A DEVELOPMENTAL CASE STUDY: IMPLEMENTING THE THEORY OF REALISTIC MATHEMATICS EDUCATION WITH LOW ATTAINERS

by

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ABSTRACT

The research documented in this report had a twofold purpose. Firstly, it was to design and implement an intervention based on the theory of Realistic Mathematics Education (RME) aimed at improving the mathematical understanding of learners in two Grade 8 remedial mathematics classes, by revisiting the key number concepts of place value, fractions and decimals. In doing so, a second purpose was to investigate the viability and emerging characteristics of an intervention based on the theory of RME in such a setting (i.e. with *low attainers* to *revisit key number concepts*). Pending the realisation of these immediate outcomes, more distant outcomes in subsequent research would be: that learners' understanding and academic performance in mathematics improves and to develop a local instruction theory in using the RME theory to revisit the concepts of place value, fractions and decimals with low attaining learners in order to improve their understanding in this regard.

Grade 8 low attainers were selected as the target group for this research as a result of the pending implementation of Mathematical Literacy as a compulsory subject for all learners, possibly from 2006. Currently in South Africa, learners who are not meeting the required standard by the end of their Grade 9 year are able to elect not to take mathematics through Grades 10, 11 and 12. When the new Further Education and Training (FET) policy is implemented, this will no longer be the case. All learners, who do not elect to take mathematics as a subject, will have to take Mathematical Literacy as a compulsory subject throughout Grades 10, 11 and 12. Although less detailed and abstract than the subject mathematics, the Mathematical Literacy curriculum still requires learners to have an understanding of key number concepts and also contains a substantial amount of algebra. As Grade 8 is when learners start working with algebra more formally, and is also their first year at secondary school, it was decided that this would be an appropriate year to try and diagnose and remediate problems in learners' understanding of the key number concepts, if and where possible. The intention was that this would then equip learners with a more appropriate structure of conceptualised knowledge of the above-mentioned concepts on which they could further construct their understanding of algebra.

The study was carried out at a local urban high school in South Africa and the research design of this study was informed by two development research approaches (van den Akker & Plomp, 1993; Gravemeijer, 1994). Also, the study was only implemented with a small number of participants, within a bounded setting and without the intention to generalise the results. It was therefore regarded as a development *case study*. The results appear to indicate that it is viable to apply the theory of RME with low attaining Grade 8 learners in order to revisit the key number concepts of place value, fractions and decimals.

Key words:

Low attainers; Realistic Mathematics Education (RME); remedial mathematics classes; low achievers; development research; remediation; Special Educational Needs; key number concepts; mathematics intervention

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LIST OF ACRONYMS

DOE	Department of Education
RME	Realistic Mathematics Education
SEN	Special Educational Needs
FETC	Further Education and Training Certificate
SPSS	Statistical Package for the Social Sciences
MCQ	Multiple Choice Questions
TIMSS	Third International Mathematics and Science Study

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CHAPTER ONE

1 Introduction

1.1 Overview

This report recounts the process and results of a development case study, comprising 12 individual cases, carried out in a local urban high school in South Africa. In the study an intervention for low attaining Grade 8 mathematics learners was implemented in an attempt to improve the understanding of the participants with regard to place value, fractions and decimals, and to identify characteristics of this type of intervention and potential design principles that could be applied in similar interventions.

The curriculum in South Africa requires learners to start their formal learning of algebra at Grade 8 level. The proposition of this study was therefore: by providing learners with a more appropriate structure of conceptualised knowledge (Skemp, 1989) regarding the key number concepts of place value, fractions and decimals, that they would experience more successful learning and understanding of algebra. With the pending introduction of the compulsory subject of "Mathematical Literacy" up to Grade 12 in South Africa, the need for learners to gain as much conceptual understanding as possible when the basis of algebra is laid, was a motivating factor for this research. The literature pertaining specifically to learners with learning disabilities or low attaining learners indicates that these learners demonstrate on average a greater percentage of systematic errors (misconceptions), than higher achieving learners (e.g. Cox, 1975; Woodward & Howard, 1994). Analysis of the error patterns revealed that many of the errors occur due to limited conceptual understanding of the algorithms and strategies taught to learners.

The theory of Realistic Mathematics Education (RME) was selected as the vehicle to drive the design and implementation of the intervention, specifically with reference to the instructional design principle of "guided reinvention through progressive mathematisation" (Gravemeijer, 1994). In this case however, it was applied as "***re***-guided reinvention" as these learners had previously been taught the concepts dealt with in the intervention. Usually the theory of RME is applied with learners who are learning new concepts for the first time. The setting was therefore somewhat different to those contained in most studies where the RME approach had been used, but the RME instructional principles remained the same and existing knowledge in this regard was therefore drawn on to inform the development and implementation of the intervention with low attainers in a South African context.

This chapter provides an introduction to the study by first examining the background to, context of and rationale for the inquiry. The problem statement and objectives of the study are then introduced before looking at the method employed in carrying out the research. Finally the scope and limitations of the study are provided and the content of the rest of the report is briefly outlined.

1.2 Background and context of the inquiry

Mathematics is regarded as a critical subject at school, which imparts at a basic level the necessary arithmetic and reasoning skills for everyday life. At secondary school level it is seen as an important subject to pass in order to gain entrance into a growing number of professional and academic courses at tertiary institutions. At a social and emotional level it is often identified as an indicator of implicit intelligence and an agent of anxiety levels (e.g. Resnick, Viehe & Segal, 1982; Ma, 1999). Mathematics is one of those subjects of which people will openly express their fear, dislike and their sense of awe, usually without hesitation (Cockcraft, 1982). It is also a common perception that it is a difficult subject to master. Although this remains a perception, it appears to have been endorsed in South Africa by the declining number of learners in recent years enrolling to write the Senior Certificate Examination on Higher Grade mathematics (Department of Education [DoE], 2002). In addition to this, recent international studies have indicated a low level of performance by South African learners in mathematics when compared to results of other countries (Howie, 2001).

For the past number of years in South Africa, learners who really struggled with mathematics were able to discontinue it at the end of their Grade 9 year by not selecting it as one of their optional four subjects (the inclusion of two languages is compulsory). However, it was recently decided by government that a subject called "Mathematical Literacy" would become compulsory for all learners in obtaining the new Further Education and Training Certificate (FETC) that will replace the Senior Certificate Examination, probably in 2006. The focus of the Mathematical Literacy subject is more on developing financial skills such as bond repayments, interest rates, etc., but also contains a fair amount of abstract work within the domain of mathematics, that in turn requires an understanding of the fundamental mathematical number skills.

The pending implementation of Mathematical Literacy, the low level of performance of many learners in mathematics in South African schools, the increasing number of learners taking extra mathematics classes and my own experience as a mathematics educator prompted the start of this study. Prior to my joining the university in 2002, I taught mathematics at a secondary school (where this study was carried out) over a period of 6 years. During this time I presented numerous extra mathematics lessons to learners in the afternoons. These lessons were usually a repeat of topics taught during the school day. However the lessons taught in the afternoon enjoyed a slower pace in order to give learners more personal and individual attention to specific problems they encountered. Furthermore there was a strong focus on repetition and reinforcement. At the time when I joined the university, I embarked on postgraduate studies and wanted to revisit the presentation of extra mathematics classes as these mentioned above in order to attempt to shift the focus away from repetition and reinforcement and more towards a constructivist approach that would promote and advance the conceptual understanding of learners who struggle in mathematics.

This is how the initial idea for this study was conceived and I approached the school to discuss the possibility of a systematic inquiry. It became clear during my discussions with the principal and the mathematics staff, that a growing number of learners were taking extra mathematics lessons after school, not only as part of the academic support programme offered at the school, but also with private tutors and at other schools elsewhere. The decision was then taken to conduct a survey amongst the learners to ascertain approximately how many learners attended these extra mathematics classes after school hours, as well as their reasons for doing so and the format of these extra lessons (such as individual or group tuition).

This initial survey was carried out as part of a "preliminary investigation" (van den Akker, 1999) or "front-end analysis" (Nieveen, 1999; Plomp, 2002) that is usually carried out in the initial stages of development research (as defined by van den Akker & Plomp, 1993). I hoped that through this survey of current practices, the needs and problems of the intended population for the study could be better understood (van den Akker, 1999). The results of this survey in fact initiated a change in the initial intended population of the study.

The results of the survey did not directly contribute to answering the research question of the study and are subsequently not included in this report. In the front-end analysis however, I found that even in a successful and well-resourced urban school, a significant number of learners still struggled with mathematics. Even more disconcerting and concerning, was that with all learners having to do Mathematical Literacy as a compulsory subject in the next few years, more and more learners may start resorting to extra mathematics lessons after school in order to successfully pass the subject at the end of Grade 12. It then becomes a matter of fiscal privilege; those who can pay, attend. The following questions came to mind: What will then become of those learners who are already performing significantly below the required standard in the lower secondary school, who still need to continue to do mathematics (in the form of Mathematical Literacy) until Grade 12, and who cannot afford to invest in extra mathematics tuition? Should we not embark on trying to design interventions that intercede in a diagnostic manner? Such interventions could perhaps assist these learners in remediating and improving their conceptual understanding where necessary in order to try and prevent an excessive need for extra mathematics lessons or continued failure on the part of the learners. It is against this background that the course and focus group of this study proceeded to change.

In response to the questions that arose from the front-end analysis, I decided to engage in the following process: Instead of working with learners who had chosen to attend extra mathematics lessons after school, I decided to concentrate on the learners struggling to pass mathematics in Grade 8. *I therefore designed and implemented an intervention within the structure that the school already had in place for these learners.* This structure existed in the form of remedial mathematics and English classes for Grade 8 and 9 learners that the school had created within the time table of selected learners in lieu of a third language. These classes are explained in Chapter 3.

1.3 Rationale of the inquiry

The reason for conducting this inquiry was to establish whether the theory of Realistic mathematics Education could be used with these low attaining Grade 8 learners in the remedial classes to improve their understanding. Furthermore it aimed to investigate possible characteristics and design principles of using RME in such a setting in order to inform the development and implementation of similar investigations. As mentioned, the key concepts dealt with in the intervention included place value, fractions and decimals.

Grade 8 learners were specifically chosen as the target group for the following reasons:

- According to the curriculum, it is the first year that they encounter algebra in a more formal manner and start performing calculations with it (which is usually when a lack of conceptual understanding in this regard is revealed).
- Currently learners are able to drop mathematics as a subject at the end of Grade 9 when they select their six subjects to take through to Grade 12. The fact that this will no longer be a possibility when Mathematical Literacy is introduced as a compulsory core subject in 2006, reiterates the importance of paying attention to remediation at Grade 8 level.
- In the past some learners coming from primary school failing mathematics, have continued to fail it throughout Grades 8 and 9 and, provided they met all the other requirements to pass, were promoted through to Grade 10. This meant that some learners could pass through 9 years of formal schooling acquiring very limited mathematical knowledge and skills.

In general, remedial and compensatory programmes are used internationally for a number of reasons, including amongst others: to provide additional instruction and tutoring for disadvantaged youth, to provide extra instruction for tertiary education preparation (Flaxman, 1985), to assist learners who need help in meeting the expected standards (National Association of State Boards of Education [NASBE], 2002) and to prevent or amend the academic problems resulting in school failure (Flaxman, 1985). Providing remedial interventions in mathematics is by no means a new concept although providing successful remediation for low achievement remains a challenge. The traditional approach in South Africa in dealing with learners who were not meeting the required level was to retain these learners in the same grade and have them take the required instruction over again. This approach however, does not have a high success rate and increases the probability of a learner not completing school (NASBE, 2002). Recent policy in South Africa has addressed this issue and stipulates that learners may not repeat a year more than once in a phase (DoE, 2002). No policy has been put in place yet on remedial or compensatory programmes or interventions for such learners.

This inquiry is therefore an important start to investigating the effects of an intervention (using the RME approach) that aims to assist low performing learners to improve their understanding, specifically with regard to key number concepts such as place value, fractions and decimals. This study serves as the exploratory phase in designing a prototype that can be reviewed and refined through subsequent cycles of implementation that occur in development research (Gravemeijer, 1994; van den Akker, 1999), and also assist in contributing to the potential development of future policy in this regard.

1.4 Problem statement

Initially the problem was to design and implement an intervention that could potentially assist the Grade 8 learners in the remedial classes to improve their understanding of key number concepts in mathematics such as place value, fractions and decimals. It was decided to provide learners with an appropriate schema (a structure of conceptualised knowledge) of these concepts on which to further construct their understanding of algebra (Skemp, 1989), which is formally taught during their Grade 8 year. The concept "understanding", is appropriately explained by Skemp (1971, p. 46 as cited in Skemp, 1989): "to understand something means to assimilate it into an appropriate schema."

Based on discussions with the Grade 8 mathematics teachers, the mathematics results of learners in the remedial classes, and the fundamental errors that these learners made in the tests, I assumed that many of them did not have "appropriate schemas" in place with regard to their understanding of the key number concepts. They would therefore continue to struggle in their learning of algebra without an appropriate schema into which to assimilate the new concepts they would be confronted with. Many of these learners also appeared to be inclined to be engaging in what Skemp (1989) refers to as "habit learning" rather than the preferred form of learning in mathematics, which he refers to as "intelligent learning". The following differentiation is offered between the two types of learning:

> *Habit learning keeps the learner dependent on being told what to do in every new situation, with little confidence in his own ability to cope if left on his own. Intelligent learning develops the learner's confidence in his own ability to deal with any situation which can be understood in relation to his existing knowledge, and encourages perception of the teacher as someone who can help him to increase this knowledge, and thereby his power of understanding. (Skemp, 1989, pp. 44, 45)*

The problem was therefore to design and implement an intervention that would improve learners' understanding of the key concepts mentioned and in doing so provide learning situations in which learners could achieve this understanding largely by their own endeavours, resulting in intelligent, as opposed to habit learning (Skemp, 1989). This meant designing and implementing an intervention that would not teach new content to learners as such, but rather revisit topics they had already been taught at primary school with the purpose of also gaining an understanding of what the *characteristics* are of such an intervention. In addition to this, design principles that could be taken into account if others were to design a similar intervention for another topic or school context could also perhaps be generated. The following sub-section 1.4.1 first gives an overview of the process involved in selecting the theory chosen to guide the design and implementation of the intervention, before outlining the theoretical framework. Section 1.4.2 then presents the key research question that was formulated to guide the research process as outlined in this report.

1.4.1 Theoretical framework for the intervention

In order to start designing the intervention, literature and previous research relating to learning mathematics, and more specifically, the teaching and learning of mathematics to low attaining learners (and other terms used to characterise these learners - see Chapter 2) was reviewed. From the literature, five common aspects were identified to be included in the instructional approach of the intervention (these are elaborated on in Chapter 2):

- *More focus on relational and conceptual understanding* as opposed to learning by rote and memorisation (instrumental understanding)
- *Creating meaningful learning contexts* that actively involve learners
- *Greater emphasis on problem-solving* and less emphasis on computation and arithmetic skills

- The *importance of social interaction* in the learning process (i.e. group work, reciprocal teaching, games etc.)
- The *importance of language development and discussion* with and between learners in teaching mathematics

The theory of Realistic Mathematics Education (RME) was subsequently selected as the domain-specific theory to guide the design and implementation of the intervention. This decision was based primarily on the fact that RME encompasses the five aspects identified above and embraces development research in its approach to the creation and refinement of its instruction theory and materials.

International studies (Treffers, 1987; Streefland, 1991; de Lange, 1996; Vos, 2002), including studies from other developing countries such as Indonesia (Armanto, 2002; Fauzan, 2002) have shown that the Realistic Mathematics Education (RME) theory is a promising direction to improve and enhance learners' understanding in mathematics. RME has its roots in Hans Freudenthal's interpretation of mathematics as a human activity (Freudenthal, 1973; Gravemeijer, 1994) and accentuates the actual activity of doing mathematics. This is an activity, which he envisaged should predominantly consist of organising or mathematising subject matter, taken from reality. Learners therefore learn mathematics by mathematising subject matter from real contexts and from their own mathematical activity, contrary to the traditional view of presenting mathematics to them as a ready-made system with general applicability (Gravemeijer, 1994).

1.4.2 Research question

In the previous sections I described the context and complexities of this investigation. I now pose the following key research question, which guided this inquiry:

> *How can the theory of Realistic Mathematics Education be used with low attaining learners in mathematics in order to improve their understanding with relation to the key number concepts of place value, decimals and fractions, and what characteristics emerge from implementing the intervention in such a setting?*

In order to operationalise this question, it was broken up into the following three sub-questions:

a) *Is it viable to apply the theory of Realistic Mathematics Education in such a setting? (i.e. with* low attainers *to* revisit concepts *they had already previously learnt)*

b) *How can it be used to improve learners' understanding of place value, fractions and decimals?*

c) *What possible design characteristics emerge from applying the theory of RME in such a setting?*

1.5 Research approach

This formative type of research that is related to design and development work is described in the literature under various labels such as (van den Akker, 1999, p.3):

a) *Design* studies; *Design* experiments; *Design* research;
b) *Development*/*Developmental* research;
c) *Formative* research; *Formative* inquiry; *Formative* experiments; *Formative* evaluation

The term used throughout this report is *development research* (van den Akker & Plomp, 1993; Gravemeijer, 1994) Two types of development research informed this study.

The first approach is one mentioned by van den Akker (1999), van den Akker and Plomp (1993) and Plomp (2002) and appears in a variety of sub-domains, such as curriculum, media and technology, learning and instruction as well as teacher education and didactics (van den Akker, 1999). This study, however, was restricted to the domains of curriculum and, learning and instruction.

Regarding curriculum design, van den Akker and Plomp (1993, p. 20) defined development research by its twofold purpose:

(i) supporting the development of prototypical products (including providing empirical evidence for their effectiveness),

(ii) generating methodological directions for the design and evaluation of such products.

The scope of this study is contained mainly within first purpose above and reports on the design and implementation of an intervention (comprising instructional activities). From the data collected during the study (that examines the characteristics and effectiveness of the intervention), a prototype can be developed that can be further evaluated, implemented and refined in a later study. While some methodological directions for the design of such an intervention in a similar setting are offered in Chapters 4 and 6 of this report, these are not intended to be generalisable but rather emerge as guidelines for the next phase of developing and implementing the prototype. As mentioned, a front-end analysis (Nieveen, 1999) was also carried out prior to developing the intervention as an initial stage in this type of development research.

In the sub-domain of learning and instruction, the emphasis is on developmental work relating to designing effective learning environments, formulating relevant curricula and assessing achievements of cognition of learning (Greeno, Collins & Resnick, 1996 as cited in van den Akker, 1999). The development of these learning environments was guided by the second approach outlined below.

The second approach is one proposed by Freudenthal (1991) and Gravemeijer (1994) and is contained more specifically within the context of instructional practice in mathematics education. The core of this approach to development research is formed by classroom teaching experiments that focus on the development of instructional sequences and the instructional theories that underpin them (Gravemeijer, 1994). During the course of the study, as the researcher and designer of the intervention, I developed sequences of intended instructional activities that were based on a combination of my own experience, relevant literature pertaining to learners who struggle with mathematics, as well as related literature about how learners learn mathematics (Gravemeijer & Cobb, 2002). These instructional sequences were then implemented with a total of 12 learners over a period of three academic school terms in order to analyse the actual process of learning that took place during the course of the lessons.

The two development research approaches outlined above were then combined in this research project. How this was done, is explained in the research design chapter (Chapter 3) where a visual model integrating the two approaches is also offered.

The intervention was implemented with a small number of learners (n=12) in order to investigate its effectiveness and potential for further development and implementation, and to gain further insight into the characteristics of the intervention that should be included in the prototype. The study is therefore classified as a development case study to try and find out whether or not the theory of RME could be used to improve low attaining learners' understanding of the key number concepts of place value, fractions and decimals.

Furthermore, this research was not conducted however, to improve either my practice (as the presenter) or to transform the practice of the teachers involved. It was done with a view to establish "what does or doesn't work" within the initial design and implementation of the intervention and to try and investigate "why or why not" with a view to identifying characteristics that can be tested in further studies. This therefore places the study outside of the immediate classification of action research.

1.6 Scope

Literature pertaining to the teaching and learning of mathematics in general and also, more specifically with regard to low attainers and the key number concepts, was first investigated in order to identify common aspects that could be included in an instructional approach for the intervention. The five aspects identified have already been mentioned in section 1.4 and the research literature in the domain of RME was then studied prior to and during the intervention in order to guide its design and implementation.

Although embedded in the theory of RME and drawing on the relevant literature with regard to low attainers, the intervention was not a "ready-made" package that already existed as implementation began. The objectives, instructional approach and guiding theory (RME) were in place but the instructional sequences, that constituted the intervention, evolved throughout the study. During this time, my own knowledge and understanding of RME was also evolving and subsequently the three cycles of implementation that took place during the second, third and fourth academic terms of 2003, each became more in line with the heuristic design principles of RME as the study progressed.

Initially the demarcation of the three cycles (first one in term 2, second one in term 3 and third one in term 4) was only in terms of the academic terms and the different key concepts that I had planned to address in each of the cycles. The way the study evolved though, the cycles also became further characterised by a specific focus on one or more of the heuristic design principles. While the first two cycles remain exploratory cycles, the third one was more controlled in that a formal attempt was made to evaluate the actual implementation of the RME instructional approach against the intended implementation thereof. Chapter 4 of this report gives an account of the three cycles, taking a more reflective approach in examining the first two cycles and an evaluative approach in recounting the third cycle.

1.7 Limitations

As already mentioned, one of the limitations in the study was my partial understanding of the theory of RME at the start of the intervention. This resulted in some instructional activities being implemented during the first cycle that could have been improved upon and more in line with the theory. The first cycle dealt mainly with the key concept of place value and it is my opinion that this cycle in particular would need to be substantially revised in future applications of the intervention. This therefore limits the use of the data in the first cycle to directly answer the research question, although it did prove to be valuable in informing the development of the next two cycles.

Although the intervention was implemented over three academic terms, learners only attended the remedial mathematics classes for three 45-minute lessons every ten-day cycle. This meant that I would have classes with them once in one of the two weeks of the ten-day cycle and twice in the alternate week. This often resulted in as much as a week between follow-up lessons, which meant that if an activity or problem had been carried over from the previous week, that the first ten minutes of the lesson would need to be spent getting back into the problem or activity. Sometimes public holidays and days taken up by sport or changes in the timetable resulted in me not seeing a class for up to two weeks. These infrequent classes also meant that it took a number of weeks to get the class used to me as the presenter and also to establish some semblance of a new classroom culture for these particular lessons, especially as the approach differed somewhat, and in one class even contradicted, the approach learners were accustomed to in their daily mathematics lessons with their regular mathematics teacher.

Another limitation that needs to be acknowledged is the danger of, as Bennie, Olivier and Linchevski (1999) concluded, grouping learners according to their ability. This can in fact even provide a barrier to their achievement due to impoverished learning environments, the possible lowering of the teachers' expectations and increased time being spent on managing the learners' behaviour. For logistical and practical reasons, this study did group low attainers together (with the exception of one of the learners who was there on the request of her parents and not because of poor performance in mathematics), but it is not necessarily assumed that these were all low ability learners. Without grouping them together, this particular study would have not been possible and it is believed that the learning environment was continually controlled in order to ensure that it was not "impoverished".

The fact that this study was only conducted with low attaining learners within one school, which is an all girls school and with only two groups, made the study very vulnerable towards contextual and accidental influences, especially as my roles included developer, presenter and researcher of the intervention. Where possible these have been acknowledged and discussed.

A final consideration is that I previously spent almost six years teaching at the school where the case study was conducted. This ensured familiarity with the workings of the school and the mathematics teachers, although I had not previously had any contact with any of the learners. Careful attention therefore needed to be paid to avoid the influence of bias due to this. On the other hand this also proved to be somewhat of an advantage as the school and the teachers involved were quite comfortable with my presence.

1.8 Summary

This chapter has given an overview of the study in terms of its origins, development, guiding question and subsequent methodology. The study is being presented as a development case study that aimed to improve the understanding of the learners participating in the intervention, while also investigating the characteristics of implementing the theory of RME to revisit key number concepts with low attaining learners. This was done with a view to improving learners' structures of conceptualised knowledge in mathematics with regard to the key number concepts of place value, fractions and decimals and to empower learners to make more use of intelligent as opposed to habitual learning (as defined by Skemp, 1989). With improved understanding regarding the basic key number concepts, it was anticipated that learners would have a more appropriate schema into which to assimilate the algebra they are taught at a more formal and abstract level in Grade 8. A distant outcome of this was that their academic performance at school would also subsequently improve. However, it is not possible within the scope of this study to be able to prove or justify that any change in their academic performance was due to the intervention.

The domain-specific theory selected to drive the design and implementation of the intervention was the theory of Realistic Mathematics Education that has it roots in Hans Freudenthal's interpretation of mathematics as a human activity. The theory encompassed all the aspects identified and incorporated into the desired instructional approach, which was set up for the intervention through the process of a literature review on the teaching and learning of mathematics to low attaining learners. Subsequently the main research question that guided the study was put forward and the rest of this document reports on the implementation of the study in an attempt to research, document and answer this question.

1.9 Outline and organisation of this report

The report is divided into six chapters, each serving an individual purpose, but overlapping and intertwining nonetheless. The first chapter has been summarised in section 1.8 and serves as an introduction to the study and its origins. Chapter 2 reports on the literature review, during which the theory of Realistic Mathematics Education was selected as a domain-specific theory to drive the design and implementation of the intervention. An overview of the theoretical underpinnings of RME is presented and reasons substantiating why the theory of Realistic Mathematics Education was chosen are elaborated on. Chapter 3 serves as the research design chapter. It firstly establishes the epistemological paradigm of the study before discussing the methodology (development case study) and elaborating on the mixture of qualitative as well as quantitative methods used to collect and analyse data. The site and sample of the study are also further introduced in this chapter and ethical issues as well as issues of validity and reliability are dealt with. The fourth chapter of this report gives an account of the three cycles of the intervention. A reflective approach is used to describe first two cycles, drawing on the data collected during this time, which mainly consisted of work samples from learners, observations from teachers and an assistant researcher where possible, and interviews conducted with learners. The portrayal of the instructional approach in the third cycle takes an evaluative stance. Chapter 5 is the data analyses of the overall performance of learners during the course of the intervention and provides an individual profile for each of the twelve learners. The final chapter reflects on the study and its research process as a whole before making final conclusions and recommendations.

CHAPTER TWO

2 RME as a theoretical framework

2.1 Introduction

In this chapter, the theoretical framework for the intervention is discussed and reasons for selecting the theory of Realistic Mathematics Education (RME) for the study are put forward. The decision to use the theory of RME was not one that that was made before the study began, but one that was taken once a literature review that looked at the teaching and learning of mathematics to learners who fall into the category of performing below the required standard, had been embarked on. Five common aspects emerged from the reviewed literature on the teaching of mathematics to learners in this category. Once these aspects had been identified, a theory in mathematics education was sought that encompassed these five aspects. The theory of RME was subsequently selected as the theoretical framework to drive the design and implementation of the intervention.

In order to simulate the process of this reasoning for the reader, this chapter first clarifies the use of the term *low attainers* to refer to the target population of the study throughout this report. A review of the literature on the teaching and learning of mathematics to low attainers is then summarised, with a specific focus on the possible causes and characteristics of low attainment and how the literature suggests we address these. The five aspects that were identified are then discussed in detail, before the theory of RME is introduced and explained. RME is also then looked at in relation to other global innovations in mathematics education so that the reasons for choosing the theory can be illuminated. The chapter closes with a discussion of how the theory of RME applies to low attainers.

2.2 Low attainers

Many terms or descriptions are used in the literature to refer to learners in this category. These include terms such as: remedial, disadvantaged, special needs, under-achievers, slow-learners and low achievers (e.g. Denvir, Stolz & Brown, 1982; Haylock, 1991; Swanson, Hoskyn & Lee, 1999; Kroesbergen & van Luit, 2003), which are used in schools to refer to children with undefined problems. As mentioned earlier on, the school where this particular case study is situated, additional mathematics and English classes provided for learners with problems during school are called the *remedial classes*. The learners in such remedial classes, however, have not necessarily been formally identified as being special needs or remedial learners or having any form of learning disability. The decision of the teacher to place them in the remedial class was made based on their final mark for mathematics in Grade 7, their mathematics mark for their first term of Grade 8 and the results of a baseline assessment administered to all learners in their first few weeks of high school.

For this report, the term *low attainer* has been chosen to refer to the learners in these remedial classes. This implies that the observable performance of the learners are described, without implying a cause (Denvir et al, 1982). The term *low attainer* should therefore not be seen as a judgment (Haylock, 1991) or as a label, and no direct effort is made in this study to identify whether the low attainment is because of the learners' ability or not. Working with the concept of "mathematical ability or abilities" has been avoided due to the inherent problematic associations with this term. Firstly the concept of "ability" is difficult to adequately define and measure, and includes related concepts such as capacity, aptitude, skills and achievement (Kilpatrick, 1980). Secondly, as Wheeler (2001) points out, there is a danger in labeling a learner's ability and then focusing on the labels rather than on the learner and allowing the labels attached to the learners' ability to dominate the educational practices and interaction with the learner.

2.3 Teaching and learning Mathematics (with specific reference to low attainers)

From a critical review of the literature on mathematics interventions and programmes for learners with ***mathematical difficulties*** (e.g., Baroody & Hume, 1991; Dockrell & McShane, 1992; Mercer & Miller, 1992;), ***learning disabilities*** (e.g., Cawley & Parmar, 1992; Swanson, et al, 1999; Dunlap & Thompson, 2001; Geary & Hoard, 2001), ***Special Educational Needs (SEN)*** (e.g., Daniels & Anghileri, 1995; Kroesbergen & van Luit, 2003; Magne, 2003) and ***low attainers*** (e.g., Hart, 1981; Denvir et al, 1982; Trickett and Sulkie, 1988; Haylock, 1991), there appears to be considerable evidence that arithmetic computation and basic mathematics skills are the dominant domains. The definition of mathematics provided in the New Revised

National Curriculum Statement for Grades R-9 in South Africa (DoE, 2002, p.1) broadens the scope of mathematics far beyond this though. The definition states that:

> *Mathematics is a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves. Through this process, new mathematical ideas and insights are developed. Mathematics uses its own specialised language that involves symbols and notations for describing numerical, geometric and graphical relationships. Mathematical ideas and concepts build on one another to create a coherent structure. Mathematics is a product of investigation by different cultures – a purposeful activity in the context of social, political and economic goals and constraints.*

The document also outlines the interrelated knowledge and skills included in the scope of mathematics, and stresses the importance of mathematical literacy to enable persons to "contribute to and participate with confidence in society" (Ibid p.2):.

The teaching and learning of mathematics can enable the learner to:

- *develop an awareness of the diverse historical, cultural and social practices of mathematics;*
- *recognise that mathematics is a creative part of human activity;*
- *develop deep conceptual understandings in order to make sense of mathematics; and*
- *acquire the specific knowledge and skills necessary for:*
  - *the application of mathematics to physical, social and mathematical problems,*
  - *the study of related subject matter (e.g. other Learning Areas), and*
  - *further study in mathematics" (Ibid p.4).*

In this context, Daniels and Anghileri (1995) identify the fundamental aim of teaching mathematics to equip learners with the strategies, skills, knowledge and most importantly the confidence to use their mathematics to solve problems that learners will encounter throughout their lives. If mathematics teaching does not result in providing learners with these skills, then an important part of their preparation for life is missing and they have been denied access to a basic human right (DoE, 2002).

Also, Denvir et al (1982) categorise mathematical aims under three broad headings, i.e.:

Useful*: as a tool for the individual and society, e.g. social competence, vocational skills.*
Cultural: *as part of our culture of which all pupils should have knowledge and experience.*
Pleasurable*: as a potential source of enjoyment.*

They add that the aims for low attainers do not differ from those stated above, although the priorities may differ depending on the needs of the learner. If the experiences in the classroom are not resulting in the learner gaining in any of these categories above, there remains little justification for keeping learners in the mathematics classroom. With the pending implementation of Mathematical Literacy in South Africa (DoE, 2002) which will result in all learners needing to pursue this subject until they leave school, we need to confirm that we do indeed have sufficient justification for keeping all learners learning mathematics. We need to ensure that even the low attainers will profit from the scope and aims of mathematics as outlined above.

Although I acknowledge that in practice computation has been interpreted as a prerequisite to any other mathematical knowledge (Parmar & Cawley, 1991), by continually focusing too much on this domain, are we allowing low attaining learners the full benefit of the definition and scope of mathematics? Daniels and Anghileri (1995, p. 23) suggest the following in response to this rhetorical question:

> *To bring SEN pupils to an understanding of the relationships and patterns that constitute mathematics itself, they will need to be involved with practical tasks,* applying *mathematics to 'real-life' problems, exploring and investigating their findings and discussing their thinking with peers and teachers.*

The rest of this section suggests ways we can address this, but first examines possible characteristics and causes of low attainment in mathematics.

2.3.1 Possible characteristics and causes of low attainment

Kroesbergen and van Luit (2003), draw on the work of Goldman (1989), Mercer, (1997) and Rivera (1997), and offer some general characteristics of learners who have difficulty in learning mathematics, which I believe are useful for this particular inquiry. These include: memory deficits, inadequate use of strategies for solving mathematics tasks, and deficits in generalisation and transfer of learned knowledge to new and unknown tasks. In this regard Haylock (1991) adds the following to this list: reading and language problems, perceptual problems and poor spatial discrimination, social problems and mathematics anxiety. This is not to say that all low attainers exhibit most or even many of the characteristics outlined above, but that these are general observations from research within this field.

In their book entitled, Low Attainers in Mathematics 5 - 16: Policies and Practices in Schools, Denvir et al (1982) offer the following list as likely causes of low attainment: physical, physiological or sensory defects; emotional or behavioural problems; impaired performances due to physical causes such as tiredness, drugs and general health; attitude, anxiety, lack of motivation; inappropriate teaching; too many changes of teachers (lack of continuity); general slowness in grasping ideas; cultural differences, English not first language; impoverished home background; difficulty in oral expression or in written work; poor reading ability; gaps in education, absence from school, frequent transfers from one school to another; immaturity, late development, youngest in the grade; low self-concept leading to a lack of confidence (Ibid p.19).

They further subdivide these factors into three categories, which include: factors beyond the control of the school, factors partly within the school's control and factors that are directly within the control of the school. The causes, which they then identify as controlled by the school, include (p. 21):

- inappropriate teaching methods or content;
- lack of suitable materials;
- lack of responsiveness to pupil's problems or lack of teacher's time to reflect on the pupil's difficulties and plan suitable work;
- a teacher's lack of detailed knowledge of the mathematics being taught, including a knowledge of which skills, concepts, etc are involved;
- a teacher's inability to motivate and involve pupils and organise work efficiently;

Also, Feuerstein (1980) has suggested that many different reasons, ranging from genetic to environmental factors, explain low cognitive performance. Abel (1983) takes the standpoint that environment rather than innate ability may be a key factor in learners' performance in mathematics. Referring to research reported by Ginsberg, Klein and Starkey (1998) and Gouws (1992) as examples, Reusser (2000) proposes that there is convincing evidence that most observed failures and low performances in mathematics are due to insufficient teaching-learning environments and not due to genetic factors at all. He also states that learning difficulties that have a neuropsychological diagnosis are "substantially reinforced and shaped by environmental influences such as insufficient measures taken by the instructional and educational support systems" (p. 1). Baroody and Hume (1991) agree and make a case that most children who experience learning difficulties are recipients of instruction not suited to how children think and learn. This in turn puts the onus on the curriculum and instructional techniques (the environment) as opposed to the learner.

In my opinion, these possible characteristics and causes identified in the preceding paragraphs, suggest that low performance or attainment in mathematics is something that can be "treated". In most cases, it is not an incurable condition that learners are born with, but something that develops as a result of the type of instruction learners receive and the teaching-learning environment (Reusser, 2000) within which they experience mathematics. The implications of this for the inquiry I carried out were, that the instructional approach and teaching-learning environment to be applied in the intervention became central to the literature review and the subsequent choice of a theoretical framework.

2.3.2 Improving teaching and learning Mathematics for low attaining learners

I therefore agree with Abel (1983), Baroody and Hume (1991) and Reusser (2000) and work on the assumption that the environmental aspects of the mathematics teaching and learning can affect a learners' performance. In order to identify the environmental aspects that might make a difference, literature by experts in the field of mathematics education and more specifically low attainment in mathematics was further reviewed. This was done to ascertain whether or not there were any common aspects that could be recognised within the literature. Aspects suggested by various experts are fore-grounded (using italics) in the paragraphs below and the common aspects that emerge are summarised in the final paragraph of this sub-section.

In their book entitled Secondary Mathematics and Special Educational Needs, Daniels and Anghileri (1995) examine the benefits of environmental aspects such as *appropriate practical work, problem-solving, games* in the mathematics classroom, *group work, co-operative learning, reciprocal teaching* and the *active participation of learners* during lessons. They also stress the point that *learning needs to be relevant* to the lives of the low attaining learners in order for it to be *meaningful*. This does not however mean that all mathematics problems should be based in real-life contexts though, as puzzles, games, patterns and brainteasers can also be used.

I here want to refer to specific aspects that are relevant for creating conducive learning environments; For instance, Denvir et al (1982) encourage teachers to embrace the role of experimenters and to try out ideas developed by themselves and their colleagues. In doing so, they encourage teachers to observe the low attainers in order to gain some insight into their "strengths and weaknesses, present state of knowledge, and to probable causes of the low attainment…" (p. 50). This allows the teacher to plan suitable work for individuals that can be extended, adjusted or abandoned, depending on how effective it turns out to be. They in turn warn against continued emphasis on computations (arithmetic skills) at secondary school and motivate this with the indication from research that learners in the 12 to 15 age range show little improvement in their performance in this regard (Hart, 1981). Denvir et al (1982) also propagate the value of *learners discussing their work* as well as the advantages of *engaging in problem solving* with low attainers. Due to the poor memory for facts and procedures that many low attainers appear to have, the research discourages the use of instrumental instruction that relies heavily on memory, and instead encourages more *emphasis on relational understanding*. In doing so, they refer to the work of Skemp (1971; 1989) relating to understanding.

Skemp (1971, 1989) differentiates between *relational* and *instrumental* understanding. Instrumental understanding on the one hand, he suggests is "rules without reasons" in that learners may possess the necessary rules, and ability to use them, without actually comprehending why or how that rule works. Often learners will need to memorise more and more of these rules in order to avoid errors and this type of understanding therefore encompasses a "multiplicity of rules rather than fewer principles of more general application" (1991, p. 5). Relational understanding, on the other hand, involves integrating new ideas into existing schemata and understanding both "what to do and why". Although lower ability learners may need more substantial support than other able learners in constructing their own meanings and connections, this building up of a schema (or conceptual structure) becomes an intrinsically satisfying goal in itself and the result is, once learnt, more lasting. Skemp (1989) uses an analogy of a stranger in a town to differentiate between the two types of understanding. One could have a limited number of fixed plans that take one from particular starting locations to particular goal locations in the town. He provides this as an example of instrumental understanding. On the other hand one could have a mental map (schema) of the town, from which one can produce, when needed, an almost infinite number of plans to guide one from a starting point to a finishing point, provided only that both can be imagined on the mental map (relational understanding).

The work of Haylock (1991) is significant because it discusses factors associated with low attainers, drawing on classroom-based research, and proposes a strategy for teaching learners in this regard. Although focused on learners who are between 8 and 12 years old, Haylock's book (1991) on Teaching Mathematics to Low Attainers, 8-12 can still be considered relevant for lower secondary learners (aged between 13 and 15). Haylock's work foregrounds the following main themes:

- the *development of understanding* as opposed to the learning of routines and procedures,
- the importance of tending to *language development* in teaching mathematics,
- the need to specify *realistic and relevant objectives* for the learners,
- the aspect of *numeracy and the basic ability to use a calculator* effectively,
- the use of *small group games* and finally,
- the need to identify "*purposeful activities in meaningful contexts*" (p. 5).

Haylock is of the opinion that it is necessary to maintain a balance between providing learners with success through the attainment of *set objectives* while also providing them with activities in *meaningful contexts* that they find *relevant* and *purposeful*.

Baroody and Hume (1991) suggest that in order for mathematics instruction for low attainers to improve, it needs to *focus on understanding*, encourage *active and purposeful learning*, *foster informal knowledge*, *link formal instruction to informal knowledge*, *encourage reflection and discussion* and include *Socratic teaching* (which involves a combination of the afore-mentioned elements).

Parmar and Cawley (1991) challenge the "routines and passivity that characterise arithmetic instruction for children with mild handicaps" (p. 1). They suggest that more approaches which encourage *learners to be active*, *productive learners* and allow them the opportunity to *demonstrate the extent of their thinking and creativity* are needed in special education classes.

Looking through the aspects above that pertain to the teaching of low attainers mentioned in this sub-section, one that appears repeatedly is the aspect relating to a greater involvement on the part of the learner in the learning process (i.e. the learner being more active). It is suggested more than once that in order to do this, learners need to be engaged in more meaningful or purposeful contexts, such as problem solving and games. Other aspects referred to by more than one scholarly source are: the need to focus on the development of understanding and the importance of discussions, both between learners themselves and with the teacher.

Using these common aspects from the literature, and drawing on my own experience as a mathematics educator, a list of five aspects to include in the instructional approach to use in the intervention, was compiled. The following section outlines and examines these aspects in more detail.

2.4 Relevant environmental aspects in an instructional approach for low attaining learners

In the previous section, the process that was used to identify the aspects explained in this section was illustrated. A clear demarcation between these aspects is however not intended, as they do overlap on a number of features. The five aspects that would be focused on in the instructional approach of the intervention are:

- *More focus on relational and conceptual understanding* as opposed to learning by rote and memorisation (instrumental understanding)
- *Creating meaningful learning contexts* that actively involve learners
- *Greater emphasis on problem-solving* and less emphasis on computation and arithmetic skills
- The *importance of social interaction* in the learning process (i.e. group work, reciprocal teaching, games etc.)
- The *importance of language development and discussion* with and between learners in teaching mathematics

2.4.1 More focus on understanding

As demonstrated by Skemp's (1971) differentiation between relational and instructional understanding, a chasm may exist between what learners are able to do and what they in fact understand. Knowing what to do in a specific situation, but not necessarily understanding why it works, may limit the transfer of that procedure or skill. The increasing number of procedures that learners need to commit to memory in mathematics often results in learners in secondary school becoming confused or partly remembering and trying to apply procedures they have never fully understood (Daniels & Anghileri, 1995). Understanding on the other hand promotes remembering and enhances transfer owing to the reduced number of bits of knowledge that need to be simultaneously held in the short-term memory (Hiebert & Carpenter, 1992). The understanding that comes from making connections, seeing how things fit together, relating mathematics to real situations and articulating patterns and relationships also carries with it a satisfaction which can further motivate low attaining learners (Haylock, 1991). Also relating to this point are the fundamental misconceptions that learners might have and the necessity to reveal these in the learning process in order to facilitate further understanding (Hart, 1981; Daniels & Anghileri, 1995). Adapting to a teaching and learning style that encourages understanding therefore also requires the study of learner errors that occur while solving mathematical tasks (Reusser, 2000). This observation and analysis of errors provides a powerful means for analysing learner understanding as well as being a valuable source of information when used as diagnostic tools (Booth, 1984; Resnick et al, 1989). Rather than being seen as indicators of failure, errors should be viewed as "learning opportunities and as challenges to clarify conceptual misconceptions" (Reusser, 2000, p. 21).

2.4.2 Involving the learner through the use of meaningful contexts

It is a common understanding that most people are less resistant to learning something new when they can see the purpose or meaning of it. This is equally important for children at school, especially with regard to mathematics. Many people in fact currently hold an instrumentalist view of mathematics, which Ernest (1988) proposes:

> *…is the view that mathematics, like a bag of tools, is made up of an accumulation of facts, rules and skills to be used by the trained artisan skilfully in the pursuance of some external end. Thus, mathematics is a set of unrelated utilitarian rules and facts. (p. 10)*

In order to not restrict low attaining learners to this view but to instead meet the challenge of giving learners a full experience of what mathematics is, as defined by the Revised National Curriculum (see section 2.3), we need to seriously consider the purposefulness of activities that we require learners to engage in. When committed to a task that makes sense to them, there is a good chance that low attainers will surprise us with what they can achieve in mathematics (Haylock, 1991). For this purpose, the teacher should take on the roll of learning facilitator and assist in piquing the curiosity of the learners in order to actively engage them in the task. Active involvement can be regarded as any situation that creates questions or cognitive conflict in children's minds and that further encourages them to rethink their views (Baroody & Hume, 1991).

Denvir et al (1982) suggest that low attainers may learn incidentally when they become involved in an absorbing activity and actively participate in the "struggle". They also add that through this activity children may learn because they spot inconsistencies in their thinking, which they then try to resolve. De Korte (1995) lists learning as being "situated" as one of the major features of effective learning processes in mathematics. By this, he means that "learning essentially occurs in interaction with social and cultural context and artefacts, and especially through participation in cultural activities and practices" (p. 41).

2.4.3 Greater emphasis on problem-solving

As already mentioned, many mathematics interventions currently focus on improving computation skills of low attaining learners. From a number of observations made during school visits, Denvir et al (1982) concluded that some of the children who do not master arithmetic skills at primary school, spend most of their secondary school repeating this computation with very little success. Compounding this is the fact that problem-solving is often seen as an activity that is considered unsuitable for low attainers as, amongst other reasons, there are so many other skills to be practised that no time is left for such a luxury (and here clearly views differ on what is regarded as luxury and necessity)! Another reason cited for this is that the basic mathematical knowledge of low attainers is so weak that they will not be able to apply it to the solution of problems. This raises the question as to the usefulness and purpose of this basic mathematical knowledge if it cannot be used when required to solve a problem! As noted by the Cockcraft Report (1982, para. 249):

> *Mathematics is only 'useful' to the extent to which it can be applied to a particular situation, and it is the ability to apply mathematics to a variety of situations to which we give the name 'problem-solving'.*

Some of the benefits of the problem solving approach for low attaining learners as identified by Trickett and Sulkie (1988) include "better ability and willingness to question, to transfer and apply their mathematics, and to sort out even quite difficult problems" (as cited by Daniels & Anghileri, 1995, p. 66).

However, the understanding and solving of even simple mathematical word problems is a complex process that requires skilful interaction of at least three kinds of knowledge: linguistic, situational and mathematical (Reusser, 2000). Learners who are therefore severely lacking in the relevant types of knowledge and skills may instead adopt coping strategies that bypass the logic of mathematical sense-making activities. Such learners in turn need the guidance of "effective pedagogical settings" (p. 23). This includes presenting problems in contexts that are more familiar, realistic and therefore also meaningful to the learner, while also providing the necessary instruction and strategies to help low attainers to analyse, reflect and practice the overall required sequences in understanding and solving different types of problems.

2.4.4 Social interaction as part of learning

Cobb and Bauersfeld (1995) identify two general theoretical positions on the relationship between social process and psychological development. While one favours the social and cultural processes (collectivism), the other gives priority to the individual autonomous learner (individualism). One of the most well known theories relating to collectivism is that of Vygotsky (1979 as cited in Cobb & Bauersfeld, 1995) where "mathematical learning is viewed primarily as a process of acculturation" (p. 3). Individualism on the other hand is exemplified by neo-Piagetian theories, where the focus is on the individual, autonomous learner as he or she takes part in social interactions. While there appears to be an apparent opposition between these two views, both social and cognitive processes have their place in the learning of mathematics. Cobb and Bauerfeld (1995, p 7) cite the following quote from Saxe and Bermudez (1992):

> *An understanding of mathematical environments that emerge in children's everyday activities requires the coordination of two analytic perspectives. The first is a constructivist treatment of children's mathematics; Children's mathematical environments cannot be understood apart from children's own cognizing activities…The second perspective derives from socio-cultural treatments of cognition….Children's construction of mathematical goals and sub-goals is interwoven with the socially organized activities in which they are participants. (pp. 2-3)*

Without getting further into these theories, it suffices to say that social interaction remains an integral part of learning. Both interactions with peers and teachers can enhance learning through creating opportunities for learners to share understandings and verbalise thought processes (Daniels & Anghileri, 1995). Some suggested forms of this are group work, reciprocal teaching, sharing of strategies and games.

Schoenfield (1985) supports the use of small ***group work*** for the following reasons: opportunities for teacher assessment, an opportunity for learners to practice collaboration, less secure learners can watch more capable peers struggle, and decision making in a group facilitates the articulation of reasoning and knowledge.

Palinscar and Brown (1988) share an additional instructional procedure for small groups that they refer to as ***'reciprocal teaching'***. This mode of cooperative learning assumes the form of a discussion between the members of the instructional group and the teacher (or another facilitator which could also be a learner) who acts as a leader and a respondent. Four strategies are used to direct the discussion. The leader first frames a question to which the group members respond. A piece is then read and the leader summarises the gist of that piece. The group then comment and elaborate on the leader's summary and any necessary points are clarified. Finally, the leader prepares to move onto the next portion of text by making predictions about the upcoming content. Reciprocal teaching is underpinned by the premise that expert-led social interactions can provide a major impetus to cognitive growth (following along the lines of Vygotsky). It therefore plays an important role in learning and has been used by Palinscar and Brown (1988) as a strategy for collaborative problem solving.

