Chapter 4

Estimating Residual Life based on Failure Intensities

4.1 Introduction

Chapter 4 deals with Residual Life Estimation (RLE) of items based on observed FOMs (in the non-repairable case) and peril rates (in the repairable case). The intention was not to develop new theory for RLE but to apply existing theory to the combined models that were developed in Chapter 3. Three steps in the process of estimating residual life are identified:

(i) Prediction of covariate behaviour because covariates are assumed to be time-dependent.

(ii) Calculating residual life based on observed FOMs or peril rates with the assistance of covariate behaviour predictions.

(iii) Presentation of results in a comprehensible manner.

These steps are discussed thoroughly in the remainder of the chapter.

In Step (i) covariates are assumed to be time-dependent for more generality but also because of practicality. This assumption requires prediction of future covariate behaviour to be able to estimate residual life. Various approaches can be followed to perform the covariate behaviour prediction and it is discussed in the first part of this chapter.

Residual life calculations are based on the models for items' FOMs or peril rates developed in Chapter 3, i.e. equations (3.13) and (3.30). A detailed literature survey was done on this subject and subsequent procedures were based on the literature survey. Step (ii) is the ultimate objective of this thesis.

To make this study more useful, a method to present the results of calculations to maintenance practitioners in a user-friendly manner, is proposed at the end of this chapter to conclude

Step (iii). This was identified as one of the main research objectives in Section 1.6 and should be achieved to make a contribution to practical reliability modeling.

4.2 Covariate characteristics and behaviour prediction

Section 4.2 covers covariate characteristics and the prediction of covariate behaviour. Before techniques for covariate behaviour prediction can be considered, three covariate characteristics need to be discussed. Covariates can be either,

(i) time-dependent or time-independent;
(ii) internal or external; or
(iii) stochastic or non-stochastic.

These characteristics are discussed in the next three subsections after which techniques are discussed to predict stochastic and non-stochastic covariate behaviour. The section ends with formal assumptions on covariate characteristics (in Section 4.2.6) that is applied in the remainder of the thesis.

4.2.1 Time-dependent vs. time-independent covariates

In this thesis it is consistently assumed that covariates are time-dependent, not only for generality but for more practicality. If covariates were time-independent, the section on prediction of future covariate values would be unnecessary since covariate values would be known and would remain constant. PIMs and residual life calculations with time-independent covariates are special cases of models that allow for time-dependent covariates.

4.2.2 Internal vs. external covariates

Covariates can be either internal or external. This subject is discussed in detail by Fahrmeir and Tutz (1994). External covariates can be measured on a system regardless of whether an event has occurred on the system and the value of an external covariate is not changed materially by the occurrence of an event. An example of an external covariate is the ambient temperature close to a system.

Internal covariates, on the contrary, can only be meaningfully measured on a system before an event has occurred. The value of an internal covariate generally changes dramatically after the event occurs and is generally uninterpretable after the event. An example from the biomedicine field is a living organism's heartbeat, which is by definition zero after the event of death.

The type of covariate, i.e. internal or external, does not play a mathematical role in predicting covariate behaviour but it is an important aspect to consider when constructing a model for covariate behaviour. In the case of internal covariates for example, it is important to study the effect of event-type on covariates before it is attempted to model the covariate behaviour.

4.2.3 Stochastic vs. non-stochastic covariates

Covariates can be either stochastic or non-stochastic. For the stochastic case, covariates can only be predicted within certain confidence bounds and an exact prediction is not possible. At least three techniques exist to model stochastic covariates, i.e. time series analysis, state space models and Markov chains. Both time series analysis and state space models requires large quantities of data (observations in this context) to produce reasonable models. Markov chains are less dependent on large data sets and because the case study of Chapter 5 deals with a fairly small data set, only Markov chains are considered in Section 4.2.4. For more detail on the theory and application of state space models, see Cmiel and Gurgul (2000), Christer, Wang, and Sharp (1997) and Wang, Wang, and Mao (1999). Harvey (1981) and Chatfield (1980) provide an introduction to time series analysis.

Non-stochastic covariates can, by definition, be predicted with reasonable accuracy. Two cases of non-stochastic covariates exist however. In the first case, covariates are known from the origin of time and observations are unnecessary in intensity models, e.g. the complete peril rate of a system would simply be a complex parametric formulation of a NHPP as a function of time. In the second case, information about a covariate's behaviour up to a point x or t is required to be able to predict covariate behaviour beyond x or t. These cases are considered in Section 4.2.5.

4.2.4 Predicting stochastic covariate behaviour

Markov chains have been used by, amongst other, Makis and Jardine (1992), Makis and Jardine (1991), Vlok (1999) and Vlok, Coetzee, Banjevic, Jardine, and Makis (2001) to predict covariate behaviour. Other authors that have applied Markov chains in reliability include, Lagakos, Sommer, and Zelen (1978), Ng (1999), Billard and Meshkani (1995), Collins (1973) and Zhang and Love (2000). Christer and Wang (1995) oppose the use of Markov chains to model covariate behaviour because present covariate levels are in practice more often than not dependent on immediately preceding levels. Ross (1990) and Hines and Montgomery (1980) discuss Markov chains in detail. In this section an overview of the theory required to predict covariate behaviour with Markov chains is presented.

