A MULTIOBJECTIVE OPTIMIZATION MODEL FOR OPTIMAL PLACEMENT OF SOLAR COLLECTORS

by

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SUMMARY

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Department:	Electrical, Electronic and Computer Engineering
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Keywords: fixed solar collectors, multi-objective optimization, weighted sum approach, Pareto front, genetic algorithm

The aim and objective of this research is to formulate and solve a multi-objective optimization problem for the optimal placement of multiple rows and multiple columns of fixed flat-plate solar collectors in a field. This is to maximize energy collected from the solar collectors and minimize the investment in terms of the field and collector cost. The resulting multi-objective optimization problem will be solved using genetic algorithm techniques.

It is necessary to consider multiple columns of collectors as this can result in obtaining higher amounts of energy from these collectors when costs and maintenance or replacement of damaged parts are concerned. The formulation of such a problem is dependent on several factors, which include shading of collectors, inclination of collectors, distance between the collectors, latitude of location and the global solar radiation (direct beam and diffuse components). This leads to a multi-objective optimization problem. These kind of problems arise often in nature and can be difficult to solve. However the use of evolutionary algorithm techniques has proven effective in solving these kind of problems. Optimizing the distance between the collector rows, the distance between the collector columns and the collector inclination angle, can increase the amount of energy collected from a field of solar collectors thereby maximizing profit and improving return on investment.

In this research, the multi-objective optimization problem is solved using two optimization ap-

proaches based on genetic algorithms. The first approach is the weighted sum approach where the multi-objective problem is simplified into a single objective optimization problem while the second approach is finding the Pareto front.

OPSOMMING

N MULTIOBJECTIVE OPTIMALISERING MODEL VIR OPTIMALE PLASING VAN SONKRAG VERSAMELAARS

deur

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Sleutelwoorde:	vaste sonkollektors, multi-objektiewe optimalisering, geweegde som benadering, Die Pareto voor, genetiese algoritme

Die doel van hierdie navorsing is om ’n multi-objektiewe optimalisering probleem vir die optimale plasing van verskeie rye en verskeie kolomme van vaste plat plaat sonkollektors in ’n veld te formuleer en op te los. Dit is die energie versamel uit die sonkollektors te maksimeer en die belegging in terme van die veld en versamelaar koste te verminder. Die gevolglike multi-objektiewe optimalisering probleem opgelos sal word met behulp van genetiese algoritme.

Dit is wat nodig is om verskeie kolomme van versamelaars te oorweeg as dit kan lei tot die verkryging van hoÃńr hoeveelhede energie van hierdie versamelaars koste en onderhoud of vervanging van beskadigde dele. Die formulering van so ’n probleem is afhanklik van verskeie faktore, wat insluit die skadu van die versamelaars, die neiging van versamelaars, afstand tussen die versamelaars, breedte van die plek en die globale sonstraling (direkte balk en diffuse komponente). Dit lei tot ’n multi-objektiewe optimalisering probleem. Hierdie soort van probleme ontstaan âĂŃâĂŃdikwels in die natuur en kan dit moeilik wees om op te los. Maar die gebruik van evolusionÃłre algoritme tegnieke effektief in die oplossing van hierdie soort probleme het bewys. Die optimalisering van die afstand tussen die versamelaar rye, die afstand tussen die kollektor kolomme en die versamelaar inklinasiehoek, kan verhoog die hoeveelheid energie wat versamel is van ’n gebied van sonkollektors sodoende die maksimum wins en die verbetering van die opbrengs op belegging.

In hierdie navorsing word die multi-objektiewe optimalisering probleem opgelos deur gebruik te maak van twee optimalisering benaderings op grond van genetiese algoritmes. Die eerste benadering is die geweegde som benadering waar die multi-objektiewe probleem in ’n enkele doel optimalisering probleem vereenvoudig word, terwyl die tweede benadering is om die Pareto-front.

LIST OF ABBREVIATIONS

MOO	Multi-objective Optimization
MOOP	Multi-objective Optimization Problem
SOOP	Single Objective Optimization Problem
EA	Evolutionary Algorithm
GA	Genetic Algorithm

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CHAPTER 1 INTRODUCTION

This aim of this chapter is to introduce the research topic and proffer reasons for carrying out this research. Here, the research gap and certain questions to be answered during this study are identified. The research approach, goals and contribution are also stated.

This dissertation formulates a new multi-objective optimization (MOO) model to solve for the optimal placement of fixed solar collectors in a field in order to maximize the energy collected and minimize the field and collector cost which is also the investment cost. This formulation results in a constrained multi-objective optimization problem (MOOP). This problem is solved using two methods: The first method is the weighted sum approach which transforms the MOOP into a single objective optimization problem (SOOP) and the other method is finding the Pareto front made up of Pareto optimal solutions. Both methods will be solved by genetic algorithm (GA), which is a branch of evolutionary algorithm (EA) techniques. The final decision on which of the solutions is to be chosen will be dependent on the interpretation of their relative importance by the decision maker.

1.1 PROBLEM STATEMENT

Presently, the demand for energy in various sectors is growing at a rate that energy supply cannot meet. Also the demand for land is on the increase which limits its availability for construction of power plants. Owing to the increase in the unit cost of available land coupled with the inability to meet the growing energy demand, there is the need to optimally place multiple rows and multiple columns of solar collectors in a solar field that will maximize the energy collected from the field and minimize the investment (field and collector cost).

1.1.1 Context of the problem

Typical collector sizes range between 1.5 to 4 m^2, each costing between £3000 and £5000 [1] or $ 200 per m^2 [2]. These individual collectors are combined to form an array where the length of an array is proportional to the cost. Logically, shorter array length collectors will be preferable as they are cost effective and require less maintenance. The choice of smaller array length collectors is also enhanced if the energy obtained is large enough and sufficient. This research will provide a technical perspective and aid better solar collector and field design.

1.1.2 Research gap

The research gap is that the optimal placement of multiple rows and multiple columns of solar collectors to maximize energy collected and minimize investment cost in a field has not yet been considered together in literature using a Pareto-based multi-objective approach. Bridging this gap can go a long way in helping to improve solar field and fixed collector design.

1.2 RESEARCH OBJECTIVE AND QUESTIONS

The objective of this research is to formulate a multi-objective optimal control model for the optimal placement of fixed solar collectors in a multi-row, multi-column field array. This model is to optimally place the solar collectors thereby maximizing the energy collected from such a field and reducing the investment cost of purchasing the field and collectors while it satisfies all the necessary constraints involved.

The problem lies in finding an optimal inclination angle for the fixed collectors, an optimal distance between the collector rows as well as an optimal distance between the collector columns. Optimizing these variables will ensure the objectives of maximizing energy collected and minimizing the investment (field and collector) cost are achieved.

The following research questions will be answered in this work:

- Can more energy be obtained from this model compared to previous models by optimizing the chosen variables?
- Can the investment cost of this model be reduced compared to previous models?

- Are the optimization methods prescribed in this research able to obtain acceptable solutions?

1.3 HYPOTHESIS AND APPROACH

The hypothesis is that the solar collector model formulated in this research which contains all the necessary parameters will maximize energy collected and minimize investment cost in a field of multiple rows and multiple columns of solar collectors.

The approach of the study is as follows.

1. A brief background study on solar energy and multi-objective optimization is done.
2. A study of existing solar collector models will be done. Their drawbacks will also be considered.
3. A MOOP will be defined for the solar collector placement problem with all the necessary constraints such as field and economic constraints and a variable not previously considered is added to the newly formulated solar collector model. This MOOP is solved using two methods namely: weighted sum approach and the Pareto front.
4. A comparison between the results obtained from this model and that of a previous model will be done using the same parameters. This comparison is to highlight the effectiveness of the new model when compared to other models.

1.3.1 Motivation for the use of multi-objective optimization in solar collector placement problem

Most real-world problems involve simultaneous optimization of several objective functions. Generally, these functions are often competing and conflicting objectives. MOO having such conflicting objective functions gives rise to a set of candidate optimal solutions, instead of one optimal solution. Here no solution can be considered to be better than any other with respect to all objectives. These candidate optimal solutions are known as Pareto-optimal solutions. Classical optimization methods can at the best find one solution in one simulation run. Therefore these methods are inconvenient to solve multi-objective optimization problems. EAs, on the other hand, can find multiple candidate optimal solutions in one single simulation run due to their population based approach [3].

In MOO, the decision maker seeks to simultaneously optimize several objectives for his problem. An improvement in one objective often results in the deterioration of other objectives and trade-offs are necessary. A diagram for MOO is shown in Figure 1.1.

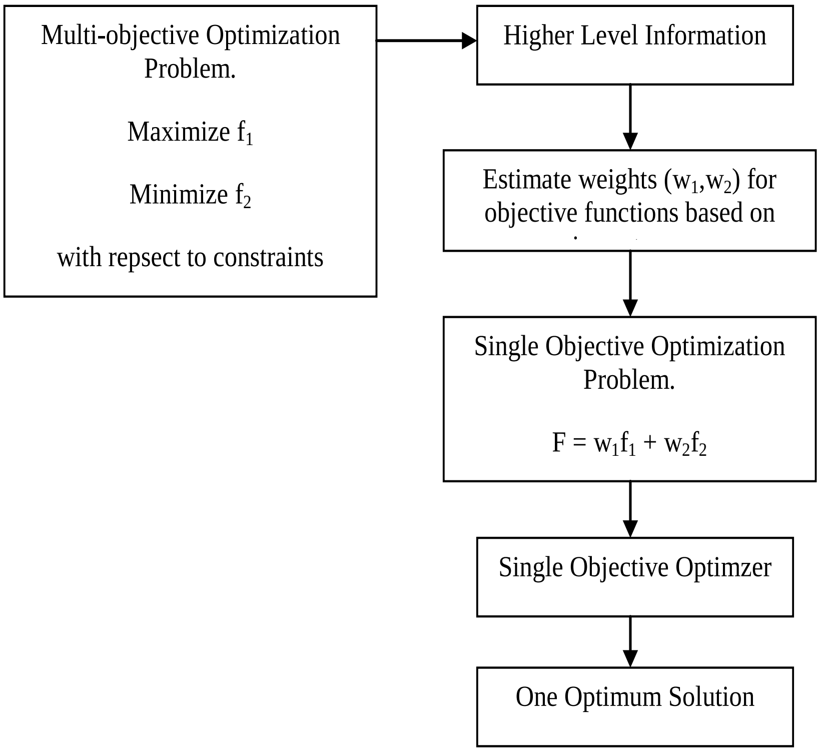

Figure 1.1: Schematic for multi-objective optimization via weighted sum approach

MOO has useful characteristics for obtaining optimal solutions for solar collector optimization problems. The most important characteristic in this research is that it gives the decision maker multiple optimal solutions to choose from depending on the objectives.

Therefore using MOO in solving a solar collector placement problem will introduce a new approach in solar collector design and field array optimization.

1.3.2 Research process and modeling

The following sequential steps are to be followed in this research:

1. The existing models are studied to identify the various methods that have been used to optimize the solar collector field design, their advantages and disadvantages.

2. A study of multi-objective optimization is done.

3. A multi-objective model using energy and cost objective functions is formulated. The model has three variables: distance between collector rows, distance between collector columns and inclination angle of collector. The bounds are then added to the model.

4. The two methods for solving MOOP using GA are then used to solve the problem.

5. A comparison is done between both methods and also with previous models.

1.3.3 How this approach addresses current issues

The current issues that are addressed by this research approach include:

1. Available land area: This constrains the problem when land available is limited. Hence by taking this into consideration, the model can be applied in such instances.

2. Available budget: This is customer dependent and must also be added as a constraint to the problem.

3. Based on the above items, a number of candidate optimal solutions will be obtained from which a choice can be made. This choice is dependent on the relative importance of each solution with respect to the decision maker's preference.

4. Time period for optimization: This varies depending on the problem and the intended output. The selected approach optimizes for the time period of a whole year.

1.3.4 Limitations and challenges of selected approach

The limitations and challenges of this approach are:

1. The GA uses a random initial population, therefore the result is different for each run. This limitation is resolved by utilizing the best solutions from each run in obtaining the optimal solutions.

2. Constraining the problem makes the modeling and optimization very complex but realistic.

3. The results are limited by the solar radiation data used.

The aim however is to add to the existing body of knowledge.

1.4 RESEARCH GOALS

The research goals are:

1. To formulate a multi-objective optimal control model for the optimal placement of fixed solar collectors in a multiple row and multiple column field array and to solve this model using MOO and GA techniques.

2. To maximize the energy collected from such a field.

3. To reduce the investment cost of purchasing the field and collectors whilst satisfying all the necessary constraints involved.

1.5 RESEARCH CONTRIBUTION

The major contributions of this research are:

1. A multi-objective optimization model has been developed for optimizing collector placement in a solar field design. A highlight of this model is the addition of multiple columns of solar collectors to existing models.

2. Some insights and deep understanding have been obtained for multi-objective problems and GA through the use of the optimization framework in solar field design.

1.6 OVERVIEW OF STUDY

This research is to formulate a collector row and column placement problem with economic consideration and derive optimal solutions using evolutionary algorithm techniques. This research presents a multi-objective optimization model that maximizes solar energy collected from a field array of solar collectors using measured solar radiation data while minimizing the investment (field and collector cost). Nature abounds with a vast number of these kind of multi-objective problems. Solving these problems are an on-going concern. Evolutionary algorithms have proven effective in solving these kind of problems. A MOOP can be solved in two ways; the first one is to solve it by transforming the MOOP into SOOP using positive weights (for objectives) and penalties (for constraints), and the other one is to obtain Pareto optimal solutions which gives the decision maker a suitable range of choices to adjust trade off between different objectives [4].

This chapter introduces the background to the research problem and briefly describes the research approach. Chapter 2 is a literature review which includes, the description of the research problem, studies of different solution techniques that have been applied to solve this kind of problem, the research approach and the contributions of this research are covered. In Chapter 3, a multi-objective optimization model for optimal placement of the solar collectors in a field array is formulated. This MOO model is then evaluated using both methods of weighted sum approach and obtaining the Pareto front. Chapter 4 gives the results obtained from the application of this model. Data from previous models is applied to the current model and the solutions are compared. Chapter 5 is a discussion of the results then Chapter 6 concludes and makes recommendations for further research.

CHAPTER 2 LITERATURE STUDY

The literature survey is covered in this chapter. It gives a background on solar energy and reviews the related existing work. A literature study is also done for the proposed research approach.

2.1 CHAPTER OBJECTIVES

The objective of this chapter is to provide a review of previous and related works on the aspect of solar collector field design. A brief analysis of each work and the major contributions as well as the drawbacks will also be discussed along with the motivation for research.

2.2 AN INTRODUCTION TO SOLAR ENERGY

The world's marketed energy consumption is projected to grow by 53 % from 2008 to 2035. Forecast also suggest that total world energy use will rise from 505 quadrillion British thermal units (Btu) in 2008 to 619 quadrillion Btu in 2020 and 770 quadrillion Btu in 2035 (Figure 2.1). Much of this growth in energy consumption occurs in countries outside the Organization for Economic Cooperation and Development (non-OECD nations), where demand is driven by strong long-term economic growth. Energy use in non-OECD nations increases by 85 % in the reference case, as compared with an increase of 18 % for the OECD economies [5]. The world energy data consumption is shown in Figure 2.1.

Solar energy is the radiant energy that is produced by the sun. The sun emits more energy per second than the world has used since time began. It takes the sun's energy about eight minutes to travel the 93 million miles to Earth at the speed of light. Only a small part of the radiant energy that the sun emits into space reaches the Earth, but it is more than enough to supply all our energy needs. The amount of solar energy that reaches the Earth daily is enough to supply a nation's energy needs for a year [6].

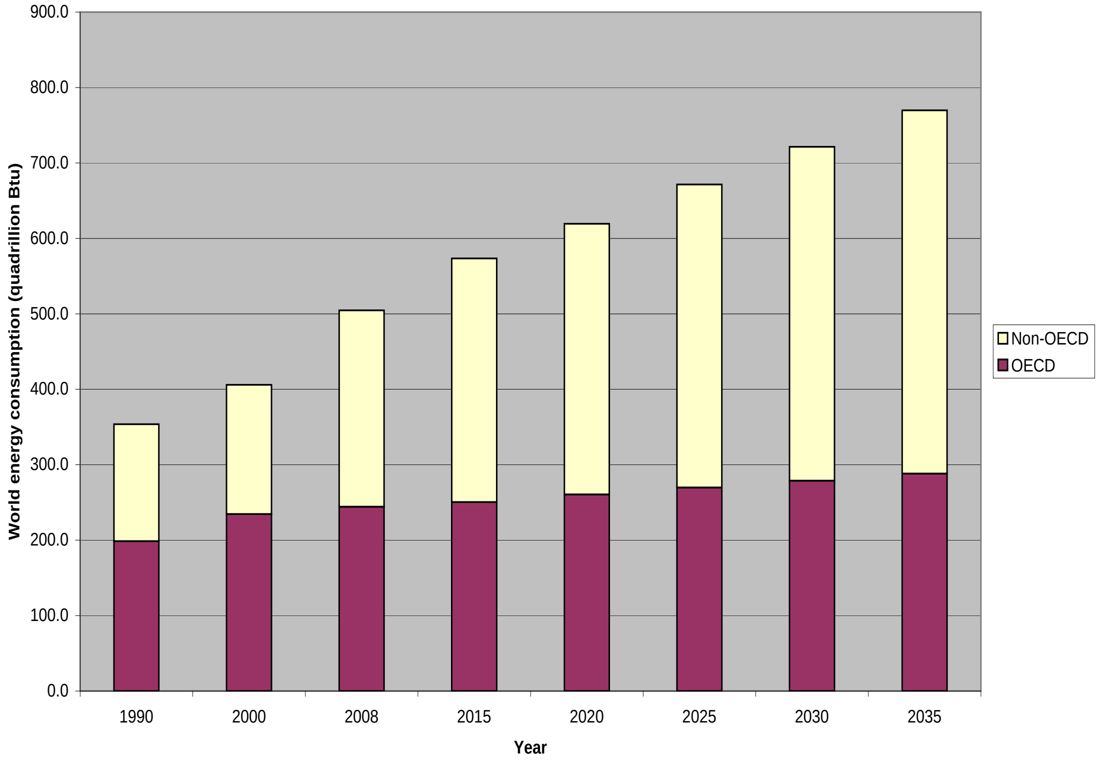

Figure 2.1: World Energy Consumption from 1930 - 2035 [5]

2.2.1 Solar collectors

Solar energy is a potential source of renewable energy for most countries especially those located in the solar belt north and south of the equator [7]. Capturing solar energy for useful work is difficult because the solar energy reaching the Earth is radiated over a large area since the sun does not deliver much energy to any one place at any given time. The amount of solar energy an area receives depends on the time of day, the season of the year, the cloudiness of the sky, and how close the area is to the Earth's equator. The use of a solar collector is one way to capture sunlight and convert it into usable heat energy. A simple example of a solar collector is a closed car on a sunny day. As sunlight passes through the car's windows, it is absorbed by the seat covers, walls, and floor of the car. The absorbed light changes into heat. The car's windows let light in, but does not allow all the heat out [6]. Figure 2.2 shows a field of solar collectors. Optimum inclination of flat-plate solar collectors will assure maximum collection of this natural solar energy [7].

