Finite element analysis of vibration models with interface conditions

by

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Title	Finite element analysis of vibration models with interface conditions
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Synopsis

We are concerned with vibration models with interface conditions. Due to interaction or damage, one is confronted by interface conditions or dynamical boundary conditions instead of classical boundary conditions.

For the analysis as well as implementation of the finite element method, the model problems must be written in variational form. We found the process more manageable if we start with the equations of motion and the constitutive equations. Consequently, we formulate each problem specifying the equations of motion and constitutive equations separately. For the sake of completeness and comparison with the literature, the models are also given in terms of the displacement, i.e. a partial differential equation with boundary and interface conditions. Due to the fact that our problems are not standard, it is necessary to discuss these aspects in some detail.

Model problems with interface conditions is a relatively new subject, and we could not find adequate derivations of the variational form in standard references. Hence we found it necessary to present rigorous derivations.

For the finite element analysis it is necessary to consider product spaces. The basis of the finite dimensional subspace for the Galerkin approximation consists of ordered pairs or triplets of functions instead of ordinary functions.

Our main concern is error analysis. As a result finite element interpolation had to be adapted for product spaces. A projection operator is defined and the approximation error derived from the interpolation error.

We consider three typical problems: Equilibrium problem, Eigenvalue problem and Vibration problem. In each case we show that the convergence theory can be adapted for product spaces.

We also present two case studies namely the model problems for a damaged beam and plate beam. The finite element method is used to find approximations for equilibrium problems, eigenvalue problems and vibration problems. The results show that the method is highly effective and that the errors correspond to the predictions of the theory.

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Samevatting

Die ondersoek handel oor vibrasie modelle met tussenvlakvoorwaardes. As gevolg van interaksie of skade ontstaan tussenvlakvoorwaardes of dinamiese randvoorwaardes in plaas van die klassieke randvoorwaardes.

Vir die analise sowel as implimentering van die eindige element metode, moet die wiskundige modelle in variasievorm geskryf word. Ons het gevind dat die proses meer hanteerbaar is as ons met die bewegingsvergelykings en die samestellingsvergelykings begin. Gevolglik het ons vir elke probleem die bewegingsvergelykings en die samestellingsvergelykings afsonderlik gegee. Vir volledigheid, en ook om met die literatuur te vergelyk, word elk van die modelle ook in terme van die verplasing gegee, naamlik 'n parsiële differensiaalvergelyking met randvoorwaardes en tussenvlakvoorwaardes. Aangesien die probleme nie standaard is nie, is dit nodig om van hierdie aspekte in besonderhede te bespreek.

Wiskundige modelle met tussenvlakvoorwaardes is 'n betreklik nuwe onderwerp en ons kon nie voldoende afleidings van die variasievorm in standaard bronne kry nie. Om hierdie rede het ons dit nodig geag om wiskundig korrekte afleidings aan te bied.

Produkruimtes is nodig vir die eindige element analise. Die basis van die eindig dimensionale deelruimte vir die Galerkin benadering bestaan uit geordende pare of geordende drietalle van funksies, in plaas van die gewone funksies.

Ons hoofbelangstelling is die foutanalise. Gevolglik moes eindige element interpolasie aangepas word vir produkruimtes. 'n Projeksie operator is gedefinieer en die benaderingsfout word afgelei in terme van die interpolasiefout.

Ons beskou drie tipiese probleme: Ewewigsprobleem, Eiewaardeprobleem en Vibrasieprobleem. In elk van die gevalle toon ons aan dat die konvergensieteorie aangepas kan word vir produkruimtes.

Ons bied ook twee gevalle studies aan, naamlik die wiskundige modelle vir die beskadigde balk en die plaat-balk. Die eindige element metode word gebruik om benaderings vir die ewewigsprobleem, eiewaardeprobleem en vibrasieprobleem te kry. Hierdie resultate toon dat die metode hoogs effektief is en dat die foute ooreenstem met die voorspellings uit die teorie.

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