Finite element analysis of plate and beam models

by

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Submitted in partial fulfillment of the requirements for the degree

Philosophiae Doctor

in the Faculty of Natural and Agricultural Sciences

University of Pretoria

Pretoria

July 2006

DECLARATION

I, the undersigned, hereby declare that the thesis submitted herewith for the degree Philosophiae Doctor to the University of Pretoria contains my own, independent work and has not been submitted for any degree at any other university.

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Summary

We consider linear mathematical models for elastic plates and beams. To be specific, we consider the Euler-Bernoulli, Rayleigh and Timoshenko theories for beams and the Kirchhoff and Reissner-Mindlin theories for plates.

The theories mentioned above refer to the partial differential equations that model a beam or plate. The contact with other objects also need to be modelled. The equations that result are referred to as "interface conditions".

We consider three problems concerning interface conditions for plates and beams: A vertical slender structure on a resilient seating, the built in end of a beam and a plate-beam system.

The vertical structure may be modelled as a vertically mounted beam. However, the dynamics of the seating must be included in the model and this increases the complexity of a finite element analysis considerably. We show that the interface conditions and additional equations can be accommodated in the variational form and that the finite element method yields excellent results.

Although the Timoshenko model is considered to be better than the Euler-Bernoulli model, some authors do not agree that it is an improvement for the case of a cantilever beam. In a modal analysis of a two-dimensional beam model, we show that the Timoshenko model is not only better, but it provides good results when the beam is so short that one is reluctant to use beam theory at all.

In applications, structures consisting of linked systems of beams and plates are encountered. We consider a rectangular plate connected to two beams. Combining the Reissner-Mindlin plate model and the Timoshenko beam model can be seen as a first step towards a better model while still avoiding the complexity of a fully three-dimensional model. However, the modelling of

the plate-beam system is more complex than in the case of the classical theory and the mathematical analysis and numerical analysis present additional difficulties.

A weak variational form is derived for all the model problems. This is necessary to apply general existence and uniqueness results. It is also necessary to apply general convergence results and derive error bounds. The setting for the weak variational forms are product spaces. This is due to the complex nature of the model problems.

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