Investigation of factors influencing the determination of discount rate in the economic evaluation of mineral development projects

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Dissertation summary

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For the evaluation of mineral development projects, it is imperative to consider the risks involved in mineral exploration and development and to bear in mind that an adjustment for these risks is a common practice which implies raising the minimum discount rate

A company may for instance use different discount rates depending on the different risks involved so as to compensate for the variability of success.

In determining a discount rate, an organization should follow this rule "The greater the risk, the higher the discount rate should be.

The discount rate will have a great influence on the economic evaluation of mineral projects. All other factors used for calculating the NPV (Net Present Value) being equal, the project at hand may be accepted or rejected depending upon the discount rate, and the fluctuation of the NPV from positive to negative.

It must be pointed out that the determination of the discount rate is the most difficult and vital aspect of "cash-flow analysis." In practice however the discount rate is usually fixed by top management and then delegated to the respective departments responsible for actual economic evaluation of the investment alternatives

A major problem in determining the appropriate discount rate is that it effectively depends more on subjective perception of the degree of risk or other past experience factors than on a systematic approach.

By using a risk-free rate of return, plus a subjectively determined risk premium, a discount rate may be developed, which is expected to compensate the investor for the extra risk involved.

In practice the selection of risk-free rate of return is relatively simple. In most cases, the yield on government bonds, under non-inflationary conditions, is adopted as the risk-free rate of return.

The real problem lies in the choice of the risk premium which must be adequately adapted to compensate for the additional risks associated with the investment under consideration.

Consideration of proper conditions in respect of a specific project under economic evaluation should help to determine the risk premium. The risk premium should be entirely dependent on the risks influencing the mineral development project

All possible risks affecting a mineral development project under consideration should be taken into account, when determining an appropriate risk premium. This is a stupendous task and will imply a large number of risks, which will no doubt make the determination very difficult to tackle and use. Furthermore, there are naturally numerous difficulties in structuring an analysis with many factors, because it is complex and multi-faceted.

In order to facilitate the implementation of the determination, there are usually a definite number of key risks to be observed. Risks, crucial for success of the mineral development project, are classified as follows:

- Technical risk– reserve, completion, production
- Economical risk –price, demand, foreign exchange
- Political risk – currency conversion, environment, tax, nationalization

From the review of factors influencing the determination of discount rate carried out (Section 4), it is concluded that the quantitative methodology for discount rate should be a process of identifying potential factors (risks), analyzing factors to determine those that have the greatest impact on mineral development, and determining discount rate. It is therefore imperative to find a method whereby all mining risks, together with their probability and impact, and an understanding of the combined effect of all risks attached to the cash flow and the rate of return. Thus then a way of a procedure calculating risk scores is required. Existing knowledge should therefore be used optimally to determine discount rate.

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1 Introduction

1.1 Introduction and background

Mining is based on the minerals on or buried in the ground. These are unchangeable facts and this different nature of mining creates a larger risk while requiring heavy capital investment with relatively long payback period when compared with other business sectors.

Thus it takes more careful assessment and sensitive decisions when investing in mining in order to reflect such distinctive characteristics in mining. Investment decision in mining projects is usually made after economic evaluation which is common in every business application. However, result of assessment on any mine depends and varies on the purpose of the evaluators regardless of type of methods used for such assessments.

Purpose of evaluations on mines can be divided into two categories such as profit and social. Profit oriented evaluations by investors seek to assess possibilities of realizing profit or prevent any economical loss by operating mines. Social oriented evaluations can be referred to as political assessments in order to achieve the best use of limited resource for the benefit of the entire society in general. Investors may conduct the evaluation depending on the purpose such as purchasing or selling of the assets. Either conservative or liberal approaches may be taken depending on the surrounding environment. However, political assessments are not restricted in terms of taking conservative or liberal approaches since its goal would be focused on the efficient utilization of the resources for the social benefit.

The construction of a realistic investment model is required in the evaluation of a proposed mine project. This investment model should include significant variables that are not fixed or known with certainty; e.g. the length of time and the cost not only to obtain necessary permits, but the actual development of the mine and plant and whether the ore deposit is an economically recoverable mineral. The actual grade, recovery and price for minerals mined, operating costs and tax burdens are also uncertain factors in the process of evaluation.

Owing to the fact that we do not have a comprehensive projection of the possible relevant variables, we are therefore obliged to estimate these in the decision making process. To arrive at a solution to the project evaluation problem, we will need to determine the level of discount rate for each project, within an acceptable margin of error.

When determining a discount rate, a golden rule should be followed by "The greater the risk, the higher the discount rate should be".

There is no doubt that the determination of the discount rate to be implemented for a given project, which is typically achieved by using risk free market rates plus a management fee and risk adjustment, is the most difficult aspect of cash-flow analysis. In practice, however the discount rate is still subjective and dependent on corporate or other experience factors. This is usually determined by top management and then handed down to the departments responsible for the immediate evaluation of projects.

This study will address the technical, economic and political risk factors which might influence the determination of the discount rate, with particular reference to the role and impact there of on the economic evaluation. This study will also pinpoint the procedure of determining discount rate by using aforementioned factors while considering various mine evaluation methods used for the investors purposes.

1.2 Problem statement

When evaluating investment opportunities, one should consider the risks associated with mineral exploration and development. These are commonly classified as technical, economic, and political risks, and are accounted for in the investment decision by changing the discount rate. Thus, a company may use different discount rates associated with varying risks in order to compensate for the variability of success.

The discount rate has a tremendous effect on the economic evaluation of mineral projects. Even when all other factors used as inputs for calculating the NPV (Net Present Value) are equal, the project under consideration may be accepted or rejected depending upon the discount rate, and the fluctuation of the NPV from positive to negative.

Some companies reject all possible projects that did not yield an expected rate of return of at least 20 percent. But, other companies would be more than satisfied if they could commence on projects yielding two thirds of that rate.

Determining a realistic discount rate for a given project is therefore the most difficult and important aspect of cash-flow analysis. It should be determined with the consideration of proper technical, economic and political conditions surrounding the specific project undergoing economic evaluation.

One key problem for determining the appropriate discount rate is that it typically depends more on subjective perception of the degree of risk or other experience factors than on a systematic approach.

Thus, the main aim of this study is to identify, analyze and document the type, role and impact of risk factors influencing the determination of discount rates. The potential and significance of a discount rate as a method of risk adjustment is emphasized, together with some guidelines as to how the discount rate should be determined.

1.3 Objectives of the study

The objectives of this study are to:

1) Outline the nature and scope of risk and uncertainty factors influencing the determination of the discount rate for feasibility studies of projects in the mineral industries,

2) Develop the most useful analytical techniques for determining the appropriate discount rate for use in the economic evaluation of mining projects,

3) Present the potential and significance of the discount rate as a method of risk adjustment; and finally,

4) Introduce the basic concept of identifying and analyzing risks and uncertainty and show how to reduce or eliminate risk.

2 Meaning of mine evaluation and procedures

2.1 Mine evaluation

Investment usually means deploying reserved capital created by saving current expenses in order to create future value for the capital used. The main purpose for the investors is to create profit. However, it is fairly difficult to set the definitive term for the investor's purpose since the purpose depends on various purposes theoretically such as profit, economic, welfare, service and balance. Solomon's market economy states that the role of the investors or businesses are, broadly speaking, the society's agents for producing and meeting the society's desire by using limited social resources with responsibilities for the society in general. In micro economics, such responsibilities are met through maximizing the profit which is regarded as the businesses' action principal. In other words, it is inevitable for the businesses to pursue the most efficient utilization of the resources in order to maximize the profit by increasing profit under given certain circumstances while avoiding any loss. These activities of achieving maximum economic value are quite simple principle, however these activities bring the most efficient utilization of resources while promoting maximum social welfare under competitive market.

