Chapter 4

Vibration Covariate PHM Application

1 Introduction

The only way to truly contribute to the reliability modelling field with this dissertation is to apply the theory discussed in Chapter 3 successfully to an applicable situation in the industry. In Chapter 4, data collected from the industry is analyzed and modeled with the Proportional Hazards Model to make such a contribution.

Up to date, no successful case study on vibration covariates in the PHM has been published, mainly due to a lack of suitable data. While searching for suitable data in South Africa, a number of serious shortcomings in vibration data recording practices were discovered in general vibration monitoring programs. From the shortcomings it was possible to compile a structured list of data requirements that have to be fulfilled before vibration covariates can be used in the PHM.

Data satisfying the determined requirements was found at SASOL Coal's Twistdraai plant at Secunda[1]. The Twistdraai plant is a coal beneficiation plant that seperates raw coal into different coal products according to client specifications. In September 1996 the plant was formally started up and ever since a vibration monitoring maintenance strategy has been used on 8 Warman® axial in, radial out pumps used to circulate a water and magnetite solution which is used in the washing process. Data recorded from these

[1] SASOL Coal has granted full permission to publish their name, data obtained from them as well as modelling results.

pumps during their operation was retrieved from the plant's meticulous Computerized Maintenance Management System (CMMS) and is used in this research project.

The data was modeled and analyzed in detail according to the theory described in Chapter 3 with close involvement of the vibration technicians who are monitoring the vibration of the pumps at the plant. Experience of these technicians was utilized in the mathematical modelling process by including their knowledge in the selection of covariates. Results obtained from mathematical models were also continually presented to the technicians for their interpretation and comments for improvement to make the final model truly useful in practice.

2 Preliminaries for PHM Analysis

Several futile searches for suitable data were undertaken before the data at SASOL Coal was discovered. During these searches a structured list of requirements for a PHM analysis was set up and used to assess the potential of a possible data source effectively and quickly. In this section the list of requirements is presented together with the shortcomings in general vibration data recording practices that were identified in the industry.

2.1 Requirements for a Vibration Covariate PHM Analysis

Requirements are defined under two main headings: (1) The suitability of an item for a vibration covariate PHM analysis; and (2) The availability of certain observations (data) throughout the item's working life.

2.1.1 Identification of a Suitable Item

First of all, a suitable item must be important enough for periodic diagnostic data collection, i.e. vibration measurements must be taken periodically (preferably at fixed intervals). If the item is important enough to be included in a vibration monitoring program, the cost of unexpected failure is usually considerably higher than the cost of preventive renewal. This is only a rule of thumb and it is not true in all cases. For situations where this is true, the optimal renewal time will be very distinct compared to a much more insensitive optimum for other scenarios.

The specific item must have been renewed on a number of occasions in the past, preferably because of failure. (The renewal assumption is thus made implicitly). Failure does not necessarily refer to a physical shutdown or destruction of the item, but to any condition where the item was unable to perform according to requirements, whereafter it had to be renewed. The preference of failure does not mean that preventive renewals are not important and all available data should be included in an analysis and handled suitably. If certain parts of available data is ignored, important information could be lost and biased estimates of the life time distribution will be the result, such as underestimation of the mean time to failure.

2.1.2 Required Information

Two main types of information have to be available:

(i) The operational age of an item at significant events (explained below) as well the event type. An operational age instant can be expressed in any suitable use parameter – in this case time will be used.
(ii) Diagnostic information (vibration levels) at every significant event.

Significant events mentioned above are any of the following:

(i) The moment when the item is brought into service.
(ii) Every point where diagnostic information is available.
(iii) Points in time where minor maintenance is done that could affect (usually reduce) covariate values, for example realignment, increased lubrication or balancing. The information on expected covariate values at these points should also be included in the data.
(iv) Time of renewal and the state of the item at renewal, i.e. failed or suspended.
(v) Data cutoff date where all operating units will be treated as calendar suspensions.

2.2 Shortcomings

Numerous shortcomings in data collection practices and data retrieving mechanisms of companies were discovered while searching for suitable data. Some of the major shortcomings most often encountered, are:

(i) Unfriendly or improperly organized Computerized Maintenance Management Systems or Enterprise Asset Management Systems.

(ii) Only the calendar age of a component is recorded and not the operational age, i.e. the real usage of the component.

(iii) Irregular inspections.

(iv) Records of maintenance done on a component that could have influenced its vibration levels are not recorded.

(v) The state of the item at the time of renewal is seldom recorded, i.e. whether a preventive renewal or failure renewal was performed.

(vi) A general lack of commitment exists regarding proper vibration monitoring documentation amongst managers of vibration monitoring programs.

The shortcomings mentioned above are all direct, major impairments of a successful vibration covariate PHM analysis, although improvements to these shortcomings could hold benefits for conventional vibration analysis techniques as well.