Covariate states have to be defined for the covariates before it can be modeled with Markov chains. For this reason, every range of covariate values is divided into appropriate intervals or bands and every covariate band is defined as a covariate state. Covariate bands are then used as boundaries for the transition probabilities in the Transition Probability Matrix (TPMX). For numerical convenience, 4 or 5 bands are usually selected between upper and lower bands except for the last band which does not have an upper bound.

Following the covariate states, suppose that $\{X_0, X_1, X_2, ...\}$ is a multidimensional Markov process which makes up an item's event history such that $X_k = (z_{k1}(x), z_{k2}(x), ..., z_{km}(x)) \in \mathbb{R}^m$, where m is the number of covariates, and $z_{ki}(x)$ is the k^{th} observation of variable i before an event, performed at time $x = k\Delta$ where Δ is a fixed inspection interval. A stochastic process $\{X_0, X_1, X_2, ...\}$ is assumed to be Markovian if, for every $k \geqslant 0$,

$$P\{X_{k+1} = j | X_k = i, X_{k-1} = i_{k-1}, X_{k-2} = i_{k-2}, ..., X_0 = i_0\} = P\{X_{k+1} = j | X_k = i\} \tag{4.1}$$

where $j, i, i_0, i_1, ..., i_{k-1}$ are defined states of the process, in this case the covariate bands.

The transition probability for any covariate in state i to undergo a transition to state j for a given inspection interval Δ is,

$$P_{ij}(k) = P_{ij}(k, \Delta) = P(X_{k+1} = j | X > (k+1)\Delta, X_k = i) \tag{4.2}$$

where X denotes time to event as before and i and j denote any two possible states.

Suppose a sample $X_0, X_1, X_2, ...$ is observed and let $n_{ij}(k)$ denote the number of transitions from state i to j at k throughout the sample, where the sample may contain several histories, i.e.

$$n_{ij}(k) = \#\{X_k = i, X_{k+1} = j\} \tag{4.3}$$

Similarly, the number of transitions from i at time $k\Delta$ to any other state can be calculated by,

$$n_i(k) = \#\{X_k = i\} = \sum_j n_{ij}(k) \tag{4.4}$$

It is hence possible to estimate the probability of a transition from state i to state j at time $k\Delta$ with the following relationship derived with the maximum likelihood method,

$$\hat{P}_{ij}(k) = \frac{n_{ij}(k)}{n_i(k)} \tag{4.5}$$

If it is assumed that the Markov chain is homogeneous within the interval $a \leqslant k \leqslant b$, i.e. $P_{ij}(k) = P_{ij}(a)$, the transition probability can be estimated by,

$$\hat{P}_{ij}(k) = \frac{\sum\limits_{a \leqslant k \leqslant b} n_{ij}(k)}{\sum\limits_{a \leqslant k \leqslant b} n_i(k)} \tag{4.6}$$

It would also be possible to assume that the entire Markov chain is homogeneous, then $P_{ij} = P_{ij}(k)$, for $k = 0, 1, 2, ...$ and hence the transition probabilities are estimated by,

$$\hat{P}_{ij} = \frac{n_{ij}}{n_i}, \quad \text{where } n_{ij} = \sum_{k \geqslant 0} n_{ij}(k), \quad n_i = \sum_j n_{ij} \tag{4.7}$$

As mentioned before, covariates are assumed to be time-dependent by default. For this reason continuous time is divided into u intervals, $(a_1, a_2], ..., (a_u, \infty)$, in which the transition probabilities are considered to be homogeneous. This manipulation simplifies the calculation of the TPMX considerably without loosing much accuracy.

The estimations of the TPMX above assumed that the inspection interval Δ was constant. In practice, this is rarely the case. This would mean that recorded data with inspection intervals different from Δ have to be omitted from TPMX calculations, thereby loosing valuable information about the covariates' behavior. To overcome this problem a technique utilizing transition densities (or rates) is used. Assume that the Markov chain is homogeneous for a short interval of time. The probability of transition from $i|_{x=0} \rightarrow j|_{x=x}$ is $P_{ij}(x) = P(X(x) = j|X(0) = i)$ and the rate at which the transition will take place is $D_x[P_{ij}(x)] = \lambda_{ij}$ $(i \neq j)$. For the case where $i = j$ the transition rate can be derived with the following argument. Suppose the system is in state $i|_{x=0}$ and state $j|_{x=x}$ with r possible states. If the sum over all probabilities over x is taken,

$$\begin{aligned} P_{i0}(x) + P_{i1}(x) + P_{i2}(x) + ... + P_{ir}(x) &= 1 \\ \sum_j P(X(x) = j|X(0) = i) &= 1 \\ \text{or } \sum_j P_{ij}(x) &= 1 \end{aligned} \tag{4.8}$$

If the time derivative is taken,

$$\begin{aligned} \sum_j \frac{\partial}{\partial x}\left[P_{ij}(x)\right] &= 0 \\ \therefore \quad \lambda_{i0} + \lambda_{i1} + \lambda_{ii} + ... + \lambda_{ir} &= 0 \\ \lambda_{ii} &= -\sum_{i \neq j} \lambda_{ij} \end{aligned} \tag{4.9}$$