Dockrell and McShane (1992) differentiate between learners being able to use a strategy and knowing when to use it. They hold the view that children are often unaware of the effectiveness of a strategy in relation to a particular problem and therefore do not make adequate use of it. However, when learners are encouraged to ***share their strategies*** and receive feedback that indicates the positive effect of the strategy, they tend to increase their use of it. The authors also argue the dynamic relation between a knowledge base and strategies. They suggest that:

> *Strategies often play a vital role in establishing a knowledge base, but once acquired, the role of strategies may become less important within the domain, because the relevant knowledge is available for retrieval. In cases of learning difficulties, it is often the case that the acquisition of knowledge is an issue. Thus, the use of strategies becomes a critical factor. Strategies require a knowledge base that provides the appropriate information on which the strategy can operate. In considering strategy training it is important to consider, as a first step, whether or not the child's knowledge base contains the information required for successful execution of the strategy. (p. 188)*

In the extensive meta-analysis of interventions for students with learning disabilities carried out by Swanson et al (1999), they classify studies within the analysis into two general approaches, namely direct instruction and strategy instruction. Strategy instruction includes verbal interaction between the teacher and the learners and the learner is viewed as a collaborator in the learning process. The teacher also provides individual feedback and makes use of verbal modelling and "think-aloud" models to solve a problem. From their first tier of analysis it was concluded that: "strategy instruction produces larger effect sizes than those studies that do not use such procedures" (p. 220). Sharing of strategies can therefore be included as an important aspect that can contribute to effective learning taking place in the teaching of low attainers.

Games are often regarded as primary school activities or something that can be used to fill up time or as an end of term activity. The United Kingdom in particular have recognised the powerful environments created through a game; so much so that they have recently incorporated games that enable assessment into their National Curriculum Assessment. Some of the benefits of games are that they provide the opportunity for learners to practise and consolidate routine procedures and number skills in a motivating environment that is neither threatening nor monotonous (Daniels & Anghileri, 1995). They also enable learners to develop problem solving strategies and aid in the acquisition and development of concepts. The opportunity is also created for teachers to observe their learners' thinking strategies and to interact with learners on a less formal level (Ernest, 1986; Haylock, 1991; Daniels & Anghileri, 1995).

2.4.5 The importance of language development and discussion

The effect of language on the learning of mathematics is a widely researched and debated topic not only internationally but also in South Africa (e.g. Howie, 2002; Setati, 2002). While there is no magic formula or solution as to how this issue should be addressed, specifically with regard to low attainers, it nonetheless remains a pertinent issue when designing programmes or interventions for these learners. Poor language skills such as reading, writing and speaking are often associated with low attainment in mathematics and in addition to that, mathematics has its own set of language patterns, symbols and vocabulary. A major part of developing an understanding of mathematics involves learning to handle these and make connections between symbols and their corresponding terminology and meaning (Haylock, 1991). Daniels and Anghileri (1995) stress that speech and written language are the tools of mathematical dialogue. The development of some aspects of mathematical thought may be constrained through a lack of access to these tools. As Dockrell and McShane (1992) point out, when solving a problem it is crucial that the learner first understands the problem before planning and executing a method for solving it.

> *Understanding is based on the child's cognitive and linguistic skills; planning a method involves constructing a mathematical representation of the problem; carrying out the plan involves executing the mathematical procedures that have been selected…Difficulties can arise in the comprehension of the problem, the construction of the mathematical model, or in the execution of strategies in solving word problems. However, it seems to be the complexity of the text of the word problem and the availability of a suitable basis for its mathematical representation that are the major determinants of performance (p. 139).*

Both the phrases "complexity of text" and "mathematical representation" in the quote above relate to use of language, in different senses however. The first relates to the written and spoken language of, for example, English. The second refers to mathematics as a collection of symbols, notation and terminology and how these all connect. Difficulties in either (and in many cases both) of these will indeed affect learners' performance and possible development in mathematics.

It would be useful if interventions aimed at assisting low attainers could therefore include components that can be used to diagnose and address the complexity of mathematics as a language and language as a tool for mathematics. This process can be assisted by the use of discussions in the classroom where learners are encouraged to verbalise their understanding, thoughts, solutions and ideas on the problems and tasks presented to them. This is not a simple task, however. In research carried out by Baxter, Woodward and Olson (2001), it was indicated that whole-class discussions are often dominated by verbal, capable learners, while the low attainers tend to remain passive. When they do in fact respond, their answers are typically simple and at times incomprehensible (Ball, 1993; Chard, 1999 as cited in Baxter, Woodward, Voorhies & Wong, 2002). Baxter et al (2002) report on the results and dilemmas that emerged during a year-long case study they carried out that focused on ways to include these learners in class wide discussions of problem-solving. One of the major dichotomies they allude to is that remedial environments that bring together only low achieving learners are not likely to result in rich, learner-centered discussions, while regular education classrooms may not provide the most optimal solution to the problem. They therefore suggest the use of small group work and sharing of strategies (without necessarily identifying the best solution) as possible interventions to alleviate the problem.

This section has presented the five aspects and examined each of them in more detail. Knowing that these aspects were to be the focus of the instructional approach in the intervention, a theoretical framework was sought that would accommodate all of them. The domain-specific theory of Realistic Mathematics Education (RME) from the Freudenthal Institute in The Netherlands was selected as the most appropriate theory to accomplish this task and the theoretical underpinnings of RME are first provided in the section below followed by an explanation of why RME was selected for working with these low attainers.

2.5 The theory of Realistic Mathematics Education (RME)

Realistic Mathematics Education has its roots in Hans Freudenthal's interpretation of mathematics as a human activity (Freudenthal, 1973; Gravemeijer, 1994). To this end, Freudenthal accentuated the actual activity of doing mathematics; an activity, which he propagated should predominantly consist of organising or mathematising subject matter taken from reality. Learners should therefore learn mathematics by mathematising subject matter from real contexts and their own mathematical activity rather than from the traditional view of presenting mathematics to them as a ready-made system with general applicability (Gravemeijer, 1994). These real situations can include contextual problems or mathematically authentic contexts for learners where they experience the problem presented as relevant and real.

The verb *mathematising* or the noun thereof *mathematisation* implies activities in which one engages for the purposes of generality, certainty, exactness and brevity (Gravemeijer, Cobb, Bowers & Whiteneack, as cited in Rasmussen & King, 2000). Through a process of progressive mathematisation, learners are given the opportunity to reinvent mathematical insights, knowledge and procedures. In doing so learners go through stages referred to in RME as horizontal and then vertical mathematisation (see Figure 2.1). *Horizontal mathematisation* is when learners use their informal strategies to describe and solve a contextual problem and *vertical mathematisation* occurs when the learners' informal strategies lead them to solve the problem using mathematical language or to find a suitable algorithm (Treffers, 1987). For example, in what we would typically refer to as a "word sum", the process of extracting the important information required and using an informal strategy such as trial and error to solve the problem, would be the horizontal mathematising. Translating the problem into mathematical language through using symbols and later progressing to selecting an algorithm such as an equation could be considered vertical mathematisation, as it involves working with the problem on different levels.

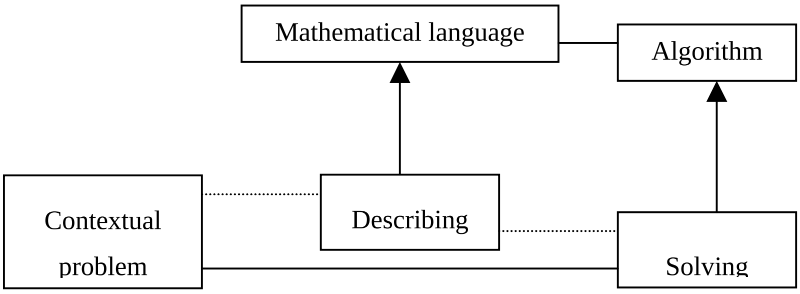


Figure 2.1 Representation of horizontal and vertical mathematisation

Horizontal mathematisation ( ); Vertical mathematisation ( )

Source: Adapted from Gravemeijer, 1994.

The traditional formal and authoritarian approach to teaching mathematics that has dominated in South African classrooms for a number of years has not afforded learners many opportunities to make use of horizontal mathematisation. Mathematics lessons are often presented in such a way that the learners are introduced to the mathematical language relevant to a particular section of work and then shown a few examples of using the correct algorithms to solve problems pertaining to the topic before being given an exercise of worksheet to complete (Venter, Barnes, Howie, & Janse van Vuuren, 2004). The exercises or worksheets are usually intended to allow learners to put the algorithms they have been taught into practice and may even contain some contextual problems that require the use of these algorithms. According to the RME model depicted in Figure 2.1, this type of approach places learners immediately in the more formal vertical mathematisation process. The danger in this is that when learners have entered that process without first having gone through a process of horizontal mathematisation, a strong possibility exists that if they forget the algorithms they were taught, they do not have a strategy in place to assist them in solving the problem. As pointed out in the literature in the previous section, this is especially prevalent with low attainers. This experience can be equated to someone being shown and told what is on the other side of a river and being expected to use what is there for their own benefit. However, they are not given or shown the bridge that assists one in crossing to the other side in order to make proper use of what is there. The horizontal mathematisation process provides this bridge.

This section began with an overview of the theoretical underpinnings of RME. The two sub-sections below endeavour to expand on this synopsis by briefly introducing two important tenets of the theory of RME, namely: the role of developmental research in continually developing and refining the theory, and the instructional design principles that the theory encompasses. Both sub-sections are given further attention in subsequent chapters of this report. Section 2.5.1 is once again referred to and expanded on in section 3.2.3, while more about the principles referred to in section 2.5.2 can be found in Chapter 4.

2.5.1 Developing Realistic Mathematics Education

The RME theory is one that is constantly "under construction", being developed and refined in an ongoing cycle of designing, experimenting, analysing and reflecting (Gravemeijer, 1994). Developmental research plays a central role in this process and, in contrast to traditional instructional design models, focuses on the teaching-learning process, focusing in specifically on the mental processes of learners (Rasmussen & King, 2000). Cyclic processes of thought experiments and instructional experiments form the crux of the method of developmental research and serve a dual function (see Figure 2.2). They both clarify researchers' learning about learners' thinking and address the pragmatic affairs of revising instructional sequences (Gravemeijer, 1999). Instructional sequences are designed by the curriculum developer who starts off with a thought experiment that imagines a route learners could have invented for themselves. The lesson is implemented and the actual process of learning that takes place in relation to the anticipated trajectory is analysed. This analysis can then provide valuable information in order to revise the instructional activities.

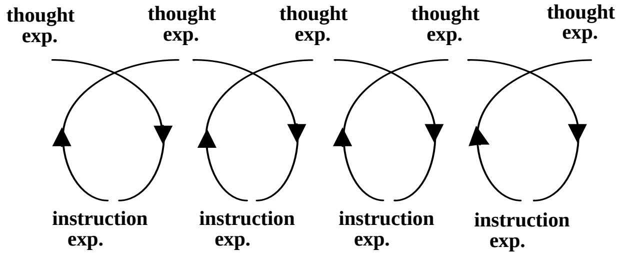


Figure 2.2 Developmental research, a cumulative cyclic process (Gravemeijer & Cobb, 2002)

2.5.2 RME instructional design principles

Gravemeijer (1994, 1999) identifies three key heuristic principles of RME for this process of instructional design, namely:

- Guided reinvention through progressive mathematisation
- Didactical phenomenology
- Self developed or emergent models

Guided reinvention through progressive mathematisation

This is the dominant principle being explored and applied throughout this study. During the first cycle this principle was exclusively focused on as this was the initial attempt to try and design and implement instructional tasks and activities, based on the RME theory, but in a remedial context for low attaining Grade 8 learners. In the RME literature that exists, this principle is most often applied in an attempt to teach learners something new in the domain of mathematics. In this study however, I was not intending to teach the learners anything new; the sections that we dealt with were all familiar to them (in terms of the fact that they had already learnt the content in previous years). However, in the pre-tests that learners wrote prior to the commencement of the intervention, many were unable to correctly answer a number of the items (see Appendix A).

As a result of an analysis of the data from these tests and interviews with the classroom teachers of the participants, it was concluded that the low performance of the learners was a result of their limited understanding relating to the concepts assessed, namely place value, fractions, decimals and basic algebra. It was evident from the tests, some preliminary work with the learners and discussions with teachers that the learners possessed a fair amount of formal knowledge relating to these topics, but that they were not able to apply this formal knowledge. As has previously been observed in the literature, the learners therefore either "improvised" or mixed up the algorithms, strategies and/or notation.

The principle of *guided reinvention* requires that well-chosen contextual problems be presented to learners that offer them opportunities to develop informal, highly context-specific solution strategies (Doorman, 2002). These informal solution procedures may then function as foothold inventions for formalization and generalization, a process referred to as "progressive mathematizing" (Gravemeijer, 1994). The reinvention process is set in motion when learners use their everyday language (informal description) to structure contextual problems into informal or more formal mathematical forms (Armanto, 2002). The instructional designer therefore tries to compile a set of problems that can lead to a series of processes that together result in the reinvention of the intended mathematics (Doorman, 2002).

The idea is not that learners are expected to reinvent everything on their own though but that Freudenthal's concept of "guided reinvention" should apply (Freudenthal, 1973). This should in turn allow learners to regard the knowledge they acquire as knowledge for which they have been responsible and which belongs to them. With guidance, the learners are afforded the opportunity to construct their own mathematical knowledge store on this basis. The word "realistic" in the RME theory does not indicate however that everyday contexts need to be continuously sought or used to motivate learners to reinvent the mathematics. Rather, the contexts selected for use in the process of instructional design should be experientially real for learners, relevant and challenging in order to act as a catalyst for progressive mathematisation (Freudenthal, 1973; Gravemeijer, 1994; Treffers, 1987).

The principle of Didactical Phenomenology

This principle was advocated by Freudenthal (1973) and implies that in learning mathematics, one has to start from phenomena meaningful to the learner, and that implore some sort of organizing be done and that stimulate learning processes.

According to Treffers and Goffree (1985) this principle should fulfill four functions:

- Concept formation (to allow learners natural and motivating access to mathematics),
- Model formation (to supply a firm basis for learning the formal operations, procedures, and rules in conjunction to other models as the support for thinking),
- Applicability (to utilize reality as a source and domain of applications),
- Practice (to exercise the specific abilities of learners in applied situations).

With respect to this principle, an instructional sequence was designed for the second cycle of the intervention made up of contextual problems that could also fulfill the above four functions alluded to by Treffers and Goffree (1985). This was done in an attempt to realise the intended implications of this principle. More about how this was done and whether or not the principle was realised can be found in Chapter 4 in section 4.5.1.

The principle of emergent or self developed models

This third principle for instructional design in RME plays an important role in bridging the gap between informal and formal knowledge (Gravemeijer, 1994). In order to realise this principle, learners need to be given opportunities to use and develop their own models when solving problems. The term "model" is understood here in a dynamic, holistic sense and learners enhance their models by using their former models and their knowledge about mathematics. As a consequence, the symbolizations that comprise the model and those rooted in the process of modelling can change over time. Learners therefore progress from what is termed a "model-of" a situated activity to a "model-for" more sophisticated reasoning (Gravemeijer & Doorman, 1999 as cited in Kwon, 2003).

This is quite different from the former (and in many instances still current) practice in South Africa, where learners are presented with a model or algorithm by the teacher and then given repeated opportunities and problems to practice using that model. This resulted in most of the learners in this study already having a degree of formal knowledge in place. This knowledge was not necessarily supported by models that the learners had progressed to or even understood but rather ones that were presented to them as "ready made" models. An attempt was therefore made to revert to the learners' informal knowledge in this regard so that learners could be encouraged to develop their own models (whilst moving through the levels), which they understood. The first challenge was getting the learners to "abandon" any "useless" models or formal knowledge that had been highlighted and to make use of their own models to solve problems. The second challenge was to then design a series of lessons that would allow them to progress through the levels from their informal knowledge back to a formal level. This is further expanded on in section 4.6.1.

2.6 Why RME for low attainers

In the preceding sections in this chapter, literature on the teaching and learning of mathematics to low attaining learners was examined and common environmental aspects that could be incorporated into the instructional approach of the intervention were identified. The theory of Realistic Mathematics Education was then proposed as a possible theory to drive the design and implementation of the intervention. The theoretical underpinnings of RME were subsequently outlined in 2.5. This section expands on these underpinnings and the identified aspects in order to substantiate the choice of RME. To facilitate this argument, RME is discussed in relation to three other global trends in mathematics education in order to highlight some of the unique features, which make it the recommended theory for this research.

2.6.1 RME in relation to other global innovations in Mathematics Education

Treffers (1987) identifies three global trends in mathematics education, which he refers to as the arithmetical, structural and empirical trends. The didactical approach of the *arithmetical trend* (also known as 'New Math') is similar to that of drill and practice instruction in the past with the main objectives being the teaching of certain arithmetic routines, notations and rules and the transfer of knowledge. The influence of the arithmetical trend on RME includes, amongst others, the inclusion of puzzles, practice games and ideas about learning basic operations.

The mathematical activity in the *structural trend* is mainly directed towards the construction of formal mathematical structures and aims less at the relationships with the reality of everyday experience. The approach is best expressed by the work of Dienes and makes use of "imagined" reality and "artificial surroundings" as a basis for mathematical analysis and exploration of mathematical structures. Treffers (1987) presents the shortcoming of this approach as being the large gap between the constructed world in which the mathematics takes place and everyday reality. This makes it almost impossible to connect the two. In spite of this criticism, influences from the structural trend are visible in the work of RME, for example, in the use of arrows and '"machines" in the basic operations, in the approach to problems of reasoning via arrow diagrams and the attention paid to structuring aids such as number line, charts, grids, diagram and graphics.

In contrast to the structural trend, the *empirical trend* takes it subjects for mathematics study almost exclusively from the biological, physical or social reality, which means that the starting point for mathematical activities lies within "the neighbourhood of the child's every day experience" (p. 10). The lack of a mathematical source of inspiration and strict methodological structure sometimes results in a badly organised collection of activities though, and it becomes problematic to ensure that children are not repeating the same experience at different stages of their school life (Biggs, 1971 as cited in Treffers, 1987). Some similarities between RME and the empirical trend include the use of charts, graphs and materials, the connection with actuality and the attention paid to the measuring aspect of number in early mathematics education. One of the main differences between the two, however, is that while RME draws on everyday contexts, the use of "imagined" realities is also subscribed to, which is not the case in the empirical approach.

The main purpose for presenting this background has been to indicate how elements of global trends, such as these, have influenced the development of the theory of RME. As previously mentioned though, the main thrust of RME is that of viewing mathematics as a human activity (Freudenthal, 1973) and the subsequent central element of *mathematisation* (Treffers, 1987). This central element of RME is now further investigated and discussed in relation to the other three global trends.

2.6.2 Mathematisation for low attainers

Treffers (1987) describes *mathematising* as "…the organising and structuring activity in which acquired knowledge and abilities are called upon in order to discover still unknown regularities, connections, structures." (p. 247) Further more mathematising is directed towards:

> *the acquisition of factual knowledge, the learning of concepts, the attainment of skills and the use of language and other organising skills in solving problems that are, or are not, placed in a mathematical context. (pp. 52, 53).*

This process or activity alone already accommodates most of the aspects suggested for inclusion in the instructional approach of the intervention for low attainers. To place the instructional approach within one of the other three trends would not allow all five of the suggested aspects to be included. To take this a step further, let us look more closely at the differentiation Treffers (1987) makes between horizontal and vertical mathematisation, as referred to previously. In his words,

> *In general one can say that 'horizontal mathematisation' consists of a schematisation of the area that makes it possible to attack the problem by mathematical means. The activities that follow and that*

are related to the mathematical process, the solution of the problem, the generalization of the solution and the further formalisation, can be described as 'vertical mathematistion' (p. 71).

Treffers admits that an exact distinction is hard to make but that the distinction is meaningful in that it demonstrates how activities such as constructing, experimenting and classifying also fit into the process of mathematising along with the more common ones of symbolising, generalising and formalising. Making a schematic comparison between the other three trends and RME, in relation to the use of horizontal and vertical mathematisation, is also a helpful way of demonstrating why RME is being suggested as the domain-specific theory for this study. In this regard, Treffers (1987) presents the following table:

Table 2.1 Classification by Treffers of inclusion of horizontal and vertical mathematisation in four different mathematics education trends

Trends	*Mathematising*	
	Horizontal	**Vertical**
Mechanistic (Arithmetic)	-	-
Empiricist	+	-
Structuralist	-	+
Realistic	+	+

In the mechanistic (or arithmetic) trend, no real phenomenon is used as a source of mathematical activity, little attention is paid to applications and the emphasis is on rote learning. This results in weaknesses in both horizontal and vertical mathematisation. The empiricist trend places a strong emphasis on horizontal mathematisation in that the emphasis is on environmental rather than on mental operations. Formal mathematical goals do not feature as a high priority and there is little pressure for learners to pass to a higher level, thus demonstrating the weakness with relation to vertical mathematisation. In structuralist instruction, where mathematical structures are emphasised, the vertical component is dominant. This is evident in this approach in that the principal part of the mathematical activity operates within the mathematical system. Instead of real phenomena, embodiments and materialisations of mathematical concepts or structures or structural games are used to create a concrete basis for learners from which to work and real phenomena subsequently do not function as models to support operating within the mathematical system. In realistic mathematics instruction however, careful attention is paid to both components. As Treffers (1987) puts it,

> *This means that the phenomena from which the mathematical concepts and structures arise are implicitly used both as source and domain of application. This, according to the tenet of the theory, creates for the learner the possibility of concept attainment by orienting himself to a variety of phenomena, which benefits the building of formal mathematical concepts and structures and their application (p. 251).*

From the literature reviewed in relation to low attainers, it appears that a lot of the teaching and learning in this domain has tended towards the mechanistic (arithmetic) and structuralist trends. The focus of the instruction and assessment has therefore been in the vertical component of Table 2.1, which could explain the dominance of instrumental rather than relational understanding. The major activities in this component are symbolising, formalising and generalising. As low attainers often struggle with these, they may have experienced repeated failure with continued emphasis on this component. Misconceptions may also be hampering them within the vertical component and may have developed due to a lack of adequate exposure to constructing, experimenting and classifying, which lie in the horizontal component. In order to rectify this, it therefore seems necessary to select an instruction theory that will pay careful attention to both components. Learners are thereby also afforded more opportunities to bridge the gap between their informal understanding and formal knowledge. This is not a once-off or linear process however, and should be viewed as a continual cycle. The desired outcome is that learners acquire the cyclical strategy of moving between horizontal and vertical mathematisation in order to assist them in improving their understanding and subsequently their performance in mathematics.

As previously mentioned, the intervention has not been designed with the intention of teaching new topics or content to the learners but rather to improve their understanding of place value, fractions, decimals and basic algebra. The instruction theory of RME was be new to them and provided a fresh approach to evaluate, challenge and would hopefully improve their understanding of these "not so fresh" topics, while also providing them with a strategy (that of mathematising) to take forward with them.

From the discussion above, it should be clear that RME provides *more of a focus on relational and conceptual understanding* (see section 2.4.1) as opposed to rote learning. In order to do this, *meaningful learning contexts* (see section 2.4.2) are created (which can be from everyday situations or "imagined" reality) that facilitate the process of progressive mathematisation. This means that learners are *actively involved in solving problems* (see section 2.4.3) and constructing their own meaning and understanding. By continual use of horizontal and vertical mathematisation, learners are using mathematical symbols and language interchangeably and hence tending to the *importance of language development* (see section 2.4.5). One of the general principles of progressive mathematisation, that has not yet been mentioned, is that of "interactivity" (Treffers, 1987). According to this principle, learners are confronted with the constructions and productions of their peers, which:

> *…can stimulate them to shorten their learning path, to help themselves up on procedures of others, to become aware of the drawbacks or advantages of their own productions, and that copying others' work slavishly will not aid their own progress. In brief, the learning process is part of the interactive instruction where individual work is combined with consulting fellow students, group discussion, collective work reviews, presentation of one's own productions, evaluation of various constructions on various levels and explanation by the teacher. (p. 249)*

This principle satisfies the *importance of social interaction* aspect that was suggested in the instructional approach in 2.4.4. The central theme of RME, mathematising, therefore adequately incorporates all the aspects suggested in the instructional approach in section 2.4.

2.7 Conclusion

Low attainers were the target group of this study, which aimed to develop an intervention that could improve their understanding with regard to place value, fractions, decimals and basic algebra. Realistic Mathematics Education was selected as the theoretical framework for the study, which therefore also informed the design and implementation of the intervention. The intervention did not aim to teach learners any new content but to revisit the topics (as mentioned) that they have already dealt with in previous years. In terms of a literature review, this presented some challenges. In the existing literature, RME is used in the context of teaching learners something new (in terms of the mathematics content) as opposed to revisiting former topics. RME is also mainly applied in mainstream classrooms and although some work has been done with SEN learners, again it is in the sense that learners are being introduced to new topics.

In this chapter, the choice of the term "low attainer" was explained and related terminology mentioned. Primary sources of literature in this domain were identified and consulted in order to present some general characteristics and causes of low attainment. These sources were also examined for common environmental aspects and practices that emerged to be included in the instructional approach of the intervention. These aspects were listed and explained in 2.4. The theory of Realistic Mathematics Education (RME) was then suggested as the theoretical framework to drive the design and implementation of the intervention. The instructional approach suggested for low attainers is therefore embedded in this domain-specific theory of RME. The theoretical underpinnings of RME were then discussed and RME was examined as an instructional approach to teaching mathematics in relation to three other global trends in this domain. Through this comparison, it was shown how RME is able to satisfy all the aspects suggested in the instructional approach for low attainers.

CHAPTER THREE

3 Method and Research Design

3.1 Introduction

As mentioned in the first chapter, the research done in this study encompassed the design and implementation of an intervention with two groups of low attaining learners in Grade 8 in order to improve their understanding of certain key concepts (whole numbers, decimals and fractions) in mathematics. Literature pertaining to low attainers and research conducted in this domain was first reviewed in order to establish a framework consisting of aspects that could be included in the design of the intervention. The theory of Realistic Mathematics Education (Freudenthal, 1973; Treffers, 1987; Gravemeijer, 1994) was selected as the domain-specific theory to guide the design and implementation of the intervention. Through the development and implementation process the following was investigated: if and how the RME approach could be used with low attainers in order to revisit key number concepts (they had already previously learnt) in such a way as to improve their understanding relating to these concepts. This process, which took place during 2003, was guided by the main research question as presented in the Chapter 1.

This chapter details the research design and approach that guided the study. The research is labelled as a *development case study* and this is further explained and substantiated in section 3.2, where the two development research approaches that informed the study are also elaborated on. Section 3.3 presents the pragmatic paradigm from within which this research was conducted and the subsequent mixed methods methodology that was used in the design. The site and sample for the case study are introduced in section 3.4 before the data collection procedures are elaborated on in section 3.5. A brief description of how the data were analysed is included in section 3.6, followed by a discussion of validity and reliability issues in section 3.7. Ethical considerations that were taken into account during the course of the study, are outlined in section 3.8 and the chapter ends with the conclusion.

3.2 Research design

This study was first implemented with a small group of low attaining learners in order to explore the potential and characteristics of using RME in such a setting, and to develop an initial prototype that could possibly be refined, implemented, and further evaluated in another study, depending on the outcome of this inquiry. As alluded to in Chapter 1 (see section 1.5), this formative type of research related to design and development work occurs in the literature under various labels (van den Akker, 1999). The term *development research* is the term used in this report to refer to this research design. Two approaches to this type of research informed this study and these are elaborated on and combined in the sub-sections 3.2.2, 3.2.3 and 3.2.4 of this section.

For this inquiry into developing and implementing an initial prototype, the setting chosen was one that Cobb, Confrey, diSessa, Lehrer and Schauble (2003) refer to as a one-on-one (teacher-experimenter and learners) design experiment. In such a bounded setting, the researcher acts as the teacher and conducts a series of teaching sessions with a small number of learners in order to create a small-scale version of a learning ecology (prototype) so that it can be studied in depth and detail (Cobb & Steffe, 1983; Steffe & Thompson, 2000 as cited in Cobb et al, 2003). This setting simulates that of a case study as defined in the literature on research designs (e.g. Adelman, Jenkins & Kemmis (1980); Guba & Lincoln, 1981; Merriam, 1988; Cohen, Manion & Morrison, 2000). Merriam (1988) cites definitions from various authors who support this, such as a case study being defined as "the examination of an instance in action" (MacDonald & Walker, 1977, p. 181) and a process "which tries to describe and analyse some entity in qualitative, complex and comprehensive terms not infrequently as it unfolds over a period of time" (Wilson, 1979, p. 448). The context of this inquiry was also dynamic and provided a unique example of real learners in a real situation (Cohen et al., 2000).

In light of this classification as a case study, as well as the formative and developmental nature of this inquiry, the research design presented here can be labeled as a *development case study*. In order to further unpack this label, the following sub-sections firstly elaborate on the choice of and classification of this inquiry as a case study before explaining the role of the two developmental approaches and how they combine to create the particular research design embarked on within this study.

3.2.1 Case study

In addition to the definitions that Merriam (1988) provides from various sources on what a case study is, she also draws on several case study characteristics from five separate sources (Guba & Lincoln, 1981; Helmstadter, 1970; Hoaglin & others, 1982; Stake, 1981; Wilson, 1979) in order to further define a case study by its special features. She summarises the following four characteristics as essential properties of a qualitative case study: particularistic, descriptive, heuristic and inductive. These are dealt with separately to illuminate their scope within this particular study in order to substantiate its classification as a case study.

The ***particularistic*** characteristic means that a bounded system (case) can be identified as the focus of the study. This case can be in the form of a programme, an event, a person, a process, an institution or a social group (see for example Merriam, 1988; Cohen et al., 2000). In this particular inquiry, the bounded system was in the form of a group of 12 Grade 8 learners who attended remedial mathematics lessons at the school that provided the context for this case study. Two out of ten of the remedial classes were selected for the study (see section 3.4). One of the classes (8X) contained 5 learners, while the other (8Y) contained the other 7 learners. The classes differed with respect to their mathematics teachers, the classroom culture that prevailed as a result of the teachers' classroom practices as well as the context of the classroom and the average performance of learners in the two different classes. Initially the intention was that they complete exactly the same activities and instructional sequences. But owing to the developmental nature of this inquiry and the differences between the two classes, it was not always possible to adhere to this intention and 8Y adapted to the approach used in these lessons quicker, worked at a faster pace and subsequently completed more of a range of activities and instructional sequences than 8X did, although both classes had approximately the same number of lessons. It could perhaps therefore be suggested that these could have been two case studies but all learners had the same presenter (myself) and experienced the same approach (RME) in their lessons. In order not to complicate issues, this is therefore regarded as one case study of a group of 12 individual learners.

Another point to be considered is that perhaps the intervention was the actual case for this study as opposed to the learners involved. Although the study sought to investigate the potential of implementing this intervention (comprising a sequence of instructional activities) with low attainers, this had to be done within an actual setting with learners so that the possible outcomes and characteristics could be explored. The intervention was implemented with the participants in order to investigate *their thinking* and if *their understanding* of the key concepts could be improved. Data relating to the learners were therefore the focus of the final analysis, making them the case under study. Through that data, yielded from the case study, a better understanding of the characteristics of such an intervention and indication of possible guidelines in developing similar interventions was obtained.

The ***descriptive*** characteristic that Merriam (1988) refers to means that the inquiry should result in a rich description of the phenomenon under study, include as many variables as possible and portray their interaction. In section 3.4 of this chapter, the context of the school that the learners attend is first established, followed by a description of the varying contexts, teachers and members of the two classes (8X and 8Y). Some of the data collected from the preliminary investigation (prior to the implementation of the intervention) are also included and briefly discussed in that particular section. In Chapters 4 and 5, data collected during the intervention are presented and discussed although the data is mainly, but not limited to, qualitative data (see section 3.5). Documentation presented in Chapters 4 and 5 include quotes, samples and discussions of work samples from learners and descriptions of events (Wilson, 1979 as cited in Merriam, 1988).

The ***heurisitc*** characteristic (Merriam, 1988) is appropriate in the sense that in this case it means that this study should illuminate the reader's understanding of the 12 learners and their response to the intervention. This is addressed in Chapters 4 and 5 where the data analyses are presented.

Lastly, according to Merriam (1988), another characteristic that should be evident in case studies is that they ought to be, for the most part anyhow, ***inductive***. This means that generalizations, concepts or hypotheses mostly emerge from an examination of data that are grounded in the context. In this inquiry however, there was a working hypotheses at the start of the inquiry. In this particular case, the main hypothesis being explored was, that by employing the theory of RME to revisit key number concepts with learners identified as low attainers, their understanding of these concepts would improve. This would in turn assist them in establishing a more appropriate schema into which they could assimilate the new concepts and knowledge they are presented with pertaining to algebra. Although the objective of the study was partly to test these working hypotheses in order to explore the potential of further employing RME in similar settings, the intention was also that the case study would lead to a discovery of new relationships, understanding and emerging concepts. As the study proceeded, these expectations were challenged, revisited, subject to reformulation and in some instances confirmed (Merriam, 1988). This study therefore did include this final characteristic.

3.2.2 Development research in curriculum

The two main "schools of thought" on development research that were followed for this study both have their origins in the Netherlands, originating at the University of Twente and the Freudenthal Institute, and in this study are used in the sub-domains of curriculum and learning and instruction in RME, respectively.

The major goal of this type of research is to inform the decision making process during the development of a product or programme (which in this study is in the form of an intervention in mathematics). This is done with a view to improving and optimising the intervention and also to advance the developer's capabilities to create interventions of this nature (i.e. to identify the characteristics of effective interventions and how to design and develop and these). The intention is not to implement a complete intervention but to go through a cyclical or spiral process of developing, implementing, testing and revising successive prototypes that increasingly meet the innovative aspirations and requirements (van den Akker, 1999). The scope of this study included the first three of these four processes and leaves the latter process as well as the implementation, testing and revising of successive prototypes for a study of a larger scale. Smith (1991, p. 42), as cited in Nieveen (1999), provides a definition of a prototype as a "preliminary version or a model of all or a part of a system before full commitment is made to develop it."

On an abstract level, the general aim of development research is to lessen the uncertainty of decision making in designing and developing educational interventions. This can serve two purposes - the first oriented towards practical ends in a given situation and the second resulting in a production of knowledge. The first purpose therefore entails providing ideas, in the form of suggestions and directions (i.e. characteristics) that will enhance and optimise the quality of the intervention. The second purpose is more concerned with generating, articulating and testing design principles (van den Akker, 1999). Although this study did have some tentative design principles in mind at the outset and also yielded a few emerging design principles, these were not the main focus of the inquiry and the prototype will need revision and successive cycles in order to result in production of knowledge that is of a generalisable nature. This study was therefore more concerned with the first purpose of identifying the characteristics of an intervention, based on input from relevant literature and my own experience and ideas, in order to try and improve the remedial learners' understanding of the mentioned key concepts.

Four defined stages are presented (by van den Akker, 1999) as being part of development research. The *preliminary investigation* (van den Akker, 1999) or *front-end analysis* (Nieveen, 1999; Plomp, 2002) involves a systematic investigation of tasks, problems and context in an attempt to find more accurate and explicit connections between the analysis and relevant literature. In this study the front-end analysis took the form of a questionnaire to learners, a literary review on low attainers and RME, an overview of examples of materials that could perhaps be adapted and contextualised, and a collection of factual and academic data on the participants. Some of the results of this front-end analysis are presented in sections 3.4 and 5.3 while findings of the literature review have mainly been worked into Chapter 2.

Van den Akker (1999) identifies the second defined stage as one of *theoretical embedding*. This means applying available knowledge and literature in articulating the theoretical rationale for the design choices. This aspect was partially covered in Chapter 2 of this report. Additional cycles of implementation are however necessary to do this stage justice. The third stage is known as *empirical testing* and implies collecting and delivering clear and empirical evidence about the practicality and effectiveness of the intervention when implemented in an actual setting with the intended target group. This is touched on in the data collection (3.6) and data analysis (3.7) sections in this chapter and further elaborated on in Chapters 4 and 5, where the data is discussed. The fourth and final stage (of the iterative) process is the *documentation, analysis and reflection* on the process and outcomes stage. Obviously this report serves as the documentation and partial analysis and reflection part of this stage but further analysis and reflection will need to be carried out in order to produce a revised prototype for further implementation with multiple cases and additional variables such as the teachers as presenters.

Plomp (2002) offers the model in Figure 3.1 as a slightly different representation of these stages, to include the immediate and distant outcomes as well as the hypotheses. This study was confined to the stages enclosed within the shaded area; as mentioned the design was not the focus and although the distant outcome of improved academic performance is mentioned, it was not within the scope of this inquiry to investigate that.

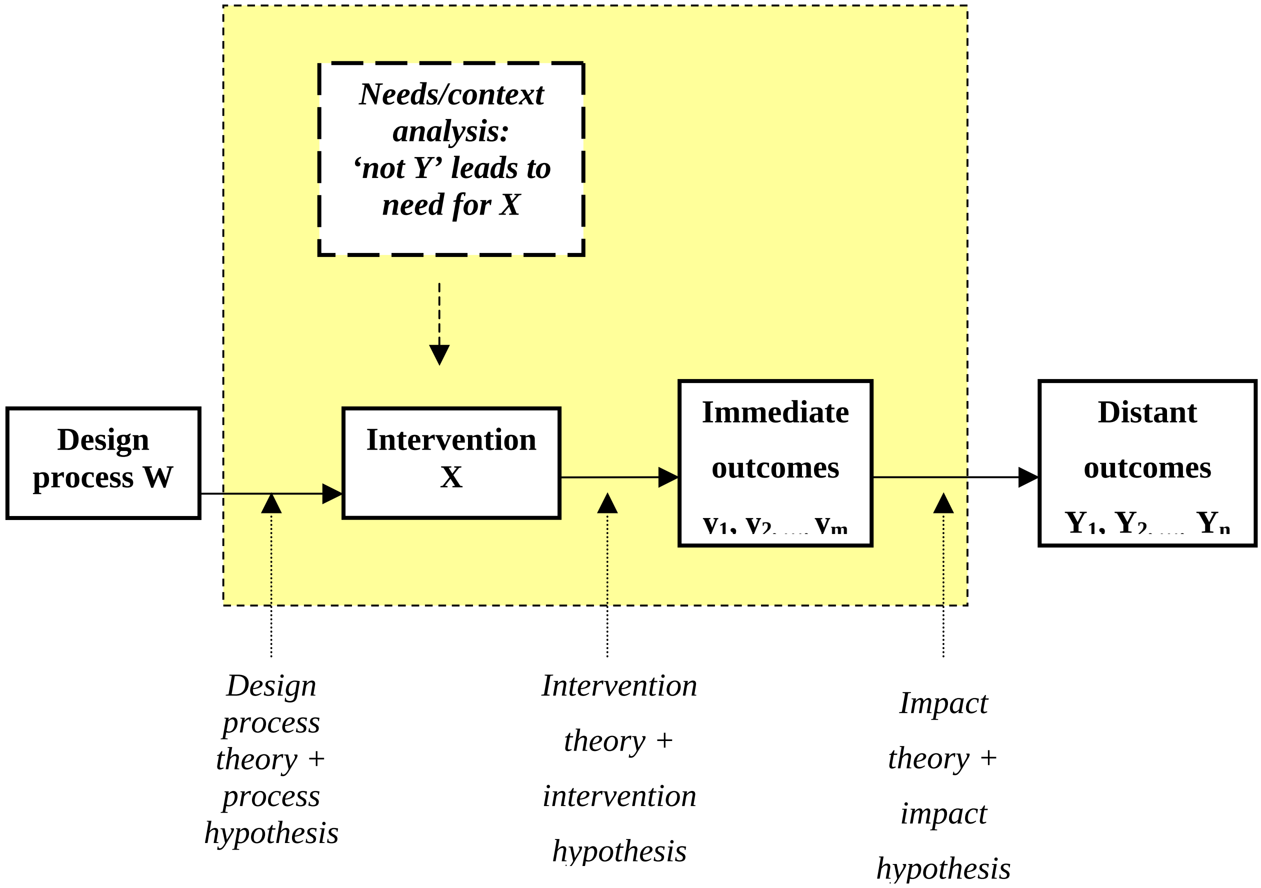


Figure 3.1 Model of stages of development research (Plomp, 2002)

3.2.3 Development research in RME

Gravemeijer (1998) distinguishes between three main perspectives that present research approaches can be categorised into. The first two can be captured under the headings, "explanation" or "understanding" (Bruner, 1994 as cited in Gravemeijer, 1998), while the third one is termed "transformational research" as suggested by the Research Advisory Committee (NCTM, 1988 as cited in Gravemeijer, 1998). Development research falls into the third category and deals more broadly with what "ought to be" as opposed to "what is". Gravemeijer actually uses the term "developmental research" (as does Freudenthal, 1991) when referring to this type of approach, but to avoid confusion the chosen term of "development research" will be adhered to throughout this report.

Although also viewed as a mixture of curriculum development and educational research, this school of thought on development research relates primarily to the domain of mathematics education as opposed to the more general nature of the previous approach, alluded to by van den Akker and Plomp (1993) and van den Akker (1999). The two approaches share a number of commonalities but this approach carries the particular focus of developing instructional activities as a means to explicate, elaborate, test, adjust, refine and expand an instructional theory relating to a specific mathematics topic. The teaching-learning process and especially the mental process of the learners are central to this approach and the research should not only produce curriculum products but also finally result in an instructional theory and a justification thereof (Gravemeijer, 2001). This therefore requires evidence of research activities like theory development, corroboration and reporting research findings so that the research community can "grasp the arguments and weigh the empirical evidence" (Gravemeijer, 1994, p. 113).

According to Gravemeijer (1994, 1998), development research can be seen as taking place at three levels that make different hierarchical contributions in terms of theory. Cyclic processes can be discerned at each level however. The first level is that of *instructional activities*, which contribute to establishing *micro theories*. As this is the level pertaining to this study, it will be further explained in a separate paragraph. The next level concerns the development of *a series of instructional activities into an instructional sequence that make up a course* concerning a specific mathematical topic (for example fractions, see Streefland, 1991) and results in a *local instructional theory*. This is done through successive cyclic processes of testing and adapting subsequent versions of an instructional course in order to provide empirical evidence and justification for the instructional theory. The local instruction theory differs from the instructional sequence itself in that it focuses on the rationale for the choice of the instructional activities. The third level can be constructed or reconstructed by analysing local instruction theories in order to develop a domain-specific instruction theory. RME instruction is an example of such a domain-specific theory (Treffers, 1987). This means that the theory of RME is one that is constantly under construction and informed by the development of local instruction theories. The main aim of the RME theory though, remains to develop mathematics education that corresponds with Freudenthal's (1973, 1991) view of "mathematics as a human activity" and the core principle is that "mathematics can and should be learned on one's own authority, through one's own mental activities" (Gravemeijer, 1998).

This study looked at developing a set of instructional activities to revisit the key number concepts of place value, fractions and decimals with learners in Grade 8 who are struggling in mathematics. The long-term goal of such research (through successive studies) would be to develop a local instruction theory; not on the above-mentioned key concepts as such (this has already been done - see for example Streefland, 1991; van den Brink, 1989; Gravemeijer, 1994 & 1998), but to develop one relating to using RME to revisit these key concepts in order to improve learners' understanding and provide them with a more appropriate schema into which to assimilate their understanding of algebra. As previous research directly relating to such an instructional theory could not be found, I needed to test the potential and viability of using RME in such a setting and in doing so also explore possible characteristics of such an intervention. This entailed developing and implementing instructional activities (collectively referred to as the intervention) in order to produce some empirical evidence on the teaching-learning process and the mental processes of learners in this regard.

This study was therefore situated in the first of the three levels outlined above and concerned with testing a unit of instructional activities at a micro-level. Gravemeijer (1994) calls this a small-scale empirical cycle, which is also a collection of mini-cycles (viz. each of the lessons as a mini-cycle) as depicted in Figure 2.2 (in Chapter 2). These mini-cycles take the form of cyclic processes of thought experiments and instruction experiments (Freudenthal, 1991) that are also the "backbone" of the method of this type of development research (Gravemeijer, 2001). In order to do this, one first envisions how the teaching-learning process will proceed (thought experiment) and then tries out the instructional experiment in the classroom. Signs are then searched for that either confirm, challenge or reject the thought experiment. This feedback can then be fed into new thought experiments that continuously fuel the cyclic process of deliberating and testing. Central to these thought and instruction experiments are the learners' mental activities. The micro theories should describe how the instructional activities provoked the perceived mental activities of the learners, and how these mental activities in turn contribute to the presumed (or anticipated) growth in mathematical understanding (Gravemeijer, 1998). The micro theories evolving from this study are dealt with in the discussion in Chapter 6.

Gravemeijer (1994, 1998) identifies two stages in the design of a development research project. The phase of *preliminary design* (similar to the front-end analysis) involves making an analysis of the situation as to why the existing curricula are unsatisfactory. Through this process, the demands of the new course become visible and a general concept of a course must develop before the actual experiments can begin. In this study, dissatisfaction with the repetitive and mechanistic nature of the remedial mathematical classes led to available research literature on low attainers being consulted. This resulted in the development of the desired instructional approach, which in a sense highlighted the demands of the new course. Using RME to improve learners' understanding relating to the key number concepts of place value, fractions and decimals became the general concept of the course. Explication and justification of this general concept were provided in Chapter 2.

The second stage is referred to as the *elaboration of the course design.* Here the overall preliminary design is expanded and adapted through the cyclic process of thought and instructional experiments (see Figure 3.3) as previously explained. The collection of hypotheses (see section 3.2.2) are therefore tested during this process and the heuristic design principles explained in section 2.5.2 (Chapter 2) indicate the criteria that are used by the researcher to make assessments and carry out adjustments (Gravemeijer, 1998). Further discussion regarding this stage is provided in Chapter 4.

As a final point on this approach, the following statement by Gravemeijer (1998, p. 25) is offered (which is hopefully fulfilled in the scope of this report):

> *Crucial for this method is the explication of the learning process. The researcher has to explain and justify his findings in such a way that it enables outsiders to retrace this process. In this way a basis is laid for a scientific discussion, that may lead to intersubjective agreement.*

3.2.4 Development case study approach in this study

Figure 3.3 is a combination of Figures 3.1 and 3.2 in order to represent the design approach used in this study. The dotted blocks represent the *preliminary* stages of the study and the cyclic process of thought and instruction experiments signifies the duration of the intervention. The study is viewed as the first small-scale empirical cycle to investigate the potential and viability of using RME to improve learners' understanding relating to these concepts. By observing the mental activities of learners during the instructional activities, micro-theories were documented and discussed in relation to the heuristic instructional design principles of RME (see Chapter 4) in order to establish an initial prototype for using RME in this setting. The initial prototype, however, was not intended to be (and is not) an instructional sequence. It remains a unit of instructional activities that have been implemented in an attempt to improve learners' understanding of place value, fractions and decimals. Through the intervention, empirical data were collected in order to test the hypotheses and report on emerging micro theories. Further reflection and design of the initial prototype would be necessary before entering a second cycle of research where the prototype could be implemented with more cases.

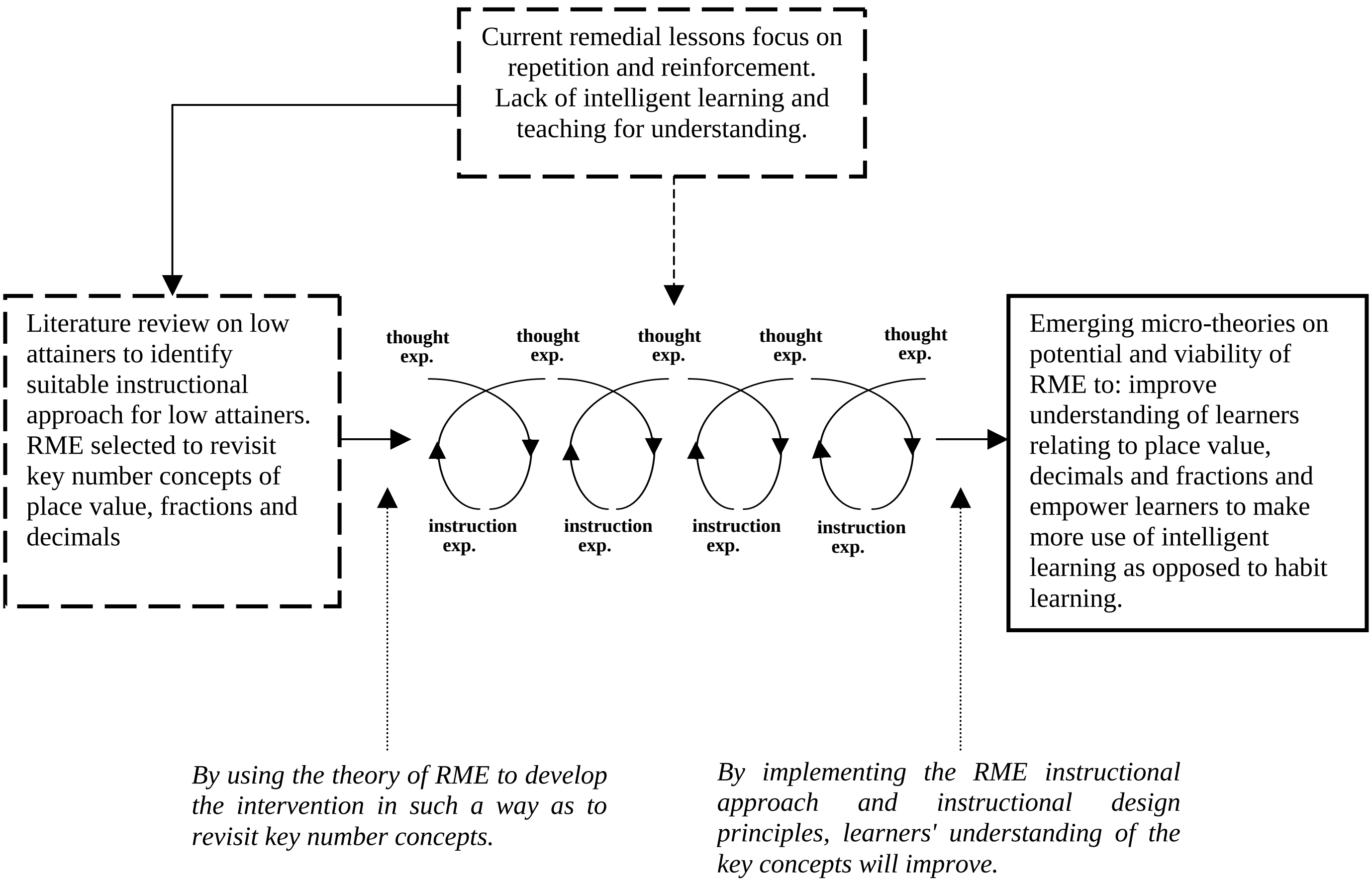


Figure 3.2 Adapted model from Plomp (2002), incorporating Gravemeijer (2001) for representing development case study design used in this study

3.3 Research paradigm

A case study contained within a combination of the two development research approaches outlined above has been chosen as the research design to guide this inquiry, and substantiation of this was provided in the previous section. By engaging in this form of development research, this study is positioned within a pragmatic paradigm in that the usefulness and workability of the intervention and the consequences thereof (as experienced by the 12 learners) are the focus, rather than its scientific infallibility (Donovan, 2000). The term pragmatism is derived from the Greek word "pragma" meaning "an act or a deed" and the term was coined by Pierce to emphasise the fact that words acquire their meanings from actions and not intuitions (Newman & Holzman, 1993). This section first provides some background on the foundations of pragmatism, followed by a justification of the selection of the paradigm for this particular study. The mixed methods methodology resulting from the pragmatic nature of this research is then also elaborated on.