2.3 Concluding Remark

The information requirements stated in section (2.1.2.) were derived from the PHM theory described in Chapter 3. These requirements are defined for a best case scenario. It does not mean if these requirements are not met flawlessly that a PHM analysis is totally impossible. Mathematical manipulations and approximations allow for some deviation of the requirements as was also described in Chapter 3. Section (2.1.2.) should thus rather be used by analysts not familiar with PHM theory as detailed guidelines for a PHM feasibility analysis in his/her situation, than as strict prerequisites for a successful PHM analysis.

3 SASOL Data

Useful data was found at one of SASOL Coal's coal beneficiation plants, a part of the Twistdraai mine, situated at Secunda. The data does not strictly satisfy all the requirements outlined above – the main deviation being that no regular inspection frequency is used, although this is not a rigid prerequisite. This data set was the best one found following a fairly extensive search for suitable data in the South African industry.

The Twistdraai plant was started up in September 1996 and is thus still relatively new. Data was collected from September 1st, 1996 to November 1st, 1998 which gives an analysis time horizon of 791 days. The information recorded over the 791 days was used to estimate the PHM and for finding the optimal decision policy. Thereafter a second data set was collected from November 1st, 1998 to February 28th, 1999 that was used to evaluate the model's performance as if it was used to make renewal decisions in a real life situation. (Other techniques, apart from the second data set, were also utilized to test the optimal policy).

3.1 Background

A total of 8 identical axial in, radial out, Warman® pumps are used in a specific section of the plant to circulate a water and magnetite solution. These pumps are very important in the washing process and significant production losses are suffered when one of the pumps breaks down. All 8 pumps work under exactly the same conditions and it was assumed that renewals on the various pumps were generated by the same renewal process. Figure 3.1. below shows the pump installation layout, with the 8 pumps. Figure 3.2 shows a close-up of one of the pumps.

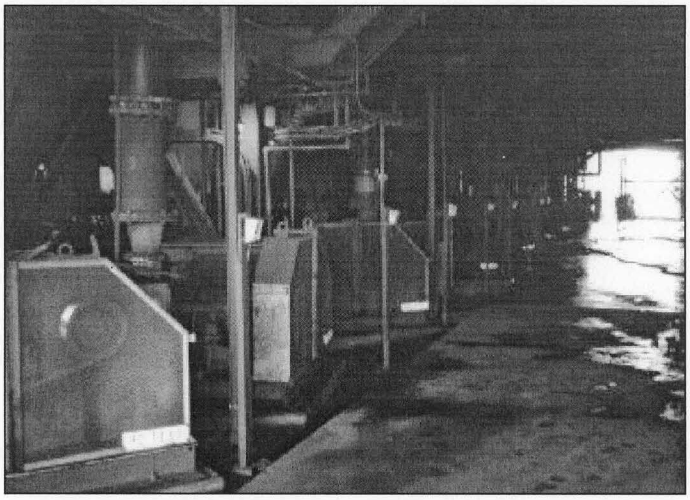

Figure 3.1.: *Pumps in operation*

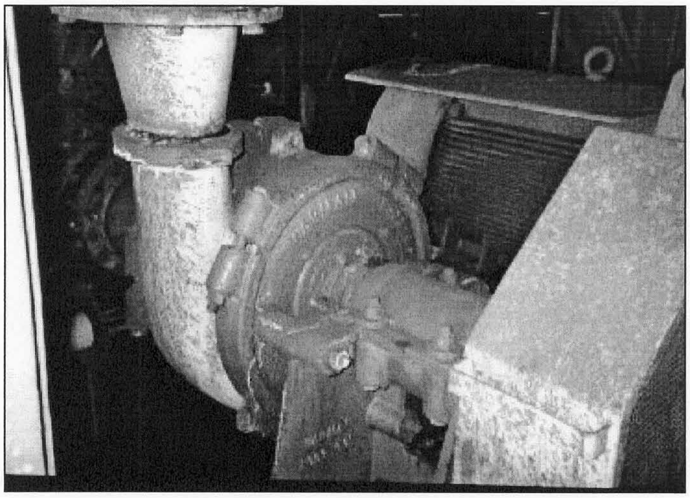

Figure 3.2.: *Warman pump*

When there is referred to a *pump* in this chapter, all the elements visible in Figure

3.2. are implied, except for the 220 kW electrical motors used to drive the pumps. A pump consists of an impeller housing, impeller, bearing housing, 2 SKF 938 932 bearings, a drive shaft, V-belt pulley and seals.

Because of the aggressive nature of the fluid being circulated and the robust environment of the pumps, total destructive failures are encountered frequently. These destructive failures often occur very abruptly, i.e. a pump's state literally change overnight from being in an acceptable condition to being completely failed. Functional failures are usually caused by one (or a combination) of the following:

i. Complete bearing seizure.
ii. Broken or defective impeller.
iii. Damaged or severely eroded pump housing.
iv. Broken drive shaft.

When a pump has failed due to one of the reasons above, it is overhauled completely to an as-good-as-new condition regardless of the amount of work that needs to be done. This may include replacement of bearings, repair or renewal of impeller, repair or renewal of impeller housing or replacement of the main shaft. No complete spare pumps are stocked at the plant but only spare parts, since some parts tend to fail more often than others.