The value of any λ_{ij} $(i \neq j)$ can be estimated by,

$$\hat{\lambda}_{ij} = \frac{n_{ij}}{\Omega_i}, \quad n_{ij} = \sum_k n_{ij}(k) \tag{4.10}$$

where, k runs over the given interval of time and Ω_i is the total length of time that a state is occupied in the sample. The calculation of the transition rates can be generalized for the system from any state i to j at any time x with,

$$P'_{ij}(x) = \sum_l P_{il}(x)\lambda_{lj} \tag{4.11}$$

Equation (4.11) provides a system of differential equations that has to be solved to obtain the transition probability matrix. A solution to the system of differential equations solution is,

$$P(x) = \exp(A \cdot x) \tag{4.12}$$

where $P(x) = (P_{ij}(x))$ and $A = (\lambda_{ij})^*$. This can be calculated by the series,

$$P(x) = \sum_{n=0}^{\infty} A^n \frac{x^n}{n!} \tag{4.13}$$

which is fast and accurate. Statistical tests (such as χ^2) can be used to confirm the validity of the homogeneity assumption over the given time intervals.

4.2.5 Predicting non-stochastic covariate behaviour

When selecting a parametric function to predict future covariate behaviour, the first option should be to select a function that has a physical relationship to the observed phenomenon. In vibration analysis, for example, it is expected that the spectral component related to unbalance would increase quadratically with increasing rotational velocity according to Rao (1995). If this spectral component is used as a covariate, its future behaviour should typically be modeled by a parametric function of some parabolic type. Selecting an appropriate parametric function should as far as possible not be a curve fitting exercise but rather a physical interpretation of the actual situation.

Rao (1980) formulated a few basic parametric functions that could be used to predict covariate behaviour†. These functions with solutions to their parameters are summarized in Table 4.1.

*Brackets denote matrices.

†These parametric functions are general and were not intended to predict covariate behaviour.

Table 4.1: Parametric functions suitable to predict covariate behaviour

	Linear curve
Form	$y = ad + b$
Solution	$a = \dfrac{(d_1 - \overline{d})(y_1 - \overline{y}) + (d_2 - \overline{d})(y_2 - \overline{y}) + \ldots + (d_n - \overline{d})(y_n - \overline{y})}{(d_1 - \overline{d})^2 + (d_2 - \overline{d})^2 + \ldots + (d_n - \overline{d})^2}$ $b = \overline{y} - a\overline{d}$
	Quadratic curve
Form	$y = ad^2 + bd + c$
Solution	$\sum_{i=1}^{n} y_i = a \sum_{i=1}^{n} d_i^2 + b \sum_{i=1}^{n} d_i + cn$ $\sum_{i=1}^{n} d_i y_i = a \sum_{i=1}^{n} d_i^3 + b \sum_{i=1}^{n} d_i^2 + c \sum_{i=1}^{n} d_i$ $\sum_{i=1}^{n} d_i^2 y_i = a \sum_{i=1}^{n} d_i^4 + b \sum_{i=1}^{n} d_i^3 + c \sum_{i=1}^{n} d_i^2$
	Hyperbolic curve
Form	$y = a/d + b$
Solution	$\sum_{i=1}^{n} y_i = a \sum_{i=1}^{n} \frac{1}{d_i} + bn$ $\sum_{i=1}^{n} \frac{y_i}{d_i} = a \sum_{i=1}^{n} \frac{1}{d_i^2} + b \sum_{i=1}^{n} \frac{1}{d_i}$
	Exponential curve
Form	$y = ab^d$
Solution	$\sum_{i=1}^{n} \log y_i = n \log a + \log b \sum_{i=1}^{n} d_i$ $\sum_{i=1}^{n} d_i \log y_i = \log a \sum_{i=1}^{n} d_i + \log b \sum_{i=1}^{n} d_i^2$
	Geometric curve
Form	$y = ad^b$
Solution	$\sum_{i=1}^{n} \log y_i = n \log a + \log b \sum_{i=1}^{n} \log d_i$ $\sum_{i=1}^{n} d_i \log y_i = \log a \sum_{i=1}^{n} d_i + \log b \sum_{i=1}^{n} \log d_i^2$

A simple technique to test the goodness of fit of a straight line to a particular data set, is to calculate the correlation coefficient, R, by

$$R = \frac{(d_1 - \overline{d})(y_1 - \overline{y}) + (d_2 - \overline{d})(y_2 - \overline{y}) + ... + (d_n - \overline{d})(y_n - \overline{y})}{\sqrt{[(d_1 - \overline{d})^2 + ... + (d_n - \overline{d})^2]\,[(y_1 - \overline{y})^2 + ... + (y_n - \overline{y})^2]}} \tag{4.14}$$

To be able to calculate R for the non-linear functions in Table 4.1, data should be linearized first. A summary of the linearization procedure is shown in Table 4.2.

Table 4.2: Linearization of data to calculate correlation coefficient

Curve	d_i	y_i
Quadratic	d_i	$y_{i+1} - y_i$
Hyperbolic	$1/d_i$	y_i
Exponential	d_i	$\log y_i$
Geometric	$\log d_i$	$\log y_i$

Rao (1980) suggests that for an acceptable fit, $0.88 < R < 1$ for 5 samples, $0.28 < R < 1$ for 50 samples and $0.20 < R < 1$ for 100 samples.