Maximization of profit from theological point of view and maximization of the value in real world has two faces. Profit itself is not important but, its value is more important. Such economical theory has market value, subjective value and book value. Market value means the actual value of an asset when sold in the market. Subjective value means the compensation level when one would like to sell the asset. Book value means any irrecoverable asset created in the accounting point of view.

In real world when transactions are made, evaluation of a value is made by the buyer or seller's subjective view. Market value is made through the negotiations of the two parties. Therefore, evaluations for investment purposes are made on the subjective view. In order to make profit upon deciding to invest, a company needs a capital in order to buy various necessities which require large costs which need to be funded with capital.

In short, investment is a sacrificing of a current value in order to create a future value that is subject for change. Such activity requires time and risk. Therefore, investors pursue interest rate in order to compensate for the time, the opportunity cost and risks associated with the investment. Interest rate is a connection between the capital and the profit. Such interest rate can be recorded as the cost of capital or a premium for a future value while always having a quantitative value.

Investment evaluation from a company's point of view is an assessment to see if enough compensation can be made by planning on possessing a certain asset while consuming the time and risk associated with such asset.

2.2 Characteristics of mining

Mining possesses a special characteristic when compared with other industries since the minerals are buried under ground. These characteristics should be considered while assessing feasibility of investment in mining. This research compares those different characteristics with other industries.

1) Limited availability

General manufacturing business can choose the materials freely while controlling the quality and quantity to utilize. In mining, minerals that can be referred to as materials are given and set by the nature. This given nature of mining such as grades and tonnages are unchangeable. Even if there was high grade and large tonnage mine which can be developed and operated profitably, the reserve is only subject to decrease as mining activities are carried out. This leads to the depletion of the mine, forcing the mine to close within certain number of years. Other businesses may renovate the necessary facilities in order to run for continuous years, mining ends with the depletion of the mineral reserves.

Machineries and plants can be replaced and recovered through depreciation and amortization, however minerals can not be replaced once mined out. Therefore, consumptive business such as Mining carries out depletion in addition to the depreciation and amortization. Depletion is done through actual mined out quantity to calculate the remaining life of the project regardless of the time flow.

2) Realization of the value

In case of closing of the manufacturing business, materials and machineries still possess their own values whereas minerals in mining does not possess any value unless they are mined and produced. In addition, almost all assets naturally increase in their values with the time and the development of the society however, values of the mining permits or areas are not increased unless they are developed. Also, machineries and other facilities placed in the mine seem to lose their values upon closing of the mine regardless of their age.

3) Investment pattern

In order to develop a mine, sufficient exploration activities are carried out in order to confirm the reserves and the grades of the ore body. Pre-production period follows in order to carry out the construction activities before commencing production whereas, normal manufacturing business only requires investment in constructing facilities required before production. Mining requires heavy investment in order to define the ore body before the investment in facilities required for production.

Further investments such as bolting, transportation, ventilation, drainage system for production are unavoidable while replacing equipments and installing new facilities are required in order to keep the same level of productivity as the increase in depth of mine and decrease in grades are met. Mining also requires reclamation fees for environmental reasons upon closing in order to preserve the environment.

Leading time up to production in mining ranges from minimum of 2 years to 4 or 6 years upon commencing the development of a mine. Although the mine possessed strong economic viability during the construction phase due to high mineral prices, sudden change of climate in mineral prices in the production years may cause unexpected loss for operating a mine.

4) Risks

Mining is usually regarded as a business with very high risks associated with it. These risks widely range from geological risk to social environmental risk. These risks can be largely divided into four main areas such as risks generated from the ore body characteristic, development environment, market environment and social environment.

First of all, geological risk is mainly a possible estimation error of reserves and grades due to faults, dykes, and change in the foot wall, sudden change in strike and dip, changes in the contained metal quantity.

Second of all, the development risk is associated with natural hazards such as possible explosion of gases, bursting of underground water, fire hazards and cavings of drifts or any other developing in the mine. In addition, default in mine designs or increase in operating costs for using inappropriate equipments and delays of construction development due to natural hazards can be the examples of the development risks.

Thirdly, market environment risks include falling price of produced metal, management failure, lack of labour, union strikes, developments of other competitive mines and trade regulations.

Lastly, social environment risks are environment regulations, excessive taxation, revolution, negative public opinion on mining, government interference and nationalization of the mineral properties.

Mining has distinct characteristics in each area explained above such as limited availability, realizing value, patterns of investment and risks. Oded Rudawsky (1986) categorized mining characterization as follows.

- The heterogeneous distribution of deposits.
- The depletion of individual deposits and fields as ore are extracted from them.
- The high degree of uncertainty and risk associated with exploration for and production of mineral deposits.
- The long lead time between discovery and full operation of deposits.
- The availability of byproduct and co-product in same deposit or fields.
- The need for intensive capital investment for development of deposits.

- The spatial aspects of potential reserves, sometimes requiring extensive development of economic infrastructure.
- The increasing role of international trade in ores, mineral, and energy commodities and the political and social implications of such activities.
- The apparent conflict between mineral development or utilization and the environment.
- The availability of scrap as a secondary source of supply, sometimes supplementing yet sometimes competing with primary sources.
- The increasing role of governmental involvement in the industry in the form of regulations, taxation, stockpiling, price controls, and even production.
- The national security concern, requiring a viable indigenous mineral and energy industry.
- The problem of conservation and preservation of resources vs. their rate of use.
- The apparent conflict between maximizing private interest gains or social benefits in the development and use of mineral resources.

2.3 Mine evaluation procedure

As described above, mining investments are largely different with investments in other sectors because the target of mining investment is minerals that are unseen in the ground. However, mining evaluation is similar to other sectors since mine is evaluated on the present value of possible generation of cash flows associated with certain time and risks by incorporating certain interest rate that seems proper for such investment.

In order to evaluate a mine, reserve needs to be calculated to estimate how much mineral is buried underground, then decide the capacity of production based on a certain mining method that suits the ore body. Finally, estimation on the possible revenue generated by selling the produced mineral product has to be made. Therefore, when evaluating mines, the information on the mineral property needs to be collected and analyzed first before calculating possible cash flows, then a certain discounted rate needs to be chosen to calculate the net present value of the mine based on the investor's evaluation standard.

Purposes of the mine evaluation are to ①sell a property ②obtain loans from a bank ③assess necessary amount of capital investment ④evaluate the value of the assets. There are many other purposes for evaluating a mine and there can be drastic difference in the outcome depending on who is evaluating the subject. For an example, manipulation of information data may be possible in order to maximize the value if the owner of a mine was conducting the evaluation. Conservatively minimizing the value may be favorable to minimize the risk on loss if lending institutions such as banks conduct the evaluation.

Therefore, increasing reliability on the evaluation by conducting the study with objective view by collecting information while proving correctness of such data by visiting the actual site is important.

In real applications, a standard procedure is adopted to evaluate a prospective new mine or pre-existing mine. This procedure can be generally described as the following steps.

1) Procedure on evaluation of a new mine
- Reserve and Grade calculation
- Calculation of Mineable Reserve
- Calculation of Mine recovery
- Set production rate
- Calculation of Mine Life
- Estimation of Net Smelter Return
- Estimation of operating cost
- Calculation of total revenue
- Calculation of depreciation, amortization, reserve depletion
- Calculation of taxation
- Calculation of Income after Tax
- Calculation of cash flows from each year
- Calculation of return on investment with discount rate
- Calculation of discounted cash flow

2) Procedure on evaluating pre-existing mines
- Collection of Information showing past record
- Evaluate mineable reserve and additional exploration potential
- Set production rate
- Estimation on the remaining Mine life estimation
- Assessment of operating cost (for the last 5 years)
- Obtain Net Smelter Return for the past 5 years
- Investigate other costs/expenditures

- Calculation of yearly depreciation, amortization
- Forecast future Net Smelter Return
- Calculation of profit
- Analysis of risk, appropriate discount rate, Rate of Return
- Discounted Cash Flow

Mine evaluation procedures can be divided into three modules of reserve evaluation, optimization decision and feasibility study.
Reserve evaluation module includes steps from collecting drilling samples to reserve calculation. Optimization decision module consists of selecting appropriate mining techniques suitable for a specific ore type while calculating mine life and optimum level of production. Feasibility study module includes an economical analysis of a proposed funding plan along with financial analysis for the cash flow projection as well as thorough review of risks that may affect the project such as politics and environmental issues. However, these modules may not necessarily occur in chronological order as they can be carried out separately and concurrently.

