coefficient of the estimated residuals from this model is then used in estimating ρ , that is

$$\hat{\rho} = \sum_{t=2}^T \hat{u}_t' \hat{u}_{t-1} / \sum_{t=1}^T \hat{u}_t' \hat{u}_t$$

This estimator is then used in the Cochrane-Orcutt transformation matrix (eq. 6), \mathbf{P}_{CO} , to obtain the Cochrane-Orcutt estimator:

$$\hat{\beta}_{CO} = (\mathbf{X}'\mathbf{P}'_{CO}\mathbf{P}_{CO}\mathbf{X})^{-1}\mathbf{X}'\mathbf{P}'_{CO}\mathbf{P}_{CO}\mathbf{y} \quad (5)$$

where

$$\mathbf{P}_{CO} = \begin{bmatrix} -\hat{\rho} & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & -\hat{\rho} & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & -\hat{\rho} & 1 \end{bmatrix}_{((T-1) \times T)} \quad (6)$$

A second round of residuals $\tilde{u}_t = (y_t - \mathbf{X}_t\hat{\beta}_{CO})$ can then be obtained and used in calculating the second round estimates of ρ and β . This procedure is then repeated iteratively until both values of $\hat{\rho}$ and $\hat{\beta}_{CO}$ converges. The Cochrane-Orcutt two stage estimator based on the second round estimates is however considered to be a consistent estimator in practice.

The Cochrane-Orcutt estimator is based on the transformation matrix that disregard the first observation. Prais and Winsten (1954) noted that this transformation procedure can have a significant impact on the efficiency of the obtained estimator, particularly for small sample sizes and in trended time series where the first observation could be very different from the average observation of the series.

They then suggested a modification of the Cochrane-Orcutt procedure where the transformation is based on the following matrix

$$\mathbf{P}_{PW} = \begin{bmatrix} \sqrt{1-\widehat{\rho}^2} & 0 & 0 & 0 & \dots & 0 & 0 \\ -\widehat{\rho} & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & -\widehat{\rho} & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \dots & -\widehat{\rho} & 1 \end{bmatrix}_{(T \times T)} \quad (7)$$

and $\widehat{\rho}$ is obtained by using the Cochrane-Orcutt procedure.

Thus, the Prais-Winsten estimator is given by:

$$\widehat{\boldsymbol{\beta}}_{PW} = (\mathbf{X}'\mathbf{P}'_{PW}\mathbf{P}_{PW}\mathbf{X})^{-1}\mathbf{X}'\mathbf{P}'_{PW}\mathbf{P}_{PW}\mathbf{y}. \quad (8)$$

Durbin (1960), on the other hand, proposed a procedure where the estimate of ρ is obtained by the coefficient of the lagged dependent variable in the model

$$\mathbf{y}_t = \rho\mathbf{y}_{t-1} + \boldsymbol{\beta}\mathbf{X}_t - \rho\boldsymbol{\beta}\mathbf{X}_{t-1} + \mathbf{v}_t. \quad (9)$$

The least squares estimate of ρ in (eq. 9) is then used to replace the Cochrane-Orcutt two stage estimate of ρ in the matrix (eq. 7) to obtain the transformation matrix \mathbf{P}_D . Therefore, the Durbin estimator is given by

$$\widehat{\boldsymbol{\beta}}_{DE} = (\mathbf{X}'\mathbf{P}'_D\mathbf{P}_D\mathbf{X})^{-1}\mathbf{X}'\mathbf{P}'_D\mathbf{P}_D\mathbf{y}. \quad (10)$$

An alternative to the estimators considered above is a non-linear estimator obtained by maximum likelihood estimation procedures that estimate $\boldsymbol{\beta}$ and ρ simultaneously. Beach and Mackinnon (1978) suggested a maximum likelihood estimator that incorporates the first observation and stationarity of the error process. Under the assumption that the error terms are normally distributed, the estimate of $\boldsymbol{\beta}$, conditional on ρ is obtained by maximising the log-likelihood function given by:

$$\mathcal{L} = \text{const} \cdot \frac{1}{2} \log(1 - \rho^2) - \frac{T}{2} \log[(1 - \rho^2)(\mathbf{y}_1 - \mathbf{X}_1\boldsymbol{\beta})^2 + \sum_{t=2}^T (\mathbf{y}_t - \mathbf{X}_t\boldsymbol{\beta} - \rho\mathbf{y}_{t-1} + \rho\mathbf{X}_{t-1}\boldsymbol{\beta})^2].$$

