

FORECASTING THE SOUTH AFRICAN ECONOMY WITH VARs AND VECMs

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Abstract

The paper develops a Bayesian Vector Error Correction Model (BVECM) of the South African economy for the period 1970:1-2000:4 and forecasts GDP, consumption, investment, short and long term interest rates, and the CPI. We find that a tight prior produces relatively more accurate forecasts than a loose one. The out-of-sample-forecast accuracy resulting from the BVECM is compared with those generated from the Classical variant of the VAR and VECM and the Bayesian VAR. The BVECM is found to produce the most accurate out of sample forecasts. It also correctly predicts the direction of change in the chosen macroeconomic indicators.

1. INTRODUCTION

This paper compares the ability of Vector Autoregressive (VAR) Models and Vector Error Correction Models (VECM), both Classical and Bayesian in nature, in forecasting the South African economy. For this purpose, we estimate these models using quarterly data on consumption, the Consumer Price Index (CPI), Gross Domestic Product (GDP), investment, and measures of short and long term interest rates, the 91 days Treasury Bill Rate and 10 years and longer government bond rates, respectively, for the period of 1970 to 2000. And then, in turn, we compare the out-of-sample forecast errors generated by these models over the period of 2001:1 to 2005:4. We also investigate the capability of the VARs and the VECMs in predicting the turning points, if any, of the chosen macroeconomic variables over the period of 2004:1 to 2005:4.

Though the Classical and the Bayesian VARs have been widely used¹ in forecasting national and regional economies as well as the housing market, the use of the ECMs and VECMs for forecasting purposes is relatively recent.² In general, the multivariate BVAR

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¹ For example, Amirizadeh and Todd (1984), Kuprianov and Lupoletti (1984), Hoehn *et al.* (1984), Hoehn and Balazsy (1985), Kinal and Ratner (1986), Gruben and Long (1988a, b), Lesage (1990), Gruben and Hayes (1991), Shoesmith (1992), Dua and Ray (1995), Dua and Smyth (1995), Dua and Miller (1996), Dua *et al.* (1999), Banerji *et al.* (2006) and Gupta and Sichei (2006).

² See LeSage (1990), Shoesmith (1992), LeSage and Pan (1995), Dowd and LeSage (1997), LeSage and Krivelyova (1999) for use of both Classical and Bayesian variants of the VECMs, and Chacra and Kichian (2004) and Smith *et al.* (2006) for the use of Classical VECMs in forecasting inventory investment in Canada and South Africa.

models have been found to produce the most accurate short and long term out-of-sample forecasts relative to the univariate and unrestricted Classical VAR models. Moreover, the BVAR models are also capable of correctly predicting the direction of change of the macroeconomic variables.

However, the relative dearth of the use of VECMs, especially the classical version, is surprising. This is surprising when one realizes that two decades back Granger (1986) had stressed that the use of long-run equilibrium relationships from economic theory in models used by time-series econometricians to explain short-run dynamics of data, in other words, the ECMs, should produce better forecasts in the short run and certainly in the long run. Engle and Yoo (1987) corroborated Granger's (1986) faith in these models, when they provided theoretical support for the superior forecasting ability of the ECMs over unrestricted VAR models. They also presented a small simulation exercise confirming the same. LeSage (1990), using industrial and labour market data from the state of Ohio also showed that VECMs outperform the VARs.

As far as the sparse use of the Bayesian version of the ECM models is concerned, two reasons can be identified. Firstly it is probably due to the concerns of Lutkepohl (1993, p. 375) and Engle and Yoo (1987) regarding the use of the Bayesian VECM (BVECM) for forecasting. They pointed out that these models are misspecified in terms of the Granger Representation Theorem, since they impose random walk restrictions.³ However, a series of recent work by LeSage (1990), Dua and Ray (1995), LeSage and Pan (1995), Dowd and LeSage (1997) and LeSage and Krivelyova (1999) have made some progress in allaying these fears to some extent. They indicate that, given that BVECM allows the forecaster to control for the balance of the short-run dynamics and the long-run influences in the model depending on the specification of the prior, the same, in fact, can produce better forecasts in comparison to the Classical VECMs, especially in the longrun. Hence, it is not surprising that these models, until recently, lacked the confidence of the forecasters.

The second reason is mostly computational and, perhaps, the more important of the two. The technical issue surrounding the relatively modest use of BVECMs in forecasting is, in our opinion, related to the difficulty associated with coding the likelihood functions involved in Bayesian estimation. To the best of our knowledge, until Professor James P. LeSage at the University of Toledo, developed the Econometric Toolbox for MATLAB,⁴ Regression Analysis of Time Series (RATS) was the only other software that had a built-in ability to handle Bayesian estimations. However, with RATS only capable of carrying out estimations of BVARs, the lack of BVECMs in the forecasting literature is not surprising.

With the theoretical concerns involved in the use of ECM models for forecasting sorted out, and with computer codes now available to estimate both Classical and Bayesian VECMs, we compare the abilities of VARs and VECMs in forecasting six important variables of the national economy of South Africa. To the best of our knowledge, this is the first attempt to simultaneously analyze the role of Classical and Bayesian VARs and VECMs in making economy-wide forecasts, given that the studies of LeSage (1990), LeSage and Pan (1995), Dowd and LeSage (1997), and LeSage and Krivelyova (1999) were only regional in nature. Finally, unlike most studies in the forecasting literature using BVARs, we also check for the robustness of our analysis by

³ See Section 2 for further details.

⁴ The Toolbox is available for free download from <http://www.spatial-econometrics.com>.

specifying alternative values of the hyperparameters for the Bayesian priors, available in the literature. Our study can thus be viewed as an attempt to extend the existing literature on forecasting by incorporating the role of the Classical and Bayesian variants of VECMs, besides the VAR counterparts of the same.

Finally, it must be pointed out that our analysis should be viewed as complementary to a recent study on forecasting of the South African economy by Gupta and Sichei (2006). The authors develop a Bayesian vector autoregressive (BVAR) model of the South African economy for the period of 1970:1-2000:4 to forecast GDP, consumption, investment, short-term and long-term interest rates, and the CPI. They found that a tight prior produces relatively more accurate forecasts than a loose one. The out-of-sample-forecast accuracy resulting from the BVAR model was compared with the same generated from the univariate and unrestricted VAR models. The BVAR model was found to produce the most accurate out of sample forecasts. The same model was also capable of correctly predicting the direction of change in the chosen macroeconomic indicators. In such a backdrop, our paper, thus, attempts to check whether we can produce better forecasts, relative to the VAR and BVARs, for the same set of variables estimated over the same period using the VECMs and the BVECMs, used in the above mentioned paper. In addition to this, using the Gupta and Sichei (2006) study, we show that the out-of-sample forecasts and, hence, forecast errors, are sensitive to the choice of algorithm used for recursive estimation of the models over the forecast horizon.⁵ The rest of the paper is organized as follows. Besides the introduction and the conclusions, section 2 discusses the advantages of using VARs and VECMs versus a structural model,⁶ and also describes the parameters required to specify a BVAR model. The technicalities involved in the Classical and Bayesian VECMs are also laid out in this section. Section 3 sets out the model for the South African Economy, while section 4 compares the accuracy of the out-of-sample forecasts generated from alternative models. Section 5 discusses, in detail, the performance of the alternative models used for forecasting, in terms of their ability to predict the turning points in the economy, if any.

