CHAPTER 3

THE THEORETICAL FOUNDATION OF PARTIAL EQUILIBRIUM MODELLING

3.1 INTRODUCTION

This chapter introduces the theoretical foundation for the structure and closure of an econometric regime-switching model within a partial equilibrium framework. The first section presents the domestic supply and demand components of a partial equilibrium model as they appear in the pre-existing BFAP sector model. The uniqueness of this study lies in the application of the regime-switching methodology in the modelling of a recursive closed system of equations. This regime-switching methodology was not applied in the previous version of the sector model. From a modeling perspective the technique that is used to "close" a simultaneous or recursive simulation model determines the manner in which market equilibrium is achieved in the model. Many different model closure techniques exist. The choice of closure technique will depend on the equilibrium pricing condition in a specific market, specifically on which market regime prevails in the market.

The concepts of model closure and price formation under various market regimes are introduced in the second section of this chapter in the form of a P-Q space and a flow diagram. This graphic depiction is complemented by a discussion of the underlying theory of the redesigned trade and price linkage components for the white and yellow maize and wheat industries. Together with the existing domestic demand and supply components, the trade and price linkage components complete the partial equilibrium framework. Finally, the modelling procedures, the estimation process and the validation of the models are discussed.

3.2 THE DOMESTIC DEMAND AND SUPPLY COMPONENTS OF THE EXISTING PARTIAL EQUILIBRIUM MODEL

3.2.1 DOMESTIC SUPPLY

Total domestic supply consists of production plus beginning stocks/inventory. Production is calculated as the total area harvested multiplied by the average yield.

3.2.1.1 PRODUCER SUPPLY

According to neo-classic theory, the producer is assumed to be a maximiser of profit or net returns, subject to some technical and institutional constraints. In this regard, economic theory suggests that the supply of products to the next highest level of the market channel depends on the expected profits accruing to the decision maker. Varian (1984) referred to the firm's production plan as the firm's technical constraints, which define the physical relationship between factor inputs and the maximum output level for the given technology, per unit of time. To illustrate this physical relationship between output and factor inputs, consider a farm that uses land -L, labour -W, and other inputs (fertiliser and capital) -K, in the production of the specific commodity.

Equation 3.1: $$Q = F(L, W, K)$$

If the input- and output prices are taken into account, let p denote the expected output price, *l* the rental cost for land L, *w* the cost of labour W, and *k* the cost of other inputs K. Assume that output and the output prices are independently distributed random variables and that the farmer is risk neutral. The objective of the farmer is to maximise profit, which is the difference between total revenue from the sale of outputs and the expenditure on all factor inputs. The farmer's profit function is algebraically defined as follows:

Equation 3.2: $$Max \prod = p\,Q - C(L, W, K)$$

thus

$$\prod(p, l, w, k, TFC) = Max \prod_{L,W,K} \left[pF(L, W, K) - lL - wW - kK - TFC \right]$$

The expected revenue is represented by pF(L,W,K), *l*L denotes the costs of land rental, *w*W represents the costs of labour, *k*K refers to the costs of capital and other inputs, and *TFC* is the total fixed costs. The profit maximisation or cost minimisation approach can now be used to derive the output supply response from the profit function by means of the first order conditions.

Dynamic relationships are particularly important in the modelling of supply and demand in the agricultural sector. Biological delays and cycles are inherent in the agricultural production process. In some cases producers base their decisions on expectations. Time may be introduced explicitly in supply functions in several ways. The two most common approaches are the partial adjustment approach and the adaptive expectations approach. These are two distinct approaches for the specification of dynamic output supply response.

The partial adjustment methodology is based on the assumption that movements from the current level of supply and demand to new equilibrium levels consequent to changes in economic or technical conditions may not be instantaneous. The partial adjustment model is commonly used to model the gradual adjustment of agricultural producers to changes within the total production environment (Sadoulet and de Janvry, 1995). The partial adjustment model is based on the principle that the change in a variable, for example supply (S) from one period to the next, can be expressed as some portion of the difference between the current level of supply and the desired level of supply. In other words, in each period actual output is adjusted in proportion to the difference between the output desired in the long-run equilibrium and the actual output. This can be illustrated as follows:

Equation 3.3:

$$S_t - S_{t-1} = \delta \, (S_t^* - S_{t-1}) + u_t$$

or

$$S_t = (1-\delta)\, S_{t-1} + \delta \, S_t^* + u_t$$

S_t^* denotes the desired long-run equilibrium level of output, S_t represents the current level of output, and S_{t-1} signifies the level of output from the previous year. δ is an adjustment factor with a numerical value of between 0 and 1. If δ =1, then a complete adjustment in the level of output has taken place from the previous period to the current period. However, if δ =0, then no adjustment has taken place and $S_t^* = S_{t-1}$.

However, the problem with equation 3.3 is that it cannot be estimated since the long-run equilibrium output level, S_t^*, is unobservable. This level of output needs to be estimated as a function of some observed variable. For simplicity, assume the following relationship:

Equation 3.4: $$S_t^* = \alpha + \beta P_t^e$$

Equation 3.4 can now be substituted back into equation 3.3 and the result can be presented as follows:

Equation 3.5: $$S_t = \alpha\delta + (1-\delta) S_{t-1} + \delta\beta P_{t-1} + u_t$$

The adjustment coefficient (δ) can now be used to calculate a short- and long-term price effect. The short-term price effect is the estimated coefficient of the price variable ($\delta\beta$) and the long-term price effect (β) is obtained by dividing the short-term price effect by the adjustment coefficient. From these price effects, short- and long-term price elasticities can be calculated.

Adaptive expectation models are based on the assumption that agricultural producers base their decisions on certain expectations regarding the future values of relevant prices. Hence, cropping decisions are based on the expected prices at the time of harvest.