2.3.1 Reserve evaluation module

There are many methodologies available in order to carry out the described tasks above. ID (Inverse Distance Method), IDS (Inverse Distance Square Method), WM (Weight Method) and PM (Polygon Method) are some traditional ways to analyze the samples taken from the drillings in order to define the ore zone or ore bearing geological structures however, they are known to contain a large error posting low confidence level when compared with the real values. Recently, Kriging method is more common and thought to generate more reliable result than the methods based on the interpretations of the distances and grades of each sample to explain the ore zone. Kriging method is a statistical analysis of Variogram which can identify the continuity of ore zone by investigating distributions of the grades combined with the direction of sample groups.

Variogram is a model that defines the relation of the samples of more than two. Kriging which was named by Matheron finds the best linear unbiased estimator between the subject blocks and the samples. From the geological point of view, the practice of Kriging is based on assuming continued mineralization between measured values. Assuming prior knowledge encapsulates how minerals co-occur as a function of space. Then, given an ordered set of measured grades, interpolation by Kriging predicts mineral concentrations at unobserved points.

Kriging is very well known among the scholars in the world and it can be used not only for the mineral ore bodies but also for the oil and underground water interpretations as well.

2.3.2 Optimum production level decision module

As the mining scale gets larger the annual cash flow may increase when designing mine plan however, the capital investment will also increase along with the mine capacity while shortening the mine life. On the other hand, smaller mine scale will decrease the capital investment while increasing its mine life however, the cash flow will also shrink which in return will decrease the value of the mine when considering time value of money.

Therefore, insufficiently small scale of mine will provide no guarantee on a favorable rate of return on investments while too big of a scale mine can trigger the over spending on the capital which may exceed the value buried under ground.

Two extreme cases described above will definitely not be economically viable and favorable. Thus, there is a need to find the level of production between the two extremes which optimizes the rate of return by balancing the cash-ins and cash outs. Recently, optimum model with assumptions of linear relationships between the production level and the production cost as well as between the production level and the capital investment is being developed in order to find the optimum level of production capacity.

2.3.3 Feasibility study module

a) Acquiring the evaluation elements

When investing in any kind of business, investors' first and the most concern would be to eliminate any possible risks in the future. Collecting information and analyzing seems the first right step in order to meet the goal explained the above.

In mining, investors also collect needed information however, in much larger quantity and scope due to the special nature of mining businesses. This required information can be divided into three large groups of ore type, market and the social related information.

* Ore type

Underground ore body is not something that can be changed or manipulated. Ore body characteristics are naturally given and such physical restriction is very important factor in investing in mining.

Thus, right investment can be made only with correct understandings for such restrictions. Information regarding the ore body type can be separated again by the two groups of direct and indirect information relative to ore body. Some examples of directly related information are shape, grade and continuity, geological information including geological structure, ore characteristics, hardness, chemical composition information, and depth of the deposit, exploration history, and reserve estimate. Indirectly related information includes mine location, topography, surface condition, access roads, power, water supply source and local human resource availability.

Based on all of the information mentioned above, suitable mining technique, production capacity, mine life and scope of capital expenditure can be decided for the new mine. For studying a operating mine, this kind of feasibility study gives feed back on the currently adopted mine plans.

*Market Attribute

Thorough study of market condition is also critical since the market condition decides whether a particular mine is economical or not even if a particular mine has good characteristics of ore body or ample amount of reserves. Factors related to the value of assets, revenue source and cost items are the main information required to assess market condition.

Namely, any information relative to marketing of goods such as type of saleable products, market location, possible replacement, demand, market price along with other relevant trends and availability of labour, trend of labour rate increase, history on labour relations, purchasing price of required equipments, power usage cost, operating cost along with other relevant cost information are needed. Required information list mentioned above can contribute to forecast the future profit.

* Social Attribute

Social attributes are the intangible factors that affect the investment. They are political information such as fiscal regime or environmental regulations and public opinion on mining. Investment becomes impossible if social attribute is not good even if the mine itself possesses great economic potential.

In today's environment where globalization, namely making overseas investment, is the world's trend, relevant country's investment environment, control of media and information, nationalization of properties, fiscal regime and public opinion on mining must be checked.

Although it is ideal and practical to collect all of the information mentioned so far in order to evaluate the economic viability of a mine, it is very hard to collect all of the data and evaluate them to adopt the results into the evaluation model. Therefore, filtering for the most important information is needed. Filtered information used to carry out the evaluation process and other non-filtered material can then be used as supporting information to make decision.

The most important information include reserve, grade, production capacity, capital expenditure scope, operating cost, depreciation, amortization, interest rate and net smelter return (revenue).

b) Calculation of the future cash flow

Previously it was explained that the value of mine asset is an expected return by owning the asset. According to Hoskold method, this kind of expected return is revenue generated by subtracting operating cost from NSR (net smelter return) or cash generated from marketing activity.

In DCF (discounted cash flow) method, an expected return takes all of the cash related items into consideration to calculate the possible cash flow. Hoskold method is simpler since the return on investment is revenue minus cost. Cash Flow needs further consideration in addition to the Hoskold method.

Revenue is a reflection of a company's performance for the accounting purposes however, cash is more important in terms of operating a company in real world.

Cash flow is Cash inflow minus Cash outflow. Cash flow can be consistent each year as assumed in Hoskold method. However, it varies each year in most cases.

Each year's cash outflow items are all required investment amount covering capital expenditure, working capital, operating expenditures and all other expensed items and their magnitude can range depending on the production capacity and specifications of equipments.

Cash inflow is mainly a return from the smelter by selling concentrates (product) or cash received from the marketing activities by selling produced metal.

When calculating Cash flow, total revenue, operating cost,

depreciation, amortization, interest, tax, tax credits are the main required items. Total revenue is a market price multiplied by the quantity of the products sold.
Operating cost include costs for materials, labour, interest on debt and working capital interest needed for activities such as drift development, mining, bolting, processing, transportation, electricity and all relevant items need to operate the mine.

Depreciation and amortization is a compensation provided to cover the losing value nature of equipments as the time passes. They are calculated by dividing the original cost of items over a certain number of periods set by the law.

Calculation of the cash flow can be done automatically if relative information can be assumed correctly. Net Income can be calculated by subtracting operation cost from the total revenue generated by selling the product. Net Income before tax can be calculated by subtracting depreciation, amortization, interest and any carried loss from the Net Income. Net Income after tax can be simply calculated by subtracting the tax payment set by the fiscal regime. Cash flow can be created by adding depreciation, amortization and any carried loss items to the Net Income after tax.

c) Decision on the capital expenditure

Among the DCF methods, NPV (Net Present Value) method converts all of the future cash flows into one present value by applying a certain discount rate and the project is thought to be acceptable if the present value is larger than zero.