2. ADVANTAGES OF USING VAR OVER STRUCTURAL MODELS

Generally, economy-wide forecasting models are in the form of simultaneous-equations structural models. However, two problems often encountered with such models are as follows: (i) the correct number of variables needs to be excluded, for proper identification of individual equations in the system, which are however often based on little theoretical justification (Cooley and LeRoy, 1985), and; (ii) given that projected future values are required for the exogenous variables in the system, structural models are poorly suited to forecasting.

The Vector Autoregressive (VAR) model, though 'atheoretical', is particularly useful for forecasting purposes. Moreover, as shown by Zellner (1979) and Zellner and Palm (1974), any structural linear model can be expressed as a VAR moving average (VARMA) model, with the coefficients of the VARMA model being combinations of the structural coefficients. Under certain conditions, a VARMA model can be expressed as a VAR and

⁵ See Section 3 for further details.

⁶ This section of the paper relies heavily on the discussion available in Dua and Ray (1995), Banerji *et al.* (2006), LeSage (1999) and Ground and Ludi (2006).

a VMA model. Thus, a VAR model can be visualized as an approximation of the reduced-form simultaneous equation structural model.

An unrestricted VAR model, as suggested by Sims (1980), can be written as follows:

$$y_t = C + A(L)y_t + \xi_t \quad (1)$$

where y is a $(n \times 1)$ vector of variables being forecasted; $A(L)$ is a $(n \times n)$ polynomial matrix in the backshift operator L with lag length p , i.e., $A(L) = A_1L + A_2L^2 + \dots + A_pL^p$; C is a $(n \times 1)$ vector of constant terms, and ξ is a $(n \times 1)$ vector of white-noise error terms. The VAR model, thus, posits a set of relationships between the past lagged values of all variables and the current value of each variable in the model.

Focusing on the practical case, of y_t being a vector of n time series that are integrated⁷ to the order of 1 ($I(1)$),⁸ the ECM counterpart of the VAR, given by (1), is captured by a VECM as follows:⁹

$$\Delta y_t = \pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \varepsilon_t \quad (2)$$

n where $\pi = - \left[I - \sum_{i=1}^p A_i \right]$ and $\Gamma_i = - \sum_{j=i}^p A_j$.

CC and P each with rank r such that $K = CCp$ and $f\beta y_t$ is $I(0)$. Note r is the number of cointegrating relations (the cointegrating rank) and each column of β is the cointegrating vector, and the elements of CC are known as the adjustment parameters in the VECM. CC is also known as the loading matrix and has a dimension $n \times r$. Since it is not possible to use conventional OLS to estimate CC and β , Johansen's (1988) full information maximum likelihood estimation is used to determine the cointegrating rank of K , using the r most significant cointegrating vectors to form β , from which a corresponding CC is derived. Note that the specification in (2) is in line with the Engle and Granger (1987) Representation Theorem.

Thus, a VECM is a restricted VAR designed for use with non-stationary series that are known to be cointegrated. While allowing for short-run adjustment dynamics, the VECM has cointegration relations built into the specification so that it restricts the long-run behaviour of the endogenous variables to converge to their cointegrating relationships. The cointegration term is known as the error correction term because the deviation from long-run equilibrium is corrected through a series of partial short-run adjustments, gradually.

Note the VAR model, generally, uses equal lag length for all the variables of the model. One drawback of VAR models is that many parameters are needed to be estimated, some of which may be insignificant. This problem of overparameterization, resulting in multicollinearity and a loss of degrees of freedom, leads to inefficient estimates and

⁷ A series is said to be integrated of order q , if it requires q differencing to transform it to a

zero-mean, purely non-deterministic stationary process.

⁸ LeSage (1990) and references cited therein for further details regarding most macroeconomic time series being $I(1)$.

⁹ See, Dickey *et al.* (1991) and Johansen (1995) for further technical details.

possibly large out-of-sample forecasting errors. One must remember that in the VECMs, besides the parameters corresponding to the lagged values of the variables, the parameters corresponding to the error correction terms are also estimated. So the problem of overparameterization, in this case, might be acute enough to outweigh the advantages, in terms of smaller forecasting errors, emanating from the use of long-run equilibrium relationships from economic theory to explain the short-run dynamics of the data. One solution, often adapted, is simply to exclude the insignificant lags based on statistical tests. Another approach is to use near VAR, which specifies an unequal number of lags for the different equations.

However, an alternative approach to overcoming this overparameterization, as described in Litterman (1981), Doan *et al.* (1984), Todd (1984), Litterman (1986), and Spencer (1993), is to use a Bayesian VAR (BVAR) model. Instead of eliminating longer lags, the Bayesian method imposes restrictions on these coefficients by assuming that they are more likely to be near zero than the coefficients on shorter lags. However, if there are strong effects from less important variables, the data can override this assumption. The restrictions are imposed by specifying normal prior distributions with zero means and small standard deviations for all coefficients with the standard deviation decreasing as the lags increase. The exception to this is, however, the coefficient on the first own lag of a variable, which has a mean of unity. Litterman (1981) used a diffuse prior for the constant. This is popularly referred to as the ‘Minnesota prior’ due to its development at the University of Minnesota and the Federal Reserve Bank at Minneapolis. Note that, as described in (2), an identical approach can be taken to implement a Bayesian variant of the Classical VECM based on the Minnesota prior.

Formally, as discussed above, the Minnesota prior means and variances take the following form:

$$P_i \sim N(0, \sigma_i^2) \text{ and } p_j \sim N(0, \sigma_j^2) \quad (3)$$

where σ_i^2 denotes the coefficients associated with the lagged dependent variables in each equation of the VAR, while σ_j^2 represents any other coefficient. In the belief that lagged dependent variables are important explanatory variables, the prior means corresponding to them are set to unity. However, for all the other coefficients, σ_j^2 's, in a particular equation of the VAR, a prior mean of zero is assigned, to suggest that these variables are less important to the model.