It would also be possible to fit a n^{th} order polynomial through the covariate history and then predict future covariate values. This should preferably only be done for fixed inspection intervals. A n^{th} order polynomic approximation for a particular future covariate value, y_i at instant d_i is given by,

$$y_i = a_n d_i^n + a_{n-1} d_i^{n-1} + ... + a_1 d_i + a_0 \tag{4.15}$$

Ellis and Gulick (1990) describe numerical methods to solve for the coefficients of non-linear systems such as (4.15).

4.2.6 Assumptions on covariate characteristics

According to Banjevic (2001), assumptions on covariate characteristics should be based primarily on the specific situation that is considered since there are too many different possible scenarios to generalize these assumptions. The assumptions on covariate behaviour made in this thesis are however generalized to a certain extent, since it is valid for the majority of situations where condition monitoring measurements are used as covariates. A summary of the assumptions is presented in Table 4.3.

Table 4.3: Summary of assumptions on covariate behaviour

Covariate Characteristic	Assumption
Time dependency	Time-dependent covariates are assumed. The majority of diagnostic parameters measured on equipment in industry are functions of time. Constant covariates are special cases of time varying covariates.
Internal vs. external	No assumption needs to be made with regards to this characteristic but it will be considered when postulating the method of covariate behaviour prediction.
Stochasticity	Non-stochastic covariates are assumed for two reasons. Firstly, it is believed that covariate behaviour in a condition monitoring environment can be predicted with reasonable accuracy provided information about the covariate is available up to a certain time x or t. Secondly, it is shown in Section 4.5 that this approach has more appeal to maintenance practitioners.

4.3 Residual life estimation based on an observed FOM

Section 4.3 discusses RLE of a system based on an observed FOM. Relevant literature is presented in Subsection 4.3.1 after which the most applicable approach is applied on equation (3.13) in Subsection 4.3.2.

4.3.1 Literature survey

Residual life is defined as the time from some current point in time, x, until the following event. This concept is not unique to reliability modeling. In reliability modeling the event corresponds to failure or suspension, in queuing theory it could correspond to the time from a customer arrives until he/she is served (see Gross and Harris (1985)) or in inventory management theory it could be the time to the reorder point (see Tijms (1976)).

The vast majority of literature found on RLE[‡] based on observed FOMs, deals with the situation where covariates are not observed or recorded. A possible reason for this is the complexity of estimating the conditional survival function of a system where the system is a function of time-dependent covariates. Percy, Kobbacy, and Ascher (1998) confirm this statement by describing the procedure as "tricky" and mention it as a possible subject for

[‡]RLE in a reliability modeling context is considered in the remainder of this section.

future research.

Many authors studied the relationship between the FOM and RLE, for example, Ghai and Mi (1999), Ruiz and Navarro (1994), Guess and Prochan (1988) and Ebrahimi (1996). The univariate residual life, μ, of a system at time x is defined as a conditional expectation, i.e.

$$\mu(x) \equiv E[X - x | X \geqslant x] = \frac{\int_x^{\infty} R_X(x)\mathrm{d}x}{R_X(x)} \tag{4.16}$$

$R_X(x)$ is related to $h_X(x)$ by,

$$R_X(x) = \mathrm{e}^{-\int_0^x h_X(s)\mathrm{d}s} \tag{4.17}$$

Tang, Lu, and Chew (1999) describe the discrete relationship between FOM and residual life and Baganha, Geraldo, and Pyke (1999) propose a simple algorithm with which the conditional expectation can be solved for discrete relationships.

Rausand and Reinertsen (1996) believe that the probability distribution selected for RLE should primarily be based on knowledge of the underlying failure mechanism in the system. Event data should only be used to quantify parameters. Lee, Chung, Kim, Ford, and Andersen (1999) follow this school of thinking with some examples from the nuclear power generation industry. Huang, Miller, and Okogbaa (1995) describe these approaches as proof that data in industry is too limited to estimate residual life.

Guess and Park (1988), Guess and Park (1991) and Mi (1995) address the possibility that the residual life of a system is not constant. This is not done by allowing for the effects of covariates but by assuming a bathtub-curved FOM and then base RLEs on this assumption. It is also proved in these publications that by minimizing the FOM, the residual life is not necessarily maximized. Lim and Park (1995) considered a similar scenario. They propose a procedure for testing constant residual life against increasing or decreasing residual life, assuming that the proportion of the population that fails at or before the change point of the residual life function, is known.

In Pulkkinen (1991), the residual life is calculated as the time until the degree of wear of a system reaches a certain threshold level. The estimate is formed by updating the distribution of a stochastic process by describing the degree of wear. The updating procedure is based on successive application of Bayes' formula. Pulkkinen concludes that even though analytical calculation is difficult, the approach is promising. Shimizu, Ando, Morioka, and Okuzumi (1991) used a similar approach but based estimates on a threshold reliability level.

Karpinski (1988) developed a general method to determine the distribution of residual life (conditional expectation) of a system after some partial failures. The RLE starts after the first partial failure and the approach is based on the knowledge of a special distribution of component lives and system life. This method was applied with success on systems operating in nuclear power plants.

Nair and Nair (1989) and Kulkarni and Rattihalli (1996) extended the common univariate residual life concept to the bivariate case where two random variables are observed at each event. This extension could be useful in cases where survival times of different parts on the same system are dependent because of common environmental influences. Zahedi (1985) and Arnold and Zahedi (1988) generalized the bivariate approached further by introducing multivariate residual life estimation. No practical examples of the bivariate or multivariate approach were found in the literature.