Equation 3.6: $$S_t = \alpha + \beta P_t^e + u_t$$

S_t denotes the current level of output and P_t^e represents the expected price prevailing at time t. In the adaptive expectation model prices of the previous period prevail and expectations are revised each period, with the revision proportional to the error in the previous expectations. This revision can be presented as follows:

Equation 3.7: $$P_t^e - P_{t-1}^e = \gamma (P_{t-1} - P_{t-1}^e)$$

or

$$P_t^e = \gamma\, P_{t-1} + (1-\lambda)\, P_{t-1}^e$$

Equation 3.7 illustrates the revision for period t. γ is called the coefficient of expectation. If γ=0, then the actual prices will have no effect on the expected prices, and if γ=1, then expected prices will be equal to the last period's actual prices. This implies that the actual prices of the previous period have prevailed perfectly. The expected price at time t can now be expressed as a function of previous actual prices over a longer period of time.

Equation 3.8: $$P_t^e = \gamma\, P_{t-1} + (1-\lambda)\, P_{t-2} + \gamma(1-\lambda)^2\, P_{t-3} + \gamma(1-\lambda)^3\, P_{t-4} \ldots\ldots$$

Equation 3.8 shows that producers base their price expectations solely on an extrapolation of past prices.

In the Nerlovian supply model the partial adjustment model and the adaptive expectation model are combined. The Koyck transformation is used to obtain the final form of the equation. In its simplest form, the model assumes that there is a desired level of supply (S_t^*), which depends on an expected price level (P_t^e). Algebraically, it can be presented as follows:

Equation 3.9: $$S_t^* = \alpha + \beta P_t^e$$

Furthermore, it is also assumed that actual supply, S, adjusts towards the desired level according to the partial adjustment model (equation 3.5) and the adaptive expectations model (equation 3.8) is used to determine the expectations regarding the prices.

Equation 3.10: $$S_t = (1-\partial)\, S_{t-1} + \partial\, S_t^* + u_t$$

Equation 3.11: $$P_t^e = \gamma\, P_{t-1} + (1-\lambda)\, P_{t-1}^e$$

The first step is to substitute S_t^* into S_t. This will yield the following equation:

Equation 3.12: $$S_t = \alpha\delta + (1-\delta)\, S_{t-1} + \delta\beta\, P_{t-1}^e + u_t$$

The second step is to substitute equation 3.11 into equation 3.12. This substitution is presented in equation 3.13:

Equation 3.13: $$S_t = \alpha\delta + (1-\delta)\,S_{t-1} + \delta\beta\left[P_{t-1} + (1-\gamma)P_{t-2} + ...\right] + u_t$$

One can argue that both the partial adjustment model and the adaptive expectation model can be applied in the South African grain market. However, careful analysis and discussions with industry experts suggest that the adaptive expectation approach might be more relevant under the current free market conditions and that the partial adjustment approach was the correct approach to use under the regulated market environment. The reason for this is simply because farmers make increasing use of the future market and base their production decisions on expected prices. Equation 3.8 shows that producers base their price expectations solely on an extrapolation of past prices.

In the existing sector model total producer supply is derived from area harvested multiplied by yield. The producer has to make the initial decision on the size of the area to be planted. Due to the unavailability of data on area planted, it has been common practice to begin crop modelling with area harvested, since area harvested is normally a good proxy for the area planted. Using the area harvested in the determination of potential supply does, however, also have some problems, as the total area planted is not always harvested. In South Africa, there has traditionally been little difference between the area planted and the area harvested and the differences that do occur appear randomly.

A feature of the existing model is that all the supply equations are driven by expected gross return-type variables. For each commodity the real expected gross return is calculated as the trend yield per hectare multiplied by the expected price and deflated by the consumer price index for food products. The total grain area harvested ($TGAH_t$) is estimated as a function of the weighted sum of the all the crops' expected real gross returns ($EGRT_{All}$), rainfall (R_t) and the price of inputs ($P_{I,t}$). Gross returns are weighted according to each crop's share of the total area harvested. The total acreage response function can be presented as follows:

Equation 3.14: $$TGAH_t = f(EGRT_{All}, R_t, P_{It})$$

The area harvested for each crop is expressed as a share ($AHSH_t$) of the total area harvested and estimated as a function of the expected real gross returns of the own crop divided by the sum of expected real gross returns for the rest of the crops. A typical grain acreage share response function, specified according to the Nerlovian approach, can be postulated as:

Equation 3.15: $$AHSH_t = f(\frac{EGRT_t}{SUM(EGRT_t)})$$

Equation 3.16 shows how the acreage share for each commodity is multiplied by the total area harvested to calculate the area harvested for each crop.

Equation 3.16: $$GAH_t = AHSH_t * TGAH_t$$

This methodology is applied to six crops in the existing model, namely white maize, yellow maize, wheat, sorghum, sunflower and soybeans. This approach has major advantages for estimating the substitution effect between various crops and the usefulness of this approach is illustrated in chapter 5 with the calculation of a supply elasticity matrix for all the crops.

After the producer has decided to plant, the yield, which is also influenced by weather conditions, will determine the total production of the crop. Equation 3.17 relates yield to rainfall and a trend variable. It is argued that in many cases farmers increase inputs, for instance fertiliser, as the output prices increase. However, empirical evidence suggests that in the case of South Africa yields are not a function of the expected output prices. Different regions produce maize and wheat, and maize is a summer crop and wheat is a winter crop. Therefore, the rainfall variables used in the model reflect the regions and specific months that influence the area planted and production of each crop. Typically, rainfall from October to December influences the decision on the maize area and rainfall from December to March influences maize production. For wheat these two periods are April – July and July – October, respectively. The inclusion of a trend variable can be motivated by the rapid improvement in technology that has occurred over the past decade.

Equation 3.17: $YIELD = f(RAIN, TREND)$

Finally, producer supply (domestic production) can be expressed as follows:

Equation 3.18: $PROD = GAH_t * YIELD_t$

3.2.1.2 BEGINNING STOCKS

In this study, ending stocks are modelled as a behavioural equation and, therefore, beginning stocks equal lagged ending stocks. Ending stocks are discussed in section 3.2.2.3.