Figure 2.2: Field of Solar Collectors [8]

2.2.2 Solar radiation

Solar systems, like any other system, need to be operated with the maximum possible performance. This can be achieved by proper design, construction, installation, and orientation [9]. The solar radiation plays an important role in any solar system performance. Solar radiation data is usually measured in the form of global and diffuse radiation on a horizontal surface at the latitude of interest. Flat-plate solar collectors are tilted so that they capture the maximum radiation and the problem of calculating solar radiation on a tilted surface is in determining the relative amount of beam and diffuse radiation contained in the measured horizontal global radiation [10]. The incident solar radiation reaches the earth's surface without being significantly scattered and coming from the direction of the sun, is called direct normal irradiance (or beam irradiance). Some of the scattered sunlight is scattered back into space and some of it also reaches the surface of the earth. The scattered radiation reaching the earth's surface is called diffuse radiation. Some radiation is also scattered off the earth's surface and then re-scattered by the atmosphere to the observer. This is also part of the diffuse radiation the observer sees. This amount can be significant in areas in which the ground is covered with snow. The total solar radiation on a horizontal surface is called global irradiance and is the sum of incident diffuse radiation plus the direct normal irradiance projected onto the horizontal surface. If the surface under study is tilted with respect to the horizontal, the total irradiance is the incident diffuse radiation plus the direct normal irradiance projected onto the tilted surface plus ground reflected irradiance that is incident on the tilted surface [11].

2.2.2.1 Components of solar radiation

1. Direct beam solar radiation: Beam radiation is the solar radiation propagating along the line

joining the receiving surface and the sun. It is also referred to as direct radiation. Therefore, direct beam radiation comes in a direct line from the sun. For sunny days with clear skies, most of the solar radiation is direct beam radiation. On overcast days, the sun is obscured by the clouds and the direct beam radiation is zero [12], [13], [14].

2. Diffuse solar radiation: Diffuse solar radiation is that portion of solar radiation that is scattered downwards by the molecules in the atmosphere. This radiation is scattered out of the direct beam by molecules, aerosols, dust and clouds; it does not have a unique direction [14]. Because it comes from all regions of the sky, it is also referred to as sky radiation. During days with clear skies, the magnitude of diffuse radiation is about 10 to 14% of the total solar radiation received at the earth's surface and up to 100% for cloudy skies. This means that only diffuse radiation may reach the earth's surface during extremely cloudy days [12], [13], [15].

3. Reflected solar radiation: When the solar radiation irradiates upon a surface which is opaque, a portion of radiation is absorbed and the remaining portion is reflected in diffuse or specular nature depending on the roughness of the surface [13], [15].

4. Global solar radiation: The sum of the direct beam, diffuse, and ground-reflected radiation arriving at the surface is called total or global solar radiation. Although the radiation reflected by the surface in front of a collector contributes to the solar radiation received, it must be noted that unless the collector is tilted at a steep angle from the horizontal and the ground is highly reflective (e.g., snow), this contribution is small. Therefore, the total or global solar radiation striking a collector has two components, direct beam radiation and diffuse radiation [12], [13], [14], [16], [17].

Most of the published meteorological data give the total radiation on horizontal surfaces. Therefore correlation procedures are required to obtain insolation values on tilted surfaces from horizontal radiation. Monthly average daily total radiation on a tilted surface (H_T) is normally estimated by individually considering the direct beam (H_B), diffuse (H_D) and reflected components (H_R) of the radiation on a tilted surface. Thus for a surface tilted at a slope angle from the horizontal, the incident total radiation is given by the relation in equation (2.1) [11, 18, 19]:

$$H_T = H_B + H_D + H_R \tag{2.1}$$

Several models are proposed by various investigators [20, 21, 22, 23, 24, 25, 26, 27] to calculate global radiation on tilted surfaces from the available data on a horizontal surface. Based on the assumptions made, the estimation models can be classified into isotropic [28] and anisotropic [20] ones. The daily beam radiation received on an inclined surface can be expressed as in equation (2.2) [11, 18, 19]:

$$H_B = (H_g - H_d)R_b \tag{2.2}$$

where H_g and H_d are the monthly mean daily global and diffuse radiation on a horizontal surface, and R_b is the ratio of the average daily beam radiation on a tilted surface to that on a horizontal surface. The daily ground reflected radiation can be written as equation (2.3) [11, 18, 19]:

$$H_R = H_g \rho \frac{1 - \cos\beta}{2} \tag{2.3}$$

where β is the tilt angle of the solar panel and ρ is the ground albedo.

In the absence of any solar radiation measurements, we employ models using meteorological data such as cloudiness and minutes of sunshine to estimate solar radiation. Although much less accurate, this is often the only option that exists for locations where solar radiation is not measured.

2.2.3 Applications of solar energy

1. **Solar Space Heating**: Solar energy can be used to heat the space inside a building. Today, many homes use solar energy for space heating. A passive solar home is designed to let in as much sunlight as possible. It acts like a big solar collector. Sunlight passes through the windows and heats the walls and floors inside the house. The light can get in, but the heat is trapped inside. A passive solar home does not depend on mechanical equipment, such as pumps and blowers, to heat the house [29].

2. **Solar Water Heating**: Solar energy is commonly used to heat water. Heating water for bathing, dishwashing, and washing clothes is the third largest home energy cost. A solar water heater works a lot like solar space heating. A solar collector is mounted on the roof where it can capture sunlight. The sunlight heats water in a tank. The hot water is piped to faucets throughout a house, just as it would be with an ordinary water heater. Today, more than 1.5 million homes and businesses use solar water heaters [29].

3. **Solar Electricity**: Solar energy can also be used to produce electricity. Two ways to make electricity from solar energy are photovoltaic and solar thermal systems [29].

   (a) Photovoltaic Electricity: In simple terms, this means converting light to electricity. Solar-powered toys, calculators, and roadside telephone call boxes all use solar cells to convert sunlight into electricity. Solar cells can supply energy to anything that is powered by batteries or electrical power. Electricity is produced when sunlight strikes the solar cell, causing the electrons to become mobile. The action of the electrons starts an electric current. The conversion of sunlight into electricity takes place silently and instantly and has the advantage of not having mechanical parts to wear out. Compared to other ways of making electricity, photovoltaic systems are expensive hence there are not many photovoltaic power plants today. In 2009, the DeSoto Next Generation Solar Energy Center in Florida opened. It is the largest photovoltaic plant in the US, generating 25 megawatts of electricity - enough to power 3,000 homes. In US, it costs between 10 to 20 cents a kilowatt-hour to produce electricity from solar cells. Most people pay their electric companies about 12 cents a kilowatt-hour for the electricity they use, and large industrial consumers pay less. Today, solar systems are mainly used to generate electricity in remote areas that are far from electric power lines [29].

(b) Solar Thermal Electricity: Like solar cells, solar thermal systems, also called concentrated solar power (CSP), use solar energy to produce electricity, but in a different way. Most solar thermal systems use a solar collector with a mirrored surface to focus sunlight onto a receiver that heats a liquid. The super-heated liquid is used to make steam to produce electricity in the same way that coal plants do. There are nine solar thermal power plants in the Mojave Desert that together produce 360 MW of electricity [29].

Solar energy has enormous potential for the future applications. Desirable characteristics of solar energy include: it is a non-pollutant, it is free, and its supplies are unlimited. The nature of its technology is such that it cannot be controlled by any one nation or industry. If this technology of harnessing solar energy can be improved, energy shortages will be a thing of the past and long forgotten.

2.3 REVIEW OF RELATED LITERATURE

As the world's supply of fossil fuel shrinks, there is a great need for clean and affordable renewable energy sources in order to meet the growing energy demands. Achieving sufficient supplies of clean energy for the future is a great societal challenge [30]. There is a large effort in raising their efficiency and in developing novel solutions to decrease the cost per watt of produced power [31].

The optimization of an energy system design consists of modifying the system structure and component design parameters according to one or more specified design objectives [32]. Solar energy can be obtained from solar fields using an array of flat-plate solar collectors. Solar collectors are usually deployed in a number of rows having a North-South orientation on a horizontal plane. Economic considerations, such as the cost of land and limited available area, demand that the field design be compact, which leaves limited spacing between the collectors for maintenance purposes. Consequently, a collector in one row may be obscured from the sun's rays at some time during the day, thus decreasing the quantity of energy collected [33]. Optimizing the distances between the rows and columns of collectors in the field is important because it reduces shading and maximizes the energy obtained from these collectors. The shading of collectors is affected by the spacing between the collector rows and columns, the collector height, and the inclination angle. An optimal placement of collectors will reduce the project cost and ensure effective utilization of the solar collector field. Therefore, this research aims to optimally place multiple rows and multiple columns of solar collectors in a fixed field area so as to maximize the energy collected from the field and minimize the investment (field and collector) costs. Currently, multiple row optimization of solar collectors has been considered in some literature [4], [34], [35], [36] but there has been no study in literature on multiple row and multiple column optimization of a fixed solar field to maximize energy and simultaneously minimize the field and collector investment cost.

The shading effect depends on the spacing between the collector rows, the collector height, and the inclination angle, to some extent on the row length and on the latitude of the solar field. Therefore, there is an optimal deployment of collectors in a field yielding different objectives which may be based on energy or economic criteria [34].

Optimal design of the solar energy collector plays a critical role in the efficient collection of solar energy. Flat-plate collectors can be designed in applications that require energy delivery at moderate temperatures (up to 100^oC above ambient temperature). These collectors use both beam and diffuse solar radiation, and do not need to track the sun. They are simple to manufacture and install, with relatively low maintenance costs, which makes this kind of solar collector popular [35].

Figure 2.3 shows a multi-row, multi-column array of flat-plate solar collectors in a given area. Increasing the number of solar-collector rows and columns will definitely increase the total collector area thereby increasing the amount of radiation energy received from the sun; however, it also increases the shading area (darker area shown in Figure 2.3), which will reduce the amount of radiation energy

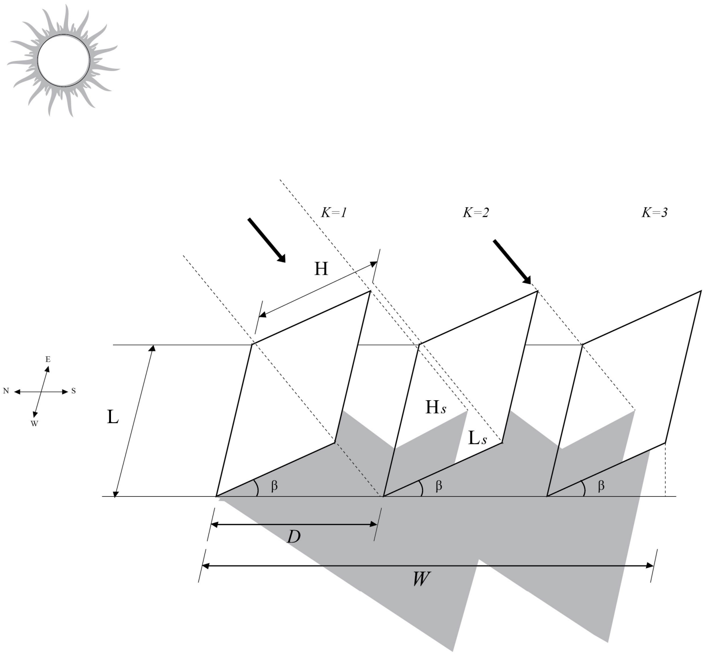

Figure 2.3: Solar collectors in a field

received from the sun.

The design of stationary solar collectors in a field involves relationships between the field and collector parameters and solar radiation data. Field and collector parameters contain field length L, which is also called the collector length, field width W, distance between collector rows D, collector height H, inclination angle β, and geometric limitations of these parameters. At a given time, the shaded height H_s and length L_s on the collector, is shown in figure 2.3. The rest of the collector area is un-shaded [4].

The optimization of solar systems has been solved using meta-heuristic methods such as artificial neural networks [37], particle swarm optimization [38], [39] and genetic algorithms [37], [40], [41] and [42]. For a theoretical background on types of solar collectors and applications, the reader is referred to the likes of [30], [43].

2.4 MAJOR CONTRIBUTIONS OF THE REVIEWED LITERATURE

The optimal placement of collectors in a solar field may be formulated mathematically as a constrained optimization problem and solved by applying available optimization algorithms for a single column of solar collectors as in [4] and [44]. For example, generic methods are formulated to analyze the effect of shading in [33], [45]. In [33], general expressions for shading and insolation are derived for different scenarios including shading caused by collectors, stepped collectors and a fence, as well as direct, diffuse and global insolation. A new measure, the shadow efficiency, is introduced in [45] to determine the acceptable distance between collectors as a function of parameters that effect shading in fixed tilt, half-tracking and full-tracking collectors.

The economic analysis of the optimization carried out in [4] is done in [36] for minimum plant cost and minimum cost of unit energy. The influence of parameter variation is also investigated. To obtain the highest incident energy from the field, the optimization tries to minimize the shading between the collector rows. This is obtained by increasing the row length and the distance between rows. Such a field produces 4.18% more energy, the periodic plant cost is less by 0.939% and the energy cost is less by 1.92% relative to a field with land constraints.

A MOO using game-theory approach and probabilistic uncertainty is done for one column of collectors in [35], [46] for three objectives, maximization of the annual average incident solar energy, maximization of the lowest month incident solar energy and minimization of costs. Also a thermoeconomic analysis, by applying a multi-objective approach to determine the complete spectrum of solutions that satisfies the economic objective as well as the energetic one is done [32].

The amount of solar energy incident on a solar collector in various time scales is a complex function of many factors including the local radiation climatology, the orientation and tilt of the exposed collector surface [11]. The performance of a solar collector is highly influenced by its orientation and its angle of tilt with the horizontal. This is due to the fact that both the orientation and tilt angle change the solar radiation reaching the surface of the collector [47]. Over the last few years, many authors have presented models to predict solar radiation on inclined surfaces [20] - [27]. Some of these models apply to specific cases; some require special measurements and some are limited in their scope. These models use the same method of calculating beam and ground reflected radiation on a tilted surface. The only difference exists in the treatment of the diffuse radiation [11].

The optimization of photovoltaic (PV) fields was done in [34]. Shadow variation on PV collectors was investigated in [48]. The results indicate the favourable interconnection between modules with similar incident levels of energy and illuminated at similar periods of time.

The spacing analysis of an inclined solar collector field was carried out in [49]. A computer code is developed to predict the change in incident energy on the collectors for various spacing distances between them. The code couples general shading models with the local weather data. The results show the variation of shadow area of the collectors for various spacing distance and presented for various time periods. The results are given for three US locations and economic implications are discussed.

The performance evaluation of solar thermal electric generation systems is carried out in [50] where a unified model is developed using a thermo-hydrodynamic model of a steam generator combined with traditional steam power house. The model is used to study the performance of different collector field and power house arrangements under Australian conditions. As spacing between the collectors affects the piping network and the pressure drop in the collector field, the spacing must be reduced as much as possible. The appropriate spacing of collector arrays in the field is calculated considering the shading between collectors.

The concept of multi-tower solar array is put forth in [51]. This involves alternately arranging solar collectors to point to different receivers thereby reducing shading. A layout optimization is done for a heliostat field in a solar thermochemical processing application using GA and Nelder-Mead algorithm [52].

2.5 DRAWBACKS OF THE REVIEWED LITERATURE AND MOTIVATION FOR RESEARCH

In the reviewed literature, single column multi-row flat-plate solar collectors are considered. As earlier mentioned, typical collector sizes range between 1.5 to 4 m^2, each costing between £3000 and £5000 [1] or \$ 200 per m^2 [2]. These individual collectors are combined to form larger length collectors where the length of such a collector is proportional to the cost. To utilize collectors with smaller dimensions in a field design of similar area, then multiple columns have to be considered. This has not yet been done in literature. A model that can work with multiple columns of smaller array length collectors may prove advantageous to solar field design for the following reasons:

1. These array collectors with smaller lengths are potentially cheaper than collectors with bigger dimensions.
2. They are easier to maintain when operational so obtainable energy levels can be maintained.

3. The cost of replacing damaged parts is less and this will also reduce the amount of energy that can be lost.

4. The potential exists for a reduced investment cost with the use of collectors with smaller dimensions to produce either the same quantity of energy or more energy in a given area than collectors with larger dimensions.

These reasons resonate with the objective in this research which is to maximize energy and minimize the investment cost.

This work presents an optimization model that maximizes solar energy collected while minimizing the investment cost in a multi-row, multi-column field of solar collectors. This model results in a MOOP which will be solved in two ways as earlier stated: the first one is to transform the MOOP into a SOOP using positive weights for objectives and penalties for constraints, and the other one is to obtain the Pareto optimal solutions which give the decision maker a range of choices to adjust trade-offs between different objectives [4]. The benefit of solving this problem will be the introduction of multiple columns of solar collectors to an existing single column field thereby increasing the amount of energy collected.

2.6 TERMINOLOGIES OF OPTIMAL SOLAR COLLECTOR PLACEMENT

Optimal placement of fixed solar collectors in a field array involves arranging solar collectors with an optimal distance between the rows and columns of the collectors to prevent shading thereby maximizing the solar energy collected, minimizing the cost and fulfilling all the necessary constraints.

2.6.1 Objective functions

Objective functions are equations describing the problem to be solved. In optimization, these objective functions are either of the minimization type, maximization type or a combination of both. Previously, the objective functions considered are shown in the following subsections.

2.6.1.1 Energy objective function

The energy objective function is given in equation (2.4) [4]. This objective function focuses on maximizing the energy collected from the solar collectors in the field array while still optimizing the variables and satisfying the bounds [4], [33], [35] and [36].

$$Q = H \times L \times [q_b + q_d + (K-1)(q_b^{sh} + q_b^{sh})] \tag{2.4}$$

where

Q is the yearly incident solar energy, H and L are the height and length of each solar collector respectively, q_b is the yearly beam irradiation per unit area of an unshaded collector (first row), q_d is the yearly diffuse irradiation per unit area of an unshaded collector (first row), q_b^{sh} is the average yearly beam irradiation per unit area of shaded collectors ((K-1) rows), and q_d^{sh} is the average yearly diffuse irradiation per unit area of shaded collectors ((K-1) rows).

2.6.1.2 Economic objective function

The economic objective function can be divided as given in equations (2.5) [4] and (2.6) [35]. This objective function aims to minimize the cost of the field array set up while optimizing the distance between the collectors to achieve efficient use of the field area [36].

$$\begin{aligned} C_p &= [C_{land} + C_{collectors} + C_{electricbackup} + C_{heatexchanger} + C_{stands} \\ &+ L_{pi}C_{pipe} + L_t C_{tank} + L_{pu}C_{pump}]\lambda_p + C_{backupenergy} + C_{maintenance} \end{aligned} \tag{2.5}$$

where

C_p is the cost of the plant, C_{land} is the cost of land, $C_{collectors}$ is the cost of collectors, $C_{electricbackup}$ is the cost of the electrical installation of the backup system, C_{stands} is the cost of the structure of the stands for the collectors, $C_{heatexchanger}$ is the cost of heat exchanger, C_{pipe} is the cost of the water piping, C_{tank} is the cost of the water tank, C_{pump} is the cost of the water pump, $C_{backupenergy}$ is the cost of the energy from the grid for backup, and $C_{maintenance}$ is the maintenance cost of the plant.

$$Cost = c_1 LW + c_2 LHK \tag{2.6}$$

where

c_1 is the unit cost of the land, c_2 is the unit cost of the collector, H is the height of each solar collector, L is the length of each solar collector and also the length of the field, W is the width of the field and K is the number of collector rows.