An approach fundamentally different to the conventional conditional expectation was proposed by Zahedi (1991). Zahedi constructed a proportional mean remaining life model, analogous to the PHM proposed by Cox (1972), i.e.

$$\mu(x, \boldsymbol{z}) = \mu_0(x) \cdot \exp(\boldsymbol{\gamma} \cdot \boldsymbol{z}) \tag{4.18}$$

where $\mu_0(x)$ is a baseline residual life function dependent on time which is influenced by a functional term containing covariates. Regression parameters are determined in a similar manner as with the model of Cox. Zahedi mentioned in this publication that practical tests were done on the model and that results would be published shortly. No further publication on this approach could be located.

4.3.2 Application of residual life theory on the combined model for non-repairable systems

In the previous section it became evident that a conditional expectation approach is most suitable for this application. This approach is a natural extension of the FOM (which is the conditional probability of failure) and has been proven in many reliability applications.

Following the assumptions made in Section 4.2.6, it is required to substitute equation (3.13) into equation (4.16). This yields,

$$\mu(x, \boldsymbol{\theta}) = \frac{\int_x^\infty R(\phi, \boldsymbol{\theta}) \mathrm{d}\phi}{R(x, \boldsymbol{\theta})} \tag{4.19}$$

where,

$$R(x, \boldsymbol{\theta}) = \exp\left[-\int_0^x \sum_{l=1}^{n} \zeta_s^{k_l} \left(\frac{\beta_s^{k_l}}{\eta_s^{k_l}} \left(\frac{\psi_s^{k_l}\left(\phi - \tau_s^{k_l}\right)}{\eta_s^{k_l}} \right)^{\beta_s^{k_l}-1} \cdot e^{\sum_{j=1}^{m_l} \gamma_{s_j}^{k_l} \cdot z_{i_j}^{k_l}} + e^{\sum_{j=1}^{m_l} \alpha_{s_j}^{k_l} \cdot z_{i_j}^{k_l}} \right) \mathrm{d}\phi \right] \tag{4.20}$$

In (4.19) and (4.20), x corresponds to the time of the last observation in the history of a system currently in operation. To be able to integrate these functions to infinity, the covariate behaviour should be extrapolated with suitable techniques up to the point where $R(x, \boldsymbol{\theta}) = 0$.

A 95% confidence bound can be constructed around the mean residual life of (4.20) to quantify the certainty of an estimate. First it is assumed that the density corresponding to the FOM in (3.13) is given by,

$$f(x, \boldsymbol{\theta}) = \mathrm{D}_x \left[1 - \exp\left(-\int_0^x h(x, \boldsymbol{\theta})\mathrm{d}x\right)\right] \tag{4.21}$$

The lower limit for the residual life is $\underset{\sim}{\mu}(x, \boldsymbol{\theta}) = \underset{\sim}{x} - x$, where $\underset{\sim}{x}$ is calculated from,

$$\int_x^{\underset{\sim}{x}} \frac{f(x, \boldsymbol{\theta})}{\int_x^\infty f(x, \boldsymbol{\theta})\mathrm{d}x}\mathrm{d}x = 0.025 \tag{4.22}$$

Similarly, the upper limit $\tilde{\mu}(x, \boldsymbol{\theta}) = \tilde{x} - x$ is found where,

$$\int_x^{\tilde{x}} \frac{f(x, \boldsymbol{\theta})}{\int_x^\infty f(x, \boldsymbol{\theta})\mathrm{d}x}\mathrm{d}x = 0.975 \tag{4.23}$$

Both equations (4.22) and (4.23) are solved numerically.

4.4 Residual life estimation based on an observed peril rate

Section 4.4 discusses RLE of a system based on an observed peril rate. Relevant literature is presented in Subsection 4.4.1 after which the most applicable approach is applied on equation (3.30) in Subsection 4.4.2.

4.4.1 Literature survey

Publications on RLE of repairable systems are not nearly as common as for non-repairable systems. Reinertsen (1996) supports this statement. All the publications found on this particular subject follow a conditional expectation approach, similar to what was described in Section 4.3.1.

Bhattacharjee (1994) investigated RLE for repairable systems and concluded that the time to first failure cannot adequately reflect its degradation over time because the aging property is influenced by maintenance. He developed a framework that attempts to formulate appropriate ratios of aging under repair and the corresponding implications. This formulation is based on conditional expectation of the next time to failure.

Calabria, Guida, and Pulcini (1990) propose a point estimation procedure for the m^{th} future failure of a repairable system based on the observation of n preceding failures. The procedure is based on the conditional expectation of the next failure calculated by maximizing the likelihood. Monte Carlo simulations done to evaluate the approach produced good results.

4.4.2 Application of residual life theory on combined model for repairable systems

Following the approaches in the literature study, a conditional expectation approach will be used to estimate residual life of a system based on an observed peril rate. According to Banjevic (2001) this is in general complicated but for the NHPP it is simple because of the definition of the NHPP (see Section A.3.3).

