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# DYNAMIC RESIDUAL LIFE ESTIMATION OF INDUSTRIAL EQUIPMENT BASED ON FAILURE INTENSITY PROPORTIONS

By

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The Author
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# OPSOMMING

## Dinamiese Oorblywende Lewe Skatting van Industriële Toerusting Gebaseer op Falingsintensiteit Verhoudings

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Daar is 'n wêreldwye strewe na optimering van instandhoudingsbesluitneming in 'n meer mededingende vervaardigingsindustrie. Voorkomende instandhouding is dikwels die mees georganiseerde en koste effektiewe strategie om te volg, maar 'n besluit moet steeds geneem word oor die tydstip waarop die voorkomende instandhouding gedoen word. Gebruiksgebaseerde instandhoudingsbesluitneming is tot 'n groot mate geoptimeer deur statistiese analise van falingsdata, terwyl voorspellende voorkomende instandhouding (toestandsmonitering) geoptimeer word deur van meer gesofistikeerde tegnologie gebruik te maak. Baie min werk is egter al gedoen om die voordele van hierdie twee denkwyses te kombineer. Hierdie proefskrif het ontstaan na 'n besef van die moontlike verbetering in instandhoudingspraktyk deur gebruiksgebaseerde instandhoudingsoptimeringstegnieke te kombineer met hoë tegnologie toestandsmonitering.

In hierdie proefskrif word 'n benadering ontwikkel waarmee oorblywende lewe van industriële toerusting dinamies geskat word deur statistiese falingsanalise en gesofistikeerde toestandsmoniteringstegnieke te kombineer. Die benadering is gebaseer op falingsintensiteitverhoudings wat bereken word uit historiese oorlewingstye en die dienooreenkomstige diagnostiese inligting verkry uit toestandsmoniteringsresultate. Gekombineerde Proporsionele Intensiteitsmodelle (PIMe) vir nie-herstelbare en herstelbare stelsels, wat die meeste konvensionele verbeterings op PIMe as spesiale gevalle bevat, asook numeriese metodes om die regressie koeffisiënte te bepaal, is ontwikkel.

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Saam met die oorblywende lewe skatting benadering, is 'n gebruikersvriendelike grafiese metode waarmee oorblywende lewe skattings vertoon kan word, ontwikkel. Hierdie metode is natuurlik selfs vir onervare data analiste maklik verstaanbaar. Die oorblywende lewe skatting benadering is toegepas op 'n tipiese datastel verkry van 'n Suid-Afrikaanse industrie en resultate is vergelyk met resultate verkry van 'n soortgelyke, bestaande instandhoudingsbesluitnemingstegniek. Die vergelyking toon aan dat die benadering ontwikkel in hierdie proefskrif relevant en prakties is en volgens sekere kriteria marginaal beter is as die genoemde bestaande instandhoudingsbesluitnemingstegniek.

SLEUTELWOORDE: Oorblywende lewe, Proporsionele gevaar, Falingsintensiteit, Voorwaardelike gemiddelde

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# SUMMARY

## Dynamic Residual Life Estimation of Industrial Equipment based on Failure Intensity Proportions

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There is a world-wide drive to optimize maintenance decisions in an increasingly competitive manufacturing industry. Preventive maintenance is often the most organized and cost efficient strategy to follow, but a decision still has to be made on the optimal instant to perform preventive maintenance. Use based preventive maintenance decisions have been optimized through statistical analysis of failure data while predictive preventive maintenance (condition monitoring) has been optimized by utilizing more sophisticated technology. Very little work has however been done to combine the advantages of the two schools of thought. This thesis originated from a realization of the potential improvement in maintenance practice by combining use based preventive maintenance optimization techniques with high technology condition monitoring.

In this thesis an approach is developed to estimate residual life of industrial equipment dynamically by combining statistical failure analysis and sophisticated condition monitoring technology. The approach is based on failure intensity proportions determined from historic survival time information and corresponding diagnostic information such as condition monitoring. Combined Proportional Intensity Models (PIMs) for non-repairable and repairable systems, containing the majority of conventional PIM enhancements as special cases, with numerical optimization techniques to solve for the regression coefficients, are derived.

In addition to the residual life estimation approach, a user-friendly graphical method with which residual life estimates can be presented was also developed. This method is natural

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and easy to comprehend, even by inexperienced data analysts.

The residual life estimation approach is applied to a typical data set from a South African industry and results are compared to those obtained from a similar, established maintenance decision support tool. This comparison showed that the approach developed in this thesis is relevant, practical and marginally better than the established decision support tool for certain criteria.

KEYWORDS: Residual life, Proportional hazards, Failure intensity, Conditional expectation

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# LIST OF ABBREVIATIONS

| | |
|---|---|
| AFTM | Accelerated Failure Time Model |
| AHM | Additive Hazard Model |
| AIM | Additive Intensity Model |
| AR | Auto Regressive |
| ARIMA | Auto Regressive Integrated Moving Average |
| AROCOF | Average Rate of Occurrence of Failure (See ROCOF) |
| BAO | Bad-as-old |
| BFGS | Broyden-Fletcher-Goldfarb-Shanno |
| BOWN | Better than old but worse than new |
| BPP | Branching Poisson Process |
| CM | Condition Monitoring |
| CMF | Cumulative Mean Function |
| CMMS | Computerized Maintenance Management System |
| EHRM | Extended Hazard Regression Model |
| FOM | Force of Mortality |
| GAN | Good-as-new |
| HFD | High Frequency Domain |
| HPP | Homogeneous Poisson Process |
| IID | Independent and Identically Distributed |
| KS | Kolmogorov-Smirnov |
| LCC | Life Cycle Cost |
| LLP | Log-linear Process |
| LNF | Lifted Noise Floor |
| MLE | Maximum likelihood estimates |
| MRL | Mean residual life |
| MTBR | Mean Time Between Renewals |
| NDT | Non-destructive testing |
| NHPP | Non-homogeneous Poisson Process |
| OEM | Original Equipment Manufacturers |
| PAR | Proportional Age Reduction |
| PAS | Proportional Age Setback |
| PDF | Probability Density Function |
| PHM | Proportional Hazards Model |