3.3.1 Foundations of pragmatism

Charles Sanders Pierce is commonly regarded as the "Father of Pragmatism"; a philosophical movement that developed in the United States during the late 19th century (The Radical Academy, 2002; Maxcy, 2003). For Pierce, pragmatism was a way of solving the problem of meaning; this meaning in turn being interpreted through conduct. He viewed Science as pursuing evidence while inquiry fixed or established a belief. For Pierce, the methodology or study of best methods for arriving at beliefs was central to his system of inquiry. This system combined the product of inquiry with the laws of Science, hence subscribing to inductive as well as deductive means of investigation in the procedure of inquiry (Maxcy, 2003).

William James expanded on the work of Pierce and is seen as the person who launched pragmatism publicly and made it popular (The Radical Academy, 2002; Maxcy, 2003). He considered pragmatism to be both a method and a theory (The Radical Academy, 2002). As a method he regarded it useful in solving problems. As a theory of truth, he equated what is true with what is fruitful or helpful in a given situation (Maxcy, 2003). From this he deduced that meanings of theories are ultimately found in their capacity to solve problems (Newman & Holzman, 1993) and should be judged in terms of their practical consequences for human conduct (The Radical Academy, 2002). As Maxcy puts it:

> *For James, pragmatism was the natural purposiveness of the stream of consciousness as it was directed rationally in the cause of making certain an idea was right. The search or method of inquiry was conceived of as looking at followed and not toward some ideal or some historical precedent. (p. 69)*

Another leading pragmatist that warrants mention in the development of this paradigm is John Dewey, as he established a link to the social and behavioral aspects of experience and his influence in this is still prevalent today. Dewey (1938 as cited in Maxcy, 2003) offered the following definition for pragmatic:

> *…the function of consequences as necessary tests of the validity of propositions, provided these consequences are operationally instituted and are such as to resolve the specific problem evoking the operation. (pp. iii-iv)*

He saw problematic situations as arising out of ordinary experiences, solutions being sought through a range of techniques deemed appropriate by the investigator, and then being tested and verified by reflecting on directly experienced matters (Maxcy, 2003). He regarded decisions as hypothetical and tentative until the expected consequences adopted in solving a particular problem were congruent with the actual consequences. These decisions were made in response to seeking the best actions to take in solving a specific problem (Dewey, 1948 as cited in Maxcy, 2003). He therefore made inquiry, rather than truth or knowledge, the essence of logic in his theory of scientific inquiry, thus equating his theory with what we would term "investigation" or "problem-solving" (The Radical Academy, 2002; Maxcy, 2003).

A number of other authors also had a profound influence on pragmatism. These include, amongst others, George Mead, Arthur Bentley, Abraham Kaplan, Richard Rorty, Richard Bernstein, Micahael Patton, Cornel West and Cleo Cherryholmes (Tashakkori & Teddlie, 1998; Creswell, 2003; Maxcy, 2003). Further discussion relating to this is however beyond the scope of this report and a description and discussion of pragmatism as a research paradigm for this study are the focus of the rest of this section.

3.3.2 Justification for the research paradigm

Resulting from the foundations as discussed in section 3.3.1, research done within the pragmatic paradigm focuses an inquiry on the connection between thinking and doing (Newman & Holzman, no date), and both the meaning and the truth of any idea are viewed as a function of its practical outcome (The Radical Academy, 2002). Therefore, for pragmatists "truth is what works" (Howe, 1988, p. 14-15) and according to Cherryholmes (1992), the pragmatist's choice of one explanation over another simply signifies the ability of an approach to produce the anticipated or desired outcomes (both cited in Tashakkori & Teddlie, 1998). The following statement by Cherryholmes (1992) further illuminates this:

> *Pragmatic research is driven by anticipated consequences. Pragmatic choices about what to research and how to go about it are conditioned by where we want to go in the broadest of senses….Beginning with what he or she thinks is known and looking to the consequences he or she desires, our pragmatist would pick and choose how and what to research and what to do. (pp. 13-14)*

For pragmatists, the values of the researcher therefore also play a large role in selecting the topics and interpreting the results. The pragmatist often selects a topic that is of special interest to him and then studies it in such a way that is congruent with their value system. This includes using methodologies and methods that are most appropriate for finding an answer to their research question (Tashakkori & Teddlie, 1998; 2003). This means that the problem yields a higher status than the actual method (Creswell, 2003).

As previously mentioned, this study arose out of a problem as I experienced it, from my own knowledge of teaching as well as from discussions with relevant parties at the school where the case study was carried out. A solution was sought to the problem of improving the understanding of low attainers in the key number concepts (place value, fractions and decimals) and it was conjectured (on consulting the literature) that improving their understanding could create a more adequate schema into which they could subsequently assimilate their understanding of algebra. RME was chosen as the theory to guide the development and implementation of an intervention in the course of the inquiry, and tentative hypotheses were formed. The first was that learners' understanding would improve through the implementation of an RME instructional approach in the intervention. And the second one then followed, that through the process learners would also be empowered to make more use of intelligent rather than habit learning. Finding a solution to the initial problem by trying to test ideas (I thought might work) was more important in the inquiry than the methods, (Creswell, 2003) thereby placing the knowledge claims being sought in this inquiry in a pragmatic paradigm.

Furthermore, the combination of the two development research approaches in this study also lends itself to the pragmatic paradigm in that it is a cyclic process of continually developing, implementing, testing and reflecting on ideas in order to ascertain what works best. In the RME development approach, a local instruction theory is the desired outcome of successive implementations of learning sequences in order to provide empirical evidence, which support the use of the RME heuristic design principles in multiple settings with regard to a specific topic. In this case, the view of "theory" is similar to that of William James in that the meaning of the theory is found in its ability to produce the desired learning of the specific mathematics topic that is the focus within the local instruction theory. To arrive at this theory, a process of scientific inquiry is embarked on until the expected consequences are the same as the actual consequences, which is in line with John Dewey's theory of inquiry.

3.3.3 Mixed methods methodology

Due to the pragmatic nature of the knowledge claims for this consequence-orientated study, a mixed methods approach was used in the design of the methodology. Drawing on the work of Murphy (1990), Cherryholmes (1992) and his own interpretation, Creswell (2003) makes the following statement with regard to the knowledge claims for which pragmatism provides a basis:

> *Pragmatism is not committed to any one system of philosophy and reality. This applies to mixed methods research in that inquirers draw liberally from both quantitative and qualitative assumptions when they engage in their research…Individual researchers have a freedom of choice. They are "free" to choose the methods, techniques, and procedures of research that can best meet their needs and purposes. (p. 13)*

This paradigm therefore provided me with the liberty to select multiple methods, draw on different worldviews and assumptions and to make use of different forms of data collection and analysis (Creswell, 2003; Tashakkori & Teddlie, 2003). In this particular study, the conceptualisation, method and inference stages of the research process all drew on what would traditionally be classified as both qualitative as well as quantitative approaches (Tashakkori & Teddlie, 2003). Exploratory (qualitative) as well as confirmatory (quantitative) questions were asked and both quantitative and qualitative data were collected to answer these. Subsequently, in keeping with pragmatic foundations of Pierce (see section 3.3.1) both inductive (qualitative) as well as deductive (quantitative) investigations of analysis into the inquiry were utilized in the inference stage to form a meta-inference at the end. This type of methodology is referred to by Tashakkori and Teddlie (2003, p. 689) as a "fully integrated mixed model design". The remaining sections of this chapter outline the site, sampling and data collection and analysis stages within the study to shed further light on this process.

3.4 Site selection and sampling

It has already been made clear earlier on in the report that I taught at the school where this study was conducted, prior to joining the university. The school was therefore selected mainly due to its accessibility, in terms of easy access to do research there as well as its close proximity to the university. Although the fact that it is an all girls school holds an implicit limitation as referred to in Chapter 1, the study was a small scale one, intending to test the hypotheses alluded to, rather than to try and prove their validity for a large and generalisable population. More details regarding the school (and the options available to learners struggling with mathematics) are provided in this section as well as an outline of the two classes that constituted the sample for the study.

3.4.1 The site

The site providing the context for this study is a public, all girls secondary school (Grades 8 - 12) with approximately 1300 learners, in an urban area. Most of the learners come from the local feeder areas in and around the city as well as from some of the surrounding former township areas. The school has an excellent academic record regarding Grade 12 results and is locally known as one of the best schools in the area. It has a boarding facility for approximately 140 learners and large grounds with an extensive array of extra mural activities on offer to learners in the afternoons. These include sporting activities, cultural clubs and activities, community and outreach programmes and academic support programmes. In terms of after-school academic support, the school has a number of optional systems in place to provide learners with opportunities for extra mathematics lessons after school. In addition to this the school also has an arrangement in place whereby certain Grade 8 and 9 learners who really struggle are able to attend three additional mathematics lessons during school over a ten-day cycle.

Extra mathematics lessons after school hours

As already mentioned, a number of options for supplementing classroom mathematics lessons are available to learners at the school. Extra lessons are offered by some of the teachers at the school in the afternoons either free of charge, or for a minimal fee. Since the beginning of 2002 the school has also had a franchise of "Kumon Mathematics" operating on the property. Learners attend one lesson per week at the centre (which they pay for) while they are given worksheets to take home and complete before returning the following week. In addition to this there is a peer tutoring system in place whereby a senior volunteer learner who is viewed as competent within mathematics is allocated to assist a junior or fellow senior learner who has requested help. Besides the options offered at the school, some of the learners prefer to attend extra lessons offered at other schools or opt for private tuition with an extra mathematics teacher. Finally some learners are referred to "Master Maths" centres in and around the city where learners work mostly independently on computers through topics they struggle with and where tutors are on standby to assist them with further explanations on request.

The Remedial mathematics lessons during school hours

In 1998 the school took the decision that certain learners were achieving such low results in mathematics and English that it would be in the best interests of these learners to allow them to attend additional lessons in these two subjects (known as remedial mathematics and English). In order to accommodate these lessons within the school day, and to reduce the workload of these learners, they were not required to take a third language, which is usually compulsory for all learners in Grades 8 and 9. The mathematics and English teachers respectively of these learners were responsible for teaching the remedial mathematics and English lessons, and each teacher decided what instructional methods, content and material to use for their own classes. No specific policy or methodology was therefore being used for these remedial lessons. The time was created on the timetable and certain learners were identified (by their poor academic performance) as needing the remedial assistance and the task of the teacher was to provide this. From my own personal experience, as well as validation from the remedial teachers at the school, it emerged that the remedial lessons remained a repeat of the normal mathematics lessons rather than a remedial intervention to diagnose and address the mathematical difficulties experienced by the learner. An agreement was therefore reached with the school that a case study would be conducted with some of the learners in these classes through the implementation of an intervention, aimed at identifying their difficulties relating to number concepts and addressing these in an attempt to improve their conceptual understanding in this regard.

3.4.2 The sample of learners

In 2003 the school had ten Grade 8 mathematics classes with approximately 30 learners in each class. Within every class, a small number of remedial learners were identified by their mathematics teacher as needing remedial assistance in mathematics (which made up the total of approximately 45 remedial learners). The decision of the teacher to classify them as remedial, was made on the basis of their final mark for mathematics in Grade 7, their mathematics mark for their first term of Grade 8 and the results of a baseline assessment administered to all Grade 8 learners. It was not compulsory for learners to accept the remedial option, but parents of all the identified learners were contacted and strongly advised to allow permission for their children to join the remedial programme in place of taking a third language. This provided learners with five 40 - 45 minute lessons every ten-day cycle, of which two were used to attend remedial English lessons and the other three were used for remedial mathematics. This resulted in an approximate total of 14 mathematics lessons per academic term in theory with each class, but once public holidays, changes of times, examinations and other interruptions, such as sports days etc. were taken into account, each class had on average between 8 and 10 lessons per term. At times these would be two days apart and at other times, consecutive lessons could be as much as a week apart. An outline of the dates and times of the lessons are attached in Appendix B.

For practical reasons and time constraints, the intervention was only implemented with two of the 10 small groups, which had 5 and 7 learners respectively in their remedial classes. Two of the teachers at the school indicated a desire for their classes to be used, so these groups were "purposively selected" by the willingness of their teachers to involve their classes in the study. Both teachers agreed to take on the role of observers and to collaborate with me (as the researcher and presenter) in an advisory role during the intervention. The learners were informed of the study and all agreed to participate. Permission forms were sent home to the parents of the participants and classes began at the start of the second academic term in April 2003. The classes will be referred to as 8X and 8Y and the next section acts as an introduction to these two classes who constituted the group of participants that made up the case for this study.

3.4.3 The classes

As already mentioned, the participants were selected by the willingness of their teachers to take part in the study. The two classes were clearly different in their composition and context and the teaching styles of their mathematics teachers also differed. The larger context (in the form of the school) has already been outlined and the structure in place for the remedial programme has been explained. This section therefore serves as an orientation to the classroom context that the learners experienced in their daily as well as remedial mathematics lessons. It also briefly introduces the learners comprising each class by providing factual details about them such as their ages, home languages etc, as well as some background to their academic performance at primary school and their first term at high school.

Class 8X

This class initially consisted of four learners. During the third week of the intervention, a fifth learner (Nomsa) decided to join the classes and stopped doing German as a third language. This decision was taken on the basis of her first term mathematics results. Although initially identified as needing remedial assistance, it appears as if the learner was intent on initially pursuing a third language until the end of the first term when her results were below the required standard. She subsequently made the decision herself to make the change. She was therefore positive about this exchange and immediately started contributing positively to the lessons.

The classroom in which 8X had their lessons was the same room in which they attended their usual mathematics lessons. This classroom is situated in the old original part of the school building and therefore has wooden floors and quite a formal setting. The classroom is not very big in relation to the amount of desks that were squeezed into it and there was therefore not much room between the desks for movement. The single desks were permanently in an array of disorganised rows, facing the board and the teacher's desk. At the board was a platform for the teacher to stand on that elevated her slightly above the learners while writing on the board. There were some posters up at the back of the classroom and some information up on the sidewalls of the class. These details are mentioned as they are seen as part of the context that contributed to the classroom culture and environment in which the lessons took place. They also contrast somewhat with the context of the classroom that will be described further on for 8Y.

8X's mathematics teacher, Mrs X, has been teaching at the school for 8 years and has been teaching mathematics for a total of 16 years. Her approach can be classified as traditional in that she subscribes to mostly explaining new work by showing learners an example on the board before giving them exercises or activities to complete from the textbook. She is very gentle and patient and always makes herself available to provide learners with additional assistance or to answer questions. She has no prior experience or training in teaching remedial classes and repeatedly expressed her concern about this. As a trained secondary school teacher, she felt unequipped to deal with teaching basic concepts such as place value or fractions from the beginning as these are normally skills and knowledge that learners have in place when they arrive at high school. During discussions, she disclosed that her approach with these classes during the first term was to normally repeat the work done in the usual lesson that day, but on a simpler level, and to help the learners with the homework she had given them during their usual mathematics lesson. When the intervention began, at the start of the second term, learners appeared to be quite used to and dependent on the already established routine and would constantly seek approval from Mrs X for what they were doing. The routine that had been established during the first term by Mrs X with this class was therefore quite different from the general format used in lessons during the intervention and it is being suggested that this factor may have had an effect on the response of the learners to the intervention. This point is further elaborated on in Chapter 4, where the implementation of the intervention is described in more detail.

In order to give an overview of factual details regarding the learners, the information is represented in Table 3.1 below. As already mentioned, this data were obtained from learners' personal files made available to me by the school. The names of learners have been changed and appear in no particular order. Where cells have been left empty, the information was not available in their files at school.

Table 3.1 Details of learners in Class 8X

	Date of Birth	Home language	Age on 01-01-2003	Primary school(s) attended	Grades repeated
Klokkie	1989-06-27	Setswana	13 years 6 mths	Laerskool Kwaggasrand	None
Emelie	1989-11-27	Pemba and English	13 years 1 mth	WEM Primary school Sunnyside Primary	None
Mary	1987-04-28	Sepedi	15 years 8 mths	Marakoma Primary	?
Nomsa	1988-03-12	English	14 years 9 mths	Middelburg Primary	Grade 4
Zwanela	1988-05-02	Venda	14 years 8 mths	Arcadia Primary	Grade 3

From the table, the variety of home languages spoken in this small group of learners is evident. However, the primary schools that they attended indicate that most of them received instruction for the majority of their school years in English, the exceptions being Mary and Klokkie who attended primary schools where the language of instruction was Sepedi and Afrikaans respectively. Out of the 5 learners, 3 of them have repeated a grade at some stage or another. Both Nomsa and Zwanela repeated a grade early on in primary school and their files indicate several references to them both frequently requiring additional help in English and mathematics. Although from Mary's age, it appears evident that she repeated a grade (perhaps even more than one), no indication of which grade(s) she repeated could be found in her records. A decision was taken not to try and get this information from the learner, owing to the sensitive nature of the issue.

In Table 3.2 below, information relating to 8X learners' academic records in mathematics from their last few years at primary school (where available) is presented along with their performance in mathematics at the end of their first term at high school. The format of the Grade 7 reports from the different primary schools varied extensively. A new system of assessment was being used during 2002, relating to specific outcomes and attainment levels and no standardised format was used by primary schools for this purpose. Marks were also not always provided, making it difficult to draw a standard level of comparison. Where possible however, levels of attainment and marks have been included.

For their first term report in Grade 8, learners were also not allocated marks, but were allocated a level for each of the four outcomes mentioned. If learners did not meet the minimum level, they were allocated a NAS. Where a negative sign (-) appears before the level, it indicates that the learner had almost, but not yet quite reached that level. A positive sign (+) after the level indicates that the learner had achieved slightly more than that level but not enough to be classified under or just short of the following level.

Table 3.2 Academic background and performance of learners in Class 8X in mathematics

	Primary school				First term of Grade 8			
	Grade 4 (%)	*Grade 5 (%)*	*Grade 6 (%)*	*Grade 7*	*Numbers and algebra*	*Measurement*	*Shape and space*	*Mathematical processes and communication*
Klokkie	65	67	72		NAS	- 4	NAS	- 4
Emelie	-	-	55	Partially achieved and not competent	NAS	- 4	NAS	4
Mary	-	-	-	Required level attained	NAS	- 4	NAS	NAS
Nomsa	(1998) 31 (1999) 46	51	50	35 %	NAS	- 5	NAS	NAS
Zwanela	42	39	30	21 %	NAS	- 4	NAS	NAS

As the learners all came from different primary schools, a comparison of their performance on the basis of their marks is therefore not possible. The table does provide an overview of the learners' exposure to success or failure in mathematics over the years, however, and their teachers' evaluation of their skills and knowledge in the various outcomes mentioned after their first term at high school. An observation that was made when going through the learners' files was the very different evaluation of some learners' mathematical performance by the primary school and the high school. This is disconcerting, especially in cases such as Mary, where her Grade 7 report attests to her attaining the required level in all the outcomes in mathematics but when assessed at high school, she does not meet the minimum required level. While the standards at different schools do vary, one must wonder how such large discrepancies do occur and what effect this has on the self-esteem of the learner when faced with failure after seeming success at primary school. The table also appears to allude to the ongoing struggle with succeeding in mathematics that Nomsa and Zwanela experience. As is evident in the table, all 5 of these learners did not meet the minimum required level in either the *Numbers and Algebra* outcome or the *Shape and Space* outcome. The intervention however, focussed mainly on the first one.

Class 8Y

8Y also gained an additional member during the course of the intervention but this happened in the second week. Circumstances were different though. Liya was really enjoying German and was not all happy when her mathematics teacher recommended that she consider dropping it in order to attend the remedial classes on the basis of her first terms results. Liya's guardian (her stepmother) gave permission and Liya was "forced" to make the change. As can be expected, Liya did not join the class with the same amount of enthusiasm that Nomsa had in 8X, and for the first few lessons this made the atmosphere a little more noticeably tense than it had been originally. This changed during the course of the second week when Liya approached her mathematics teacher with a smile to say that she realised that this was going to benefit her and that she would make more of an effort. The lessons were not always easy with regard to Liya though but this is elaborated on in the following chapter.

8Y had their mathematics lessons in one of the newly built classrooms that had been added onto the school when renovations were done in 1997. The classroom is some distance from the main formal building and has a modern look about it with tiled floors and red brick walls. It is also slightly bigger than those in the main building. A mobile divider wall separates it from the classroom next door, making it quite noisy at times. The single desks were always neatly grouped into rows of four facing the board and the teacher's desk. A wide passage was created in the middle that allowed for movement between the desks. In the newly built classrooms, boards were installed at a lower height so that platforms were no longer needed to elevate the teachers. The classroom was meticulously decorated with pictures and posters and colourful examples of learners' work on investigations such as tessellations.

Mrs Y's teaching experience extends over 11 years and she has been at the school since 1998. She usually makes use of investigative activities, group work and discussions when introducing new topics and often responds to learners' questions with another leading question rather than an answer. She is enthusiastic and always looking for new ways to make the mathematics accessible to her learners. Mrs Y also has no prior training or experience with remedial classes and expressed her desire to try and diagnose the problems learners have in order to address them. She mentioned her frustration at not knowing where to start such a process and therefore also spent most of the remedial lessons in the first term revisiting content covered in class and encouraging class discussions to try and further their understanding. The general approach to lessons in this class appeared to be more congruent with those offered in the course of the intervention and the learners seemed to adapt more easily, with more enthusiasm and with less resistance to the format of the lessons in the second term than was the case with 8X.

General information relating to the learners in 8Y is provided in Table 3.3 below. Again a variety of home languages are spoken in this group, with most of the learners also having attended primary schools where the language of instruction was English. Connie, however, spent her first three years at Afrikaans primary schools and Violet spent her first five years at a Sepedi primary school.

Table 3.3 Factual information of learners in Class 8Y

	Date of Birth	Home language	Age on 01-01-2003	Primary school(s) attended	Grades repeated
Liya	1989-11-25	English	13 years 1 mth	Northridge Primary	None
Connie	1988-03-28	Afrikaans	14 years 9 mths	Laerskool Marble Hall Theresa Park Laerskool Sunnyside Primary	Grade 8
Mpho	1989-06-27	Venda	13 years 6 mths	Lynwood Ridge Hamilton Primary	None
Gloria	1989-06-05	Setswana	13 years 7 mths	Brooklyn Primary	None
Violet	1989-03-11	Sepedi	13 years 10 mths	Mbuduma Primary Sunnyside Primary	Grade 6
Patience	1989-08-24	Zulu	13 years 3 mths	Barberton Primary Sunnyside Primary	None
Leratho	1989-02-23	Sepedi	13 years 10 mths	Shanan Christian	None

Connie and Violet are also the only two learners in 8Y to have repeated a grade. Connie failed Grade 8 at this particular school in 2002 and was therefore repeating it in 2003. Her mark for her final exam in mathematics at the end of her first year doing Grade 8 was 12 % and her results for her other subjects were also below the required level. This is quite surprising when one looks at her results in mathematics in the lower grades (see Table 3.4). It is suspected that she perhaps had difficulty adjusting to high school initially but this remains speculation as the school was not able to offer any confirmation of this and again the decision was taken to not enquire about it from the learner. Violet repeated Grade 6, which also happened to be the year she moved from a Sepedi to an English school. Her academic record repeatedly mentions a need for additional assistance in English and mathematics throughout primary school. Learners' academic background in mathematics from primary school (where possible) is provided in Table 3.4 along with their results from their first term at high school.

Table 3.4 Academic background and performance of learners in Class 8Y in mathematics

	Primary school				First term of Grade 8			
	Grade 4 (%)	*Grade 5 (%)*	*Grade 6 (%)*	*Grade 7*	*Numbers and algebra*	*Measurement*	*Shape and space*	*Mathematical processes and communication*
Liya	45	46	50	47 %	4 +	4	- 6	4
Connie	78	70	73		4 +	- 4	- 5	4 +
Mpho	49	40	50		4 +	4	- 5	4
Gloria	42	44	35	35 %	4 +	- 4	- 5	4 +
Violet	40	17	(2000) 31 (2001) 41		4 +	- 4	- 5	4 +
Patience		45	36	Not competent	4	- 4	- 5	4
Leratho	77	83	76		- 5	- 4	- 5	- 5

As mentioned in the section on 8X, although a comparison of the learners is not intended on the basis of marks (due to the different standards of schools), it is evident from the table that certain learners have struggled more with mathematics since the lower grades already than others. Liya, Mpho, Gloria, Violet and Patience appear to have struggled more than Leratho and Connie in this regard. On the basis of her Grade 7, baseline assessment and first term results Leratho did not need to be in the remedial class. Her mother had requested that she rather do this than attend a third language, however, as they were afraid that she would not cope with the level of English and mathematics at high school. She was therefore in this class by their request as opposed to by the suggestion of her mathematics teacher. She did not seem to mind though and proved to be a positive member of the class.

3.5 Data collection

The data collection in this study took part in two phases. The first phase involved the front-end analysis, which, as already mentioned, included a questionnaire to learners, a literature review on low attainers and RME, an overview of examples of materials that could perhaps be adapted and contextualised, and a collection of factual and academic data on the participants. The results from this first phase are not presented as isolated data that were analysed in answering the research question. They were drawn on as a resource to provide additional information regarding the participants (see sections 3.4 and 5.3) and literature pertaining to RME and low attainers.

The second phase involved the development and implementation of the intervention and was further divided into three cycles according to the terms (terms 2, 3 and 4) of the academic year. Besides the natural demarcation of the cycles according the school terms, each cycle was also characterized by certain characteristics and stages within the research design. These are outlined in Chapter 4 where the intervention is discussed. The data collection during this phase was done through a mixture of both quantitative as well as qualitative methods, in the form of document analysis, achievement tests, classroom assessments interviews and observations. Details of these are provided in the sub-sections that follow.

3.5.1 Document analysis

Documents analysed during the course of the intervention include the personal files of learners available at the school and examples of learners' work collected during the implementation of the intervention in some of the remedial lessons. The *personal files* of learners were used to gather the nominal data that have already been presented in section 3.4.3, where the learners in the case study were introduced. As mentioned, these were not presented for comparison purposes, but rather to offer some background information on each of the learners and their academic records relating to learning mathematics. *Learners' work samples* were also retained from some of the lessons in order to provide empirical evidence of their thinking during the course of the instructional activities as well as provide data that could provide some indication of their understanding of the concepts being dealt with during the intervention.

3.5.2 Cognitive achievement tests

According to Gay and Airasian (2003), *cognitive achievement tests* provide information about how well takers of the test have learnt what they were taught and Cohen et al (2002) add that they measure achieved performance in given content areas. The cognitive achievement instruments in this study were designed to act as indicators of any change in the learners' understanding of the key concepts being revisited during the intervention.

A pre- and post-test of 30 and 31 items respectively (see Appendix A) were given to learners at the beginning and at the end of the intervention. The tests were identical, with the exception of an additional item in the post-test, which was a contextual problem that was not used in the quantitative data analysis of the tests, but rather classified as an additional sample of learners' work. This item was then included in the document analysis referred to in the previous section (3.5.1) and was used for diagnostic purposes in identifying the level of the solution used by each learner for that particular item.

The pre-test was given during the middle of April 2003 and the post-test was administered during early December 2003. This means that there was at least seven months between the two tests. Learners did complete diagnostic assessments at the end of each cycle, which included some of the items from the pre- and post-tests though, but the results of these assessments were mainly used to ascertain which concepts still required additional attention and how individual learners were progressing. These results are reported on in the analyses as learners' work samples.

The tests were "paper-and-pencil" based and non-parametric in that they were specifically designed for the population contained within this study, in order to provide quick, relevant and focused feedback on learners' performance (Cohen et al, 2000). They were also self-developed (Gay & Airasian, 2003) but did draw on released items from the Third International Mathematics and Science Study (TIMSS) 1995 and 1999 that are available on their website (http://www.csteep.bc.edu/timss). Some items were also adapted from the ColorMathPink.com website. Both "selection" as well as "supply" items were included in these tests. *Selection* items include Multiple Choice Questions (MCQ), True and False type questions and match the column type questions (Gay & Airasian, 2003), while *supply* questions require short or extended answers and explanations from learners. The selection items within the cognitive achievement tests were however limited to MCQ. The frameworks for tests are included in Appendix A. All tests were applied either by myself or by the teacher under formal test conditions and learners were not restricted to a specific time limit in which to complete the tests.

3.5.3 Mainstream classroom assessments

These assessments consisted of a standardised test (written by all the Grade 8's during October 2003) and the final end of year mathematics examination (written in December 2003) that the learners completed at school within their mainstream mathematics classrooms (see Appendix B). The mathematics staff at the school developed and implemented these assessments as part of the usual school curriculum and I therefore had no input into the content, structure or nature of the assessments. The teachers kindly retained copies of the learners' assessments to give to me as well as copies of the question papers and a memorandum. However, any improvement learners' demonstrated in these assessments could not scientifically be attributed to the intervention as there were obviously other factors also affecting their performance. While the marks and performance of learners were therefore rather used as an indicator of whether not there had been any impact during the course of the intervention, specific questions within their classroom assessments (relating to the key concepts covered during this time) were extracted and further analysed. This was done in an attempt to firstly identify any possible links to the intervention and secondly to compare learners' responses within these assessments to their responses in similar items within the cognitive achievement tests where possible.

3.5.4 Interviews

One interview per learner was conducted during the course of the intervention with all of the participants, except for one, who was too shy to do the interview. Prior to the interviews, learners were given the option to rather answer the questions in the written format of a questionnaire but they all agreed to rather do the interviews. When it came to Zwanela's turn, however, she kept saying she was too shy that week but that she would do it the following week. I did not force the issue and chose to rather let it go. The interviews were done on a one-to-one basis (Creswell, 2003) and all audio-taped with the permission of the learners.
This form of data collection was used for two main purposes; firstly with a view to enabling the participants to discuss their interpretations of certain key concepts handled during the intervention and secondly to allow them to express their opinions, experience, attitude towards and perceptions of the intervention and of mathematics (Cohen et al., 2000; Gay & Airasian, 2003). It was hoped that through the interviews, additional data that could not be collected through the tests or observations could be obtained and also used for triangulation purposes.

With these aims in mind, I constructed an interview protocol that took the format of what Patton (1980) refers to as the "Interview guide approach" (as cited in Cohen et al., 2000) and what Krathwohl (1993) calls "Partially structured" interviews. In this type of interview, the area of discussion is chosen and the questions are formulated in advance, but the interviewer decides on the sequence of the questions during the course of the interview. The questions are mainly open-ended and the interviewer also has the liberty to add questions or to modify them as she sees fit, depending on the responses of the participants. The types of questions asked took a mixture of both direct and indirect forms and were mostly what Kvale (1996 as cited in Cohen et al., 2000) terms "process questions". These are questions that either directly or indirectly ask for information that follow-up on a topic or ideas, that probe for further information or responses and that ask respondents to specify and provide examples.

The larger of the two remedial classes involved in the intervention (8Y) was interviewed (referred to as Interview one) at the end of cycle one. The interview protocol included content-based questions regarding their understanding of place value as well as questions on their opinion about the intervention (see Appendix C) and also about mathematics as a subject. The smaller class (8X) were not interviewed at the end of cycle one due to time constraints but four of the learners were interviewed at the end of cycle two using a similar interview schedule as was used in Interview one, with the exception that the content-based questions pertained to fractions and decimals. The questions in the interview specifically pertaining to the mathematics content were not used in the final data analysis as there were only two questions (which therefore did not provide sufficient information to make any conclusions) and the two classes were interviewed on different content (which meant they could not be used for comparative purposes either). The interviews were therefore finally only used for the second intended purpose stated above.

At the end of the intervention, the intention was to interview all the learners again, but due to unforeseen circumstances, this was not possible. The limitation of such single interviews to provide depth of data is therefore acknowledged (Gay & Airasian, 2003) as well as the possible influence of my role as both the presenter of the intervention, the researcher and the one who conducted the interviews. Due to time constraints, someone else was unfortunately not available to conduct them though.

Although formal interviews were not audio-taped with the teachers of the two classes or with research assistants who observed some of the lessons, informal discussions were often held with them before, after or at times even during a lesson. Relevant points from these conversations were recorded by me in the form of field notes, as discussed in section 3.5.3 where data collected from observations is addressed.

3.5.5 Observations and field notes

The naturalistic observations done during the intervention were recorded in the form of field notes, taken by myself, the mathematics teacher of the class and an assistant researcher (when available). In my case I was very much a participant observer (Gay & Airasian, 2003) or what Creswell (2003) more specifically refers to as the "observer-as-participant", in that I fully engaged in activities during the intervention but my role as researcher was known to the learners. As the researcher, I attempted to keep a journal of lessons in the intervention that contained both emic as well as etic data. *Emic* data is descriptive and gives an account of what is seen and heard, while *etic* data is of a reflective nature and includes the researchers' thoughts or ideas about the description (Gay & Airasian, 2003). These notes were used initially to inform my subsequent instructional activities and thought experiments and to record events in the classes that had stood out. Any relevant points noted during informal discussions with the teacher or researcher at some stage in or after the lesson were also recorded. Where possible, these notes were also compared to the field notes and observation schedules of the teacher and the additional researcher for triangulation purposes. The data obtained served as an indication of the extent of the alignment (or lack thereof) between the planned trajectory and the implemented instructional activity. This also contributed to establishing the implementation and application of both the heuristic as well as emergent design principles within the third cycle.

The teachers took on a slightly less participant role, in that they mainly observed the lessons, but did get involved at times in assisting learners or contributing to the class discussion. The research assistants, however, took on the role of "non-participant observer" (Gay & Airasian, 2003) and simply sat somewhere in the classroom without ever contributing or getting involved in the activities. During the first and second cycles, the observation procedures and schedules were not very stringent or consistent. The teachers observed approximately 90% of the lessons but only provided written observations for around 50% of those sessions. On some occasions, an assistant researcher was also available to attend the lessons. These observations were initially unstructured in order to allow the observers to decide in the situation what they deemed relevant to record that would have significance to the particular research question in this study (Cohen et al., 2000).

While the qualitative data obtained from these observations was interesting and useful in the ongoing development of thought and instructional experiments, I needed them to be more structured and standardised for the third cycle in order to specifically gather data relating to the implementation of RME during the lessons. An observation schedule was therefore compiled for use in the third cycle and this focused on observers (the teacher and a research assistant) assessing the implementation of the RME design principles as well as the emerging design principles in the instructional sequences during the third cycle. The observation schedule (see Appendix C) contained pre-determined statements, grouped into observation categories, with which the observers could "strongly agree", "agree", "disagree" or "strongly disagree". A "not applicable" option was also included if the statement did not have any relevance to a particular lesson. The statements in the observation schedule were mostly taken and adapted from the "teaching profile checklist" used by Armanto (2002). He in turn adopted them from Thijs (2000), who had tried out and used the checklist in several African schools and produced valid and reliable data.

Also attached to the observation schedule was a section where the teachers could comment on anything relevant or noticeable about each learner during the lesson that they deemed relevant to the study. The classes were small (5 and 7 respectively) so this was not a problem for the teachers, who knew the learners by name. As the additional researchers did not know the learners by name, they were not necessarily expected to comment on the behaviour of individual learners as such, but did sometimes make comments using sketches to indicate the relevant learners' seating places. Although an assistant researcher was present at all of the lessons during the third cycle, it was not always possible for the same assistant researcher to be present so multiple assistants shared this role.

3.6 Data analyses

The methodology employed in this study has already been identified as a *fully integrated mixed method design* (Tashakkori & Teddlie, 2003, p. 689). This means that both qualitative and quantitative aspects are present throughout all the stages of the study. In the data collection phases of the fieldwork, both qualitative as well as quantitative data were collected in the form of personal documents, achievement tests, classroom assessments, learners' work, interviews and observations. For the data analyses in such a mixed methods model, Tashakkori and Teddlie (1998) suggest the use of what they call "alternative analytic strategies" that "enable the researcher to use both of the traditional types of analysis simultaneously or in a sequence in the same study" (p. 125). In this particular study, the data analyses done were predominantly qualitative (with the exception of part of the analysis of the achievement tests), but both types of analyses were used simultaneously. This results in what is referred to as "concurrent mixed data analysis", specifically in this case a "parallel mixed analysis", which is also known as triangulation of data sources (Tashakkori & Teddlie, 1998). This parallel analysis that occurred can be combined under the traditional umbrella term of *content analysis*, which is further subdivided into two types; *manifest* and *latent* (Tashakkori & Teddlie, 1998). These are now examined in more detail in relation to the data collected during this study and how they were analysed and used to answer the research question central to the study.

3.6.1 Manifest Content Analysis

According to Tashakkori and Teddlie (1998), this particular type of content analysis was defined by Berelson (1952) as "…a research technique for the objective, systematic, and quantitative description of the manifest content of communication" (p. 18). It is therefore associated mostly with quantitative techniques (often used on qualitative data) due to the use of standardised measurements that are applied to metrically defined units and used to characterise and compare documents (Manning & Cullum-Swan, 1994 as cited in Tashakkori & Teddlie, 1998). Categories are therefore predetermined and coding procedures are standardised to the highest degree possible. In this study, the data that were compared through the use of this type of analysis were the achievement tests (pre- and post-test) completed by learners, the classroom assessments completed in their mainstream classrooms, a selection of work samples from learners and the observation schedules that were completed by the teachers and assistant researchers during the third cycle of the intervention. This was done with a view to represent and analyse any changes in learners' understanding of the key concepts of place value, fractions, decimals and basic algebra during the intervention, in order to address the confirmatory aspect of the research question and to investigate the characteristics of using the theory of RME in designing and implementing the intervention (the more exploratory aspect). For this type of analysis, Miles and Huberman (1994) suggest the use of matrices (developed before or during data collection) that the data can be placed into. As suggested by Miles and Huberman (1994), the raw data was first simplified and then transformed using identified codes.

For the ***achievement tests***, this was done by first entering the learners' responses for each item in the test as a code into a statistical programme known as the Statistical Programme for the Social Sciences (SPSS). The key concepts mentioned above were classified as categories and the items relating to each of the four categories were also specified from the frameworks from which the test had been constructed. After programming SPSS to mark the items as correct or incorrect (according to their allocated codes) and to also produce a total score for each of the categories (by grouping together the scores of specified items) in each of the tests, a matrix representing any change in learners' understanding relating to the five categories could be displayed. Part of such a matrix is included in Table 3.5 as an example.

Table 3.5 Matrix representing any change in learners' understanding as measured by the achievement tests

Learner	Place value	Fractions	Decimals	Basic algebra	Other
Klokkie					
Mary					

The ***highly structured observation schedules*** completed by the teachers and the assistant researchers during lessons in the third cycle of the intervention were also analysed using SPSS. Codes were assigned to the "Strongly agree", "agree", "disagree", "strongly disagree" and "not applicable" columns and the data were captured into SPSS. Statements were grouped into five categories (statements relating to the *principle of guided reinvention*, statements relating to the *principle of didactical phenomenology*, *overall implementation of RME instructional approach*, *working with low attainers*, *behaviour and responses of learners*) and the mean for each statement and category was represented and discussed.

Regarding the ***classroom assessments****;* questions relating to only three of the categories (i.e. place value, fractions and decimals) were identified and searched for in learners' final examinations (that had been marked by their mainstream mathematics teachers). These scores were manually coded and captured onto a data matrix (see table 3.6) for each learner that represented a summary of a selection of learners' work. Basic algebra was not looked at mainly because it was problematic to classify questions in the final examination as relating to basic algebra. Another reason for not including the category in this analysis was because it was not one of the main topics of the intervention (although it was hoped that learners' improved understanding in place value, fractions and decimals would improve their performance in algebra). One of the questions from the standardised test was also selected as being one that most learners should be able to access (this was based on a discussion between the teachers and myself, drawing on our experience of teaching Grade 8 learners). Although content wise it did not directly pertain to the work dealt with during the intervention, it did allow for both formal and informal strategies. The analysis of learners' solutions to this question was also included in the data matrix.

Data extracted from the *diagnostic assessments* that learners wrote at the end (or start) of a cycle as well as *work samples* collected from the learners during the remedial classes were also represented in the data matrices profiling each learner as explained above. This was also done according to categories but not the key concept categories previously referred to. These five categories (*use and level of strategies, appropriateness of solution, final correctness of solution, major mathematical errors, minor mathematical errors)* focused more on the level of, appropriateness and correctness of strategies. Any references to individual learners, that were made in observations or field notes during lessons from which the samples of work had been selected, were also included in the matrices. The data matrix summarising each learners' profile took the format as represented in Table 3.6 below.

Table 3.6 Data matrix summarising selection of learner's work

	S	A	FS	ME	me	Comments
Cat pills						
Insulin						
DA 2 (19)						
DA 2 (20)						
Overseas						
Ratio problem						
Standardised test						
Calculating mark						
Post-test						
Final mainstream exam						

NOTE:
S = Levels and use of strategies; A = Appropriateness of solution; FS = Correctness of final solution; ME = Major mathematical error(s); me = minor mathematical error(s)
FL = Formal level; GL = General level; RL = Referential level; SL = Situational level
A = Appropriate; PA = Partially appropriate; NA = Not appropriate; T&E = Trial and error
✔ = Correct; PC = Partially correct; ✖ = Incorrect
1/5 indicates that mark obtained for that question was one out of a possible 5 marks
DA 2 = diagnostic assessment 2

3.6.2 Latent Content Analysis

Tashakkori and Teddlie (1998) state that the latent content analysis of a text "is determined by a subjective evaluation of the overall content of the narrative" (p. 122). This means that the scheme for analysing the themes associated with the content emerge during the analysis itself and are not predetermined as is the case with the manifest content analysis (Tashakkori & Teddlie, 1998). This is in line with what is also known as inductive analysis (Miles & Huberman, 1994; Creswell, 2003; Gay & Airasian, 2003) where the researcher constructs patterns that emerge from the data in order to make sense of them. In such an analysis one usually starts with a large set of issues and, through an iterative process, progressively narrows them down into small important groups of key data. From this data variables are then identified through further examination and analysis that can be interpreted and discussed. This therefore creates a multistage process of organising, categorising, synthesising, interpreting and reporting on the available data (Gay & Airasian, 2003). In this study, the data that were analysed in this manner were the documents of learners' work, the observations (in the form of field notes and observation schedules) and the interviews. The rest of this section reports on the procedure used to analyse these documents as informed by Creswell (2003), Gay and Airasian (2003) and Miles and Huberman (1994).

The data was first arranged and labelled according to type of notes, sequence and dates. This was done in order to check the data for completeness and to start the analysis process. Gay and Airasian (2003) call this the "organising" stage while Miles and Huberman (1994) refer to it as "formatting" the data. Once this had taken place, the data could be further examined for themes, patterns, regularities and issues that emerged or that had been noted in the ongoing data analysis of documents and field notes that was done throughout the intervention. The actual process of the intervention as it took place was then described in relation to the context, actions and reactions of the participants as it played out through the collection of field notes (see Chapter 4). The next step involved classifying (Miles and Huberman call this indexing) similar ideas and concepts, from the description and further examination of the data, into categories that could be discussed in relation to answering the exploratory aspect of the research question. This aspect of the question required a response as to how RME could be used revisit key number concepts with a view to improving learners' understanding of these concepts and to getting them to make more use of intelligent as opposed to habit learning.

As already mentioned, a description of the three cycles of the intervention is presented in Chapter 4 and the resulting data and analyses are presented in Chapter 5. This includes the separate presentation of data analysed through the two types of content analysis outlined above. These deductive and inductive inferences were then combined in a meta-inference (Tashakkori & Teddlie, 2003), used to draw the final interpretations and conclusions alluded to in the final chapter, Chapter 6. Before concluding this chapter however, the steps put in place during the study to tend to issues of validity and reliability are dealt with so that these are made clear before the final two chapters that wrap up the essence of the study are presented.

3.7 Validity and reliability

As this is a mixed methods study, the terms used to depict validity and reliability are not limited to these two terms, which are more commonly associated with quantitative data. When working with qualitative data, the terms trustworthiness, dependability, transferability and credibility are also used. Each of the two development research approaches being used in the design of this study also has particular terminology that they make use of to refer to these concepts. These include terms such as trackability, practicality and effectiveness. To try and simplify the process of taking cognisance of all of these, this section has been divided into three sub-sections, which deal with the triangulation, the instruments and the quality criteria for the intervention separately. Issues relating to validity and reliability in relation to each of these sub-sections are then outlined therein.

3.7.1 Triangulation

Cohen et al. (2001) define triangulation as the "use of two or more methods of data collection in the study of some aspect of human behaviour" (p. 112). For Denzin (1994), the concept of triangulation involved the combination of data sources to study the same social phenomenon. According to Lin (1976 as cited in Cohen et al., 2000) triangulation helps to ensure that data generated are not relics of one specific method of collection, by using different methods of data collection that substantially produce the same results. And for Gay and Airasian (2003), triangulation involves using different data sources to confirm one another. These explanations sound similar to statements in typologies that are offered for classifying mixed method approaches and models. In fact, the mixed method designs were initially defined under the general heading of method of triangulation (Creswell, 2003). Triangulation is therefore intrinsic to this study by the very nature of its design.

Denzin (1994) distinguished between four basic types of triangulation; three of which have been applied in this study. These include data triangulation, investigator triangulation and methodological triangulation. *Data triangulation* involves the use of a variety of data sources in a study and in this particular case; these were present in the form of observations, interviews and document analysis. While not all the sources were simultaneously employed to answer the same part of the research question, more than one data source was always used in drawing conclusions relating to the exploratory and confirmatory aspects of the question. For example, in an attempt to answer the exploratory portion of the question (the how), observations of the lessons, interviews conducted with the learners as well as samples of learners' work were all used in the process. In addressing the confirmatory segment of the question, the four cognitive achievement tests and the relevant questions from classroom assessments, as well as samples of learners' work were all employed. The observations included field notes from the teacher, a research assistant and myself. This therefore satisfies the criteria for *investigator triangulation* where more than one observer is required. Finally in *methodological triangulation*, the use of multiple methods is used to study a research problem. This process has been explained throughout the previous sections within this chapter, where the mixed data collection and analyses procedures were outlined.

Owing to the fact that I adopted multiple roles in the study, in the form of developer and presenter of the intervention as well as the researcher, these three types of triangulation play an important part in ensuring that my multiple roles do not threaten the validity and reliability of this report. However, as Fielding and Fielding (1986 as cited in Cohen et, 2000) warn, methodological triangulation does not necessarily increase validity, reduce bias or bring objectivity. Patton (1980) also states his concern about multiple data sources not necessarily ensuring consistency or replication. The validity and reliability aspects of the instruments as well as approaches within development research used to address these criteria are also outlined in this section, in an attempt to tend to this counsel offered by Paton and Fielding and Fielding.

3.7.2 Instruments

This sub-section outlines the validity and reliability procedures put in place to try and ensure that the data obtained from the cognitive achievement tests were as valid and reliable as possible, acknowledging that these aspects are never absolute states within a study and should rather be viewed as a matter of degree (Gronlund, 1981 as cited in Cohen et al., 2000). The types of validity dealt with are *content validity* and *criterion-related validity*. In terms of reliability, the *stability* and *internal consistency* of the instruments are reported on.

Validity

Content validity requires that the instrument must show that it fairly and comprehensively covers the domain or items that it purports to cover (Cohen et al., 2000). Gay and Airasian (2003) state that content validity is achieved when the two components, namely *item validity* and *sampling validity*, contained therein are both met. Item validity deals with whether the test items are relevant to the measurement of the intended content area, while sampling validity is concerned with how well the test samples the total content area that is being tested. This is mainly determined by expert judgement where an "expert" in the field reviews the process of compiling the test as well as the content of the test (Gay & Airasian, 2003).

In an attempt to ensure item and sampling validity for the cognitive achievement instrument, a framework was first constructed (see Appendix A) to ensure that the key concepts were all measured and that varying skills under each concept were measured. The items for the test were then selected and adapted from publicly available websites (the TIMSS website and the colorpink website) and where necessary new items were written. The framework and test were sent to an expert in test construction as well as one working in the domain of mathematics education and suggested changes or comments were adhered to before the instrument was finalised. Although it is recognised that the validity and reliability of the instrument could probably have been increased by first piloting it, the scope of this study did not allow for such an endeavour.