An algorithm for maximising this likelihood function is given by Beach and Mackinnon (1978). Judge et al. (1985) assert that all the FGLS estimators discussed above have the same asymptotic properties. However, for small samples the performance of the FGLS estimators will be influenced by the accuracy of the estimates of ρ . Monte Carlo evidence from Rao and Griliches (1969) suggests that all the estimators of ρ are biased in small samples, though the Durbin estimator of ρ is significantly less biased for positive values of ρ . They conclude that the Prais-Winsten estimator based on the Durbin estimate of ρ is likely to be more efficient over a wide range of parameters compared to other estimators. Park and Mitchell (1980) also compared the efficiency of the various estimators, specifically for trended time series. Their results show that the Cochrane-Orcutt procedure has very low efficiency among all the FGLS estimators. Furthermore, they found that the Prais-Winsten estimator only performs slightly better than the maximum likelihood estimator.

3. Risk functions

Suppose $\widehat{\boldsymbol{\beta}}$ is an estimator of the unknown parameter vector $\boldsymbol{\beta}$. The LINEX loss takes the form

$$\mathcal{L}_{LINEX}(\widehat{\boldsymbol{\beta}}; \boldsymbol{\beta}) = c[\exp[\mathbf{a}'(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta})] - \mathbf{a}'(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}) - 1] \quad (11)$$

where $\mathbf{a} = (a_1, \dots, a_k)'$ and $a_i \neq 0$ for $i = 1, \dots, k$ and $c > 0$.

The LINEX loss was proposed by Varian (1975) and its properties are discussed therein and are further illustrated in Figure 1 which presents the graphs for the loss function for selected values of the loss parameters c and \mathbf{a} , where $k = 2$. The parameter c is a scale parameter and the vector \mathbf{a} determines the shape of the loss function. The signs of the a_i 's determine and reflect the direction of asymmetry and their magnitudes reflect the degree of asymmetry. The LINEX loss is quite asymmetric when the absolute values of the a_i 's are large and almost symmetric for smaller absolute values. For simplicity, we will assume equal values of the a_i 's in our analysis.

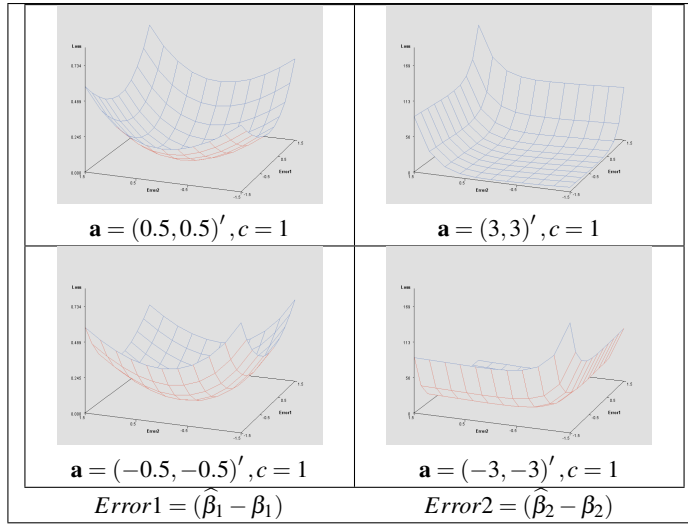


Figure 1: Graphs of LINEX loss (see eq. 11) for selected parameter values.

3.1. Risk function of OLS estimator

From (eq. 2) and (eq. 11), it follows that the risk function of the OLS estimator under the LINEX loss is defined by

$$\begin{aligned}
 \mathfrak{R}_{LINEX}(\hat{\beta}_{OLS}; \beta) &= E\{c[\exp(\mathbf{a}'(\hat{\beta}_{OLS} - \beta)) - \mathbf{a}'(\hat{\beta}_{OLS} - \beta) - 1]\} \\
 &= c[E[\exp(\mathbf{a}'\hat{\beta}_{OLS})] \exp(-\mathbf{a}'\beta) - \mathbf{a}'(E[\hat{\beta}_{OLS}] - \beta) - 1] \\
 &= c[\exp(-\mathbf{a}'\beta) M_{\hat{\beta}_{OLS}}(\mathbf{a}) - \mathbf{a}'(\beta - \beta) - 1] \\
 &= c[\exp(-\mathbf{a}'\beta) \exp(\beta'\mathbf{a} + \frac{1}{2}\mathbf{a}'\Sigma_{OLS}\mathbf{a}) - 1] \\
 &= c[\exp(\frac{1}{2}\mathbf{a}'\Sigma_{OLS}\mathbf{a}) - 1]
 \end{aligned}$$

where $M_{\hat{\beta}_{OLS}}(\cdot)$ is the moment generating function of $\hat{\beta}_{OLS}$ and $\Sigma_{OLS} = \sigma_e^2(\mathbf{X}'\mathbf{X})^{-1}$.