IRR (Internal Rate of Return) method provides a comparable rate of return to the investors' hurdle rate in order to make the investment. IRR is a discount rate when NPV equals zero. If IRR exceeds the investors' hurdle rate then, the project is qualified for making investment.
Therefore, choosing discount rate and IRR which are the focal points of making investments is very important and difficult when evaluating the economic viability of the projects. Such discount rate can be presented in various formats depending on the object of its applications. Of such discount rates, all of Minimum rate of return (minimum requirement for the investment decision), Cut off rate (objection rate against the capital expenditure), Hurdle rate (guaranteeing justification of capital expenditure), Target rate (purpose of the investment) and Cost of capital (capital required to facilitate financing investment) mean minimum compensation for the time and risk required by the investors.

Then, what are the necessary things to consider when deciding the discount rate?
Eugene Logrant suggests that possible usage of invested money and its sources, opportunity costs, risks, life of the investment and currency value (interest rate) are needed to be considered when choosing the minimum rate of return. Solomon defines target rate as same as the opportunity rate. Therefore it can be generally concluded that the discount rate should be a reflection of the risk associated with the investment, opportunity cost and the currency value.
However, discount rate is more of a discretionary figure since it

reflects investors' subjective point of view while it is also an objective matter from the company's point of view. Normally, Weighted Average Cost of Capital in accordance with company's financial structure is used when deciding such capital cost. In recent years, application of capital asset price model which uses rate of return as a projection of capital cost in accordance with risk level that each capital possesses when the financial market consisted of risky asset and risk free asset is balanced is being progressively used. Therefore it can be concluded that the rate sought by the capital asset cost model represents a company's cost of capital in the financial market in accordance with the company's exposure to risk.

However in mining, weighted average cost of capital method is more used since the capital asset price model is currently being further researched in order to eliminate the model's non-flexible assumptions.

3 Methods of investment appraisal

3.1 Introduction

The normal procedure when evaluating investment opportunities is to compare the benefits of a presented opportunity with the costs involved, and then to invest in those projects that are worth more than the costs.

But what factors should be considered when estimating benefits and costs? How should the cost associated with exploration and development brought about in the early years of the typical mineral projects be compared with the benefits of mineral production of later years which are nevertheless uncertain? The answers to these questions differ considerably according to the context. Now we get the principles of investment analysis for private and public organizations, which provide a basis for the more specialized, subsequent discussions of the economic evaluation of exploration projects and mineral deposits.

The most important key to making investment in mines is to choose accurate data while applying appropriate evaluation technique because of complex nature of mining such as high level of risk, technical complexity, large capital investment and long pay back period.

There are a vast number of evaluation techniques as enumerated in many books and papers and as adopted in the mining industry, available to solve the investment decision problem

3.2 Traditional methods

Since the first time that the mines were developed in history, there should have been a method for evaluating such properties. However, it was late in the 19^{th} century when the first official qualitative analysis based on measurements was beginning to be used. Philosophy, methodology and general approach to evaluating mines developed during the 19^{th} century were being used for almost 100 years with minor modifications until the major change occurred in 1960's.

Until 1960s, investors viewed mining as a high risk business. All of the mines were developed by the company's internal capital since it was impossible to borrow the capital from other investors. Hoskold method and Payback Period method are developed based on the key concepts of public's view of mining as a high risk with limited life.
Payback Period method until 1960s was used as a primary basis for evaluating mining investment and the projects with payback period of greater than 5 years were regarded as unfavorable projects. However, using the Payback Period method as the first basis for making investment decisions proved many errors. Currently, this method is being widely used as a part of other evaluating techniques as a supporting or non-primary basis.

Hoskold method was developed by Henry d. Hoskold in 1888 in order to compensate the mining's high risk and limited life. It was being used for almost 100 years although it had mathematical problems. In Hoskold method, a certain portion of the Net Income was set aside as a dividend to shareholders each year in an attempt to mitigate the high risk associated with mining investment while another small portion of the Net Income is also retained at a low safety rate as a mean to recover the capital invested. Hoskold method introduced an idea of mine evaluation based on such risk rate and safety rate in order to calculate the value of a mine. For this reason it is highly regarded as the method dedicated much in mining industry although it had an equation error.

3.2.1 Payback Period (PB)

The payback method is most commonly used as a guide and is especially useful when evaluating very risky projects.

Payback Period method involves cash flow from operating activities becoming eventually the same with the capital investment made in the project. The payback period is simple to calculate and is thus a useful preliminary indicator of a project's economic attractiveness. But the evaluation completely ignores cash flows after the payback period (both revenues and expenditures) and the time value of money during and after the payback period.

The method provides no guidance in the choice of an acceptable payback period. The choice is essentially arbitrary; one company may choose 3 years, for example, while another chooses 6 years under the exact same set of circumstances. Thus, payback period should not be used as the primary determinant of an investment decision.

The annual cash flows are summed up arithmetically until they equal or exceed the initial capital investment. Thus, this method uses cash flow, but not any interest compounding or discounting, even though the emphasis is on time, probing how rapidly is the investment recovered.

Mathematically:

$$PB = E + P + D + O$$

Where: E = Years Spent Doing Further Exploration

P = Years Spent Obtaining Necessary Permits

D = Years Spent in Development Prior to Start of operation

O = Years of Operations Necessary for Cash Flow from Operations to Equal Total Investment

Companies that evaluate mineral projects by payback period accept investments with payback period less than an arbitrarily defined length of time.

Pros)

- Simple calculation, easy to understand.
- Projects with large cash flow in relatively short time period are preferred, thus acting as mitigation mechanism relative to investment risk.
- Easy for application, lower management in organization can carry out the activity.

Cons)

- Hard to judge economic viability since investment option is evaluated from time perspective as supposed to profit oriented perspective.
- Disregarding cash flow after pay back period
- Disregarding of time value of currency
- Difficult to evaluate multiple cases of investment options with different scale.
- Focus around the short term profit as supposed to long term profit.

Range of Application)

- Can be used as a supportive mechanism to another evaluation technique rather than being used as an independent methodology.

3.2.2 Hoskold method

Hoskold developed his innovative unique mine evaluation equation considering mining's high level of risk and limited time nature of mine life in 1877 through publication in the Engineer's Valuing Assistant. His publication was the first equation oriented approach to evaluating mines which was used in practice for almost a century. During this era, a few scholars made attempts to make improvements on errors in the Hoskold's equation however, their improvements also were kept within Hoskold equation's boundary.

Assumptions made in Hoskold equation are as the followings,

- Constant annual income
- Annual Income is reserved for the dividend and capital recovery
- Constant rate of return on capital
- Invested capital recovered through the retained income at the end of mine life

Characteristics of this theory is that compensation for the risk is made through the form of dividend (risk rate multiplied by the income) on a yearly basis while a certain portion (safe rate multiplied by income) of annual profit is retained till the end of mine life in order to recover the invested capital.

Risk rate and Safe rate in Hoskold equation is different from each other. Interest rate for the safe rate is relatively low since the retained money is reserved at a low risk institution such as banks. Risk rate is relatively set at higher interest rate in order to compensate for the high risk involved with mining investment.

Looking at the derivation of Hoskold equation, if

Vp: Present value

Ir : Risk rate for the capital invested

Is : Safe rate for recovering the capital

A : Annual Income

N : mine life

At beginning of the pre-production years, Vp amount of capital is invested in order to generate A amount of income for n years. Value accumulated within Vp is represented by the future A form and vice versa.