3.2.2 DOMESTIC DEMAND

The "law of demand" states that the higher the price, the less of a given good will be purchased (Ferris, 1998). This implies that the demand curve is downward sloping. For the ultimate buyer of food, demand could relate retail prices to amounts that will actually be consumed within a given time frame. However, the final consumer is not the only actor on the demand side. We can distinguish between two main categories of domestic demand, namely demand for direct use and inventory demand. The demand for direct use consists of primary as well as derived demand. Primary demand is the demand at a retail level where the individual consumer can make decisions based on price and preference. Derived demand can also be referred to as intermediate demand, for example the demand of wheat for baking bread or the demand for grain as a livestock feed. Inventory demand strongly reflects expectations and consists of the demand for storage and the demand for speculation. Expectations are determined by expected utilisation, product availability, market prices and factors such as agricultural policies.

In the demand block, human consumption, feed and seed consumption, exports, and ending stocks determine the total demand for South African maize and wheat. White maize and wheat are mainly utilised in the human consumption market, while yellow maize is mainly consumed in the feed market. The data that report on seed use are unreliable. As a result, two categories, viz. human and feed consumption, are estimated by means of behavioural equations. Seed use is included as an exogenous variable in the calculation of total demand.

3.2.2.1 CONSUMER DEMAND

To enable the derivation of the consumer demand function we have to assume that the consumer has a rational, continuous, and locally non-satiated preference relation, and we take $U(x)$ to be a continuous utility function representing these preferences (Mas-Colell, Whinston, and Green, 1995). Suppose the consumer is faced with the problem of choosing a bundle of goods in order to maximise his or her utility subject to given prices and the level of income. Hence, the consumer will purchase a combination of goods, which will provide him with the highest level of satisfaction. This is also referred to as "the rational behaviour hypothesis". The utility maximisation problem can be presented mathematically as follows:

Equation 3.19:

$$MAX\ U\,(x_1, x_2, ...x_n)$$
$$subject\ to$$
$$m = \sum_{i=1}^{n} p_i x_i$$

$U\,(x_1, x_2, ...x_n)$ is the consumer's utility function. $m = \sum_{i=1}^{n} p_i x_i$ represents the budget constraint and consists of m, the consumer's total available budget and p_i, the unit price of commodity x_i. The utility function is a strictly quasi-concave and twice differentiable (Mas-Colell *et al*, 1995). This problem is solved through the use of the Lagrange Multiplier. This method starts by defining an auxiliary function known as the Lagrangian.

Equation 3.20:

$$L = U(x_1, x_2, ..., x_n) - \lambda\,(\Sigma\, p_i x_i - m)$$

The new variable, λ, is called the Lagrange Multiplier since it is multiplied by the budget constraint. According to the Lagrange theorem an optimal choice or utility maximisation must satisfy the First Order Condition (FOC), which involves the partial derivation of equation 3.20 with respect to x_i and λ.

Equation 3.21:

$$\frac{\partial L}{\partial x_i} = \frac{\partial U(x_i)}{\partial x_i} - \lambda p_i = 0 \quad \text{with } i = 1,2,.....n.$$

Equation 3.22: $$\frac{\partial L}{\partial \lambda} = (\Sigma\, p_i x_i - m) = 0$$

The FOC simply sets the derivatives of the Lagrangian with respect to x_i and λ each equal to zero. Hence, equation 3.21 is merely the budget constraint that is set equal to zero. Solving the (n+1) FOC equations we can show that λ is equal to marginal utility divided by price for all commodities, which indicates the increased rate of satisfaction derived from spending an additional rand on a particular commodity. The Lagrange Multiplier can thus be interpreted as the marginal utility of income.

The simultaneous solution of equation 3.21 and equation 3.22 yields the demand function of x_i, which is an implicit function of own prices, the prices of complementary or substitute goods, and consumer income. The demand function of x_i can be presented as follows:

Equation 3.23: $$x_i = x_i\,(p_1, p_2, ..., p_i, m), \quad i = 1,2..., n$$

This demand function represents the demand for x_i of every individual consumer and is homogeneous of degree zero in prices and income. The aggregated retail demand for x_i is calculated by multiplying the individual demand for x_i by the number of consumers in the market. In this study total human consumption is divided by the total population to obtain the *per capita* consumption of maize and wheat. *Per capita* consumption is estimated as follows:

Equation 3.24: $$PCC_t = f\,(P_{G,t}, P_{s,t}, INC_t, G)$$

PCC_t denotes the *per capita* consumption in period *t*, $P_{D\ t}$ denotes the domestic price of the grain, $P_{s\ t}$ denotes the price of a range of commodities that can be used as substitute or as complementary products in the human market, *INC* denotes the level of disposable income *per capita*, and *G* denotes government policies. The existing model structure consists of human consumption equations for white maize, yellow maize, wheat and sorghum. It is important to note that for food demand, symmetry does not hold for Marshallian equations (like the ones estimated in the model), but rather for Hicksian responses representing pure substitution effects holding utility constant.

3.2.2.2 FEED DEMAND

The demand for grain in the feed sector is derived from the profit maximisation condition of the livestock sector. Yellow maize is the dominant feed grain in South Africa by far. White maize and wheat can be regarded as a substitute product for yellow maize. For the sake of simplicity, assume that the quantity of livestock production is a function of the quantity of white maize, yellow maize and wheat. The livestock production function can thus be represented as follows:

Equation 3.25: $$Q_L = f(Q_{WM}, Q_{YM}, Q_{WH})$$

where Q_L denotes the production of livestock products and Q_{WM}, Q_{YM} and Q_{WH} represent the quantities of white maize, yellow maize and wheat utilised as feed in the feed market.

For example, the derived demand of yellow maize in the feed market can now be determined in a similar fashion as the derived demand for x_i in equation 3.21. By setting the FOC equal to zero and solving the system of equations simultaneously, the following derived demand function for yellow maize can be determined.

Equation 3.26: $$Q_{Feed} = g_1(P_L, P_G, P_S)$$

Therefore, the derived demand for white maize, yellow maize and wheat in the feed sector is a function of the price of the livestock product (P_L), the own price (P_G), and the price of the substitute commodities (P_S).