In this dissertation, two objectives are considered. The energy objective which focuses on maximizing the energy collected from the solar collectors in the field array while still optimizing the variables within the set bounds and the economic objective which aims to minimize the cost of the field array set up while optimizing the distance between the collectors to achieve efficient use of the field area. Before now, these objectives are evaluated independently. There are bounds that the objective functions are subjected to, for the problem to reach its optimal solutions.

2.6.2 Variables

Following from the above section, the variables previously considered are the number of collector rows K, the collector inclination angle β , the distance between the collector rows D and collector height H [4], [35]. In this dissertation however, the variables considered are only the collector inclination angle β and the distance between the collector rows K. The number of collector rows K is related to the distance between the collector rows so it is excluded as a variable. Also a new variable, the distance between the collector columns U, is added to the problem to determine the optimal number of columns.

2.6.3 Bounds

The bounds are conditions imposed on the variables that must be satisfied for an optimal solution to be obtained. In optimization, these bounds can be formulated as equality or inequality constraints.

2.6.4 Optimal solar collector placement modeling

The design of stationary photovoltaic and thermal solar collectors in a field involves relationships between the field and collector parameters and solar radiation data. In addition, shading and masking affect the collector deployment by decreasing the incident energy on collector plane of the field. The use of many rows of collectors densely deployed, in a limited field, increases the field incident energy but also increases the shading. The optimal design of a solar field may be formulated mathematically as a constrained optimization problem and the solution may be based on applying available optimization algorithms like those described in literature [4].

2.7 TECHNIQUES USED TO EVALUATE SOLAR COLLECTOR PLACEMENT PROBLEMS

There are two major techniques that have been used to solve this type of optimization problem. One technique is based on branch and bound programming and the other is the game-theory approach. The main problem with these techniques is that they produce a singular optimal solution that requires extensive computational efforts which increases with the complexity of the problem. In this research, multi-objective optimization based on genetic algorithm will be used to solve this kind of problem.

2.7.1 Branch and bound optimization

This method was first proposed by A. H. Land and A. G. Doig in 1960 for discrete programming [53]. The branch and bound method does a systematic search in the space of all feasible solutions to find the maximum (negative of minimum). It achieves this by partitioning into smaller subsets, the space of all feasible solutions and calculates an upper bound on the value of the objective function associated with the solutions that lie within a given subset. After each partitioning, those subsets whose upper bound are less than the best known feasible solution are excluded from further partitioning and are discarded. The partitioning continues until the value of the objective function for the best feasible solution is not less than the upper bound of any subset. The optimal solution is the best feasible solution. This is the technique applied in [4] and [36] to obtain maximum energy for a given field area. The results are obtained for different field sizes and are summarized in Table 2.1.

Table 2.1: Design results from [4]

Field (m)	K	β (deg)	D (m)	H (m)
L = 7.5, W = 12	6	48.19	0.8	2
L = 100, W = 200	80	31.24	0.8	2

The usefulness of this technique is that it excludes infeasible and non-optimal subsets from further search without them being fully expanded. Figure 2.4 shows important steps in a typical branch and bound technique.

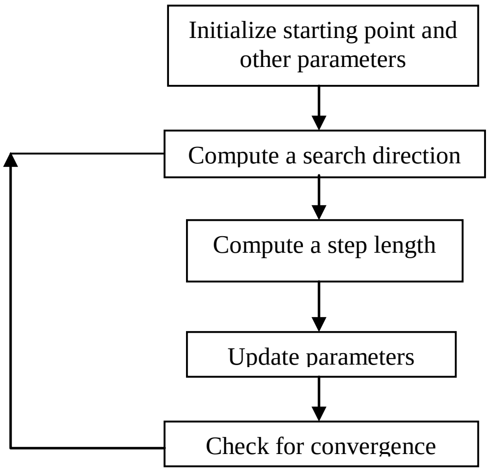

Figure 2.4: Typical branch and bound technique

2.7.2 Game-theory approach

Game theory did not really exist as a unique field until John von Neumann published a paper in 1928. His paper was followed by his 1944 book Theory of Games and Economic Behavior, with Oskar Morgenstern, which considered cooperative games of several players [54].

Game theory is mainly used in economics, political science, and psychology, and other, more prescribed sciences, like logic or biology. Today, however, game theory applies to a wide range of class relations, and has developed into an umbrella term for the logical side of science, to include both human and non-humans, like computers. Classic uses include a sense of balance in numerous games, where each person has found or developed a tactic that cannot successfully better his results, given the other approach.

Game theory is the formal study of decision-making where several players must make choices that potentially affect the interests of the other players. Game theory is also the formal study of conflict and cooperation. Game theoretic concepts apply whenever the actions of several agents are interdependent. These agents may be individuals, groups, firms, or any combination of these. The concepts of game theory provide a language to formulate, structure, analyze, and understand strategic scenarios.
A game is a formal description of a strategic situation while a player is an agent who makes decisions in a game. A game in strategic form, also called normal form, is a compact representation of a game

in which players simultaneously choose their strategies. The resulting payoffs are presented in a table with a cell for each strategy combination.
In a game in strategic form, a strategy is one of the given possible actions of a player. In an extensive game, a strategy is a complete plan of choices, one for each decision point of the player.
A mixed strategy is an active randomization, with given probabilities, that determines the player's decision. As a special case, a mixed strategy can be the deterministic choice of one of the given pure strategies.
A Nash equilibrium, also called strategic equilibrium, is a list of strategies, one for each player, which has the property that no player can unilaterally change his strategy and get a better payoff.
A payoff is a number, also called utility, that reflects the desirability of an outcome to a player, for whatever reason. When the outcome is random, payoffs are usually weighted with their probabilities. The expected payoff incorporates the player's attitude towards risk.

This approach is used in [35] for multi-objective optimal design of stationary flat-plate solar collectors and the results in Table2.2 are obtained when F_l = 30m, W = 200m, $C_{UC} = \$100/m^2, C_{UF} = \$100/m^2$.

Table 2.2: Design results for individual objective functions from [35]

Objective	H(m)	L(m)	D (m)	$\beta(^o)$	K	Average incident solar solar energy (W/h)	Lowest month incident solar energy (W/h)	Cost (10^6\$)
1	2	30	2.51	35.1915	82.4823	1.3689	1.1003	1.0949
2	2	30	2.51	53.4313	100.8248	1.3407	1.1135	1.2049
3	1.6453	25.1457	1.0859	50.8246	79.2061	0.8213	0.6496	0.7322

To obtain an optimal solution using this approach, only one of the multiple objectives is considered at a time similar to the previous technique. Hence the need for a more Pareto-based approach in solving this kind of problem.

2.8 MULTI-OBJECTIVE OPTIMIZATION

The general multi-objective optimization problem is posed as in equations (2.7) and (2.8) [55]:

$$Minimize_x\ F(x) = [F_1(x), F_2(x), \cdots, F_k(x)]^T \tag{2.7}$$

subject to

$$g_j(x) \leq 0, j = 1,2,\cdots,m,\ h_l(x) = 0, l = 1,2,\cdots,e, \tag{2.8}$$

where

k is the number of objective functions, m is the number of inequality constraints, and e is the number of equality constraints. $x \in E^n$ is a vector of design variables (also called decision variables), and n is the number of independent variables x_i. $F(x) \in E^k$ is a vector of objective functions $F_i(x) : E^n \rightarrow E^1$. $F_i(x)$ are also called objectives, criteria, payoff functions, cost functions, or value functions and $g_j(x)$ and $h_l(x)$ are constraints [55].

For multiple-objective problems, the objectives are generally conflicting, preventing simultaneous optimization of each objective. Many, or even most, real engineering problems actually do have multiple objectives, i.e., minimize cost, maximize performance, maximize reliability, etc. These are difficult but realistic problems. There are two general approaches to MOO. One is to combine the individual objective functions into a single composite function or move all but one objective to the constraint set. In the former case, determination of a single objective is possible with methods such as utility theory, weighted sum method, etc., but the problem lies in the proper selection of the weights or utility functions to characterize the decision makers preferences. In practice, it can be very difficult to precisely and accurately select these weights, even for someone familiar with the problem domain. Compounding this drawback is that scaling amongst objectives is needed and small perturbations in the weights can sometimes lead to quite different solutions. In the latter case, the problem is that to move objectives to the constraint set, a constraining value must be established for each of these former objectives. This can be rather arbitrary. In both cases, an optimization method would return a single solution rather than a set of solutions that can be examined for trade-offs. For this reason, decision makers often prefer a set of good solutions considering the multiple objectives [56].

The second general approach is to determine an entire Pareto optimal solution set or a representative subset. A Pareto optimal set is a set of solutions that are non-dominated with respect to each other. While moving from one Pareto solution to another, there is always a certain amount of sacrifice in one objective(s) to achieve a certain amount of gain in the other(s). Pareto optimal solution sets are often preferred to single solutions because they can be practical when considering real-life problems since the final solution of the decision maker is always a trade-off. Pareto optimal sets can be of varied sizes, but the size of the Pareto set usually increases with the increase in the number of objectives [56].

In many real life applications the objective function has multiple, often times conflicting, goals. The set of points that bounds the bottom of the feasible region is known as the Pareto front. The two approaches to multi-objective optimization considered in this research are: weighted sum method and finding the Pareto front.

2.8.1 Constrained multi-objective optimization

A constraint is a logical relation among several variables, each taking a value in a given domain. A constraint thus restricts the possible values that variables can take, it represents some partial information about the variables of interest [57].

Several methods have been proposed for handling constraints by GA's for optimization problems. [58], [59] grouped these methods into four categories:

1. Methods based on preserving the feasibility of solutions. The idea behind the method is based on specialized operators which transform feasible parents into feasible offspring.

2. Methods based on penalty functions. Many evolutionary algorithms incorporate a constraint-handling method based on the concept of exterior penalty functions which penalize infeasible solutions.

3. Methods which make a clear distinction between feasible and infeasible solutions. There are a few methods which emphasize the distinction between feasible and infeasible solutions in the search space.

4. Other hybrid methods. These methods combine evolutionary computation techniques with deterministic procedures for numerical optimization problems.

Most constrained problems can be handled by the penalty function method. A measure of the constraint violation is often useful when handling constraints.

2.8.2 Evolutionary algorithms

Evolutionary algorithm (EA) represents a group of stochastic optimization methods that simulate the process of natural evolution whose origins can be traced back to the late 1950s. The 1970s saw an emergence of several evolutionary methodologies, mainly genetic algorithms, evolutionary programming, and evolution strategies [60]. All of these approaches operate on a set of candidate solutions and using strong simplifications, this set is subsequently modified by selection and variation. Selection mimics the competition for reproduction and resources among parents, while variation imitates the natural capability of creating new offsprings by means of recombination and mutation. Although these algorithms seem basic, they have proven themselves as a general, robust and powerful search mechanism. In particular, they possess several characteristics that are desirable for problems involving multiple conflicting objectives and intractably large and highly complex search spaces. This is a rapidly growing area of interest [61].

Evolutionary algorithms (EA's) such as evolution strategies and genetic algorithms have become the method of choice for optimization problems that are too complex to be solved using deterministic techniques such as linear programming or gradient (Jacobian) methods. The large number of applications and the continuously growing interest in this field are due to several advantages of EA's compared to gradient based methods for complex problems. EA's require little knowledge about the problem being solved, and they are easy to implement, robust, and inherently parallel. To solve a certain optimization problem, it is enough to require that one is able to evaluate the objective function for a given set of input parameters. Because of their universality, ease of implementation, and fitness for parallel computing, EA's often take less time to find the optimal solution than gradient methods. However, most real-world problems involve simultaneous optimization of several often mutually concurrent objectives. Multi-objective EA's are able to find optimal trade-offs in order to get a set of solutions that are optimal in an overall sense. In multi-objective optimization, gradient based methods are often impossible to apply. Multi-objective EA's, however, can always be applied, and they inherit all of the favorable properties from their single objective relatives [62].

2.8.3 Genetic algorithm

The concept of GA was developed by Holland and his colleagues in the 1960s and 1970s [63]. GA are inspired by the evolutionist theory explaining the origin of species. In nature, weak and unfit species within their environment are faced with extinction by natural selection. The strong ones have greater opportunity to pass their genes to future generations via reproduction. In the long run, species carrying the correct combination in their genes become dominant in their population. Sometimes, during the slow process of evolution, random changes may occur in genes. If these changes provide additional advantages in the challenge for survival, new species evolve from the old ones. Unsuccessful changes are eliminated by natural selection [56].

The GA is an optimization and search technique based on the principles of genetics and natural selection. A GA allows a population composed of many individuals to evolve under specified selection rules to a state that maximizes the “fitness" (i.e., minimizes the cost function) [64]. As a heuristic optimization approach, it initially discretizes the allowed parameter space and creates a number of parameter vectors (called population) distributed randomly over the parameter space. Afterwards, the value of the objective function (called fitness) for each individual parameter vector is calculated. From this population, a certain number of individuals with the best fitness are selected, recombined, and subject to random mutation to form the subsequent generation. The random number generator is based on an initial arbitrary seed value and therefore allows creating different optimization runs within the same parameter space. The termination criterion is a given population size and number of generations [52].

In GA terminology, a solution vector $x \in X$ is called an individual or a chromosome. Chromosomes are made of discrete units called genes. Each gene controls one or more features of the chromosome. GA operates with a collection of chromosomes, called a population. The population is normally randomly initialized and as the search evolves, the population includes fitter and fitter solutions until it is dominated by a single solution. GA use two operators to generate new solutions from existing ones: crossover and mutation. The crossover operator is the most important operator of GA. In crossover, generally two chromosomes, called parents, are combined together to form new chromosomes, called offspring. The parents are selected among existing chromosomes in the population with preference towards fitness so that offspring is expected to inherit good genes which make the parents fitter. By iteratively applying the crossover operator, genes of good chromosomes are expected to appear more frequently in the population, eventually leading to convergence to an overall good solution. The mutation operator introduces random changes into characteristics of chromosomes. Mutation is generally applied at the gene level. In typical GA implementations, the mutation rate (probability of changing the properties of a gene) is very small and depends on the length of the chromosome. Therefore, the new chromosome produced by mutation will not be very different from the original one. Mutation plays a critical role in GA. As discussed earlier, crossover leads the population to converge by making the chromosomes in the population alike. Mutation reintroduces genetic diversity back into the population and assists the search escape from local optima. Reproduction involves selection of chromosomes for the next generation. In the most general case, the fitness of an individual determines the probability of its survival for the next generation. There are different selection procedures in GA depending on how the fitness values are used. Proportional selection, ranking, and tournament selection are the most popular selection procedures [56].

The working principle of a basic GA is as shown in the flowchart of figure 2.5. The objective functions first have to be defined along with the variables and the GA parameters. The next step for the GA is to randomly generate the initial population.

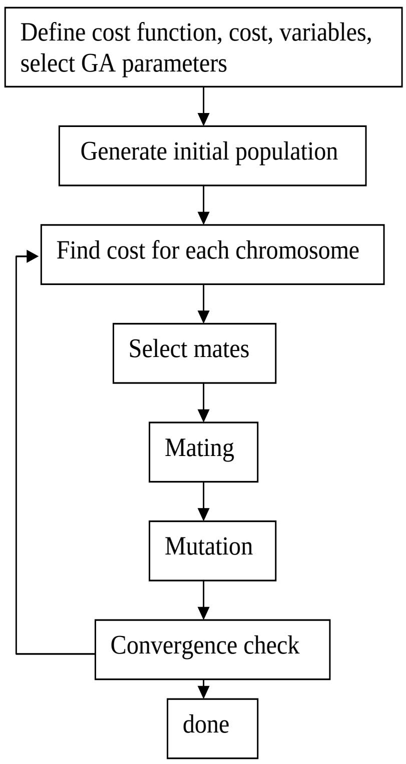

Figure 2.5: Flowchart for Pareto optimization using GA

Some of the advantages of a GA include that it:

1. Optimizes with continuous or discrete variables,

2. Does not require derivative information,

3. Simultaneously searches from a wide sampling of the cost surface,

4. Deals with a large number of variables,

5. Is well suited for parallel computers,

6. Optimizes variables with extremely complex cost surfaces (they can jump out of a local minimum),

7. Provides a list of candidate optimal solutions and not just a single solution,

8. May encode the variables so that the optimization is done with the encoded variables, and

9. Works with numerically generated data, experimental data, or analytical functions.

These advantages are intriguing and produce stunning results when traditional optimization approaches fail miserably [64].

2.8.3.1 Weighted sum method

The classical approach to solve a multi-objective optimization problem is to assign a weight w_n to each normalized objective function f_n so that the problem is converted to a single objective problem with a scalar objective function as follows:

$$objective = \sum_{n=1}^{N} w_n f_n, \tag{2.9}$$

where

f_n is objective function n and $0 \leq f_n \leq 1$,
w_n is weighting factor and $\sum_{n=1}^{N} w_n = 1$.

This is called the priori approach since the user is expected to provide the weights. Solving a problem with the objective function (2.9) for a given weight vector $w = (w_1, w_2, \ldots, w_k)$ yields a single solution, and if multiple solutions are desired, the problem must be solved multiple times with different weight combinations. The main difficulty with this approach is selecting a weight vector for each run.

2.8.3.2 Finding the Pareto front

Pareto-ranking approaches explicitly utilize the concept of Pareto dominance in evaluating fitness or assigning selection probability to solutions. The population is ranked according to a dominance rule, and then each solution is assigned a fitness value based on its rank in the population, not its actual objective function value. Note that herein all objectives are assumed to be minimized. Therefore, a lower rank corresponds to a better solution in the following discussions. The first Pareto ranking technique was proposed by Goldberg [65] as follows:

Step 1: Set $i = 1$ and $TP = P$.

Step 2: Identify non-dominated solutions in TP and assig or set them to F_i.

Step 3: Set $TP = TPF_i$. If $TP = \oslash$ go to Step 4, else set $i = i+1$ and go to Step 2.

Step 4: For every solution $x \in P$ at generation t, assign rank $r_1(x,t) = i$ if $x \in F_i$.

In the procedure above, $F_1, F_2, \ldots$ are called non-dominated fronts, and F_1 is the Pareto front of population P.

2.9 CHAPTER SUMMARY

In previously enumerated techniques, the emphasis has been placed on single objective optimization of a single column of solar collectors in a field array. The single objective optimization has been done for either maximizing the energy or minimizing cost. In the instant where multi-objective optimization was attempted, different choices of weights were not considered and there was no Pareto front obtained. However this was still based on a single column of solar collectors in a field array. These gap in solar collector field design brought about the need to formulate a new multi-objective model that considers multiple columns of solar collectors in a field array and optimizes for two objectives: maximum energy and minimum cost of investment, using both approaches of different weights for objective functions and finding a Pareto front.

There is no literature on solar collector problems with multiple columns in a field array and optimizing for two objectives: maximum energy and minimum cost, using both approaches of different weights and a Pareto front. This model resembles a real world situation and the solutions obtained from this model has the potential to be more robust.