However, dividend R is

R = Vp x Ir ···(1)

thus compensation for the investment. Therefore retained S would be S = A – Vp x Ir ·······································(2)

Retained S is reserved until the end of mine life. Retained amount of S has to equal the capital invested at the closing of the mine. Annul rate of Is while S is the value of retained income at the end of mine life If Vf is the

$$Vf = S \times (1 + Is)^{n-1} + (1 + Is)^{n-2} + \cdots + 1$$

$$= S \times \frac{(1+Is}{} \frac{(1=Is)^n - 1}{Is} \cdots\cdots(3)$$

However Vf by assumption has to be the same as the kept (or retained capital)

Since it involves any one, equation 2 has to be incorporated in to

$$= S \times \frac{(1=Is)^n - 1}{Is}$$

$$= (A - Vp \ \times Ir) \times \frac{(1 = Is)^n - 1}{Is} = Vp$$

Therefore, $Vp(1 + Ir \times \frac{(1 = Is)^n - 1}{Is}) = A \frac{(1 = Is)^n - 1}{Is}$

$$Vp = A \frac{\frac{(1+Is)^n - 1}{IS}}{1 + Ir \times \frac{(1+Is)^n - 1}{Is}} \quad \cdots\cdots(4)$$

$$= A \frac{1}{(Ir + \frac{Is}{(1+Is)^n - 1}} \quad \cdots\cdots(5)$$

Is the final derivation of the Hoskold equation.

☐ Problem in the Hoskold equation

As previously explained, difficulties in its application because of the assumptions made in the Hoskold equation exist. The summary of the problem is described below.

-During the operating years, the sum of the evaluated mine value and the retained money up to that point in time differs from the original evaluation value.

-Evaluation value is only the present value of the annual incomes as supposed to the cash flow.

-Retained fund for recovery of the invested capital and its application does not match with modern company's financial activities' behaviour.

-Dividends to compensate for the risk is calculated by the initial investment amount. They are not deducted from the initial investment amount even if the capital is repaid..

-Time value of the currency is not concerned.

-Hoskold method seems to under estimate the mine value.

3.3 DCF Method

During the mid 1950s, there had been two major trends that affected the overall decision for making mining investment.

First, complication of income tax separated income from the cash flow. Thus cash flow became more important.

Income was actual flow of the currency however, cash flow involves many items such as investment incentives, depreciation, amortization, interest payment, carried loss, income tax, depletion in addition to revenue although it depends on the accounting rules used.

Secondly, theories regarding capital investment have been equationed and expanded. Well known scholars such as Solomon, Van Horen tried to prove that the Interest rate that equalizes the present value of the cash flows with the present value of the cash out flows is the true rate of return. Such rate of return is called internal rate of return (IRR) and Canes called it as a limited efficiency of the capital in 1936.

Mine evaluation technique showed a great turn around in 1960~70s as above scholars utilizing new evaluation techniques were hired and involved in the mineral industry.

DCF method in wide range is considered as the valuation technique that turns future cash flow into the present value however, in many cases it is called as the NPV method and IRR method.

3.3.1 Net Present Value (NPV)

The net present value method is probably the most common evaluation technique and is used as the primary basis for investment decisions.

The net present value (NPV) is a discounted cash flow method that examines the present value of all the cash flows from a project after they have been discounted at the required rate of return.

The net present value calculation requires a discount rate to be specified. This discount rate indicates how much interest must be earned on the outstanding investment in each year of the project's life in order to justify making the investment.

The NPV is merely the sum of the net cash flows for every year in the life of the project, after being discounted at the specified rate.

The investment period must be included along with the anticipated operational life. The NPV includes present values of exploration and investment outlays made prior to production, as well as the present values of the cash flows from operations.

The NPV may be used as an indicator of project worth. A positive NPV signifies that the investment expenditure will earn a higher return than that specified by the discount rate. If the NPV is equal to zero, the investment will earn the exact return specified by the discount rate. Finally, if the NPV is negative, the investment does not earn as large a return as specified by the discount rate. Therefore; a positive NPV indicates an acceptable project, an NPV of zero indicates marginal project, and a negative NPV indicates an unacceptable project.

The discount factor for each year can be calculated by adding the discount factor to 1.00, take the nth power of this sum (where n is the year desired) and divide the result into 1.00.

Mathematically (if, discount rate is 20%):

Discount factor @20% = $1/(1+0.20)^n$ where: n = year desired.

The net present value of a stream of revenues and costs is given by the following formula:

$$NPV = (R0 - C0) + \frac{(R1 - C1)}{(1+R)} + \frac{(R2 - C2)}{(1+R)^2} + \cdots \frac{(Rn - Cn)}{(1+R)^n}$$

Mathematically:

$$NPV = \sum_{i=0}^{n} \frac{CFi}{(1+r)^i}$$

Where : NPV = Net Present Value

n = Total Life of the Project Including Investment Period and Operation Period

CF*i* = Cash Flow for the *i* th Year in The Project Life

r = Discount Rate

i = Range Consecutively from 0 to n and Denotes Each Year in the Life of The Project

Pros)

- Theoretically better than pay back period method and the average rate of return method
- All of cash flows and the life of the project are reflected
- Time value of money is considered

- Easier calculation than the internal rate of return method
- Cash flow in near future years are preferred

Cons)

- Company's capital cost or the decision maker's minimum rate of return must be set prior to the calculation
- Difficult for the lower management employees to share the work without proper education since application is harder than the pay back period method and the average rate of return method
- Assumption of cash flow being reinvested as capital cost

3.3.2 Internal Rate of Return (IRR)

The IRR is a discount rate at which NPV equals zero. It can be used to evaluate a proposed investment. The decision rule is that a firm should undertake those projects with IRRs greater than its minimum-acceptable rate of return (equivalent to discount rate).

If the IRR is less than the required discount rate, the project will not earn the required return. If the IRR is greater than the discount rate, the project will earn a better return than required and the investment should be made. Note that when a particular stream of cash flows yields a positive (negative) NPV, the IRR will be greater than(less than) the discount rate used in the NPV calculation.

This method also takes the cash flows and the time value of money into account. The IRR method has several drawbacks, in that it does not consider the size of the investment under consideration, the length of the project, or timing and size of annual cash flows. Thus, use of this method could possibly lead to sub-optimal choices for investment if it is used as a means of ranking for purpose of project selection.

The internal rate of return calculation requires setting the NPV to zero by adjusting the discount rate. The discount rate arrived at is the internal rate of return. Mathematically:

$$NPV = 0 = \frac{(R1 - C1)}{(1 + IRR)} + \frac{(R2 - C2)}{(1 + IRR)^2} + \cdots \frac{(Rn - Cn)}{(1 + IRR)^n}$$

Mathematically:

$$NPV = 0 = \sum_{i=0}^{n} \frac{CFi}{(1 + IRR)^i}$$

Where : R = revenues, C=costs.

n = Total Life of The Project Including Investment Period and Operation Period

CF*i* = Cash Flow for The *i* th Year in The Project Life

i = Range Consecutively from 0 to n and Denotes Each Year in The Life of The Project

IRR = Internal Rate of Return for the Project.

IRR of x % has the same meaning as the invested principal amount growing at x % compound rate basis

The IRR must be found iteratively by the trial and error method. One must solve the net present value equation by using different discount rates, getting progressively closer to the real IRR each time. This process is done iteratively until sufficient precision for the IRR has been achieved.

Whitney (1979) proposed a rough approximation to the IRR which can be determined by dividing the total investment expenditures by the average annual cash flow, and dividing the result into 0.7.

Estimate IRR = (0.7)÷{(Investment Expenditure) / (Average Cash Flow)}

The rough estimate determined in this manner is not sufficiently accurate for evaluation purposes, but it does serve as a useful starting point for subsequent trial and error refinement.

Pros)

Theoretically better than the pay back period method and the average rate of return method

- All of cash flows and the life of the project are reflected
- Time value of money considered
- Comparison of IRR with Capital Cost makes it easier to evaluate the investment value
- Cash flows in near future is preferred
- Tool to project the capital cost

Cons)

- Requires estimation of the company's capital cost
- Complex calculations hard to achieve without computational help
- Frequent change of sign (positive and negative) in cash flow may cause overlapping errors
- IRR must be used for evaluation of mutually exclusive multiple

investment analysis. Therefore, a project which has higher IRR than another does not necessarily mean it is the better project.

3.3.3 Profitability Index (PI)

NPV returns more favorable result as the investment size and scale gets larger since the method itself views profit as an absolute value rather than the cost basis. This makes it hard to compare two projects with different investment size.