The prior variances σ_i^2 and σ_j^2 , specify uncertainty about the prior means $\mu_i = 1$, and $\mu_j = 0$, respectively. Because of the overparameterization of the VAR, Doan *et al.* (1984) suggested a formula to generate standard deviations as a function of small numbers of hyperparameters: w , d , and a weighting matrix $f(i, j)$. This approach allows the forecaster to specify individual prior variances for a large number of coefficients based on only a few hyperparameters. The specification of the standard deviation of the distribution of the prior imposed on variable j in equation i at lag m , for all i, j and m , defined as $S(i, j, m)$, can be specified as follows:

$$S(i, j, m) = [w \times g(m) \times f(i, j)] \frac{\hat{\sigma}_j}{\hat{\sigma}_i} \quad (4)$$

with $f(i, j) = 1$, if $i = j$ and k_j otherwise, with $(0 < k_j < 1)$, $g(jn) = \frac{1}{n^d}$, $d > 0$. Note that σ_i is the estimated standard error of the univariate autoregression for variable i . The ratio σ_i / σ_j scales the variables so as to account for differences in the units of measurement and, hence, causes specification of the prior without consideration of the magnitudes of the variables. The term w indicates the overall tightness and is also the standard deviation on the first own lag, with the prior getting tighter as we reduce the value. The parameter $g(m)$ measures the tightness on lag m with respect to lag 1, and is assumed to have a harmonic shape with a decay factor of d , which tightens the prior on increasing lags. The parameter $f(i, j)$ represents the tightness of variable j in equation i relative to variable i , and by increasing the interaction, i.e., the value of k_j , we can loosen the prior.¹⁰

The Bayesian variants of the Classical VARs and VECMs are estimated using Theil's (1971) mixed estimation technique, which involves supplementing the data with prior information on the distribution of the coefficients. In an artificial way, the number of observations and degrees of freedom are increased by one, for each restriction imposed on the parameter estimates. The loss of degrees of freedom due to over-parameterization associated with a VAR model is, therefore, not a concern in the BVAR model.

Given the structure of the Bayesian prior, we can now discuss the issue of misspecification involved with the BVECMs, as referred to in the introduction, in more detail. Lutkepohl (1993, p. 375) has claimed that the Minnesota prior is not a good choice if the variables in the system are believed to be cointegrated. He bases his argument on the interpretation of the prior as suggesting that the variables are roughly random walks. Moreover, Engle and Yoo (1987) argued that with the Minnesota prior, a BVAR model approaches the classical VAR model with differenced data, and, hence, would be misspecified for cointegrated variables without an error correction term.

But Dua and Ray (1995) indicate that the suggestion of the Minnesota prior being inappropriate, when the variables are cointegrated, is incorrect. They point out that the prior sets the mean of the first lag of each variable equal to one in its own equation and sets all the other coefficients equal to zero, thus implying that if the prior means were indeed the true parameter values, each variable would be a random walk. But at the same time the prior probability that the coefficients are actually at the prior mean is zero. The Minnesota prior, indeed, places high probability on the class of models that are stationary. Alternatively, if a model specified in levels is equivalent to one in differences, then the sum of the coefficients on the own lags will equal to one, while the sum of the coefficients on the other variables exactly equals zero. Though this holds for the mean of the Minnesota prior, used in this paper, the prior actually assigns a probability of zero to the class of parameter vectors that satisfy this restriction. Lesage (1990) and Dua and Ray (1995), however, point out that if a very tight prior is specified, the estimated model will be close to a model showing no cointegration. With the Minnesota priors, chosen in practice, being not too tight to produce the forecasts, concerns of misspecification with cointegrated data are, therefore, misplaced.

¹⁰ For an illustration, see Dua and Ray (1995).

3. A BVAR AND BVECM MODEL FOR THE SOUTH AFRICAN ECONOMY

Along the lines of Litterman (1986), Ni and Sun (2005) and Gupta and Sichei (2006), we estimate a BVAR model and a BVECM model for the South African economy for the period of 1970:1 to 2000:4, based on quarterly data. We then compute an out-of-sample one through eight-quarters-ahead forecasts for the period of 2001:1 to 2005:4, and then compare the accuracy of the forecast relative to the forecasts generated by an unrestricted VAR and a VECM, as in LeSage (1990). The variables included are real GDP, consumption, investment, the 91 days Treasury Bill rate, 10 years and longer government bond rates, and the CPI. All data are seasonally adjusted in order to, *inter alia*, address the fact, as pointed out by Hamilton (1994:362), that the Minnesota prior is not well suited for seasonal data. All data are obtained from the Quarterly Bulletin of the Reserve Bank of South Africa. Note that the real variables correspond to the values of the variables at year 2000's prices.

In each equation of the BVAR there are 25 parameters including the constant, given that the model is estimated with four lags for each variable, as in Dua and Ray (1995).¹¹ While, in the BVECM we have 26 parameters, including the constant, as one cointegrating relationship was found, which, in turn, led to the inclusion of one error-correction term.¹² All variables, except for the measures of the short- and long-term interest rates, have been measured in natural logarithms. Note that Sims *et al.* (1990) indicate that with the Bayesian approach entirely based on the likelihood function, the associated inference does not need to take special account of nonstationarity. This is because the likelihood function has the same Gaussian shape regardless of the presence of nonstationarity. Given this, the variables have been specified in levels.¹³

The so called, 'optimal' Bayesian prior is selected on the basis of the Mean Absolute Percentage Error (MAPE) values of the out-of-sample forecasts. Specifically, the six-variable BVAR and the BVECM are estimated for an initial prior for the period of 1971:1 to 2000:4 and, then, we forecast for 2001:1 through 2005:4. Since we use four lags, the initial four quarters of the sample, 1970:1 to 1970:4, are used to feed the lags. We generate dynamic forecasts, as would naturally be achieved in actual forecasting practice. During each quarter of the forecast period, the models are estimated in order to update the estimate of the coefficient before producing 8-quarters-ahead forecasts. This iterative estimation and 8-step-ahead forecast procedure was carried out for 20 quarters, with the

¹ Hafer and Sheehan (1989) find that the accuracy of the forecasts from the VAR is sensitive to the choice of lags. Their results indicated that shorter-lagged models are more accurate, in terms of forecasts, than longer lag models. Therefore, as in Dua and Ray (1995), for a 'fair' comparison with the BVAR models, alternative lag structures for the VAR and VECM were also examined. When we reduce the lag length to 3 and then to 2, we find marginal improvements in the accuracy of all six variables, but the rank of ordering, resulting from the alternative forecasts remained unchanged.

¹² The cointegrating relationships are based on the trace statistics compared to the critical values at the 95 per cent level. From the results of the test, we observed that the null hypothesis of $r < 1$ was rejected at the 95 per cent level because the trace statistic of 65.333 is less than the associated critical value of 69.819.