CHAPTER 3 SOLUTION METHODOLOGY

3.1 CHAPTER OVERVIEW

This chapter formulates the multi-objective solar collector placement problem with the necessary bounds. It also elaborates on the optimization techniques; weighted sum approach and Pareto front; used to obtain the results.

3.2 MODELING THE PROBLEM

Figure 3.1 shows a multi-row-multi-column of flat-plate solar collectors in a given area. In theory, any increase in the number of rows and columns will increase the total collector area thereby increasing the amount of radiation energy that can be received from the sun. Consequently, this increases the shaded area on the collector surfaces which reduces the amount of radiation energy that can be received from the sun. The design of solar collectors in a fixed field involves relationships between the field and collector parameters and solar radiation data. Field and collector parameters contain field length F_l, collector length L, field width W, horizontal distance D between collector rows, vertical distance U between collector columns, collector height H, inclination angle β, solar elevation angle α, solar azimuth angle γ, number of rows K, number of columns M, and geometric limitations of these parameters. At a given time, the shaded height $H_s(t)$ and length $L_s(t)$ on the collector are shown in Figure 3.1. The rest of the collector area is unshaded.

3.2.1 Multi-objective problem formulation

The objective is to formulate a collector row placement problem with economic consideration and derive an optimal solution using GA. This research presents an multi-objective optimization model that maximizes solar energy collected from a field array of solar collectors using measured data while

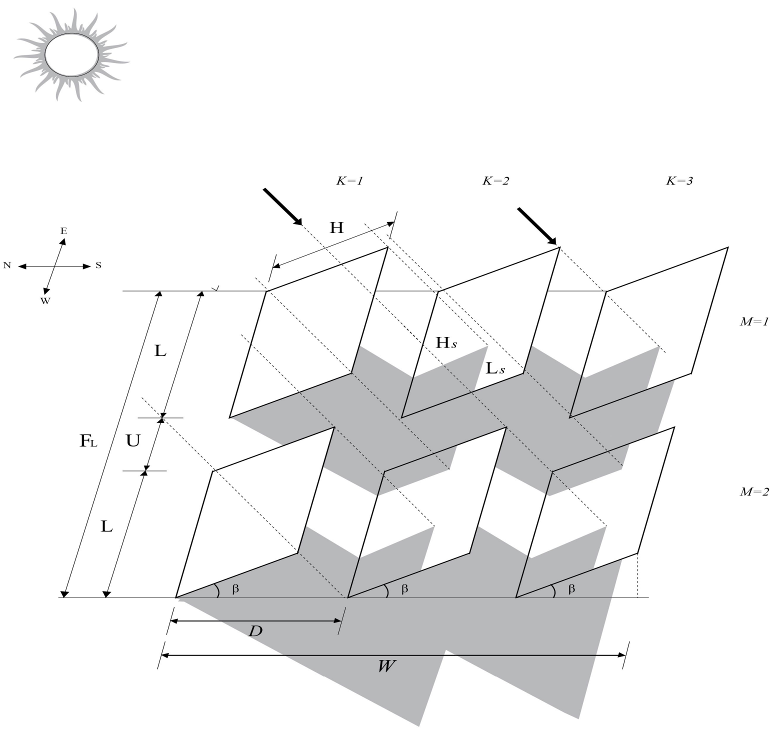

Figure 3.1: Model of solar collectors in a field

minimizing the associated cost of investment for producing such energy. In this research, the MOOP is solved firstly by transforming the MOOP into a SOOP using weights (for objectives) and penalties (for constraints), and also by obtaining the Pareto-optimal solutions that presents the decision maker with a suitable range of choices to adjust trade off between different objectives [4].

The multi-objective problem is formulated as shown below.

3.2.2 Variables

Determination of the optimum inclination angle is a prerequisite for proper installation of any solar system [7]. Also reduction of shading by proper spacing of collector rows is important. The variables to be optimized are the distance (D) between rows of the collectors, the distance (U) between columns of the collectors and the collector inclination angle (β) as shown in Figure 3.1. Optimizing these variables can ensure that maximum amount of energy is collected. The distances between the rows and columns of the collectors affect the investment.

3.2.3 Objective functions

1. Energy Objective: This objective is to maximize the total energy collected from the field. From Figure 3.1, more energy may be obtained from the first row than the other rows because of the absence of shading. The energy input to the whole field is given by equation (3.1).

$$\max\ E_{total} = \sum_{i=1}^{M} E_i. \tag{3.1}$$

This energy E_{total} is dependent on the solar radiation and the unshaded area of the collectors which vary with the time of the day. E_i in equation (3.2) is given by [33, 4].

$$E_i = \sum_{t=1}^{T} G(t)A_{Ti}(t), \tag{3.2}$$

$$G(t) = G(t) = G_b(t)R_b + G_d(t)R_d, \tag{3.3}$$

Equation (3.3) represents the global solar radiation taking into account the direct beam and the diffusion components on a horizontal surface [33, 4, 12].

$$A_{Ti}(t) = HL + (K-1)S_i(t), \tag{3.4}$$

$$S_i(t) = \begin{cases} HL - H_s(t)L_s(t), & i = 1, \\ HL - H_s(t)L_s(t) - H_s(t)(L - U - L_s(t)), & i \geq 2, \end{cases} \tag{3.5}$$

where

E_{total} is total energy obtained from field in Watt hour (Wh),

E_i is the total energy obtained from column i,

$G(t)$ is hourly global solar radiation in W/m^2,

$A_T(t)$ is total unshaded collector area in m^2 at time t,

$G_b(t)$ is the direct beam hourly solar radiation in W/m^2 on a horizontal surface,

$G_d(t)$ is the diffuse solar radiation in W/m^2 on a horizontal surface,

R_b is the configuration factor that transforms the direct beam component of the solar radiation from the horizontal surface to an inclined plane,

R_d is the configuration factor that transforms the diffusion component of the solar radiation from the horizontal surface to an inclined plane,

H, L are the height and length of each collector in m at time t,

$H_s(t), L_s(t)$ are the shaded height and shaded length of each collector in m at time t,

$S_i(t)$ is the unshaded area of each collector in m^2 at time t,

K is an integer and it represents the number of collector rows,

M is total number of collector columns,

F_l is the field length,

$\alpha(t)$ is solar elevation angle in degrees at time t,

$\gamma(t)$ is solar azimuth angle in degrees at time t,

T is the total number of hours with sunshine in a year, and

t is the current time in hours.

The total energy that can be obtained from the first column E_1, can be obtained from equation (3.2) by substituting equations (3.3) and (3.4).

$$\begin{aligned} E_1 &= \sum_{t=1}^{T} G(t)\{HL+(K-1)(HL-H_s(t)L_s(t))\} \\ &= \sum_{t=1}^{T} G(t)\{KHL-(K-1)H_s(t)L_s(t)\}. \end{aligned} \tag{3.6}$$

Also the total energy from the second to the M-th column is given in equation (3.7).

$$\sum_{i=2}^{M} E_i = \sum_{t=1}^{T} G(t)(M-1)\{KHL-(K-1)H_s(t)(L-U)\}. \tag{3.7}$$

Substituting equations (3.6) and (3.7) in equation in (3.1) gives the total energy in (3.8).

$$\begin{aligned} E_{total} &= \sum_{t=1}^{T} G(t)\{KHL-(K-1)H_s(t)L_s(t)\} \\ &+ G(t)(M-1)\{HL+(K-1)(HL-H_s(t)(L-U)\} \\ &= \sum_{t=1}^{T} G(t)\{KHL-(K-1)H_s(t)L_s(t) \\ &+ (M-1)KHL-(M-1)(K-1)H_s(t)(L-U)\} \\ &= \sum_{t=1}^{T} G(t)\{MKHL-(K-1)H_s(t)(L_s(t)+(M-1)(L-U))\}, \end{aligned} \tag{3.8}$$

where $H_s(t)$ and $L_s(t)$ are calculated from [33, 4] as

$$H_s(t) = H\left(\frac{\sin\beta\cos\gamma(t)-(\frac{D}{H}-\cos\beta)\tan\alpha(t)}{\cos\beta\tan\alpha(t)+\sin\beta\cos\gamma(t)}\right), \tag{3.9}$$

$$L_s(t) = L - \frac{D\sin\beta\sin\gamma(t)}{\cos\beta\tan\alpha(t)+\sin\beta cos\gamma(t)}, \tag{3.10}$$

K = $\lfloor W/D \rfloor$, [1] $M = \lfloor F_l/(L+U) \rfloor$ and F_l is the field length. Equation (3.1) is therefore trans-

[1]The main idea where $\lfloor\ \rfloor$ appears is to take the lower integer value only at the final solution after the computation has been carried out with the actual values.

formed to (3.11).

$$E_{total} = \sum_{t=1}^{T} \{G_b(t)R_b + G_d(t)R_d\} \times \\ \left\{\lfloor \frac{W}{D} \rfloor HL(\lfloor \frac{F_l}{L+U} \rfloor) - (\lfloor \frac{W}{D} \rfloor - 1)H\left(\frac{\sin\beta\cos\gamma(t) - (\frac{D}{H} - \cos\beta)\tan\alpha(t)}{\cos\beta\tan\alpha(t) + \sin\beta\cos\gamma(t)}\right) \times \\ \left(L - \left(\frac{D\sin\beta\sin\gamma(t)}{\cos\beta\tan\alpha(t) + \sin\beta\cos\gamma(t)} + \left(\lfloor \frac{F_l}{L+U} \rfloor - 1\right)(L-U)\right)\right)\right\}. \tag{3.11}$$

The objective function of equation (3.11) represents the maximum energy E_{total} that can be collected from the field. It is the hourly sum of the product of the global solar radiation $G_b(t)$ in watt per m^2 and the total unshaded collector area $A_{Ti}(t)$ in m^2 in the field in a year.

2. Investment Objective: This objective minimizes the investment which is the cost of the total collector area and the field area with respect to the variables. This investment is represented by:

$$C = HLKMC_{UC} + WF_lC_{Uf}.$$

The objective to be minimized can be written as in (3.12).

$$\min\ C = HL\lfloor \frac{W}{D} \rfloor \lfloor \frac{F_l}{L+U} \rfloor C_{UC} + WF_lC_{UF} \tag{3.12}$$

where

C is the total investment,

C_{UC} and C_{UF} are collector and land costs in unit square meter respectively,

W and F_l are the width and length of the field respectively.

The objective function C in equation (3.12) is the investment which is the sum of the cost of the total collector area and the cost of the total field area. This objective has to be minimized.

3.2.4 Constraints

The following constraints described in equations (3.13) to (3.17) is added to the problem.

1. The constraint on the inclination angle in equation (3.13) implies that the collectors are neither parallel nor perpendicular to the ground.

$$0 < \beta(t) < 90^o. \tag{3.13}$$

2. Equations (3.14) and (3.15) ensure that there is always enough distance between the rows and columns of the collectors for maintenance and cleaning purposes.

$$D_{min} \leq D, \tag{3.14}$$

$$U_{min} \leq U. \tag{3.15}$$

3. Equation (3.16) constrains the total energy E_{total} obtained from the field to be greater than or equal to a certain minimum value E_{min}.

$$E_{total} \geq E_{min}, \tag{3.16}$$

where E_{total} and E_{min} are the total energy and minimum energy to be obtained from the field respectively.

4. Equation (3.17) keeps the total cost for the setup to be within the available budget.

$$C \leq C_{budget}, \tag{3.17}$$

where C and C_{budget} are the investment for the solar setup and the available budget respectively.

3.2.5 Penalty function

The idea for penalty functions was introduced in [66]. It is the earliest and most commonly used approach in the EA community to handle constraints. In this method a constrained optimization problem is transformed into an unconstrained problem by adding a penalty factor to the objective function value of each infeasible individual so that such individual is penalized for violating one or more of the constraints. Static penalty and dynamic penalty are types of penalty functions. The former depends only on the degree of violation, while the later depends also on the current generation count. In adaptive penalty, information gathered from the search process will be used to control the amount of penalty added to infeasible individuals. Penalty-based constraint handling techniques for multi-objective is similar to single objective except that the penalty factor is added to all the objectives instead of only one objective. A self adaptive penalty function is proposed by [67] to solve constrained multi-objective optimization problems using evolutionary algorithms. The method keeps track of the number of feasible individuals in the population to determine the amount of penalty added to infeasible individuals. If there are a few feasible individuals in the whole population, a larger penalty factor is added to infeasible solutions. Otherwise, a small penalty factor is used. Self adaptive penalty function uses modified objective function values instead of using the original objective values. The modified objective value has two components: distance measure and adaptive penalty [68, 69, 70].

In this research, a static penalty function is used. The penalty function penalizes the total energy E_{total} obtained from the field when it goes below a certain minimum value E_{min} as shown in the constraint equation (3.16). It also penalizes the total cost for the setup when it exceeds the available budget as given in constraint equation (3.17).

3.3 METHODS OF SOLVING MULTI-OBJECTIVE PROBLEMS

In multi-objective optimization (MOO), there is no single best solution to the conflicting objectives. Two approaches are used to solve the multi-objective optimization problem (MOOP).

3.3.1 Weighted sum approach

The easier of the two approaches to multi-objective optimization (MOO) is to assign a weight to each function and sum them as given in equation (2.9). The problem with this method is how to determine the appropriate values of w_n. Different weights produce different results for the same f_n. This approach requires assumptions on the relative worth of the objective functions prior to running the GA. This approach is not computationally intensive and results in a single best solution based on the assigned weights.

To solve the above multi-objective problem, weights are assigned to the two objectives as in equation (3.18) below. Suitable values are then chosen for the weights and the resulting function is then maximized.
In this work, three different set of weights are chosen where each set add up to unity and their results compared.

$$\max\ P = \lambda_1 E_{total} - \lambda_2 C, \tag{3.18}$$

where P is the resulting fitness value, E_{total} is the energy obtained, and C is cost in Rands (R) of collector and field.

The solar optimization problem is of the form:

$$\min_x\ \mathbf{f}(x). \tag{3.19}$$

subject to inequality constraints in the form

$$\mathbf{A}\, x \leq \mathbf{b} \tag{3.20}$$

and equality constraints in the form

$$\mathbf{Aeq}\, x = \mathbf{beq} \tag{3.21}$$

and boundary constraints of the form

$$lb \leq x \leq ub \tag{3.22}$$

$\mathbf{x} = (D, u, \beta)^T$ [71].

The objective function f is defined as in equation (3.17).

In order to maximize f(x), minimize $f(x)$, because the point at which the minimum of $f(x)$ occurs is the same as the point at which the maximum of f(x) occurs. In this paper, the objective to be maximized is assigned a negative value. The objective functions are weighted and the constrained optimization problem is converted to an unconstrained problem through the use of a penalty function. The constraints are included in the penalty function and penalize the objectives when they violate the constraints.

3.3.2 Pareto optimization using GA

In multi-objective optimization (MOO), there is usually no single solution that is optimum with respect to all objectives. Consequently in this research, there are a set of optimal solutions, known as Pareto optimal solutions, non-inferior solutions, or effective solutions. Without additional information, all these solutions are equally satisfactory. The goal here is to find as many of these solutions as possible. If reallocation of resources cannot improve one objective or cost without deteriorating another objective (cost), then the solution is Pareto optimal.

The GA used in this research is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution. The GA repeatedly modifies a population of individual solutions. At each step, the GA selects individuals at random from the current population to be parents and uses them to produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution. The GA can also be applied to solve a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, non-differentiable, stochastic, or highly nonlinear.

The GA uses three main types of rules at each step to create the next generation from the current population:

1. Selection rules select the individuals, called parents, that contribute to the population at the next generation.

2. Crossover rules combine two parents to form children for the next generation.

3. Mutation rules apply random changes to individual parents to form children.

To solve the problem in this paper using Pareto-based GA optimization, the following flowchart in Figure 3.2 is followed.

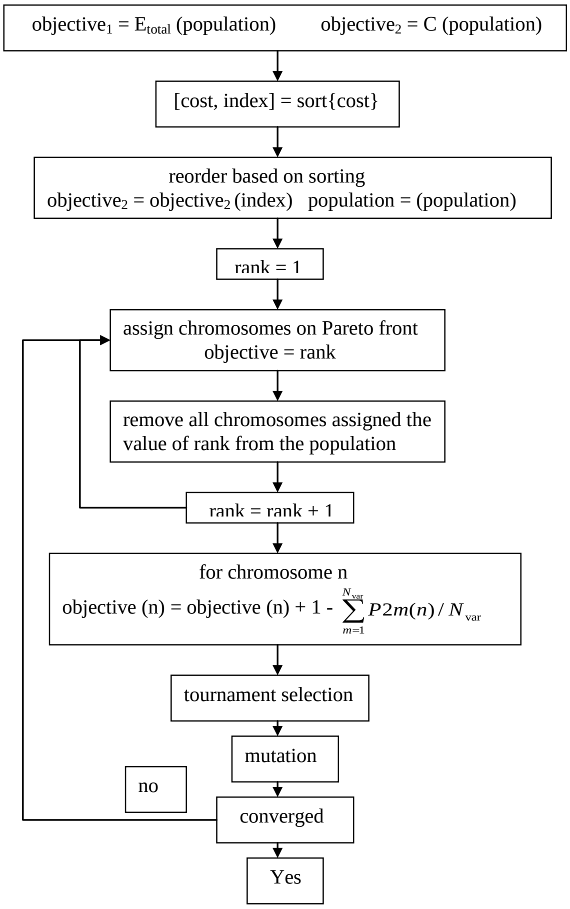

Figure 3.2: Flowchart for Pareto optimization using GA

The objective functions are as defined in equations (3.11) and (3.12). The initial population is defined as a N_{pop} by N_{var} matrix, where N_{pop} is the population size and N_{var} is the number of variables.

The objective functions are then evaluated using the initial population and sorted in order of non-dominance. The dominated solutions are discarded and the non-dominated solutions are kept and used to select mates to produce offspring for the next generation. Crossover and mutation are also carried out on the resulting population to maintain diversity. These procedure is continued for a specified number of generation until convergence occurs or the stopping criteria is reached.

3.4 DATA FOR SIMULATION

The following data gives variations of the solar radiation for the year 2011 as supplied in the Agrometeorology Hourly Data Reports by the Agricultural Research Council-Institute for Soil, Climate and Water (ARC-ISCW) Pretoria in South Africa. The necessary angles used in the simulation are also given.

1. Average daily solar radiation data: This is the average solar radiation per day from January to December 2011. The average daily values in watts are as shown in tables 3.1 and 3.2. The actual hourly values from January to December 2011 are available in the Appendix tables A.1 to A.4.

2. Solar radiation data (G_b): This is the direct beam component data as reported in the Agrometeorology Hourly Data Reports by the Agricultural Research Council-Institute for Soil, Climate and Water (ARC-ISCW) Pretoria in South Africa with latitude 25.45^o S. The data used are hourly solar radiation data from January 1 to December 31, 2011. The average of these hourly values are calculated monthly resulting in 12 typical days representing the whole year as shown in Table 3.3. These average values are used in this simulation.

3. Solar radiation data (G_d): This is the diffusion component data sourced from Solar Radiation Research Laboratory (SRRL) Baseline Measurement System (BMS) with latitude 39.74^o N [72]. The data are from January 1 to December 31, 2011 and are used in a similar way to G_b. Table 3.4 shows the values used in this simulation.