In the existing model structure the feed demand equations are taken one step further by linking the feed grain demand to the level of livestock production by means of a weighted total feed demand. The weighted total feed demand is derived from the level of livestock production and the inclusion rate of grains in the various feed rations and is expressed in tons. Feed grain consumption is, therefore, estimated as a function of the weighted total feed demand (*TFD*), the own price of grain (P_G), and the price of the substitute feed grains (P_S).

Equation 3.27: $$Q_{Feed} = f(TFD, P_G, P_S)$$

3.2.2.3 ENDING STOCKS/ INVENTORY

Due to the biological nature of agricultural production, many agricultural products are supplied to the market only at one specific period during a year, whereas consumption occurs throughout the whole year. Since inventories provide a constant supply of products throughout the year, they are an important component in the commodity models and play a decisive role in determining the prices of mainly agricultural goods where production and consumption are relatively inelastic. Bressler and King (1970) identified three motives for holding stock: transaction demand, precautionary demand and speculative demand.

Transaction and precautionary demand are related to domestic demand and supply. Transaction demand specifies that the level of stock is a fraction of the current production. A higher (lower) level of production implies that inventories should rise (decrease). The precautionary demand can also be referred to as the "buffer stock". In the case of maize and wheat when the marketing boards were still functioning, they retained a buffer stock to deal with uncertainties in the local food balance sheet, which could potentially occur due to unknown and unexpected demand and supply shocks. This buffer stock, also referred to as the "Joseph Rule", was sufficient to satisfy the demand for each commodity over a period of three months. Even in the absence of government policies in the deregulated market, the market will usually hold at least some grain for transaction and precautionary reasons. Whereas transaction demand is specified as a fraction of varying production, precautionary demand is usually treated as a constant. Simplistically, the first two reasons for holding stock can be presented as follows:

Equation 3.28: $$S_t = \omega_1 + \omega_2 Q_t$$

Q_t is the total production in period t, ω_2 represents the marginal fraction of production stored and ω_1 denotes a constant level of precautionary stocks.

The final reason for holding stock is speculative. It is assumed that stock operators are rational decision-makers. Due to market uncertainty, storage operators hold stock and position[1] themselves in the market so that they are able to benefit from future market conditions. Speculative demand for stocks is thus based on expected prices in the next period t+1. Hence, expected prices also need to be included in the specification of stock behaviour. In summary, speculative commodity stock holdings can be specified as follows:

Equation 3.29: $$S_t = f(S_{t-1}, Q_t, P_{t+1})$$

In equation 3.29 stock holdings are expressed as a function of beginning stock[2], the expected price in the next period and current production. In the current sector model ending stocks are estimated as follows:

Equation 3.30: $$ENDS_t = f(ENDS_{t-1}, PROD_t, P_{D,t})$$

Ending stocks in period t depend on the beginning stocks in period t, local production and the market price. Ending stocks in period t are equal to the beginning stocks for period t+1. In the flow diagram (figure 3.1 and 3.2), a dotted line is used to denote the lagged effect between ending stocks in period t and beginning stocks in period t+1.

Since the deregulation of the markets, speculative stock holding has become a major factor in the South African grain market. The level of uncertainty surrounding speculative stocks has increased with the increasing popularity of on-farm storage facilities. In many cases farmers base their expectancy of higher prices on the seasonal nature of agricultural production. Many agricultural products trace out fairly definable and consistent seasonal patterns. This is primarily due to the seasonal nature of agricultural production, but may also relate to seasonal demand factors. In the case of grains, it is generally expected that prices are at their lowest level at harvest time and increase as time passes. Hence, farmers and storage operators tend to carry stocks at harvest time and sell the grain at a later stage. Opportunity costs also play a major part in stock holding behaviour. Once opportunity costs are perceived as being too high, stock holders will consider selling their grain, even if prices have not increased.

[1] Storage operators hedge their positions in the market by making use of future markets
[2] Beginning stock is equal to the ending stock of the previous year

3.3 MODEL CLOSURE AND PRICE FORMATION UNDER SWITCHING MARKET REGIMES

The previous section presented the theoretical foundation of domestic supply and demand components of the previous sector model. However, a partial equilibrium model also consists of trade and price components. These are the components that have to be redesigned for the new regime-switching model. Price and trade are instrumental for a model to reach equilibrium. In an equilibrium framework, total demand has to equal total supply. The technique that is used to “close” a recursive simulation model determines the manner in which market equilibrium is achieved in the model. Many different model closure techniques exist. The choice of technique will depend on the equilibrium pricing condition in a specific market, specifically on which market regime prevails in the market. This section makes use of flow and price-quantity (P-Q) diagrams to provide easy guidance towards the understanding of important economic and biological relationships. These diagrams also distinguish between the model closures under different market regimes. This discussion leads to the theoretical foundation of the trade and price components of an equilibrium model.

3.3.1 THE FLOW DIAGRAM AND THE PRICE QUANTITY (P-Q) DIAGRAM

Flow diagrams portray the elements of the supply, demand, trade, and price linkage blocks and the relationship between them. The supply block consists of the function determining total area harvested, yield, production and beginning stocks/inventory. The demand block consists of human, feed and seed consumption, and ending stock.

Figure 3.1 and 3.2 show the flow of a typical grain, like maize or wheat, through the market channel from the producer to the ultimate consumer of the product. While the model cannot replicate all the decisions occurring within the industry, the major behavioural relationships are captured. The dashed lines represent lagged relationships between variables. As explained in the first section of this chapter, the farmers’ decision to plant is influenced by the lagged price of the product, the weather, and the lagged price of substitute products and inputs. Yield is also influenced by the weather. Beginning stocks equal the ending stocks of the previous season. The current price influences domestic consumption and ending stocks.