Although this report keeps referring to the key number concepts of place value, fractions and decimals being revisited, the instrument contains a number of items pertaining to learners' understanding of basic algebra. The reason for this is that the initial general design of the intervention had intended to deal with revisiting place value during the first cycle of the intervention, fractions and decimals during the second cycle and then deal with basic algebra during the third cycle. However, the number of lessons needed to focus on fractions and decimals turned out to be more than originally anticipated, so the third cycle continued to focus on these concepts. Basic algebra was therefore not taught as a "dedicated" concept within one of the cycles but was alluded to on occasions during the lessons and was handled during the mainstream mathematics classes. This is not seen as a threat to the validity of the data, however, as the items relevant to the concept of basic algebra could be excluded in drawing conclusions on learners understanding of the other three key concepts or be used for further comparisons.

The possibility of "testing effects" occurring is acknowledged, however. This means taking into account the fact that changes on respondents' scores on later administrations of the test may be due to the fact that they took the same test at an earlier time (Johnson & Turner, 2003). An attempt was made to minimise this effect by ensuring that there was a gap of at least three months between learners' exposure to any of the items and there was also a time period of 7 months between learners writing the pre and the post-test. Learners would therefore probably not have remembered the items, although it is acknowledged that learners' exposure to some items more than others (in the diagnostic assessments) may have had an impact on the results. It would have been preferable to rather have a large database of similar and comparable items available, from which the diagnostic assessments could be constructed, so that the same items were not used more than once in between the pre- and post-tests. Unfortunately due to the limited scope and time of this study, this was not possible, but will be addressed, should the study continue into a second phase.

A second limitation of the test (used as the pre- and post-test), that will need attention for future research, was that it contained 26 conventional mathematical problems and only 4 contextual problems. A problem-solving approach was used in the intervention though, so there should have been more contextual problems in the achievement tests to align the assessment more with the teaching and learning approach. On the other hand, if the learners' understanding did improve during the course of the intervention, then learners should also have been better equipped to answer or solve conventional mathematical questions. Learners' performance on these items can therefore still be regarded as an indicator of learners' understanding, but it is my opinion that the quality of the data would have been higher with more of a balance between the amount of conventional and contextual problems.

The cognitive achievement tests were not used as the only measure of learners' understanding either. To try and improve the validity of data used to indicate change in their understanding, classroom assessments completed by learners during their mainstream mathematics classes were also used as a source of data in this regard. Questions within the classroom assessments relating to the key concepts of place value, fractions and decimals were identified and learners' responses on the achievement tests were compared to their responses in the classroom assessments, which took the form of standardised tests and their final examinations. This is in line with what is commonly known as *criterion-related validity* (Cohen et al., 2000; Gay & Airasian, 2003) and endeavours to relate the results of one particular instrument (the cognitive achievement tests) to a second test or measure (in this case questions in the classroom assessments).

Reliability

Reliability is a synonym for consistency and replicability over time, over instruments and over groups of respondents and is therefore concerned with precision and accuracy (Cohen et al., 2000). The two types of reliability measured in relation to the cognitive achievement instruments were the *stability* of the instrument and the *internal consistency* of it. The post-test was administered seven months after the pre-test. The two sets of scores were correlated, rendering a correlation coefficient of 0.781, significant at a 0.01 level, which points to a high level of **stability** between the two tests (Gay & Airasian, 2003).

In order to determine an estimate of the **internal consistency** of the instruments, the Cronbach Alpha was calculated. This measures how all the items on a test relate to all the other test items and to the total test score, thereby giving an indication of its internal consistency. The Cronbach Alpha for the pre-test was 0.8542 and for the post-test was 0.7873 indicating a high internal consistency in both the pre- and post-tests (Cohen et al., 2000).

3.7.3 Quality criteria for the intervention

Owing to the mixed methods model that both the development research approaches used in this design subscribe to, both advocate the use of triangulation as outlined in section 3.7.1. In addition to this, the development research approach subscribed to by van den Akker (1999) places a lot of emphasis on continuously developing and improving the quality of the intervention itself by focusing on three main characteristics inherent in high-quality products.

These include validity, practicality and effectiveness and are further addressed individually, below, drawing on the work of Nieveen (1999) and van den Akker (1999). The development research approach propagated by the Freudenthal Institute lays its emphasis regarding criteria for quality on the trackability of process of design and implementation and is discussed below with specific reference to Gravemeijer (1998, 2001) who also draws on the work of Smaling (1990).

Validity, practicality and effectiveness

The *validity* of the intervention is the extent to which its design is based on "state-of-the-art" knowledge (they equate this with content validity) and the fact that all the components are consistently linked to each other (equated with construct validity). This characteristic is determined by expert appraisal. For the purpose of this study, an attempt was made to ensure the validity of the intervention by using the theory of RME to guide the design and implementation of the intervention and by discussing examples of thought experiments and the results thereof with RME experts when possible (i.e. Maarten Dolk, Jeffery Choppin, Pauline Vos, Cyril Julie) and with mathematics educators and other experts familiar with this domain (amongst others Tjeerd Plomp, Andy Begg, Chris Breen). It is acknowledged, however, that the validity of the intervention gradually improved throughout the intervention as my knowledge and understanding of RME improved and that this validity can still be enhanced in the development and testing of further prototypes.

The *practicality* of an intervention is the extent that users (and other experts) consider the intervention to be usable and easy to apply in a way that is largely compatible with the developer's intentions. This aspect can be established through micro evaluations and try-outs. While my intention is to produce an initial prototype to try out in further studies, based on further reflections, workshops and results from this report, the intervention as it was implemented was not a prototype but rather a collection of instructional activities, as mentioned before. One can therefore say that the instructional activities have been "tried out" and that micro evaluations on them have been done, but reporting on the practicality of the intervention as a whole is not yet possible from this study. During the third cycle of this study though, the teacher and an assistant researcher completed a highly structured observation schedule during each lesson. One of the observations they needed to comment on, was the "usability" of the lessons for others and more about the *practicality* of the individual lessons in cycle 3 can be found in Chapter 5.

The third characteristic of a high quality product (in this case the intervention) alluded to by Nieveen (1999) and van den Akker (1999) is that of *effectiveness*. This is an indication of the extent that the experiences and outcomes of the intervention are consistent with the intended aims, and is measured through field tests. The main immediate outcome of this intervention was that it would improve the mathematical understanding of the participants with regard to the key concepts of place value, fractions and decimals. The extent to which this was realised (in other words the effectiveness of the intervention to meet its intended aim) is reported on in the concluding chapter.

Trackability

As already mentioned, *reliability* is often defined as reproducibility, and refers to the absence of accidental errors. To account for this in qualitative research means reporting the research in such a way that it can be reconstructed or virtually replicated by other researchers. In this regard Smaling (1990 as cited in Gravemeijer, 1998) uses the term "trackability", which is established by reporting on "failures and successes, on the procedures followed, on the conceptual framework and on the reasons for the choices made" (p. 6). As Gravemeijer (2001) notes, this in turn links up well with Freudenthal's concept of development research, which he explains as:

> *…experiencing the cyclic process of development and research so consciously, and reporting on it so candidly that it justifies itself, and this experience can be transmitted to others to become like their own experience. (Freudenthal, 1991)*

In this regard, this dissertation has sought to reproduce and track the processes followed in the course of this study, by firstly outlining and summarising the literature consulted and the theory used (Chapter 2), by giving a detailed description of the research design and methodology employed (Chapter 3), and by reporting on the three cycles of the intervention (Chapter 4) and the data collected and analysed during that time (Chapter 5), before drawing its final conclusions (Chapter 6).

3.8 Ethical considerations

As previously mentioned, I taught at the school where I carried out this research for a number of years prior to joining the university. I subsequently had a good relationship with the principal of the school as well as the mathematics teachers who volunteered their classes to be used in the study. Once consent had been obtained by the principal of the school to conduct the study and two teachers had offered their classes as samples, the learners in those classes were approached by me and as suggested by Gay and Airasian (2003), the following steps were followed:

- My position as a lecturer at the university was explained to the learners and the purpose and an outline of the intervention was provided.
- It was made clear to learners that their remedial lessons would now be taught mainly by me and that the focus would no longer be on what they had done in their mainstream mathematics class that day, but would revisit the key number concepts.
- It was emphasised that participation was voluntary and that their parents would also be informed of the study.
- Learners were promised full confidentiality on events that took place during the lesson (although they knew that their teacher would be present).

The teachers were also promised confidentiality in that their names would not be divulged in the report or any data pertaining to them be given to their superiors. The learners and the teachers were comfortable with the agreement and a letter of consent was subsequently drawn up with the principal and sent out to parents (see Appendix D). The letter contained a reply slip and these were signed by the learners' parents or guardians and returned to me.

With these steps in place, the intervention began and no ethical dilemmas or problems were encountered during the course of the study. As already mentioned, the learners were given the opportunity to fill in anonymous questionnaires regarding their experience of the intervention, but they chose to rather do interviews. Zwanela admitted to being too shy to do an interview, wanting to do it later and this issue was not pursued, thereby respecting her right and avoiding making her self-conscious of the issue. Where learners' personal records did not contain all the information required to report accurately on their academic success or failures in mathematics, the decision was taken not to question learners on this due to the sensitive nature of the issue.

The university also has an "Ethics Committee" who review all proposals of studies that will be taking place and this study was given the consent to continue as planned.

3.9 Conclusion

This chapter covered the research design and methodology pertaining to this study in detail. The intention of the study was to see if and how the theory of RME could be used in this setting to improve the learners' understanding of the key number concepts of place value, fractions and decimals. This therefore placed the emphasis more on what works than on which methods to use, thereby situating the study within a pragmatic paradigm. Consequently some of the development of pragmatism as a philosophy (and later as a research paradigm) was referred to and justification for placing this research within a pragmatic epistemology was provided. The resulting research design chosen to guide the study was identified as a development case study and this term was further unpacked. This was done by firstly defending its classification as a case study and secondly by providing a description of the two schools of thought on development research that informed the design.

The main research question driving the study was both confirmatory (to confirm whether the RME approach can make a difference in learners' understanding) and exploratory (asking how this can be done within this study) therefore giving it both quantitative as well as qualitative dimensions. The data collection was done in two phases and these were further outlined and explained. Procedures used during data collection also included a mixture of both quantitative and qualitative approaches in the form of document analysis, interviews and observations. Finally, both inductive and deductive data analyses approaches were utilised, usually associated with qualitative and quantitative analysis respectively, in order to compile a meta inference from which conclusions could be drawn. This mixing of the two approaches in the conceptualisation, method and inference stages of the research resulted in what Tashakkori and Teddlie (2003) refer to as a *fully integrated mixed model design*.

Lastly, this chapter also dealt with the issues of validity and reliability, and how they can be treated within a mixed methods design and from the development research perspective. Possible shortcomings and limitations of the instruments were identified and the triangulation aspect built into the study was elaborated on. The chapter concluded with a brief discussion pertaining to ethical issues involved in this type of educational research.

Now that the design and methodology of the study have been represented and defended, a more detailed description of the three cycles of the intervention that took place during the second phase of the fieldwork is presented in Chapter 4.

CHAPTER FOUR

4 The Intervention

4.1 Introduction

This chapter provides an overview of the development of the intervention that was implemented during the course of the study, and represents a data analysis of the intervention according to data obtained from observations (by the teachers and an assistant researcher where possible), interviews conducted with the learners, field notes or logs (that I kept throughout the intervention) and work samples from learners. The analysis is done with a view to giving an account of the intervention in terms of: the overall design, the type of instructional activities used, the instructional approach and the implementation of the heuristic design principles as set out by the theory of RME. As the first two cycles were mainly exploratory cycles in trying to implement the RME instructional approach and design principles, they are reflected on in a more informal manner, using the data mentioned above. The third cycle was more controlled and is therefore more formally evaluated. The data analysis in Chapter 5 focuses on the individual learners and their performance throughout the intervention. From these analyses of the intervention and the learners' performances, conclusions are drawn in the final chapter.

The development and implementation of the intervention was divided into three cycles according to the last three school terms (terms 2, 3 and 4). Initially it was planned that the first cycle would focus on revisiting place value, the second cycle on fractions and decimals and the third cycle could be dedicated to looking at basic algebra. However, during the course of the study, the learners' misconceptions and conceptual shortcomings relating to the topics designated to the second cycle, as well as their difficulty in solving arithmetic contextual mathematics problems, were evident. This highlighted the need to continue focusing on further improving their sense and understanding of numbers (whole numbers, integers and rational numbers) before further embarking on algebra. As mentioned in Chapter 3, basic algebra and the symbolic relations and meanings therein, were related to during the course of the intervention where possible though, and also in relation to work that the learners were busy with in their mainstream mathematics classes.

Besides the natural demarcation of the three cycles according to the school terms, each cycle became characterised by certain characteristics and stages within the research design, although this was not the initial intention but something that evolved as the study progressed. These three cycles are elaborated on and discussed in the rest of this chapter. This is done for each cycle by first depicting their instructional designs, discussing examples of instructional activities from each cycle and then reflecting on and evaluating what transpired against the intended design and objectives. Before looking at the individual cycles however, the role of this intervention in developing future prototypes relating to revisiting the key number concepts of place value, fractions and decimals with low attaining learners, is first outlined. This is followed by an overview of the general approach used in the lessons throughout the intervention and finally by the analysis of each of the three cycles.

4.2 The role of this intervention in the development cycles

The development research approach of van den Akker and Plomp (1993) suggests that the type of formative research that was carried out in this study takes place through an iterative process of big cycles. Each of these cycles involves the design, implementation, evaluation and refinement of a prototype in an attempt to continually improve the quality of the prototype while also generating knowledge in the form of design principles, which can guide the development and implementation of future interventions. In light of this explanation, it is necessary to differentiate between the three smaller cycles being referred to within the intervention of this study and the bigger iterative cycles of implementation of development research.

I view the research being reported on in this report as the first big cycle (containing three smaller cycles) in the research process described above. However, I would not say that the intervention as it evolved in this study could already be classified as a prototype but should rather be regarded as a collection of instructional activities, with the aim of constructing a prototype from the data collected. My main reason for making this statement is due to the informal nature in which the intervention evolved during the course of the study, and the emphasis of the study not being based on a systematic design process as such but rather on an exploration whether an intervention theory could be developed that may result in the desired immediate outcomes (see section 3.2.4).

Drawing on the second development approach as explained by Gravemeijer (1994; see section 3.2.3), the collection of instructional activities for the entire intervention were not all thought out and planned in advance, although an overall plan with immediate outcomes for the intervention did exist (see section 3.2.4). Some instructional activities were planned to start off with (in the form of thought experiments) and these were then implemented as instructional experiments in order to inform the development of further instructional activities as the intervention progressed. Data relating to the learners' work, their performance and perceptions as well as observations (by the teacher, assistant researcher on occasion and myself) of the learners and the instructional approach used during the lessons were being collected throughout the intervention. These data informed the development of further thought activities during the course of the study. Through this process the focus was continually on trying to improve the learners' understanding of the key concepts being dealt with while also trying to advance the quality of the instructional activities and their implementation in relation to their adherence to the RME theory.

Conclusions drawn from the analyses of the learners' performance and the instructional activities (that make up the intervention) can at a later stage then be used to inform the design of an actual prototype that can be implemented and evaluated in a second big cycle of the development research approach as explained by van den Akker and Plomp (1993). While the conclusions from the analyses are presented in Chapter 6 of this report, the design of the resulting prototype for the second big cycle is beyond the scope of this study. An account of the actual intervention implemented in this first big cycle is now reported on in the remainder of this chapter though.

4.3 The instructional approach to lessons in the intervention

The RME instructional approach used in the intervention was quite different to the traditional approach still used in many mathematics classes in South Africa, where a lesson often consists of an example being shown by the teacher and the learners then being given an exercise to complete. In most South African mathematics classes learners are seldom required to solve problems related to their lives, although an increase in the use of class discussions has been noted (Howie, 2002; Howie, Barnes, Cronje, Herman, Mapile & Hattingh, 2003; Venter et al., 2004).

On the whole, most instructional activities in this intervention included a problem that learners were first introduced to and then given an opportunity to go about solving, either individually or in groups. Learners were encouraged to keep notes of their thinking and strategies and then to share these with the rest of the class either verbally or by writing them up on the board. If time permitted, we discussed some of the solutions in terms of their adequacy and efficiency and where possible reflected on the solution from a mathematical point of view. I also tried to reinforce correct notation and terminology. Often though, the 40-45 minute lesson ended before we could discuss all the solutions and these would need to be resumed the following lesson. As previously mentioned, these lessons were at times even a week apart, which made picking up where we had left off quite problematic. I usually collected learners' work at the end of a lesson so that it could be returned to them the following week to continue (I learnt early on that they forgot to bring them back themselves) or so that I could keep them for diagnostic and data purposes.

Learners were also encouraged to work in groups at times, usually pairs, but on occasion even the whole class would decide to work on a problem together. There were lessons where they chose to go about solving problems on their own or some worked individually while others collaborated. In some lessons whole class discussions were pursued, while on other days, depending on the dynamics of the class at that time, smaller groups discussions were facilitated by me moving around the class, with the help of the teacher if she chose to assist. Games were also used, mostly in the beginning with place value, so that learners would become more at ease with myself and also with each other, while hopefully also fostering a positive attitude towards the classes and mathematics through their enjoyment.

4.4 Cycle one

Cycle one began at the end of April 2003, approximately three weeks after the start of the second academic term. The original intention was to begin the intervention at the beginning of the academic year. However, discussions with the teachers at the school convinced me that starting in the second term would be better for two reasons. Firstly the new Grade 8's were still finding their feet at their new school and settling into the routine of things. Secondly, the remedial classes usually grew in number at the end of the first term, when learners' first term reports were drawn up and learners who had perhaps not fared badly in Grade 7, but who were evidently not making the required grade in Grade 8, were identified. On the other hand, my concern with starting in the second term was that learners would perhaps become accustomed to a certain approach in the remedial classes during the first term, which would make it more difficult for them to adapt to the RME instructional style to be applied in the intervention.

The second term began on the 7th April but the first three weeks of the second term were used for administrative purposes; the school was still identifying learners who needed to join the remedial classes as a result of their poor performance during the first term and permission needed to be obtained from the learners' parents. The pre-tests were also conducted during this time and learners completed an information questionnaire about themselves (see section 3.5). The first actual lessons of the intervention therefore began during the last week of April and could not continue beyond the end of May, due to the mid year examinations starting on Friday 30 May 2003. This resulted in 8X and 8Y each having 6 and 7 lessons respectively during this first cycle.

The rest of this section gives an overview of the instructional design for cycle one, followed by a discussion of an instructional activity that was illustrative for this cycle. A reflection of the cycle, based on data from observations, interviews and my own logs is finally presented along with revisions that needed to be considered for the second cycle.

4.4.1 Instructional design for the first cycle

In general the first cycle was mainly an exploratory stage during which I got to know the learners and try to make them more comfortable with me. During this phase, I also tried to establish (or rather negotiate) a particular classroom culture of explicit norms and practices that would be followed for the duration of the intervention. These included making learners aware that their thinking and thought processes in solving a problem or doing a calculation were more important than the actual answer or use of a prescribed procedure, and that they would be expected to explain and justify their solutions and make an effort to understand the solutions of others (Gravemeijer, 1994). It also included making them comfortable and confident to share their solutions and strategies with myself and with the class, regardless of whether they were right or wrong. Learners really appeared to enjoy writing on the board at the front of the class, so they were often given the opportunity to write their solutions on the board. I made a point of initially discussing solutions without reference to the person who had written it up on the board so that it became more about the mathematics and less about "whose mathematics".

Besides getting to know the learners better and establish a classroom culture during this first cycle, I also became more acquainted with the theory and principles of RME while embarking on the first set of what Freudenthal (1991) and Gravemeijer (2001) describe as cyclical processes of thought experiments and instructional experiments as depicted in Figure 2.2. As previously mentioned, these serve a dual function in that they both clarify researchers' learning about learners' thinking and address the pragmatic affairs of revising instructional sequences. Instructional activities are designed by the curriculum developer (in this case myself) who starts off with a thought experiment that imagines a route learners could have invented for themselves. A lesson is implemented and the actual process of learning that takes place in that lesson in relation to the anticipated trajectory is analysed. This analysis can then provide valuable information in order to revise the instructional activities for the next lesson. During the first cycle of this study, the instructional activities were mainly developed around the topic of place value of whole numbers up to seven digits long and on improving the learners' conceptual understanding of our denary system.

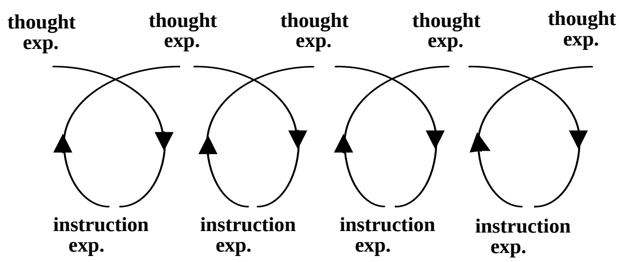


Figure 4.1 Developmental research, a cumulative cyclic process (Gravemeijer & Cobb, 2002)

Gravemeijer (1994, 1999) identifies three key heuristic principles of RME for the process of instructional design, namely: *guided reinvention through progressive mathematisation, didactical phenomenology* and *self developed or emergent models.* The dominant principle being explored and applied throughout this study was the principle of *guided reinvention through progressive mathematisation* (see section 25.2). During this first cycle I focused exclusively on this principle though as this was the initial attempt to try and design and implement instructional tasks and activities based on the RME theory. However, it needs to be emphasised that this was done by *revisiting* certain key number concepts in a remedial context for low attaining Grade 8 learners, as opposed to introducing these concepts to learners for the first time. This meant that I had to keep in mind that these concepts had been part of the curriculum taught to these learners in almost all of the grades they had completed since starting school, especially in the case of place value. Freudenthal (1991) refers to this process of "recalling old learning matter" and reviewing it from a higher stance or a broader context as "retrospective learning" (p. 118). He sees this type of learning as serving a dual purpose; to root the new matter in the old one and to strengthen the old root. In this intervention, the intention was to revisit old matter in order to strengthen the old roots so that new matter (that learners were being taught in their mainstream mathematics classes) could be rooted within more secure roots. In order to do this, one tries to make the learner "more conscious of a complex of less previously conscious pieces of knowledge and abilities and of their interrelatedness" (Freudenthal, 1991, p. 118). I therefore tried to design an activity that would provide learners with an opportunity to mathematise the concept of place value of whole numbers in a base ten system through a process of retrospective learning. I had to take care to avoide using an activity though that would simply make them feel like they were back in a lower grade relearning work they had already been taught.

The main objective of the learning activities pertaining to place value was to take learners through a mathematisation process that would enable them to see and understand the development and of the pattern of **units**, **tens** and **hundreds** moving onto **thousands**, **tens** *of thousands* and **hundreds** *of thousands* and finally into one million. It was hoped that this would improve their knowledge and understanding of whole numbers in order to advance their performance in reading and writing whole numbers up to seven digits, as well as their calculations involving whole numbers (such as subtraction, rounding off etc). This was predominantly done through a series of lessons that involved working with Dienes blocks (an example of one of these is provided in Box 1), interspersed with playing a game pertaining to place value and an instructional activity that included a contextual problem on ordering the salaries of various professionals and purchasing houses (see Appendix E).

4.4.2 Example of an instructional activity from the first cycle

Box 1 below outlines one of my first attempts at designing an instructional activity to revisit place value with the learners. A critical discussion of the instructional experiment (in retrospect) with regard to the theory of RME is also provided, followed by a description of the actual learning that took place in relation to this trajectory in each of the classes.

For the instructional activity explained in Box 1 (that took up two lessons), I chose to work with Dienes blocks in order to provide the learners with material from which to further develop the denary system. Although the theory of RME does enable the use of structuring aids such as number lines, charts, grids, diagrams and graphs (Treffers, 1987), it does not encourage the use of base ten Dienes blocks (Freudenthal, 1991; Gravemeijer, 1994). Gravemeijer (1994) relates the use of them back to the "mainstream information processing approach" (p. 77) that uses representational models and manipulatives to introduce, exemplify and learn abstract mathematical knowledge. Treffers (1987) classifies this as the "structural" approach in his classification of approaches to the teaching of mathematics (see section 2.6.1). This form of teaching with Dienes blocks (base ten blocks) is seen in the RME approach as being too prescriptive, providing too much structure and as an approach that requires learners to use the blocks according to a set of rules determined by the researcher (Gravemeijer, 1994). They argue too that the teaching of written algorithms for addition and subtraction are the main concern of such an approach as opposed to the teaching of place value. Cobb (1987 as cited in Gravemeijer, 1994) also criticises the information processing approach in failing to make an adequate transition from concrete action to abstract, conceptual knowledge. He argues that the learners in fact need the mental representation (which learners are usually being expected to develop through using the blocks) in order to be able to interpret the concrete representation.

From the discussion above, it appears as though the main opposition to the use of the blocks relates more to the function of the blocks as opposed to the actual blocks themselves. This can be reinforced by the fact that RME researchers do use materials such as stones, tokens and an abacus in developing the denary system with children. Gravemeijer (1994) argues, however, that their materials are relatively unstructured and that the concepts (such as tens and hundreds) are not illustrated by the material. He adds that, "in so far as the material is structured, the structurization is directed at eliciting certain mental activities"s (p. 69).

My understanding of the manner that I used the Dienes blocks (their function) in this instructional activity was not in the structured or information processing way Gravemeijer (1994) and Cobb (1987) refer to. My reasons for saying this are as follows: Firstly the learners were not given any indication of what the blocks represented. The blocks were given to them to interpret and use in representing a selection of numbers and they were not given any rules to follow. Secondly, the intention was not to develop or teach the learners any written algorithms (they have learnt these already at primary school), but to rather improve their understanding of the workings of the algorithms they already know and eventually take them through a process of re-guided reinvention to mathematise the development of these algorithms (which they might have known but not understood). The blocks were therefore not the focus, but rather a tool through which they could demonstrate and discuss their thinking. Finally, I am of the opinion that the learners already had a mental representation of place value and were therefore expected to be able to use the concrete blocks in order to demonstrate to me their mental representations of working with base ten rather than learn anything from the blocks themselves. This is then in line with Gravemeijer's thinking when he states that "…concrete embodiments do not convey mathematical concepts" (p. 80).

An outline and description of the lesson conducted with 8X and 8Y with the Dienes blocks is now presented. This is taken directly from my log entries, written after each of the lessons. These are followed by a critical reflection on the learning trajectory in relation to the anticipated thought experiment that preceded the lessons. The worksheet referred to in the box can be found in Appendix E.

Box 1

Lesson using the Dienes blocks

Dienes blocks were taken along in order to allow the learners to work with concrete objects to further their understanding of place value. The learners were instructed to get themselves into groups and were handed a worksheet with two parts to it. The first part of the worksheet required them to the use the blocks to demonstrate various numbers such as 12, 21, 38, 123, etc. The second activity on the worksheet required them to use the blocks to demonstrate and explain the calculation (using the blocks) to each other of calculations such as 32 + 41, 48 + 36, 94 + 18, etc. They were given the blocks (a number of units, tens, hundreds and a few thousand blocks) with no further guidance as to what the blocks represented or which blocks to use. As the facilitator, I wandered between the two groups listening to their communication and sometimes asking them to explain or justify their decisions. Some groups worked faster than others so a whole class discussion was not held as intended at the end of the lesson, but discussions with the groups individually and the facilitator were rather embarked on before dismissing the class.

Tuesday 29 April 2003 - 8Y

The learners were instructed to get themselves into groups of two or three and start Activity one on the worksheet. They were given the blocks with no further guidance as to which blocks to use. Every group in this class immediately picked up a "ten block" and added two unit blocks to display a value of 12. The groups worked through the activity fairly quickly realising the value of each block and using them accordingly. When I asked one of the groups how they knew that the "hundred block" in fact represented 100, they showed me how they had counted the number of rows (10) and the number of columns (10) and multiplied them to get 100. One learner argued at first that the "thousand block" cube represented 600 (she explained to us how she counted the sides) but the rest of her group quickly explained/showed her the composition of the thousand block from 10 hundred blocks and how you had to count the ones "inside" too. There were not enough "thousand blocks" so the groups used their initiative and used the excess of hundreds blocks to help them display the values in part 4 of Activity one.

Two of the groups were initially calling the "tens blocks" "tenths" when questioned by me on what they were doing. We entered into a short discussion about the difference of tenths and tens but not extensive as it was something I still wanted to address with the whole class when talking of the decimal system in the next lesson.

The groups were not encouraged to continue onto Activity two of the worksheet on completion of Activity one. Instead they were shown two ten blocks and a unit block (by me) and asked what they saw. This was done individually with each group. All the groups immediately responded that they saw "twenty one". The groups were then asked to generate as many different ways as they could think of to write the number twenty one other than as "21". At first they were hesitant and not sure of what I meant. I asked them again what they saw and at least one in each group realised that the two ten blocks and one unit block could be written as "two tens plus one unit" and that got them started. A learner from one of the groups initially thought that 21 could also be written as $\frac{2}{20}$ and one. When I questioned her it appeared that she thought that the fraction represented the "two twenties and one" making twenty one. I'm not sure if it was due to my continued questioning around this or the reaction and discussion from other members in the group, but she said she thought actually she was wrong. I asked her to spend time during the week trying to find out if and why it was wrong or right. Each group was then asked to write the different ways they had generated up on the board and I have compiled a list of what came out below:

2 x 10 + 1

10 + 10 + 1

two tens and one

20 + 1

30 – 9

2 tens + 1 unit

<u>2 tens</u>

1 unit

$\frac{21}{1000}$ (this was done by the same learner - Violet - who wrote 21 as $\frac{2}{20}$ plus 1 but she also voiced her suspicion that this may be an incorrect representation)

twenty one

21 units

$\frac{21}{1}$

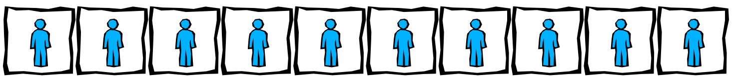
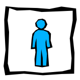
T	U
2	1

7 x 3

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

$1\overline{)21}$

10 + 10 + 10 – 9

 x 2 + 

Masome tharo (which translated from Sepedi means two tens and three – I think they meant to write masome tee which means two tens and one)

Een en twintig (which translated from Afrikaans means twenty one)

21 ÷ 1

Unfortunately there was not time to discuss all of these during the class as the bell went soon after they had written them up. I copied them down off the board in order to discuss during their next class in one weeks' time. Although the teacher was present during this first session, she was not asked to do any formal written observation.

Friday 2 May 2003 - 8X

I started this session in the same way as I had with 8Y on Tuesday. As there are only four learners, they got themselves into groups of two, and it appeared that they had done so according to where they were already sitting in the classroom. The interesting observation in this class was (is) the vast difference between the two groups. The one group (Klokkie and Zwanela) immediately realised what each block represented and starting using the blocks to display the numbers as the worksheet instructed. The other group (Mary and Emelie) however really struggled through the first activity. To represent 12, Emelie (who seemed to take the lead) immediately got together twelve units and said that was how to display it. When questioned on whether that was the easiest way they could see that it was 12, they started realizing that they could perhaps make use of the other blocks and counted the ten block to find that it was 10. They then used the ten block and two unit blocks to display 12. When I later returned to Emelie and Mary, they were still busy trying to display "123". They had one hundred block, two tens blocks and five unit blocks. When asked to explain how they had compiled their display, Emelie pointed to the hundred block and explained how they had counted it and found out there were 98 blocks on it. Then she counted the ten blocks (giving her 118) and then added the five unit blocks to get 123. I then asked them to demonstrate how they had counted the hundred block (which was being referred to as the "big slab" at this stage). Emelie again took the lead and started counting aloud and fast, not realising each time that when she got to the end of a row, she was at a multiple of ten. Mary sat quietly and let Emelie finish. This time, in her haste during the last row, Emelie came out at 102. I asked Mary if she agreed with Emelie and she said no, but that she thought there were actually 100 blocks in the "big slab". When asked to justify why she thought so she showed us how when Emelie had been counting, she had realised that there were ten blocks in each row. She then counted with her finger along the rows "twenty, forty, sixty, eighty, hundred" as she counted the block. Emelie smiled and agreed with her.

During this time, the other group had immediately realised how to use the blocks and had identified the hundred block as having 100 units in it by counting each side (10) and multiplying them to get 100. They were able to successfully represent all the numbers on the first part of the worksheet. While one group did experience this activity as a real problem, it did not present a real challenge to the other group, although they did demonstrate some worrying misconceptions during discussions with them in this lesson on their conceptual understanding of basic algebra. While they had no problem ascertaining that 1T (ten) + 1 T (ten) = 2 T (tens), when asked to give the value of $1x + 1x$ (with $x = 10$), they interestingly enough were initially convinced that it was "twenty tens." It has since become obvious that many of these learners battle to differentiate between the digit in a place holder (such as three tens, four hundreds etc) and the value of that number (i.e. twenty).

4.4.3 Summary and reflection of the first cycle

This cycle focused mainly on the concept of place value and aspired to implement the first heuristic design principle subscribed to by RME of *guided reinvention.* The main aim of the set of instructional activities implemented during this cycle was to take the learners through a process of progressive mathematisation relating to the building up of the denary system and the development of the addition and subtraction algorithms they were taught at primary school, in order to improve their understanding in this regard. This was also done to facilitate the development of more relational as opposed to instructional understanding (Skemp, 1989 - see section 2.3.2) regarding the relationships and patterns in the denary system so that learners would improve their performance in reading and writing whole numbers up to seven digits, and in doing calculations involving whole numbers. Through the use of the base ten Dienes blocks and the learners' prior knowledge and mental representations of the denary system, it was hoped that they could be helped to make an adequate connection between the syntax of the written algorithms and the principles that lie behind the column algorithms for addition and subtraction and the application thereof (Gravemeijer, 1994). In order to do this, the learners were required to first represent certain numbers using the Dienes blocks (using their own interpretation of what the blocks stood for) and to then visually demonstrate while verbally explaining to each other how to do some addition and subtraction calculations with the blocks. As mentioned, a game relating to place value was also played now and then and learners were given a worksheet in the final two lessons of the cycle that involved ordering a range of salaries earned by people in different careers, and answering questions relating to the salaries.

Through this I anticipated that they would go through a process of reinventing the algorithms they were already familiar with (but did not necessarily always understand). Only 5 of the 11 learners who wrote the pre-test had chosen the correct answer for Item 5 (see Appendix A). My intention was not to teach them to use new or alternative strategies or algorithms for doing these calculations but rather to lead them to a point of understanding the application and mathematisation of the algorithms they had been taught to apply at primary school. Any notational errors that emerged from discussions, such as the one represented in the lesson above with 8Y, were reviewed and discussed. Learners were busy at the time with exponential notation, including scientific notation in their mainstream mathematics classes so when we discussed the denary system, we also referred back to this notation that they were learning about in class.

Looking through my current lens of understanding of the principle of guided reinvention and progressive mathematisation, and at the actual learning that took place during the instructional activity presented in above in section 4.4.2, my conclusion must be that this was not a very good example to realise the intended principle. For example, the mathematical context of the problem illustrated in Box 1 was not necessarily experientially real for all the learners, although it did elicit some misconceptions demonstrated by learners, which could be discussed and hopefully rectified. The problem was also not a true contextual problem as such, as the context provided was situated within the mathematical structures as opposed to within a real context out of which the mathematics could be formalised and generalised.

In June 2003, I was fortunate enough to have a meeting with a visiting researcher from the Freudenthal Institute and in discussions with him, I realised how using the open number line and drawing on the history of our number system in designing the thought experiments may have been more optimum (Dolk, 2003). A future consideration therefore in designing an instructional sequence that revisits place value, may be rather to draw on knowledge of the history of our number system as a resource to create the intermediate steps by which the intended mathematics (in this case of our base ten number system) could be reinvented. Gravemeijer (1994) suggests this as also being one of the ways of realising this principle, but unfortunately I only discovered that once I'd already embarked on this first cycle.

This cycle did however have some positive outcomes. Based on comments from the observations done by teachers and myself it appeared that learners became more comfortable with me as the presenter and more willing to share their solutions for the purpose of discussions. They also enjoyed playing the game on place value and writing their solutions up on the board. The classroom culture that I had set out to establish during this cycle was achieved, although the particular series of instructional activities would need to be revised when the initial prototype is put together. Some extracts from observations provided by the teachers and an assistant researcher are provided below to substantiate these claims.

Teacher X made the following comment in her unstructured observation done on the ***7th May 2003*** (the underlined words were underlined by the teacher):

> *Pupils were given an opportunity to write on the chalkboard.*
>
> *- seemed very comfortable - (all were at the front)*
>
> *- wrote diff [different] ways of writing 21 & added no's [numbers] loudly - revealed slowed response of adding - but encouraged by teacher.*
>
> *Pupils encouraged to find their own ways of adding/representing numbers rather than given a method. Seemed to make more sense to them.*

Teacher X also made the comment below during another lesson with 8X on the ***21 May 2003:***

> *Pupils are really encouraged to 'speak' the numbers - everyone says it aloud - its wonderful to see them concentrating on what they're saying without worrying about what anyone else in the class thinks…Pupils found 'the game' exciting - everyone had an equal chance of participating (not always possible in larger groups). They felt equally important!*

In lessons done with 8Y on the 6th May and the 14th May 2003, the following extracts are taken from observations recorded by Teacher Y and an assistant researcher respectively.

6 May (Teacher Y):

> *Everybody waits for the first one to say something and then they talk like parrots afterwards…It is as if what you teach them is all new to them.*

14 May (assistant researcher)

> *Learners had to explain to the teacher how they got the answer. Learners are not corrected for a wrong answer but given an opportunity to correct themselves…I like role swopping activities [where learners had to 'teach' each other].*

Only the learners from 8Y were interviewed at the end of cycle one, using the interview schedule in Appendix C. A latent content analysis (see section 3.6.2) was used to analyse the transcriptions and the dominating two themes that emerged were: overall learners had enjoyed the lessons and felt that their understanding of place value had improved during the course of the cycle. There were mixed feelings about what they liked most and what they disliked about the lessons. Two of them enjoyed "being the teacher" most while another two disliked that role the most. Writing on the board, playing the game and working with the blocks were the other activities that learners cited as liking the most. The following quotes from the transcriptions of the interviews are offered regarding learners' opinions of the classes:

Liya:

> *They help but it's still a bit confusing…They've been fun, some of them and others have been serious and not so fun.*

Connie:

> *They are fun cause you have to like, um you have to like work it out and then you like decide if it's right or wrong so it's actually fun…The stuff we do in actual maths class, it's not fun because it's like you try and work it out but when you get the real answer, it's still wrong. But like in the classes with you, it's fun because then you like actually know what's going on.*

Mpho:

> *I think um, they've helped me a lot…well it's easier now than it was before because then before the numbers of the millions and stuff like that confused me but now they don't because then I just count what's before the zeros.*

Gloria:

> *They are cool cause they are easy to work out than just listening to a teacher talking about it, like using blocks.*

Violet:

> *I think that the classes are really helping me cause I'm improving. I didn't understand, I didn't even know what's after units and things and with the stuff I was a bit confused.*

Patience:

> *Think I'm doing quite better now…they helping [the classes]...beginning of the year they did help but then it came to class tests and stuff and like they didn't help but then now we know the place value, like in class we are doing chapter five, notations and stuff [scientific notation], that was easy for me to… cause I know the number, um ja place value.*

Leratho:

> *They nice... They much nicer than like normal maths cause there we don't get enough time to go over something and get it stuck in your head…They've been interesting and fun.*

In conclusion: in terms of the instructional design principles of RME, the instructional activities pertaining to place value will need substantial revision. This statement is based on the reasons provided above in the reflection as well as on the fact that learners did not show as much progress in this topic as had been hoped. For example on Item 5 of the pre- and post-test, as mentioned, five of the learners chose the correct option in the pre-test. In the post-test, this frequency only increased by one. More about learners' performance relating to the concept of place value can be found in the following chapter.

On the other hand, it appears as though the intended classroom culture that accompanies the RME instructional approach was established and that learners mostly enjoyed the lessons and felt they had benefited from them. This encouraged me to work at maintaining the set classroom culture while trying to improve the design of the instructional activities in relation to the RME principles in the following cycle.

4.5 Cycle two

Cycle two began at the start of the third academic term, which commenced on the 15th July 2003. The term ended on the 17th September, thereby allowing for approximately ten lessons with each class (once one disregards the lessons that were lost due to sport's days or timetable changes) during the second cycle of the intervention.

In terms of mathematical content, the concepts of fractions and decimals were mainly concentrated on during this cycle. As the learners were in Grade 8, they were already expected to know and understand the concept of a fraction and a decimal, be able to carry out calculations with them using the four main operations and to apply them in solving contextual problems. Most of the learners' performance on the items relating to fractions and decimals within the pre-test indicated that this was not the case. For example, on the item in the pre- and post-test (Item 14) that required learners to shade $\frac{3}{8}$ of a rectangle containing 24 blocks, only two of the eleven learners that wrote the pre-test correctly shaded 9 blocks. On another item requiring learners to identify the smallest fraction (Item 6) out of the options $\frac{1}{6}$; $\frac{2}{3}$; $\frac{1}{3}$; $\frac{1}{2}$, six of the learners correctly chose the first option, while two of the learners identified $\frac{1}{2}$ another two $\frac{1}{3}$ and one of the learners $\frac{2}{3}$ as the smallest fraction. Finally, on the item (Item 12) that required learners to choose the correct decimal for of the fraction $\frac{3}{5}$, from the following options 0,3; 0,8; 0,5; 0,6 only three of the learners correctly identified 0,6 as the correct option. Unfortunately a contextual problem involving fractions was not included in the pre-test so no information regarding learners' responses to such a problem at the beginning of the intervention can be offered. However, from the responses to items in the pre-test, my own experience in teaching mathematics at Grade 8 level and findings from other research, I made the deduction that although learners may have been able to carry out certain calculations involving fractions and decimals, many of them did not appear to have a good understanding of the concepts. From the results of the 1995 and 1999 TIMSS (Third International mathematics and Science Study), Howie (2002) makes the comment that South African Grade 8 learners who took part in the study had considerable difficulty dealing with fractions questions and that they "lacked the basic mathematics knowledge expected at Grade 8 level" (Howie, 2002, p. 110).

The rest of this section examines the objectives of the second cycle and the intended design that was implemented in order to attain these objectives. This includes a detailed look at how the RME principles of *guided reinvention* and *didactical phenomenology* steered and informed the design of instructional activities within this cycle. Examples of some instructional activities from this cycle are provided and discussed, with work samples of learners' strategies and solutions to instructional activities also being displayed. Finally this second cycle is then summarised and reflected on, once again drawing on data from observations of teachers and research assistants, my logs, interviews with learners (this time with 8X) and work samples from learners.

4.5.1 Instructional design for the second cycle

The main objective of this cycle was to improve learners' understanding of the concepts of fractions and decimals to enable them to solve real life problems that require this understanding. In line with the RME approach, the idea was not to do this in the order it is written in the previous statement (i.e. improve their understanding so that they could then solve real life problems) but to make use of real contextual problems to improve their understanding of the concept of fractions while simultaneously improving their ability to tackle contextual problems involving fractions and decimals. As the learners already had been taught these concepts for some years in the past, I did not think that it was realistic to expect them to go through a process of mathematisation that would progressively develop the formal rules for operations with fractions and decimals. In a sense they already knew what these were through having previously learnt the algorithms. However, they did not always use them correctly, owing to a lack of understanding I assumed. So the intention of the instructional activities designed around these concepts was to provide the learners with contextual problems that would encourage the use of their informal knowledge and strategies in solving the problems rather than the formal knowledge they should already have in place in this regard. Once they had gained some confidence in their informal strategies, the idea was to then guide them through a reinvention process to link these strategies with their formal knowledge of the algorithms. The focus was therefore initially concentrated on providing learners with problems that enabled them to embark on horizontal mathematisation (see section 2.5) rather than vertical mathematisation, which they were more accustomed to. I hoped that even if there was not enough time to accomplish all of this, that learners would gain enough confidence in their informal strategies to rather use them when confronted with a real life problem than to not be able to solve the problem at all, or make incorrect use of formal algorithms or routines they did not understand. An attempt to design contextual problems that could stimulate the use of informal strategies by the learners and allow for a variety of solutions was therefore concentrated on during this cycle.

In relation to the RME principles, more effort was made to realise the principle of *guided reinvention through progressive mathematisation* while some attention was also given in the instructional design to the second principle of *didactical phenomenology*.

The principle of guided reinvention through progressive mathematisation

In further examining this principle, I think it is useful to once again highlight the difference between the views of mathematics from the traditional information processing approach and the RME approach and how these differences translate into teaching and learning practices, as explained by Gravemeijer (1994). In the former approach, mathematics is viewed as a ready-made system with general applicability. Consequently, mathematics instruction is seen as a process of breaking up formal mathematical knowledge into learning procedures and then learning to use them accordingly. In the RME approach, however, mathematics is seen as an activity and learning mathematics subsequently means doing mathematics.

When embarking on solving a contextual problem using formal mathematical knowledge, the following steps are usually followed. First the problem needs to be translated from its contextual state into mathematical terms. Available mathematical means are then drawn on in order to solve the problem, which then needs to be translated back into the original context. This process can be illustrated by the following example:

The problem: 17 people are trapped on a mountain and need to be rescued by helicopter. The helicopter can take a maximum of four passengers at a time, in addition to the pilot. How many trips will the helicopter need to make?

Translation into mathematical terms: 17 people / 4 per trip

Mathematical means to solve problem: $17 \div 4 = 4$ remainder 1 or $4\frac{1}{4}$ or 4,25

Translation back into original context: Helicopter will need to make 5 trips

On the other hand, in the RME problem-centered approach, the problem, rather than the use of a specific mathematical tool, is the actual aim. Instead of trying to formalise the problem into mathematical terms, the learners are encouraged to describe the problem in a way that makes sense to them. This can involve using their own self-invented symbols or pictures and identifying the central relations in the problem situation. In this way the problem is also simplified for the learner. Because the symbols are meaningful for the problem-solver, further translation and interpretation of the problem is easier and using a standard procedure is not mandatory. In Box 2 below is an example of one of the first contextual problems presented to learners during the second cycle of the intervention and an example of a learner's response is provided and discussed in relation to the instructional approach of RME as outlined above.

Box 2

Lesson on the Cat's pills

My cat's recent diagnosis of diabetes initiated this problem, which served as an introductory contextual problem in revisiting the concept of fractions. A discussion on what diabetes is and how it occurs in humans and cats was first embarked on with learners as an introduction (learners had little knowledge of diabetes and were not easy to convince that cats also get diabetes). Learners were then presented with and asked to solve the following problem (either in groups or individually; they were given the choice):

Problem:

My cat needs to take two types of pills and an insulin injection twice a day to control its diabetes. The cat takes half a big pill in the morning and again in the evening and a quarter of a small pill also in the morning and again in the evening. Firstly, the vet has given me 17 big pills (8X were told 18) and 27 small pills (8X were told 10) to start off with, how many days will these pills last me for before I have to go back to get more? Secondly, how many of each pill should I buy each month so that they last me for a whole month?

About ten minutes before the end of the lesson, some learners were asked to demonstrate and explain their solutions to the class and a short whole class discussion on these explanations was held before the class was dismissed.

The solutions offered by Liya, from 8Y (in Figures 4.2 and 4.3) to the contextual problem provided in Box 2, are included below to exemplify this process of problem solving. There are two parts to the solution as Liya first tried Part A (Figure 4.2) and then realised from her answers that something was not right and then proceeded to do the solution in Figure 4.3. It is also interesting to note how her Part A solution more closely resembles the steps usually followed when using formal mathematical knowledge in an information processing approach. As these learners were probably more accustomed to using that approach, and obviously had some formal knowledge in place regarding fractions, they initially tried to often go through the steps of translation into mathematical terms, searching for an adequate mathematical procedure to solve the problem and then translating it back to the context. In doing so though, I noticed that some learners did not really have a grip on which mathematical procedure to use and even when they chose the correct one (sometimes by chance as they could not justify their decisions), they made mistakes in executing them (as can be seen in Part A of Liya's solution).

I therefore continually made an effort to encourage them to go through the RME approach of simplifying the contextual problem by first representing it in their own symbols and/ or words and then further solving and interpreting it from there. When some of them started to do this, they found that they could more often solve the problem, using their informal strategies rather than formal procedures they were unsure of. This does not mean that they never used formal procedures or any mathematical means but that they were expected to only use them at a point in the problem-solving process when they could justify the use thereof and demonstrate an understanding of the application in that regard.