Table 3.1: The average daily solar radiation (Watts) from January to June 2011

Day	Jan	Feb	Mar	Apr	May	Jun
1	341.79	265.77	329.48	104.5	152.14	357.02
2	287.49	299.89	280.8	29.2	376.88	297.36
3	162.42	230.86	238.51	228.84	79.45	358.91
4	214.74	239.63	195.89	118.47	301.23	343.74
5	75.35	307.94	157.49	72.02	233.05	362.35
6	248.69	286.14	265.37	133.64	234.39	358.75
7	310	265.33	323.14	166.11	333.07	358.02
8	230.51	296.17	319.04	162.18	341.25	272.03
9	180.21	321.36	300.92	142.24	247.37	173.34
10	311.42	345.58	261.69	245.18	294.42	121
11	307.16	305.04	244.23	130.55	305.89	119.59
12	257.67	321.48	292.49	210.22	305.58	167.52
13	238.57	327.51	290.26	300.95	260.52	167.03
14	255.07	108.08	241.78	239.71	279.53	173.55
15	280.11	249.01	231.39	230.15	354.09	175.21
16	249.09	225.82	203.51	206.68	373.16	178.05
17	206.28	254.42	234.77	203.84	279.63	174.47
18	151.44	267.94	288.11	261.19	269.73	176.77
19	262.95	327.16	166.8	290.06	266.95	175.24
20	168.33	278.33	201.76	191.07	273.18	170.15
21	111.5	332.82	239.59	294.91	321.2	170.43
22	226.9	275.97	163.3	271	332.99	168.85
23	269.39	236.54	255.28	194.96	331.57	162.26
24	204.82	295.5	89.75	96.23	306.67	159.6
25	206.68	68.87	236.78	151.62	362.72	163.02
26	159.03	217.89	221.44	98.77	309.66	156.79
27	151.03	190.35	152.47	104.49	302.61	156.98
28	287.77	305.53	170.75	343.03	346.27	84.26
29	216.56	0	86.61	248.77	301.23	83.04
30	169.87	0	227.24	321.45	187.37	135.94
31	228.47	0	163.42	0	218.25	0

Table 3.2: The average daily solar radiation (Watts) from July to December 2011

Day	Jul	Aug	Sep	Oct	Nov	Dec
1	136.4	175.85	215.07	197.13	225.93	222.5
2	95.56	186.32	216.78	199.43	242.96	171.42
3	160.34	181.51	208.39	195.51	246.2	149.75
4	101.85	185.13	220.36	196.89	196.27	260.46
5	94.71	183.28	226.19	192.5	205.76	225.39
6	154.08	173.9	216.54	199.37	182	216.8
7	152.72	173.72	204.6	203.74	176.16	140.46
8	140.29	179.65	209.11	216.41	208.44	218.28
9	157.47	188.51	208.58	211.84	78.96	248.92
10	170.36	198.43	193.15	204.38	256.21	252.36
11	170.61	198.91	138.81	217.34	223.65	249.35
12	172.65	194.2	227.69	190.51	240.86	75.38
13	151.47	190.22	231.81	154.87	242.64	89.02
14	171.66	194.24	185.76	108.17	189.73	101.57
15	155.8	207.01	195.84	242.24	102.05	41.03
16	176.17	200.56	177.53	233.9	168.07	146.75
17	179.68	196.74	167.5	170.14	155.68	278.92
18	176.48	191.45	194.4	233.56	32.82	205.18
19	174.77	186.26	199.2	169.63	131.83	242.32
20	175.08	167.13	190.69	242.88	237.81	229.61
21	173.68	205.63	146.49	233.45	242.46	204.88
22	139.29	188.63	210.7	194.12	111.47	243.03
23	158.66	188.69	190.65	170.82	229.96	238.41
24	173.76	188.55	183.19	143.99	151.97	230.4
25	138.22	187.52	195.05	254.29	189.61	278.99
26	140.91	182.24	196.21	260.33	161.78	185.74
27	168.58	199.65	149.24	236.66	235.37	176.11
28	173.55	196.58	95.52	130.35	199.97	268.18
29	169	189.84	193.89	194.92	246.12	191.49
30	150.27	207.19	198.88	240.71	108.73	129.66
31	150.52	220.96	0	192.4	0	219.03

Table 3.3: Average direct beam solar radiation in different months (Watts/m^2)

Month	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12
1	0.026	0.025	0.024	0.032	0.037	4.819	56.177	220.212	346.680	459.825	652.087	640.704
2	0.028	0.028	0.023	0.034	0.037	0.708	35.816	237.351	435.576	666.396	768.580	817.756
3	0.031	0.027	0.027	0.037	0.041	0.050	22.825	172.382	368.343	509.525	664.086	714.984
4	0.037	0.033	0.033	0.045	0.047	0.050	11.161	148.756	427.636	678.904	497.049	317.110
5	0.035	0.031	0.035	0.041	0.048	0.049	5.260	118.909	451.317	911.986	1,185.356	218.526
6	0.027	0.024	0.030	0.030	0.034	0.037	1.265	76.084	281.779	522.299	726.386	802.124
7	0.025	0.021	0.025	0.027	0.029	0.030	0.898	65.002	221.111	389.756	513.780	579.958
8	0.017	0.020	0.020	0.022	0.027	0.029	5.362	93.371	279.480	454.127	593.070	675.818
9	0.015	0.018	0.015	0.020	0.023	0.169	27.554	155.564	316.948	471.351	591.833	659.065
10	0.016	0.016	0.017	0.018	0.022	3.766	66.582	206.795	347.046	500.564	577.531	654.830
11	0.020	0.020	0.017	0.022	0.028	10.056	75.010	207.739	345.641	469.269	565.292	619.686
12	0.019	0.017	0.016	0.024	0.026	9.727	69.869	213.668	333.985	441.889	518.421	584.957

Month	12 - 13	13 - 14	14 - 15	15 - 16	16 - 17	17 - 18	18 - 19	19 - 20	20 - 21	21 - 22	22 - 23	23 - 24
1	758.550	689.758	650.459	461.896	298.895	131.823	24.618	0.36	0.027	0.027	0.028	0.03
2	875.604	807.096	637.835	507.627	385.673	181.824	24.923	0.065	0.029	0.031	0.033	0.031
3	757.077	699.667	621.719	494.597	305.015	138.842	7.240	0.022	0.027	0.035	0.034	0.032
4	384.751	329.425	481.906	608.980	283.715	53.342	0.270	0.026	0.033	0.035	0.034	0.039
5	312.987	829.751	1,183.700	808.054	287.060	22.538	0.023	0.032	0.038	0.036	0.037	0.035
6	453.913	771.199	617.710	401.718	137.022	8.430	0.022	0.030	0.031	0.028	0.029	0.026
7	573.567	534.193	422.238	290.559	119.483	8.833	0.016	0.026	0.026	0.026	0.026	0.025
8	694.567	647.374	539.926	376.228	188.338	26.394	0.034	0.019	0.022	0.021	0.020	0.019
9	673.716	622.888	508.619	364.246	193.094	44.758	0.246	0.018	0.022	0.020	0.018	0.019
10	664.194	637.250	511.813	379.629	211.591	61.664	1.700	0.013	0.019	0.020	0.019	0.016
11	619.152	539.271	428.625	325.925	201.641	81.637	8.027	0.016	0.019	0.019	0.022	0.020
12	607.978	613.373	557.526	399.691	260.829	119.084	15.600	0.112	0.018	0.021	0.021	0.021

Table 3.4: Average diffuse solar radiation in different months (Watts/m^2)

Month	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12
1	0	0	0	0	0	0	0	21	53	92	121	135
2	0	0	0	0	0	0	6	31	70	134	161	158
3	0	0	0	0	0	2	31	74	113	154	200	232
4	0	0	0	0	0	21	57	104	146	201	239	256
5	0	0	0	0	8	32	91	139	186	219	236	240
6	0	0	0	0	13	41	82	118	140	149	180	209
7	0	0	0	0	10	32	67	83	120	125	165	209
8	0	0	0	0	0	19	44	71	99	120	170	185
9	0	0	0	0	0	12	35	73	120	153	156	157
10	0	0	0	0	0	0	20	48	83	106	102	115
11	0	0	0	0	0	0	8	32	76	108	120	121
12	0	0	0	0	0	0	0	22	54	95	118	120

Month	12 - 13	13 - 14	14 - 15	15 - 16	16 - 17	17 - 18	18 - 19	19 - 20	20 - 21	21 - 22	22 - 23	23 - 24
1	136	128	101	65	32	4	0	0	0	0	0	0
2	184	167	131	96	61	27	0	0	0	0	0	0
3	242	227	177	144	93	41	9	0	0	0	0	0
4	248	228	196	148	103	60	24	0	0	0	0	0
5	249	265	208	174	126	80	29	5	0	0	0	0
6	228	259	221	198	150	90	48	18	0	0	0	0
7	219	200	178	155	120	81	38	14	0	0	0	0
8	196	216	191	156	118	74	29	2	0	0	0	0
9	167	165	154	119	83	41	8	0	0	0	0	0
10	122	130	112	71	43	14	0	0	0	0	0	0
11	136	113	87	59	29	0	0	0	0	0	0	0
12	115	108	84	52	22	0	0	0	0	0	0	0

4. Solar elevation angle: is the angle between the direction of the geometric center of the suns apparent disk and the (idealized) horizon [2, 33]. It can be calculated using the following formula in equation (3.23).

$$\sin\alpha = \sin\phi\sin d + \cos\phi\cos d\cos\omega \tag{3.23}$$

where ϕ is latitude of location, d is declination angle and ω is the azimuth angle.

5. Declination angle: is the angular distance at solar noon between the sun and the equator, north positive. It depends on the day of the year [73], and is calculated from equation (3.24) below [74, 75]:

$$d = 23.45\sin(360\frac{284+N}{365}) \tag{3.24}$$

where N is the number of days from January 1 (the Julian day).

6. R_b: This factor transforms the direct beam component of the solar radiation from the horizontal surface to an inclined plane. It is represented mathematically as shown in equation (3.25) [10, 18, 19, 47, 76, 77]:

$$R_b = \frac{\cos\theta}{\cos\theta_z}, \tag{3.25}$$

where θ and θ_z are the incident angles for beam radiation on tilted and horizontal surfaces, respectively and $\cos\theta_z$ is given by equation (3.26) [4, 33]:

$$\cos\theta = \cos\beta\sin\alpha + \sin\beta\cos\alpha\cos\omega. \tag{3.26}$$

7. R_d: This factor transforms the diffusion component of the solar radiation from the horizontal surface to an inclined plane. It is represented mathematically as shown in equation (3.27) [10, 11, 18, 77].

$$R_d = \frac{1+\cos\beta}{2} \tag{3.27}$$

3.5 CHOICE OF OPTIMIZATION TIME PERIOD

The time period is 1 year which is 12 months. However this time is averaged into 12 typical days where each day represent 1 month. This is done to reduce computational time and effort. This approximation is good because a day is not chosen at random in a month but the each day in the month contributes to this approximation. Hence the error in the results are insignificant.

3.6 SOLVING THE PROBLEM WITH MATLAB

The MOOP is solved with the Matlab environment. Matlab has a GA programming function which makes it easier to use. GA works with random population guesses and so does not give the same results each time but each result is a very good approximation of the true optimal solution. The algorithm can be summarized from Figure 3.2.

The Matlab environment is summarized in Table 3.5.

Table 3.5: The Matlab simulation environment

Component	Description
Computer	Intel C2Q Top End System
Processor	Intel Core2, Quad CPU Q8200, 2.33GHz
Random access memory	2GB
Operating system	Windows XP
Matlab Version	Version R2008a

CHAPTER 4 RESULTS

4.1 CHAPTER OVERVIEW

The optimal design results obtained using both weighted sum approach and Pareto optimization are presented in this chapter. The results are obtained over a time period of 1 year and the solar radiation data is the monthly hourly average for 2011. Figure 4.1 shows the hourly solar radiation average throughout 2011 while Figure 4.2 shows the solar elevation angle and the monthly hourly average solar radiation indicated by 12 typical lines representing 24 hours for each month.

For summer months (represented by the bold colored lines) there is high solar radiation and for winter months (represented by the thinner colored lines), the solar radiation fluctuates drastically between high and low values. In Pretoria, South Africa, the ambient temperature is affected during winter but the solar radiation remains fairly constant.

4.1.1 Results from the weighted sum approach

The optimization results using the weighted sum approach are summarized in Table 4.1. The following general parameters which are typical of a real world scenario are used:
$L = 30m, F_l = 100, W = 200m, H = 2m, C_{UC} = \$200/m^2, C_{UF} = \$150/m^2$ and the time frame for optimization is 1 year.

The constraints for the variables and the objective functions in this optimization are defined as follows:

1. The distance between the rows and the columns of the solar collectors are constrained to equa-

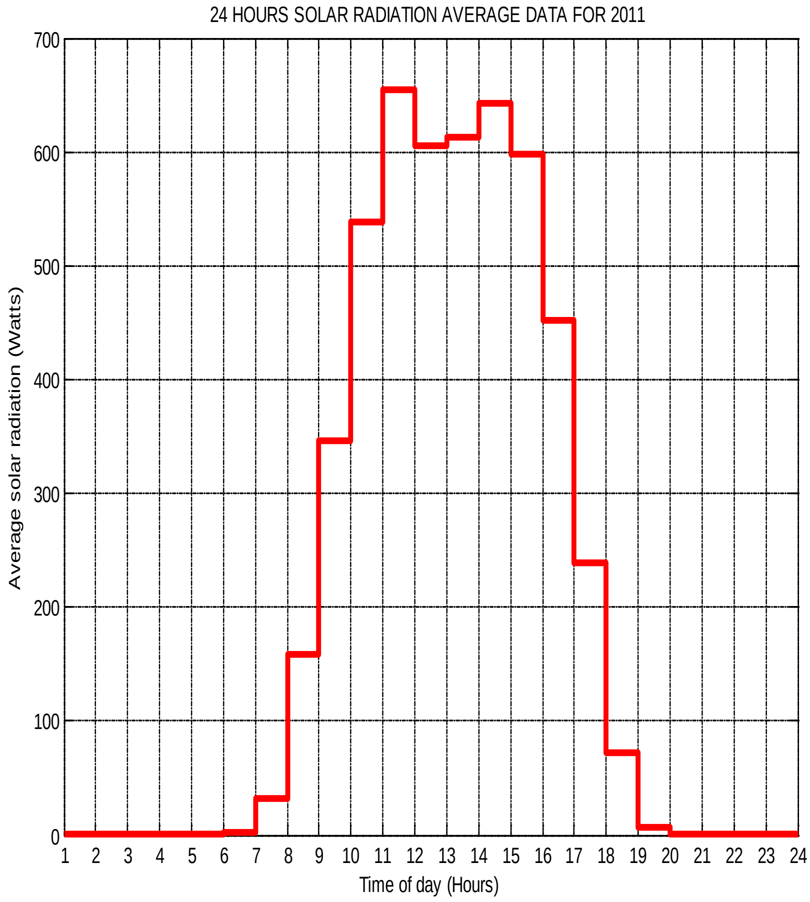

Figure 4.1: Variation of average hourly solar radiation for 2011

tions (4.1) and (4.2),

$$0.8m \leq D \leq 2.5m, \tag{4.1}$$

$$0.8m \leq U \leq 2.5m. \tag{4.2}$$

As indicated earlier, the lower bound is to ensure that there is always enough distance between the rows and columns of the collectors for maintenance and cleaning purposes while the upper bound a land constraint to ensure effective utilization of the land area.

2. The energy objective is constrained to equation (4.3),

$$E_{total} \geq 1MWh. \tag{4.3}$$

1MWh is the minimum amount of energy that can be produced during periods of non-zero solar radiation. If this constraint is violated the objective function is penalized.

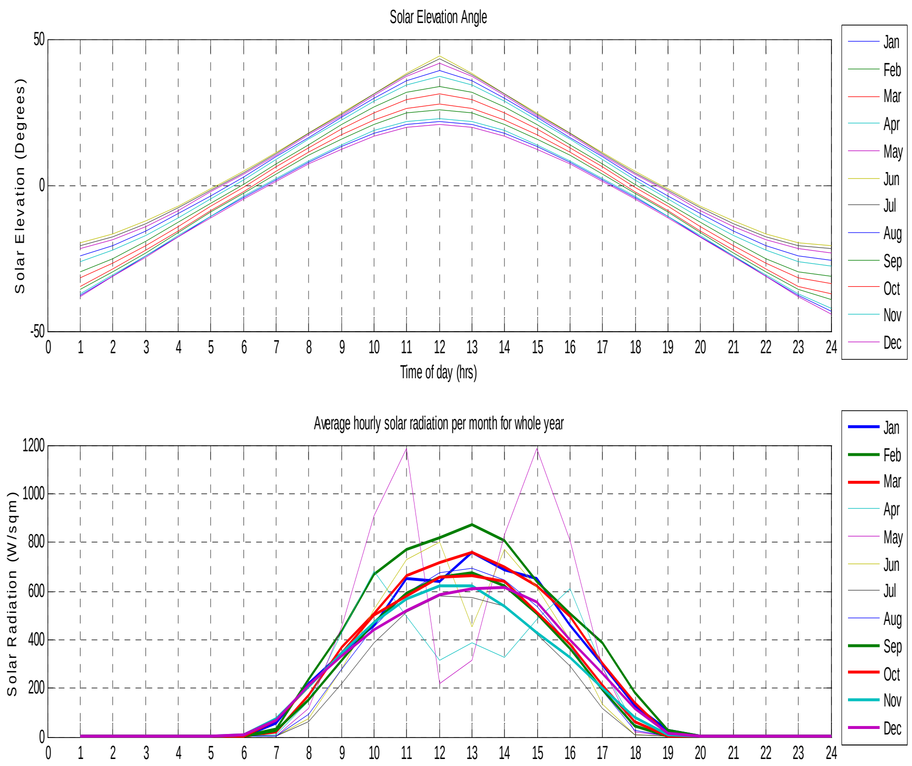

Figure 4.2: Average hourly solar radiation per month for 2011

3. The cost objective must never exceed the value as specified by equation (4.4),

$$C \leq \$6.5 \times 10^6. \tag{4.4}$$

If the objective function exceeds this maximum cost value during runtime, it is penalized.

Table 4.1: Numerical results for the weighted sum approach

λ_1	λ_2	D (m)	U (m)	$\beta(^o)$	K	M	Energy (GWh)	Investment $(10^6)\$$
0	1	2.4158	2.0621	22.315	82	3	0.8862	3.5772
0.1	0.9	2.2101	1.4942	20.463	90	3	1.0392	3.7621
0.3	0.7	2.1414	1.8844	29.776	93	3	0.93548	3.7576
0.5	0.5	2.1342	1.495	25.022	93	3	0.91633	3.7536
0.7	0.3	2.0652	1.2466	27.739	96	3	1.07894	3.8596
0.9	0.1	2.0098	1.1926	21.357	99	3	1.1138	3.9141
1	0	1.9888	1.1828	23.765	100	3	1.1147	4.0609

The hourly results for the energy objective function with each set of weights are presented in Tables 4.2 to 4.8 while the cost or investment objective is as given in the last column of Table 4.1.