Figure 3.1 illustrates model closure and, therefore, the equilibrium pricing condition under near-autarky. Strictly speaking, under the definition of autarky no trade takes place as domestic prices trade at levels where no arbitrage for trade is triggered. However, as previously explained, in the South African white and yellow maize market some level of trade does occur with neighbouring countries at price levels which suggest that the market is trading under a type of regional autarky (in this study referred to as "near-autarky") isolated from world markets. Since significant trade occurs under near-autarky, experts argue that although domestic prices are mainly determined by domestic demand and supply, trade does have an impact on the domestic equilibrium price. Because net trade is modelled as a function of the world price and the exchange rate, these variables subsequently have an impact on the domestic price. The two-directional arrow between net trade and the domestic price illustrates this point. The block arrows versus the two-directional line arrow make a clear distinction between the impacts of domestic supply and demand, and trade respectively.

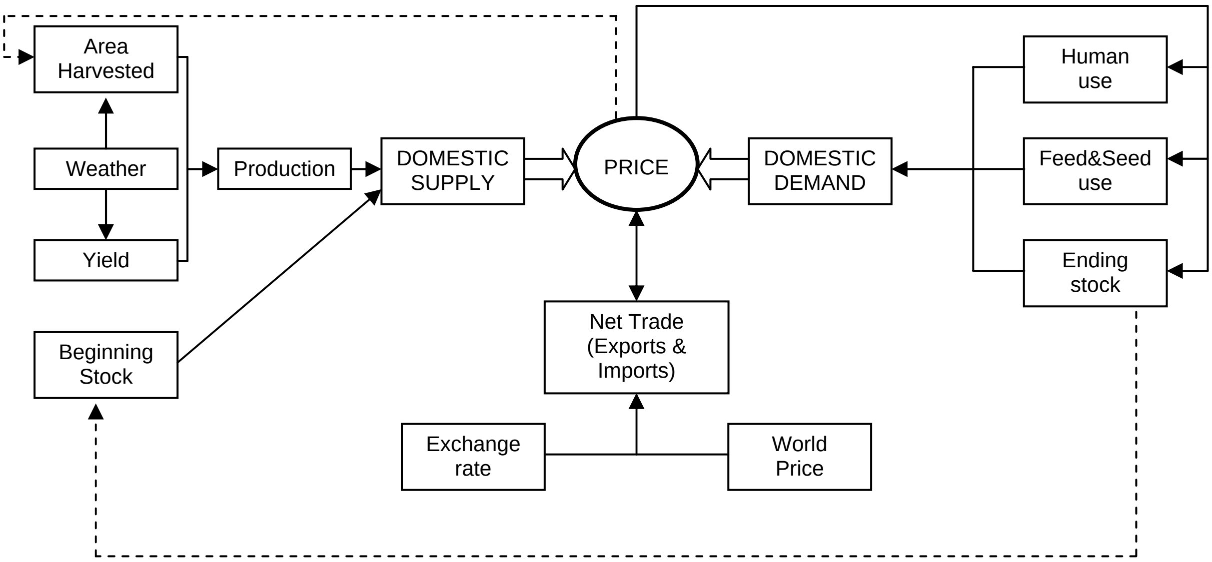

Figure 3.1: Flow diagram of SA grain market in near-autarky

In this type of model the equilibrium price is simulated by setting demand equal to supply in a price equilibrator framework. Price is thus solved endogenously in the domestic market and not as an endogenous variable in a behavioural equation.

Figure 3.2 represents model closure under an import parity or export parity regime. Under the import and export parity regimes, the domestic price is modelled as a function of the import and export parity price respectively and can, therefore, be regarded as predetermined in the system of equations. The exchange rate is factored into these prices. This is also referred to as the price linkage equation. Thus, under this trade regime it can be expected that the correlation between world prices, exchange rate and domestic prices is high and the market should thus be integrated into the world market. If the estimated coefficients of the price linkage equations are equal to one, then the law of one price holds. Net trade (either net exports or net imports) is used to close the model in the form of an identity. Block arrows show how domestic demand and supply determine the level of trade.

The domestic price is also influenced by the level of trade. This is contrary to what particular applications of economic theory suggests for a small, open economy trading in the world market, but industry experts are of the opinion that in the South African market exports to neighbouring countries also have an impact on the domestic price. It is important to note that whereas South Africa can be regarded as a large nation in the Southern African region, it is a small nation with respect to the world. Three possible motivations for trade affecting prices are, firstly, the regional issues as discussed, secondly, the possibility of transaction costs rising as quantities increase, and thirdly, goods may not be perfect substitutes, so a wider price gap is required to encourage the movement of products across borders.

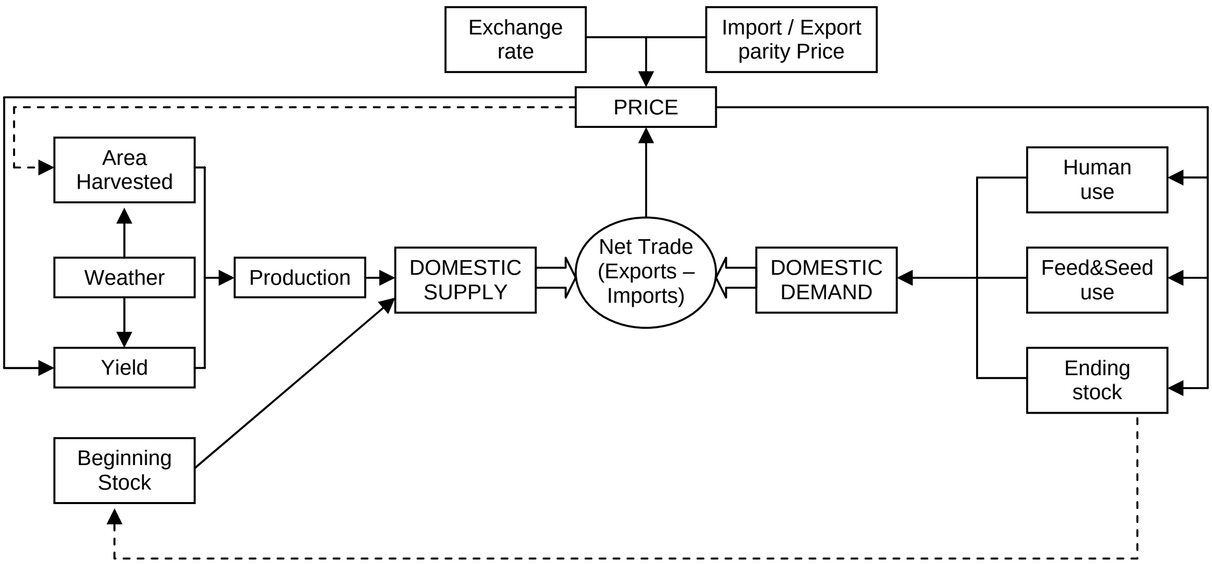

Figure 3.2: Flow diagram of a typical grain market in net export or net import parity