PI method is similar to NPV method however, its evaluation is based on the index developed from cash flow thus eliminates the effect of the investment size on the value of a project.

PI is calculated by dividing the present value of cash inflows by the present value of the cash out flows.

$$PI = \frac{\sum_t Qt(1+I)^{-t}}{\sum_t Ct(1+I)^{-t}}$$

If NPV is greater than zero, PI becomes larger than1 and vice versa. Therefore, standard evaluation basis for the NPV has to be PI being greater than 1.

3.3.4 Weighted Average Cost of Capital (WACC)

The weighted average cost of capital (WACC) is used in finance to measure a firm's cost of capital. This has been used by many firms in the past as a discount rate for financed projects, as the cost of financing (capital) is regarded by some as a logical discount rate (required rate or return) to use. The weighted average cost of capital (WACC) is the minimum return a firm must earn on existing assets to keep its stock price constant and satisfy its creditors and owners.

Corporations raise money from two main sources: equity and debt. Thus the capital structure of a firm comprises three main components: preferred equity, common equity and debt (typically bonds and notes). The WACC takes into account the relative weights of each component of the capital structure and presents the expected cost of new capital for firm.

WACC is calculated by multiplying the cost of each capital component by its proportional weight and then summing:

$$WACC = \left(\frac{E}{V}\right) \times \mathrm{Re} + \left(\frac{D}{V}\right) \times Rd(1 - Tc)$$

Where:

Re = cost of equity

Rd = cost of debt

E = market value of the firm's equity

D = market value of the firm's debt

V = E + D

E/V= percentage of financing that is equity

D/V= percentage of financing that is debt

Tc = corporate tax rate

This equation describes only the situation with homogeneous equity and debt. If part of the capital consists, for example, of preferred stock (with different cost of equity Re), then the formula would include an additional term for each additional source of capital.

Broadly speaking, a company's assets are financed by either debt or equity. WACC is the average of the costs of these sources of financing, each of which is weighted by its respective use in the given situation. By taking a weighted average, we can see how much interest the company has to pay for every dollar it finances.
A firm's WACC is the overall required return on the firm as a whole and, as such, it is often used internally by company directors to determine the economic feasibility of expansionary opportunities and merger. It is the appropriate discount rate to use for cash flows with risk that is similar to that of the overall firm.

WACC is a special way to measure the capital discount of the firms gaining and spending.

3.3.5 Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) is used in finance to determine a theoretically appropriate required rate of return (and thus the price if expected cash flows can be estimated) of an asset, if that asset is to be added to an already well-diversified portfolio, given that asset's non –diversifiable risk. The CAPM formula takes into account the asset's sensitivity to non-diversifiable risk (also known as systematic risk or market risk) as well as the expected return of the market and the expected return of a theoretical risk-free asset.

The CAPM is a model for pricing an individual security (asset) or portfolio. For individual asset perspective, we made use of the security market line (SML) and its relation to expected return and systematic risk (beta) to show how the market must price individual securities in relation to their security risk class. The SML enables us to calculate the reward-to-risk ratio for any security in relation to that of the overall market. Therefore, when the expected rate of return for any security is deflated by its beta coefficient, the reward-to-risk ratio for any individual security in the market is equal to the market reward-to-risk ratio.

The CAPM model says that the expected return of a security or a portfolio equals the rate on a risk-free security plus a risk premium. If this expected return does not meet or beat the required return, then the investment should not be undertaken.

The CAPM formula is:

Expected Security Return = Riskless Return + Beta × (Expected Market Risk Premium)

or:

$$K_c = R_f + Beta(K_m - R_f)$$

Where:

K_C = the expected return rate (the risk-adjusted discount rate)

R_f = the rate of a risk-free investment

K_m = the return rate of a market benchmark

$Beta$ = the sensitivity of the asset returns to market returns

The general idea behind is that investors need to be compensated in two ways: time value of money and risk. The time value of money is represented by the risk-free rate (R_f) in the formula and compensates the investors for placing money in any investment over a period of time. The other half of the formula represents risk and calculates the amount of compensation the investor needs for taking on additional risk. This is calculated by taking a risk measure(beta) that compares the returns of the asset to the market over a period of time and to the market premium($K_m - R_f$).

3.3.6 Comparison of NPV and IRR

When comparing two investment opportunities, decision must be made by evaluating each opportunity while placing the priority over the opportunities.

Either of NPV or IRR can be used when evaluating an individual investment opportunity. If IRR is greater than the cost of capital, the present value of this investment opportunity becomes the right value where as if NPV is greater than zero, IRR becomes greater than the cost of capital.

On the other hand, NPV and IRR can post dramatically different results when used for placing priority purposes. NPV posts favorable results for the projects with larger scale with long investment life while IRR posts opposite results. This is because the IRR is the ratio of the average values while NPV is an absolute value. More precise reason is that the assumptions for the reinvestment rate of the cash inflows are based on different ground. Reinvestment rate of IRR is the internal rate of return for the investment while NPV's become the cost of capital. In most cases, NPV is thought to be a better measure than the IRR because of the following reasons.

First, assumption of reinvesting at the cost of capital seems more appropriate than reinvesting at the IRR. Normally, companies reinvest at the cost of capital when ample amount of cash is generated. It is irrelevant when and from which investment the high profit is generated since companies make investments when they can yield at higher rate that the cost of capital.

Secondly, one's reinvestment rate varies since investment opportunities post different rate of return when IRR method is used. On the other hand, one reinvests cash generated from any other investment which was already invested at the minimum rate of return, namely cost of capital, when using the NPV

Thirdly, good investment opportunities post great rate of return than the cost of capital. So, it is overly too positive to reinvest at the same rate of return. Overly positive forecast and plans can bring a crisis in a company's future.

Fourth of all, NPV suits well with a company's object. IRR opposes with investment opportunity with rate of return (IRR>profitability>capital cost) between the IRR and the cost of capital. In reality, increase in value may be achievable by a company accepting such investment opportunity. On the other hand, NPV accepts all opportunities with rate of return being greater than the cost of capital. IRR is thought to have flaws in assuming the reinvestment rate.

However, a minor difference in the discount rate in NPV method can also revert the priority of the investment opportunities. This is why the cost of capital needs to be measured precisely. Therefore, low confidence level for the cost of capital may make NPV method lose its present value. NPV method may create an error to value the larger size projects more favorable.

Therefore, NPV method is more adequate if a condition where a company's cost of capital and the cash flow of the project can be measured quite accurately. On the other hand, if availability of such data and reliability of the data is fairly low, then the IRR method is more appropriate.

3.4 Other methods

3.4.1 Accounting Rate of Return (ARR)

The accounting rate of return (ARR) is a non-discounted evaluation method. It fails to take into consideration the life of the project or the time value of money. It has another severe drawback for mining companies; it fails to recognize the benefits afforded by the percentage depletion allowance.

Yet, the ARR method is still one of the most common conventional investment evaluation methods in use by the mining industry and elsewhere.

The ARR on capital is defined as the ratio of an average accounting profit (net of depreciation and depletion allowance) to the investment capital. This ratio is then compared with the firm's cost of capital to evaluate profitability.

There are two common accounting rates of return methods. The first method takes average income after taxes as a percentage of average investment.

A second method takes average income after taxes as a percentage of total investment. This is less consistent, but usually gives a closer approximation to internal rate of return than using average investment. These two methods are illustrated below since they are both commonly used.

Method1: $ARR1 = \frac{AINC}{AINV}$

Method2: $ARR2 = \frac{AINC}{TINV}$

Where: ARR = Accounting Rate of Return

AINC = Average Income After Taxes

AINV = Average Investment over Project Life

TINV = Total Investment for Project

Note: AINC is total income after taxes for the life of the project divided by number of years in the project life.