3.2. Risk functions of GLS and FGLS estimators

Using (eq. 3) and (eq. 11), the GLS risk expression can be derived as

$$\begin{aligned}
\mathfrak{R}(\tilde{\beta}_{GLS}; \beta) &= E\{c[\exp[\mathbf{a}'(\tilde{\beta}_{GLS} - \beta)] - \mathbf{a}'(\tilde{\beta}_{GLS} - \beta) - 1]\} \\
&= c[E[\exp(\mathbf{a}'\tilde{\beta}_{GLS})] \exp(-\mathbf{a}'\beta) - \mathbf{a}'(E[\tilde{\beta}_{GLS}] - \beta) - 1] \\
&= c[\exp(-\mathbf{a}'\beta) M_{\tilde{\beta}_{GLS}}(\mathbf{a}) - \mathbf{a}'(\beta - \beta) - 1] \\
&= c[\exp(-\mathbf{a}'\beta) \exp(\beta'\mathbf{a} + \frac{1}{2}\mathbf{a}'\Sigma_{GLS}\mathbf{a}) - 1] \\
&= c[\exp(\frac{1}{2}\mathbf{a}'\Sigma_{GLS}\mathbf{a}) - 1]
\end{aligned}$$

where $M_{\tilde{\beta}_{GLS}}(\cdot)$ is the moment generating function of $\tilde{\beta}_{GLS}$ and $\Sigma_{GLS} = \sigma_e^2(\mathbf{X}'\phi^{-1}\mathbf{X})^{-1}$. It follows directly from the GLS risk expression that the risk function for the FGLS estimator, (eq. 4), is given by

$$\mathfrak{R}(\hat{\beta}_{FGLS}; \beta) = c[\exp(\frac{1}{2}\mathbf{a}'\Sigma_{FGLS}\mathbf{a}) - 1]$$

where $M_{\hat{\beta}_{FGLS}}(\cdot)$ is the moment generating function of $\hat{\beta}_{FGLS}$ and $\Sigma_{FGLS} = \sigma_e^2(\mathbf{X}'\hat{\phi}^{-1}\mathbf{X})^{-1}$.

4. Numerical evaluation and discussion

Following Beach and Mackinnon (1978), the data used in the Monte Carlo study are generated by a single explanatory variable such that

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

where $X_t = \exp(0.04t) + w_t$, $w_t \sim N(0, 0.009)$, $\beta = (\beta_1, \beta_2) = (1, 1)$ and the sample size $T = 20$. The values of the explanatory variable are held fixed in repeated samples. The u_t are then generated by the first order autoregressive error process

$$u_t = \rho u_{t-1} + e_t \quad e_t \sim N(0, 0.0036)$$

and $N = 1000$ replications of the experiment were used. We consider values of ρ from 0 to 0.9. The risks for the estimator $\hat{\beta}$ are calculated by (eq. 12) and normalised by dividing by the GLS risks $\mathfrak{R}(\tilde{\beta}_{GLS}; \beta)$ to obtain the relative risks.

$$\mathfrak{R}_{LINEX}(\hat{\beta}; \beta) = \frac{1}{N} \sum_{j=1}^N c(\exp[\mathbf{a}'(\hat{\beta}_j - \beta)] - \mathbf{a}'(\hat{\beta}_j - \beta) - 1) \quad (12)$$

All computations were done in SAS 9.2.

We have evaluated the risk functions for loss parameters $c = 1$; $\mathbf{a} = (-0.5, -0.5)'$, $\mathbf{a} = (-3, -3)'$ and $\mathbf{a} = (-8, -8)'$. Representative results from our analysis are presented in Table 1 and Figure 2. As expected for $\mathbf{a} = (-0.5, -0.5)'$, which represents a relatively lower degree of asymmetry, we find that the results from our analysis are consistent with those given by Rao and Griliches (1969). That is, the OLS estimator is risk superior to all FGLS for $\rho \leq 0.2$, the GLS estimator strictly dominates