The P-Q diagram (figure 3.3) and the flow diagram are closely related. The P-Q diagram reflects the different layers of the market. The P-Q diagram also consists of the supply and demand blocks. The supply block consists of the functions that determine total area harvested, average yield, production, and beginning stocks. The sum of these components equals domestic supply. The demand block consists of human and feed consumption, ending stocks, and net trade. It is important to note that the P-Q diagram depicts the economic relationships among the dependent and explanatory variables at different layers in the commodity markets, for example the production layer, the consumption layer, and the trade layer. In addition to the relationship between own-price and quantities, the impacts of other variables are depicted by means of arrows (shifters). A rightward shifter is used to explain a positive relationship between the dependent and independent variable, i.e. the expected sign of the parameter associated with the variable in the estimated equation is positive. A negative sign is expected for a leftward shifter.

The P-Q diagram is constructed according to scale and illustrates the price elasticities at the different layers in the market. This implies that the various sections all add up to the equilibrium market condition. The area harvested is perfectly inelastic (vertical line) towards the current price because it is a function of the lagged price. It is expected that there is no relationship between yield and price and therefore yield is vertical (perfectly inelastic) with respect to the current price. It is expected that production of agricultural products is inelastic due to the seasonality and the biological nature of production. Once you have planted, the level of production is mainly determined by the weather. Production and the beginning stocks equal the total domestic supply (oa + ob = oc). Beginning stocks are equal to ending stocks and are thus estimated on the demand side of the model.

Human and feed consumption are both downward sloping. A positive relationship between income, population and human consumption is expected. Feed consumption is positively related to the feed index, which is derived from the size of livestock operations. Ending stocks are downward sloping, which indicates the negative relationship between ending stocks and prices, as discussed in the first section of this chapter.

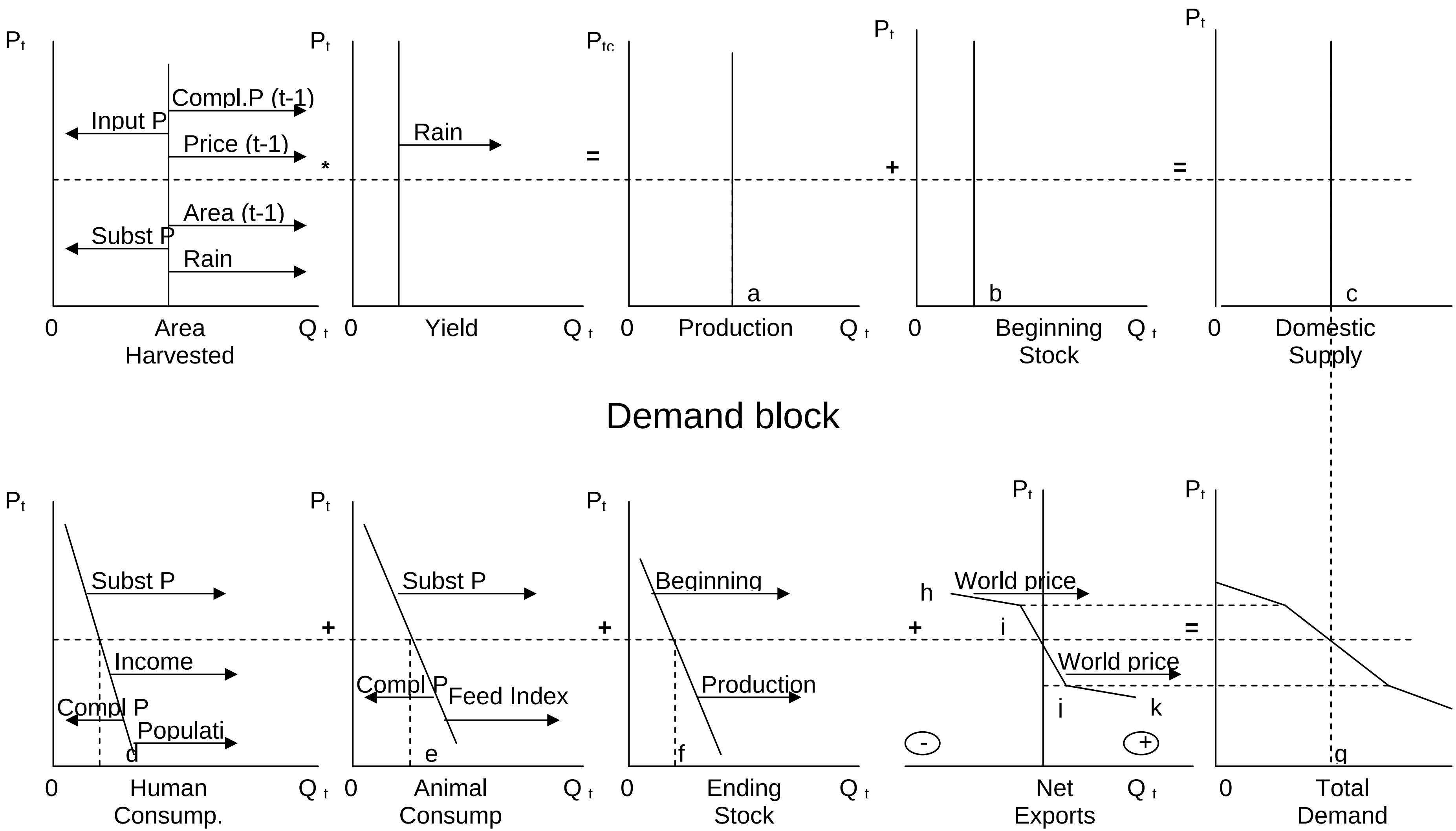

Figure 3.3: P-Q diagram for three different trade regimes.