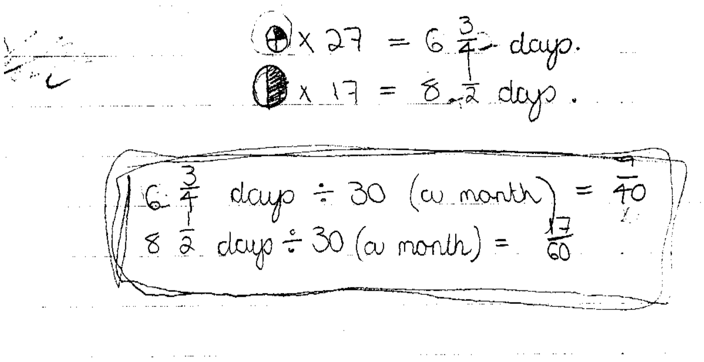


Figure 4.2 Part A of Liya's solution to the cat pills problem

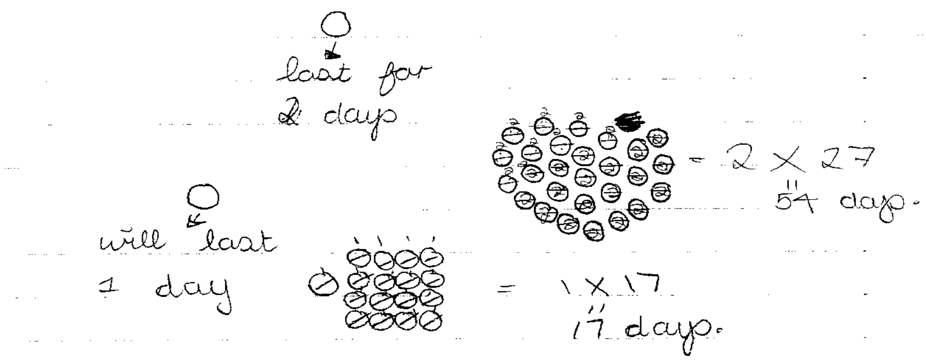


Figure 4.3 Part B - Liya's second attempt at solving the cat pills problem

Gravemeijer (1994) explains that by getting learners to solve a sequence of similar problems, another process is induced. The problem descriptions develop into an informal language, which is further simplified and formalised into a more formal mathematical language eventually. A similar process could be experienced in terms of the solving procedure, where solving similar kinds of problems becomes routine and actual algorithms take shape. Through this learning process, formal mathematical knowledge itself can be constructed, or in the case of this study, reconstructed. Treffers (1987) distinguishes between horizontal and vertical mathematisation in this regard (see also section 2.5). The description of the problem and use of more informal strategies are regarded as processes within *horizontal mathematisation* while the formalisation or mathematisation of mathematical matter is called *vertical mathematisation*. The reivention principle is therefore played out through the sequence of instructional activities (in this case contextual problems) that result in mathematics as a product (Gravemeijer, 1994). Figure 4.4 provides Gravemeijer's (1994) visual model of this process. During this cycle a concerted effort was made to encourage learners to make more use of horizontal mathematisation in order to represent the problem in a manner that made sense to them so that solving it would be more easily facilitated.

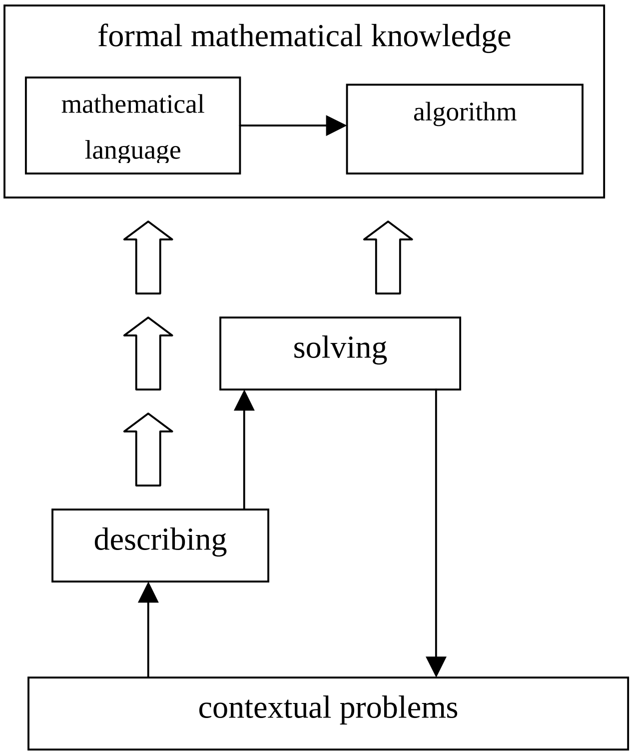


Figure 4.4 Reinvention (Gravemeijer, 1994)

The principle of didactical phenomenology

This principle was advocated by Freudenthal (1973; 1983) and implies that in learning mathematics, one has to start from phenomena meaningful to the learner that implore some sort of organising be done and that stimulate learning processes. In the phenomenology of mathematics, a mathematical structure is dealt with as a *cognitive product* in the way it describes the objects. In didactical phenomenology a mathematical structure is dealt with as a learning and teaching matter, in other words as a *cognitive process* (Freudenthal, 1983). For this design principle, knowledge of mathematics and its applications as well as knowledge of instruction are necessary.

The implications of this principle for the designer/instructional developer are to provide learners with contextual problems that are real and meaningful to them. This does not imply though that these need to be really "real" situations in their surroundings but that "the context in which a problem is situated should be experientially real to the learners in that they can immediately act intelligently within the context" (Gravemeijer, 1999). The goal of the phenomenological investigation is therefore to create situations in which learners can collectively renegotiate increasingly sophisticated solutions to experientially real problems by individual activity and whole-class discussions (Gravemeijer, Cobb, Bowers & Whitenack, 2000 as cited in Kwon, 2003).

In his publication entitled Didactical Phenomenology of Mathematical Structures, Freudenthal (1983) examines this principle of didactical phenomenology in relation to a number of mathematical topics. One of the topics he looks at is fractions or rational numbers. In doing so, he mentions the following views of fractions:

- Fractions in everyday language (such as a half an hour, R3,99 per litre of petrol etc.)
- The fraction as fracturer (such as dividing up a whole into equal parts)
- Fractions as comparers (such as a bench being half the height of a table)
- Decimal fractions (in relation to measurement, percentages, place value)

He also distinguishes between the fraction appearing in an *operator* or in a *relation*. For example if one is halving a cake, the fraction acts as an operator. However, if one makes the comment that your piece of cake is half as big as your friend's cake, the fraction is appearing in a relation.

Using the views of fractions presented by Freudenthal, (1983) I tried to find problem situations that would give rise to solution procedures specific to these views of fractions. The lesson presented in Box 2, for example, dealt with the use of fractions in everyday language, but also with the fraction as a fracturer, in that the whole pills needed to be divided up into halves and quarters. Subsequently some similar problems relating to these first two views were also given to learners that included sharing apples, baking tarts, etc. and initially this was to try and get them to make use of horizontal mathematisation (describing the problems and using informal strategies) in order to provide a better basis for subsequent vertical mathematisation. Some instructional activities aimed at addressing the last two views of fractions as suggested by Freudenthal were also included in this cycle, although not to the extent that the first two were dealt with, mainly because time was limited and the designing was challenging.

In Gravemeijer (2001, p. 8), he offers the following comment from Streefland (1990) regarding the principle of *didactical phenomenology*:

> *The didactical phenomenological analysis may orient the researcher/developer towards applied problems that can be presented to students who do not know the mathematics in question yet. The spontaneous solutions of the students may show strategies, notations, and insights that can be used in the sequel of the learning process.*

As these learners already in a sense "knew" the mathematics in question, some were able to solve the problems by applying formal solution procedures that they knew. As previously mentioned though, some learners tried to continually search for or recall formal solution procedures that they knew should assist them in solving the problems, but which they did not understand. They then either used the incorrect procedure or used the correct procedure incorrectly. This made it more difficult initially to use learners' work in designing the sequel of the learning process as suggested by Streefland (1990 as cited in Gravemeijer, 2001). As the intervention progressed though, more learners started becoming comfortable with trying their own informal strategies and a study of these solutions would make it easier to implement this principle of didactical phenomenology in a further study.

4.5.2 Examples of instructional activities from the second cycle

One example of an instructional activity from this cycle has already been presented in Box 2 in the preceding section. As mentioned, that example was related to fractions in everyday language and fractions as part of a whole. An additional two examples are provided in this section; the examples in Boxes 3 and 4 (only 8Y were given this problem and not 8X) are similar problems to the one in Box 2. Some of the learners' solutions provided in response to these problems are provided and discussed in relation to the two principles implemented in this cycle.

Box 3.

You decide to start making banana bread to sell in order to earn some extra money. To start off with, you decide to make 5 loaves of banana bread. According to the recipe, each loaf requires $4\frac{1}{2}$ *bananas. How many bananas will you need to make the 5 loaves of banana bread? Show your working out in the space provided below.*

Both classes were given this problem to complete (as part of a diagnostic assessment so they worked individually) and a solution from one learner in each class is provided and discussed. The first solution (Figure 4.5) is one done by Zwanela from 8X and the second one (Figure 4.6) is from Violet's work in 8Y.

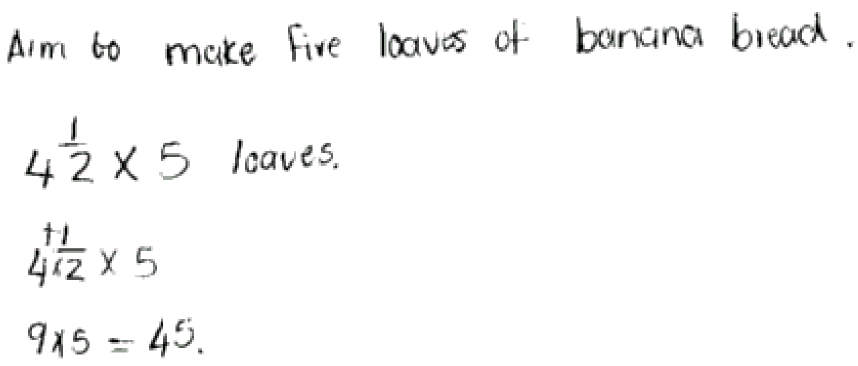


Figure 4.5 Zwanela's solution to the banana bread problem

As can be seen in Zwanela's solution (Figure 4.5), she correctly selected multiplication as her strategy. However, when she carried out the actual multiplication, she tried to change the mixed number into an improper fraction and in doing so "lost" the denominator and got 9 instead of $\frac{9}{2}$,rendering her final answer incorrect. In contrast to Zwanela's more formal solution, the use of horizontal mathematisation is more evident in the solution from Violet (Figure 4.6). Zwanela more often resorted immediately to vertical mathematisation in that she searched for the "correct" formal procedure to apply.

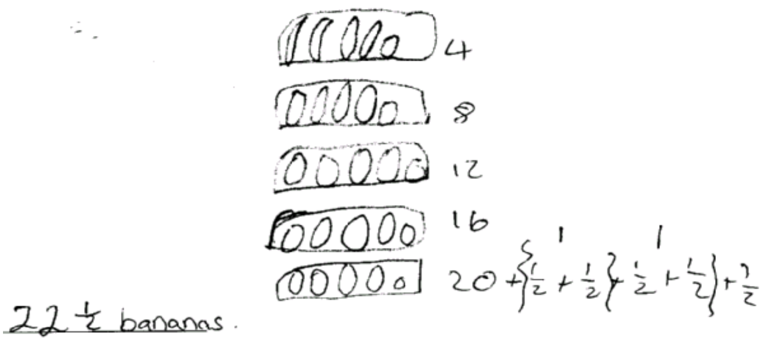


Figure 4.6 Violet's solution to the banana bread problem

Violet and Zwanela both often demonstrated the same incorrect conceptions regarding fractions in the concept tests (see Figure 4.7 for an example). However, Violet's use of horizontal mathematisation appears to have assisted her more in overcoming this lack of understanding in order to still arrive at the correct solution when solving contextual problems involving fractions. Zwanela on the other hand continually demonstrated a dependency on algorithms and an aversion to trying to understand and represent the problems in her own way throughout the intervention. Her biggest obstacle in fact was the number of alternative conceptions in mathematics that she brought with her. It is as though she had collected a "mental toolbox" of mathematical instruments over the years, without fully understanding how they really work or what they do. As soon as she attempted a mathematical problem, she would immediately start searching through her toolbox trying out the tools she thought might be most suitable. This method was often done on a trial and error basis, and failing to find the right tool and not knowing which one to choose resulted in much frustration for her. She often found the correct tool though and then proceeded to operate it incorrectly, hence the alternative conceptions coming to the fore in any case, allowing for these to then be addressed.

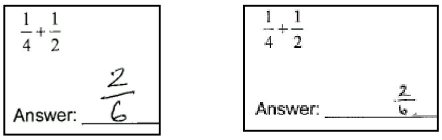


Figure 4.7 Violet and Zwanela's respective responses to one of the items from a diagnostic assessment done at the end of cycle 2

Box 4.

A recipe that you find for making apple tarts states that you need $\frac{3}{4}$ *of an apple to make one apple tart. You want to make 10 apple tarts. How many apples do you need?*

The solution below (Figure 4.8) belongs to Leratho from 8Y. As mentioned in Chapter 3, she was in the remedial class on the request of her mother rather than due to poor performance, as was the case for the other 11 learners in the intervention. Academically she also scored the highest throughout the year on all the assessments done in their regular mathematics classes in relation to her other classmates taking part in the intervention. What is interesting about Leratho's solution here, is how she took herself through a process of trying quite an abstract level of vertical mathematisation, then resorted to horizontal mathematisation before moving onto a more basic attempt at vertical mathematisation. She started off by applying an "algorithm" to try and get the answer. She realised she should make use of multiplication but multiplied by one tenth instead of ten. She then proceeded to make use of equivalent fractions to obtain a common denominator in order to add the fractions (instead of multiplying them). She was not satisfied with her answer of $\frac{17}{20}$ and resorted to horizontal mathematisation in order to check this. She did this by drawing the number of apple tarts and counting the number of quarters required (a total of 30). She then correctly translated this into 7 whole apples and another two quarters of an apple being necessary. Immediately she then went on to use a basic form of vertical mathematisation to verify this solution and by doing so corrected her own first mistake of multiplying by one tenth instead of ten. This in itself is an explicit example of how a learner has used the process of horizontal mathematisation to check a solution and to then correct the answer initially reached. Resorting back to vertical mathematisation then allowed the learner to identify and correct the source of the error.

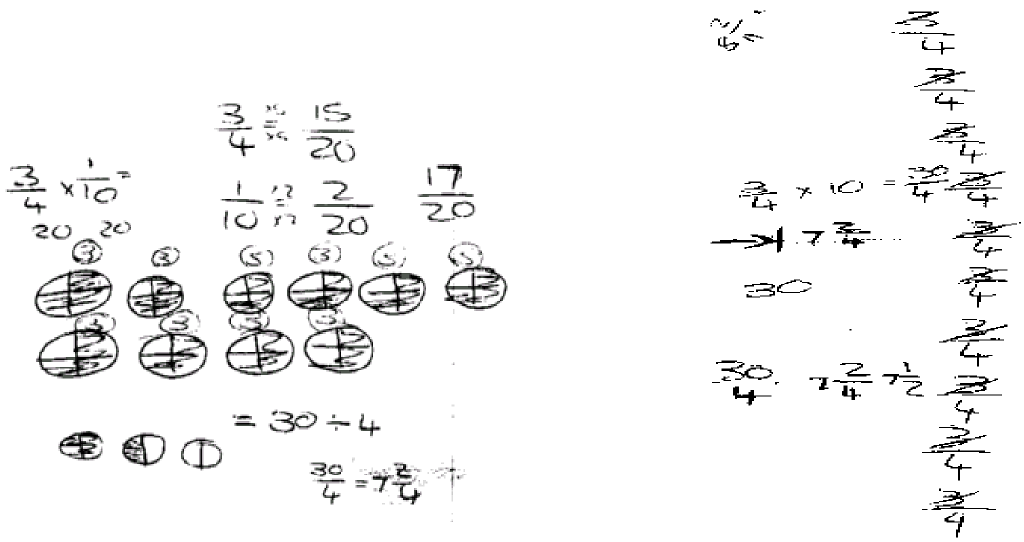


Figure 4.8 Leratho's solution to the "apple tart" problem in Box 4

In the example below (Figure 4.9), Liya correctly made use of horizontal mathematisation by drawing the ten apple tarts. She then correctly established that there would be one whole additional (extra) apple resulting from every four tarts (as only $\frac{3}{4}$ of an apple is used in a tart leaving $\frac{1}{4}$ over from each of the four tarts) and a half an apple over from two tarts. For ten tarts she therefore had two and a half additional apples if she had ten apples. Her only mistake was then to incorrectly add this value onto 10 instead of taking it away. The benefit of this solution for both the learner and the teacher (or in this case for me as the researcher) was that it was immediately obvious to ascertain where the mistake had been made and to then deal with this conception. Solutions of two other learners in the class yielded a similar error with slightly different visual representations.

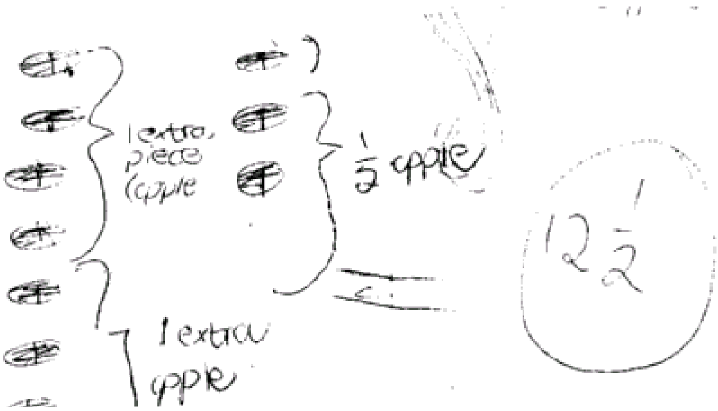


Figure 4.9 Liya's solution to the "apple tart" problem in Box 4

4.5.3 Summary and reflection of the second cycle

Examples of lessons and learners' work provided above suggest that the principle of *guided reinvention through progressive mathematisation* was better realised in the design of these instructional activities within cycle two compared to the previous cycle. The problems were more contextual and based in real life, while initiating a range of solutions and allowing for informal strategies. Many of the learners also started engaging in horizontal mathematisation while some moved onto the use of vertical mathematisation as well, or switched between the two (as demonstrated in Leratho's solution to the apple tart problem). Comments from the observations (of the teachers and assistant researchers) such as the ones presented below appear to support this reflection (underlining provided by me to indicate emphasis):

25 July 2003 (Instructional activity on "Cat Pills" problem in Box 2) Teacher X:

- *home related - pupils attention captured, therefore close to home/ <u>real life relevance makes sense to listen</u>*
- *pupils to solve a problem - allowed to work together*

5 August 2003 (Instructional activity on "Insulin" problem in Box 5 further on)

Teacher X:

- *The calibrated syringe was passed around and the pupils had to draw it and fill in missing parts [values] and indicate where 0,08 is.*
- *Pupils are now forced to link number concept e.g. what/how much is 0,08 with the real thing (0,08 ml) - makes more sense to them.*
- *There's better association with decimals and the real thing when they visualise it for themselves.*

5 August 2003(Assistant researcher):

- *It seemed to me that this was the first time they actually sat down and thought about increase and decrease of volume. Correlating numbers to this, e.g. realising that 0,1 ml is larger than 0,01 ml.*
- *I liked that they were involved - they had to construct their own knowledge.*

12 August 2003 (Instructional activity that preceded "Insulin" problem in Box 5) Assistant researcher observing 8Y:

- *using syringe (in place of a number line) as a mechanism for demonstrating locating 0,09.*
- *Hayley [researcher] uses questioning to establish fundamental concepts. Builds on students response to frame next question.*
- *Context engages students in the use of a tool (syringe) to consider the magnitude of decimal quantity*

Through providing instructional activities that required learners to solve a sequence of similar problems (such as those in Boxes 2, 3 and 4), it is my opinion that the principle of didactical phenomenology was implemented within this cycle. However, these problems did not always follow directly after each other and I initially tried to accomplish too much with one problem, instead of rather developing a series of problems that gave rise to more situation-specific solution procedures and created more of a basis for vertical mathematisation (Gravemeijer, 2001). For example the following problem presented in Box 5 was designed in a thought experiment to address the domain of calculations in decimal fractions, moving from repeated subtraction towards division.

Box 5

Over and above the pills that the cat needs to take for his diabetes, he also needs to have two insulin injections each day. With each injection he gets 0,08 ml of insulin. If there is 2,5 ml of insulin in a bottle, how many days will the bottle last me?

What emerged from the instructional experiment were some informal strategies such as repeated subtraction or building up by addition of the decimals (see Figure 4.10) or an immediate move to divide (see Figure 4.11), but in both cases learners needed to use calculators to calculate these correctly. In fact when given an actual syringe to work with, most of the learners could not even identify 0,08 ml on the syringe, so more lessons could have rather been spent on this type of relational thinking regarding decimals and measurement. I don't think learners gained much "mathematics of decimals" that could be improved on or refined in a series of similar problems so that learners would gain a better understanding of decimals. In a subsequent problem, some learners did use more efficient strategies but this was a development in their move from repeated subtraction towards division as opposed to any progress in relation to understanding decimals. Perhaps in compiling the initial prototype, a series of similar problems with more careful attention given to the amounts used in the progressive problems should be also considered. Through designing a series of similar problems with the intent of making a pattern emerge in the eventual division of decimals by 10, 100 and 1000 may be more useful for learners.

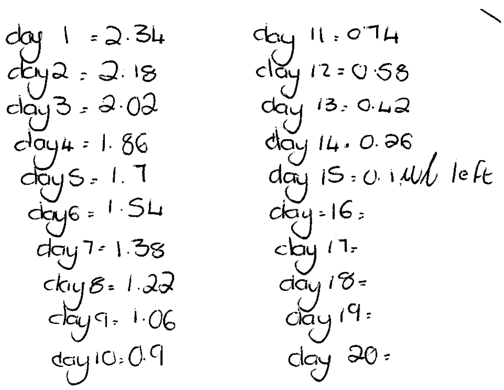


Figure 4.10 Mpho's solution to insulin problem in Box 5

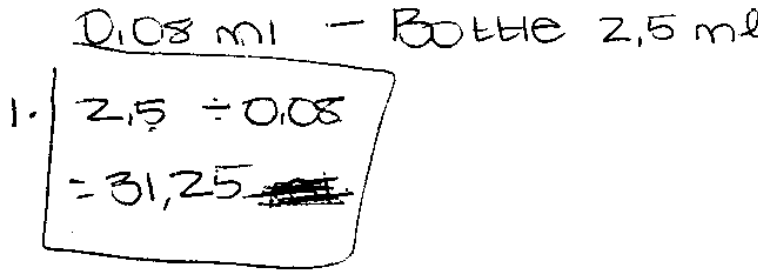


Figure 4.11 Klokkie's solution to insulin problem in Box 5

With 8X the time that learners took to solve the problems always seemed to take up more time and learners often required more individual attention than the learners in 8Y. Less instructional activities were subsequently done with this class and when it came to decimals, they seemed to struggle so, even to execute simple tasks such as indicating 0,08 ml on a syringe that it seemed pointless to progress with more difficult problems requiring calculations with decimals. The work relating to decimals done with 8X and 8Y was therefore different in that 8Y continued with similar types of problems to the one presented in Box 5, while after trying the lesson in Box 5 with 8X, I decided to work with something more familiar to them, in the form of money. This was also done in an attempt to get them to make more use of informal strategies. For two of the lessons (as they did not complete the task in the first lesson), the class were given a bag filled with various South African coins including 1c, 2c, 5c, 10c, 20c, 50c R1,00, R2,00 and R5,00 coins. Their instruction was that we needed to find out what the total value of all the coins in the bag was and that as a class they had to do this as quickly and efficiently as possible. The coins were all mixed up in the bag and there must have been around 400 coins. They were also not allowed to use calculators.

It was hoped that they would allocate the responsibility of counting different denominations of money to different individuals in the class to calculate and that a range of informal strategies in sorting, representing and counting the money would emerge. This did in fact happen and these informal strategies (and the more formal offered one by Klokkie) were discussed. Although this type of problem (as was the case with the "insulin" problem in Box 5) did allow for a range of informal strategies and gave them the opportunity to carry out calculations with decimals, it did not assist learners in furthering their relational understanding of the place value of decimal fractions and their relationship to fractions. It is therefore suggested that more instructional activities relating to this aspect be included in the prototype.

At the end of this cycle (actually during the first week of the third cycle), interviews were conducted with four of the learners from 8X. As previously mentioned, an interview was not conducted with Zwanela, initially due to her own request to do it later and then from my side not wanting to force the issue. Nomsa did complete an interview but unfortunately the data was accidentally erased during the transcriptions, by which time it was too late to try and conduct another interview, as Nomsa left the school at the end of the year. An analysis of the transcriptions from the remaining three interviews suggested that learners experienced the classes as helpful. This was deduced from comments cited by the learners, such as:

Klokkie:

> *They are very nice and they helping us on the tests in classes. Some of the work we also do in maths class and extra maths [remedial classes]. According to me you helping us a lot because some of the things I didn't understand, I now do.*

Emelie:

> *Uh uh some of them [the classes] were hard. Some were fine…when we started I didn't know anything but now it's going higher.*

Mary:

> *I think its [the classes] nice…cause you teach us a lot and we know, you help us with our maths and now we know what to do with maths and we don't have a problem at all.*

In conclusion: the mathematical content focused on during this cycle was related to fractions and decimals. Given the scope of these particular concepts and my increased knowledge and confidence in applying RME theory in instructional design, the instructional experiences within this cycle yielded learning that was more congruent to the thought experiments than had been experienced in cycle one. Data collected from the learners' work and from observations from observers as well as from my logs, indicate that the principle of *guided reinvention through progressive mathematisation* was better realised and that the principle of *didactical phenomenology* was also implemented. The problems relating specifically to decimals will need substantial revision however, in compiling the initial prototype, and a point should be made of creating a set of more focused instructional activities that are similar and yield more situation-specific procedures "that can function as (paradigmatic) solution procedures that can be taken as the basis for vertical mathematisation" (Gravemeijer, 2001, p. 8). Learners did appear to find the lessons helpful though and felt that they benefited from them.

As mentioned earlier on in this chapter, the initial plan was to continue with basic algebra during the third cycle. However, based on the learners' performance in their diagnostic assessment and their responses in the classroom, as well as the fact that not enough attention had been paid to the *views of fractions as comparers* and *decimal fractions*, I took the decision to continue working with fractions and decimals. I also wanted to continue working at improving the implementation of the first two instructional design principles as well as pay attention to attempting to employ the third principle of *emerging or self-developed models*. Although the unstructured observations from the first two cycles had been useful to me in the process of designing and revising instructional activities, I decided that a more formal and controlled evaluation was necessary for the third cycle in order to ensure the RME instructional approach was being adhered to. A highly structured observation schedule (see section 3.5.5) was therefore prepared for the teachers and research assistants to use during the third cycle.

4.6 Cycle three

This was a short cycle, due to the two and a half weeks allocated for final exams at the end of year. The fourth academic term (marking the third cycle of the intervention) commenced at the beginning of October 2003 and ended in the middle of November, making it only 6 weeks, which ended up providing 8 lessons per class. As previously mentioned, the decision to continue working on the concepts of fractions and decimal instead of moving on to basic algebra, was taken. During this cycle, learners were busy with the section on ratio and rate in their mainstream classes so an attempt was made to link up with this while also trying to focus more on implementing the third heuristic design principle of RME; that of *self-developed models* (in addition to the first two principles). The observations of lessons done by the teachers and a research assistant were highly structured throughout this cycle, as both the teacher and a research assistant each completed an observation schedule for almost every lesson (see Appendix C for a copy of the observation schedule). These schedules were designed to focus on assessing the implementation of an RME approach. The rest of this section provides an overview of the intended instructional design of the third cycle, some examples of instructional activities from the cycle and an evaluation of the actual implementation that transpired in relation to the intended design.

4.6.1 Instructional design for the third cycle

Relating to content, three of the four views of fractions namely, *fractions in everyday language, fractions as fracturer* and *decimal fractions,* were focused on during cycle two. While these three views were also included in instructional activities in the third cycle, *fractions as comparers* and *the fraction as fracturer* were the two views that received the most emphasis during the design. The intention was to develop instructional activities relating to ratio and rate in particular, as these concepts were also being covered in the Grade 8 mainstream mathematics classes at the time.

The first two design principles of *guided reinvention* and *didactical phenomenology* were discussed in the previous cycle. The intention was to continue to employ them in this cycle with regard to fractions and decimals, while monitoring their actual implementation through the observation schedules. More of an attempt was made during this cycle to realise the principle of *self-developed models* and more about this is now presented.

The principle of self- developed or emergent models

According to Gravemeijer (1994, 1999, 2001), the use of models in RME versus a more traditional information processing approach differs in the role and character of the models. In the latter mentioned approach, models are primarily used as a departure point for developing formal mathematics. Formal expert knowledge is taken as the source for didactical models and embedded in concrete models, through manipulatives. The assumption is also made that, once learnt, formal mathematical knowledge has general applicability and the application thereof is usually embarked on and practiced once the necessary formal mathematics has been learnt. In the previous syllabus used by mathematics teachers in South Africa for example, the formal mathematics of solving algebraic equations would be taught to learners after which they would be given some "word problems" to solve that required the application of these formal strategies they had learnt. From my own experience as a teacher, I observed how often the majority of learners were able to successfully execute the formal procedures required to solve the equations but then struggled to solve the "word problems". Their main struggle appeared to be translating the given contextual problem into a mathematical equation, which could then be solved using their formal procedures. In other words learners battled to connect their informal, situated knowledge with the formal mathematics they were learning in this information processing approach that can also be viewed as a top-down approach (Gravemeijer, 1994).

In contrast to this, RME encourages learners to construct models for themselves and to then use these models as a basis for developing their formal mathematical knowledge. The formal mathematics is therefore regarded as something that grows out of the learner's activity and not something that is embedded in expert mathematical knowledge. The aim is to provide learners with the opportunities to experience the use of formal mathematics in a way that is synonymous with the manner in which they are accustomed to using their informal strategies. This can therefore be regarded as a more bottom-up approach in that learners develop formal mathematical knowledge by mathematising their own informal mathematical activities (Gravemeijer, 1994). In RME, models can be viewed as progressing in their character and function through four levels; situation specific models, models for mathematical reasoning (referential level), models of mathematical reasoning (general level) and finally as formal knowledge. These are not fixed but relative to the starting point of the learners' work. Figure 4.12 below depicts the different models, as represented by Gravemeijer, 1994.

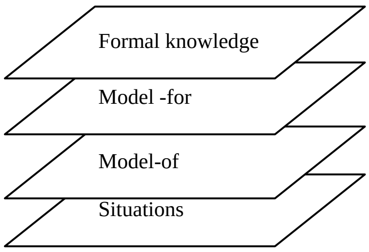


Figure 4.12 Emerging and self-developed models (Gravemeijer, 1994)

In general terms the different models can also be viewed in terms of levels (Gravemeijer, 1994). The first level is usually associated with real-life activities that involve being in the actual context to solve the problem. For example, when the problem on the cat pills presented in Box 2 in section 4.5.1 was first given to learners, the actual pills were taken to class so that the learners could physically work with them if they needed to or wanted to. This would be representative of the first level, also known as the *situational level*. Most learners however, opted to work on paper using drawings or symbols for the pills (see Part B of Liya's solution in Figure 4.3 as an example). They therefore represented the problem for themselves using paper and pencil (as opposed to the actual materials) and through doing this proceeded to solve the problem. This is characteristic of the second level, also known as the *referential* level, "where models and strategies refer to the situation which is sketched in the problem" (Gravemeijer, 1994, p. 101). The following problem in Box 6 of a contextual problem given to learners during the third cycle is presented as a departure point to further illustrate and discuss the *referential*, *general and formal* levels.

Box 6.

You write a test out of 50 and you get $\frac{4}{5}$ of the total marks. What is your mark for the test?

The third level, also known as the *general level*, is less dependent on the situation and has more of a mathematical focus in that the learner deals more with numbers and relies less on diagrammatically representing the context. An example of Connie's solution to the problem in Box 6 is provided in Figure 4.13 as an example of a model situated within this general level. Also included is an example of Violet's solution (Figure 4.14), which I would regard as being more representative of the referential level. While Connie's model for calculating her mark for the test is similar to that of Violets, Connie's strategy was slightly more formal in that she also used numbers as opposed to relying solely on the diagrammatic representation.

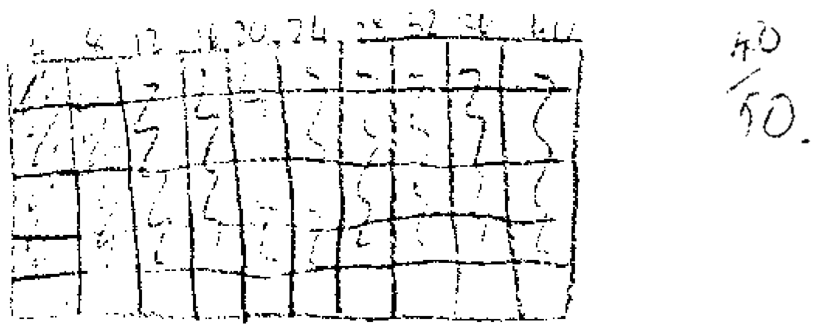


Figure 4.13 Connie's solution to the test problem in Box 6

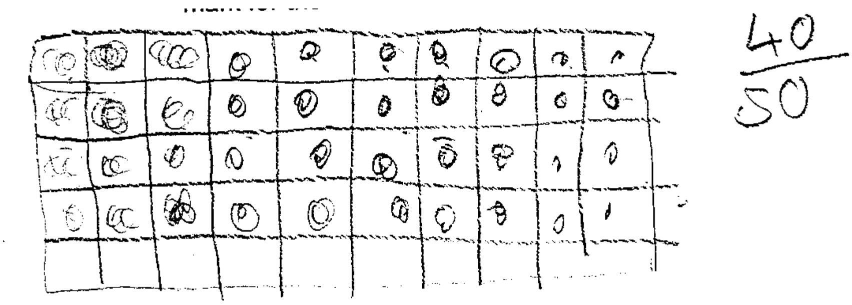


Figure 4.14 Violet's solution to the test problem in Box 6

The fourth and most formal level is characterised by the use of conventional procedures or standard algorithms and notation. The solutions of Patience (Figure 4. 15) and Leratho (Figure 4.16) to the problem in Box 6 are provided as illustrations of models characteristically used in this level.

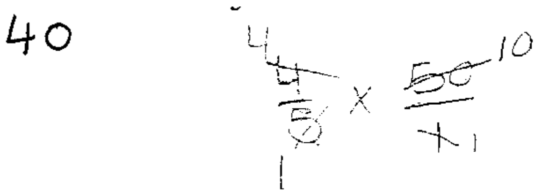


Figure 4.15 Patience's solution to the test problem in Box 6

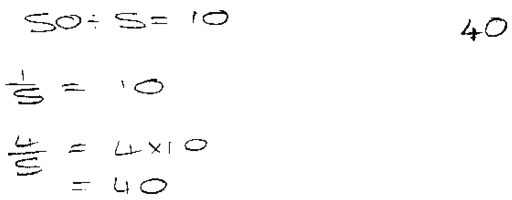


Figure 4.16 Leratho's solution to the test problem in Box 6

The difficulty with implementing the instructional design principle of *emergent models* in this study was the fact that the learners had already been exposed or introduced to the formal mathematical knowledge of the concepts the intervention was addressing. Judging by the performance of the majority of them in their mainstream classes as well as in the pre-test, a gap still existed between their situational knowledge (or common sense as Freudenthal, 1991 puts it) and the formal mathematical knowledge relating to the key number concepts being dealt with in this particular cycle, namely fractions and decimals. The challenge was therefore to try and get them to first return to making use of their own situational and referential knowledge so that this informal knowledge of theirs could then be generalised and finally formalised through progressive mathematisation. It was hoped that this would help to bridge the gap between their informal and formal knowledge, thereby increasing their understanding of and ability to use the formal procedures they had already been introduced to previously more effectively, while also improving their ability to solve contextual problems.

4.6.2 Examples of instructional activities from the third cycle

Another of the problems similar to those presented in Boxes 2, 3 and 4 was provided in Box 6 in the preceding section. The intention was that learners would start to formalise their models through the mathematisation process required, from solving the four problems relating mainly to fractions as a fracturer. The problem in Box 6 was also designed with the intention of starting to move the learners towards working with fractions as a comparer or a ratio, before moving onto the concept of rate. The problem in Box 7 was given to learners during the same instructional activity as the one in Box 6 and is provided as a problem that views fractions in terms of everyday language, as a fracturer and as a comparer.

Box 7.

Your grandfather leaves an inheritance of R6 000 to share between his three grandchildren. The oldest grandchild gets one half of the total amount. The second oldest grandchild gets one third of the total amount and the youngest grandchild gets one sixth of the total amount. How much money does each grandchild get? Is there any money left over?

By this stage the learners who were able to solve this problem all used solutions that can be regarded as general or formal models. Nomsa did not solve this problem as she was absent for this instructional activity. Zwanela as usual began searching for a formal procedure and could not find one. She was still unable to use an informal procedure and needed a lot of guidance and individual attention in order to solve the problem. Mary was also able to solve it with a great deal of individual attention and guided questioning. Examples of solutions from three of the learners are provided in Figures 4.17 - 4.19. The instructional activities then moved on to focus more on rate and working more with decimals.

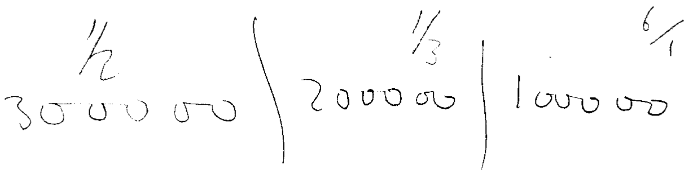


Figure 4.17 Emelie's solution to the inheritance problem in Box 7

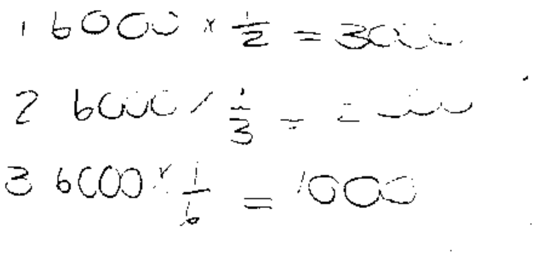


Figure 4.18 Gloria's solution to the inheritance problem in Box 7

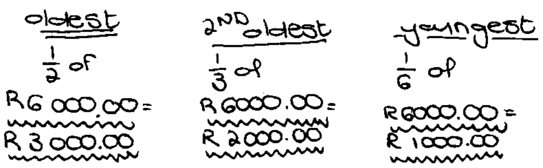


Figure 4.19 Liya's solution to the inheritance problem in Box 7

As mentioned, during their fourth academic term, the Grade 8 learners were dealing with the concept of ratio and rate in their mainstream mathematics classes. In order to keep their remedial lessons in some way relevant to these mainstream classes, I decided to develop a problem that involved working with a foreign exchange rate but that would also allow for a range of informal strategies to be used in solving it as well as for some learners to go from informal to more formal models regarding their calculations with decimals. I therefore designed the worksheet included in Box 8 below and introduced the lesson to the first class (8X) by handing out some examples of coins of pounds and pence to them from the UK as well some Euros and Euro cents. What I thought would then be a five minute introduction discussion on which countries these coins come from and a little more information on each of the countries, turned out to be a discussion for almost the entire period with little time left to embark on the actual mathematics. This is further discussed after the box.

Box 8.

Travelling overseas - A visit to the United Kingdom and The Netherlands

In the United Kingdom we will be spending a weekend in London. The currency used is the Pound Sterling (£) and the current exchange rate is approximately R12,10 to £1,00. This means that you need to exchange R12,10 to get One British Pound.

You arrive in London on Friday afternoon and leave on Sunday afternoon and during that time you spend the following:

2 nights hotel accommodation: (includes breakfast)	*£40,00 per night*
1 Trip on the London Eye:	*£10,00*
1 visit to the Tower of London:	*£10,00*
Tube ticket for the weekend:	*£13,00*
1 pizza:	*£7,00*
1 cup of coffee:	*£2,00*
2 McDonald's Happy Meals:	*£2,00 per meal*
A meal at an Indian restaurant:	*£16,00*
1 Coke:	*£1,50*
1 Aero chocolate:	*£0,50*

In The Netherlands we will be spending a day in Amsterdam. The currency used is the Euro (∈) and the current exchange rate is approximately R8,40 to ∈1,00. This means that you need to exchange R8,40 to get One Euro.

You arrive in Amsterdam on Sunday evening and leave on Tuesday afternoon and during that time you spend the following:

2 nights hotel accommodation: (Breakfast included)	*∈70,00 per night*
A trip to Anne Frank's house:	*∈5,00*
A boat trip on the canal:	*∈8,00*
A meal at a "Pannekoek Huis":	*∈7,50*
A cup of coffee:	*∈1,75*
Hire of a bicycle for the two days:	*∈32,00*
A meal at an Indonesian Restaurant:	*∈13,00*
Bread and cheese for lunch:	*∈3,20*
1 Coke:	*∈1,80*

How much (in Rands) did you spend during your time in:

a) London
b) Amsterdam

Which country turned out to be the most expensive for you as a South African?

As previously mentioned, very little mathematics actually got done in the lesson with the first class where this worksheet was given to the learners. In this particular class (8X), none of the learners had ever seen any of these coins so they were firstly a source of fascination and curiosity for a few minutes. One learner was not aware that other countries even used currencies that differed from ours. When I began to question the learners as to where England

and The Netherlands are, their answers (or lack thereof in some instances) brought on the realisation that this problem I had thought would be so experientially real to them did in fact not present phenomena to them that they could immediately start interacting intelligently with (hence not realising the *didactical phenomenology* principle). There was silence when they were asked where the Netherlands or Holland (as we often refer to it as) is and no-one in the class could correctly name any country in Europe. Some thought Europe was in America and one suggested Brazil might be in Europe. As to the location of England, most then said "in Europe" and so the lesson went on. Eventually I tried to explain to them that England was above Africa almost directly in line with Cape Town (without the aid of a map). The issue of what languages are spoken in these countries was then also embarked on and general facts about the countries and the people who live there. The concept of the exchange rate also consumed a large portion of the lesson and the implication for us as South Africans wanting to visit one of these destinations. Only about five minutes remained for learners to start engaging with the actual problem and they then also had difficulty getting used to the symbols for Pounds and Euros.

When this lesson was done with the second class (8Y), I went armed with a map of the world, fact sheets on the two countries, pictures of some landmarks and concepts in the countries (such as the London Eye) and a real Dutch person from the Netherlands (who was actually observing the lesson) in an attempt to make the context more experientially real to the learners so that they could engage with the problem and the mathematics. This appeared to provide learners with more of a feeling of really visiting the places and they were able to embark on the mathematics more quickly than the other class. What transpired in this class regarding the actual mathematisation was also interesting though. Learners were initially told not to use calculators (in the hope that they would practice their calculations with decimals). In some cases this affected the strategy that the learners used as some were not confident multiplying with decimals. On their worksheets they would write down the multiplication calculation and then realise they do not know (or could not remember) how to use the long multiplication algorithm for multiplying decimals. Some then resorted to using repeated addition as can be seen in the examples below and those still using repeated addition as their model stayed with it, probably due to the daunting task of trying anything else with decimals if one is not confident calculating with them. I therefore allowed the learners to use calculators for the second part of the task (the visit to Amsterdam) and to check their answers. With this, some learners did move to more vertical mathematisation and the use of more formal models.

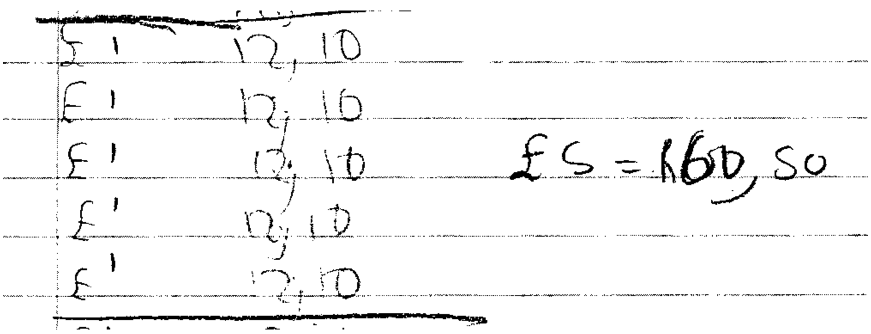


Figure 4.20 Part of Emelie's solution to the weekend trip problem

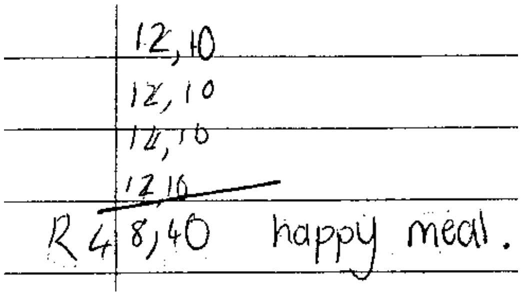


Figure 4.21 Part of Zwanela's solution to weekend trip problem

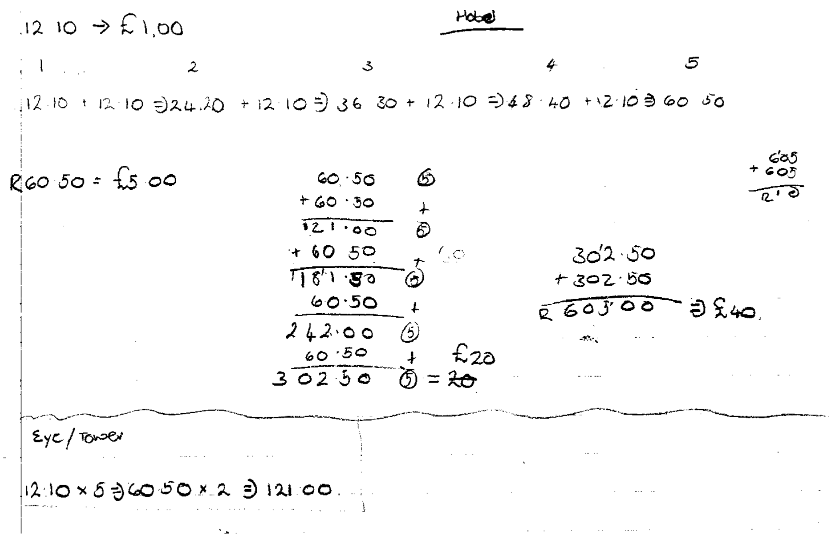


Figure 4.22 Part of Connie's solution to the weekend trip problem in Box 8

4.6.3 Evaluation of the third cycle

The evaluation of this cycle is done in two parts. The first part looks at whether the instructional design principle of *self-developed or emergent models* was implemented within this cycle, based on the information provided in 4.6.1 and 4.6.2 (which is taken from my logs as well as work samples from learners). The observations from the teachers and the assistant researchers were not included in this first part of the evaluation as the design principle for the following reasons: Firstly the principle is not one that can easily be observed in an isolated lesson, as it is something that develops through a series of instructional activities. Secondly observers mainly remained in one place throughout the lesson, which meant they were not often able to see what individual learners were doing unless they wrote it up on the board or explained it. Finally, if one has not been exposed to the theory behind RME and the design principles, one does not necessarily know what to look for to identify the principles. As I was designing and presenting the lessons, not much time was spent specifically discussing the three heuristic design principles of *guided reinvention, didactical phenomenology* and *self-developed or emergent models* with the teachers or the assistant researchers. The statements on the observation schedule were discussed in depth with them though to ensure that we shared a common understanding of their meaning, but the schedule focuses more on the instructional approach rather than on the design of the instructional activities. While certain statements on the schedule could reflect the actual implementation of the two principles of *guided reinvention* and *didactical phenomenology*, without the observers knowing that these statements necessarily pertained to these principles, this was not the case for the third principle of *self-developed or emergent models*.

The second part of the evaluation of this cycle draws on the results of the observation schedules that were quantitatively analysed using a manifest content analysis. This analysis gives an indication of the adherence of the lessons in the third cycle to the intended RME instructional approach, the implementation of the first two principles of *guided reinvention* and *didactical phenomenology* and the subjective opinion of the two observers with regard to the general impression of each of the lessons in terms of their usefulness, how interesting and easy they were to apply and how enjoyable they were. An open section for remarks or comments was also provided at the end of the schedule but these remarks pertained mainly to individual learners and are therefore incorporated into the analysis in the chapter that follows.

Part one of the evaluation

It is my opinion that the instructional design principle of *self-developed or emergent models* was realised to a sufficient extent within this cycle, by following on some of the instructional activities from the previous cycle with the focus on fractions used in everyday language, as fracturers and as comparers. The problems in Boxes 2, 3, 4 and 6 are examples of contextual problems that tried to achieve this. The extent to which this principle of *self-developed models* was realised in this cycle is also limited to fractions though, and I believe that the instructional activities on decimals will need substantial revision and additional similar contextual problems in order to offer learners opportunities to move through the levels in order to connect their formal mathematical knowledge with their more informal situational knowledge. The scope of knowledge relating to decimals is a wide domain and more contextual problems will be needed to try and cover the different aspects such as place value of decimals, operations with decimals, rate, converting from fractions to decimals, percentages, measurement etc. Additional contextual problems where fractions act as operators will also be required for the initial prototype as this aspect appears to have been neglected in the instructional activities implemented during this cycle.

