

RELIABILITY MODELLING OF COMPLEX SYSTEMS

by

ALIFAS YEKO MWANGA

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## SUMMARY

Two well-known methods of improving the reliability of a system are

- (i) provision of redundant units, and
- (ii) repair maintenance.

In a redundant system more units are made available for performing the system function when fewer are required actually. There are two major types of redundancy - parallel and standby. In this thesis we are concerned with both these types.

Some of the typical assumptions made in the analysis of redundant systems are

1. the life time and the repair time distributions are assumed to be exponential
2. the repair rate is assumed to be constant
3. the repairman is assumed to be perfect, and hence go with only one repairman
4. the repair facility can take up a failed unit for repair at any time, if no other unit is undergoing repair
5. the system under consideration is needed all the time
6. usage of only conventional methods for the analysis of the estimated reliability of systems.



However, we frequently come across systems where one or more of these assumptions have to be dropped. This is the motivation for the detailed study of the models presented in this thesis.

In this thesis we present several models of redundant systems relaxing one or more of these assumptions simultaneously. More specifically it is a study of stochastic models of redundant repairable systems with non-exponential life time and repair times, varying repair rate, different types of repairmen, intermittent use and the use of time series in reliability modelling.

The thesis contains seven chapters. Chapter 1 is introductory in nature and contains a brief description of the mathematical techniques used in the analysis of redundant systems.

In chapter 2 assumption (1) is relaxed while studying two models with the assumption of life times and repair times to follow bivariate exponential distributions. Various operating characteristics have been obtained and the confidence limits have been established analytically for the system measure, availability for both the models.

Reliability analysis of a two unit standby system with varying repair rate is studied in chapter 3, by relaxing the assumption (2). In this chapter a similar study of chapter 2 is studied with assumption that the repair time distribution is generalised Erlangian.

Assumption (3) is relaxed in chapter 4, and we introduced two repairman (one regular repairman and the other expert repairman) to so that the system will be more efficient. The asymptotic confidence limits are obtained for the study state availability of such a system.

A three-unit system in which the "preparation time" is introduced, and hence

the assumption (4) is relaxed in this chapter 5. The difference-differential equations for the state probabilities are derived. The confidence limits for the steady state availability are obtained analytically and illustrated numerically.

In chapter 6, assumption (5) is relaxed. An intermittently used  $k$  out of  $n:F$  system with a single repair facility is considered with the assumption that failures will not be detected during a noneed period. Identifying regeneration points expressions are derived for the survivor function of the time to the first disappointment and the mean number of disappointments and the system recoveries in an interval. Expressions are also deduced for the stationary rate of occurrence of these events.

Chapter 7 presents an unconventional but powerful method for the analysis of the estimated reliability of systems constituted of subsystems (components) operating in series and/or in parallel under varying operational and environmental conditions. In this chapter assumption (vi) is relaxed. The proposed method construes the estimated reliability data as time series which are analysed using the well-known time series techniques.