Meeker and Escobar (1998) describe RLE of repairable systems modeled by NHPPs in detail. It is required to calculate the mean of the distribution of failure times of a repairable system that experienced its most recent failure at time T_i and is currently operating at time t where $t > T_i$. The residual life μ for a repairable system is $\mu = T_{i+1} - t$ and for a NHPP with covariates it is expected to be,

$$\mu(t,\boldsymbol{\theta}) = \frac{\int_t^\infty \vartheta \cdot \mathrm{D}_\vartheta \left[1 - \exp\left(-\int_t^\infty (\rho(s,\boldsymbol{\theta}) - \rho(t,\boldsymbol{\theta}))\,\mathrm{d}s\right)\right]\mathrm{d}\vartheta}{1 - \exp\left(-\int_t^\infty (\rho(s,\boldsymbol{\theta}) - \rho(t,\boldsymbol{\theta}))\,\mathrm{d}s\right)} - t \tag{4.24}$$

A 95% confidence bound can be constructed around the mean residual life of (4.24) to quantify the certainty of an estimate. First it is assumed that the density corresponding to the relevant portion of the peril rate is given by,

$$f(t,\boldsymbol{\theta}) = \mathrm{D}_t\left[1 - \exp\left(-\int_{T_i}^t (\rho(s,\boldsymbol{\theta}) - \rho(t,\boldsymbol{\theta}))\,\mathrm{d}s\right)\right] \tag{4.25}$$

The lower limit for the residual life is $\underset{\sim}{\mu}(t,\boldsymbol{\theta}) = \underset{\sim}{t} - t$, where $\underset{\sim}{t}$ is calculated from,

$$\int_t^{\underset{\sim}{t}} \frac{f(t,\boldsymbol{\theta})}{\int_t^\infty f(t,\boldsymbol{\theta})\mathrm{d}t}\mathrm{d}t = 0.025 \tag{4.26}$$

Similarly, the upper limit $\tilde{\mu}(t,\boldsymbol{\theta}) = \tilde{t} - t$ is found where,

$$\int_t^{\tilde{t}} \frac{f(t,\boldsymbol{\theta})}{\int_t^\infty f(t,\boldsymbol{\theta})\mathrm{d}t}\mathrm{d}t = 0.975 \tag{4.27}$$

Both equations (4.26) and (4.27) are solved numerically.

4.5 Presentation of results to maintenance practitioners

Up to this point, objectives (i) and (ii) of the problem statement (see Section 1.6) were addressed. But, if these results are not presented in a user-friendly and comprehensible manner, the contribution of the thesis to practical reliability modeling will be small. It is necessary to "sell" the concept of RLE to maintenance practitioners on two levels. The first level of people is the middle and upper level maintenance managers. This would typically include maintenance supervisors, general maintenance managers and engineering managers. The second level of people will be referred to as end-users of the RLE methodology and could include maintenance planners and highly skilled maintenance technicians responsible for a limited scope of equipment.

Different approaches should be followed to promote the RLE methodology amongst the two identified levels of maintenance practitioners because each level will evaluate the methodology differently. Middle and upper level maintenance practitioners will be interested in the process-flow of the concept and how the concept will be integrated with conventional practices and processes.