Part two of the evaluation

During cycle three, a total of six lessons for each of the classes were observed by an assistant researcher and 8X's teacher also observed all six of those lessons while the teacher for 8Y was available for five out of the six lessons. During each of these lessons, the assistant researcher as well as the teacher each completed their own observation schedule. The schedule (see Appendix C) used in cycle 3 was made up of a list of statements relating to the introduction, body and conclusion of the lesson as well as a section where some general statements were provided and an overall impression of the lesson could be given. The observers were required to strongly agree, agree, disagree or strongly disagree with each statement. Where they did not feel that a particular statement could be commented on within a lesson, they could opt for the "not applicable" option. This *Likert* type scale was then scored with a value of between 0 and 4. If the “not applicable” column was ticked, it was assigned a value of 0 as it was assumed that the statement had not been dealt with sufficiently so no opinion could be provided relating to it. The other columns were allocated the following values: Strongly agree: 4; Agree: 3; Disagree: 2; Strongly disagree: 1. The more the observers therefore agreed with the statement, the higher the value attributed to that statement. These values were entered into

SPSS so that the tables provided below could be generated to depict the observers' opinions of the lessons during the third cycle in relation to the statements on the observation schedule. The statements were classified into five categories, namely:

- Statements relating to the principle of *guided reinvention*
- Statements relating to the principle of *didactical phenomenology*
- Overall implementation of RME instructional approach
- Working with low attainers
- Behaviour and responses of learners

The first category looked at statements that could collectively reflect on the implementation of the *principle of guided reinvention*, with the focus being on the presenter's approach and not on the learners' responses. Table 4.1 below shows the mean calculated over the six observed lessons in each of the classes, for each of the statements in this group. To obtain this mean, the values attached to the applicable statements as indicated by both the teacher and the assistant researcher for each of the lessons were totalled and divided by the number of observations.

Table 4.1 Implementation of principle of guided reinvention in cycle 3

#	*Statement from observation schedule*	*8X*	*8Y*
1	Teacher asks relevant and guided questions to introduce the lesson	3.67	3.90
2	Teacher asks learners for their own ideas and encourages learners to share them	3.42	3.75
3	Teacher responds to learners' ideas	3.50	3.89
4	Teacher uses and discusses learners' ideas	3.25	3.50
5	Teacher allows learners to choose their own approach	3.17	3.64
6	Teacher asks learners guiding questions, but does not directly provide the answers	3.50	3.55
7	Teacher allows learners to draw own conclusions	3.33	3.38

NOTE:
(Scale with 4 = strongly agree to 1 = strongly disagree; 0 = not applicable; $N_x = 12$; $N_y = 11$)

The mean values for all of the statements contained in Table 4.1 are over 3 indicating that the observers in both classes mostly "agreed" or "strongly agreed" with these statements. Collectively these statements represent an instructional approach indicative of the principle of *guided reinvention* and these data therefore indicate that the principle of *guided reinvention* was well implemented during the third cycle. Additional evidence to support this is also contained in Boxes 6, 7 and 8 in the previous two sections where some contextual problems are provided as well as work samples from learners in solving the problems. The use of both horizontal mathematisation (e.g. in Figures 4.13, 4.14 and 4.20) and vertical mathematisation (e.g. in Figures 4.15, 4.16 and 4.18) can be seen in the solutions provided by some of the learners.

The second category analysed included statements from the observation schedule that related to the use of problems within the lesson. As the *principle of didactical phenomenology* focuses on the use of problems, these statements were seen as an important indicator to demonstrate the observers' opinion about this principle.

Table 4.2 Implementation of principle of didactical phenomenology in cycle 3

#	***Statement from observation schedule***	***8X***	***8Y***
1	Teacher clearly introduces and formulates the problems	3.42	3.44
2	Problem presented is clearly within the frame of reference of the learners	3.00	3.64
3	Problem presented is within the zone of proximal development of learners	3.08	3.36
4	Teacher familiarises learners with the context of the problem if necessary	3.50	3.30
5	Learners understand and are able to engage with the context of the problem	3.42	3.25
6	Learners experience the problem being formulated as real and meaningful	3.42	3.56
7	Learners explore problems in groups or individually	3.33	3.55

NOTE:
(Scale with 4 = strongly agree to 1 = strongly disagree; 0 = not applicable; $N_x = 12$; $N_y = 11$)

This table (4.2) indicates that the observers felt that problems were overall perceived as being effectively used within the lessons conducted in cycle 3, and that learners were mostly able to engage with the problems. It is interesting to note the difference between the mean values of 8X and 8Y relating to the second and third statements in Table 4.2. They both refer to the presentation of the problem in relation to the accessibility of that problem by the learners. In both cases the mean for 8X is lower than that of 8Y. It was often my experience that 8Y seemed to access the problems more easily than 8X and that 8X needed more familiarisation with the problem. It often happened though that 8X's lessons would take place before those of 8Y in which case a thought experiment would first be implemented with them and depending on the results thereof, it would be adapted before being implemented with 8Y. This may well have been a contributing factor to these mean values. According to the observers, this was mostly dealt with in a way that still allowed the learners to understand and engage with the problem (statement #5). This table therefore indicates that the principle of *didactical phenomenology* appears to have been implemented well during the third cycle.

The third group of statements that were classified together were those relating to the implementation of the *RME instructional approach* within the classroom, but not directly linked to the use of problems. These included statements about the socio mathematical norms in the lessons in terms of the presenter's interaction with the learners as well as the instructional style of the presenter (which in this case was myself) in relation to questioning and discussion techniques. Again the mean values from the observations done by the assistant researcher and teacher have been calculated for each statement and are presented in Table 4.3.

Table 4.3 Implementation of RME instructional approach in cycle 3

#	*Statement from observation schedule*	*8X*	*8Y*
1	Teacher often encourages learners to ask questions	3.00	3.57
2	Teacher interacts with learners during the lesson	3.75	4.00
3	Teacher encourages learners to discuss with peers in their groups	2.17	3.22
4	Teacher asks several groups/individuals to report their results to the class	2.42	3.43
5	Teacher invites and encourages learners to comment on their outcomes	2.42	3.40
6	Teacher asks critical open-ended questions regarding the outcomes	2.83	3.71
7	Teacher compares learners' outcomes and their differences or discrepancies	1.75	3.57
8	Teacher acknowledges learners' ideas	3.25	3.63
9	Teacher summarises learners' answers	2.92	3.00
10	Teacher asks open-ended questions to individual learners	3.50	3.63
11	A classroom atmosphere prevails that encourages learners to ask and answer questions	3.50	3.63

NOTE:
(Scale with 4 = strongly agree to 1 = strongly disagree; 0 = not applicable; $N_x = 12$; $N_y = 11$)

Overall the mean values for this group of statements relating the implementation of an RME instructional approach were 2.86 and 3.53 for 8X and 8Y respectively. This indicates that observers were of the opinion that the socio mathematical norms propagated by the theory of RME were well implemented in 8Y during the course of the lessons in the third cycle of the intervention, but slightly less successfully implemented with 8X. Two instructional practices that were found to be occurring in both classes less than most other practices were: encouraging learners to discuss their solutions in groups (statement #3) and summarising their answers (statement #3). In 8X the mean values for statements relating to the presentation, questioning and discussion of learners' outcomes were noticeably lower than those in 8Y (statements # 4, 5 & 6).

This may be due to the fact that the learners in 8X often struggled far longer to reach outcomes and needed far more guidance in doing so than the majority of learners in 8Y. I also often needed to work more closely with individuals or pairs in 8X, leaving little time for further whole class discussion or comparison of solutions. In general most of the 8X learners (with the exception of Klokkie) battled to work independently. Zwanela often seemed "paralysed" to continue when she couldn't find the correct formal procedure to apply, and Mary mostly wrote nothing down without direct questioning or assistance from the teacher or myself. Emelie was always highly motivated to attempt the problems and share her solutions with the rest of the class, but her strategies remained highly situational and often contained major errors. This was therefore a problematic class in terms of engaging in class discussions regarding learners' strategies and outcomes. The problems also often took longer to introduce with 8X and learners spent more time trying to solve them, leaving little time for discussion. Within 8Y, learners mostly produced a wider variety of solutions and outcomes though that could be compared and discussed more easily. Hence the higher mean values probably in that class with regard to those statements (#'s 4, 5 & 6).

The fourth group of statements in the classification related specifically to instructional activities pertaining to working with *low attaining learners*. These included activities such as emphasising crucial aspects, assisting learners where necessary, focusing on notation and terminology and helping learners to understand their discrepancies and draw conclusions. These statements were classified into their own group as some of them are not emphasised or even encouraged in the RME theory. However, from the literature on low attainers these emerge as important instructional activities and were therefore included. Table 4.4 shows the mean values for this group of statements.

Table 4.4 Impression of instructional activities for low attainers used during the third cycle

#	*Statement from observation schedule*	*8X*	*8Y*
1	Teacher often guides the learners to the conclusion	2.83	3.22
2	Learners are encouraged to work together with each other	2.50	3.22
3	Teacher focuses learners' attention on crucial aspects	2.92	3.33
4	Teacher draws attention to and re-emphasises the relevant mathematical notation and terminology relevant to the lesson	3.67	3.78
5	Teacher assists learners when necessary	3.67	3.73
6	Teacher guides learners to understand discrepancies in their solutions	2.92	3.29
7	Teacher draws conclusions from the activity with the learners	2.67	2.83

NOTE:
(Scale with 4 = strongly agree to 1 = strongly disagree; 0 = not applicable; N_x = 12; N_y = 11)

The most commonly employed strategies for both classes in this regard appear to have been assisting learners and re-emphasising relevant mathematical notation and terminology (statements #4 & 5). On the other hand, guiding learners to the conclusion and drawing conclusions for the learners appears to have been less well handled. This may have been due to my own uncertainty in finding the balance between guiding learners to conclusions and giving them the conclusions one wants them to reach. It appears as though I therefore sometimes simply avoided committing to actually doing either. The limited time available in most lessons was also a contributing factor in this though. A noteworthy difference also exists between the mean values for the 8X and 8Y lessons with regard to encouraging learners to work together with each other (statement #2). This may be due to the fact that by cycle 3 the learners in 8X were in such different places with regard to their approach to problems (as explained in the discussion following the previous group of statements) that the dynamics in that class simply did not lend itself to a great deal of co-operation between learners. Perhaps this should have been encouraged despite the dynamics in order to adhere more strictly to the suggested strategies for both low attainers and the RME instructional approach in this regard, which both support learners working together.

The final group that statements were classified into, related to the *behaviour and response of the learners* within the lessons as observed by the teacher and assistant researcher. These assisted me in gaining some feedback on the learners' participation in the intended socio-mathematical norms. Table 4.5 summarises these mean values.

Table 4.5 Learners' behaviour and responses in cycle 3

#	*Statement from observation schedule*	*8X*	*8Y*
1	Learners interact with the teacher	3.17	3.70
2	Learners share their ideas willingly	2.75	3.45
3	Learners appear bored and disinterested	1.33	1.00
4	Learners appear interested in the work	3.08	3.30
5	Learners actively make use of their knowledge	3.42	3.44
6	Learners discuss the operation employed in the problems	2.42	3.29
7	Learners ask questions during the lesson	2.75	2.88
8	Learners understand and are able to engage with the context of the problem	3.42	3.25
9	Learners experience the problem being formulated as real and meaningful	3.42	3.56

NOTE:
(Scale with 4 = strongly agree to 1 = strongly disagree; 0 = not applicable; $N_x = 12$; $N_y = 11$)

Overall the behaviour and responses of the learners in 8X to the RME approach were less in line with the intended socio mathematical norms than those of the learners in 8Y. This suggests that the learners in 8X struggled more than those in 8Y to adapt to the instructional approach used in the intervention. It is my opinion that this may have been influenced by the more traditional information processing instructional approach employed by 8X's mainstream mathematics teacher, although evidence to support this is beyond the scope of this paper. The obvious difference between the willingness of the learners in 8X and 8Y to share their ideas can be seen in the disparity between the mean values of the two classes regarding the second statement. Learners in 8X also discussed the operation employed in their problems less and asked fewer questions during the lessons (statements #6 & 7). Observers from both classes agreed that on the whole learners were mostly interested in the work (statements #3 & 4) and appeared to experience the problems used in instructional activities during the third cycle as real and meaningful (statement #9).

Table 4.6 shows the overall mean values for the *general impression* of the six lessons observed. The observers had to circle an amount of 1, 2, 3, 4 or 5 with the "1" indicating a negative general impression of the lesson (for example, "not useful") and 5 indicating a positive impression.

Table 4.6 General impression of lessons in cycle 3

Attribute	*8X*	*8Y*
Useful	4.75	4.78
Interesting	4.58	4.89
Easy to apply	4.50	4.78
Enjoyable	4.33	5.00

NOTE:
(Scale with 5 = very useful to 1 = not useful; 0 = not applicable; N_x = 12; N_y = 11)

Impressions of the lessons, observed during the third cycle, are highly favourable. Overall the classes for 8Y seem to have been more enjoyable. Examining my own logs, this is also something that emerges and reasons for this (listed in my logs as the intervention progressed) included: the dynamics amongst the learners in the classes, the vast difference of pace and progress between the learners in 8X whereas 8Y didn't seem to have quite such a range, the different approaches to teaching mathematics that the classes experienced in their mainstream classes, the greater dependency in general of 8X learners on a teacher figure for constant guidance and their lack of perseverance in attempting to solve problems.

In conclusion: the concepts of fractions and decimals were continued during this cycle with an emphasis on rate and ratio. The data obtained from the highly structured observation schedules, completed by the assistant researchers and teachers during the lessons in cycle three, my own logs and work samples from learners, indicate that lessons implemented during this cycle were on the whole in line with the problem-solving approach suggested by the theory of RME and that the intended principles of *guided reinvention, didactical phenomenology* and the principle of *self-developed or emergent models* were fairly well implemented. Certain principles relating to teaching low attainers were better adhered to than others and the challenge of guiding learners to conclusions needs additional attention. Observations of learners' behaviour and responses to the lessons suggest that while in general they appeared interested in the work, the learners in 8X struggled more to adapt to the RME instructional approach than those in 8Y. In general the observers perceived the lessons as being useful, interesting, easy to apply and enjoyable.

4.7 Conclusion

This chapter has sought to provide the reader with an account of the intervention as it took place over the three cycles. The first cycle ended up being more of an exploratory cycle where an attempt at implementing the first heuristic design principle of *guided reinvention* was not very successful. However, a rapport with the learners in the study was formed and it provided the opportunity to establish expected classroom norms and practices that would be followed in the remedial classes for the duration of the intervention. 8Y appeared to adapt to this new classroom culture more easily than 8X and it is my opinion that this could be due to a combination of the learners in the class and the vast difference in the approach to instruction used by their mainstream mathematics teacher and the more problem-based approach of the intervention.

The second cycle was successful in terms of more correctly implementing the first heuristic design principle of *guided reinvention* and to a lesser degree realising the principle of *didactical phenomenology*. Some learners started to demonstrate an increased use of their own informal strategies in solving the contextual problems, and clearer examples of both horizontal and vertical mathematisation became evident in the learners' work and thinking. An attempt was made to address the four views of fractions as expressed by Freudenthal (1983), but it was felt that the first two (fractions in everyday language and fractions as fracturers) received more attention and were more successfully dealt with than the latter two of fractions as comparers and decimals fractions.

The third and final cycle was the shortest but most controlled cycle in terms of observations from the teachers and an assistant researcher for almost each lesson. These observation schedules aimed to evaluate the implementation of the RME heuristic instructional design principles during each lesson as well as the occurrence of suggested emerging design principles that had been identified in the former two cycles. An attempt was made to implement the third instructional design principle of *self-developed models* during this cycle to enable learners to move from using mostly informal and situational strategies to employing more formal ones. This was accomplished to some degree but will need additional work in compiling the initial prototype. Overall the observers gave a good general impression of the lessons and the mean values produced from their observations indicate that the RME instructional approach appears to have been fairly well implemented.

Looking at the intervention as a whole, it is my opinion that I was too ambitious in terms of the key concepts I planned to address. If one looks at the actual total number of lessons in the intervention that transpired, once all the holidays, sports days and timetable changes had been taken into account, there were only approximately 24 lessons for each class. The dilemma was trying to get through as much as possible, knowing that these learners (with the exception of Leratho) already had a backlog in terms of their knowledge and understanding of mathematics. In retrospect, with the limited number of lessons, it may have been more beneficial for the learners, and certainly would have improved the design, if only one of the key concepts had been dealt with. From my part, more time could have been spent on refining and improving the design of one concept, while ensuring that the instructional design principles of RME were being implemented. Instead I ended up doing "bits" of each concept, which runs the threat of falling into an empirical approach (see section 2.6.1) according to Treffers, 1987. In redesigning such an intervention, I would therefore allow for more time in each concept to allow learners to work through and discuss more contextual problems. I would also do this to give them more opportunities to try their own informal strategies and to gain confidence in these before working towards refining their solutions towards more formal models.

CHAPTER FIVE

5 Learner performance

5.1 Introduction

This chapter draws together the data collected during the course of the intervention, pertaining specifically to how learners' performance developed, and presents it in the form of two main sections, namely the achievement tests and learner profiles. As explained in the research design in Chapter 3, the conceptualisation, method and inference stages of the research process within this study all drew on both qualitative as well as quantitative approaches. The data that are the focus of this particular chapter are the data gathered from the achievement tests, the observations (by the teachers, research assistant and myself as researcher and presenter), work samples of learners (completed during the remedial classes) and classroom assessments (completed in their mainstream mathematics classes).

Section 5.2 first discusses the quantitative analysis of the data obtained from the achievement tests, before a detailed profile of each learner is constructed in section 5.3 from a range of documents (further detailed in that section). References to individual learners made during the observations are included in their profiles as well as data from a questionnaire that learners completed prior to the commencement of the intervention. Section 5.4 concludes by pulling together data from sections 5.2 and 5.3.

5.2 Achievement tests

The analysis in this section is restricted to learners' performance in the pre- and post-tests. As mentioned in Chapter 3 (section 3.5.2) the tests contained 30 items (the post-test contained 31 items but the last item is not included in this quantitative analysis but rather in the learner profiles in the next section). The data were generated using the computer software programme SPSS and are discussed in relation to the tables presented below. As previously mentioned, Nomsa did not write the pre-test as she only joined the 8X two weeks after the start of the intervention. Liya, on the other hand did not write the post-test as she was away from school for the last month of the fourth term in order to receive psychological counselling for problems she was experiencing at home due to her father's recent death. Their scores are therefore not included in the data presented in this section, with the exception of Table 5.2 where a breakdown of the scores that Liya and Nomsa obtained in the pre- and post-tests respectively are provided as an indication of at least one of their performances in the cognitive achievement test. However, deductions relating to their performance based on these achievement tests cannot be drawn. Samples of their work, observations and interviews in subsequent sections are used in commenting on any change in their understanding of the concepts addressed in the intervention.

The first table below (Table 5.1) is an overall description of the learners' achievements in the pre- and post-tests. A visual representation of this table is also presented in Figure 5.1. Here the learners' scores on the tests are categorised into six levels of performance namely lower 1 and 2, middle 1 and 2 and upper 1 and 2. The table indicates the number (and percentage) of learners in each level and the levels are categorised as follows:

Scores ranging between 0 and 5 fall into lower 1
Scores ranging between 6 and 10 fall into lower 2
Scores ranging between 11 and 15 fall into middle 1
Scores ranging between 16 and 20 fall into middle 2
Scores ranging between 21 and 25 fall into upper 1
Scores ranging between 26 and 30 fall into upper 2

Table 5.1 Description of learners' overall scores in pre- and post-tests

Levels	**Pre-test**		**Post-test**	
	N	***%***	***N***	***%***
Lower 1	1	10	0	0
Lower 2	1	10	1	10
Middle 1	3	30	1	10
Middle 2	2	20	3	30
Upper 1	3	30	3	30
Upper 2	0	0	2	20

NOTE:
N = number of learners in that level (Total = 10)
% = percentage of total number of learners (10) who wrote both tests

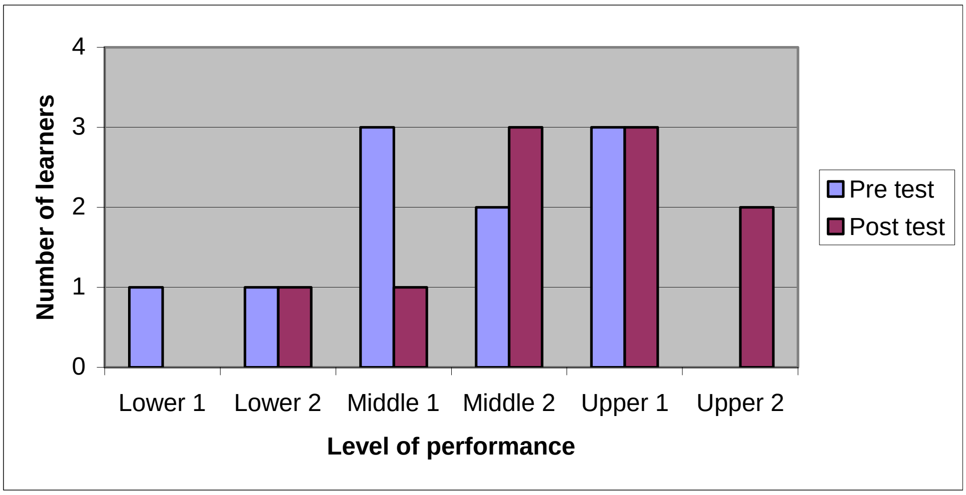


Figure 5.1 Learners' overall scores in pre- and post-tests

From Table 5.1 the shift between the six levels is evident on this small sample. The mean score for these ten learners for the pre-test was 15 (with a standard deviation of 7) and for the post-test it was 19 (with a standard deviation of 5). None of the learners scored between 25 and 30 in the pre-test, whereas two of them (Klokkie and Leratho) achieved scores within this group in the post-test. In the pre-test half of the learners (50 %) scored between 0 and 15 for the test. In the post-test this percentage dropped to 20 % with the remainder getting more than half of the total number of items correct. This indicates an initial overall positive outcome of the intervention for the majority of learners, but before jumping to conclusions based on limited data, these scores are elaborated on in terms of learners' individual performances.

Table 5.2 below represents the total scores obtained by learners on the pre- and post-tests, and a breakdown between their scores on the conventional and contextual items. To calculate these totals, each of the 30 common items in the tests was scored as correct (1) or incorrect or omitted (0) and totalled. From the test, items 4, 17, 27 and 29 were regarded as contextual problems while the rest were viewed as conventional type questions.

Table 5.2 Comparison of learners' total scores (out of 30) and breakdown of their scores on conventional and contextual items.

Class	Learner	Conventional items		Contextual items		Total scores	
		Pre	*Post*	*Pre*	*Post*	*Pre*	*Post*
8X	Klokkie	13	22	3	4	16	26
	Emelie	9	11	2	3	11	14
	Mary	2	7	1	2	3	9
	Zwanela	11	13	3	3	14	16
	Nomsa	-	16	-	3	-	19
8Y	Liya	11	-	1	-	12	-
	Connie	14	18	4	4	18	22
	Mpho	17	19	4	3	21	22
	Gloria	18	16	3	3	21	19
	Violet	9	18	2	3	11	21
	Patience	9	15	1	3	10	18
	Leratho	22	22	3	4	25	26

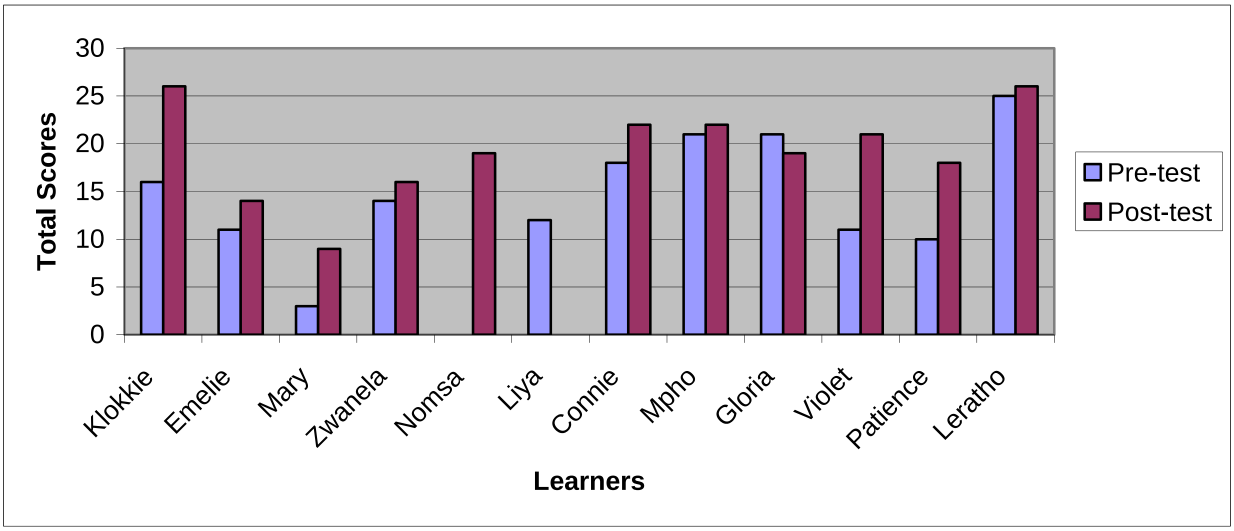


Figure 5.2 Comparison of learners' total scores in the pre- and post-tests

The table (Table 5.2) and the visual display in Figure 5.2 indicate that most of the learners improved their total scores on the post-test. Out of the ten learners who wrote both the pre- and post-tests, nine of them showed a positive improvement while Gloria scored better in her pre-test than in her post-test. The two learners who improved the most were Klokkie and

Violet (10 marks improvement in their totals), followed by Patience (8 marks improvement in her total) and Mary (6 marks improvement). The other learners who improved only did so by a difference of between 1 and 4 marks (Leratho, Emelie, Zwanela, Connie and Mpho). Most of the learners, with the exception of Gloria and Leratho, improved their performance on the conventional items and six of the ten learners who wrote both tests, also improved their scores on the contextual items.

As mentioned in Chapter 4, Leratho was not in the class as a result of her performance in mathematics. She was therefore not regarded as a low attainer, but her data have been included in the results as she is still discussed in the study as one of the cases. Further discussion relating to possible explanations for learner's increase (or decrease in the case of Gloria) in these achievement tests is reserved for a later section, once the next section on learners' profiles has been done, so that a clearer picture of each of the learners has been painted by then.

As a final representation of the analysis of the achievement tests, Table 5.3 is presented. It shows any change (positive or negative) in the learners' scores between the pre- and post-tests according to the key concepts that were dealt with during the intervention. For this comparison, the items in the test were classified into the following groups: place value of whole numbers, fractions, decimals, basic algebra (although this was not designated to a specific cycle). Items that did not fit "neatly" into any of these four groups were classified into a group called "other". These included problems on integers as well as one pertaining to patterns and a contextual problem, which was not related specifically to any one of the key concepts but involved an integration of skills. The classification of the items into groups is presented first, followed by the table.

Items relating to place value of whole numbers:	Items 1, 2, 4, 5, 7, 20
Items relating to concept of fractions:	Items 6, 8, 9, 13, 14, 23
Items relating to concept of decimals:	Items 3, 10, 11, 12, 29
Items relating to concept of algebra:	Items 15, 16, 18, 19, 22, 25, 28, 30
Items classified as other:	Items 17, 21, 24, 26, 27

Table 5.3 Change in learners' scores between pre- and post-tests according to key concepts

Class	Learners	Place value	Decimals	Fractions	Algebra	Other
	Klokkie	2	-1	4	3	2
	Emelie	0	-1	3	0	1
8X	Mary	3	0	2	1	0
	Zwanela	3	1	-1	-1	0
	Nomsa					
	Liya					
	Connie	0	3	1	0	0
	Mpho	-1	-1	0	2	1
8Y	Gloria	0	0	-1	0	-1
	Violet	2	1	4	3	0
	Patience	2	-1	2	4	1
	Leratho	0	2	-1	0	0

Overall, learners appear to have shown the most significant improvement in the fractions section and the section on algebra. 8X seem to have benefited more from the instructional activities relating to place value (as was suspected from the lessons that transpired from the thought experiments where 8X were more challenged by the instructional activities in cycle one than 8Y). The learners' performance on decimals appears to demonstrate that this is the section that learners benefited least from, with the exception of Connie who improved most with regard to this key concept. Four of the learners however, performed worse in the post-test than in the pre-test, two learners demonstrated no change, and the rest made a slight improvement. As mentioned, in the previous chapter, this section would need substantial revision when compiling the initial prototype. Once again, discussions of individual performance are left to a later discussion when other data are also considered.

The quantitative data analysed and presented in this section indicates that most of the cases in this study made some progress in their performance in the key number concepts as examined by the pre- and post-tests. This is regarded as preliminary data though and as an indication that such an intervention for low attainers, based on the theory of RME, may well be worth pursuing. It is again emphasised that this is an exploratory study aimed at investigating whether or not using the theory of RME in the setting described in this study is in fact viable and how it may be used to assist learners in improving their understanding. From the data obtained from the achievement tests, it does appear to be feasible to develop an initial prototype that can be implemented and put through an iterative process of further development and refinement in order to produce a local instructional theory on using RME to revisit key number concepts with low attaining learners. Further investigation and evidence of this is provided in the sections that follow, where other data collected during the fieldwork are also further explored.

5.3 Learner profiles

A quantitative representation of learners' performance in the cognitive achievement tests was presented in the previous section. This gave some indication of the learning that took place during the course of the intervention. The cognitive achievement tests were only one of the indicators of learning however, and data provided by the achievement tests were enriched by the more detailed profile of each learner presented in this section. In order to compile a profile for each learner, a variety of documents and data sources were analysed. These included:

- A selection of learners' work completed in class during the course of the intervention. (This selection was made on the basis of samples of learners' work that were collected from learners from some of the lessons during the second and third cycles of the intervention. The work samples selected were ones that were also discussed in Chapter 4 for the purpose of continuity.)
- Two contextual questions from the diagnostic assessment, which was written during the first week of the third cycle.
- A contextual question (item 31) from the post-test that was not included in the pre-test (and therefore not quantitatively analysed in the previous section on achievement tests)
- A standardised test that learners wrote in their mainstream mathematics classes.
  (Question 9 was selected from this test due to its accessibility for most of the learners and its potential to allow for more informal as well as formal procedures - this specific topic of simple and compound interest was not dealt with in the intervention, however.)
- The summative mathematics examination that learners wrote at the end of their Grade 8 year in their mainstream mathematics classes.
  (The questions pertaining to the various concepts of place value, fractions and decimals were identified and learners' scores for these questions were calculated)

- Field notes kept by me during the course of the intervention
- Observations carried out by the teachers and assistant researchers
- Interviews conducted with learners as outlined in section 3.5.4.

Table 5.4 below gives a summary of the selection of learners work, the contextual items and mainstream classroom assessments in order to provide the chronological order of these as well indicate the appendices where the questions or items can be found.

Table 5.4 Summary of data analysed in this section

Source	Date	Item(s) Question(s)	Appendix
Cat pills lesson	July 2003	Box 2 in Section 4.5.1 in Chapter 4	
Insulin lesson	Aug 2003	Box 5 in Section 4.5.3 in Chapter 4	
Diagnostic assessment	Sept 2003	Items 19, 20	E
Overseas lesson	Oct 2003	Box 8 in Section 4.6.2 in Chapter 4	
Ratio problem	Oct 2003	Box 7 in Section 4.6.2 in Chapter 4	
Standardised test	Oct 2003	Question 9	B
Calculating mark problem	Nov 2003	Box 6 in Section 4.6.1 in Chapter 4	
Post-test	Dec 2003	Item 31	A
Final mainstream exam	Dec 2003	Questions from exam relating to: Place value: 1.3; 4.1; 4.6; 4.7.1; 10.3 (7 marks) Decimals: 2.3; 4.2.1; 4.7.2 (4 marks) Fractions: 1.1; 1.2; 4.3; 4.4; 7.1.3; 10.1; 10.2.1 (16 marks)	B

The data matrices presented in the sub-sections 5.3.1 to 5.3.12 were then constructed and completed for each learner from this variety of documents. The deductive analysis was carried out according to 5 categories (*use and level of strategies, appropriateness of solution, final correctness of solution, major mathematical errors, minor mathematical errors*) and any additional comments from field notes, observations or interviews that pertained to that example of learner's work were also included. A synopsis from each data matrix was then completed.

In the deductive analysis, the *use and level of strategies* category included analysing the type of strategies learners used to solve problems, in terms of their own informal strategies or more formal procedures and algorithms they had previously been taught. The object of this analysis was also to assess the learners' solutions and strategies according to the four levels (situational, referential, general or formal) of self-developed models as explained by Gravemeijer (1994), discussed in this report in the previous chapter in section 4.6.1. The *appropriateness of the strategy* refers to whether or not the strategy selected by the learner in solving a particular problem was appropriate or not. For example, if a learner decided to multiply when if fact they should have been dividing, the strategy was deemed inappropriate. The *final correctness of the solution* looked at whether or not learners were able to reach a final solution to a problem and at the mathematical correctness of the solution. The final two categories (major mathematical errors and minor mathematical errors) were differentiated between by my own opinion, drawing on 8 years experience in teaching Grade 8 mathematics. For example, thinking that $\frac{1}{2}$ is equal to 1.2 was regarded as a major mathematical error whereas calculating 6 x 9 = 57 was regarded as a minor mathematical error. When a learner chose a strategy that was classified as not being appropriate, this was indicated under the *appropriateness of strategy* column and therefore not marked off again as also being a major mathematical error.

The introductory section (preceding the data matrix) on each learner in the following sub-sections was compiled from the transcriptions of the interviews that were conducted with each learner (see section 3.5.4), as well as from a questionnaire that each learner completed prior to the start of the intervention. The questionnaire was adapted from the TIMSS 1999 background questionnaire (Howie, 2002) and given to learners in an attempt to gain some understanding of their attitude towards mathematics and their perceptions of themselves as mathematics learners. The questionnaires were not used for data analyses purposes but some information from them is rather included in order to provide the reader with additional insight into each learner.

5.3.1 Klokkie (8X)

Klokkie did not experience herself as usually doing well at mathematics at school, although she felt that her friends did ask her for help when they struggled with mathematics. It was her perception, however, that her parents do not think she is good at mathematics. According to Klokkie, she likes mathematics, "…but then I hate it sometimes when I do not understand, the work, when it gets too difficult."

Table 5.5 Data matrix summarising selection of Klokkies' work

Source	S	A	FS	ME	me	Comments
Cat pills	FL RL	A	✔			"Knew she had to × ÷ by the numbers - but battled to see the picture." ***(Teacher)***
Insulin	FL	T & E	PC			
Diagnostic assessment (19)	FL GL	A	✘	1		
Diagnostic assessment (20)	FL GL	A	PC		1	
Overseas	FL GL	A	✔			
Ratio problem	FL	A	✔			
Standardised test	FL	A	1/5		1	
Calculating mark	FL	A	✔			
Post-test	FL GL	A	✘	2		
Final mainstream exam			P: 3/7 D: 3/4 F: 7/16			

NOTE:
S = Levels and use of strategies; A = Appropriateness of solution; FS = Correctness of final solution; ME = Major mathematical error(s); me = minor mathematical error(s)
FL = Formal level; GL = General level; RL = Referential level; SL = Situational level
A = Appropriate; PA = Partially appropriate; NA = Not appropriate; T&E = Trial and error
✔ = Correct; PC = Partially correct; ✘ = Incorrect
1/5 indicates that mark obtained for that question was one out of a possible 5 marks

Synopsis

Klokkie always used formal strategies to start off with in attempting to solve problems. She did make use of more informal strategies after that if she was not confident in carrying out a calculation using a formal procedure (such as long multiplication of decimals where she rather used repeated addition) or if she wanted to check a solution. Her solutions were therefore predominantly formal although some contained a mixture of both formal and informal strategies. In terms of the levels, her strategies were mostly classified as falling into the formal and general levels. She very seldom produced solutions that could be classified as referential.

Although Klokkie mostly used formal procedures, she remained unsure at times which algorithm to use in solving a particular problem. She would then resort to trial and error in trying to identify the correct one or resorted to an informal strategy to assist her, but this was the exception rather than the rule. For example in wanting to find out how many days 2,5 ml of insulin would last if you used 0,16 ml per day, she would first try 0,16 - 2,5, then 2,5 x 0,16 before settling on 2,5 - 0,16. Although repeatedly encouraged to make more use of informal strategies when she was unsure of which formal procedure to use, she was mostly determined to continue searching for the "correct" algorithm. This resulted in her solving problems incorrectly at times and demonstrated a continued dominance of instructional as opposed to relational understanding, especially with regard to the addition of fractions. On more than one occasion, she applied a suitable strategy to a problem involving fractions, but then incorrectly added the fractions (see Figures 5.3 and 5.4). In the final examination she scored 48 % on the questions pertaining to the relevant key number concepts addressed during the intervention, which according Teacher X was a good performance on Klokkie's behalf.

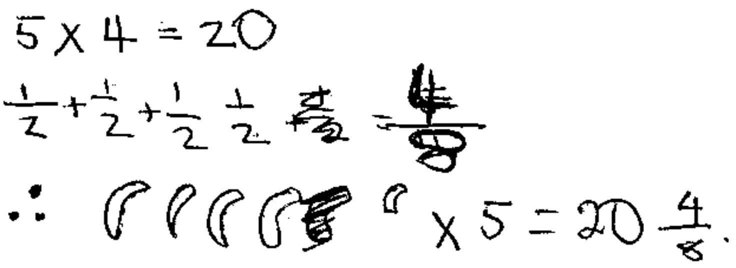


Figure 5.3 Klokkie's solution to Item 19 of the diagnostic assessment

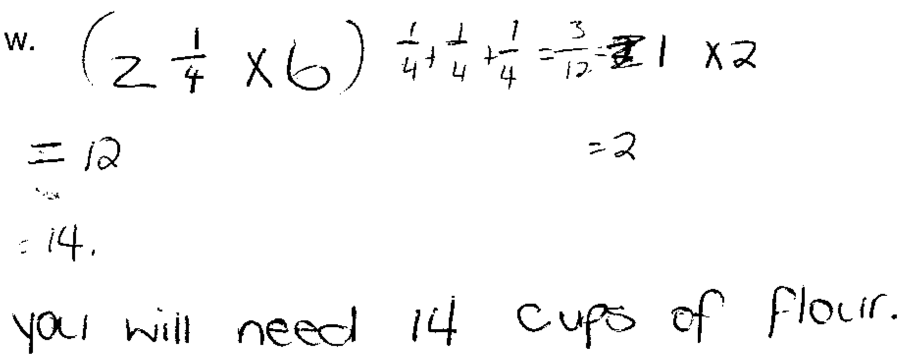


Figure 5.4 Klokkie's solution to Item 31 of the post-test

5.3.2 Emelie (8X)

Emelie also did not feel that she usually did well in mathematics. She stated that her friends did not usually ask her for help when they struggled, although her mother thought that she was good at mathematics. In response to the question of whether or not she liked mathematics, she responded, "At first I didn't like maths but now, it is interesting for now cause we are learning about fractions and how to draw…."

Table 5.6 Data matrix summarising selection of Emelie's work

Source	S	A	FS	ME	me	Comments
Cat pills	RL	A	PC			"Emelie used 'sticks' to calculate." ***(Teacher)***
Insulin	SL GL	A	PC			"Emelie was confused as to where 0,1 goes (top/bottom)…Emelie still cannot understand where does 0,1 start top/bottom of syringe. Confused with 0,8 and 0,08 and where it is on the needle." ***(Teacher)***
Diagnostic assessment (19)	GL	A	✔			
Diagnostic assessment (20)	GL FL	A	PC		1	
Overseas	GL	A	PC		1	"Emelie struggles to work methodically." ***(Researcher)***
Ratio problem	FL	A	✔			
Standardised test	GL	A	5/5			
Calculating mark	FL	A	✔			"Used equivalent fractions." ***(Researcher)***
Post-test	GL	A	✔			
Final mainstream exam			P: 2/7 D: 1/4 F: 2/16			

NOTE:
S = Levels and use of strategies; A = Appropriateness of solution; FS = Correctness of final solution; ME = Major mathematical error(s); me = minor mathematical error(s)
FL = Formal level; GL = General level; RL = Referential level; SL = Situational level
A = Appropriate; PA = Partially appropriate; NA = Not appropriate; T&E = Trial and error
✔ = Correct; PC = Partially correct; ✘ = Incorrect
1/5 indicates that mark obtained for that question was one out of a possible 5 marks

Synopsis

Emelie often battled to work methodically. She usually worked all over the page (when she had an entire one at her disposal) and her writing and solutions were illegible at times. Her strategies as indicated in the matrix above were predominantly of a general level although she initially made use of diagrams to assist her in solving problems. There was a distinct movement in her solutions though from the more referential level during the second cycle to ones that could be classified as being more on the general level during the third cycle.

The strategies and solutions Emelie used throughout the course of the intervention to solve problems, demonstrate a definite improvement and refinement from long winded solutions with lots of graphics to more refined ones where she also made use of some mathematical notation. Her improved relational understanding of fractions and approaching contextual problems in general was especially evident. This was a slow process though and for Emelie it would have sufficed to only cover one of the key concepts. She continued to struggle with decimals and it appears as though she could have benefited from continued work on place value. Her level of understanding of even basic key mathematical concepts appeared to be well below that of the required standard for a Grade 8 learner, so although she demonstrated some progress during the intervention and on occasion in questions in her mainstream classroom assessments, she remained unable to improve her performance significantly in the final exam at the end of the academic year (she scored 19% on the identified questions in the final exam).

5.3.3 Mary (8X)

A large portion of Mary's written questionnaire was not completed, and Teacher X and myself came to the conclusion that there was a misunderstanding on Mary's part in how the questionnaire should be answered, probably due to her poor comprehension of English. Each section in the questionnaire contained several statements that learners needed to respond to (on a scale of strongly agree, agree, disagree or strongly disagree). Mary, however, only responded to one of the statements in each section, as one would do with a multiple-choice questionnaire.

Table 5.7 Data matrix summarising selection of Mary's work

Source	S	A	FS	ME	me	Comments
Cat pills	FL RL	T & E	PC			"…has problems with English - cannot even say 'I cut the big pill in half'. Afraid to speak English." ***(Teacher)***
Insulin	SL FL	T & E NA	✗			"Hayley [researcher] had to explain to Mary how the syringe actually works - to pull up water to 0,2 ml, the back part must be on that." ***(Teacher)***
Diagnostic assessment (19)	FL	NA	✗	3		
Diagnostic assessment (20)	FL	A	PC	2	1	
Overseas	RL GL	A	PC	3	5	"Mary required a great deal of individual questioning and attention." ***(Researcher)***
Ratio problem	FL	A	✓			"Mary still struggles with 'word sums'…needs constant guidance". ***(Teacher)***
Standardised test	FL	NA	0/5	3	1	
Calculating mark	FL	A	✓			
Post-test	FL	A	✗	1	1	
Final mainstream exam			P: 1/7 D: 0/4 F: 1/16			

NOTE:
S = Levels and use of strategies; A = Appropriateness of solution; FS = Correctness of final solution; ME = Major mathematical error(s); me = minor mathematical error(s)
FL = Formal level; GL = General level; RL = Referential level; SL = Situational level
A = Appropriate; PA = Partially appropriate; NA = Not appropriate; T&E = Trial and error
✓ = Correct; PC = Partially correct; ✗ = Incorrect
1/5 indicates that mark obtained for that question was one out of a possible 5 marks

Synopsis

Most of Mary's work contains a mixture of both formal as well as informal strategies but these were mostly incorrectly applied or represented. When using a formal procedure, she would either select the incorrect one or apply the correct one incorrectly. Her informal strategies seldom demonstrated a correct representation of the problem either. Mary therefore seemed to really struggle to understand the problem and even when she did, she seldom appeared to be able to represent the information diagrammatically or with the use of mathematical notation (see Figure 5.5). She needed a great deal of guidance and individual attention in the form of clarifying the problem and questioning her in order to guide her to any form of solution. There were almost always mathematical errors in her solutions, even when carrying out routine procedures such as basic adding or subtracting and she battled to correctly use a calculator. Even when given a syringe to work with, to try and assist her in reaching a situational solution, she showed difficulty in handling the syringe in terms of co-ordination.

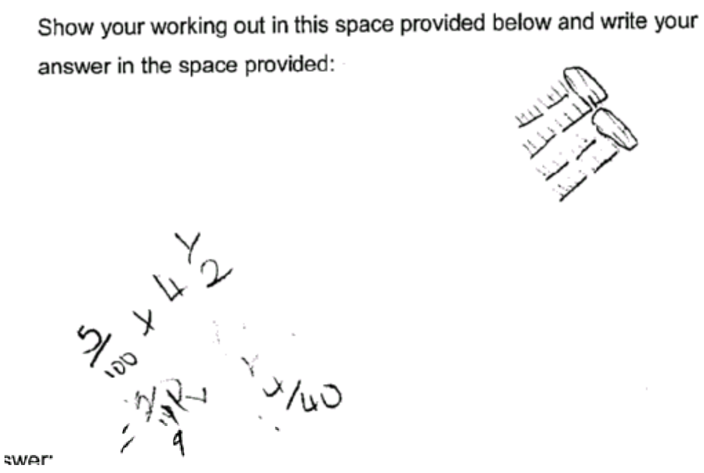


Figure 5.5 Mary's solution to the banana bread problem

Evidence of Mary's progress was more evident through verbal interaction with her than through her written work. On an individual basis, she became more confident asking and answering questions but was mostly unable to work independently in producing strategies and solutions that she could explain or that could be understood. Mary showed signs of potentially having a learning disability and also more specifically, a mathematical disability, but except for making the school aware of this, it was beyond the scope of this study to try and confirm this. Her trouble in understanding and speaking English also presented a barrier to her showing any marked improvement in her academic performance at school (7% for the identified questions in the final exam). Even during the intervention though, when I would sometimes try to explain the problem to her in her own mother tongue (Sepedi), she was still not able to find a way, either using pictures, symbols or numbers to represent it. When her solutions were completed and correct, this was usually as a result of a great deal of individual attention and leading questioning to assist her in arriving at the conclusion. It is therefore difficult to conclude whether or not Mary made any progress in her relational understanding of the key concepts.

5.3.4 Zwanela (8X)

Zwanela disagreed with the statement about her usually doing well at mathematics and was of the opinion that her parents did not think she was good at mathematics either. She did indicate on her questionnaire that she enjoyed learning mathematics though.

Table 5.8 Data matrix summarising selection of Zwanela's work

Source	S	A	FS	ME	me	Comments
Cat pills	FL	T & E PA	PC	1	1	"…uses words like 'I'm confused'. Does not know fractions - confused with $\frac{1}{2}$ and 1,2. Thinks 1,2 = $\frac{1}{2}$ (corrected by teacher [researcher])." ***(Teacher)***
Insulin	FL	T & E NA	✗			"Zwanela seemed frustrated (especially if she doesn't know)." ***(Teacher)***
Diagnostic assessment (19)	FL	A	✗	1		
Diagnostic assessment (20)	FL	A	PC		2	
Overseas	GL	A	PC		3	"Used long lists of repeated addition." ***(Researcher)***
Ratio problem		T & E PA	PC	2		"Zwanela got very frustrated with not being able to correctly 'guess' the solution to this problem. Questioned her individually right into break to guide her to solution so that she wouldn't leave still frustrated." ***(Researcher)*** "Not sure of herself." ***(Teacher)***
Standardised test	FL	A	4/5		1	
Calculating mark	FL	A	✔			
Post-test	FL	A	PC		1	
Final mainstream exam			P: 1/7 D: 2/4 F: 1/16			

NOTE:
S = Levels and use of strategies; A = Appropriateness of solution; FS = Correctness of final solution; ME = Major mathematical error(s); me = minor mathematical error(s)

FL = Formal level; GL = General level; RL = Referential level; SL = Situational level
A = Appropriate; PA = Partially appropriate; NA = Not appropriate; T&E = Trial and error
✔ = Correct; PC = Partially correct; ✖ = Incorrect
1/5 indicates that mark obtained for that question was one out of a possible 5 marks

Synopsis

Zwanela continually relied on formal procedures in solving problems both in the intervention and in her mainstream mathematics assessments (see Figure 5.6). This was often done through trial and error in an attempt to find the "correct" procedure and she often became frustrated and despondent while searching for it. If and when she did find the correct formal procedure or algorithm to apply, she often made mathematical errors that prevented her from finding the correct solution. An example of this was provided in Figure 4.5 in the previous chapter and an additional illustration is given in Figure 5.6 where her solution to the "cat pills" problem is discussed. Only on one occasion (when they were not permitted to use calculators) did she use a strategy that can be classified as falling into the more general rather than formal level (in the form of repeated addition). The rest were formal and, with only one exception from the selection represented above (the calculating mark problem), all contained some sort of fundamental or calculation error.

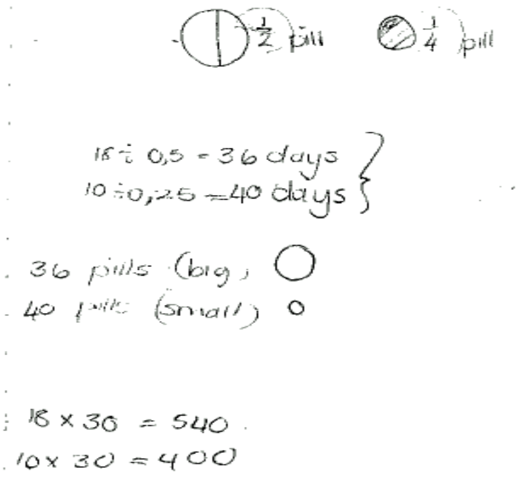


Figure 5.6 Zwanela's solution to the cat pills problem

In Zwanela's work above in Figure 5.6, if one looks at her representation of $\frac{1}{4}$ in the diagram she drew, she divided the pill into four parts that clearly cannot be equal with that type of division. Another error that she made, which is not evident from the example as it now stands (as Zwanela erased and corrected this after discussion), is the following.