Table 4.2: Average energy per hour for the whole year (MW)using weights [0 1] for objectives

Month	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12
1	0	0	0	0	0	0	0.43055	2.5431	5.1518	8.0005	12.388	12.525
2	0	0	0	0	0	0	0.17306	2.1317	5.4629	10.179	13.062	14.379
3	0	0	0	0	0	0	0.026113	0.86392	3.0791	5.5704	8.3346	9.3667
4	0	0	0	0	0	0	0.011059	0.71928	3.4914	7.2802	6.1299	4.0842
5	0	0	0	0	0	0	0.025600	1.072	5.6758	13.961	20.184	3.8498
6	0	0	0	0	0	0	0.0097074	0.87911	4.1886	9.0893	13.801	15.683
7	0	0	0	0	0	0	0.0060807	0.70238	3.1511	6.5844	9.5342	11.094
8	0	0	0	0	0	0	0.014628	0.6334	2.8732	5.8885	8.692	10.294
9	0	0	0	0	0	0	0.010209	0.63982	2.3325	4.6454	6.7616	7.8807
10	0	0	0	0	0	0	0.19533	1.449	3.6499	6.614	8.6103	10.142
11	0	0	0	0	0	0	0.51772	2.268	4.9591	7.9673	10.534	11.9
12	0	0	0	0	0	0	0.53261	2.4606	4.9548	7.68	9.8407	11.427
E_{total}	0	0	0	0	0	0	1.9527	16.362	48.97	93.46	127.87	122.63

Month	12 - 13	13 - 14	14 - 15	15 - 16	16 - 17	17 - 18	18 - 19	19 - 20	20 - 21	21 - 22	22 - 23	23 - 24
1	14.41	12.001	9.6660	5.3342	2.2908	0.21656	0	0	0	0	0	0
2	14.88	12.328	7.9996	4.5590	1.8635	0	0	0	0	0	0	0
3	9.5017	7.6491	5.1971	2.4787	0.34895	0	0	0	0	0	0	0
4	4.7449	3.5326	3.9344	2.9446	0.28111	0	0	0	0	0	0	0
5	5.3296	12.702	14.886	7.2851	1.397	0	0	0	0	0	0	0
6	8.6244	13.421	9.1822	4.6417	1.0512	0.013939	0	0	0	0	0	0
7	10.644	9.0245	6.0174	3.1397	0.80929	0.004102	0	0	0	0	0	0
8	10.18	8.3942	5.5507	2.5522	0.51379	0	0	0	0	0	0	0
9	7.6971	6.1389	3.743	1.4981	0.071543	0	0	0	0	0	0	0
10	9.9024	8.42	5.3827	2.66	0.62075	0	0	0	0	0	0	0
11	11.537	9.1557	6.1497	3.5583	1.3917	0.051216	0	0	0	0	0	0
12	11.541	10.66	8.2711	4.6028	1.9883	0.1888	0	0	0	0	0	0
E_{total}	118.99	113.43	85.98	45.254	12.628	0.474617	0	0	0	0	0	0

Table 4.3: Average energy per hour for the whole year (MW)using weights [0.1 0.9] for objectives

Month	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12
1	0	0	0	0	0	0	0.43006	2.5439	5.1552	8.0073	12.4	12.539
2	0	0	0	0	0	0	0.17304	2.1329	5.4672	10.188	13.074	14.394
3	0	0	0	0	0	0	0.026209	0.86482	3.082	5.5757	8.3429	9.3763
4	0	0	0	0	0	0	0.011106	0.72005	3.4947	7.2872	6.1359	4.0884
5	0	0	0	0	0	0	0.025598	1.0726	5.6802	13.974	20.204	3.8539
6	0	0	0	0	0	0	0.0096963	0.87936	4.1914	9.097	13.815	15.699
7	0	0	0	0	0	0	0.0060757	0.70265	3.1533	6.5901	9.5435	11.106
8	0	0	0	0	0	0	0.014644	0.63391	2.8757	5.8939	8.7006	10.305
9	0	0	0	0	0	0	0.010329	0.6406	2.3348	4.6499	6.7683	7.8887
10	0	0	0	0	0	0	0.19552	1.4501	3.653	6.6201	8.6188	10.152
11	0	0	0	0	0	0	0.51727	2.2688	4.9626	7.9741	10.544	11.913
12	0	0	0	0	0	0	0.53201	2.4613	4.9581	7.6866	9.8503	11.439
E_{total}	0	0	0	0	0	0	1.9516	16.371	49.008	93.544	128	122.76

Month	12 - 13	13 - 14	14 - 15	15 - 16	16 - 17	17 - 18	18 - 19	19 - 20	20 - 21	21 - 22	22 - 23	23 - 24
1	14.424	12.011	9.6725	5.3358	2.2882	0	0	0	0	0	0	0
2	14.895	12.339	8.0059	4.5616	1.8634	0	0	0	0	0	0	0
3	9.5111	7.6565	5.202	2.4813	0.35023	0	0	0	0	0	0	0
4	4.7496	3.536	3.9382	2.9478	0.28232	0	0	0	0	0	0	0
5	5.3348	12.714	14.898	7.2892	1.3969	0	0	0	0	0	0	0
6	8.6327	13.432	9.1883	4.643	1.05	0	0	0	0	0	0	0
7	10.654	9.0323	6.0217	3.1409	0.80864	0	0	0	0	0	0	0
8	10.19	8.402	5.5555	2.5543	0.51436	0	0	0	0	0	0	0
9	7.7047	6.1448	3.7467	1.4999	0.072382	0	0	0	0	0	0	0
10	9.9121	8.4278	5.3873	2.662	0.62133	0	0	0	0	0	0	0
11	11.548	9.1636	6.154	3.5596	1.3905	0	0	0	0	0	0	0
12	11.552	10.669	8.2766	4.6042	1.9861	0	0	0	0	0	0	0
E_{total}	119.11	113.53	86.046	45.279	12.624	0	0	0	0	0	0	0

Table 4.4: Average energy per hour for the whole year (MW)using weights [0.3 0.7] for objectives

Month	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12
1	0	0	0	0	0	0	0.42736	2.5276	5.1219	7.955	12.319	12.456
2	0	0	0	0	0	0	0.17194	2.1188	5.4307	10.12	12.986	14.297
3	0	0	0	0	0	0	0.025997	0.85863	3.0602	5.5365	8.2841	9.31
4	0	0	0	0	0	0	0.011013	0.71487	3.4699	7.2358	6.0926	4.0594
5	0	0	0	0	0	0	0.025434	1.0656	5.6423	13.88	20.068	3.8278
6	0	0	0	0	0	0	0.0096355	0.87376	4.1643	9.0376	13.724	15.596
7	0	0	0	0	0	0	0.0060375	0.69813	3.1328	6.5467	9.4803	11.032
8	0	0	0	0	0	0	0.014545	0.62957	2.856	5.8534	8.6406	10.234
9	0	0	0	0	0	0	0.010196	0.63584	2.318	4.6167	6.72	7.8323
10	0	0	0	0	0	0	0.19421	1.4402	3.628	6.5747	8.5595	10.082
11	0	0	0	0	0	0	0.51402	2.2542	4.9302	7.9217	10.474	11.834
12	0	0	0	0	0	0	0.52867	2.4456	4.926	7.6364	9.7856	11.364
E_{total}	0	0	0	0	0	0	1.9391	16.263	48.68	92.914	127.13	121.92

Month	12 - 13	13 - 14	14 - 15	15 - 16	16 - 17	17 - 18	18 - 19	19 - 20	20 - 21	21 - 22	22 - 23	23 - 24
1	14.33	11.933	9.6099	5.3017	2.2738	0	0	0	0	0	0	0
2	14.794	12.256	7.9525	4.5315	1.8515	0	0	0	0	0	0	0
3	9.4441	7.6026	5.1653	2.4636	0.34741	0	0	0	0	0	0	0
4	4.7161	3.511	3.9103	2.9265	0.27994	0	0	0	0	0	0	0
5	5.2988	12.628	14.799	7.2412	1.388	0	0	0	0	0	0	0
6	8.5761	13.344	9.1289	4.6134	1.0434	0	0	0	0	0	0	0
7	10.584	8.9729	5.9824	3.1206	0.80355	0	0	0	0	0	0	0
8	10.119	8.3443	5.5174	2.5368	0.51088	0	0	0	0	0	0	0
9	7.6498	6.101	3.7197	1.4888	0.071451	0	0	0	0	0	0	0
10	9.8439	8.37	5.3505	2.6439	0.61717	0	0	0	0	0	0	0
11	11.472	9.1034	6.1139	3.5367	1.3818	0	0	0	0	0	0	0
12	11.476	10.6	8.2231	4.5748	1.9736	0	0	0	0	0	0	0
E_{total}	118.3	112.77	85.472	44.98	12.543	0	0	0	0	0	0	0

Table 4.5: Average energy per hour for the whole year (MW)using weights [0.5 0.5] for objectives

Month	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12
1	0	0	0	0	0	0	0.43298	2.5617	5.1953	8.0733	12.505	12.645
2	0	0	0	0	0	0	0.17495	2.1572	5.5305	10.307	13.228	14.563
3	0	0	0	0	0	0	0.027572	0.88454	3.1405	5.674	8.4852	9.5356
4	0	0	0	0	0	0	0.011778	0.73711	3.5627	7.4181	6.2425	4.159
5	0	0	0	0	0	0	0.025878	1.0848	5.7457	14.136	20.44	3.899
6	0	0	0	0	0	0	0.0097621	0.88553	4.224	9.1719	13.932	15.832
7	0	0	0	0	0	0	0.0061225	0.70847	3.1816	6.6517	9.6343	11.212
8	0	0	0	0	0	0	0.014944	0.64426	2.9192	5.9803	8.8262	10.454
9	0	0	0	0	0	0	0.012045	0.65855	2.3852	4.7406	6.8948	8.0352
10	0	0	0	0	0	0	0.19922	1.4729	3.7068	6.7149	8.7407	10.296
11	0	0	0	0	0	0	0.52118	2.2872	5.0062	8.0474	10.643	12.025
12	0	0	0	0	0	0	0.53564	2.4787	4.997	7.7504	9.9342	11.537
E_{total}	0	0	0	0	0	0	1.9527	16.362	48.97	93.46	127.87	122.63

Month	12 - 13	13 - 14	14 - 15	15 - 16	16 - 17	17 - 18	18 - 19	19 - 20	20 - 21	21 - 22	22 - 23	23 - 24
1	14.547	12.11	9.7478	5.3733	2.3037	0.36823	0	0	0	0	0	0
2	15.07	12.483	8.0986	4.6136	1.8839	0	0	0	0	0	0	0
3	9.6734	7.7913	5.3008	2.5379	0.36845	0	0	0	0	0	0	0
4	4.8321	3.5995	4.0148	3.0176	0.2994	0	0	0	0	0	0	0
5	5.3971	12.862	15.07	7.3718	1.4122	0	0	0	0	0	0	0
6	8.7058	13.543	9.2597	4.6756	1.0571	0.023634	0	0	0	0	0	0
7	10.755	9.1167	6.0756	3.1669	0.81486	0.014768	0	0	0	0	0	0
8	10.337	8.5251	5.6396	2.596	0.52491	0	0	0	0	0	0	0
9	7.8487	6.2647	3.8277	1.542	0.084412	0	0	0	0	0	0	0
10	10.052	8.5485	5.4667	2.7039	0.63311	0	0	0	0	0	0	0
11	11.657	9.2478	6.2081	3.5884	1.401	0.14924	0	0	0	0	0	0
12	11.65	10.758	8.3415	4.6368	1.9996	0.3262	0	0	0	0	0	0
E_{total}	118.99	113.43	85.98	45.254	12.628	0.474617	0	0	0	0	0	0

Table 4.6: Average energy per hour for the whole year (MW)using weights [0.7 0.3] for objectives

Month	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12
1	0	0	0	0	0	0	0.33479	3.565	5.1687	8.0795	11.506	13.646
2	0	0	0	0	0	0	0.17465	2.1529	5.5187	10.284	13.198	14.531
3	0	0	0	0	0	0	0.026699	0.87544	3.1171	5.6374	8.434	9.4788
4	0	0	0	0	0	0	0.011336	0.72906	3.5349	7.3684	6.2035	4.1334
5	0	0	0	0	0	0	0.025835	1.0827	5.7336	14.106	20.395	3.8903
6	0	0	0	0	0	0	0.0097803	0.88666	4.2267	9.1745	13.933	15.834
7	0	0	0	0	0	0	0.0061289	0.70869	3.1809	6.6482	9.628	11.205
8	0	0	0	0	0	0	0.014810	0.64064	2.9055	5.9544	8.7894	10.41
9	0	0	0	0	0	0	0.010802	0.64933	2.363	4.7037	6.8453	7.9784
10	0	0	0	0	0	0	0.19767	1.4653	3.6904	6.6874	8.7061	10.255
11	0	0	0	0	0	0	0.52179	2.2882	5.0057	8.0441	10.637	12.018
12	0	0	0	0	0	0	0.53662	2.4818	5	7.7522	9.9348	11.537
E_{total}	0	0	0	0	0	0	1.9699	16.526	49.475	94.436	129.21	123.92

Month	12 - 13	13 - 14	14 - 15	15 - 16	16 - 17	17 - 18	18 - 19	19 - 20	20 - 21	21 - 22	22 - 23	23 - 24
1	14.548	12.114	9.754	5.3801	2.308	0.23875	0	0	0	0	0	0
2	15.036	12.456	8.0813	4.6043	1.8807	0	0	0	0	0	0	0
3	9.6151	7.7411	5.2612	2.5118	0.35678	0	0	0	0	0	0	0
4	4.8019	3.5754	3.9835	2.9846	0.28816	0	0	0	0	0	0	0
5	5.3852	12.834	15.038	7.3574	1.4099	0	0	0	0	0	0	0
6	8.7067	13.547	9.2657	4.6815	1.0591	0.015358	0	0	0	0	0	0
7	10.748	9.112	6.0743	3.1679	0.81571	0.0055948	0	0	0	0	0	0
8	10.294	8.4881	5.6131	2.5814	0.52019	0	0	0	0	0	0	0
9	7.7924	6.2159	3.792	1.5204	0.075703	0	0	0	0	0	0	0
10	10.012	8.5135	5.4426	2.6899	0.62816	0	0	0	0	0	0	0
11	11.65	9.2441	6.2075	3.59	1.4027	0.065017	0	0	0	0	0	0
12	11.651	10.761	8.3465	4.6425	2.0033	0.20886	0	0	0	0	0	0
E_{total}	120.24	114.6	86.86	45.712	12.748	0.5335798	0	0	0	0	0	0

Table 4.7: Average energy per hour for the whole year (MW)using weights [0.9 0.1] for objectives

Month	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12
1	0	0	0	0	0	0	0.45013	2.6594	5.3824	8.3542	12.933	13.078
2	0	0	0	0	0	0	0.17965	2.2139	5.6744	10.574	13.57	14.941
3	0	0	0	0	0	0	0.025183	0.881	3.1617	5.7344	8.5895	9.6565
4	0	0	0	0	0	0	0.010498	0.73246	3.5824	7.4905	6.3144	4.2087
5	0	0	0	0	0	0	0.026580	1.1135	5.8959	14.504	20.971	4.0004
6	0	0	0	0	0	0	0.010149	0.91931	4.3762	9.4913	14.409	16.375
7	0	0	0	0	0	0	0.0063483	0.73306	3.2863	6.8643	9.9382	11.566
8	0	0	0	0	0	0	0.014933	0.65269	2.968	6.0894	8.9935	10.654
9	0	0	0	0	0	0	0.0077512	0.64697	2.3853	4.7682	6.9511	8.1052
10	0	0	0	0	0	0	0.19997	1.4946	3.7727	6.8431	8.9129	10.5
11	0	0	0	0	0	0	0.54063	2.3677	5.1732	8.3078	10.982	12.409
12	0	0	0	0	0	0	0.5568	2.5728	5.1762	8.019	10.273	11.93
E_{total}	0	0	0	0	0	0	2.0286	16.987	50.835	97.04	132.84	127.42

Month	12 - 13	13 - 14	14 - 15	15 - 16	16 - 17	17 - 18	18 - 19	19 - 20	20 - 21	21 - 22	22 - 23	23 - 24
1	15.045	12.532	10.099	5.5781	2.3949	0.88598	0	0	0	0	0	0
2	15.459	12.807	8.3092	4.7349	1.9346	0.63136	0	0	0	0	0	0
3	9.7922	7.8743	5.3365	2.5278	0.33653	0	0	0	0	0	0	0
4	4.8878	3.6346	4.037	2.9986	0.26685	0	0	0	0	0	0	0
5	5.5373	13.196	15.464	7.5669	1.4505	0.079130	0	0	0	0	0	0
6	9.0041	14.014	9.5934	4.8539	1.099	0.056737	0	0	0	0	0	0
7	11.095	9.408	6.2756	3.2768	0.84491	0.050097	0	0	0	0	0	0
8	10.533	8.6807	5.7339	2.6299	0.52453	0.031235	0	0	0	0	0	0
9	7.9128	6.3012	3.8277	1.5149	0.054320	0	0	0	0	0	0	0
10	10.25	8.7117	5.5639	2.7437	0.63547	0.086556	0	0	0	0	0	0
11	12.029	9.5471	6.4153	3.7148	1.4533	0.47522	0	0	0	0	0	0
12	12.048	11.131	8.6407	4.8128	2.0786	0.79449	0	0	0	0	0	0
E_{total}	123.59	117.84	89.296	46.953	13.073	2.9303	0	0	0	0	0	0

Table 4.8: Average energy per hour for the whole year (MW)using weights [1 0] for objectives

Month	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12
1	0	0	0	0	0	0	0.45739	2.6963	5.4502	8.4534	13.082	13.227
2	0	0	0	0	0	0	0.18144	2.2335	5.7224	10.661	13.68	15.061
3	0	0	0	0	0	0	0.024003	0.87716	3.1627	5.7455	8.6116	9.6825
4	0	0	0	0	0	0	0.009874	0.72852	3.5817	7.5024	6.3288	4.2188
5	0	0	0	0	0	0	0.026847	1.1234	5.9461	14.624	21.143	4.0329
6	0	0	0	0	0	0	0.010313	0.93209	4.4313	9.6041	14.575	16.561
7	0	0	0	0	0	0	0.006442	0.74219	3.3235	6.9379	10.042	11.685
8	0	0	0	0	0	0	0.014888	0.65475	2.9816	6.1206	9.0415	10.711
9	0	0	0	0	0	0	0.0057504	0.64018	2.3792	4.7678	6.9572	8.1138
10	0	0	0	0	0	0	0.19977	1.5003	3.7916	6.8806	8.9632	10.56
11	0	0	0	0	0	0	0.54873	2.3977	5.2327	8.3983	11.099	12.539
12	0	0	0	0	0	0	0.56575	2.6084	5.241	8.1138	10.391	12.066
E_{total}	0	0	0	0	0	0	2.0512	17.135	51.244	97.81	133.91	128.46

Month	12 - 13	13 - 14	14 - 15	15 - 16	16 - 17	17 - 18	18 - 19	19 - 20	20 - 21	21 - 22	22 - 23	23 - 24
1	15.218	12.681	10.226	5.6555	2.4336	0.77613	0	0	0	0	0	0
2	15.585	12.912	8.3796	4.7768	1.9538	0.44876	0	0	0	0	0	0
3	9.8175	7.8896	5.3383	2.5167	0.32075	0	0	0	0	0	0	0
4	4.8989	3.6404	4.0363	2.9824	0.25099	0	0	0	0	0	0	0
5	5.5826	13.306	15.595	7.6345	1.465	0.056523	0	0	0	0	0	0
6	9.1078	14.181	9.7143	4.9214	1.1168	0.049718	0	0	0	0	0	0
7	11.21	9.509	6.3467	3.3176	0.85738	0.042102	0	0	0	0	0	0
8	10.589	8.7252	5.7601	2.6383	0.52293	0.0035045	0	0	0	0	0	0
9	7.9197	6.3007	3.8179	1.499	0.040298	0	0	0	0	0	0	0
10	10.308	8.7594	5.5918	2.7543	0.63486	0.021907	0	0	0	0	0	0
11	12.156	9.6511	6.4891	3.7618	1.4751	0.040208	0	0	0	0	0	0
12	12.186	11.263	8.749	4.8793	2.112	0.69481	0	0	0	0	0	0
E_{total}	124.58	118.82	90.044	47.338	13.183	2.0859	0	0	0	0	0	0

4.1.2 Results from comparisons

The results from [4], [34], [35], [36] are based on a single column of solar collectors. The model presented in this paper is based on multi-row-multi-column solar collectors. Thus for comparison to be made, solar radiation data, field and collector parameters from the previous models will be used with the proposed model.