Taking the objectives of this study into account, the most important graph in the P-Q diagram is the one for net exports. All three regimes are captured in this graph, with *hi* representing the demand for imports (negative net export demand) under an import parity regime, *ij* representing some level of negative and positive net trade under near-autarky, and *jk* representing the demand for exports under an export parity regime.

The essence of this graph lies in the portrayal of the price elasticities under the various market regimes. Under true autarky, *ij* should be vertical and thus perfectly price inelastic. However, in the South African markets some trade still occurs under near-autarky conditions and consequently the domestic price has an impact on the net trade position. As one moves from near-autarky to import parity or export parity, the elasticity increases sharply to become almost infinitely elastic. From the above discussion it becomes clear that the relationship between world market prices, trade and domestic prices varies in the case of discontinuous trade, consequently changing the model closure technique. To distinguish clearly between the various market regimes, trade and price equations have to be estimated independently for each regime..

In the case of a small nation, it is expected that the demand for imports and supply of exports are infinitely elastic towards the domestic price because the domestic market is integrated with the international market and any change in the net trade position of the small country has no effect on the world price. The elasticity of the net export demand equation depends on the domestic demand and supply elasticities. Lower elasticities will induce larger internal price changes.

A rightward shift is used to illustrate the relationship between net trade and the world price. In the case of net imports (negative net exports), a rightward shift implies a decrease in imports, illustrating the negative relationship between imports and the world price. In the case of exports, the rightward shift implies a positive relationship between the world price and the level of exports. A shift in the world price can almost be seen as a vertical shift since higher world prices increase the export and import parity prices.

Total demand equals the sum of domestic consumption, ending stocks and net trade (od + oe + of = og). In this P-Q diagram, market equilibrium is reached in the range between the import parity and export parity price where no trade occurs. Any increase (decrease) in the domestic price would trigger the demand for imports (exports), which would then have to be deducted from (added to) the total demand.

3.3.2 THE TRADE AND PRICE LINKAGE COMPONENTS UNDER SWITCHING MARKET REGIMES

From the above discussion it becomes clear that the relationship between world market prices, trade and domestic prices varies in the case of discontinuous trade, consequently changing the model closure technique. To distinguish clearly between the various market regimes, trade and price equations have to be estimated independently for each regime. The underlying methodology of these behavioural equations and identities is based on the principles that were explained by the flow diagram and the P-Q space.

3.3.2.1 NEAR-AUTARKY

When the market is in autarky, prices are used to close the model. They are solved endogenously by means of a price equilibrator. The equilibrator is based on the principle that net export demand must equal export supply. Net export demand is estimated as a function of domestic and world prices, and domestic production and consumption. The inclusion of production and consumption into the net export equation could create problems with simultaneity since all that is lacking for this equation to be an identity is the change in ending stocks. However, in some of the South African grain markets (for instance, white maize) this specification can be justified as the key decision is whether to store or export surplus production. It is important to note that one can expect the world price not to matter very much because the main factors causing some limited trade under near-autarky are more regionally demand driven by the factors mentioned above than by price movements in the world and domestic markets.

In equation 3.31 the level of net export demand is defined as a function relating the quantity of net export demand ($NEXD_t$) to the ratio of the domestic price ($P_{D,t}$) over the average of the import ($P_{IP,t}$) and export parity price ($P_{EP,t}$), and the local grain production ($PROD_t$) – consumption ratio ($CONS_t$). The exchange rate, transaction costs and government trade policies are already factored into the import and export parity price calculations[1]. According to the definition of autarky, domestic prices are expected to fluctuate between import and export parity prices and, therefore, the average of these two price levels is applied in this equation.

Equation 3.31: $$NEXD_t = f(\frac{P_t}{Avg(P_{IP,t} \& P_{EP,t})}, \frac{PROD_t}{CONS_t}, e_t)$$

Export supply EXS_t is calculated in the form of an identity

Equation 3.32: $$EXS_t = PROD_t - CONS_t - (BEGS_t - ENDS_t)$$

In order to set up the price equilibrator, the difference between $NEXD_t$ and EXS_t, due to market disequilibria, is calculated. The new market clearing price is simulated by linking the old market price to the difference between $NEXD_t$ and EXS_t, and solving the model with the help of a Gauss Seidel algorithm. The new market equilibrium price is reached once the difference between $NEXD_t$ and EXS_t is zero.

Often industry specialists and policy makers prefer distinguishing between exports and net exports. To meet this requirement, imports (IMP_t) are simply modelled as a function of $NEXD_t$, as illustrated in equation 3.33, and imports are then added to $NEXD_t$ to calculate exports (equation 3.34).

Equation 3.33: $$IMP_t = f(NEXD_t, e_t)$$

Equation 3.34: $$EXS_t = NEXD_t + IMP$$

[1] Chapter 2

3.3.2.2 IMPORT AND EXPORT PARITY

Under an import/export parity market regime domestic prices are determined by behavioural price linkage equations. These equations determine the relationship between import and export parity prices (world prices, transaction costs, and the exchange rate taken into consideration) and the domestic prices. Price linkage equations are most appropriate when domestic markets are integrated with world markets with continuous trade flow. Under these conditions, the law of one price suggests that the correlation between the world price and the domestic price equals one but the elasticities are not equal to one.