During the analysis time horizon, the plant's management prescribed a condition based preventive renewal strategy based on vibration monitoring results. No fixed inspection interval was used and vibration levels were only measured sporadically or when a notable deterioration in a pump's condition became evident, whereafter more regular inspections were done. This strategy lead to several unexpected failures.

Vibration levels of the pumps were measured on the shaft bearings in two directions, horizontally and vertically, to assess a pump's condition. Figure 3.3. on the next page shows the horizontal measuring positions.

The 'wet-end' bearing (the bearing closest to the impeller) is labeled as bearing number *3* while the 'dry-end' bearing is labeled as bearing number *4*. Measuring positions 3H and 4H are thus the horizontal measurements on bearing number 3 and 4, respectively. Only the horizontal measurements were used in this PHM analysis – reasons are presented later.

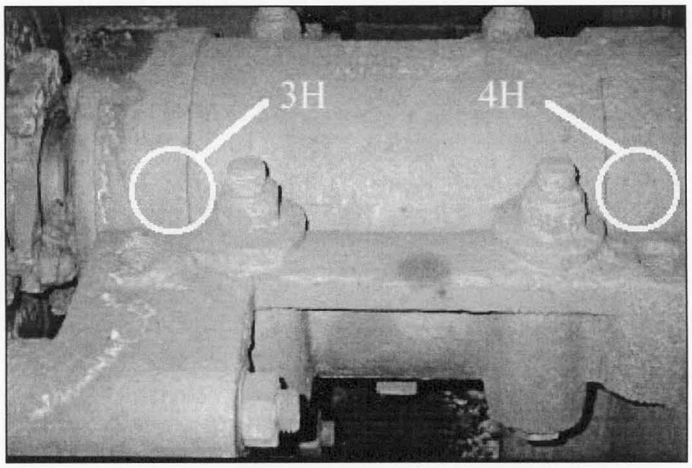

Figure 3.3.: *Monitoring positions*

As in most typical vibration monitoring programs, the renewal decisions of pumps were based on spectral vibration analysis. Several important frequencies are enveloped or benchmarked and renewal is performed as soon as two or three of the benchmarks are exceeded. Benchmarks levels were determined by a combination of technician experience and OEM specifications.

Vibration data loggers were used to capture vibration data on the pumps, from where the information was downloaded onto a dedicated computerized vibration measurement database. Data used in this research was retrieved from this database. Frequency spectrums of all measurements are stored in the database and the chosen covariate levels (discussed later) could be retrieved easily and accurately.

The vibration measurement database does not contain information regarding events during a pump's life, nor does the plant's computerized maintenance management system (CMMS). This is not considered to be a serious shortcoming for this research since the only event or action performed on a pump during its life time is additional lubrication, which probably does not effect the covariate levels too severely.

Failure analysis records obtained from the CMMS provided insight on the state of a pump when it was renewed, i.e. whether it was in the failed state or was suspended (preventively renewed).

3.2 Covariates

Covariate selection was based primarily on the experience of vibration technicians involved with the pumps at the plant. These technicians are of the opinion that the horizontal vibration measurements on the bearings alone is a good enough indication of a pump's condition and that not much additional information is obtained from the vertical measurements. This corresponds to vibration theory and hence only the horizontal vibration measurements are considered.

As mentioned earlier, the vibration monitoring program that was used on the pumps was based on spectral analysis. A number of important frequencies (as defined by theory and experience) are monitored and a pump is renewed as soon as two or three of the frequencies' amplitudes exceed certain benchmarks. It was decided to use all of these frequencies as covariates in the PHM, thereby incorporating vibration theory and prior experience with the pumps in the model. Table 3.1. summarizes the 12 selected covariates.

The biggest challenge when defining vibration covariates is to select a single quantity that describes a specific defect most clearly. A specific defect can often be identified by numerous parameters but not all parameters can be used as covariates, since the number of covariates has to be limited. Too many covariates may cause the proportional hazards model to become mathematically unstable or difficult to estimate, especially when the sample size is fairly small.

	Covariate Abbreviation	**Description**
1.	RF043H	0.4 x Rotational frequency amplitude, measured on bearing 3, indicative of a bearing defect.
2.	RF13H	1 x Rotational frequency amplitude, measured horizontally on bearing 3, indicative of unbalance in the pump.
3.	RF23H	2 x Rotational frequency amplitude, measured horizontally on bearing 3, indicative of misalignment in the pump.
4.	RF53H	5 x Rotational frequency amplitude, measured horizontally on bearing 3, indicative of cavitation in the pump.
5.	HFD3H	High frequency domain components between 1200-2400 Hz, measured on bearing 3, indicative of bearing defect. This is a subjective covariate where 1 indicates the presence and 0 the absence of the mentioned components.
6.	LNF3H	Lifted noise floor in 600-1200 Hz range, measured on bearing 3, indicative of a lack of lubrication where 1 indicates the presence and 0 the absence of the mentioned components.

