Chapter 7

Conclusion

This chapter briefly summarizes the findings and contributions of this thesis, followed by a number of ideas for further research and analysis.

7.1 Summary

This study investigated the application of the unique properties of the PSO algorithm to solve multimodal optimization problems. As a result, two new niching techniques were developed. Chapter 3 presented an overview of existing GA–based niching techniques. It was concluded that although it is not impossible to convert existing GA–based techniques for use with the PSO, the genetic representation and generational replacement used by GAs make it hard to directly implemented GA–based techniques on the PSO. Chapter 3 also tested the potential of an existing PSO global optimization algorithm, 'Stretched'-PSO (SPSO). It was found that the algorithm's performance is impaired by modifications made to the search space.

Chapter 4 introduced the *nbest* PSO niching algorithm, specifically to solve systems of equations. The use of topological neighborhoods that promote local social interaction among particles was found to be a promising extension to the PSO algorithm. Chapter 5 introduced NichePSO, a niching algorithm that uses multiple particle swarms to maintain different solutions in a single search space. In Chapter 6, the performance of both *nbest* and NichePSO was compared to two well-known GA niching techniques, *deterministic* *crowding* and *sequential niching*. It was found that both of the newly introduced PSO niching algorithms were effective niching techniques, with performance comparable and better than the GA-based techniques.

7.2 Future Research and Analysis

A number of areas can be identified where the research in this thesis can be applied, or further investigated. These include the following:

nbest Neighborhood Formulation

The neighborhood formulation of the *nbest* algorithm showed to effectively find and maintain areas where very similar equations overlap. This property can be used in multi–objective optimization problems to locate and maintain Pareto fronts, as has been recently done by [38] with a similar algorithm.

Application to Global Optimization

The goal of niching techniques is to maintain diverse solutions in a search space. It remains to be seen whether this property of niching can be used to extend or enhance existing PSO diversity improvement techniques in a way that has not yet been investigated. Maintaining several solutions in different swarms and sharing information between the different swarms, may lead to improved global optimization algorithms (such an approach would seem similar to the island GA [34], and PSO subswarm techniques, such as work done by Løvbjerg *et al* [60]).

Further Investigation of NichePSO

- *Parameter Independence of NichePSO*: The current NichePSO implementation fails to correctly locate all solutions to a multimodal function if μ is greater than the inter-solution distance. The situation could be avoided by monitoring the effect of merging on swarm fitness. Ideally, swarm fitness should remain stable or improve. If particles from different potential solutions are merged, swarm fitness will be erratic until the swarm settles on one solution. Swarms may of course not settle at all, as several potential solutions would confuse it. Alternatively, a technique similar to that of Goldberg and Wang's CSN could be utilized to remove this parameter limitation [65].

- *Swarm Sizes*: In section 5.4.4 it was found that a^2, where a is the number of optima in a multimodal function, was a conservative boundary as to the number of particles required to locate all solutions. The accuracy of this estimate warrants further investigation, specifically when applying NichePSO to multimodal functions of higher dimensions. Functions of higher dimension exist in a much larger search space. Alternative velocity vector initialization techniques could also be investigated to see whether the number of particles required per solutions can be reduced.

Ensemble Neural Networks

Section 2.9.1 investigated work presented by a number of authors where particle swarms were used to train neural networks. Ensemble architectures train a number of neural networks, either sequentially or in parallel on the same problem. Since the search space of a neural network may be highly multimodal, the use of a niching technique may be beneficial. The PSO has been shown to be an effective optimization technique for neural network training, and it seems a natural step to exploit the nature of niching algorithms and apply it to ensemble learning.

Development of Further Niching Techniques

The usefulness of existing PSO diversity improvement techniques as precursors to niching need still be investigated. The crossover operator used by Løvbjerg *et al* [60] is triggered by randomly assigning a crossover probability to particles. If this assignment is changed to use a ranking scheme, only highly fit particles will be used for crossover. Although crossover between highly fit particles in a multimodal domain is not at all beneficial, the approach is similar to deterministic crowding [61].

Section 3.2 mentioned the notion of cannibalism within a species. When using an algorithm such as NichePSO to find multiple optima in a vastly multimodal function, a potentially large number of particles must be used to ensure consistent results. The use of a cannibalism operator, that removes particles where highly similar particles occur, may serve to simplify the interpretation of results.

Application of Niching Algorithms to Dynamic Clustering

Data clustering sorts data records into groups based on their similarity. The k-means algorithm is an example of a widely used algorithm. k-means however suffers from a major drawback: The number of clusters must be known in advance. In complex data sets, the number of clusters may not be known, or may be time dependent. For such problems, clustering algorithms that dynamically determine the number of clusters are needed. A cluster centroid represents a vector of the same dimension as data records in a data set, positioned on a location within the hyper-space defined by the data. If each particle in a swarm of particles represent a potential cluster centroid, centroids may be located by finding positions in the search space subject to the following conditions:

- The variability among data records associated with each centroid must be minimized.
- The variability among data centroids must be maximized.

Such a scheme could then dynamically determine the number of clusters in a dataset through the use of a niching algorithm such as NichePSO.