A multi-objective optimization problem is solved using game theory in [35]. The number of rows K of the solar collectors is varied and the following results in Table 4.9 are obtained with F_l = 30m, W = 200m, $C_{UC} = \$100/m^2, C_{UF} = \$100/m^2$.

Table 4.9: Optimization results from [35]

D (m)	$\beta(^o)$	K	Energy (GWh)	Cost (10^6\$)
2.51	31.2403	80	0.19692	1.08
2.4790	32.9120	81	0.19703	1.086
2.4488	34.4733	82	0.19711	1.092
2.4193	35.9394	83	0.19711	1.098
2.3905	37.3222	84	0.19702	1.104

Using solar radiation data from [35] and applying it to the model presented in this paper for a single column of collectors, the result in Table 4.10 is obtained.

Table 4.10: Results from applying data from [35] to current model

λ_1	λ_2	D (m)	$\beta(^o)$	K	Energy (GWh)	Investment (10^6\$)
0	1	2.4926	23.4858	90	0.21733	1.0667
0.1	0.9	2.4256	28.0694	85	0.22055	1.0795
0.3	0.7	2.3756	22.3138	86	0.23447	1.0887
0.5	0.5	2.3385	21.3426	84	0.23807	1.097
0.7	0.3	2.3043	23.7500	82	0.24317	1.1039
0.9	0.1	2.2045	28.3652	80	0.25320	1.1281
1	0	2.2880	25.6418	87	0.25341	1.107

4.2 RESULTS FROM PARETO OPTIMIZATION USING GA

In the Pareto optimization carried out for this research, a group of solutions is obtained whose characteristic is such that a solution is better in one objective but worse in the other objective when compared to the other solutions and as such there is no better solution amongst this group of solutions. These solutions are known as Pareto optimal solutions and a line that connects this point form the Pareto front.

Some parameters used in this simulation are: the number of generations to iterate through is 100 and the population size is 100. Half of the population is discarded and replaced by new offsprings. The parents are chosen for mating through random tournament selection. The mutation rate is 10% of the population.

The Pareto optimal solutions for the three variables along with the objective functions values for energy and cost is shown in Table 4.11 and the Pareto front is plotted in Figure 4.3.

Table 4.11: Pareto-optimal solutions for the whole year

D (m)	U (m)	β^o	K	M	Energy (MWh)	Investment (10^6\$)
2.44994	2.33805	27.4783	81	3	733.05	0.286
2.48716	2.25205	26.4182	80	3	746.57	0.2875
2.46198	2.46807	25.4242	81	3	748.31	0.2891
2.36738	2.47280	24.4250	84	3	750.70	0.2917
2.44631	2.41481	24.4258	81	3	756.96	0.292
2.46317	2.45476	24.4037	81	3	757.05	0.2927
2.48911	2.18575	24.4086	80	3	765.39	0.2942
2.48764	2.05546	24.4005	80	3	768.70	0.2959
2.44463	1.99736	22.4056	81	3	783.52	0.3078
2.47258	1.99782	22.4389	80	3	785.12	0.3101

The data in Table 4.12 gives the average hourly energy values obtained per month for the whole year using the first vector of solutions from Table 4.11. The final choice of which vector of solutions to be implemented depends on which objective is given the higher priority after more information is

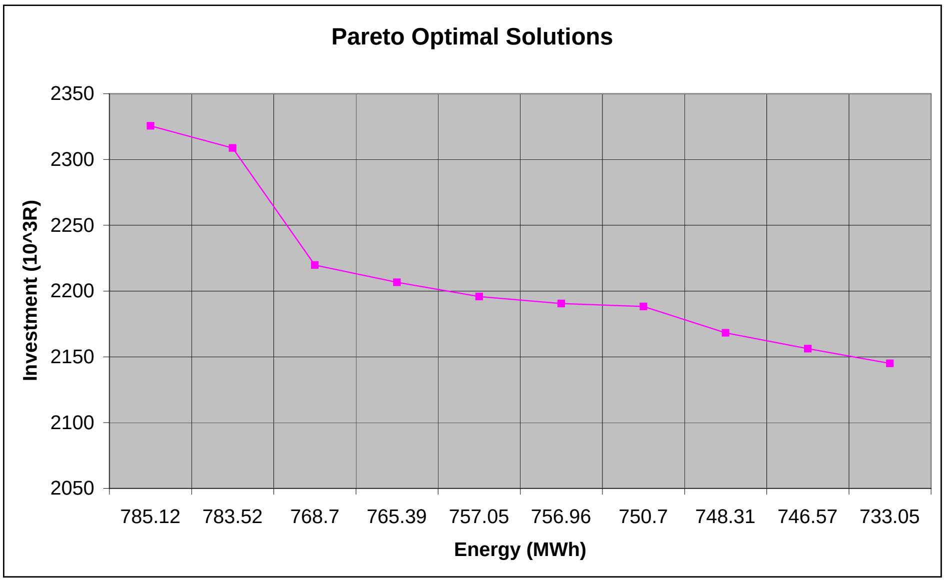

Figure 4.3: Pareto Front

provided by the decision maker.

Table 4.12: Table showing the average energy per hour for the whole year (MW)

Month	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12
1	0	0	0	0	0	0	0.071691	1.1287	2.8003	5.0818	8.099	8.2684
2	0	0	0	0	0	0	0.0697	1.5071	4.3193	8.435	11.054	12.242
3	0	0	0	0	0	0	0.0932	1.4412	4.4321	7.5677	11.042	12.323
4	0	0	0	0	0	0	0.0480	1.5402	5.9841	11.396	9.2176	6.0727
5	0	0	0	0	0	0	0.039240	1.3754	6.8014	16.193	23.052	4.3776
6	0	0	0	0	0	0	0.010181	0.91633	4.3525	9.43	14.418	16.257
7	0	0	0	0	0	0	0.007091	0.77113	3.3848	6.9959	10.077	11.709
8	0	0	0	0	0	0	0.03629	1.0229	4.0646	7.8544	11.285	13.264
9	0	0	0	0	0	0	0.13629	1.4363	4.09232	7.4171	10.365	11.945
10	0	0	0	0	0	0	0.1921	1.4654	3.7147	6.7492	8.7971	10.366
11	0	0	0	0	0	0	0.08539	1.0652	2.9667	5.2747	7.2988	8.337
12	0	0	0	0	0	0	0.2487	0.90183	2.5298	4.4871	6.1053	7.114
E_{total}	0	0	0	0	0	0	0.81439	14.472	49.443	96.719	130.65	122.39

Month	12 - 13	13 - 14	14 - 15	15 - 16	16 - 17	17 - 18	18 - 19	19 - 20	20 - 21	21 - 22	22 - 23	23 - 24
1	9.3641	7.3785	5.2541	2.1576	0.22182	0.14666	0	0	0	0	0	0
2	12.593	10.216	6.325	3.2233	0.85637	0.18412	0	0	0	0	0	0
3	12.588	10.392	7.4809	4.1352	1.2479	0.11831	0	0	0	0	0	0
4	8.1351	5.5298	6.7436	6.3052	1.731	0.03351	0	0	0	0	0	0
5	6.0868	14.733	17.838	9.3469	2.114	0.0098734	0	0	0	0	0	0
6	8.9412	13.924	9.541	4.8382	1.1024	0.0028787	0	0	0	0	0	0
7	11.249	9.5884	6.4638	3.447	0.93419	0.003579	0	0	0	0	0	0
8	13.217	11.197	7.8524	4.1218	1.272	0.014301	0	0	0	0	0	0
9	11.799	9.8017	6.5671	3.363	0.95512	0.034213	0	0	0	0	0	0
10	10.117	8.5922	5.473	2.6901	0.61178	0.059175	0	0	0	0	0	0
11	7.942	6.0615	3.679	1.6712	0.22941	0.08825	0	0	0	0	0	0
12	7.16	6.2284	4.223	1.687	0.092873	0.13516	0	0	0	0	0	0
E_{total}	118.25	113.64	87.447	46.986	11.397	0.830425	0	0	0	0	0	0

CHAPTER 5 DISCUSSION

5.1 CHAPTER OVERVIEW

This chapter discusses the results obtained in the previous chapter. It also analyzes the effects of the constraints on the results.

5.1.1 Weighted sum approach analysis

Generally, a large amount of energy results in large amounts of investment as observed from the results in Table 4.1. From the results in Table 4.1, the weights have a profound effect on the optimal values of the variables. These variable values are introduced into equations (3.11) and (3.12) to obtain values for both objective functions. For example if the energy objective is given less priority than the investment objective and the weights are chosen as 0.1 and 0.9 respectively, the solution for the variables D, U and β are obtained as 2.2101m, 1.4942m and 20.463^o respectively which result in objective function values of 1.0392 GWh for the energy and $ 3.7621 million for the investment. Also if the energy objective is given more priority than the investment objective and the weights are chosen as 0.9 and 0.1 respectively, the solution for the variables D, U and β become 2.0098m, 1.1926m and 26.8865^o respectively which result in fitness values of 1.1138 GWh for the energy and $ 3.9141 million for the investment. Therefore, the objective functions are greatly affected by the variables and it is observed that the higher the maximum energy obtained, the greater the investment.

Secondly, the complex relationship between the variables and the randomness of the GA make the results for the variables unpredictable. However the algorithm converges to an almost unique solution for the objective functions with each weight chosen. All results indicate that if the energy objective is given a higher priority than the investment or cost objective, the optimal spacing U between the columns of the collectors is reduced. This reduction allows for the addition of more columns to the field thereby increasing the amount of energy that can be produced. Conversely, when the priority is given to the cost or investment objective, the optimal spacing U between the columns of the collectors is increased. This prevents the addition of extra columns of collectors to the field thereby reducing the investment costs and the energy that can be obtained. To avoid the undesirable effects of shading, the optimal spacing U between the columns of the collectors is constrained to 1m. It is also worthy of noting that the optimal spacing D between the rows of the collectors is within the range of 1.99m $\leq D \leq$ 2.41m. This ensures that the objectives of maximizing energy and minimizing the investment or cost are both attained simultaneously without violating the constraints.

Thirdly, the component parts of the solar radiation (direct beam and diffuse) also play an important role in determining the maximum energy that can be obtained. Areas with an increased value of such radiation may see more maximum energy obtained. In addition, by varying the location of the collectors with respect to the latitude, this impacts the inclination angle β of the collectors. This is because the latitude affects the amount of solar radiation that the location receives. The closer the inclination angle is to the angle of latitude, the higher the solar radiation intensity and the greater the energy that can be obtained. This is consistent with previous findings in literature [9], [78], [79], [68]. Importantly also, by varying the weights for the objectives, this variable is affected and consequently impacts on the value of the objectives.

Fourthly, in Table 4.1, the optimization returns the number of collector rows in the field as 100 for an objective weight vector of 1 and 0, 93 for an objective weight vector of 0.5 and 0.5 and 82 for the objective weight vector of 0 and 1 respectively. This means that when more priority is placed on the energy objective, the algorithm tries to increase the number of rows in order to maximize the energy obtained and vice-versa. Also the optimization places the number of collector columns in the field at 3 for each choice of weights. These two parameters (number of collector rows and columns) are also constrained by the limitation of the field area in addition to the choice of weights. An increase or decrease in the field area will lead to a corresponding increase or decrease in the number of rows and columns.

In addition, the weights affect the results of both the energy and the investment/cost objectives. This effect can be seen from Tables 4.2 to 4.8 for the energy objective. Generally, a positive increase in the weights from 0 to 1 for the energy objective results in an increase in the energy collected hourly per month and this consequently increases the overall energy collected hourly for the whole year. Furthermore, comparing the actual energy collected hourly per month in Tables 4.2 to 4.8, it can be seen that for winter months 6 to 9 (which represent June to early September) there is a reduction in the amount of energy that is collected per hour especially in the early hours of the morning and in the late hours of the evening. This is because of the shorter days and longer nights during winter. However around noon, considerably large amounts of energy are still collected even for the winter months. This is because South Africa generally has high solar radiation even in winter months.

Summarily, a large amount of energy can be obtained from using this model even though the investment is initially high. This investment can be recovered once the project is operational. The results are obtained for different combinations of the weights depending on the assumed relative importance of each objective function on the expected outcome. Even though the energy and costs are comparable, the results show that irrespective of the weights assigned to the objective functions, more energy is always obtained. This shows that the model is robust. The variables are properly constrained to their bounds which take into consideration the criteria for maintenance purposes. The hourly results for the energy objective function with each set of weights are presented in Table 4.2 to Table 4.8. It is important to note that the cost/investment objective is only dependent on the final optimized value of the variables and hence does not vary hourly as is the case with the energy objective, hence the result for the investment or cost objective is as given in the last column of Table 4.1. The reason for this is because the energy objective is largely dependent on the average values for the solar radiation in W/m^2 which varies depending on the time of the day and the period of the year while the investment objective is not dependent on this parameter.

5.1.2 Comparisons to previous models

The result from a previous model [35] is presented in Table 4.9. The result is based on a single column of collectors and the following parameters F_l = 30m, W = 200m, $C_{UC} = \$100/m^2$,$C_{UF} = \$100/m^2$. These parameters are inserted in the formulated model and the optimization is carried out. The results are presented in Table 4.10. The variable U is 1 since its only a single column of collectors and is excluded from the table. As an example if the energy objective is given less priority than the investment objective and the weights are chosen as 0.1 and 0.9 respectively, the solution for the variables D and β are obtained as 2.4256 m and 28.0694^o respectively which results in 85 collector rows and objective function values of 0.22055 GWh for the energy and \$ 1.0795 million for the investment. Also if the energy objective is given more priority than the investment objective and the weights are chosen as 0.9 and 0.1 respectively, the solution for the variables D and β become 2.2045 m and 28.3652^o respectively which results in 80 collector rows and fitness values of 0.25320 GWh for the energy and \$ 1.1281 million for the investment. Similarly, the objective functions as well as the number of collector rows are affected by the variables, and the higher the energy obtained, the greater the investment required.

Analyzing the results from Table 4.10, it can also be seen that the weights affect the variables and this has an effect on the value of the objectives with respect to the assigned priority. If the distance D between the rows of solar collectors is increased, there is an undesirable reduction in the amount of energy that may be produced as less rows may be installed in the field but this effect sees a reduction on the investment which is desirable. Also the latitude of the location which is 25.4^o affects the inclination angle of the collectors.

Comparing the results derived from this model in Table 4.10 to that from [35] in Table 4.9 using equal priority weights 0.5 and 0.5 for both objectives and an equal number of collector rows (84) , the formulated model produces 20.83% more energy than that from [35] with an almost equal spacing between the rows of the collectors. The investment is also 0.7% less than that from [35]. This means that for an equal amount of investment, more energy can be obtained using the model proposed in this work. Conversely, for an equal amount of energy collected, the investment is reduced by using the model presented in this work. These results prove that the model presented in this work can be applied practically to produce more energy at less costs than previous models.

5.1.3 Pareto optimization analysis

From the results in Table 4.11 and Figure 4.3, a reduction in the distance between collector rows can lead to an increase in energy obtained. This is because more collector rows can be added to the field which also increases the investment. A decrease also in the distance between the collector columns results in an increase in the energy obtained. This is due to the fact that the total collector area can be increased by adding more columns to the field at an increased investment. Furthermore, a change in the collector inclination angle has minimal effect on the investment compared to the energy that can be obtained. For example, the solution with $D = 2.47258$, $U = 1.99782$ and $\beta = 22.4389$ has 80 collector rows and 3 collector columns. It also yields 0.78512 GWh of energy and \$ 0.3101 million investment. The solution with $D = 2.44631$, $U = 2.41481$ and $\beta = 24.4258$ has 81 collector rows and 3 collector columns. It also yields 0.75696 GWh of energy and \$ 0.292 million investment. The solution with $D = 2.36738$, $U = 2.47280$ and $\beta = 24.4250$ has 84 collector rows and 3 collector columns. It also yields 0.75070 GWh of energy and $ 0.2917 million investment. The first solution is better on the energy objective but worst on the investment objective than the other two solutions. The second solution is worse than the first solution and better than the third solution on the energy objective but is better than the first solution and worse than the third solution on the investment objective. The third solution is worst on the energy objective than the previous two solutions but is best on the investment objective than the previous two solutions. Thus the choice of which solution is implemented is dependent on further information available to the decision maker.

Here also, the number of collector rows and the number of collector columns are affected by the limitation of the field area and an increase or decrease in the field area will lead to a corresponding increase or decrease in the number of rows and columns.

The hourly results for the energy objective function is presented in Table 4.12. The cost or investment objective does not vary hourly as is the case with the energy objective. This is because the investment objective is not dependent on the average solar radiation which varies depending on the time of the day and the period of the year. Also from Table 4.12 a deduction similar to the weighted sum approach can be made for the winter months.

For the Pareto optimization, the results are consistent with existing knowledge. Only a few solutions however form the Pareto front. This is due to the fact that the variables are constrained rather tightly.

5.1.4 Comparison between the weighted sum approach and Pareto optimization

Reviewing the results obtained from the weighted sum approach and that from the Pareto optimization, the energy objective function converges to about 0.7 GWh using the Pareto approach while higher values are recorded for the weighted sum approach. This means that the weighted sum approach is better suited to find the global maximum for the energy objective than the Pareto approach which is able to find a local maximum. However on the investment objective, the weighted sum approach was in excess of $ 3.5 million while the Pareto optimization yielded under $ 0.3 million. This means that the weighted sum approach converged to a local minimum whereas the Pareto optimization was able to find the global minimum and thus is better suited than the previous approach in finding a global minimum. Another reason for this may be that the GA is better suited to MOOP than SOOP. Albeit, the Pareto optimization is better suited to find a global minimum more readily than the weighted sum approach.