7.	RF044H	0.4 x Rotational frequency amplitude, measured on bearing 4, indicative of a bearing defect.
8.	RF14H	1 x Rotational frequency amplitude, measured horizontally on bearing 4, indicative of unbalance in the pump.
9.	RF24H	2 x Rotational frequency amplitude, measured horizontally on bearing 4, indicative of misalignment in the pump.
10.	RF54H	5 x Rotational frequency amplitude, measured horizontally on bearing 4, indicative of cavitation in the pump.
11.	HFD4H	High frequency domain components between 1200-2400 Hz, measured on bearing 4, indicative of bearing defect. This is a subjective covariate where 1 indicates the presence and 0 the absence of the mentioned components.
12.	LNF4H	Lifted noise floor in 600-1200 Hz range, measured on bearing 4, indicative of a lack of lubrication where 1 indicates the presence and 0 the absence of the mentioned components.

Table 3.1.: *Summary of covariates*

3.3 Data

The data collected include the pump unit identification, dates of inspection, vibration frequency spectrum at each inspection, date of failure or suspension and the state at renewal, i.e. failed or suspended. Accurate inspection data was generally not available for cases where unexpected failures occurred and data was generated by extrapolating available data as appropriately as possible to the date of unexpected failure.

A total of 27 histories were compiled over the analysis horizon with 98 inspections (extrapolations included). This gives an average of 3.6 inspections per history. Approximately 50% of all inspections were done on an irregular basis either at the beginning or the end of a pump's life time.

Of the 27 histories, 11 were failures, 8 were suspensions and 8 were calendar suspensions since all 8 pumps were running at the cutoff date of the analysis horizon. The 11 failures were all unexpected and production losses were suffered following these events. The 8 suspensions were all done based on vibration measurements and were considerably cheaper than the unexpected failures. Three of the 8 suspensions were done on very short life times relative to other survival times.

The working age of the pumps was considered to be the same as the calendar age, because the pumps run 24 hours per day, 365 days per year. The pumps are very rarely shut down because of breakdowns on other parts of the plant and these times are considered to be insignificantly small.

Three events were defined for the pumps through their life times: (1) B – Begin or pump startup; (2) ES – Event suspension; and (3) EF – Event failure. Events that occurred to the pumps are listed in Table 3.2. below:

Pump Identification	Age (days)	Date	Event
PC1131	0	9/1/96	B
PC1131	397	10/3/97	ES
PC1131	397	10/3/97	B
PC1131	554	3/9/98	EF
PC1131	554	3/9/98	B
PC1131	690	7/23/98	ES
PC1131	690	7/23/98	B
PC1131	765	10/6/98	EF
PC1131	765	10/6/98	B
PC1131	791	11/1/98	ES
PC1132	0	9/1/96	B
PC1132	491	1/5/98	EF
PC1132	491	1/5/98	B
PC1132	544	2/27/98	ES
PC1132	544	2/27/98	B
PC1132	557	3/12/98	ES
PC1132	557	3/12/98	B
PC1132	751	9/22/98	EF
PC1132	751	9/22/98	B
PC1132	791	11/1/98	ES
PC1231	0	9/1/96	B
PC1231	563	3/18/98	EF
PC1231	563	3/18/98	B
PC1231	578	4/2/98	ES
PC1231	578	4/2/98	B
PC1231	791	11/1/98	ES
PC1232	0	9/1/96	B
PC1232	599	4/23/98	ES
PC1232	599	4/23/98	B
PC1232	791	11/1/98	ES
PC2131	0	9/1/96	B
PC2131	184	3/4/97	EF
PC2131	184	3/4/97	B
PC2131	470	12/15/97	ES
PC2131	470	12/15/97	B
PC2131	631	5/25/98	EF
PC2131	631	5/25/98	B
PC2131	774	10/15/98	EF
PC2131	774	10/15/98	B
PC2131	791	11/1/98	ES
PC3131	0	9/1/96	B
PC3131	450	11/25/97	EF
PC3131	450	11/25/97	B
PC3131	791	11/1/98	ES
PC3132	0	9/1/96	B
PC3132	506	1/20/98	EF
PC3132	506	1/20/98	B
PC3132	791	11/1/98	ES
PC3232	0	9/1/96	B
PC3232	563	3/18/98	EF
PC3232	563	3/18/98	B
PC3232	723	8/25/98	ES
PC3232	723	8/25/98	B
PC3232	791	11/1/98	ES

Table 3.2.: *Events table*

Detailed inspection data of all the covariate measurements between events is provided as an appendix to this chapter. Covariate values immediately after the occurrence of an event were all taken to be zero. Further detailed comments on the data are presented below:

1. Covariate RF043H recorded two unusually high values of 250 and 1200 mm/s compared to the normal range of between 0 and 5.6 mm/s. These high values were confirmed by the vibration monitoring database and vibration technicians are confident that these levels were not due to faulty monitoring equipment or human error. A further noticeable fact is that these values occurred at suspensions.