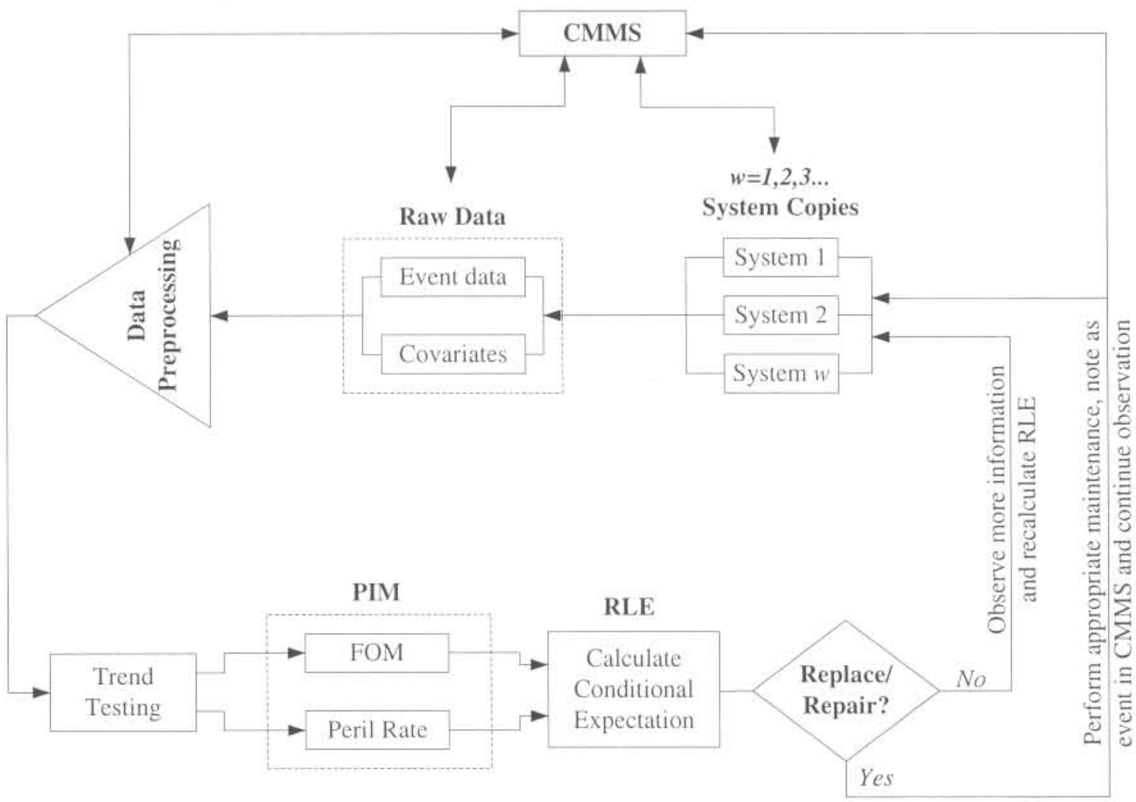

Figure 4.1: Diagrammatic overview of the process-flow of the RLE concept

Figure 4.1 shows a diagrammatic overview of the process-flow of the RLE methodology that can be used as a basis to introduce the concept to the first level of maintenance practitioners. The involvement of an organization's Computerized Maintenance Management System (CMMS) is emphasized in Figure 4.1 because data required for RLE would typically be retrieved and stored in the CMMS. Note that even though the illustration in Figure 4.1 is very conceptual, the difference between the non-repairable and repairable cases is stressed. The process is sketched as a loop to show that as soon as new information is available (a new CM inspection was done) or a maintenance action was taken, the process is repeated.

End-users of the RLE concept will not be interested in the process-flow of the methodology but rather in the outputs of the concept. For this reason, the methodology should be introduced and results should be presented in a practical (graphical) manner. End-users should understand that RLE algorithms developed in this thesis are only decision support tools and that the final maintenance decision is still up to the individual. Guidelines for decision making are covered in Section 4.6.

Two different illustrations of the output of the RLE methodology should be presented to end-users to promote the concept: one for a non-repairable system and another for a repairable system. Figure 4.2 shows the output of the RLE methodology on a non-repairable system.

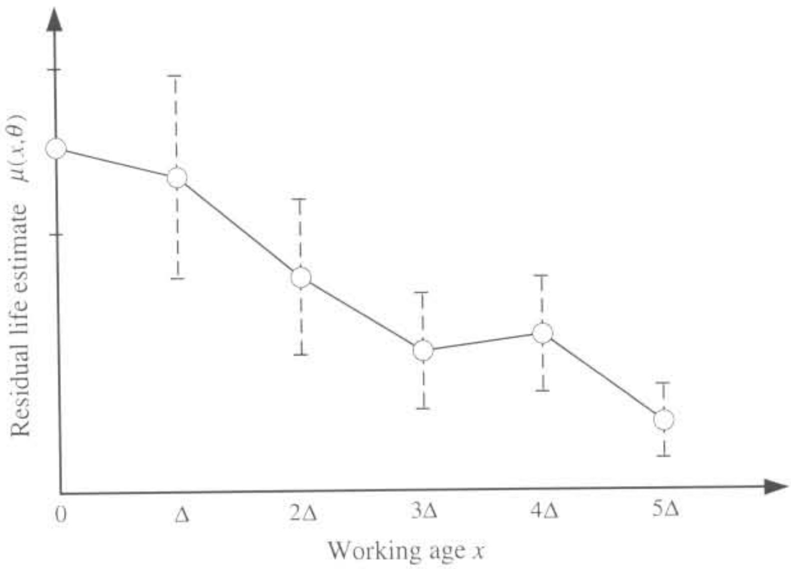

Figure 4.2: Presentation of RLE results of non-repairable systems to end-users

Suppose a fixed CM inspection interval of Δ is used to monitor a particular non-repairable system. At time $x = 0$ when the system was installed, an estimate of the residual life is made within statistical bounds (indicated as a dotted vertical lines). After Δ time units new information (covariates) are observed and the residual life is recalculated. If the system's degradation is linear and the model is accurate, the residual life estimate should reduce by Δ. It is expected that the $\mu(x, \boldsymbol{\theta})$ would decrease monotonically with wear but minor maintenance to the system could elongate the system's life and $\mu(x, \boldsymbol{\theta})$ could increase (see $x = 4\Delta$ on Figure 4.2).

The output of the RLE methodology on a repairable system is virtually the same as for the non-repairable system except that the time scale is different. See Figure 4.3.