5.2 INTERPRETATION OF RESULTS AND DECISION MAKER CHOICES

With these results, the choice of the final solution is dependent on the decision maker's interpretation of which objective is of more importance whether it is to obtain the maximum energy or to minimize the cost of investment.

CHAPTER 6 CONCLUSION AND RECOMMENDATIONS

6.1 CONCLUSION

This research sought to investigate the potential for increasing the energy obtained from a field of solar collectors at a reduced investment with the addition of columns of collectors with smaller dimensions compared to a single row of collectors with larger dimensions. Previous models were based on optimizing a single energy objective. An attempt at multi-objective optimization using game-theory is done in literature but a field area smaller than that applied in this research was considered.

In this research, a multi-objective optimization model is formulated for the optimal placement of rows and columns of solar collectors in a large field area. Two objectives are considered, namely, maximizing the energy collected and minimizing the investment/cost. Two approaches are used in this optimization: the weighted sum approach and finding the Pareto front.

The results from the weighted sum approach show that this multiple row, multiple column model produces 20.83% more energy when compared to previous models and using similar parameters. The formulated model also returns a fairly large value for the investment. However this investment is a small price to pay and can be recovered within a short time after the project is operational. Also when the investment is compared to previous models, the formulated model results in 0.7% less investment. This means that for an equal amount of investment, more energy can be obtained using the model proposed in this research. Conversely, for an equal amount of energy collected, the investment is reduced by using the model presented in this research. These results prove that the model presented in this research can be applied practically to produce more energy at less costs than previous models. The resulting number of rows and columns are acceptable along with the total energy and investment cost compared to previous models.

Pareto-optimal solutions are obtained from the Pareto optimization and make up the Pareto front. The results obtained from the Pareto optimization show that each solution that forms the Pareto front is better than the other solutions on the Pareto front in one objective but worse than the other solutions in the other objective. This means that no one solution dominates over the other solutions. A choice of which solution is to be implemented can only be made when more information is available.

Reviewing the results obtained from the weighted sum approach and that from the Pareto optimization, although the energy objective function converges to a better value using the weighted sum approach than the Pareto approach, but the costs obtained from the investment objectives is much less with the Pareto optimization than with the weighted sum approach. Thus it can be deduced that the Pareto optimization is better suited to find a global minimum more readily than the weighted sum approach.

The shortcomings of this model are:

1. In reality, the investment objective may involve some other costs not considered in this model. This however is not a major limitation of the model as the field and collector cost make up a huge part of the investment. Therefore the investment objective can be approximated as done in this formulated model.

2. The solar radiation data reported in this model is limited to the accuracy of the measuring instruments used.

3. The results are purely numerical and no experimental validation is reported as yet.

6.2 RECOMMENDATIONS

This work is an initial inquiry into multi-objective optimization and so has been limited to certain areas. It is necessary that future work on this topic should be extended to include the following areas.

1. The use of non-static or tracking solar collectors instead of fixed collectors.

2. Extra economic consideration like calculation of actual payback period.

3. Additional constraints especially the cost of other parameters not included here.

4. Increased number of objective functions such as optimizing for winter months could be added to the problem.

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APPENDIX A ADDITIONAL DATA

A.1 HOURLY SOLAR RADIATION VALUES FROM JANUARY TO DECEMBER 2011

The actual hourly solar radiation data for January and February 2011 is presented here. The data for March to December 2011 can be made available on request.

The hourly values are given in $watts/m^2$ for each month are as shown from tables A.1 to A.4. The average of these hourly values were calculated monthly, resulting in 12 typical days representing the whole year as shown in table 3.3. These average values were used in the simulation.

Table A.1: Table showing measured solar radiation for January (Watts/m^2)

Day	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12
1	0.02	0.02	0.01	0.03	0.04	15.33	110.58	208.76	673.89	859.06	1,007.18	1,087.18
2	0.02	0.03	0.02	0.03	0.04	6.63	55.35	402.43	539.69	810.9	930.67	829.18
3	0.03	0.02	0.03	0.05	0.03	5.36	36.96	100.25	246.59	285.6	335.52	483.34
4	0.02	0.03	0.02	0.04	0.04	12.3	69	110.1	283.97	539.29	1,102.50	1,036.16
5	0.04	0.02	0.02	0.06	0.03	1.11	49.05	154.95	201.55	148.58	234.91	137.41
6	0.05	0.02	0.04	0.05	0.04	8.15	62.79	192.89	347.95	385.73	878.2	721.34
7	0.03	0.02	0.02	0.05	0.03	9.74	85.04	391.58	637.69	860.91	974.23	676.62
8	0.04	0.02	0.02	0.03	0.04	2.88	59.03	180.58	161.55	193.46	528.51	731.57
9	0.03	0.02	0.01	0.02	0.03	5.64	123.99	467.04	307.01	519.55	658.79	406.84
10	0.02	0.01	0.01	0.01	0.06	6.72	57.09	410.75	562.33	518.51	854.63	1,109.81
11	0.02	0.03	0.01	0.04	0.04	9.39	86.42	266.5	603.51	767.32	957.98	1,086.78
12	0.02	0.03	0.01	0.02	0.02	6.35	63.42	367.69	568.58	664.09	993.87	783.38
13	0.02	0.02	0.03	0.01	0.04	7.02	76.56	324.55	454.19	457.7	809.66	572.98
14	0.02	0.02	0.02	0.01	0.03	2.03	36.97	215.35	487.19	492.39	739.76	863.1
15	0.03	0.03	0.02	0.05	0.06	2.46	49.68	353.45	589.15	816.4	898.89	975.18
16	0.02	0.02	0.03	0.02	0.04	7.61	52.98	400.59	390.01	600.82	1,013.35	1,000.59
17	0.02	0.03	0.04	0.03	0.02	3.01	47.15	169.49	199.32	311.1	850.93	566.42
18	0.03	0.03	0.02	0.03	0.02	8.96	83.41	82.92	98.75	107.59	250.23	130.1
19	0.02	0.02	0.02	0.04	0.04	9.21	71.4	299.09	519.28	648.76	652.44	588.89
20	0.02	0.02	0.02	0.03	0.06	1.02	9.6	58.19	227.48	352.24	569.33	460.74
21	0.04	0.02	0.02	0.03	0.03	6.51	43.76	76.16	90.55	104.54	155.46	252.23
22	0.03	0.02	0.03	0.04	0.05	0.75	73.5	177.95	337.82	525.04	679.79	508.43
23	0.03	0.02	0.03	0.04	0.04	3.58	64.07	303.01	578.52	649.17	907.29	475.28
24	0.01	0.02	0.03	0.03	0.03	0.57	53.4	295.1	341.4	326.12	286.17	424.1
25	0.02	0.03	0.04	0.02	0.02	0.12	20.27	45.12	43.55	176.25	405.8	592.43
26	0.03	0.03	0.05	0.02	0.03	3.11	47.96	204.78	186.2	348.37	396.16	437.86
27	0.03	0.04	0.02	0.03	0.04	0.14	15.48	23.2	84.1	211.39	185.45	472.77
28	0.04	0.03	0.02	0.03	0.04	2.03	45.75	296.77	443.82	660.34	938.41	815.55
29	0.02	0.03	0.02	0.04	0.05	0.46	42.77	70.03	154.94	304.84	429.01	557.13
30	0.03	0.03	0.04	0.02	0.03	1	30.76	98.25	252.67	314.49	250.39	629.83
31	0.02	0.03	0.03	0.03	0.04	0.21	17.3	79.06	133.84	294.03	339.21	448.6

Table A.2: Table showing measured solar radiation for January (Watts/m^2)

Day	12 - 13	13 - 14	14 - 15	15 - 16	16 - 17	17 - 18	18 - 19	19 - 20	20 - 21	21 - 22	22 - 23	23 - 24
1	1,067.81	806.39	772.18	801.8	614.04	150.41	28.07	0.1	0.05	0.02	0.03	0.04
2	953.3	803.59	789.09	437.82	214.12	107.89	18.71	0.05	0.03	0.02	0.03	0.03
3	491.39	643.05	382.95	358.83	354	130.83	42.68	0.37	0.03	0.03	0.03	0.03
4	634.07	651.14	467.23	162.05	24.46	51.32	9.82	0.08	0.03	0.03	0.02	0.03
5	224.35	178.89	153.86	169.6	90.17	46.9	16.73	0.13	0.04	0.02	0.04	0.04
6	632.81	671.37	857.55	627.94	429.18	141.89	10.23	0.16	0.02	0.03	0.03	0.04
7	936.9	835.07	867.97	517.35	459.73	171.66	14.97	0.16	0.02	0.04	0.03	0.04
8	848	665.74	721.41	677.71	544.2	180.61	36.34	0.31	0.01	0.03	0.02	0.03
9	634.87	474.23	462.17	73.44	87.06	88.74	15.34	0.06	0.03	0.03	0.03	0.04
10	897.2	780.28	1,015.84	617	455.71	156.34	31.43	0.28	0.03	0.02	0.03	0.02
11	1,132.73	857.84	886.81	296.94	337.48	58.11	23.41	0.29	0.03	0.04	0.03	0.02
12	1,059.76	604.16	570.75	164.45	198.55	117.36	21.32	0.19	0.02	0.03	0.02	0.02
13	320.75	749.38	869.46	741.14	260.79	30.99	50.11	0.18	0.02	0.03	0.03	0.02
14	824.29	895.61	812.23	468.52	130.5	124.34	29.16	0.12	0.02	0.02	0.03	0.02
15	758.03	855.38	643.37	379.2	235.92	120.36	44.74	0.14	0.04	0.02	0.03	0.02
16	1,022.00	770.21	437.51	67.23	55.54	132.13	25.97	1.37	0.03	0.02	0.02	0.02
17	647.59	610.93	657.77	631.5	209.67	38.66	6.82	0.04	0.03	0.02	0.03	0.03
18	707.7	610.93	690.38	186.14	102.82	186.93	40.18	0.13	0.04	0.04	0.03	0.04
19	773.24	1,008.68	730.35	689.78	225.54	84.77	9.12	0.04	0.02	0.04	0.04	0.02
20	919.08	634.53	369.63	275.64	106.08	43.15	12.22	0.63	0.03	0.03	0.03	0.02
21	446.63	477.76	168.01	378.64	276.22	172.33	26.88	0.06	0.03	0.04	0.03	0.04
22	884.12	487.3	633.44	282.67	601.6	237.4	15.3	0.1	0.02	0.02	0.03	0.05
23	1,004.53	617.61	573.58	755.29	347.06	148.75	37.37	0.04	0.03	0.03	0.03	0.03
24	539.31	590.19	635.58	761.24	534.14	104.5	23.65	0.07	0.02	0.02	0.03	0.04
25	847.05	797.53	804.69	619.2	345.89	227.69	34.35	0.06	0.02	0.03	0.01	0.02
26	804.41	680.44	557.68	120.88	11.35	11.54	3.46	2.18	0.03	0.03	0.04	0.03
27	383.19	448.86	592.15	513.92	408.96	246.29	38.36	0.28	0.01	0.02	0.03	0.03
28	763.57	772.71	891.23	733.79	444.73	82.42	12.06	2.92	0.04	0.02	0.03	0.02
29	841.3	742.62	826.57	641.88	308.2	256.71	20.5	0.21	0.03	0.03	0.03	0.02
30	733.22	393.74	373.39	438.71	348.19	175.19	36.42	0.32	0.02	0.03	0.01	0.04
31	781.86	919.28	949.41	728.49	503.85	260.3	27.45	0.09	0.03	0.02	0.02	0.03

Table A.3: Table showing measured solar radiation for February (Watts/m^2)

Day	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12
1	0.03	0.02	0.04	0.03	0.04	2.13	32.67	319.99	581.25	793.93	963.05	1,020.99
2	0.02	0.02	0.01	0.02	0.03	1.84	42.72	321.38	586.98	787.16	877.17	918.62
3	0.03	0.03	0.03	0.05	0.04	0.34	21.76	69.47	134.16	293.7	510.33	579.7
4	0.03	0.02	0.03	0.04	0.04	1.47	24.74	316.56	574.39	795.04	842.16	953.08
5	0.02	0.04	0.03	0.03	0.06	1.12	45.81	151.75	297.21	792.14	912.7	1,012.71
6	0.03	0.02	0.02	0.05	0.04	1.7	38.26	320.35	575.46	789.98	911.48	1,017.66
7	0.02	0.03	0.03	0.04	0.03	1.12	30.87	306.23	554.24	771.31	925.84	1,028.43
8	0.04	0.02	0.02	0.03	0.04	2.88	59.03	180.58	161.55	193.46	528.51	731.57
9	0.03	0.02	0.01	0.02	0.03	5.64	123.99	467.04	307.01	519.55	658.79	406.84
10	0.02	0.01	0.01	0.01	0.06	6.72	57.09	410.75	562.33	518.51	854.63	1,109.81
11	0.02	0.03	0.01	0.04	0.04	9.39	86.42	266.5	603.51	767.32	957.98	1,086.78
12	0.02	0.03	0.01	0.02	0.02	6.35	63.42	367.69	568.58	664.09	993.87	783.38
13	0.02	0.02	0.03	0.01	0.04	7.02	76.56	324.55	454.19	457.7	809.66	572.98
14	0.02	0.02	0.02	0.01	0.03	2.03	36.97	215.35	487.19	492.39	739.76	863.1
15	0.03	0.03	0.02	0.05	0.06	2.46	49.68	353.45	589.15	816.4	898.89	975.18
16	0.02	0.02	0.03	0.02	0.04	7.61	52.98	400.59	390.01	600.82	1,013.35	1,000.59
17	0.02	0.03	0.04	0.03	0.02	3.01	47.15	169.49	199.32	311.1	850.93	566.42
18	0.03	0.03	0.02	0.03	0.02	8.96	83.41	82.92	98.75	107.59	250.23	130.1
19	0.02	0.02	0.02	0.04	0.04	9.21	71.4	299.09	519.28	648.76	652.44	588.89
20	0.02	0.02	0.02	0.03	0.06	1.02	9.6	58.19	227.48	352.24	569.33	460.74
21	0.04	0.02	0.02	0.03	0.03	6.51	43.76	76.16	90.55	104.54	155.46	252.23
22	0.03	0.02	0.03	0.04	0.05	0.75	73.5	177.95	337.82	525.04	679.79	508.43
23	0.03	0.02	0.03	0.04	0.04	3.58	64.07	303.01	578.52	649.17	907.29	475.28
24	0.01	0.02	0.03	0.03	0.03	0.57	53.4	295.1	341.4	326.12	286.17	424.1
25	0.02	0.03	0.04	0.02	0.02	0.12	20.27	45.12	43.55	176.25	405.8	592.43
26	0.03	0.03	0.05	0.02	0.03	3.11	47.96	204.78	186.2	348.37	396.16	437.86
27	0.03	0.04	0.02	0.03	0.04	0.14	15.48	23.2	84.1	211.39	185.45	472.77
28	0.04	0.03	0.02	0.03	0.04	2.03	45.75	296.77	443.82	660.34	938.41	815.55
29	0.02	0.03	0.02	0.04	0.05	0.46	42.77	70.03	154.94	304.84	429.01	557.13
30	0.03	0.03	0.04	0.02	0.03	1	30.76	98.25	252.67	314.49	250.39	629.83
31	0.02	0.03	0.03	0.03	0.04	0.21	17.3	79.06	133.84	294.03	339.21	448.6

Table A.4: Table showing measured solar radiation for February (Watts/m^2)

Day	12 - 13	13 - 14	14 - 15	15 - 16	16 - 17	17 - 18	18 - 19	19 - 20	20 - 21	21 - 22	22 - 23	23 - 24
1	986.93	737.34	186.53	227.13	382.01	130.06	13.97	0.25	0.03	0.01	0.03	0.03
2	830.62	829.95	850.16	463.46	456.47	210.46	19.98	0.16	0.03	0.03	0.05	0.03
3	854.08	811.8	942.87	708.7	436.46	154.69	22.2	0.08	0.03	0.03	0.03	0.04
4	793.63	181.94	60.52	565.94	522.35	95.85	23.09	0.14	0.03	0.05	0.03	0.03
5	1,098.28	842.67	772.53	721.95	496.32	230.41	14.62	0.08	0.04	0.02	0.03	0.03
6	1,144.34	767.64	713.58	334.09	184.81	38.6	29	0.15	0.04	0.05	0.03	0.03
7	880.21	854.8	673.97	67.52	92.4	154.64	26.08	0.07	0.02	0.03	0.03	0.03
8	848	665.74	721.41	677.71	544.2	180.61	36.34	0.31	0.01	0.03	0.02	0.03
9	634.87	474.23	462.17	73.44	87.06	88.74	15.34	0.06	0.03	0.03	0.03	0.04
10	897.2	780.28	1,015.84	617	455.71	156.34	31.43	0.28	0.03	0.02	0.03	0.02
11	1,132.73	857.84	886.81	296.94	337.48	58.11	23.41	0.29	0.03	0.04	0.03	0.02
12	1,059.76	604.16	570.75	164.45	198.55	117.36	21.32	0.19	0.02	0.03	0.02	0.02
13	320.75	749.38	869.46	741.14	260.79	30.99	50.11	0.18	0.02	0.03	0.03	0.02
14	824.29	895.61	812.23	468.52	130.5	124.34	29.16	0.12	0.02	0.02	0.03	0.02
15	758.03	855.38	643.37	379.2	235.92	120.36	44.74	0.14	0.04	0.02	0.03	0.02
16	1,022.00	770.21	437.51	67.23	55.54	132.13	25.97	1.37	0.03	0.02	0.02	0.02
17	647.59	610.93	657.77	631.5	209.67	38.66	6.82	0.04	0.03	0.02	0.03	0.03
18	707.7	610.93	690.38	186.14	102.82	186.93	40.18	0.13	0.04	0.04	0.03	0.04
19	773.24	1,008.68	730.35	689.78	225.54	84.77	9.12	0.04	0.02	0.04	0.04	0.02
20	919.08	634.53	369.63	275.64	106.08	43.15	12.22	0.63	0.03	0.03	0.03	0.02
21	446.63	477.76	168.01	378.64	276.22	172.33	26.88	0.06	0.03	0.04	0.03	0.04
22	884.12	487.3	633.44	282.67	601.6	237.4	15.3	0.1	0.02	0.02	0.03	0.05
23	1,004.53	617.61	573.58	755.29	347.06	148.75	37.37	0.04	0.03	0.03	0.03	0.03
24	539.31	590.19	635.58	761.24	534.14	104.5	23.65	0.07	0.02	0.02	0.03	0.04
25	847.05	797.53	804.69	619.2	345.89	227.69	34.35	0.06	0.02	0.03	0.01	0.02
26	804.41	680.44	557.68	120.88	11.35	11.54	3.46	2.18	0.03	0.03	0.04	0.03
27	383.19	448.86	592.15	513.92	408.96	246.29	38.36	0.28	0.01	0.02	0.03	0.03
28	763.57	772.71	891.23	733.79	444.73	82.42	12.06	2.92	0.04	0.02	0.03	0.02
29	841.3	742.62	826.57	641.88	308.2	256.71	20.5	0.21	0.03	0.03	0.03	0.02
30	733.22	393.74	373.39	438.71	348.19	175.19	36.42	0.32	0.02	0.03	0.01	0.04
31	781.86	919.28	949.41	728.49	503.85	260.3	27.45	0.09	0.03	0.02	0.02	0.03