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| | |
|---|---|
| PIM | Proportional Intensity Model |
| PLP | Power-law Process |
| PMIM | Proportional Mean Intensity Model |
| POM | Proportional Odds Model |
| PWP | Prentice Williams Peterson |
| RCM | Reliability Centered Maintenance |
| RLE | Residual Life Estimation |
| ROCOF | Rate of Occurrence of Failure, i.e. the time derivative of an expected number of failures |
| ROOF | Repair only on failure |
| RV | Random variable |
| TPM | Total Productive Maintenance |
| TPMX | Transition Probability Matrix |
| TTT | Total Time on Test |
| URL | Useful Remaining Life |
| WO | Worse than old |
| WRP | Weibull Renewal Process |

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# NOTATION

| | |
|---|---|
| $\nu$ | Additive functional term |
| $\iota_\tau$ | Average intensity |
| $B(t)$ | Backward recurrence time, i.e. $t - T_{N(t)}$ |
| $\hat{c}_j$ | Correlation coefficient of lag $j$ |
| $R$ | Linear correlation coefficient |
| $C_1$ | Cost of minimal repair |
| $C_2$ | Cost of system replacement |
| $C_f$ | Cost of unexpected failure maintenance |
| $C_p$ | Cost of planned preventive maintenance |
| $F_X(x)$ | Cumulative density function (Unreliability function) |
| $\mathrm{D}_x$ | Derivative with respect to $x$ |
| $C$ | Event indicator, i.e. $C = 0$ in case of suspension and $C = 1$ in case of failure |
| $M(t)$ | Expected number of failures up to time $t$, i.e. $E[N(t)]$, for a situation modeled by the full intensity, $\iota(t)$ |
| $M_u(t)$ | Expected number of failures up to time $t$, i.e. $E[N(t)]$, for a situation modeled by the unconditional intensity, $\iota_u(t)$ |
| $W(\mathbb{D})$ | Expected time until replacement in the decision-model of Makis and Jardine, regardless whether preventive action or failure |
| $E[]$ | Expected value of a function |
| $\tau$ | Factor that acts additively on $x$ or $t$ in $g$ to represent a time jump or time setback that could be system copy- and stratum-specific |
| $\psi$ | Factor that acts multiplicatively on $x$ or $t$ in $g$ to result in an acceleration or deceleration of time that could be system copy- and stratum-specific |
| $h_X(x)$ | Force of Mortality |
| $W(t)$ | Forward recurrence time, i.e. $T_{N(t)+1} - t$ |
| $g$ | Fully parametric baseline function used in a combined PIM that could be system copy- and stratum-specific |
| $t$ | Global time |
| $H_t$ | History or filtration of a process |
| $\Delta$ | Inspection interval |
| $\{N(t), t \geq 0\}$ | Integer valued counting process |

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| | |
|---|---|
| $\iota$ | Intensity of a process (also called *full* intensity or *conditional* intensity) |
| $\iota$ | Intensity or conditional intensity |
| $T_i$ | $i^{th}$ arrival time |
| $X_i$ | $i^{th}$ interarrival time |
| $L$ | Likelihood |
| $x$ | Local time |
| $\iota_u$ | Mean intensity or unconditional intensity |
| $S$ | Mean sojourn time of a system in a particular state |
| $\lambda$ | Multiplicative functional term that acts on $g$ |
| $N(t)$ | Number of failures recorded in the interval $(0, t]$ |
| $d_s$ | Number of events observed in stratum $s$ |
| $q$ | Number of observed events |
| $n$ | Number of parts in a system |
| $w$ | Number of system copies |
| $n^*$ | Optimal number of minimal repairs before system replacement |
| $I^*$ | Optimal system replacement time under the minimal repair assumption |
| $\rho_1(t)$ | Peril rate, i.e. the ROCOF of an NHPP, modeled by a log-linear process |
| $\rho_2(t)$ | Peril rate, i.e. the ROCOF of an NHPP, modeled by a power-law process |
| $\Pr[]$ | Probability |
| $f_X(x)$ | Probability density function |
| $Q(\mathbb{D})$ | Probability that failure replacement will occur in Makis and Jardine's cost optimization decision-model |
| $\zeta$ | Random variable that acts as a frailty in a combined PIM and that could be system copy- and stratum-specific |
| $R_X(x)$ | Reliability function, i.e. $1 - F_X(x)$ |
| $\mu(x, \boldsymbol{\theta})$ | Residual life of a non-repairable system |
| $\mu(t, \boldsymbol{\theta})$ | Residual life of a repairable system |
| $\mathbb{D}$ | Threshold risk level |
| $v(t)$ | Time derivative of an expected number of failures, i.e. ROCOF |
| $\boldsymbol{\theta}$ | Vector containing all form parameters of a PIM |
| $\boldsymbol{z}$ | Vector containing covariates that may be time-dependent, i.e. $\boldsymbol{z}(x)$ in the non-repairable case and $\boldsymbol{z}(t)$ in the repairable case |
| $G$ | Warning level function |