   The most logical physical explanation for these values lies in the wear mechanism present in the bearing. (RF043H is indicative of a particular bearing defect). It could be that the bearings that produced these extreme values were able to withstand the wear associated with RF043H, i.e. did not abrade with the introduction of the RF043H vibration, which would have kept the vibration levels within normal limits. The vibration levels continued to rise up to their outrageously high values, which persuaded management to renew the pumps preventively.

2. Subjective covariates HFD3H, HFD4H, LNF3H and LNF4H indicated the presence of their associated phenomena with a simple 0 or 1. These phenomena certainly appear in different degrees of severity and it could be argued that covariates that quantify the severity could lead to a more accurate model. It is however very difficult to quantify the severity of these phenomena with a single number (covariate) because it ranges over large frequency bands. In practice, vibration technicians do not try to estimate the severity of these phenomena either but only use the presence (or absence) thereof as a supportive argument in decisions. It was hence decided that a simple 0 or 1 would suffice for this study.

   Intuitively it is expected that whenever one of the considered covariates turns to 1, it will remain 1. This is however not observed in the data, once again due to wear mechanisms present in the pumps. For example, LNF3H or LNF4H will be present in a certain inspection but will be absent in the following, only to return in subsequent inspections. LNF is indicative of a lack of lubrication. When there is a lack of lubrication, asperities induce a lifted noise floor over 600-1200Hz but the asperities are soon worn off, thereby inducing increased levels of unbalance, but a reduction in the lifted noise floor. Hence, the LNF covariate appears, diminishes and reappears.

   Interaction between the subjective covariates and the quantitative covariates is an area which should be investigated in a study such as this.

3. Failure times are distributed such that 6 failures occurred below 200 days and the remaining 5 failures above 450 days (not randomly distributed). Suspension times are randomly distributed with some being very short like 53, 15 and 13. The question is whether these renewal patterns can be explained by the covariates.

4. Covariate RF13H shows comparatively high values in the beginning of histories and then decreases gradually towards events. RF14H has a very similar pattern, although not as distinct. Technical reasons for this would be the same as discussed in (3).

Costs associated with failures and suspensions of the pumps could not be disclosed exactly by the Twistdraai plant because of company policy. The Twistdraai plant did provide scaled costs however which is proportional to the true costs. An unexpected failure cost $C_f = R\,162\,200$ will be used and a preventive renewal cost of $Cp = R\,25\,000$. These costs were average costs sustained by the Twistdraai plant over the two years over which the data was collected. No details are available.

4 Weibull PHM Fit

There is no straightforward procedure to select the most appropriate covariates for a good Weibull PHM. For this data set, a combination of backward selection (eliminating covariates with the highest p-values, one at a time), residual graphs, goodness-of-fit tests and technical experience were used to get to the best possible model.

Some important facts and guidelines concerning vibration covariates and vibration covariate selection for the PHM were discovered and established from experience gained in this research project:

(i) It is not recommended to exclude several covariates from the model in one step. This may lead to an inaccurate model.

(ii) If two covariates are highly correlated, it can produce very uncertain estimates (large standard errors) which will make them appear as insignificant, even if one of them could be a very good predictor of failure.

(iii) Some covariates can appear as insignificant, contrary to a technician's opinion, simply because of insufficient data or high variations. It is not recommended to include these in the model, because their parameters could be very inaccurate and produce a misleading model. They could be checked again when more data is collected.

(iv) Positive covariates with negative regression coefficients should be considered with special care, because it indicates that the hazard increases with decreasing covariate values (as is the case with RF13H and RF14H), which is not usually expected. In some cases it could be because some influential events, such as minor repairs, were not recorded.

(v) Some covariates can surprisingly appear as significant, without practical interpretation. This almost always indicates some data problem, particularly if wrong covariate values are reported at failures, because failure information has a large influence on the maximum likelihood. .

An extensive discussion about practical analysis of covariate data and modeling procedures can be found in [5].

The following theory, described in Chapter 3, will be used to fit the PHM:

- The numerically convenient method of maximum log-likelihood as objective function in the optimization routines. (Chapter 3, section 2.3.2.).
- For estimation of the parameters in the objective function, the Newton-Raphson optimization method because of its rapid convergence. Snyman's method will only have a supportive role in the modelling process because of its robustness. (Chapter 3, section 4.).
- Guidelines on covariate behavior and selection will constantly be referred to in the modelling process. (Chapter 3, section 3.).
- Residual plots as graphical indication of the goodness-of-fit of models because all the PHM assumptions can be evaluated by analyzing the residuals. The Kolmogorov-Smirnov test, Wald test, p-values and standard errors will be used as analytical goodness-of-fit tests. (Chapter 3, sections 5.1. and 5.2.).

To be able to recognize all patterns in the data, it was decided to model the data in three phases: (1) By a simple Weibull model; (2) By a Weibull PHM where the subjective covariates are temporary excluded; and (3) By a Weibull PHM with all covariates included from the start.

The results of the modelling procedures are presented below.

