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A DEVELOPMENTAL CASE STUDY:
IMPLEMENTING THE THEORY OF REALISTIC
MATHEMATICS EDUCATION WITH LOW
ATTAINERS

by

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ABSTRACT

The research documented in this report had a twofold purpose. Firstly, it was to design and implement an intervention based on the theory of Realistic Mathematics Education (RME) aimed at improving the mathematical understanding of learners in two Grade 8 remedial mathematics classes, by revisiting the key number concepts of place value, fractions and decimals. In doing so, a second purpose was to investigate the viability and emerging characteristics of an intervention based on the theory of RME in such a setting (i.e. with *low attainers to revisit key number concepts*). Pending the realisation of these immediate outcomes, more distant outcomes in subsequent research would be: that learners' understanding and academic performance in mathematics improves and to develop a local instruction theory in using the RME theory to revisit the concepts of place value, fractions and decimals with low attaining learners in order to improve their understanding in this regard.

Grade 8 low attainers were selected as the target group for this research as a result of the pending implementation of Mathematical Literacy as a compulsory subject for all learners, possibly from 2006. Currently in South Africa, learners who are not meeting the required standard by the end of their Grade 9 year are able to elect not to take mathematics through Grades 10, 11 and 12. When the new Further Education and Training (FET) policy is implemented, this will no longer be the case. All learners, who do not elect to take mathematics as a subject, will have to take Mathematical Literacy as a compulsory subject throughout Grades 10, 11 and 12. Although less detailed and abstract than the subject mathematics, the Mathematical Literacy curriculum still requires learners to have an understanding of key number concepts and also contains a substantial amount of algebra. As Grade 8 is when learners start working with algebra more formally, and is also their first year at secondary school, it was decided that this would be an appropriate year to try and diagnose and remediate problems in learners' understanding of the key number concepts, if and where possible. The intention was that this would then equip learners with a more appropriate structure of conceptualised knowledge of the above-mentioned concepts on which they could further construct their understanding of algebra.

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The study was carried out at a local urban high school in South Africa and the research design of this study was informed by two development research approaches (van den Akker & Plomp, 1993; Gravemeijer, 1994). Also, the study was only implemented with a small number of participants, within a bounded setting and without the intention to generalise the results. It was therefore regarded as a development *case study*. The results appear to indicate that it is viable to apply the theory of RME with low attaining Grade 8 learners in order to revisit the key number concepts of place value, fractions and decimals.

Key words:

Low attainers; Realistic Mathematics Education (RME); remedial mathematics classes; low achievers; development research; remediation; Special Educational Needs; key number concepts; mathematics intervention

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LIST OF ACRONYMS

DOE	Department of Education
RME	Realistic Mathematics Education
SEN	Special Educational Needs
FETC	Further Education and Training Certificate
SPSS	Statistical Package for the Social Sciences
MCQ	Multiple Choice Questions
TIMSS	Third International Mathematics and Science Study

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