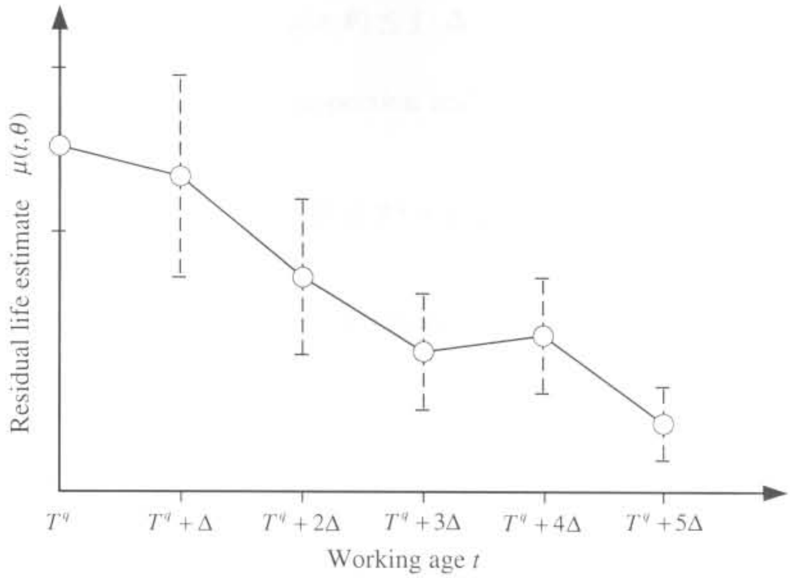

Figure 4.3: Presentation of RLE results of repairable systems to end-users

Suppose a fixed CM inspection interval of Δ is used to monitor a particular repairable system. At time T^q when the system was re-installed, an estimate of the residual life is made within statistical bounds (indicated as dotted vertical lines). After Δ time units new information (covariates) are observed and the residual life is recalculated. If the system's degradation is linear and the model is accurate, the residual life estimate should reduce by Δ. It is expected that the $\mu(t, \boldsymbol{\theta})$ would decrease monotonically with wear but minor maintenance to the system could elongate the system's life and $\mu(t, \boldsymbol{\theta})$ could increase (see $t = T^q + 4\Delta$ on Figure 4.3).

4.6 Decision making with the assistance of dynamic residual life estimates

The ideal decision rule for the RLE approach would be to take action as soon as the lower limit of the residual life estimate equals zero. This is not practical however because of two reasons. Firstly, because the lower limit of the residual life estimate is calculated from the conditional expectation of failure it implies that this estimate will only approach zero when time (local or global) approaches infinity. Secondly, inspections are most often done discretely which means that the lower limit of the residual life estimate could become zero in-between inspections and the failure could occur before the next inspection. For these reasons a simple but practical decision rule is proposed: take preventive maintenance action as soon as the lower residual life estimate is less than the time to the next inspection. Thus, for the non-repairable case it is when,

$$\underset{\sim}{\mu}(x,\boldsymbol{\theta}) \leq j \cdot \Delta - x \tag{4.28}$$

where the j^{th} inspection is the next inspection and Δ is the inspection interval. Similarly, for the repairable case it is when,

$$\underset{\sim}{\mu}(t,\boldsymbol{\theta}) \leq T^{q} + j \cdot \Delta - t \tag{4.29}$$

It is important to realize that these decision rules are influenced by the following:

(i) Quality of data
(ii) Quantity of data
(iii) Selection of the PIM
(iv) Selection of covariates
(v) Accuracy of covariate behavior prediction
(vi) Accuracy of covariate measurements

It is difficult to quantify the effect of these influences on residual life estimates and it would differ for each situation. The proposed decision rule should thus only be used as a maintenance decision support tool and the final decision should still be made by the maintenance practitioner.

4.7 Conclusion

In this chapter, the RLE process was divided into three steps: (i) prediction of future covariate behaviour; (ii) calculating residual life based on observed FOMs or peril rates with the assistance of covariate behaviour predictions; and (iii) the presentation of results in a comprehensible manner. These steps are similar to the 2-phase approach of Christer and Wang (1995) that constructed a model for the prediction of covariate behaviour first and then used this prediction as a covariate in a PIM to make maintenance decisions. Christer and Wang restricted their approach to the renewal case modeled by a simple Weibull FOM and used the results to minimize long term cost, downtime or risk (similar to Makis and Jardine (1991)).

Covariates in this thesis are assumed to be time-dependent, internal and non-stochastic, provided that the covariates have been observed up to a certain point x or t. Several possible parametric functions with their solutions that could be used for covariate behaviour prediction are summarized in Table 4.1.

A detailed literature study was done on RLE based on observed FOMs or peril rates and it was found that very few attempts have been made to calculate RLE based on an observed peril rate. Many publications were found on RLE based on FOMs. In both cases, the vast majority of authors used a conditional expectation approach to calculate residual life. Following this, it was decided to use the conditional expectation approach to calculate $\mu(x,\boldsymbol{\theta})$ and $\mu(t,\boldsymbol{\theta})$. The conditional expectation approach applied to the theory of Chapter 3 is summarized in Table 4.4.

Table 4.4: Summary of RLE calculations based on a FOM or peril rate

Non-repairable Case[§]
$\mu(x,\boldsymbol{\theta}) = \frac{\int_x^\infty R(\phi,\boldsymbol{\theta})\mathrm{d}\phi}{R(x,\boldsymbol{\theta})}$
Repairable Case[¶]
$\mu(t,\boldsymbol{\theta}) = t - \frac{\int_t^\infty \vartheta \cdot \mathrm{D}_\vartheta \left[1-\exp\left(-\int_t^\infty (\rho(s,\boldsymbol{\theta})-\rho(t,\boldsymbol{\theta}))\,\mathrm{d}s\right)\right]\mathrm{d}\vartheta}{1-\exp\left(-\int_t^\infty (\rho(s,\boldsymbol{\theta})-\rho(t,\boldsymbol{\theta}))\,\mathrm{d}s\right)}$

The RLE process used in this thesis is of a fairly complex nature and would probably not find acceptance amongst maintenance practitioners in its mathematical form. For this reason, simplified graphical representations of the approach were developed for middle to upper level management and for end-users in Section 4.5.

In this chapter, Steps (ii) and (iii) of the objectives set out in Section 1.6 were achieved. It is hence required to apply the theory and methodologies developed on an actual data set. This is done in Chapter 5.

[§]Variables for the models corresponding to the non-repairable case are declared and described in Section 4.3

[¶]Variables for the models corresponding to the repairable case are declared and described in Section 4.4