4.1 Phase 1: Simple Weibull Model

The simple Weibull model was calculated to be:

$$h(t) = \frac{1.984}{468.7} \cdot \left(\frac{t}{468.7} \right)^{0.984} \qquad (4.1.)$$

Residual plots of the model in (4.1.) yield the following:

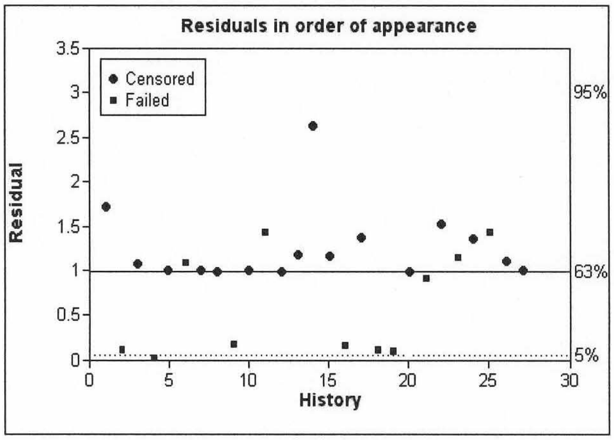

Figure 4.1.: *Residuals in order of appearance*

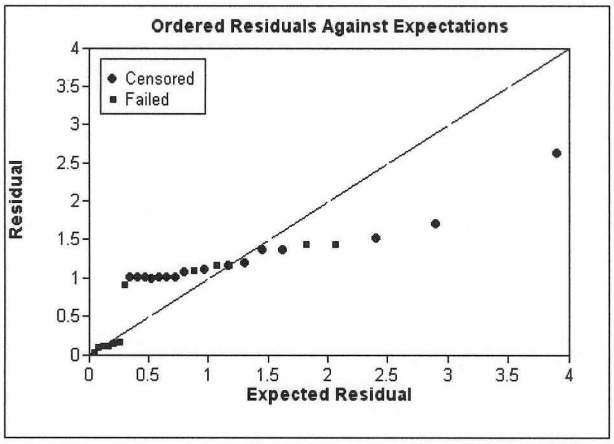

Figure 4.2.: *Ordered residuals agains expectations*

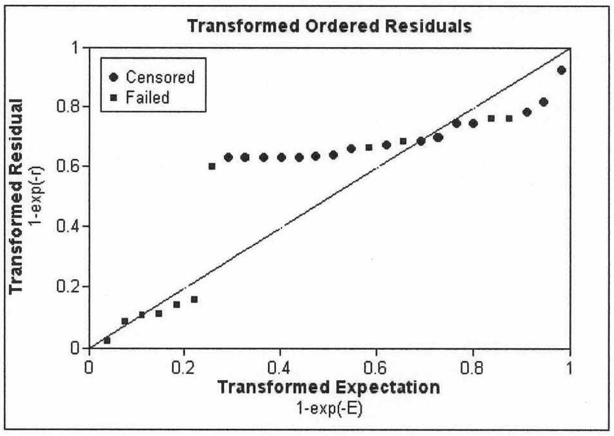

Figure 4.3.: *Transformed ordered residuals*

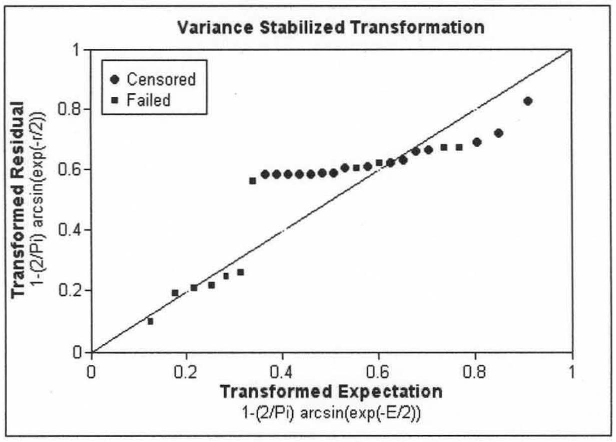

Figure 4.4.: *Variance stabilized transformation*

From the residual plots it is clear that the model does not represent the data very well, especially the suspended observations. Analytical tests revealed a standard error for the shape parameter of 0.46, which is significantly different from 1, while the standard error for the scale parameter is 71.9 days, showing that the model is not very accurate but still a useful estimate. The $MTTF = 415.5$ which is realistic. The Kolmogorov-Smirnov test (KS-test) yielded a value of $KS = 0.3949$ with a $p-\text{value} = 0.000276$, which rejects the fit at a 5 % level of significance. (A time-independent model, i.e. $\beta = 1$ fixed, was also tested but rejected based on an observed value for the Wald test of 4.57 with a $p-\text{value} = 0.0325$).

4.2 Phase 2: Weibull PHM with Subjective Covariates Excluded

For this phase, HFD3H, HFD4H, LNF3H and LNF4H were excluded from the modelling process. A reasonable model fit was obtained when using all of the remaining quantitative covariates, i.e. RF13H, RF14H, RF23H, RF24H, RF53H, RF54H, RF043H and RF044H, with negative regression coefficients for RF13H and RF14H. This is consistent with the observed behavior of the data (see comment 5 of section 3.3.).