¹³ However, using the Augmented Dickey Fuller, the Phillips-Perron tests, all the 6 variables were found to be, first-order difference stationary, *i.e.*, integrated of order 1 ($I(1)$).

first forecast beginning in 2001:1. This experiment produced a total of 20 one-quarter-ahead forecasts, 20 two-quarters-ahead forecasts, and so on, up to 20 8-step-ahead forecasts. We use the algorithm in the Econometric Toolbox of MATLAB¹⁴ for this purpose. The MAPEs¹⁵ for the 20 quarter 1 through quarter 8 forecasts were then calculated for the six variables of the model. The average of the MAPE statistic values for one- to eight-quarters-ahead forecasts for the period 2001:1 to 2005:4 are then examined. Thereafter, we change the prior and a new set of MAPE values is generated. The combination of the parameter values, in the prior, that produces the lowest average MAPE values is selected, as the ‘optimal’ Bayesian prior. Following Doan (1990) and Dua *et al.* (1999), we choose 0.1 and 0.2 for the overall tightness (w) and 1 and 2 for the harmonic lag decay parameter (d). Moreover, as in Dua and Ray (1995), we also report our results for a combination of $w = 0.3$ and $d = 0.5$. Finally, a symmetric interaction function $f(i, j)$ is assumed with $k_{ij} = 0.5$, as in Dua and Smyth (1995) and LeSage (1990).

4. EVALUATION OF FORECAST ACCURACY

To evaluate the accuracy of forecasts generated by the BVARs and the BVECMs, we need to perform alternative forecasts. To make the MAPEs comparable with the BVARs and BVECMs, we report the same set of statistics for the out-of-sample forecasts generated from an unrestricted classical VAR (the benchmark model) and the Vector Error Correction (VEC) models. The unrestricted VAR has been estimated in levels with four lags. The corresponding VECM also included four lags. In Tables 1 through 6, we compare the MAPEs of one- to eight-quarters-ahead out-of-sample-forecasts for the period of 2001:1 to 2005:4, generated by the unrestricted VAR, the VECM and the 5 alternative multivariate BVARs and BVECMs. The conclusions from these tables are as follows:

(i) The VAR versus the VECM: For all the six variables the VECM outperforms the VAR in terms of the average MAPE for one- to eight-quarters ahead out of sample forecasts. Importantly, the MAPE values from the VECM are less than those generated from the VAR model in all the steps, which, in turn, ultimately results in a lower average MAPE. (ii) BVARs versus the VAR: Unlike in the forecasting literature, we do not find overwhelming evidence of the BVARs, corresponding to alternative specification of the priors, to outperform the traditional VAR. In fact, out of the six variables in our model, the VAR produces, on average, lower out-of-sample forecast errors, in terms of the one-to eight-quarters-ahead MAPEs, for three variables (CPI, 10 years and over government bond rates and the 91 days Treasury Bill rate). In the case of consumption expenditure, the VAR model does as well as the BVAR model with the most loose prior ($w = 0.3$, $d = 0.5$). However, the BVAR model, with the most tight prior ($w = 0.1$, $d = 2$), produces lower out-of-sample forecast errors for the GDP and investment expenditures, when

¹⁴ All statistical analysis was performed using MATLAB, version R2006a.

¹⁵ Note that if A_{t+n} denotes the actual value of a specific variable in period $t+n$ and ${}_tF_{t+n}$ is the forecast made in period t for $t+n$, the MAPE statistic can be defined as

$$\left(\frac{1}{N} \sum abs \left(\frac{A_{t+n} - {}_tF_{t+n}}{A_{t+n}} \right) \right) \times 100 \quad \text{where } abs \text{ stands for the absolute value. For } n=1, \text{ the}$$

summation runs from 2001:1 to 2005:4, and for $n = 2$, the same covers the period of 2001:2 to 2005:4 and so on.

Table 1. MAPE (2001:1-2005:4): Final Consumption Expenditure by Households in logs

O	VAR	VEC	(w=0.3, d=)		(w=0.2, d=)		(w=0.2, d=)		(w=0.1, d=)		(w=0.1, d=)	
			BVA	BVEC	BVA	BVEC	BVA	BVEC	BVA	BVEC	BVA	BVEC
1	0.002	0.000	0.002	0.000	0.002	0.000	0.002	0.000	0.002	0.000	0.002	0.000
2	0.005	0.003	0.005	0.003	0.005	0.003	0.006	0.003	0.005	0.003	0.006	0.003
3	0.008	0.003	0.008	0.003	0.008	0.003	0.008	0.003	0.008	0.003	0.008	0.003
4	0.010	0.003	0.010	0.003	0.010	0.003	0.010	0.003	0.010	0.003	0.010	0.003
5	0.012	0.003	0.012	0.003	0.012	0.003	0.012	0.003	0.012	0.003	0.012	0.003
6	0.014	0.004	0.014	0.004	0.014	0.004	0.014	0.003	0.014	0.003	0.014	0.003
7	0.015	0.005	0.015	0.005	0.015	0.005	0.015	0.004	0.015	0.004	0.015	0.004
8	0.016	0.006	0.016	0.006	0.016	0.006	0.016	0.005	0.016	0.005	0.016	0.005
A	0.010	0.003	0.010	0.003	0.010	0.003	0.011	0.003	0.010	0.003	0.010	0.003

MAPE: mean absolute percentage error; QA: quarter ahead.

Table 2. MAPE (2001:1-2005:4): CPI in logs (Indices 2000 = 100)

O	VAR	VEC	(w=0.3, d=)		(w=0.2, d=)		(w=0.2, d=)		(w=0.1, d=)		(w=0.1, d=)	
			BVA	BVEC	BVA	BVEC	BVA	BVEC	BVA	BVEC	BVA	BVEC
1	0.010	0.009	0.010	0.009	0.010	0.010	0.010	0.010	0.010	0.010	0.011	0.011
2	0.023	0.014	0.023	0.014	0.023	0.014	0.023	0.014	0.023	0.014	0.023	0.014
3	0.041	0.024	0.041	0.024	0.042	0.024	0.043	0.025	0.042	0.024	0.043	0.024
4	0.051	0.023	0.051	0.023	0.052	0.023	0.054	0.024	0.053	0.023	0.054	0.023
5	0.046	0.007	0.046	0.006	0.047	0.007	0.049	0.008	0.048	0.006	0.049	0.007
6	0.037	0.014	0.038	0.014	0.038	0.013	0.041	0.011	0.040	0.013	0.042	0.012
7	0.029	0.034	0.030	0.034	0.031	0.034	0.034	0.031	0.032	0.033	0.035	0.031
8	0.018	0.059	0.018	0.059	0.019	0.058	0.024	0.054	0.021	0.057	0.025	0.054
A	0.032	0.023	0.032	0.023	0.033	0.023	0.035	0.022	0.034	0.023	0.035	0.022

MAPE: mean absolute percentage error; QA: quarter ahead.

Table 3. MAPE (2001:1-2005:4): Real GDP in logs

O	VAR	VEC	(w=0.3, d=)		(w=0.2, d=)		(w=0.2, d=)		(w=0.1, d=)		(w=0.1, d=)	
			BVA	BVEC	BVA	BVEC	BVA	BVEC	BVA	BVEC	BVA	BVEC
1	0.001	0.000	0.001	0.000	0.001	0.000	0.001	0.000	0.001	0.000	0.001	0.000
2	0.004	0.001	0.004	0.001	0.004	0.001	0.004	0.001	0.004	0.001	0.004	0.001
3	0.007	0.004	0.007	0.004	0.007	0.004	0.007	0.004	0.007	0.004	0.007	0.003
4	0.009	0.004	0.009	0.004	0.009	0.004	0.009	0.004	0.009	0.003	0.009	0.003
5	0.009	0.003	0.009	0.003	0.009	0.003	0.009	0.003	0.009	0.003	0.009	0.003
6	0.007	0.001	0.007	0.001	0.007	0.001	0.007	0.001	0.007	0.001	0.007	0.000
7	0.006	0.001	0.005	0.000	0.005	0.000	0.005	0.000	0.005	0.000	0.005	0.000
8	0.004	0.001	0.004	0.001	0.004	0.000	0.004	0.000	0.004	0.000	0.004	0.000
A	0.006	0.002	0.006	0.002	0.006	0.002	0.006	0.001	0.006	0.001	0.006	0.001

MAPE: mean absolute percentage error; QA: quarter ahead.

Table 4. MAPE (2001:1-2005:4): 10 Years and Longer Government Bond Rate

O	VAR	VEC	(w=0.3, d=)		(w=0.2, d=)		(w=0.2, d=)		(w=0.1, d=)		(w=0.1, d=)	
			BVA	BVEC	BVAR	BVEC	BVAR	BVEC	BVAR	BVEC	BVAR	BVEC
1	0.585	0.491	0.587	0.492	0.592	0.497	0.617	0.514	0.603	0.507	0.644	0.518
2	1.078	0.652	1.079	0.655	1.088	0.665	1.134	0.694	1.104	0.686	1.162	0.707
3	1.533	0.958	1.536	0.961	1.553	0.972	1.636	0.981	1.585	0.990	1.682	0.973
4	1.277	0.411	1.279	0.418	1.290	0.442	1.350	0.505	1.311	0.489	1.396	0.523
5	0.923	0.064	0.925	0.057	0.934	0.032	0.991	0.034	0.953	0.020	1.041	0.063
6	1.615	0.325	1.616	0.330	1.624	0.359	1.681	0.441	1.641	0.424	1.738	0.488
7	1.903	0.407	1.904	0.410	1.911	0.439	1.971	0.525	1.927	0.507	2.039	0.586
8	2.643	0.808	2.643	0.807	2.648	0.844	2.721	0.961	2.665	0.933	2.810	1.046
A	1.445	0.514	1.446	0.516	1.455	0.531	1.513	0.582	1.474	0.569	1.564	0.613

MAPE: mean absolute percentage error; QA: quarter ahead.

Table 5. MAPE (2001:1-2005:4): Investment Expenditure in logs

O	VAR	VEC	(w=0.3, d=)		(w=0.2, d=)		(w=0.2, d=)		(w=0.1, d=)		(w=0.1, d=)	
			BVA	BVEC	BVA	BVEC	BVA	BVEC	BVA	BVEC	BVA	BVEC
1	0.051	0.054	0.051	0.054	0.050	0.053	0.048	0.048	0.049	0.051	0.047	0.046
2	0.078	0.051	0.078	0.051	0.078	0.055	0.076	0.058	0.077	0.057	0.075	0.057
3	0.086	0.041	0.086	0.041	0.086	0.041	0.086	0.041	0.086	0.042	0.083	0.043
4	0.100	0.045	0.100	0.045	0.100	0.045	0.098	0.045	0.099	0.044	0.093	0.043
5	0.105	0.050	0.105	0.050	0.104	0.047	0.097	0.040	0.101	0.042	0.090	0.036
6	0.128	0.081	0.128	0.076	0.127	0.071	0.121	0.066	0.125	0.068	0.113	0.060
7	0.104	0.059	0.104	0.059	0.102	0.056	0.093	0.045	0.098	0.049	0.084	0.038
8	0.086	0.056	0.086	0.050	0.085	0.047	0.076	0.038	0.081	0.041	0.067	0.031
A	0.092	0.055	0.092	0.054	0.091	0.052	0.087	0.048	0.089	0.049	0.081	0.044

MAPE: mean absolute percentage error; QA: quarter ahead.

Table 6. MAPE (2001:1-2005:4): 91 Days Treasury Bill Rate

O	VAR	VEC	(w=0.3, d=)		(w=0.2, d=)		(w=0.2, d=)		(w=0.1, d=)		(w=0.1, d=)	
			BVA	BVEC	BVA	BVEC	BVA	BVEC	BVA	BVEC	BVA	BVEC
1	0.165	0.259	0.167	0.254	0.178	0.229	0.217	0.135	0.204	0.178	0.237	0.098
2	0.571	0.278	0.575	0.271	0.592	0.243	0.659	0.149	0.627	0.189	0.664	0.114
3	1.721	0.180	1.726	0.190	1.749	0.234	1.837	0.345	1.790	0.305	1.814	0.350
4	2.750	0.215	2.754	0.220	2.773	0.299	2.840	0.506	2.805	0.422	2.788	0.538
5	2.855	0.278	2.857	0.261	2.869	0.181	2.914	0.061	2.888	0.038	2.852	0.112
6	2.323	1.023	2.325	0.992	2.330	0.911	2.356	0.653	2.337	0.762	2.301	0.589
7	2.363	1.284	2.363	1.276	2.365	1.101	2.389	0.900	2.360	1.031	2.350	0.831
8	2.250	1.607	2.250	1.576	2.248	1.487	2.279	1.180	2.250	1.314	2.264	1.088
A	1.875	0.640	1.877	0.631	1.888	0.597	1.936	0.492	1.909	0.530	1.909	0.465

MAPE: mean absolute percentage error; QA: quarter ahead.

compared to the VAR. Ni and Sun (2005) find similar results with regard to tighter BVARs producing better forecasts.

(iii) BVARs versus the VECM: From the discussion above, in (i), we know that, in terms of the out-of sample one- to eight-quarters ahead forecasts, the VECM outperforms the VAR for all the six variables, so naturally, given the discussion in (ii), the former also performs better in comparison to the BVARs in terms of CPI, 10 years and over government bond rates, the 91 days Treasury Bill rate and consumption. In addition, the VECM produces lower one- to eight-quarters-ahead average MAPEs for the GDP and the investment expenditures, when compared to the BVAR with a most tight prior. (iv) BVECMs versus the VAR: In this case, it is always possible to come up with a BVAR model, based on alternative specification of priors, that produces lower out-of-sample forecast errors, measured by the average MAPE, when compared to the VAR. Except for the final consumption expenditures by households and the 10 years and longer government bond rates, the BVECM model with the most tight prior ($w = 0.1$, $d = 2$) does the best amongst the alternative specifications of the BVECMs. Final consumption expenditures and the 10 years and longer government bond rates with relatively loose priors, namely ($w = 0.3$, $d = 0.5$) and ($w = 0.2$, $d = 2$) respectively, produce the lowest average MAPEs. However, the BVECM with the most tight prior ($w = 0.1$, $d = 2$) produces the second best average MAPE for the final consumption expenditures amongst the BVECMs, which is, however, lower than that generated by the VAR. (v) BVECMs versus the VECM: Except for the 10 years and longer government bond rates, we can find a BVECM model that produces lower average MAPEs in comparison to the VECM. Moreover, as in (iv), a BVECM model with the most tight prior ($w = 0.1$,

$d = 2$) outperforms the VECM model for five of the six variables, even though the former, with the existing prior specifications, does only second best, in terms of out-of-sample forecasting errors, for the consumption expenditures of the households. Moreover, as in LeSage (1990), we observe that the BVECM, in general (except for the long-term interest rate), produces smaller values of MAPE, in comparison to the BVAR, with an increase in the size of the forecast-horizon. This result exemplifies the importance and suitability of the BVECM models, especially for long-run forecasting.

(vi) BVARs versus BVECMs: Given that we know the VECM outperforms the BVARs for all the six variables, but the former is superseded by the BVECMs for five of the six variables, it is obvious that the BVECMs produce lower out-of-sample forecast errors, for all the six variables, in comparison to the BVARs.

Thus, from (i) to (vi), we can conclude that the BVECM model with an overall tightness value (w) of 0.1 and a decay factor (d) of 2, is clearly best suited to forecast at least five (household consumption expenditures, CPI, GDP, investment expenditures and the 91 days Treasury bill rate) of the six variables of our choice, used to represent the South African economy. This is especially true for long-horizon forecasting over the period of 2001:1 to 2005:4. It is also important to emphasize that the Bayesian models with relatively tight priors are found to produce, on average, the minimum out-of-sample forecast errors relative to the Classical versions of the VAR and the VECM. Note that as far as the tightness of priors are concerned, our results are in sharp contrast with that of Dua and Ray (1995), since they find that a loose prior generally produces more accurate forecasts. However, we corroborate the findings of Ni and Sun (2005).¹⁶ But, amongst the alternative BVARs, the one with the most loose prior ($w = 0.3$ and $d = 0.5$) is optimal for the consumption expenditures of households, CPI, 10 years and longer government bond rates, and the 91 days treasury bill rate.

Given the above findings, we are now ready to compare the results of our study with that of Gupta and Sichei (2006). The authors show that it is possible to obtain a BVAR model that can produce lower out-of-sample forecast errors on average, in comparison to the unrestricted VAR model, for consumption, CPI, GDP and investment expenditures. We, however, find that a BVAR model with the most tight priors produces lower forecast errors for GDP and investment only, with the VAR doing equally as well as the 'optimal' BVAR for consumption, and better for the CPI. But, like this study, Gupta and Sichei (2006) indicate that the BVAR models that perform better than the VAR, are in fact the ones with most tight priors. The obvious question, then, is: How can two studies, using the same data-set, obtained from the same source, estimated over the same period of time with the same number of lags and the same set of priors for the BVARs, and, most importantly, using the same method of estimation, *i.e.*, Theil's (1971) mixed estimation technique, produce different results?

In our opinion, this is possibly due to the different algorithms used to estimate the model recursively over the period of 2001:1 and 2005:4, since there is no difference in the estimated values of the coefficients,¹⁷ when we estimate the VAR and BVARs in RATS – the software Gupta and Sichei (2006) use for their paper. Note that RATS uses the

¹⁶ Our choice of the priors is in line with the suggestions of Doan (2000).

¹⁷ The results from the estimation are available from the author on request.

Kalman filter algorithm for carrying out the recursive estimation, while we use the algorithm available in the Econometric Toolbox of MATLAB. Our results, thus, also highlight the importance of algorithms of alternative softwares used for generating forecasts. This problem of inconsistency, if we may call it so, is, however, not too much of a concern in our case. This is because, using the algorithm in MATLAB, we were able to come up with ‘better’ forecasting models – the VECM and the BVECMs. Note that the term ‘better’ is used in the sense that these models generated lower out-of-sample forecast errors in comparison to both the VAR and the BVARs.

At this stage, it must be pointed out that there are at least two limitations to using the BVAR and BVECM models for forecasting. Firstly, as is clear from Tables 1 to 6, the accuracy of the forecasts is sensitive to the choice of the priors. Clearly, then, if the prior is not well-specified, an alternative model used for forecasting may perform better. Secondly, in case of the Bayesian variants, one requires to specify an objective function, for example the MAPE, to search for the ‘optimal’ priors which, in turn, needs to be optimized over the period for which we compute the out-of-sample forecasts. However, there is no guarantee that the chosen parameter values specifying the prior will also be ‘optimal’ beyond the period for which it was selected.

5. TURNING POINTS: THE PERFORMANCE OF ALTERNATIVE MODELS

While, in general, the BVECM models produce the most accurate forecasts, a different way to evaluate the performance of the alternative models can be based on their ability to predict the turning point(s) in the chosen variables. In this regard, we compare the performance of the optimal BVARs and BVECMs, the (benchmark) unrestricted VAR and the VECM, with respect to the actual data. Based on the minimum average MAPE values for the one- to eight-quarters-ahead out-of-sample forecasts over the period of 2001:1 to 2005:4, the optimal priors for the BVARs and BVECMs, identified from Tables 1 to 6, are indicated below. For the BVAR models: $w = 0.3$ and $d = 0.5$ are optimal for the consumption expenditures of households, CPI, 10 years and longer government bond rates and the 91 days Treasury Bill rate, while for GDP and investment expenditures, the BVAR model with the most tight prior ($w = 0.1$ and $d = 2.0$) produces the minimum one- to eight-quarters-ahead out-of-sample forecast errors on average.

For the BVECMs: $w = 0.1$ and $d = 2.0$ are optimal for the CPI, GDP, investment and the 91 days Treasury bill rate. The BVECMs with $w = 0.2$ and $d = 2.0$ and $w = 0.3$ and $d = 0.5$ generate the minimum average values of the MAPE for consumption and the 10 years and longer government bond rates, respectively.

As is indicated by Figures 1 through 6, the VECM and the optimal BVECMs correctly predict the direction of change for all the variables over the period of 2004:1 to 2005:4. On the other hand, except in the case of the short-term interest rate measure, the VAR and the optimal BVAR models perform as well as the Classical and Bayesian variants of the VECMs. As can be seen from Figure 6, the VAR and the optimal BVAR predict an increase in the Treasury bill rate, when it has actually declined over the period concerned. Moreover, note that the forecasts from the VAR and the ‘optimal’ BVAR move very close to one another in the case of consumption, CPI, and the long- and short-term interest rate measures.

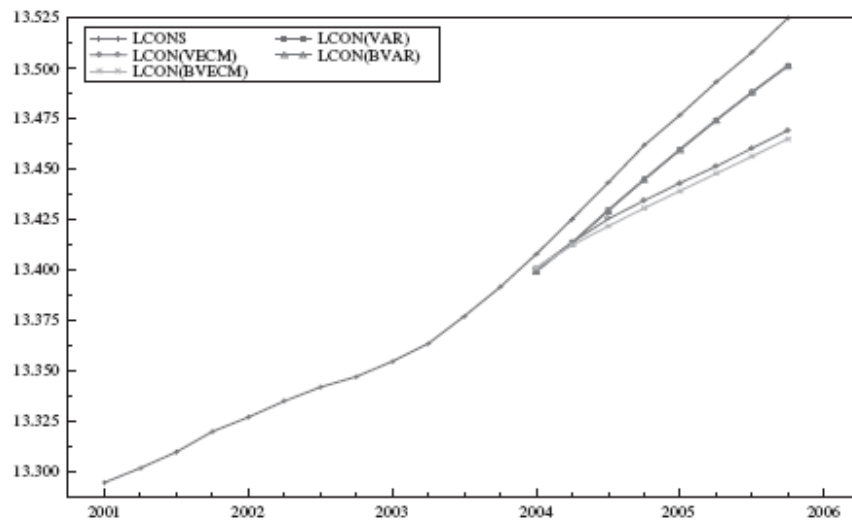


Figure 1. Household Consumption Expenditures, forecasts for 2004:1-2005:4

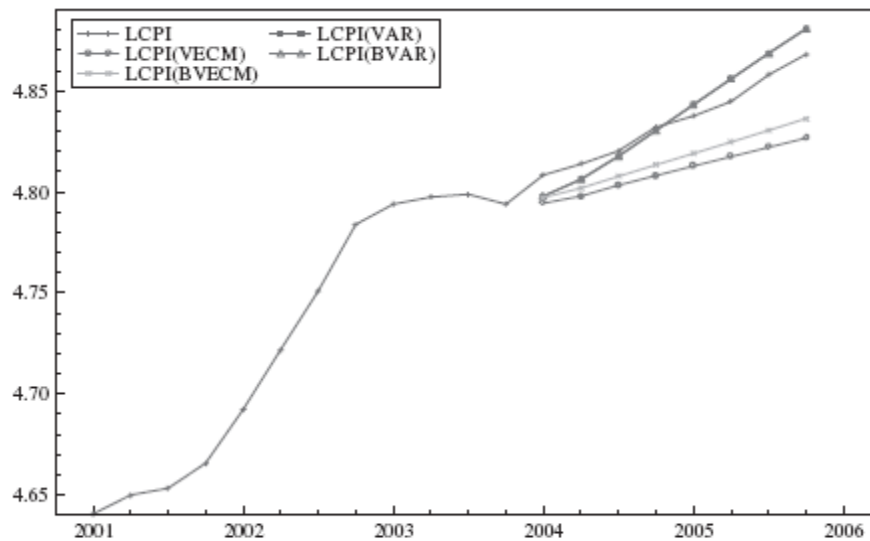


Figure 2. Consumer Price Index, forecasts for 2004:1-2005:4

6. CONCLUSIONS AND AREAS FOR FURTHER RESEARCH

This paper compares the ability of Vector Autoregressive (VAR) Models and Vector Error Correction Models (VECM), both Classical and Bayesian in nature, in forecasting the South African economy. For this purpose, we estimate these models using quarterly data on consumption, the Consumer Price Index (CPI), Gross Domestic Product (GDP), investment, and measures of short and long term interest rates, measured by the 91 days Treasury Bill Rate and 10 years and longer government bond rates respectively, for the period of 1970 to 2000. We then compare out-of-sample forecast errors generated by

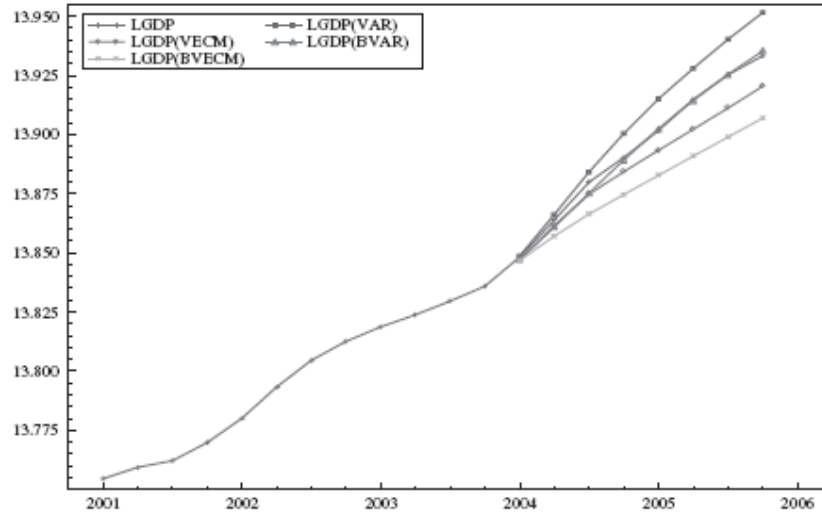


Figure 3. Real Gross Domestic Product, forecasts for 2004:1-2005:4

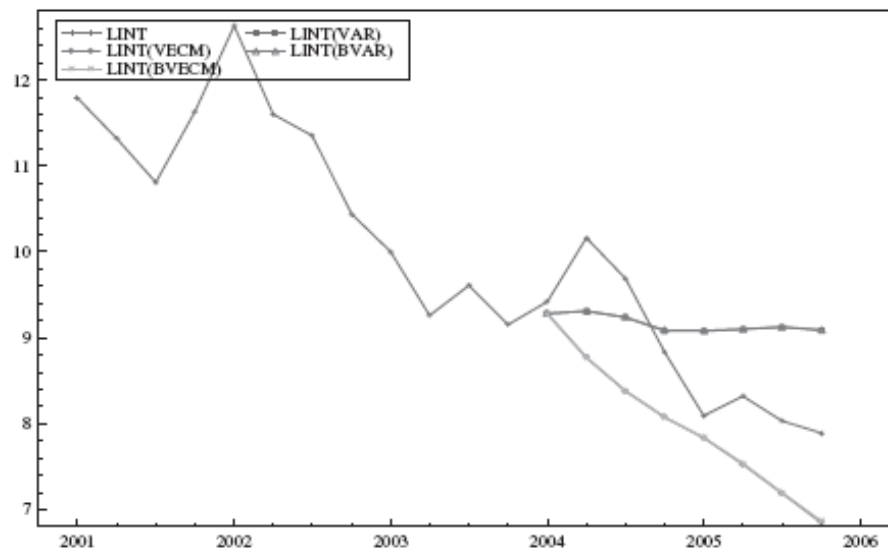


Figure 4. 10 Years and Longer Government Bond Rate, forecasts for 2004:1-2005:4

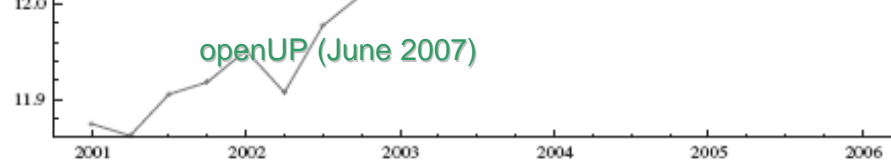


Figure 5. Investment Expenditure, forecasts for 2004:1-2005:4

of
in

macroeconomic variables over the period of 2004:1 to 2005:4.

The BVECM model, in general, except for the long-term interest rate measure, produces the most accurate forecasts relative to the alternative models. Within the class of the multivariate BVECMs, the model with the most tight prior outperform the other models, in terms of forecasting consumption, CPI, GDP, investment and the 91 days Treasury bill rate. The 'optimal' BVECMs also correctly predict the direction of change

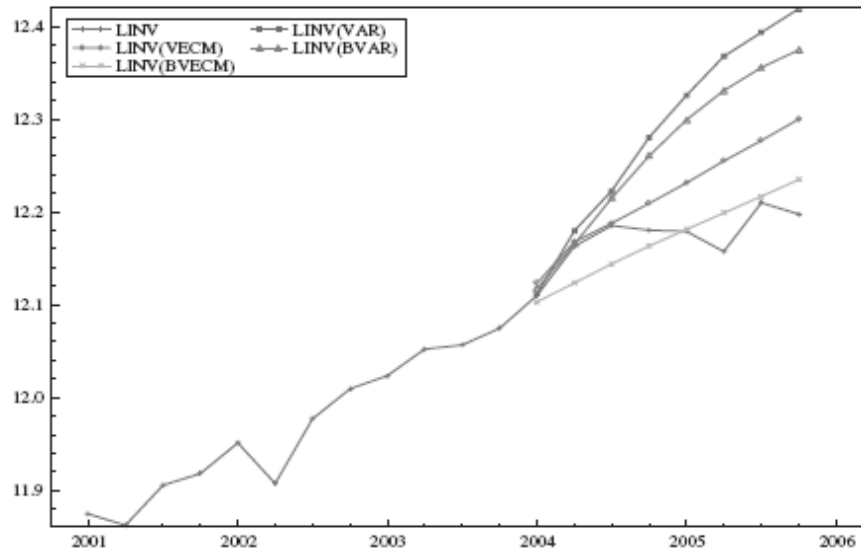


Figure 5. Investment Expenditure, forecasts for 2004:1-2005:4

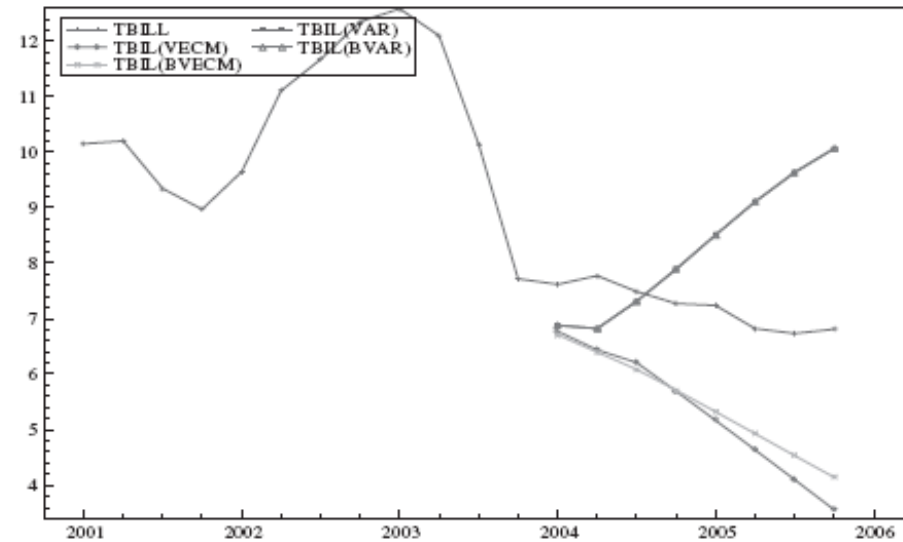


Figure 6. 91 Days Treasury Bill Rate, forecasts for 2004:1-2005:4

for the chosen macroeconomic variables. Based on our study, it seems that a BVECM with relatively tight priors is best suited for forecasting the South African economy.

As an aside, we also indicate that alternative algorithms in alternative softwares, namely RATS and MATLAB, used for generating out-of-sample forecasts in a recursive fashion, can yield different results. However, this is not a concern in our case, since we were able to generate better forecasts based on the Classical and Bayesian variants of the VECMs relative to their VAR counterparts, among which the inconsistency is observed.

There are, however, as noted earlier, limitations to using the Bayesian approach. Firstly, the forecast accuracy depends critically on the specification of the prior, and secondly, the selection of the prior based on some objective function for the out-of-sample forecasts may not be 'optimal' for the time period beyond the period chosen to produce the out-of-sample forecasts.

Besides these, there are two other major concerns which are, however, general to any traditional statistically estimated models, for example the VARs and the VECMs – both Classical and Bayesian in nature, used for forecasting business cycle frequencies. Such procedures perform reasonably well as long as there are no structural changes experienced in the economy. Such changes, whether in or out of the sample, would then render the models inappropriate. Alternatively, these models are not immune to the 'Lucas Critique'.¹⁸ Furthermore, the estimation procedures used here are linear in nature, and, hence, they fail to take into account nonlinearities in the data. One, and perhaps the best, response to these objections has been the development of micro-founded Dynamic Stochastic General Equilibrium (DSGE) models, which are capable of handling both the problems arising out of the structural changes and the issues of nonlinearities.¹⁹ The current trend in the forecasting literature is clearly dominated by the use of calibrated and estimated versions of DSGE models which, in turn, have also been found to produce better forecasts relative to the traditional forecasting models. In this regard, some studies worth mentioning are: Hansen and Prescott (1993), Ingram and Whiteman (1994), Rotemberg and Woodford (1995), Ireland (2001), and Zimmermann (2001), to name a few. Future research involving DSGE models to forecast the South African economy is hence, clearly an area to delve into.

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¹⁸ See Lucas (1976) for details.

¹⁹ For a detailed review of the literature on the use of DSGE models for forecasting, see Zimmermann (2001).

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