Initially she wrote: 18 $\div \frac{1}{2} = 15$. When I questioned her on how she got 15 as her answer, Zwanela demonstrated on her calculator how she had said: 18 ÷ 1,2 = 15. She explained how she thought that $\frac{1}{2}$ and 1,2 were equal. When questioned on what she thought $\frac{1}{2}$ meant, she replied that it meant 1 ÷2 and I asked her to try this on her calculator. She obviously then got 0,5 and not 1,2 and realised her mistake which she rectified. Zwanela also appeared to not have a real grasp of the problem in that she repeatedly mixed up the pills and the number of days. Although she did reach the correct answer regarding the number of days the pills will last, when required to work out how many pills were needed for one month, she resorted to an algorithm without considering the context. She multiplied the number of pills initially handed out by 30, instead of multiplying the number of pills needed per day by 30.

Analysing Zwanela's solutions throughout the intervention provides no substantial evidence of improved relational understanding in terms of the key number concepts addressed. Her academic performance in the final examination also supports this, where she obtained 15 % for the number strand that contained questions relating to these key concepts. She appears to be one of those "victims of rigid instruction" that Freudenthal refers to for whom it may be too late to be trying to implement this approach. On a pragmatic note, perhaps a more "drill and practice" instructional approach would have improved her performance in mathematics although this would not necessarily have assisted her in the long term in terms of her relational understanding.

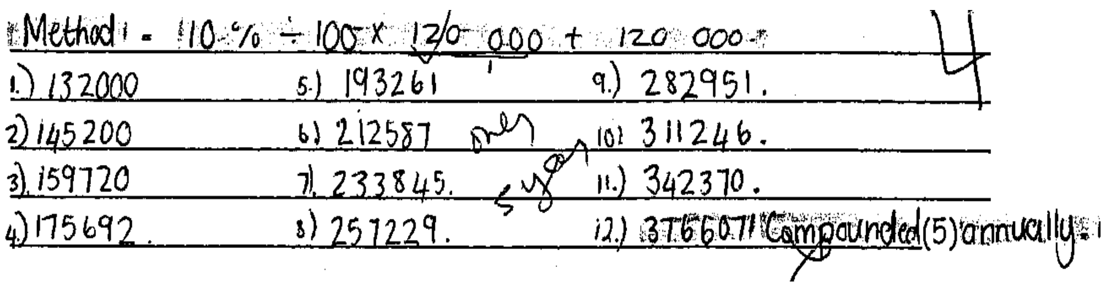


Figure 5.7 Zwanela's solution to question 9 of the standardised test

5.3.5 Nomsa (8X)

Nomsa did not complete the initial background questionnaire given to learners, as she was not yet part of the remedial group at that stage. The same attitude and background questionnaire that learners completed at the start of the intervention though was also completed towards the end of the year, in order to monitor any changes in the attitudes or perceptions of learners. While the decision was taken to not include those data in this report on the basis that they were beyond the scope of this study, Nomsa's questionnaire was used in order to obtain the information regarding her attitude towards mathematics and her perceptions of her ability.

Nomsa felt quite strongly that she does not usually do well in mathematics and that her parents do not think that she is good at it. Her friends usually ask her for help when they struggle though.

Table 5.9 Data matrix summarising selection of Nomsa's work

Source	S	A	FS	ME	me	Comments
Cat pills	RL	A	✔			"Nomsa explains answers on board. Nice clear and correct illustrations - (Did not summarise using maths - counted1 to 40 but did not link to 4 × 10) - only pictures." ***(Teacher)***
Insulin	SL RL GL	T & E	PC			"Nomsa seems to be responding positively to Hayley's [researcher's] individual questioning." ***(Teacher)***
Diagnostic assessment (19)	GL	A	✔			
Diagnostic assessment (20)	GL FL	A	✔			
Overseas	GL FL	A	PC		2	"Nomsa knew to multiply but needed to make use of a calculator." ***(Researcher)***
Ratio problem						Not done (absent)
Standardised test	FL		1/5			
Calculating mark						Not done (absent)
Post-test	GL	A	✕	1		

Source	S	A	FS	ME	me	Comments
Final mainstream exam			P: 5/7 D: 2/4 F: 10/16			

NOTE:
S = Levels and use of strategies; A = Appropriateness of solution; FS = Correctness of final solution; ME = Major mathematical error(s); me = minor mathematical error(s)
FL = Formal level; GL = General level; RL = Referential level; SL = Situational level
A = Appropriate; PA = Partially appropriate; NA = Not appropriate; T&E = Trial and error
✔ = Correct; PC = Partially correct; ✕ = Incorrect
1/5 indicates that mark obtained for that question was one out of a possible 5 marks

Synopsis

Nomsa's solutions demonstrate a variety of levels of strategies, ranging from situational (when she actually worked with and then drew the syringe) to the referential level (when she drew pictures to assist her), on to the more general level (when she used repeated addition without drawing the pictures) and one or two problems where she used formal procedures. She enthusiastically set about using informal strategies from the beginning and a clear refinement of these strategies is visible in her work on fractions (see Figures 5.8, 5.9).

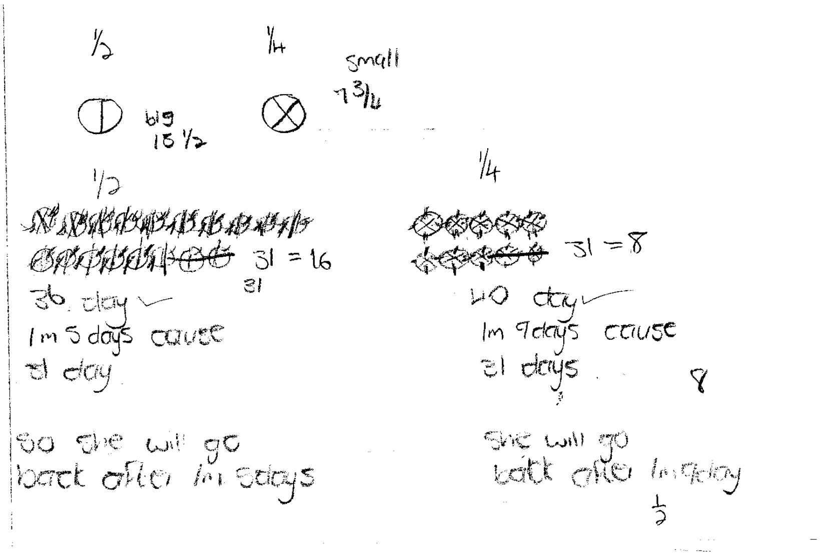


Figure 5.8 Nomsa's solution to the "Cat pill" problem

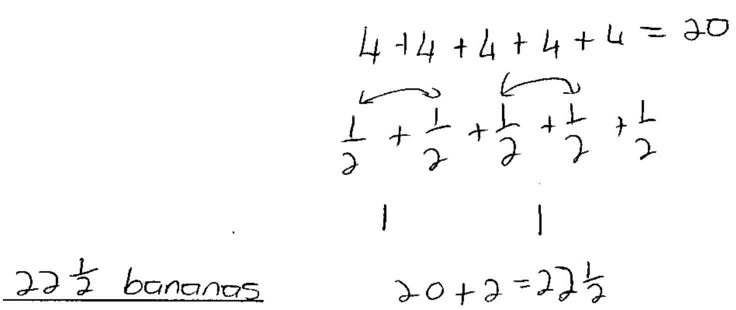


Figure 5.9 Nomsa's solution to Item 19 of the diagnostic assessment

Nomsa's solutions slowly became increasingly sophisticated and showed less dependency on the use of pictures. They were also mostly correct or partially correct due to calculation errors, with the exception of Item 31 from the post-test (see Figure 5.10) where she made a major mathematical error in adding the fractions, which creates doubt about her relational understanding regarding fractions. Her performance in the final examination in the number strand was apparently better than expected though (according to her teacher), where she scored 63 % for the identified questions relating to place value, decimals and fractions.

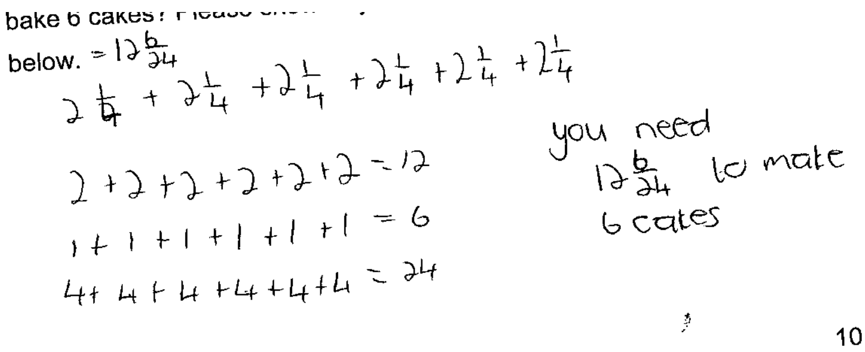


Figure 5.10 Nomsa's solution to Item 31 of the post-test

5.3.6 Liya (8Y)

Liya did not experience herself as doing well in mathematics. Her friends did not usually ask her for help and her perception was that her parents (guardians) did not think she was good at mathematics either. In responding to her opinion about the subject, she said: "I don't really like the subject at all but I think as I get to understand it the more it will be better."

Table 5.10 Data matrix summarising selection of Liya's work

Source	S	A	FS	ME	me	Comments
Cat pills	FL RL GL	PA	PC	1		"Liya went straight to multiplying without considering that 27 pills of which not even a whole one is taken per day, should at least last more than 27 days. When questioned about her answer and thought process and understanding of how many quarters there are in a whole, Liya realised her mistake and was left to try another strategy. When I returned, she presented her reworked solution, which shows a more positive attempt

Source	S	A	FS	ME	me	Comments
						at horizontal mathematisation." ***(Researcher)***
Insulin	SL	None	None			"Does Liya still take long to grasp? Or do I imagine things?" ***(Teacher)*** "…HB [researcher] returns to student L5 [Liya] who has made little progress. Student answers more confidently on this round, physically manipulating the syringe at one point to indicate her relational understanding. (After this, L5 [Liya] checks out)." ***(Assistant researcher)***
Diagnostic assessment (19)	RL	A	✔			
Diagnostic assessment (20)	GL FL	A	✔			
Overseas	GL	A	PC		1	
Ratio problem	GL	A	✔			
Standardised test	FL	NA	0/5			
Calculating mark	?	A	✔			Only answer supplied - no working out shown
Post-test						Absent
Final mainstream exam						Absent

NOTE:
S = Levels and use of strategies; A = Appropriateness of solution; FS = Correctness of final solution; ME = Major mathematical error(s); me = minor mathematical error(s)
FL = Formal level; GL = General level; RL = Referential level; SL = Situational level
A = Appropriate; PA = Partially appropriate; NA = Not appropriate; T&E = Trial and error
✔ = Correct; PC = Partially correct; ✗ = Incorrect
1/5 indicates that mark obtained for that question was one out of a possible 5 marks

Synopsis

Liya's selection of solutions to problems done in class and mainstream assessments was not as comprehensive as the other learners. This was partly due to her high absentee rate. There were also times when she was present in the class however, but unable to make any contribution, either because she was in tears for most of the lesson or on the brink of tears, so the decision was taken not to force the issue. Where she did manage to attempt some problems, her strategies can mainly be classified as referential and general. She enjoyed making her own representation of the problem and on a good day enjoyed explaining her solutions. Most of her progress was therefore contained within the second cycle. During the third cycle, she appeared to be having a particularly difficult emotional time (the guidance counsellor confirmed this) and this seemed to have an effect on her ability to work independently in trying to solve the problems. She would also not co-operate within a group during this time, and required a great deal of individual attention and questioning to get her started on solving a problem.

The available solutions from this selection that were completed by Liya, do indicate an improvement in her relational understanding, especially with regard to fractions (see Figures 4.2, 4.3, 4.9, 4.19 and 5.11). Although she mainly made use of models from the referential level in solving the problems, she was able to represent the problem in her own way and explain and justify her solutions. Her informal strategies remained rather long and cumbersome, however, and she did not progress in refining them or moving towards solutions that could be classified as formal. Her high rate of absenteeism may have affected this as well as her volatile emotional state. Guiding her was also challenging due to this as one had to try and read her mood at the start of the lesson before trying to question her on improving or explaining her solution. If it was a bad day and one showed her individual attention, she would break down in tears and this could have an effect on the dynamics of the whole class. On those days, it was therefore better to "work around" her so that the entire lesson would not be disturbed. As previously mentioned, she did not write the post-test or her final examinations at school as she spent the whole of November in a clinic receiving treatment for her depression. No results are therefore available from her final examination.

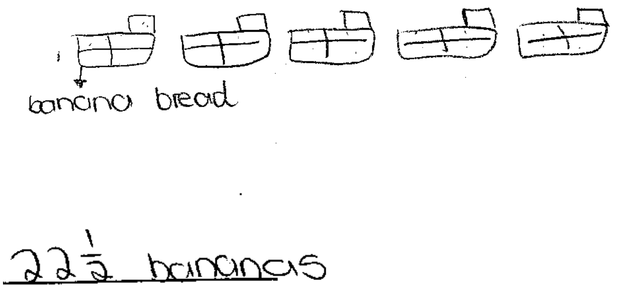


Figure 5.11 Liya's solution to Item 19 of the diagnostic assessment

5.3.7 Connie (8Y)

Connie's perception about her performance in mathematics was that she does not usually do well, that her friends do not usually ask her for help in this regard and that her parents do not think that she is good at the subject. She finds mathematics "boring, because if you like don't understand it, it takes like hours and hours to figure out one sum." She thought she might find mathematics "nice" if she understood it better.

Table 5.11 Data matrix summarising selection of Connie's work

Source	S	A	FS	ME	Me	Comments
Cat pills	FL RL GL FL	A	✔			"Connie first tries formal methods and is not satisfied with her solution. It is interesting to note how she then embarks on horizontal mathematisation by drawing the small pills and dividing them into quarters. She then splits 27 up into 10 + 10 + 7 and solves that part of the problem using three rows, which she then checks by again drawing the 27 pills in nine rows (or three columns) with three in each row." ***(Researcher)***
Insulin	FL	A	✔			
Diagnostic assessment (19)	RL GL	A	✔			
Diagnostic assessment (20)	FL	A	✔			
Overseas	GL	A	✔			"Connie still uses repetitive addition instead of multiply or divide." ***(Teacher)***
Ratio problem	FL	A	✔			
Standardised test	GL	A	5/5			
Calculating mark	RL GL	A	✔			
Post-test	RL GL	A	✔			
Final mainstream exam			P: 5/7 D: 4/4 F: 9/16			

NOTE:
S = Levels and use of strategies; A = Appropriateness of solution; FS = Correctness of final solution; ME = Major mathematical error(s); me = minor mathematical error(s)
FL = Formal level; GL = General level; RL = Referential level; SL = Situational level

A = Appropriate; PA = Partially appropriate; NA = Not appropriate; T&E = Trial and error
✔ = Correct; PC = Partially correct; ✕ = Incorrect
1/5 indicates that mark obtained for that question was one out of a possible 5 marks

Synopsis

Connie used a variety of informal and formal strategies, which in terms of the levels included the referential, general and formal levels. Towards the beginning of the intervention she would often first start off by trying to use a formal procedure. If she got stuck or realised that her solution was incorrect, she would resort back to strategies representative of the referential and general levels in order to correct or check up on her formal procedure. During the course of the second cycle, she appeared to gain more confidence in using her own informal strategies and subsequently produced solutions that were more on the referential and then general levels. She still continued to make use of formal procedures however, when she knew which one to use.

Connie's solutions to all of the problems analysed were correct. She showed an increasing ability to engage with the context of the problem by drawing or constructing her own representation of the information contained therein in order to set about solving the problem. Her tendency to revert to using strategies representative of the referential and general levels, when she was not succeeding in using a formal procedure, demonstrated her increased relational understanding with regard to the relevant key number concepts. She also began to simplify and refine her solutions as the intervention progressed, especially with regard to calculations involving decimals and repeated addition. Her performance in the mainstream mathematics exam also showed a vast improvement from her final examination she had written in Grade 8 in 2002 (for which she got 12 %) and she managed to obtain 67 % in the questions relating to number concepts in the 2003 exam.

5.3.8 Mpho (8Y)

Mpho's perception of her mathematical performance corresponded directly with Connie's; she felt that she did not usually do well, her friends did not consult her for help and that her parents did not think she was good at mathematics. Her dislike for the subject also came through quite strongly in the interview when she used the words, "maths sucks" to express her opinion. Her reasons for this attitude included the fact that she found it very difficult and because she felt that she could not "…do it as fast as most people."

Table 5.12 Data matrix summarising selection of Mpho's work

Source	S	A	FS	ME	me	Comments
Cat pills	FL GL	A	✔			"Mpho did not express the solution in words nicely even though she knew exactly what to do." ***(Teacher)***
Insulin	FL GL	A	✔			"I am worried about the way some of them pick on Mpho! It seems as if this is the only period she enjoys!" ***(Teacher)*** "Mpho starts off formally correctly using division but appears unsure of her answer. Starts doing repeated subtraction of 0,16 ml of insulin per day and labels days till she runs out of insulin." ***(Researcher)***
Diagnostic assessment (19)	RL GL	A	✔			
Diagnostic assessment (20)	GL FL	A	✔			
Overseas	FL	A	PC		2	
Ratio problem	GL	A	✔			
Standardised test	FL	NA	0/5			
Calculating mark	? (FL)	A	✔			Only answer supplied - no working out shown
Post-test	FL GL	A	✔			
Final mainstream exam			P: 4/7 D: 3/4 F: 9/16			

NOTE:
S = Levels and use of strategies; A = Appropriateness of solution; FS = Correctness of final solution; ME = Major mathematical error(s); me = minor mathematical error(s)
FL = Formal level; GL = General level; RL = Referential level; SL = Situational level
A = Appropriate; PA = Partially appropriate; NA = Not appropriate; T&E = Trial and error
✔ = Correct; PC = Partially correct; ✘ = Incorrect
1/5 indicates that mark obtained for that question was one out of a possible 5 marks

Synopsis

Mpho appeared to enjoy these classes, and according to her teacher participated far more during these lessons than during her mainstream mathematics classrooms. Her solutions showed an improvement in her ability to solve contextual problems, especially during the second and third cycles, although her determination to always do something different appeared to be to her disadvantage at times. Her motivation and determination to complete a solution also appeared to be very dependent on her mood. Some days she was more committed to finding the correct solution and then enjoyed sharing her explanation with the class. On other days, it would take a lot of encouragement and questioning to get her focused and working on the problem. When she put her mind to it, she was quite capable of engaging with the problems. The challenge was getting her to put her mind to it! The relevance of the contextual problems included in the intervention could often be judged by Mpho's inclination to either engage with them or abandon them in search of someone to distract. She achieved 59 % on the identified questions in her final Grade 8 examination.

5.3.9 Gloria (8Y)

Although Gloria thought that she did not usually do well in mathematics and had the perception that her mother also thought that she was not good at it, she was of the opinion that her friends usually asked her for help when they struggled with mathematics. During the interview she confessed to not liking the subject, explaining that it is just "…something I don't feel passionate about."

Table 5.13 Data matrix summarising selection of Gloria's work

Source	S	A	FS	ME	me	Comments
Cat pills	RL FL	A	✔			
Insulin	FL	A	✔			"I think Gloria has grown a lot in confidence but sometimes to the detriment of others." ***(Teacher)*** "L4 [Gloria] tries to explain L3's [Leratho's] solution, but cannot justify why division." ***(Assistant researcher)***
Diagnostic assessment (19)	GL	A	✔			
Diagnostic assessment (20)	FL	A	PC		1	
Overseas	FL GL	A	PC		3	"Gloria always seems to start and finish so quickly." ***(Teacher)***

Source	S	A	FS	ME	me	Comments
Ratio problem	FL	A	✔			
Standardised test	GL	A	5/5			
Calculating mark	FL	A	✔			
Post-test	GL	NA	✕		1	"Gloria used the multiplication symbol instead of the addition symbol to add the cups of flour but then does in fact add instead of multiply. Her division is then inaccurate resulting in the incorrect solution." ***(Researcher)***
Final mainstream exam			P: 5/7 D: 3/4 F: 12/16			

NOTE:
S = Levels and use of strategies; A = Appropriateness of solution; FS = Correctness of final solution; ME = Major mathematical error(s); me = minor mathematical error(s)
FL = Formal level; GL = General level; RL = Referential level; SL = Situational level
A = Appropriate; PA = Partially appropriate; NA = Not appropriate; T&E = Trial and error
✔ = Correct; PC = Partially correct; ✕ = Incorrect
1/5 indicates that mark obtained for that question was one out of a possible 5 marks

Synopsis

Gloria stuck mostly with formal procedures in her approach to the problems. She was seated in the class next to Leratho though and at times it was difficult to tell how much she was relying on Leratho to take the initiative in supplying a procedure or strategy. Gloria mostly got her solutions either correct or partially correct but was on occasion unable to explain or justify the formal procedures she had used. At times, when she working on her own, she would use a general level solution, such as repeated addition, when she could not multiply decimals for example, but she only once made use of pictures or her own representation of a problem in order to solve it.

Her attitude was often to get through the work as quickly as possible, but this did not appear to disadvantage her. However, perhaps she took the post-test is such haste that she did not demonstrate an accurate reflection of her comprehension of the concepts tested. I say this as a result of her good performance in the final examinations where she managed to achieve 74 % for the identified questions. This placed her achievement out of the 12 learners (in the final exam in those particular questions) second only to Leratho. From the selection of Gloria's work that was analysed though, it is difficult to confirm that the intervention had any effect on her above average performance in the final examination.

5.3.10 Violet (8Y)

Although Violet stated that her friends do not usually ask her for help when they struggle with mathematics, she felt that she usually fared well in the subject and that her parents also thought that she was good at it. Her opinion of mathematics was that, "Maths is really difficult but we get to have fun" and added that she felt "nice in the Maths class, but sometimes confused."

Table 5.14 Data matrix summarising selection of Violet's work

Source	S	A	FS	ME	me	Comments
Cat pills	RL GL	A	✔			
Insulin	GL	A	PC			
Diagnostic assessment (19)	RL GL	A	✔			
Diagnostic assessment (20)	GL FL	A	✔			
Overseas	FL	A	PC		2	"Violet was either determined to use multiplication or did not think to use repeated addition. She could not do the actual calculations correctly though without the use of a calculator, although she did seem to know which operation to use." ***(Researcher)***
Ratio problem	FL	A	✔			
Standardised test	GL	A	4/5		1	
Calculating mark	RL	A	✔			
Post-test	GL	A	✕	1		"Although her strategy is correct, Violet incorrectly adds the quarters resulting in an incorrect final solution. She may have benefited from checking her solution with a diagram." ***(Researcher)***
Final mainstream exam			P: 4/7 D: 2/4 F: 7/16			

NOTE:
S = Levels and use of strategies; A = Appropriateness of solution; FS = Correctness of final solution; ME = Major mathematical error(s); me = minor mathematical error(s)
FL = Formal level; GL = General level; RL = Referential level; SL = Situational level
A = Appropriate; PA = Partially appropriate; NA = Not appropriate; T&E = Trial and error
✓ = Correct; PC = Partially correct; ✗ = Incorrect
1/5 indicates that mark obtained for that question was one out of a possible 5 marks

Synopsis

From the beginning of the second cycle Violet began using her own informal strategies. She did at times require more individual attention and questioning in order to understand the problem and the context though, which may have been due to her poorer command of the English language than her peers in 8Y. Her strategies could mostly be classified as referential and general, with the occasional use of a formal strategy or part of her solution representing the formal level. Her informal solutions slowly became more refined and methodical though, especially with regard to problems involving fractions (see figures 4.6 and 4.13). The difficulty for her came, when she could not find an informal strategy and did not yet have sufficient command or understanding of the formal procedures required in solving a particular problem.

Violet appeared to demonstrate limited understanding (either instructional or relational) of place value, fractions and decimals in the initial stage of the intervention. During the course of the lessons, she began to gain more confidence in her informal strategies and often used diagrams to depict the problem for herself before starting to solve it and/or as a means to solving the problem. She readily shared her solutions with the class and was mostly able to explain and justify her informal strategies. At times the class would comment on how her informal strategy had simply been a diagrammatic representation of their formal procedure. Through this, it can be deduced that Violet did make progress in her ability to solve contextual problems regarding the relevant key number concepts as well as in her understanding of these concepts. She did not manage to make the transition from the referential and general levels to the more formal one during the time allotted to the intervention though and appeared to still demonstrate quite a number of gaps in her thinking that could have perhaps benefited from continued exposure to such an approach. In the final examination, she achieved 48 % in the identified questions, which according to her teacher (Teacher Y) was a good result for Violet. This then corresponds with her improvement as demonstrated in the pre- and post-tests.

5.3.11 Patience (8Y)

Patience felt quite strongly that she did not usually do well in mathematics and that her parents did not hold the perception that she was good at it either. She was also of the opinion that her friends seldom asked her for help when they were struggling. Like Mpho, she expressed (in no uncertain terms) her dislike of the mathematics using the phrase, "Maths sucks" and then continuing, "…you have to be clever to understand this thing! You need to sit down and think about things and there is not enough time and I can't add fractions and stuff."

Table 5.15 Data matrix summarising selection of Patience's work

Source	S	A	FS	ME	me	Comments
Cat pills	RL FL	A	✔			"Patience is a bit unwilling." ***(Teacher)***
Insulin	GL	A	PC			"Patience doesn't really work on her own. She always tries to overhear you [the researcher] and the others. She is also very conscious of the others." ***(Teacher)*** "L7 [Patience] did repeated subtraction….HB [researcher] clarifies L7's [Patience's] strategy to highlight connection [between division and subtraction]…L7 says 'division is faster'." ***(Assistant researcher)***
Diagnostic assessment (19)	GL	A	✔			
Diagnostic assessment (20)	FL	A	✔			
Overseas	FL	T&E PA	PC	1	1	"Patience was one of the two learners to first total the pounds before converting to rands, but when she tried to convert, she divided instead of multiplying." ***(Researcher)***
Ratio problem	? (FL)	A	✔			Only answer supplied - no working out shown
Standardised test	GL	A	5/5			
Calculating mark	FL	A	✔			

Post-test	FL	A	✕	1
Final mainstream exam			P: 4/7 D: 2/4 F: 12/16	

NOTE:
S = Levels and use of strategies; A = Appropriateness of solution; FS = Correctness of final solution; ME = Major mathematical error(s); me = minor mathematical error(s)
FL = Formal level; GL = General level; RL = Referential level; SL = Situational level
A = Appropriate; PA = Partially appropriate; NA = Not appropriate; T&E = Trial and error
✓ = Correct; PC = Partially correct; ✕ = Incorrect
1/5 indicates that mark obtained for that question was one out of a possible 5 marks

Synopsis

Patiences' solutions can mostly be classified as falling into the general and formal levels. On the odd occasion, she would draw diagrams, but this was usually when she was working with Mpho, who more often made use of them. She was able to justify and explain her informal procedures and they usually resulted in her solution being correct. Patiences' solutions definitely demonstrated an improvement in her ability to tackle contextual problems. As she appeared to grow in confidence throughout the intervention, her attitude and ability to persevere seemed to also improve. Through her improved explanations and written solutions, a shift from more instructional to relational understanding also became evident, regarding the key number concepts addressed. This appeared to have had a positive impact on her classroom assessments where she also at times made use of informal strategies if possible. In the final examination she managed to obtain 67% on the questions relating to number concept.

5.3.12 Leratho (8Y)

Leratho agreed that she usually did well in mathematics, although her friends did not usually seek her help when they struggled. She was also of the opinion that her parents did not think she was good at mathematics. In response to whether or not she liked the subject, she responded, "I don't really like it…cause I struggle a lot…now with the whole algebra thing."

Table 5.16 Data matrix summarising selection of Leratho's work

Source	S	A	FS	ME	me	Comments
Cat pills	FL	A	✓			"Leratho explained very well." ***(Teacher)***
Insulin	FL	A	✓			HB [researcher] has one of the students explain (L3) [Leratho] solution, which student does coherently. There seems to be a range

Source	S	A	FS	ME	me	Comments
						of ability among the 7 learners." ***(Assistant researcher)***
Diagnostic assessment (19)	FL GL	A	✔			
Diagnostic assessment (20)	GL FL	A	✔			
Overseas	FL	A	✔			"Leratho was one of the two learners that added up the pounds first before converting them to rands, and she was the only one able to multiply the decimals correctly without the use of a calculator. She did then go back and convert all the amounts first and then total them in order to check her answer." ***(Researcher)***
Ratio problem	GL	A	✔			
Standardised test	FL	A	5/5			
Calculating mark	GL	A	✔			
Post-test	GL	A	✔			
Final mainstream exam			P: 4/7 D: 4/4 F: 13/16			

NOTE:
S = Levels and use of strategies; A = Appropriateness of solution; FS = Correctness of final solution; ME = Major mathematical error(s); me = minor mathematical error(s)
FL = Formal level; GL = General level; RL = Referential level; SL = Situational level
A = Appropriate; PA = Partially appropriate; NA = Not appropriate; T&E = Trial and error
✔ = Correct; PC = Partially correct; ✖ = Incorrect
1/5 indicates that mark obtained for that question was one out of a possible 5 marks

Synopsis

The analysis of Leratho's strategies reveal an interesting pattern regarding her use of formal and informal procedures and with relation to the four levels as described by Gravemeijer (1994). Although she was not regarded as a low attainer and in the class on the request of her parents as opposed to poor performance in mathematics, she always willingly participated in the lessons and remained enthusiastic throughout the course of the intervention. Overall (and especially in the beginning) her solutions were predominantly formal and all yielded the correct solutions. As the problems relating to fractions became increasing difficult though, she started to make more use of the general level to assist her when she got stuck or if she wanted to check a solution. One of her solutions (not included in this selection - see figure 4.8 in section 4.3.2) showed strategies from all four of the levels in fact, beginning with a formal procedure (executed incorrectly), reverting to drawing diagrams for herself, then using repeated addition and finally again making use of the correct formal procedure. As she became more confident in using her own informal strategies, she would use them in conjunction with the more formal procedures and algorithms she had previously learnt.

From the analysis of Leratho's solutions, one cannot make the deduction that her formal strategies improved or that she showed an increase in her knowledge of the key number concepts addressed. I think that it would be fair though to deduce that she became more proficient at being able to estimate and control whether or not her solutions were correct, through the use of her own informal strategies. In this regard, her relational understanding in terms of solving contextual problems relating to the concepts, improved in that she was often able to identify her own mistakes and correct them. Her explanations were always clear and concise, often raising the level of solutions within the classroom discussions. Her performance in the final examination was better than the rest of the class and she achieved 78 % for the identified questions relating to the number concepts dealt with.

5.4 Conclusion

This chapter focused on the performance of learners during the course of the intervention. Results from the cognitive achievement tests were first presented followed by an individual profile of each learner regarding their strategies and solutions in solving a selection of problems done in the remedial classes as well as their performance based on assessments from their mainstream mathematics classes. A synopsis of each learner was then concluded drawing on the profile and the learner's achievement demonstrated in the achievement tests.

From this chapter it can be concluded that the intervention appeared to have different effects on the various learners. The improvement that Klokkie, Violet and Patience showed in their achievement tests was supported from data in their profiles. Mary's improvement from the pre-test to the post-test is more problematic to substantiate from the work summarised in her profile. Her difficulty to communicate in English must be taken into account though as well as the possibility that she may have a learning disability (although this could not be confirmed).

Although Zwanela and Emelie showed some progress in their performance in the achievement tests, this could not be translated into improved academic performance in their final mathematics examination. In Emelie's case, her profile does demonstrate an increased ability to solve contextual problems, although a substantial gap between the level at which she was demonstrating an improvement and the required level of Grade 8 learners exists. In my opinion, with continued remedial instruction using the RME approach, this gap could be made smaller. Zwanela on the other hand demonstrated a dependency on algorithms and an aversion to trying to understand and represent the problems in her own way throughout the intervention.

Nomsa and Liya did not write the pre- and post-tests respectively, but their profiles demonstrate an improvement in their ability to use both informal and to an extent more formal solutions in solving contextual problems, especially with regard to the concept of fractions. Nomsa's performance in her final examination also supported this, although the major mathematical error she made in adding the quarters in Item 31 of the post-test was concerning.

Gloria is a difficult case to comment on. Her performance in her cognitive achievement tests indicated a decrease in the number of items she answered correctly. However, she performed very well in the identified questions relating to the key concepts of place value, fractions and decimals in her final Grade 8 mathematics examination. Perhaps her performance in the post-test can be attributed to an arbitrary bad day.

Connie, Mpho and Leratho all appeared to show improvement in the strategies they applied in the intervention as well as a slight improvement in their cognitive achievement tests. The mixture of both horizontal and vertical mathematisation appearing in their solutions appears to indicate that they did benefit from the intervention and their above average performance in the identified questions also substantiates this.

Now that the data analyses from the intervention and the performance of learners have both been presented, some final conclusions and recommendations from the study are presented in the next and final chapter of this report.

CHAPTER SIX

6 Conclusion

6.1 Synopsis of results

This final chapter serves to draw together the research question, the process embarked on to answer it and the results, conclusions and recommendations that emerged from the study. A summary of the report is presented in 6.2, followed by a reflective discussion of the research and its results in section 6.3. The chapter closes with recommendations for policy and practice and further research and development work. Prior to the summary, a synopsis of the results in response to the main research question (which can be divided into three parts) is first offered. The research question guiding the study was:

> *How can the theory of Realistic Mathematics Education be used with low attaining learners in mathematics in order to improve their understanding with relation to the key number concepts of place value, decimals and fractions, and what characteristics emerge from implementing the intervention in such a setting?*
>
> a) *Is it viable to apply the theory of Realistic Mathematics Education in such a setting? (i.e. with* low attainers *to* revisit concepts *they had already previously learnt)*
>
> b) *How can it be used to improve learners' understanding of place value, fractions and decimals?*
>
> c) *What possible design characteristics emerge from applying the theory of RME in such a setting?*

The improved performance of the majority of learners on the cognitive achievement tests (section 5.2) indicates that *applying the theory of RME with low attaining learners to revisit the key number concepts does hold potential*. This is substantiated by the learner profiles, where again, the majority of learners showed some improvement in their strategies and a shift to more intelligent learning (section 5.3).

It appears that the use of the principle of guided reinvention played an important role in improving learners' understanding of the concepts that were revisited. This was done by *providing learners with contextual problems relating to the concepts of place value, fractions and decimals* that required them to make use of their own informal or more formal strategies in order to solve the problems. Learners were encouraged not to make use of formal procedures they did not understand though, as they would have to explain and justify their solution to others when discussing it with the class. Through this they were continually reminded *to rather try and represent and solve the problem using their own symbols and strategies*. The use of both their informal and more formal strategies in solving the contextual problems often *elicited misconceptions* (or alternative conceptions as some literature prefers to call them) that the learners held with regard to the key number concepts and their basic operations. These could then be *addressed and discussed* in an attempt to assist learners in improving their understanding (Chapter 4).

Also noticeable (sections 4.5, 4.6 & 5.3) was that some learners started to make use of informal strategies to check their solutions when they were unsure or they did not know how to continue with a more formal procedure. This provided direct evidence of the fact that some of these low attaining learners were able to make *a visible shift from dependency on the teacher to displaying more confidence in their own ability to reason.* This is not to say that all learners were able to make this shift. One of the learners clung to her use of formal procedures relentlessly and persistently searched for the "correct" algorithm or formal procedure to implement, almost always depending on the teacher for help and often growing frustrated in her attempts. However, even without her making this shift, her strategies and solutions still *elicited a number of misconceptions* with regard to her understanding of place value, fractions and decimals and the use of basic operations. One of the learners though was mostly even unable to represent the problems using her own symbols or mathematical notation, without a great deal of guidance from the teacher or another learner. Her poor command of the English language, compounded by the possibility of a learning disability, is cited as possible reasons for this. Although she did show some improvement in her performance in the cognitive achievement tests, it is difficult to say how the intervention assisted her.

A number of design characteristics that could be taken into account when developing a similar intervention, emerged from the study (Chapter 4). These are only summarised here and then elaborated on in section 6.3.3. Firstly, the fact that the learners had previously been taught the concepts that were being revisited during the course of the intervention meant that the lessons needed to be presented in a way that *would not make learners feel like they had to once again relearn work* that had been done in so many earlier grades. The RME approach therefore provided a fresh means of presenting the concepts through the use of contextual problems.

Secondly, in using the contextual problems though, one had to ensure that the context was not only meaningful to the learners (the principle of didactical phenomenology) but also that the *context was within the frame of reference of the learners*. When learners were confronted with a context that was not within their frame of reference or one which inherently required them to access a lot of new facts or knowledge, the context rather than the mathematics that should come from the problem became the focus of the lesson. Thirdly, it was found that this could be overcome though by *first familiarising the learners with the context*, if and where necessary, through the use of visual aids and discussions. Once the learners felt more a part of the context, they were able to engage with it in setting about to find a solution to the problem.

Finally, it has already been mentioned on several occasions that these learners had previously learnt the concepts being revisited, which meant that they were aware of various existing procedures (or models) that can be used to solve mathematical problems even though they may not have fully understood how or why these procedures are used. Having these tools but not knowing what they are or how to correctly use them *sometimes disempowered* these learners. They therefore needed to be continuously encouraged to *resort to horizontal mathematisation* (constituting their own informal strategies of solving problems) should they find themselves "disempowered" or caught up in searching for the "correct" algorithm.

6.2 Summary

The research documented in this report had a twofold purpose. Firstly, it was to design and implement an intervention based on the theory of Realistic Mathematics Education (RME) aimed at improving the mathematical understanding of learners in two Grade 8 remedial mathematics classes, by revisiting the key number concepts of place value, fractions and decimals. In doing so, the second purpose was to investigate the viability and emerging characteristics of an intervention based on the theory of RME in such a setting (i.e. with *low attainers* to *revisit key number concepts*). Pending the realisation of these immediate outcomes, more distant outcomes in subsequent research would be: that learners' understanding and academic performance in mathematics improves and to develop a local instruction theory in using the RME theory to revisit the concepts of place value, fractions and decimals with low attaining learners in order to improve their understanding in this regard (see 3.2.3).

Grade 8 low attainers were selected as the target group for this research as a result of the pending implementation of Mathematical Literacy as a compulsory subject for all learners, possibly from 2006. Currently in South Africa, learners who are not meeting the required standard by the end of their Grade 9 year are able to elect not to take mathematics through Grades 10, 11 and 12. When the new Further Education and Training (FET) policy is implemented, this will no longer be the case. All learners, who do not elect to take mathematics as a subject, will have to take Mathematical Literacy as a compulsory subject throughout Grades 10, 11 and 12. Although less detailed and abstract than the subject mathematics, the Mathematical Literacy curriculum still requires learners to have an understanding of key number concepts and also contains a substantial amount of algebra. As Grade 8 is when learners start working with algebra more formally, and is also their first year at secondary school, it was decided that this would be an appropriate year to try and diagnose and remediate problems in learners' understanding of the key number concepts, if and where possible. The intention was that this would then equip learners with a more appropriate structure of conceptualised knowledge of the above-mentioned concepts on which they could further construct their understanding of algebra.

Literature pertaining to the teaching and learning of mathematics to *low attaining learners* (other search terms included learners with *mathematical difficulties, learning disabilities, low achievers* and *Special Educational Needs [SEN]*), was first reviewed in order to identify common or recurring aspects from within the literature that could be included in the instructional approach of the intervention (see Chapter 2). The five aspects set apart were: m*ore focus on relational and conceptual understanding* as opposed to rote learning and memorisation (instrumental understanding), c*reating meaningful learning contexts* that actively involve learners, g*reater emphasis on problem-solving* and less emphasis on computation and arithmetic skills, the *importance of social interaction* in the learning process (i.e. group work, reciprocal teaching, games etc.) and the *importance of language development and discussion* with and between learners in teaching mathematics. Once these aspects had been singled out, the theory of Realistic Mathematics Education (RME) was chosen as the theoretical framework to guide the design and implementation of the intervention.

The study was conducted at a local urban high school and implemented with two remedial mathematics classes consisting of five and seven learners respectively. These learners received an additional (the school called it remedial) three mathematics lessons in every ten-day cycle, whilst their classmates were attending lessons in which they learnt a third language. Usually it was the mainstream mathematics teacher of each class that taught the learners attending the remedial classes, but for the purpose of this study, I took over the role of teaching the two selected classes and the teachers agreed to act as observers for the duration of the study. The sample classes were selected by the request of their teachers to be involved in the research. The intervention was implemented over nine months, which spanned the second, third and fourth school terms of 2003. Once holidays, examinations, sports days and timetable changes and disruptions were taken into consideration, each of the two classes (8X and 8Y) only received approximately 24 lessons in total throughout that time.

Two development research approaches informed this study, both originating from The Netherlands. The first approach is one that stems from the work of van den Akker and Plomp (1993) from the University of Twente, in Enschede. The second approach drawn on is the developmental research approach contained within the theory of RME (Freudenthal, 1991; Gravemeijer, 1994) and has its origins in the Freudenthal Institute in Utrecht. An explanation of how these approaches were combined in this study was presented in Chapter 3 in sections 3.2.2 - 3.2.4. The first approach was used to inform the curriculum design process that guided the development of this first implementation of the intervention in an attempt to establish an initial prototype pending the outcome of the study. It also guided the process of the three smaller cycles contained within the study that were differentiated by the three school terms as well as by the implementation of different design principles of RME (see Chapter 4). The second approach is contained more specifically within the context of instructional practice in mathematics education and is more focused on the individual instructional activities and, once implemented, how they work out in relation to the way they were planned. So while the second approach was informing the implementation and revision of the instructional activities from lesson to lesson, the first approach was used to reflect on, evaluate and revise the series of lessons in each of the three smaller cycles (in terms 2, 3 and 4) and the implementation of the intervention as a whole.

The *design hypothesis* (see 3.2.2 & 3.2.4) of the research was that by designing an intervention based on the instructional design principles of RME, i.e. the principle of *guided reinvention*, the principle of *didactical phenomenology* and the principle of *emergent or self-developed* models, to revisit the key number concepts of place value, fractions and decimals, would assist learners in improving their understanding. The RME instructional approach encourages the use of contextual problems where learners are presented with meaningful contexts that afford them the opportunity to make use of their own informal as well as more formal strategies in finding a solution. When learners make use of their own symbols or drawings to represent and solve a problem, this is known as *horizontal mathematisation*. Through a process of solving similar problems, the learners are guided to refine and formalise their informal solutions, resulting in what is referred to as *vertical mathematisation* (Treffers, 1987). The *intervention hypothesis* (see 3.2.2 and 3.2.4) was therefore that by implementing the intervention, based on the design principles and instructional approach of RME, that learners would be given the opportunity to gain confidence in first utilising their own informal strategies in solving problems (relating to the concepts of place value, fractions and decimals). They could then be taken through a process of formalising these strategies into the more formal procedures they had previously been taught, but did not necessarily understand. It was also hoped that through this process learners would start engaging more in intelligent as opposed to habit learning. Habit learning tends to render a learner dependent on the teacher for guidance in each new situation whereas intelligent learning develops the learner's confidence in her own ability, therefore drawing on the teacher as a resource if necessary (Skemp, 1989).

As mentioned, the research design of this study was informed by two development research approaches (van den Akker & Plomp, 1993; Gravemeijer, 1994). Also, the study was only implemented with a small number of participants, within a bounded setting and without the intention to generalise the results. It was therefore regarded as a development *case study*. A *fully integrated mixed methodology* (Tashakkori & Teddlie, 2003) was applied because the conceptualisation, method and inference stages of the research process all drew on what would traditionally be classified as both qualitative and quantitative approaches (see Chapter 3). The data collection included: document analysis (of personal files and work samples of learners), cognitive achievement tests, assessments from learners' mainstream mathematics classes, interviews, observations and field notes. Both inductive and deductive approaches were used to analyse the data, in the form of latent and manifest content analyses respectively.

An account of the actual intervention was then given (Chapter 4), substantiated by data that had been collected and analysed during the fieldwork. From further data analyses a learner profile for each learner was compiled and presented in the form of a data matrix (Chapter 5).

Learner performance was also indicated and compared on the basis of the quantitative analysis of the cognitive achievement tests. Conclusions were then drawn from these analyses.

It therefore appears to be viable to apply the theory of RME with low attaining Grade 8 learners in order to revisit the key number concepts of place value, fractions and decimals. Through the use of meaningful contextual problems, a fresh approach to understanding these concepts was presented to learners and some were able to start using their own informal strategies to represent and solve the problems. This in turn often revealed errors or misconceptions of learners regarding the relevant concepts, which could then be addressed. Through discussions and having to explain and justify their solutions, learners were encouraged to avoid making use of formal procedures they had previously learnt but did not understand. In some cases the previous knowledge that learners had, "disempowered" them in that they remained determined to search for the correct tool or algorithm instead of attempting their own informal strategies. In other cases, however, learners increasingly gained confidence in their own informal strategies and began to formalise these through a process of solving similar contextual problems. They were therefore able to make use of both horizontal and vertical mathematisation, which places them in a stronger position to employ intelligent as opposed to habit learning. From this report, my conclusion is therefore that it will be feasible to develop a prototype of an intervention on revisiting the concepts of place value, fractions and decimals with low attainers, and four design characteristics to take into consideration in that process were also generated from this study. These conclusions are now further discussed and reflected on in the following section.

6.3 Discussion

The previous sections presented a synopsis of the results with regard to the three sub-sections that make up the main research question and provided a summary of this report. This section reflects on the study as a whole and discusses the process and outcomes of the research. In section 6.3.1 the methodology is first critically discussed and reflected on. Section 6.3.2 then examines the results of this study in relation to other research in this area. Finally section 6.3.3 is a scientific reflection on the benefits of this research and what has been learnt with respect to implementing RME in such a setting in further research.

6.3.1 Methodological reflection

The research approach used in this study was labelled as a *development case study*. The intervention was only implemented with a small sample of learners (n = 12) in order to investigate if and how RME could be used in an intervention to improve the understanding of low attaining learners with regard to place value, fractions and decimals. Although the results suggest that it is viable to apply RME in such a setting, how applying RME in an intervention can improve learners' understanding (see 6.3.2) and characteristics that emerge from applying it in this way (see 6.3.3) are not generalisable. Subsequent iterative cycles would need to be embarked on in different contexts before actual knowledge or theory in this regard can be generated (see 6.4.2).

The two development research approaches that informed the study both had their origins in The Netherlands and complemented each other in the role that they each played within the research. The approach of van den Akker and Plomp (1993) created the outer framework of the research in that it provided the structure for the front-end analysis, the design and intervention hypotheses and the immediate outcomes (i.e. the input-output process). The other more domain-specific approach contained within the theory of RME (Freudenthal, 1991; Gravemeijer, 1994) guided the development and implementation of the intervention, which was the core part of the framework. Through a cyclical process of instructional activities, thought experiments (learning trajectories planning and anticipating what would take place during the lesson) were translated into instructional experiments (actually implementing the thought experiment to ascertain the actual learning trajectory in relation to the planned one). These in turn informed subsequent thought experiments and so the intervention evolved. The data and subsequent analyses obtained from a combination of these two approaches have provided evidence that suggests that developing a prototype for further implementation in a similar setting is worth pursuing. Characteristics to guide such a development have also been generated. It would therefore be useful to continuing using this conceptual framework in further research in this regard. The outer framework provided by the first approach (van den Akker & Plomp, 1993) could be retained as is, while the core (the intervention made up of instructional activities) will be revised into an instructional sequence (Gravemeijer, 1994) in the form of a prototype.

The results suggest though that instead of trying to cover a few instructional activities on each of the three concepts of place value, fractions and decimals, it would be more beneficial to learners' understanding to rather develop more instructional activities on each concept and if time does not permit, to rather deal with fewer concepts. Alternatively, more lessons are needed to adequately deal with all three concepts to allow learners time to gain confidence in trying their informal strategies and then refining these to the more formal procedures they are already aware of. It would also be optimum if lessons could be presented (and therefore also revised) on a daily basis but such a situation is probably not likely to be the case where schools have put structures in place to assist low attaining learners. Learners would probably need to be removed from their mainstream mathematics classes, for such a condition to be met, and this in turn may have a negative impact on their self-esteem and would also affect their exposure to the actual Grade 8 curriculum they should be learning at the time.

Another methodological reflection relates to my three roles in the study: as developer, researcher and presenter (teacher). In the actual implementation of the intervention, this did cause some conflict in terms of my role as an observer (being an aspect of my role as researcher). It was often difficult (mostly almost impossible) to take notes when important moments occurred. Logs were therefore written directly after lessons where possible but sometimes lacked details that might have been included if I had been able to make notes during the lessons. I did have the observations of the teachers and the assistant researcher to work from on occasion. As they were not very familiar with the theory of RME though, they would sometimes focus on different moments and occurrences (and Teacher Y sometimes wrote only a few sentences, as she then got involved in the lessons). This also had a positive side to it though, in that their comments often supported the fact that the RME principles and instructional approach were being implemented, even though they did not directly know what they were looking for (except during the third cycle when the observations where highly structured). Although the scope (and budget) of this research did not allow for it, a future consideration in dealing with such an issue would be to videotape the classes so that during the lesson I could focus on being the teacher and later view the video as a researcher.

Also pertaining to the multiple roles, the possibility of bias was dealt with in this study through the use of triangulation. *Data triangulation* was present in the form of more than one data source (e.g. observations, interviews and work samples) being used to draw conclusions. The use of teachers as observers and at times using an assistant researcher ensured the implementation of *investigator triangulation* where more than one observer is required. Finally *methodological triangulation* was implicit to the study through the mixed methodology that the study applied.