Using mainly backward selection with an upper Wald p-value limit of 5 %, the best possible model was obtained by using only the two covariates associated with cavitation, RF53H and RF54H. The model is presented below as (4.2.).

$$h(t,\overline{z(t)}) = \frac{1.464}{1431.8}\cdot\left(\frac{t}{1431.8}\right)^{0.464}\exp\left(0.127\cdot \mathrm{RF53H} + 0.143\cdot \mathrm{RF54H}\right) \qquad (4.2.)$$

The following results were obtained with residual analyses of (4.2.):

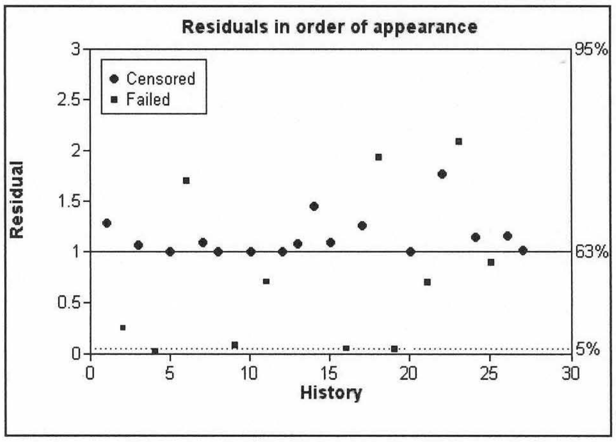

Figure 4.5.: *Residuals in order of appearance*

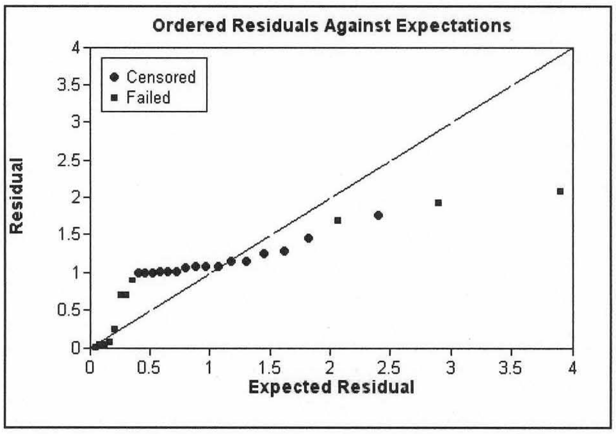

Figure 4.6.: *Ordered residuals against expectation*

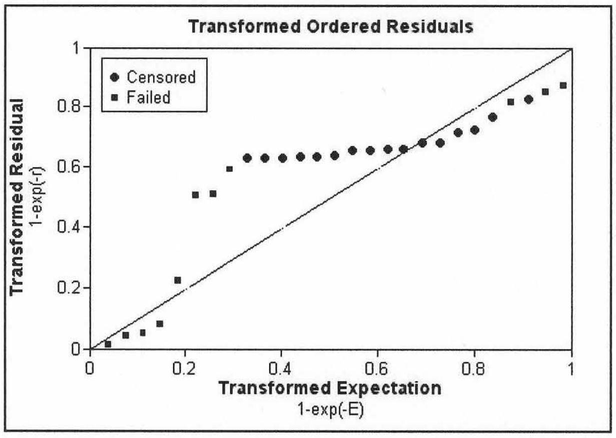

Figure 4.7.: *Transformed ordered residuals*

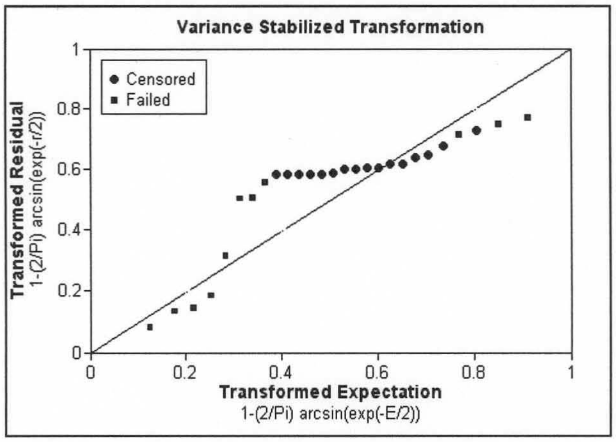

Figure 4.8.: *Variance stabilized transformation*

The results of analytical significance tests on the parameters are summarized in Table 4.1. It is clear that both RF53H and RF54H are very significant in the failure process although the shape parameter did not prove to be significant. The KS-test was determined to be $KS = 0.3180$ with a *p*-value of 0.00628, which is not an extremely good model fit.

	Parameters		
	β	**RF53H**	**RF54H**
Estimate	1.464	0.1271	0.1414
Standard Error	0.4719	0.0227	0.0569
Wald	0.9678	31.24	6.172
Wald *p*-Value	0.3252	0.000	0.013

Table 4.1.: *Results of analytical goodness-of fit tests performed on (4.2.)*

The graphs obtained from the residual analysis show that 4 of the 6 short failures (see comment 4 in section 3.3.) cannot be explained well by the model (e.g. with high covariate values). The data was analyzed and it was found that no other quantitative covariate contributed significantly to these early failures. Further analyses of the data revealed that the contribution of RF53H and RF54H to the other, longer, failures is evident. The model with RF53H only was also considered, and a better model fit was obtained $KS\ p - \text{value} = 0.145$. Still, this model did not explain the 4 short failures any better.

It was noticed that for all considered models, the shape was not significant although the hypothesis of $\beta = 1$ was never rejected with Wald *p*-values between 0.18 and 0.36, except for the model with RF53H only, where the Wald *p*-value was calculated to be 0.062. (Values of β ranged from 1.4 to 2). After this observation, models with $\beta = 1$ were hence estimated, and in all cases better model fits were obtained than with $\beta \neq 1$ (Wald *p*-values >10 %). It is important to note however that RF53H and RF54H were, as before, the only two significant covariates (after a process of backward covariate selection based on Wald *p*-values). This implies that time (working age) is not a significant variable in the model and that some of the failures could be better explained by an additional non-observed covariate. Vibration technicians do not agree with this statement and the problem possibly lies in a too short data horizon.

4.3 Phase 3: Weibull PHM With All Covariates

For this phase, the inspection data was analyzed with all the covariates. Inspection showed that subjective covariates are somehow, 'complementary' to the numerical covariates, i.e. at failures the majority of them have the value 1 if numerical covariates are low, and mostly the value 0, if numerical covariates are high.

To get a feel for the behavior of the subjective covariates, they were first analyzed separately. Only LNF4H appeared to be significant, with test model fit KS *p*-value = 0.17, which is acceptable. It was further noticed when LNF3H and LNF4H are in the model, their regression coefficients have the opposite signs and the same for HFD3H and HFD4H, as they tend to compensate each other. The data was analyzed again and the correlation coefficient for LNF3H and LNF4H was calculated to be 0.57, and for HFD3H and HFD4H to be 0.80. The high correlation between HFD3H and HFD4H could be because of the similar configuration (and operating conditions) of the bearings and a lack of lubrication will affect both bearings.

The next step was to build a model with all covariates included. Estimation procedures (both Snyman and Newton-Raphson) failed to converge initially, with the scale parameter approaching infinity. In such a case, it is not simple to decide which covariate to exclude from the model. By looking at the highest partial derivatives in the model fitting optimization routine, it was decided to exclude RF043H from the model (this could be because of the few unproportionally high observations). Still the estimation procedure would not converge and it was necessary to exclude more covariates from the model, in different combinations, to get convergence. These covariates were considered in the model at later stages, to check whether their removal was not only due to some relationship with other covariates.

A good example of this was RF53H, which was removed in one of the procedures at an early stage, and when later considered showed high significance. When both RF53H and RF54H were included in a model, they appeared as significant, but with a poor model fit, due to the large residuals when both covariates have high values. Their correlation was found to be 0.60 (one measurement with a very high value for RF53H and a very low value for RF54H was excluded). It shows that both these covariates are good predictors of failures but it still has to be decided whether it makes practical sense to include both in the final model.

When RF53H was removed from the model (either at an early stage, as mentioned above, or to improve the model fit), RF54H and LNF4H remained in the model, with a p-value for the scale parameter of 0.416, and the model fit KS p-value = 0.015, which is not very good. When β was fixed to 1, a much improved model fit was obtained (KS p-value = 0.647). From the residuals it appears that some of short failures could be better explained by this model, than by the model without LNF4H. The sum of LNF3H and LNF4H was also included as a covariate in the model with RF54H, and the model showed a very good model fit for both estimated values of β, and β fixed to 1. This definitely shows that subjective covariates could be useful in the pumps' condition diagnosis.

4.4 Final Model

The analyses showed clearly that RF53H and RF54H are the two most significant covariates in the data and will hence be used in the final model. It was also decided that the shape parameter should not be restricted in the final model although better model fits were obtained with $\beta = 1$. The only reason for the good performance of the model with $\beta = 1$ could be because of a shortage of data and it was concluded that a model with $\beta \neq 1$ would be of more practical use.

The final PHM with which the decision models will be constructed is presented below as (4.3.), previously (4.2.):

$$h(t, \overline{z(t)}) = \frac{1.464}{1431.8} \cdot \left(\frac{t}{1431.8} \right)^{0.464} \exp\left(0.127 \cdot \mathrm{RF53H} + 0.143 \cdot \mathrm{RF54H}\right) \qquad (4.3.)$$

5 Decision Model

This section describes the construction of the transition probability matrices (TPM) as well as the calculation of the optimal cost function. The policy is evaluated theoretically by applying it on the observed data but also evaluated practically on a second data set collected from November 1st, 1998 to February 28th, 1999, as if it was used in a real life situation.