Equations 3.35 and 3.36 define the price linkage equations for the import and export parity regime respectively, where the domestic price ($P_{D,t}$) is estimated as a function of the import ($P_{IP,t}$) and export parity ($P_{EP,t}$) price and net export demand ($NEXD_t$). Trade is only perfectly elastic at import or export parity if a number of assumptions hold that may not be true in the case of South Africa, like the assumptions that products are homogenous, that South Africa has a true small-country status and the supply of transportation services is infinitely elastic. Therefore, net export demand is included in these equations. Barrett and Li (2002) also argued that trade flow has to be taken into consideration when market integration is analysed. Industry specialists are of the opinion that although parity prices mainly determine the local price when the market is trading at import or export parity levels, trade flow matters, but it is expected that its influence is much smaller than that of the parity price. As previously mentioned in chapter 2, parity prices can also be referred to as "border prices". Border prices are more appropriate for the estimation of market integration than internal prices because they better represent arbitrage opportunities (Goodwin *et al*, 1990).

Equation 3.35: $$P_{D,t} = f(P_{IP,t} NEXD_t)$$

Equation 3.36: $$P_{D,t} = f(P_{EP,t} NEXD_t)$$

The price linkage equation formalises the interaction between the domestic market and the world markets. Under the parity regimes, the model is closed on net trade. In the case of the import parity regime, the model is closed on net imports, and in the case of the export parity regime the model is closed on net exports. The net trade identity can be expressed as

Equation 3.37: $$NT_t = BEGS_t + PROD_t - CONS_t - ENDS_t$$

, where net trade (NT_t) equals beginning stock ($BEGS_t$) plus local grain production ($PROD_t$) less local consumption ($CONS_t$) less ending stocks ($ENDS_t$).

3.4 ESTIMATION PROCEDURES, MODEL SOLVING AND VALIDATION

With a total of 126 equations, the BFAP sector model can be classified as a relatively large-scale, multisector commodity level econometric simulation model and in total, eight crops, five livestock and five dairy commodities are included in the current version of the model. The term "econometric" refers to statistically measured relationships between endogenous and exogenous variables that are included in the simulation framework. According to Ferris (1998), most large econometric multi-market models include statistically estimated relationships as well as equations that are transformation of technical relationships and synthetic equations that are not derived – and so the term "simulation" is added.

At this point it is worth reminding the reader that only the white maize, yellow maize and wheat models of the previous version of the sector model will be redesigned. In short, the redesigned sector model will be made up of the demand and supply components from the previous version of the sector model, redesigned price and trade equations for alternative market regimes, and most importantly, a switching mechanism that allows the model to switch between various model closure techniques that are dictated by the equilibrium pricing conditions. The re-estimation of the system of equations combines econometric methods with simulation techniques. The domestic supply and demand components will not be re-estimated, but for the purpose of completeness of this study, chapter 4 reports the actual equations that are included in the previous version of the sector model. A unique set of price and trade equations will be estimated for each of the market regimes that were identified for the three grain markets for the newly designed regime-switching model. To achieve this, a separate database has to be constructed for each of the possible regimes by distributing all the observations among the three possible trade regimes. Alternative estimation procedures are followed in some cases to find estimates that provide an accurate estimate of reality. Where necessary, synthetic parameters are imposed to ensure reasonable model behaviour. In some equations, indicator variables have been used to "dummy" out the effects of one or more observations that reflect anomalous events or that significantly change the equation elasticities from *a priori* equations. All the improvements and analysis will be undertaken within the already existing partial equilibrium framework.

After the parameters have been estimated or imposed, the next step is to simulate or solve the model. The model is solved in the form of a recursive system of equations. The prevalence of the biological lag in agriculture makes the applications of recursive econometric models most appropriate. The process of simulation can simply be referred to as the mathematical solution of a set of different equations. Whereas traditional approaches to solving a set of linear equations involved inverting large matrices, by the early 1970s large-scale model builders turned to the Gauss-Seidel technique. The Gauss-Seidel algorithm is also used in this study to solve the model's simultaneous system of equations. The procedure is a fairly simple one and involves a step-wise-and-error method to achieve an approximate solution.

Since the evaluation criteria become more complicated with multi-equation simulation models, this study embraces a broad definition of model validation as stated by Pindyck and Rubinfeld (1998): "In practice, it may be necessary to use specifications for some of the equations that are less desirable from a statistical point of view but that improve the ability of the model to simulate well". For the monthly estimations the standard statistical measures like the goodness of fit can be applied, but for the simulation model, alternative techniques have to be applied. These include techniques to determine if the model behaviour is plausible and if it can handle realistic shocks and provide reasonable results. One of the most popular techniques utilised for model validation is to plot the actual and simulated values on a graph and to conduct a visual inspection of how well the model simulates the turning points in the data. The ability of a model to pick up the turning points or rapid changes in the actual data is an important criterion for model evaluation.

Selective *ex-post* simulation tests will be conducted to determine whether the inclusion of the regime switch in the sector model improves the model's ability to track reality and produce the smallest error term for a specific year under switching market regimes. It is expected that the model closure that correlates with the market regime of a specific year, also produces the smallest error term. For the purpose of this study more emphasis will be placed on the economical significance than on the statistical significance of the simulation results. Economic significance refers to the model's ability to simulate real-world issues and salient features of the South African agricultural industry. Even more important for testing the hypothesis of this study, economic significance refers to the model's ability to generate reliable estimates and projections of endogenous variables under market-switching regimes. A number of shocks can occur that can cause a market to switch between various market regimes. It is important that the model is able to handle these shocks in the forecasting period. Before shocks can be introduced, a benchmark is needed for the forecasting period. This benchmark is also referred to as a baseline and is simulated for the next ten years. The baseline is a simulation of the South African grain, livestock and dairy sector model under agreed policy and certain macroeconomic assumptions. Shocks will be introduced in the form of scenario analyses. The forecasts under a specific scenario will then be compared to the baseline results. The model's performance in the forecasting period will be undertaken in chapter 5.

3.5 SUMMARY

This chapter has laid down the theoretical foundations for this study by presenting the theory of domestic supply and demand, trade, switching regimes, model closure, and price formation. Flow and price-quantity (P-Q) diagrams were used to provide easy guidance towards the understanding of important economic and biological relationships. Essentially, these diagrams provide a clear graphic illustration of the hypothesis of this study, namely that the model structure and closure need to be determined for each product under a specific market regime to ensure a true reflection of reality. The empirical results of the models are presented and the performance of the model validated in chapter 4.