6.3.2 Substantive reflection

Results of this study showed that using the theory of RME to revisit the concepts of place value, fractions and decimals to improve the understanding of low attaining Grade 8 learners in this regard, is a viable option to further investigate in subsequent research. Analyses of the intervention (see Chapter 4) and the learner profiles (see Chapter 5) indicate that one of the ways suggesting how this was done, was through the eliciting and addressing of learners' misconceptions. Through both the use of informal and formal strategies in solving contextual problems, mathematical errors and misconceptions held by learners pertaining to the relevant concepts were often demonstrated and could be used as departure points to further explore and improve learners' understanding in that regard, through questioning and guided reinvention.

Through the use of horizontal and vertical mathematisation, some learners became equipped during the course of the intervention to check their own strategies in an attempt to identify any errors or misconceptions or to make use of horizontal mathematisation when they were unsure of how to continue with formal procedures. When misconceptions arose, learners could make use of the elements of horizontal mathematisation to reconstruct, experiment and reclassify their knowledge (if and where necessary) as a bridge to then take them to the vertical components. The horizontal mathematisation allowed them repeated opportunities to transform problem fields into mathematical problems and the vertical mathematisation provided them with practice in processing within the mathematical system (Treffers, 1987).

As this conclusion relating to misconceptions and errors started emerging from within this research, the literature was once again reviewed with a more specific focus on this aspect, also with respect to low attainers. The theme of errors and misconceptions is prevalent in much of the literature pertaining to low attainers. A brief review on this is therefore now discussed and a reflection on how eliciting and addressing errors and misconceptions may have helped improve these learners' understanding is provided, through the lens of existing literature.

Before continuing though, let me first clarify my use of the concepts *error* and *misconception*. In order to do so I draw on the distinction that Olivier (1989) makes between slips, errors and misconceptions. He views *slips* as "wrong answers due to processing; they are not systematic, but are sporadically carelessly made by both experts and novices; they are not easily detected and are spontaneously corrected" (pp. 6,7). When referring to *errors*, the emphases are on the fact that they are systematic, applied regularly in the same circumstances and occur due to planning (as opposed to processing). Errors are caused by underlying conceptual structures (beliefs and principles in the cognitive structure) and surface as symptoms. These systematic conceptual errors are regarded as *misconceptions*. Smith, diSessa and Rochelle (1993) also support this understanding through their use of the term *misconception* to "designate a student conception that produces a systematic pattern of errors" (p. 10).

Different learning theories and perspectives hold diverse views on the role of errors and misconceptions in the process of teaching and learning. Not all see them as useful or important. In some instances, errors are regarded as an indication of failure and as mistakes that impede learning. Others suggest that errors serve as a major source of information (Dockrell & McShane, 1992) that can be used as motivational devices and starting points for mathematical explorations (Borassi & Rafaella, 1987). It is the latter view to which I subscribe within this reflection and on which I base the importance of the conclusion that misconceptions and errors were revealed and could subsequently be utilised through the use of the RME instructional approach.

Two important learning theories relating to mathematics education are behaviourism (e.g. Thorndike, 1913; Gagné, 1965 as cited in Olivier, 1989) and constructivism (e.g. Piaget, 1952, as cited in Olivier, 1989; Skemp, 1971; Cobb, Wood & Yackel, 1991). In a paper entitled "Handling Misconceptions", Olivier (1989) outlines how the theory we adopt determines how we handle errors and misconceptions in the teaching and learning of mathematics. Behaviourism assumes that knowledge can be transferred from one person to another and that learners therefore learn all of or at least part of what they are taught. The learners' current concepts are not considered as relevant to learning and so from a behaviourist perspective, errors and misconceptions are not important (Olivier, 1989) - they can be erased or replaced by teaching the correct rules. The following quote from Gagné (1983, p. 15), cited by Olivier (1989) substantiates this:

> *The effects of incorrect rules of computation, as exhibited in faulty performance, can most readily be overcome by deliberate teaching of correct rules...This means that teachers would best ignore the incorrect performances and set about as directly as possible teaching the rules for correct ones.*

A constructivist perspective on learning views the learner as an active participant in the construction of his own knowledge and ascertains that learning involves the interpretation, organisation and structuring of this knowledge into a scheme of interrelated concepts (Olivier, 1989; Smith et al, 1993). Misconceptions therefore play a vital role from this perspective as they form part of a learners' conceptual structure (scheme) and in turn interact with new concepts and influence new learning (Olivier, 1989). It is from this perspective that Borassi and Rafella (1987) suggest that misconceptions can be used as motivational devices and to foster a deeper and more complete understanding of mathematical content and nature of mathematics itself. Reusser (2000) refers to the errors that learners make as "windows into their mathematical thinking" (p. 21) and Resnick (1984) argues the importance of addressing conceptual misunderstandings (as she refers to them) during instruction. The RME instructional approach is aligned with a constructivist perspective (Gravemeijer, 1994) and it therefore makes sense that, misconceptions should play a vital role in improving learners' understanding.

The literature pertaining specifically to learners with learning disabilities or low attaining learners appears to indicate that these learners on average demonstrate a greater percentage of systematic errors (misconceptions), than higher achieving learners (e.g. Cox, 1975; Richardson, Arthur; O'Brien, Peter, 1987; Woodward & Howard, 1994). Analysis of the error patterns revealed that many of the errors occurred due to limited conceptual understanding of the algorithms and strategies taught to learners. Resnick and Ford (1981) report on a study conducted by Lankford (1972) where he selected and compared computing strategies used by a group of learners classified into "good computers" and "poor computers". While the "good" group were able to correctly execute the taught algorithms and keep track of their steps, the "poor" group appeared to have trouble remembering the conventional algorithms and subsequently "improvised" procedures. These often resulted in errors, which Lankford concluded seem to reflect "fundamental misunderstandings of procedures rather than random mistakes in carrying out basically correct procedures" (as cited in Resnick & Ford, p. 86).
The gist of the paragraph above is that low attainers make more systematic errors (demonstrate more misconceptions) than higher achievers because of their limited conceptual

understanding of the required content. This leads me to assume that if misconceptions of learners are revealed and addressed (by trying to find their origins and working with them to enable the learner to include the correct understanding in their conceptual structure), that their understanding may increase in which case they will also demonstrate fewer systematic errors. In this way, misconceptions become a possible resource through which the understanding of learners can potentially be improved. The results of this study show that the theory of Realistic Mathematics Education appears to be a vehicle through which this process was facilitated.

6.3.3 Scientific reflection

As mentioned in the summary in section 6.2, four design characteristics relating to apply RME in the setting described in this study emerged. These are once again listed and then individually discussed below:

- Concepts need to be revisited in a way that does not make learners feel like they are simply relearning work they have often done in earlier grades.
- The context of the contextual problem should be within the frame of reference of each learner.
- If and where necessary, learners should be familiarised with the context through the use of visual aids and discussions before they are required to engage in it.
- Low attainers may be disempowered by their current knowledge of formal procedures and should continuously be encouraged to resort to horizontal mathematisation.

Through the use of contextual problems, RME provided a fresh instructional approach to the concepts of place value, fractions and decimals that did not result in learners feeling like they were relearning work previously done in most of their grades throughout primary school. Learners never complained that they had done the work before and mostly saw the contextual problems as challenging and meaningful. Observers' comments, shared in Chapters 4 and 5, indicate that when this was the case for learners, then the work also "seemed to make more sense to them" (quote from Teacher X; section 4.4.3)

An experience of where the context of the problem was not within the frame of reference of learners was discussed in Chapter 4 in section 4.6.2. Learners had no idea where in the world

The Netherlands or England are and the concept of the different currencies also intrigued them. The context provided so much new information for them that it was almost as if they could not "get into" it to start engaging with it. This meant that the intended mathematics that should have emerged from that thought experiment was not realised. Learners got all caught up with (but not inside of) the context and therefore never started solving the intended problem but rather starting asking their own questions about the context.

To overcome this when the lesson was presented to the next class, and also in subsequent situations where it was anticipated that the context would be "overwhelming", visual aids were taken along and a lengthy discussion was first held to make learners more familiar with the context. This appeared to shorten their transition time from viewing the context from the outside to stepping inside and engaging with it, thereby getting to the intended mathematics quicker.

The final emerging design characteristic also deals with the fact that learners have previously learnt the concepts that were covered during the course of the intervention. The metaphor of a toolbox can perhaps be used here to illustrate the position that most of the learners in this study found themselves in. Learners who are discovering and constructing an understanding of concepts for the first time through the theory of RME can be viewed as workers who set about trying to fix things (attempting various problems) and in the process start developing their own tools (procedures/models) which they accumulate and refine throughout the process to use in their "fixing". However, the learners in this study have been given or acquired tools over the years which most of them have religiously put away in their toolbox whether they knew what it was used for or not (or perhaps at the time could use it but have since forgotten) and initially spent a lot of time in these instructional activities grasping around in their toolbox trying out some of the tools in an attempt to "fix" the problem. Having the tools but not knowing what they are or how to correctly use them *sometimes disempowered* these learners. They therefore needed to be continuously encouraged to *resort to horizontal mathematisation* should they find themselves "disempowered" or caught up in searching for the "correct" algorithm.

6.4 Recommendations

This final section of the report presents recommendations that could be considered, firstly in policy and practice relating to the teaching and learning of mathematics to low attainers, and secondly with respect to further research and development work that may be carried out in this regard.

6.4.1 Policy and practice

As no policy is yet in place in South Africa to assist low attaining learners in Grades 8 and 9 who will need to continue with Mathematical Literacy until the end of Grade 12, the Department of Education could consider designing and developing such an intervention, that could be sent out to schools coupled with a training course that teachers, who would be implementing the intervention, could undergo.

Currently the media has been launching a number of television broadcasts to assist learners with their schoolwork. This type of intervention could perhaps be considered for such broadcasts. Contextual problems could be presented both verbally and visually (to assist second language speakers) while a small studio audience of learners could be involved who share their strategies with other viewers and encourage them to phone or write in and share their ideas.

Such broadcasts or training programmes on implementing RME could also assist teachers in giving them new ideas for their instructional approaches. The RME instructional approach requires the learner to be an active participant in the learning process by participating in discussions, sharing ideas and formulating informal and formal mathematics as a means to understanding the necessary concepts and procedures. In this way, learners also take more responsibility for their learning, while the role of the teacher is to provide guidance and support. This is very much in line with the principles of outcomes based education where, according to our curriculum statement, "the outcomes encourage a learner-centred and activity-based approach to education" (DoE, 2002) and the teacher is encouraged to take on the role of facilitator, rather than a traditional "chalk and talk" stance in front of the class.

6.4.2 Further research and development work

From the results of this report that demonstrate the potential of revisiting the key number concepts of place value, fractions and decimals with low attaining learners, an initial prototype can now be constructed. The subsequent prototype could then be implemented in different contexts (i.e. with male learners, in different socio-economic areas, etc.) and with teachers as presenters, so that the quality of the prototype (in terms of its validity, practicality and effectiveness, Nieveen, 1999) could be further refined and improved on. Through the data that is collected in subsequent iterative cycles, additional design characteristics could also be generated and supported by empirical evidence, eventually resulting in *a local instruction theory on applying RME to revisit concepts with low attainers, to improve their understanding*.

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APPENDIX A - COGNITIVE ACHIEVEMENT TESTS

- Framework for the cognitive achievement tests
- Content analysis of the cognitive achievement tests
- Copy of the cognitive achievement tests
- Frequencies for each item scored by learners on the pre- and post-tests

Cognitive achievement test framework

Categories:		Number and Algebra
Focus of cognitive domains:		Knowing and Using concepts
Ratio of closed to open ended	≈	2:1
Ratio of number to algebra	≈	2:1
Length of instrument:		40 minutes
Number of items:		30
Total score:		30

Breakdown of content

Number

- Calculations using four basic operations
  - addition
  - subtraction
  - multiplication
  - division
  - terminology that indicates calculations
- Place value
  - numbers to words
  - words to numbers
  - rounding off to nearest tens, hundreds and thousands
  - understanding
- Number patterns
  - complete
  - identify
  - generalise
- Fractions
  - terminology and notation
  - four operations
  - ordering
  - conversion to decimals
  - simplification
- Decimals
  - rounding off to nearest whole number and one, two and three decimal places
  - ordering
  - four operations
- Contextual problems

- involving arithmetic
- reasoning, interpreting and decision making

- Integers
  - understanding of place value
  - four operations
  - application

Algebra

- Terminology and notation
  - variable
  - more than, less than, a certain number
  - increase, decrease, is greater than, is smaller than etc
  - exponential notation
- Generating mathematical expressions from language sentences
  - translations from sentences to mathematical expressions
- Calculations involving algebraic expressions with whole numbers
  - addition
  - subtraction
  - multiplication
  - division
- Calculations involving algebraic expressions with integers
  - addition
  - subtraction
  - multiplication
  - division
- Simplifying algebraic expressions
  - distributive law
  - collecting like terms
- Solving simple equations by trial and error

Cognitive domains

A. Knowing facts and procedures: ≈ 50%

B. Using concepts: ≈ 30%

C. Solving routine problems: ≈ 15%

D. Reasoning: ≈ 5%

Content analysis of cognitive achievement tests

Item	Multiple choice (MC) Short Answer (SA) Explanation (E)	Score	Language/ Non language	Understanding Notation	Cognitive domain	Category Number-n Algebra-a	Topic	Specific
1	MC	1	NL	N	B	n	Place value	Words to number
2	MC	1	NL	N	B	n	Place value	Number to words
3	MC	1	NL	N	B	n	Place value	Decimal - words to number
4	MC	1	L	-	A	n	Rounding off	To nearest hundred
5	MC	1	NL	-	A	n	Operations	Subtraction
6	MC	1	NL	-	B	n	Fractions	Ordering
7	MC	1	NL	-	C	n	Rounding off	Complex procedure
8	MC	1	NL	N	A	n	Fractions	Terminology/Notation
9	MC	1	NL	N	A	n	Fractions	Terminology/Notation
10	MC	1	NL	N	B	n	Place value	Decimal - words to number
11	MC	1	NL	N	A	n	Place value	Decimal - number to words
12	MC	1	NL	N	B	n	Fractions	Conversion to decimals
13	SA	1	NL	-	A	n	Fractions	Division
14	SA	1	NL	N	B	n	Fractions	Notation
15	MC	1	NL	N	A	a	Algebra notation	Notation
16	MC	1	NL	N	A	a	Algebra notation	Notation
17	SA	2	L	-	C	n	Contextual problem	Integers
18	SA	2	NL	N	A	a	Simplification	Two like terms
19	SA	2	NL	N	A	a	Simplification	Three like terms
20	MC	1	L	-	A	n	Operations	Division-complex procedure
21	MC	1	NL	-	A	n	Integers	Subtraction of integers
22	SA	2	NL	N	A	a	Simplification	Two like terms
23	SA	2	NL	N	B	n	Fractions	Order and place value
24	MC	1	NL	-	D	n	Patterns	Square numbers
25	SA	2	NL	N	A	a	Simplification	Like terms - Integers
26	MC	1	NL	-	B	n	Integers	Order
27	E	3	L	-	A	n	Rounding off	Contextual problem - estimation
28	SA	2	NL	N	A	a	Simplification	Multiplying factors
29	SA	2	L	N	D	n	Contextual problem	Addition of decimals
30	MC	1	NL	N	C	a	Equations	Trial and error
	19 MC 10 SA 1 E	40	5 L 25 NL		23 A (58%) 10 B (25%) 6 C (15%) 1 D (2%)	22 n 8 a		

Cognitive achievement test

[1]Learner's name and surname: ____________

Class: _____________

Mathematics concept test

[1] Some items taken from the released items of TIMSS 1995 and 1999 as well as from ColorMathPink.com website

General instructions

This test is designed to help your instructor and your teacher to evaluate your needs regarding your mathematics work. The test will not affect your marks in this class in any way. However, it is very important that you take it seriously and do your best.

NO CALCULATORS MAY BE USED.

There are a series of questions in Mathematics that you are required to answer.
You have 40 minutes to answer these questions.

Some are multiple choice questions and for these you are requested to circle ONE correct answer. If you decide to change an answer to a question, put an "X" over your first choice and then put a circle over the correct answer.

For other questions you will be asked to write short answers in the space provided below the question. For these questions, you may use words, drawings and numbers in your answers.

You may use the extra space on the page to do your work. Please show all your working out on the test. When an answer line is provided, place your final answer on the line.

1. Which one of the following numbers represents:

Five hundred thousand, four hundred and ninety two

A. 50 040 092
B. 5 492
C. 5 004 092
D. 500 492

---

2. Which of the following words represents:

1 086 003

A. One hundred and eighty six thousand, and three
B. One million eight hundred and sixty thousand and three
C. One million, eighty six thousand and three
D. One hundred thousand, eight hundred and sixty three

---

3. Which number is two hundred and six and nine-tenths?

A. 206,09
B. 206,9
C. 206,910
D. 2006,9

---

4. A company produced 17 175 cars in 1998. For a report, this number was rounded off to the nearest hundred. Which was the number of cars given in the report?

A. 17 000
B. 17 100
C. 17 200
D. 17 270

---

5. Subtract:

  7 004
– 4 078

A. 3 034
B. 2 926
C. 3 006
D. 3 926

---

6. Which one of these fractions is the smallest?

A. $\frac{1}{6}$

B. $\frac{2}{3}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$

---

7. The sum of 497 + 304 is closest to the sum of:

A. 400 + 300

B. 500 + 300

C. 400 + 400

D. 500 + 400

---

8. In the fraction $\frac{3}{4}$, what number represents the number of parts the whole is divided into?

A. 1

B. 3

C. 4

D. 7

---

9. In the fraction $\frac{7}{8}$, what is the numerator?

A. 7

B. 8

C. 15

D. 1

---

10. How do you write thirty-two hundredths?

A. 320

B. 3,2

C. 0,32

D. 0,032

---

11. What is 0,01?

A. One

B. One tenth

C. One hundredth

D. One thousandth

12. Write $\frac{3}{5}$ as a decimal:

A. 0,3

B. 0,8

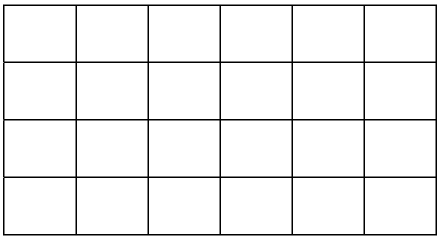
C. 0,5

D. 0,6

13. $\frac{8}{35} \div \frac{4}{5} =$

Answer: ______________________

14. Shade in $\frac{3}{8}$ of the unit squares in the grid.



15. Which of these expressions is equivalent to n x n x n for all values of n.

A. $\frac{n}{3}$

B. $n + 3$

C. $3n$

D. n^3

16. For all numbers k,

$k + k + k + k + k$ can be written as:

A. $k + 5$

B. $5k$

C. k^5

D. $5(k + 1)$

17. If you owe your mother R30 and you then pay her back R10 of that, how much do you still owe her?

Answer: ______________________________

18. Simplify the following expression:

$2x + 3x$

Answer: ___________________________

---

19. Simplify the following expression:

$x + 4x - 2x$

Answer: ________________________

---

20. What is the remainder if 87 is divided by 7?

A. 12
B. 7
C. 0
D. 3

---

21. Calculate:

$-6 - 8 =$

A. 14
B. – 14
C. 2
D. – 2

---

22. Simplify:

$3x^3 + 6x^3 =$

Answer: ___________________

---

23. Write down any fraction smaller than a half.

Answer: ____________________

---

24. Which sequence below continues the following pattern correctly:

1 ; 4 ; 9 ; 16 ; …………………………..

A. 20 ; 24 ; 28
B. 25 ; 30 ; 35
C. 25 ; 36 ; 49
D. 19 ; 26 ; 34

---

25. Simplify the following expression:

$-3x + 5x$

Answer: ________________________

---

26. – 8 is greater than:

A. – 10
B. – 4
C. – 7
D. 8

---

27. Tebogo wants to record 5 songs on tape. The length of time each song plays for is shown in the table:

Song	Amount of Time
1	2 minutes 41 seconds
2	3 minutes 10 seconds
3	2 minutes 51 seconds
4	3 minutes
5	3 minutes 32 seconds

ESTIMATE to the nearest minute the total time taken for all five songs to play and explain how this estimate was made.

Estimate: ________________

Explain:

---

28. Multiply:

$3y \times 5y =$

Answer: ______________________

---

29. A chemist mixes 3,75 millilitres of solution A with 5,265 millilitres of solution B to form a new solution. How many millilitres does this new solution contain?

Answer: ________________________

---

30. In order to make the following equation true,

$3x + 2 = 14$

the value of the x must be:

A. 14

B. 0

C. – 4

D. 4

---

31. A recipe for making a cake requires that you put $2\frac{1}{4}$ cups of flour in to make 1 cake. How many cups of flour will you need to add if you want to bake 6 cakes? Please show all your working out in the space provided below.

Frequency of learners who answered items correctly

(n = 11)

Item number	Pre-test	Post-test
1	7	9
2	8	9
3	6	5
4	5	10
5	5	6
6	6	6
7	9	11
8	5	7
9	7	11
10	4	5
11	5	6
12	3	4
13	5	9
14	2	7
15	6	7
16	4	8
17	11	11
18	7	8
19	2	8
20	7	8
21	5	6
22	4	1
23	9	8
24	5	7
25	2	4
26	3	6
27	5	5
28	1	3
29	6	9
30	8	8

APPENDIX B - DOCUMENTS FROM THE SITE

- Outline of times and dates of lessons
-
- Standardised assessment
-
- Final examination

Outline of times and dates of lessons

8X

Day 3 : lesson 2 (08:35 – 9:15)

Day 6 : lesson 4 (09:55 – 10:35)

Day 10: lesson 3 (09:15 – 09:55)

8Y

Day 1 : lesson 7 (12:30 – 13:10)

Day 5 : lesson 5 (11:00 – 11:40)

Day 9 : lesson 2 (08:35 – 09:15)

Term 2

APRIL

Date	Day	PHSG day	Lesson	Time	Class
16 April	Wednesday	5	5	11:00	8Y
17 April	Thursday	6	4	09:55	8X
24 April	Thursday	9	2	08:35	8Y
25 April	Friday	10	3	09:15	8X
29 April	Tuesday	1	7	12:30	8Y

MAY

Date	Day	PHSG day	Lesson	Time	Class
2 May	Friday	3	2	08:35	8X
6 May	Tuesday	5	5	11:00	8Y
7 May	Wednesday	6	4	09:55	8X
12 May	Monday	9	2	08:35	8Y
13 May	Tuesday	10	3	09:15	8X
14 May	Wednesday	1	7	12:30	8Y
16 May	Friday	3	2	08:35	8X
20 May	Tuesday	5	5	11:00	8Y
21 May	Wednesday	6	4	09:55	8X
26 May	Monday	9	2	08:35	8Y
27 May	Tuesday	10	3	09:15	8X
28 May	Wednesday	1	7	12:30	8Y

Term 3

JULY

Date	Day	PHSG day	Lesson	Time	Class
21 July	Monday	10	3	09:15	8X
23 July	Wednesday	1	7	12:30	8Y
25 July	Friday	3	3	08:35	8X
29 July	Tuesday	5	5	11:00	8Y
30 July	Wednesday	6	4	09:55	8X

AUGUST

Date	Day	PHSG day	Lesson	Time	Class
4 August	Monday	9	2	08:35	8Y
5 August	Tuesday	10	3	09:15	8X
6 August	Wednesday	1	7	12:30	8Y
8 August	Friday	3	2	08:35	8X

12 August	Tuesday	5	5	11:00	8Y
13 August	Wednesday	6	4	09:55	8X
18 August	Monday	9	2	08:35	8Y
19 August	Tuesday	10	3	09:15	8X
20 August	Wednesday	1	7	12:30	8Y
22 August	Friday	3	2	08:35	8X
26 August	Tuesday	5	5	11:00	8Y
27 August	Wednesday	6	4	09:55	8X

SEPTEMBER

Date	Day	PHSG day	Lesson	Time	Class
1 Sept	Monday	9	2	08:35	8Y
2 Sept	Tuesday	10	3	09:15	8X
3 Sept	Wednesday	1	7	12:30	8Y
5 Sept	Friday	3	2	08:35	8X
9 Sept	Tuesday	5	5	11:00	8Y
10 Sept	Wednesday	6	4	09:55	8X
15 Sept	Monday	9	2	08:35	8Y

Term 4
OCTOBER

Date	Day	Day	Lesson	Time	Class
1 October	Wednesday	6	4	09:55	8X
6 October	Monday	9	2	08:35	8Y
7 October	Tuesday	10	3	09:15	8X
8 October	Wednesday	1	7	12:30	8Y
9 October	Friday	3	2	09:15	8X
14 October	Tuesday	5	5	11:00	8Y
15 October	Wednesday	6	4	09:55	8X
20 October	Monday	9	2	08:35	8Y
21 October	Tuesday	10	3	09:15	8X
22 October	Wednesday	1	7	12:30	8Y
24 October	Friday	3	2	09:15	8X
28 October	Tuesday	5	5	11:00	8Y
29 October	Wednesday	6	4	09:55	8X

NOVEMBER

Date	Day	PHSG day	Lesson	Time	Class
3 November	Monday	9	2	08:35	8Y
4 November	Tuesday	10	3	09:15	8X
5 November	Wednesday	1	7	12:30	8Y
7 November	Friday	3	2	09:15	8X
11 November	Tuesday	5	5	11:00	8Y

Standardised test

MATHEMATICS TEST
GRADE 8 - CHAPTER 10

TIME: 35 min
MARK: 30

13/30

1. Calculate

1.1. 18% *of* 360 (2)

6480

1.2. $11\frac{1}{2}$% *of* 1200 (2)

13800

look for your decimal!

2. The number of elephants in a reserve decreased by 10%. If there were 1400 elephants previously, how many are there now?

$\frac{10}{100} \times \frac{1400}{1} - 1400 = 140$

There are 140 elephants now. (4)

3. The price of a certain bicycle increased from R680 to R816. Calculate by what percentage the price of the bicycle increased.

① $\frac{R680}{100} \times \frac{R816}{1}$ = R 5548,8 − 816 = 4732,8

② R816 − 680 = R136 = 4732,8 (4)

2

4. The cost price of an item is R14 and the profit made on it is 12%. What is the selling price?

① $\frac{R12\%}{100} \times \frac{R14}{1}$ = R1,68 | ② R1,68 + R14 = R15,68 (3)

3

5. Lindiwe buys shirts at R75 each. Her percentage profit is 20% on each shirt. After the calculation of profit she must also add 14% vat. Calculate the selling price of each shirt. ~~What will her total profit be if 20 shirts are sold?~~ Ignore this.

$\frac{20\%}{100} \times \frac{75}{1} + 14$ = R29 | $\frac{20}{100} \times \frac{75}{1} \times 20$ = R300 (5)

2

7. Calculate the interest if R660 is invested for $6\frac{1}{4}$ years at 12% simple interest.

$\frac{12}{100} \times \frac{R660}{1} \times 6\frac{1}{4}$ yrs = R495 | or $\frac{12}{100} \times \frac{R660}{1} \times (6\frac{1}{4} = 25)$ = 1980 (3)

2

8. What is compound interest?

It is when you've invested money in the bank. Interest over the original amount. Interest over interest. (2)

2

9. Calculate the value of a house in five years time if it is worth R120 000 now and appreciates in value at a rate of 10% per annum Compounded annually. (5)

1 way:

1st yr $\frac{10}{100} \times \frac{120\ 000}{1}$ + 120 000 = 132 000

5th $\frac{10}{100} \times \frac{120\ 000}{1}$ + 166 000 = 178 000

Other way:

$\frac{10}{100} \times \frac{120\ 000}{1}$ = 12 000

Other way

1. 1441 2000
2. 1561 2000
3. 1705 2000
4. 18 49 2000
5. 1992 2000

Final mathematics examination

1

Mathematical Literacy, Mathematics & Mathematical Science

Grade 8 December 2003

Time: 2,25 hrs Marks: 140

Strand 1: Number & algebra (Questions 1,4,7,10)

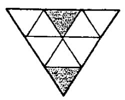
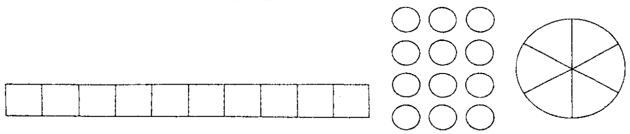
Strand 2: Measurement (Questions 2,5,8,11)

Strand 4: Space & shape (Questions 3,6,9,12)

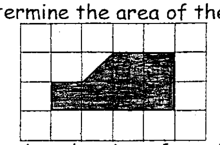
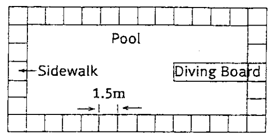
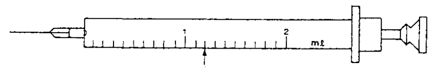
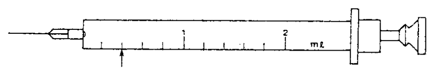
Instructions:

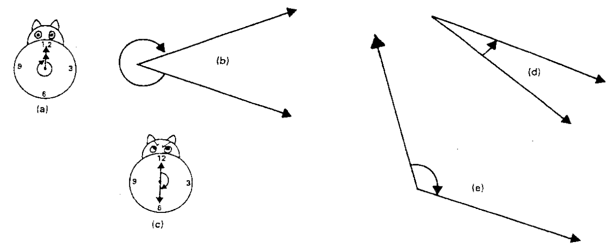
1) Calculators may be used.

2) Answer all questions on the answer sheet. Show all working out!

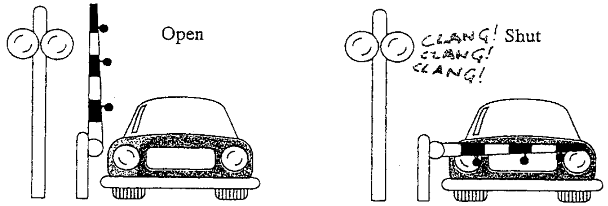
QUESTION 1	S1 L3	
1.1) What fraction of the figure is shaded? 		(1)
1.2) Three fractions and three diagrams are given. Colour in, in **pencil**, one fraction per diagram on the answer sheet. The fractions are $\frac{2}{3}$; $\frac{2}{5}$; and $\frac{1}{4}$.  Figure A Figure B Figure C		(3)
1.3) Write sixty-two thousand and sixty-three in figures.		(1)

2

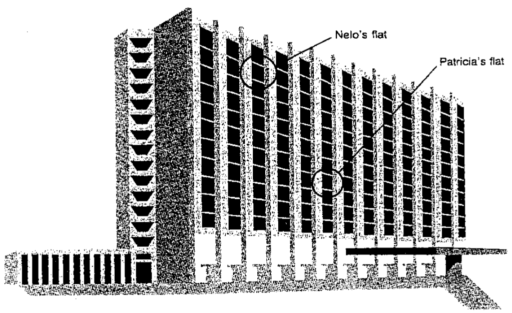
QUESTION 2	S2 L3	
2.1) Determine the area of the shaded shape. 		(1)
2.2) Here is a drawing of a swimming pool. Each square of the sidewalk paving has a length of 1,5m.  Using the drawing, find the perimeter of		
2.2.1) one square of the sidewalk paving.		(2)
2.2.2) the swimming pool (inner edge of sidewalk).		(2)
2.3) Nurses have to be very accurate when they measure out medicine. If they measure incorrectly, patients may die! Read the following dosages:-		
2.3.1) 		(1)
2.3.2) 		(1)

QUESTION 3	S4 L3	
3.1) What type of angle is each of the following? 		(5)

3.2) Through how many degrees does the railway boom gate move, from the open to the shut position? (1)



3.3) Patricia lives on the fourth floor of the block of flats, in the sixth flat from the left. Her flat number is 6-4.



3.3.1) Thamsanqua lives in flat 10-9. Indicate his flat on the grid <u>on the answer sheet</u>, using a " ✗ ". (1)

3.3.2) Nelo's flat is indicated in the picture. What is the number of his flat? (1)

QUESTION 4 S1 L4

4.1)

4.1.1) Write 387 to the nearest ten. (1)

4.1.2) Write 121 to the nearest hundred. (1)

4.2) Complete the number sequences:

4.2.1) 0,5 ; 1 ; 1,5 ; 2 ; __ ; __. (1)

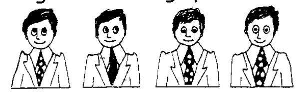
4.2.2) 6 ; 10 ; 9 ; 13 ; __ ; __. (1)

4.3) Write in ascending order: (2)

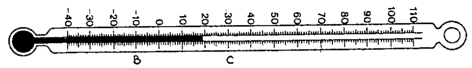
$$\frac{1}{6};\frac{5}{12};\frac{1}{12};\frac{1}{4};\frac{1}{3}$$

4.4) What fraction of these seeds has **NOT** begun to germinate? (1)

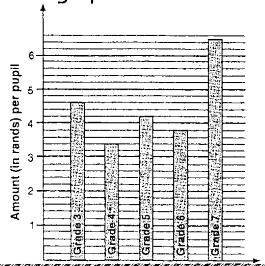
4.5) What percentage are wearing spotted ties? (1)



4.6) How much colder is the reading at B than at C? (1)

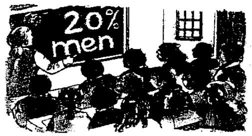


4.7) The school principal offers a prize to the class that collects the most money <u>per learner</u> for a new school bus.
Study the bar graph and answer the questions:



4.7.1) Which class collected the largest contribution per learner? (1)
4.7.2) What was the amount per learner? (1)
4.7.3) There are 30 learners in this class. How much did they collect in total? (1)

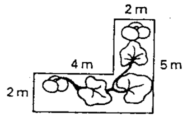
4.8) An adult education class has 14 students and 1 teacher. The teacher writes on the board that 20% of the total number of people in the room are men.



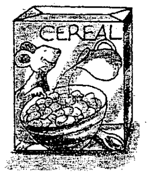
4.8.1) How many men are in the classroom? (2)
4.8.2) If 6 of the people in the room are between 20 and 30 years old, what percentage will this be? (2)

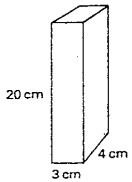
QUESTION 5 S2 L4

5.1) Calculate the perimeter of the pumpkin patch. (2)

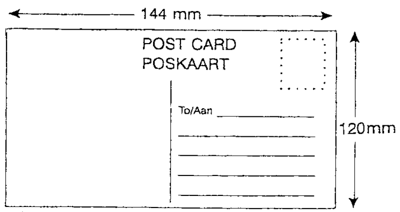


5.2) Calculate the volume of the cereal box, if it's dimensions are 3cm, 4cm and 20cm. (2)





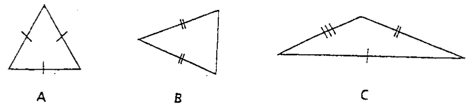
5.3) The dimensions of a postcard are shown.



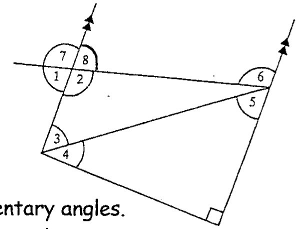
5.3.1) Calculate the area of the postcard. (2)
5.3.2) If the dimensions of a stamp are 24mm by 30mm, how many stamps will cover the postcard completely? (3)

QUESTION 6 S4 L4

6.1) Name the following triangles, according to their sides. (3)

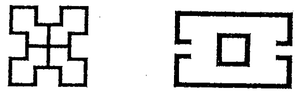


6.2) Name the angles for each of the following descriptions, using the numbered angles in the sketch.



6.2.1) 1 pair adjacent supplementary angles. (1)
6.2.2) 1 pair vertically opposite angles. (1)
6.2.3) Angles round a point. (1)

6.3) Draw in the axis/axes of symmetry (if any), for each motif. Do this on the answer sheet! (2)



QUESTION 7 S1 L5

7.1) Calculate the following, without using a calculator. 

7.1.1) $\sqrt{100-64}$ (2)

7.1.2) $\sqrt[3]{-64}$ (1)

7.1.3) $\frac{-2}{3} \div 1\frac{1}{4}$ (3)

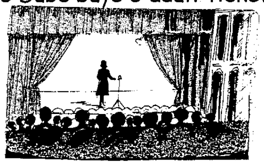
7.2) A wool jersey is advertised as 99% pure wool.

7.2.1) The price of this jersey is R280,95 but it has a 30% discount tag on it. Calculate the sale price. (2)

7.2.2) A jersey that is 100% pure wool costs 10% more than the original price of the jersey above. What will it cost? (2)

7.2.3) What does 99% pure wool mean? (1)

7.3) An adult's ticket for a concert costs R5 more than a student's ticket. Mrs Dube buys 5 adult tickets and 3 student tickets.



7.3.1) If the price of one student ticket is x rand, write an expression for the price of one adult ticket. (1)

7.3.2) Write an expression for the cost of:- (2)
7.3.2.1) 3 student tickets.
7.3.2.2) 5 adult tickets.

7.3.3) If the total cost of the tickets is R105, how much does each type of ticket cost? (use an equation) (4)

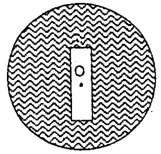
7.4) A brand new car costs R60 000.

7.4.1) It will lose 10% of its value after each year. What will its value be after 3 years? (3)

7.4.2) If after 4 years the car is sold at a give-away price of R27 000, what percentage is this of the original amount? (2)

QUESTION 8 S2 L5

8.1) The diagram shows a circular cricket field with centre O and a radius of 70 metres. The batting pitch is rectangular with measurements 22m by 2m.

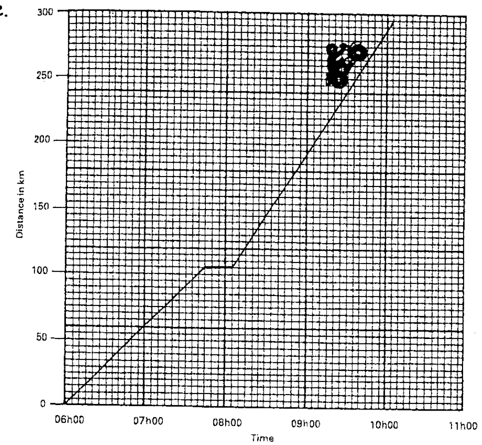


Calculate the following, **correct to the nearest metre.**

8.1.1) At the start of every practice, the team members have to run around the field 5 times. What distance does each player run? (3)

8.1.2) How many square metres of grass was planted to cover the field, excluding the batting pitch? (4)

8.2) The graph depicts a motor cyclist's journey from Durban to Dundee.



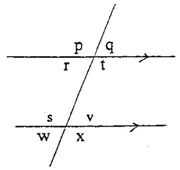
8.2.1) How long did he take to complete his journey? (1)
8.2.2) What distance did he travel from Durban to Dundee? (1)
8.2.3) For how long did he rest? (1)
8.2.4) Calculate the cyclist's speed before resting.(in km/hr) (3)

8.2.5) He increased his speed after resting. By how many km an hour did he increase his speed? (3)

QUESTION 9

S4
L5

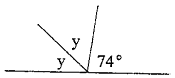
9.1) Write down one pair of angles from the diagram, for each of the following:-



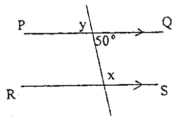
9.1.1) Co-interior angles (1)
9.1.2) Corresponding angles (1)
9.1.3) Alternate angles (1)

9.2) Determine the values of x *and* y in the diagrams. (6)

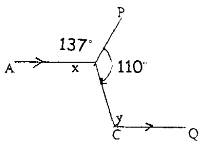
9.2.1)



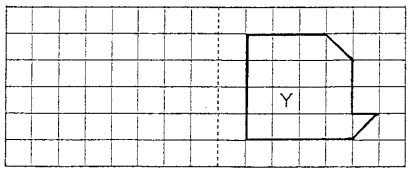
9.2.2)



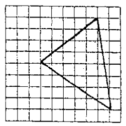
9.2.3)



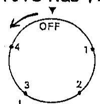
9.3) On the grid, draw the reflection of shape Y, about the dotted line. (2)



9.4) Translate the triangle two units to the left. (2)



9.5) The switch on a stove has five possible positions, equally spaced.



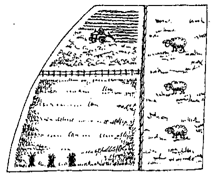
Through how many degrees does the switch turn from the OFF position to position 2? (2)

QUESTION 10

S1
L6

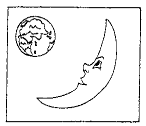
10.1) The ratio of the chemicals, Nitrogen (N), Phosphorus(P) and Potassium (K) in plant fertiliser, is 3 : 2 : 1 . The large bags of the fertiliser contain 3 750 grams of the chemical mixture.
Calculate how many grams N, how many grams P and how many grams K make up this mixture of 3 750 g. (4)

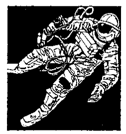
10.2) Five eighths of a farm is **arable** land **(used to plant crops)** and the rest is used for sheep farming.
Wheat is grown on $\frac{2}{3}$ of the arable land.



10.2.1) What fraction of the farm is used for wheat cultivation? (2)
10.2.2) What is the size of the farm if wheat is cultivated on 100 hectares? (2)
10.2.3) What area of the land is used for sheep farming? (2)

10.3) The moon is nearly 384 000km from the earth.





Write this distance in scientific notation. (2)

APPENDIX C - INTERVIEW AND OBSERVATION SCHEDULES

- Interview schedule for 8Y (end of cycle one)
- Interview schedule for 8X (end of cycle two)
- Observation schedule for cycle three

8Y - Semi-structured interview with learners from remedial programme

After first module on place value - end of May 2003

Purpose:

- To get the learners' viewpoints on their experience of the programme so far in terms of:
- Their understanding
- Their confidence
- Their enjoyment
- The content
- To find out more about the reasons learners chose certain answers on the pre-test of the concept test and to monitor any change in their thinking and understanding.
- To gather any comments or suggestions from the learners in relation to the next term.

Format: The interviews will take an oral format initially for the questions relating to their concept tests but learners will be offered the option of continuing orally or answering the questions in the form of a written letter to me.

Questions

1. What do you think of the module/classes so far?
   Can you describe what the classes have been like for you?

2. a) What did you like/enjoy most about the classes?

   b) What did you like/enjoy least about the classes?

3. a) Is there anything in the course you found difficult?
   What? How? Why? When? Anything else?

   b) Is there anything in the course you found easy?
   What? How? Why? When? Anything else?

4. What do you think about your understanding of place value now compared to before we started the classes?

5. What do you think about mathematics?

6. Can you make any suggestions for changes for next term?

8X - Semi-structured interview with learners from remedial programme

After second module on fractions and decimals - end of Sept 2003

Purpose:

To get the learners' viewpoints on their experience of the programme so far in terms of:

- Their understanding
- Their confidence
- Their enjoyment
- The content
- To find out more about the reasons learners chose certain answers on the pre-test of the concept test and to monitor any change in their thinking and understanding.
- To gather any comments or suggestions from the learners in relation to the next term.

Format: The interviews will take an oral format initially for the questions relating to their concept tests but learners will be offered the option of continuing orally or answering the questions in the form of a written letter to me.

Questions

1. What do you think of the module/classes so far?
   Can you describe what the classes have been like for you?

2. a) What did you like/enjoy most about the classes?

   b) What did you like/enjoy least about the classes?

3. a) Is there anything in the course you found difficult?
   What? How? Why? When? Anything else?

   b) Is there anything in the course you found easy?
   What? How? Why? When? Anything else?

4. What do you think about your understanding of fractions and decimals now compared to before we started the classes?

5. What do you think about mathematics?

6. Can you make any suggestions for changes for next term?

Teacher observation schedule

Date: Class:

		SA	A	D	SD	N/A
1.	Introduction					
1	Teacher clearly introduces and formulates the problems.					
2	Teacher asks relevant guided questions to introduce the lesson.					
3	Teacher responds to learners' ideas.					
4	Teacher asks learners for their own ideas and encourages learners to share them.					
5	Teacher often encourages learners to ask questions.					
6	Teacher often guides the learners to the conclusion.					
7	Problem presented is clearly within the frame of reference of the learners.					
8	Problem presented is within the zone of proximal development of the learners.					
9	Teacher "familiarises" learners with the context of the problem if necessary.					
10	Learners interact with the teacher.					
11	Learners understand and are able to engage with the context of the problem.					
12	Learners share their ideas willingly.					
13	Learners appear bored and disinterested.					
14	Learners appear interested in the work.					
15	Learners experience the problem being formulated as real and meaningful.					
16	Learners are encouraged to work together with each other.					

		SA	A	D	SD	N/A
2.	Body					
1	Learners explore problems in groups or individually.					
2	Teacher allows learners to choose their own approach.					
3	Learners actively make use of their knowledge.					

		SA	A	D	SD	N/A
4	Learners discuss the operation employed in the problems.					
5	Teacher focuses learners' attention on crucial aspects.					
6	Teacher draws attention to and re-emphasizes the relevant mathematical notation and terminology relevant to the lesson.					
7	Teacher interacts with learners during the lesson.					
8	Teacher assists learners when necessary.					
9	Teacher asks learners guiding questions, but does not directly provide the answers.					
10	Teacher encourages learners to discuss with peers in their groups.					
11	Teacher allows learners to draw own conclusions.					
12	Learners ask questions during the lesson.					

		SA	A	D	SD	N/A
3.	Conclusion					
1	Teacher asks several groups/individuals to report their results to the class.					
2	Teacher invites and encourages learners to comment on their outcomes.					
3	Teacher asks critical open-ended questions regarding the outcomes.					
4	Teacher compares learners' outcomes and their differences or discrepancies.					
5	Teacher guides learners to understand discrepancies in their solutions.					
6	Teacher draws conclusions from the activity with the learners.					

		SA	A	D	SD	N/A
4.	General					
1	Teacher acknowledges learners' ideas.					
2	Teacher uses and discusses learners' ideas.					
3	Teacher summarises learners' answers.					
4	Teacher asks open-ended questions to individual learners.					
5	A classroom atmosphere prevails that encourages learners to ask and answer questions					

General impression of the lesson:

Useful	5	4	3	2	1	Not useful
Interesting	5	4	3	2	1	Not interesting
Easy to apply	5	4	3	2	1	Not easy to apply
Enjoyable	5	4	3	2	1	Not enjoyable

Remarks or comments

APPENDIX D - ETHICAL CONSIDERATIONS

- Letter to parents/guardians

Letter to parents/guardians

7 April 2003

Dear ....

Ms Hayley Barnes is a lecturer from the University of Pretoria who is currently completing her Masters in Mathematics Education. The Masters involves implementing a remedial intervention for Grade 8 learners over four months to assist them in improving their confidence, conceptual understanding and academic performance in Mathematics.

Your daughter's mathematics class has been selected to be part of this study and we therefore request your permission for your daughter to be taught her remedial mathematics lessons by Ms Barnes for the next two terms of this year. Your daughter will still attend her usual mathematics lessons with her teacher and in addition to that, she will continue to attend three remedial mathematics lessons during school with Ms Barnes. Ms Barnes is a former member of our Mathematics staff and taught at Girls' High for almost seven years.

Please could you complete this form and return it to the school as soon as possible as lessons will commence next week.

Thank you for your co-operation in this regard.

Yours sincerely

APPENDIX E - FROM THE INTERVENTION

- Worksheet one (with Dienes blocks)
- Worksheet three (contextual place value)
- Item 19 from diagnostic assessment
- Item 20 from diagnostic assessment

Worksheet 1

To do this worksheet you need to use the blocks available. The blocks are called "Dienes' Blocks" after the man who invented them.

Activity one

Using the blocks display the following numbers:

1. 12
2. 123
3. 2 345
4. Five thousand and sixteen
5. One thousand, two hundred and three

Activity two

Work in pairs (or groups of three):

- Each of you have a turn at being the teacher, while the other one is the learner.
- First of all both of you have to do the calculation.
- Then the teacher must show the learner how to get the answer to the calculation using the blocks.

Calculations:

1. 23 + 46
2. 15 + 12
3. 42 + 39
4. 27 + 14
5. 59 + 44
6. 66 + 46

Worksheet 3

Activity one

Some annual salaries of people in various positions have been listed below:

Accountant:	R240 450
Lawyer:	R180 000
Personnel Manager:	R175 233
Store manager:	R210 398
Chartered Accountant:	R560 900
Computer programmer:	R490 080
Network manager:	R308 120

Which of the above salaries is the highest?

Which of the salaries above is the lowest?

How much does the accountant earn per month?

What is the difference between the salary earned by the Accountant and the one earned by the Chartered accountant?

Write down in words what the computer programmer earns per year.

Activity two

The following houses are on sale and their prices are given:

- ✓ A beautiful architectural designed house with three bedrooms and a swimming pool for a small family: **R 987 400**
- ✓ A lovely upmarket townhouse in a secure complex overlooking the mountains. **R688 400**
- ✓ A real investment for the clever homebuyer. You will not regret this one. With five bedrooms and a large family room, it's a steal! **R999 500**

Which house costs the most?

Which house costs the least?

How much would it cost to buy all three houses?

What is the price difference between the most expensive and the cheapest house?

Diagnostic assessment - Item 19

You decide to start making banana bread to sell in order to earn some extra money. To start off with, you decide to make 5 loaves of banana bread. According to the recipe, each loaf requires $4\frac{1}{2}$ bananas. How many bananas will you need to make the 5 loaves of banana bread? Show your working out in this space provided below and write your answer in the space provided:

Diagnostic assessment - Item 20

<u>The jive and eat shop</u>

Cup of Coffee:	R4, 50
Cup of tea:	R4, 00
Breakfast:	R11,50
Toasted sandwich:	R9, 80

A waiter at the "Jive and Eat shop" needs to work out the bill for a table of people that she served. In total they had:

2 cups of coffee
1 cup of tea
2 Toasted sandwiches
1 Breakfast

What is the final bill for this table? Show all your working out in the space provided below and write the answer on the line provided: