



Vibration Covariate Regression Analysis of Failure Time Data with the Proportional Hazards Model

by

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There is no doubt about the potential economical advantages of preventive renewals based on statistical failure analysis or vibration monitoring, if performed correctly. Despite their advantageous abilities to produce economical benefits, both techniques have shortcomings. Vibration monitoring strategies recommend renewal based on short term vibration information only (often by waterfall plots or other short term trending techniques) and no scientific technique exists with which long term vibration information can be included in recommendations. Conventional statistical failure analysis techniques, on the contrary, utilizes the statistical long term life cycle cost optimum to base renewal decisions on and do not take diagnostic information (like vibration information) into account. The mentioned techniques complement each other extremely well and in this dissertation a scientific method to integrate these techniques was searched for with the emphasis on practicality.

Regression models with the ability to handle covariates (explanatory variables) was found to be the most logical route to the dissertation objectives. After an extensive literature study, the Proportional Hazards Model (PHM) was selected as the most suitable model for this application because of its sound theoretical foundation, numerical tractability and previous successes. The PHM was thoroughly researched including the investigation of different model forms, numerical parameter estimation techniques, covariate behavior and goodness-of-fit tests. A decision model utilizing the PHM, with the ability to handle non-monotonic covariates (like vibration parameters) through Markovian chains, was also identified and studied.

Data obtained from a typical South African industrial concern was collected and modeled with the studied theory, with promising results.

Keywords: Proportional Hazards Model, Vibration Monitoring, Failure Analysis, Preventive Maintenance, Renewal



Opsomming

Vibrasie Verklarende Veranderlike Analise van Falings tyd Data met die Proporsionele Gevaarkoers Model

deur

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Daar is geen twyfel oor die potensiele ekonomiese voordele van voorkomende hernuwing gebaseer op statistiese falingsanalise of vibrasie monitering nie, mits dit korrek bedryf word. Ten spyte van hul vermoëns wat groot finansiële voordele inhou, het beide tegnieke tekortkominge. Vibrasie monitering strategieë beveel hernuwing aan alleenlik gebaseer op korttermyn vibrasie inligting (tipies deur waterval grafieke of ander korttermyn tendens identifiseringstegnieke) en geen wetenskaplike tegniek bestaan waarmee langtermyn vibrasie inligting in besluite geïnkorporeer kan word nie. In teenstelling hiermee, neem konvensionele statistiese falingsanalises geen diagnostiese inligting (soos vibrasie inligting) in ag in hernuwings aanbevelings nie, maar grond besluite op die minimum statistiese langtermyn lewensikluskoste. Die bogenoemde tegnieke komplimenteer mekaar uitstekend en in hierdie studie is daar gesoek na 'n wetenskaplike metode waarmee die twee tegnieke geïntegreer kan word met die klem op praktiese toepaslikheid.

Regressie modelle met die vermoë om verklarende veranderlikes te hanteer, het geblyk om die mees logiese roete na die studiedoelwitte te wees. Na 'n deeglike literatuurstudie is besluit dat die Proporsionele Gevaarkoersmodel (PGM) die mees geskikte model is vir die genoemde toepassing vanweë sy breë teoretiese fondasie, numeriese implementeerbaarheid, asook vorige suksesse wat behaal is in soortgelyke toepassings. Die PGM is in diepte bestudeer, onder andere verskillende modelvorms, numeriese parameter beringstegnieke, verklarende veranderlike gedrag en pasgehalte toets. 'n Besluitnemingsmodel wat op die PGM gebaseer is met die vermoë om nie-monotone verklarende veranderlikes (soos vibrasie parameters) te hanteer deur Markov kettings is ook geïdentifiseer en bestudeer.

Data is verkry uit 'n tipiese Suid-Afrikaanse industriële situasie en is gebruik om die bestudeerde teorie te toets, met belowende resultate.

Sleutelwoorde: Proporsionele Gevaarkoersmodel, Vibrasie Monitering, Falingsanalise, Voorkomende Instandhouding, Hernuwing



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Notation

$\bar{z} / \overline{z(t)}$	Covariate vector dependent / independent of time
$\lambda(\overline{z(t)})$	Functional term dependent on time and covariates
$z_i / z_i(t)$	i^{th} covariate dependent / independent of time
$\bar{\gamma}$	Vector of regression coefficients
\hat{P}_{ij}	Transition probability estimate
Δ	Inspection interval
β	Weibull shape parameter
η	Weibull scale parameter
λ_{ij}	Transition rate from state i to j
$[\]^T$	Transposed vector
AFTM	Accelerated failure time model
CBM	Condition Based Maintenance
C_f	Cost of unexpected failure renewal
CM	Condition monitoring
C_p	Cost of planned preventive renewal
d	Threshold risk level
f	Unreliability function / Cumulative failure density function
F	Failure density function
$g(t)$	Warning level function
h	Hazard rate
H	Cumulative hazard rate
h_0	Baseline hazard rate
IID	Independent and identically distributed
KS	Kolmogorov-Smirnov
L	Likelihood
l	Log-likelihood
LCC	Long term life cycle cost
L_e	Expected life
MTTF	Mean time to failure
NHPP	Non Homogeneous Poison Process
$P\{\}$	Probability
PDF	Probability density function
PHM	Proportional hazards model / modelling
PL	Partial likelihood
POM	Proportional odds model
PWP	Prentice Williams Peterson model
$Q(d)$	Probability of failure renewal



R	Reliability / Survivor function
R_0	Baseline survivor function
ROCOF	Rate of occurrence of failure
t	Continuous time
T_i	i^{th} Observed renewal time
T_i	i^{th} Random renewal time variable
TPM	Transition probability matrix
$W(d)$	Expected time to renewal

1 Introduction

The first part of the book is devoted to the introduction of the renewal theory, which is a branch of probability theory that deals with the study of the times between successive renewals of a system. The theory is applied to the study of the times between successive failures of a system, which is a common problem in reliability engineering. The theory is also applied to the study of the times between successive repairs of a system, which is a common problem in maintenance engineering.

The second part of the book is devoted to the study of the times between successive failures of a system. This part is divided into two chapters. The first chapter is devoted to the study of the times between successive failures of a system with a constant failure rate. The second chapter is devoted to the study of the times between successive failures of a system with a failure rate that is a function of time.

The third part of the book is devoted to the study of the times between successive repairs of a system. This part is divided into two chapters. The first chapter is devoted to the study of the times between successive repairs of a system with a constant repair rate. The second chapter is devoted to the study of the times between successive repairs of a system with a repair rate that is a function of time.

The fourth part of the book is devoted to the study of the times between successive renewals of a system. This part is divided into two chapters. The first chapter is devoted to the study of the times between successive renewals of a system with a constant renewal rate. The second chapter is devoted to the study of the times between successive renewals of a system with a renewal rate that is a function of time.