ρ		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
OLS	$\mathbf{a} = (-0.5, -0.5)'$	1.000	1.010	1.030	1.060	1.105	1.168	1.260	1.387	1.533	1.559
	$\mathbf{a} = (-3, -3)'$	1.000	1.010	1.030	1.060	1.105	1.169	1.260	1.387	1.534	1.561
	$\mathbf{a} = (-8, -8)'$	1.000	1.010	1.030	1.061	1.107	1.173	1.268	1.401	1.563	1.627
D	$\mathbf{a} = (-0.5, -0.5)'$	1.020	1.025	1.030	1.035	1.042	1.055	1.078	1.121	1.174	1.199
	$\mathbf{a} = (-3, -3)'$	1.020	1.025	1.030	1.035	1.042	1.054	1.076	1.118	1.169	1.192
	$\mathbf{a} = (-8, -8)'$	1.019	1.024	1.029	1.034	1.041	1.053	1.075	1.114	1.160	1.185
CO	$\mathbf{a} = (-0.5, -0.5)'$	1.282	1.342	1.414	1.498	1.594	1.701	1.815	1.919	1.964	1.787
	$\mathbf{a} = (-3, -3)'$	1.282	1.342	1.414	1.498	1.594	1.702	1.816	1.919	1.960	1.788
	$\mathbf{a} = (-8, -8)'$	1.286	1.346	1.419	1.506	1.608	1.726	1.853	1.970	2.031	1.901
PW	$\mathbf{a} = (-0.5, -0.5)'$	1.012	1.016	1.023	1.033	1.047	1.069	1.104	1.160	1.241	1.293
	$\mathbf{a} = (-3, -3)'$	1.011	1.016	1.023	1.033	1.047	1.069	1.103	1.158	1.237	1.293
	$\mathbf{a} = (-8, -8)'$	1.011	1.016	1.023	1.032	1.047	1.068	1.102	1.156	1.236	1.318
MLE	$\mathbf{a} = (-0.5, -0.5)'$	1.029	1.049	1.047	1.049	1.057	1.073	1.100	1.133	1.158	1.172
	$\mathbf{a} = (-3, -3)'$	1.027	1.047	1.045	1.048	1.056	1.072	1.098	1.130	1.152	1.171
	$\mathbf{a} = (-8, -8)'$	1.025	1.042	1.042	1.045	1.054	1.071	1.097	1.127	1.144	1.175

Table 1: Relative risks of the estimators under the LINEX loss.

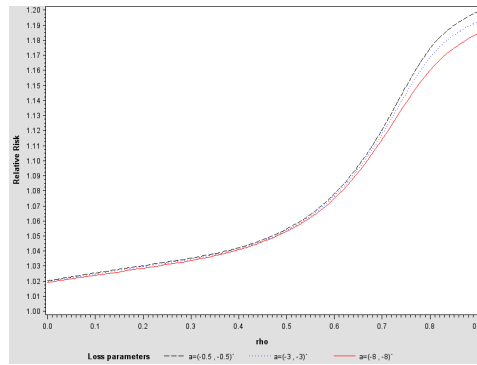


Figure 2: Relative risk functions for the Durbin estimator for different choices of LINEX loss parameters.

all estimators over the entire range of ρ and the Durbin estimator performs the best and Cochrane-Orcutt the worst among all the FGLS estimators considered.

We note that some of the key results based on the mean square error criterion, or equivalently the squared error loss, continue to hold as the degree of loss asymmetry increases. The relative ordering of the FGLS estimators' performance as well as the range of ρ over which the OLS estimator dominates all FGLS estimators remains the same. Generally, the risks of the OLS and the Cochrane-Orcutt estimators increase with an increase in loss asymmetry and most importantly the OLS estimator strictly dominates the Cochrane-Orcutt estimator over the entire range of ρ , once again showing the inferiority of the Cochrane-Orcutt estimator as an alternative to OLS. On the other hand, the risks of the Prais-Winsten, MLE and Durbin estimators decreases as the degree of loss asymmetry increases and as clearly illustrated in Figure 2 for the Durbin estimator, the decrease in the risks is more substantial for higher levels of serial correlation. The implication of this is that the risk gains of the FGLS estimators, with the exception of the Cochrane-Orcutt estimator, over the OLS estimator increase as the loss asymmetry increases, particularly for higher levels of serial

correlation.

Considering that the OLS estimator is relatively more efficient than FGLS estimators for small values of ρ , it is imperative to use serial correlation test procedures to determine the significance of the serial correlation in the error terms. The estimation strategy is then made to choose between the OLS estimator and FGLS estimator depending on the significance of the serial correlation in the error terms. Work in progress by the authors investigates this strategy from an asymmetry viewpoint. This work is based on the research supported by the National Research Foundation of South Africa for the grant TTK1206151317. Any opinion, finding and conclusion or recommendation expressed in this material is that of the authors and the NRF does not accept any liability in this regard. The authors would like to thank the anonymous referees for constructive comments that improved the presentation of the paper.

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