AINV is half of the total investment

For a mining project, it may be more appropriate to add the depletion allowance to the net after tax profits. Although this allowance serves as compensation to the operator for the use of depletory natural resources, it still leaves him or her with a larger cash-on-hand share, after the initial investment is fully recovered by annual depreciation charges. In such a case, the returns actually are cash flows, although time value of money is still ignored.

Pros)

- Easy to understand
- Easy to collect information
- Long investment life and helpful when relatively constant cash flows
- Entire project life is reflected
- Easiness makes it possible for low management employees to carry out the evaluation process

Cons)

- Time value of money is not considered
- Capital payback is proportionate to the time. Namely size and timing of the cash flow during the payback period is ignored
- Hard to make comparisons when evaluating more than one projects of different scale.

- Difficult to achieve objective evaluation since rate of return differs with the method used for depreciation, amortization and any carried losses
- Principled around the return basis

3.4.2 Wealth Growth Rate (WGR)

WGR is a rate where the future value of the investment and the future value of the cash flow become equal. This method makes an assumption that the cash flow is reinvested at WGR. Evaluating basis is that the comparison is made between the WGR and a company's reinvestment rate. WGR calculation is also similar to that of IRR.

Pros)

- Uses cash flow theory as supposed to being profit oriented
- Time value of money considered
- Reinvestment rate defined
- Entire project life is reflected
- Ratio as supposed to an absolute value is the basis which assists in placing priorities among a number of investment opportunities.

3.4.3 Growth Rate of Return (GRR)

GRR is also called as Baldwin Rate of Return. This method discounts the investment amount at a company's reinvestment rate. A point in time, it is selected. Cash flow generated by investment before time is converted to the future value. Cash flow generated onward from time is converted to its value at time. Interest rate which is the present value of the Investment at time equals the future value at time.

Evaluating basis is that calculated interest rate must be greater than a company's reinvestment rate.

Advantages of GRR are that it is similar to WGR and fairly easy to follow. However, it required estimation of the company's reinvestment rate. It also seems to down value a project's real value.

3.4.4 Real Option Valuation (ROV)

Real option valuation is a financial technique for evaluating investments under conditions of uncertainty, particularly uncertainty associated with market variables such as future product demand or the future value of an asset. Option pricing is a well-developed area of financial engineering, dealing with the valuation of puts, calls, and more complex derivatives, but when applied to non-financial assets, the term "real options" is used. In real options valuation, the general ideas from financial options pricing theory are used along with some of the mathematics. Real option valuation has already been applied to a variety of investment decisions by industry, and is widely taught as part of a modern curriculum in business investment analysis.

Basically, real option valuation is a way of capturing value that goes unrecognized in traditional NPV analysis. In particular, when the future is uncertain, there is a value in having the flexibility to decide what to do after some of that uncertainty has been resolved.

The managerial flexibility to wait, abandon, or expand on an investment opportunity is captured in a real option. The real option value of the investment opportunity, then, is what a value-maximizing firm would pay for the right to undertake the investment project with its inherent decision points.

A real option is the right, but not the obligation, to undertake some business decision, typically the option to make a capital investment. For example, the opportunity invest in the expansion of a mine's processing plant is a real option. In contrast to financial options, a real option is not often tradable- e.g. the processing plant owner can not the sell the right to extend his plant to another party, only he can make this decision: however, some real options can be sold, e.g., ownership of a vacant lot or land is a real option to develop that land in the future. Some real options are proprietary (owned or exercisable by a single individual or a company); others are shared (can be exercised by many parties). Therefore, a project may have a portfolio of embedded real options; some of them can be mutually exclusive.

Additionally, with real option analysis, uncertainty inherent in investment projects is usually accounted for by risk-adjusting probabilities (a technique known as the equivalent martingale approach). Cash flows can then be discounted at the risk-free rate. With regular DCF analysis, on the other hand, this uncertainty is accounted for by adjusting the discount rate, using e.g. the cost of capital or the cash flows (using certainty equivalents).
These methods normally do not properly account for changes in

risk over a project's lifecycle and fail to appropriately adapt the risk adjustment. More importantly, the real options approach forces decision makers to be more explicit about the assumptions underlying their projections. In business strategy, real options have been advanced by the construction of option space, where volatility is compared with value-to-cost, NPV. Latest advances in real option valuation are models that incorporate fuzzy logic and option valuation in fuzzy real option valuation models.

3.5 Overall comparison of each evaluation technique

Various mining evaluation techniques have been studied and appropriate method depending on their characteristics must be chosen carefully when evaluating investment opportunities. In comparison of the out come presentations, IRR, ARR, WGR and GRR generate rate oriented results. NPV and Hoskold methods generate absolute value oriented results while PI method generates and index form out come. Time value of money is considered in NPV, IRR, PI, WGR, GRR, ARR and Hoskold require input information of profitability while NPV, IRR, PI, WGR and GRR require cash flow. PB can make use of both types of information for the evaluation.

All of the above techniques can be used when evaluating only one investment opportunity while ARR, IRR, PI, WGR and GRR can be used for comparing two opportunities. NPV can only be applied in cases where similar size or life of the project is being evaluated.
To summarize the advantages and disadvantages of each method, ARR and PB techniques are easy to understand and more simple

to calculate although the time value of money is ignored. DCF is theoretically more advanced while considering the time value of money however, it involves relatively complex calculations and the cost of capital needs to be estimated. More advanced computer applications are eliminating complexity nature of calculations for the DCF technique.

It must be kept in mind that no technique is absolutely better than the others and it is recommended that more than one technique should be used when making evaluation.

As previously mentioned, Hoskold technique seems to down value the investment opportunities. DCF technique also is depended by evaluator's opportunity cost or minimum expected rate of return as supposed to having its distinctive absolute evaluation basis. Therefore, value of the investment opportunity is really depended on the evaluator's subjective point of view. Even if the result of the evaluation is acceptable or right at the marginal line, it is only the evaluator's subjective decision.

In all, Hoskold method applying risk rate or discount rate of 10% or DCF method posting zero value as a result are only the evaluator's point of view. The absolute value of a mine will never be zero but it is an asset where at least 10% of rate of return can be achieved.

4. Factors influencing the determination of discount rate

The magnitude of uncertainties in a mine development project is even larger than in most other manufacturing industries. On the basis of sample and geologic maps, a decision must be reached about development of a mine, its capacity in terms of rate and level of output, a processing plant, and a smelter/refinery complex.

Uncertainty can arise in the estimates of reserves and their average metallic content, in the expected demand and prices for the mineral, and in any other aspects of operation.

Future revenues and costs associated with mineral development are not known with certainty because the factors that determine these revenues and costs are impossible to know with certainty at the time of investment.

During initial exploration, for example, many outcomes are possible, ranging from no indication of commercial mineralization to geologic evidence that eventually leads to a producing mine. During the development of a deposit, initial ore reserve estimates may have to be revised, thus altering estimates of future production and revenues. During production, mineral prices may be higher or lower than predicted at the time of investment, leading to higher or lower revenues than anticipated.

These factors can be grouped into three categories of mineral-development risk according to the cause of the risk.

- Technical risks
- Economic risks
- Political risks

Table 4.1 Risks Important in Mineral Development

Category and type	Major impact on:
Technical risks	
Reserve	Mine production, costs
Completion	Mine production, costs
Production	Mine production, costs
Economic risks	
Price	Prices received for mineral output
Demand	Mineral production
Foreign exchange	Revenues, costs
Political risks	
Currency convertibility	Revenues
Environment	Costs
Tax	Costs
Nationalization	Mine production

4.1 Technical risks

The technical risks are divided into the following three sub categories:

- Reserve risk
- Completion risk
- Production risk

❶ Reserve Risk

Reserve risk, determined both by nature (the distribution of minerals in the earth's crust) and the quality of ore-reserve estimates, reflects the possibility that actual reserves will differ from initial estimates. A complete understanding of the geology of the deposit is imperative to estimate accurately the distribution, grade and tonnage contained in reserve estimates.

❷ Completion Risk

Completion risk reflects the possibility that a mineral-development project will not make it into production as anticipated because of cost overruns, construction delays, or engineering or design flaws.

❸ Production risk

Production risk reflects the possibility that production will not proceed as expected, either because of problems with equipment or extraction processes, or because of poor management. Technical risks are at least partly under the control of the organizations active in mineral development

4.2 Economic risks

The economic risks are divided into the following three sub categories:

- Price risk
- Demand risk
- Foreign – exchange risk

❶ Price risk

Price risk is the variability of possible future mineral prices. Mineral prices are normally determined by the economic law of supply and demand. Mineral prices, together with production levels, determine revenues from mining. Thus, to the extent that actual future prices differ from the prices expected at the time of the cash-flow analysis, actual revenues and profits will differ from those expected.

❷ Demand risk

Demand risk is the variability of future demand for minerals. General economic conditions directly impact on the fluctuation in demand. To the extent that actual and expected mineral demands differ, actual mine production and revenue are affected.

❸ Foreign – Exchange risk

Foreign-Exchange rate risk is the natural consequence of international operation in a world where relative currency value move up and down. Foreign-Exchange risk is the variability of foreign-exchange rates in the future. Rates of foreign exchange importantly influence the revenues of firms operating outside their home country, and the revenues of firms selling products that are priced in terms of foreign currencies, as well as the costs of firms importing equipment from outside the country of operation.

4.3 Political risks

The political risks are determined by the action of governments and reflect the possibility that unforeseen government actions will affect the profitability of an investment. Potential actions include nationalization and changes in regulations concerning, for example, the environment, taxation, currency convertibility

Political risk can be hedged in several ways, particularly when confiscation or nationalization is a concern. The use of local financing, perhaps from the government of the foreign country, reduces the possible loss because the company can refuse to pay on the debt in the event of unfavorable political activity.

- Currency convertibility
- Environment
- Tax
- Nationalization

❶ Currency convertibility

Currency convertibility affects guaranteed freedom of capital transfer.

❷ Environment

Environmental regulations affect the economics viability of mineral projects in three different ways.

First, they often increase the costs of mining and mineral processing by requiring, for example, scrubbers on smelter smokestacks that reduce the amount of sulfur dioxide emitted into the air, plastic liner at the base of tailings ponds that minimize the release of toxic heavy metals into adjoining ground and surface water

Second, environmental regulation often increase the time spent on non mining activities, such as conducting environmental baseline studies, filling environmental impact statement, and applying for mining permits and waiting for their approval.

Third, regulations often increase the risks associated with an investment in mining, because of the discretionary authority that some regulations vest in government agencies to halt development or mining even after significant expenditures have been made.

❸ Tax

The main aim of mining taxation is to earn as much government revenues as possible from the exploitation of mineral resources. Tax risk affects operating costs and reduces the profitability of mineral projects by the amount of taxes paid. Tax incentives such as tax concession for a limited period, special allowance, debt balance carried forward can be disadvantageously changed by government.

❹ Nationalization

In mineral producing countries, the nationalization aims are pursued to acquire control over foreign mining companies operating in the country, sometimes leading to complete ownership.

4.4 Treatment of risks

The traditional or market approach to the treatment and analysis of risks associated with investment projects uses the framework of the income method of valuation.

Any methods of evaluating exploration projects and mineral deposits must consider the risk involved in the investment. Investors demand to be compensated for investment in a risky project rather than in a safe alternative.

In other words, a "risky" dollar is worth less than a "safe" one. Therefore, the value of "risky" dollars must be adjusted before being compared to the value of "safe" dollars.

The following risk adjustment techniques have been suggested to improve the market-based treatment of risk and to satisfy the requirement of the traditional market-oriented investor:

The first is to raise the discount rate.
The rationale is that income from successful risky projects must be sufficiently greater than normal (or risk less) rate of return to make up for losses incurred from unsuccessful risky projects. Rather than discounting risky future cash flows by a rate reflecting the forgone opportunity to invest in safe government securities, companies should discount risky cash flow by a higher rate that reflects the extent to which the future cash flows are less certain than the interest to be received from government securities

The second way is to adjust the future cash flows, rather than the discount rate. The value of a risky future cash flow is reduced to its certainty equivalent: the smallest certain cash flow the investor would be willing to exchange for the risky cash flow. The riskier or more uncertain the future cash flow, the smaller its certainty equivalent.

The third and perhaps most widely used method is sensitivity analysis. A base case is established using the most likely values for each variable in the NPV or benefit-cost analysis. Then a range of other possible NPVs or benefit-cost ratios are calculated using other possible values of each variable. The range of other possible outcomes often includes best-case and worst-case scenarios, reflecting the best and worst combinations of other possible values of each variable influencing NPV or the cost-benefit ratio. Sensitivity analysis also may involve testing the extent to which individual variables influence the economic attractiveness of an investment, a series of calculations is made using various values of the variable under consideration while holding all other variables constant.

5. Proposed quantitative methodology for discount rate

From the review of factors influencing the determination of discount rate carried out (Section 4), it is concluded that the quantitative methodology for discount rate should be a process of identifying potential risk, analyzing risk to determine those that have the greatest impact on mineral development, and determining discount rate. It is therefore imperative to find a method whereby all mining risks, together with their probability and impact, and an understanding of the combined effect of all risks attached to the cash flow and the rate of return. Thus then a way of a procedure calculating risk scores is required. Existing knowledge proposed by Kim Heldman (2005) should therefore be used optimally to determine discount rate.

Hence, it is proposed that the quantitative methodology for discount is a process consisting of the following steps:

- Identifying risk
- Developing Rating Scales
- Determining risk values
- Calculating Risk Scores
- Determining discount rate

These steps will be discussed briefly in the following section of the research

5.1 Identifying risks

The first step in the determination of discount rate is identifying all the potential risks that might crop out in the mineral development project. The identification of risk and attitudes towards it are very important in the life of a mine. The following risks should be considered:

5.1.1 Technical risks

Possible technical risks can be divided into the following three sub categories:

- Reserve Risk
- Completion Risk
- Production Risk

5.1.2 Economic risks

The economic risks are divided into the following three sub categories:

- Price risk
- Demand risk
- Foreign – exchange risk

5.1.3 Political risks

The political risks are divided into the following four sub categories:

- Currency convertibility
- Environment
- Tax
- Nationalization

5.2 Developing rating scales

A risk scale assigns High–Medium–Low values for both probability and impact. Most risks will impact cost, revenue, time or scope to a minimum. Thus, scales for each of these constraints can be devised as shown in Table 5.1.

Table 5.1 Risk Scales Assigned by High–Medium–Low Values

Constraint	Risk	Low	Medium	High
Cost	Environmental Tax	Less than 5% increase	6–10% increase	Greater than 10% increase
Revenues	Price Foreign exchange	Less than 5% decrease	6–10% decrease	Greater than 10% decrease
Time	Completion	Less than 5% increase	6–20% increase	Greater than 20% increase
Scope	Reserve Production Demand Currency convertibility Nationalization	Insignificant change	Change to major Deliverable	Change to critical path task

5.3 Determining risk values

The second way to create a risk scale is to assign numeric values to both probability and impact so that an overall risk score can be calculated.

Risk is associated with events in the future and, therefore, to try to measure risk objectively is very difficult. To surmount this difficulty I suggest using the quantitative risk analysis method.

The quantitative risk analysis method is to assign not only High–Medium–Low values but also to assign numeric values to both probability and impact, so that an overall risk score can be calculated

Cardinal scale values are numbers expressed between 0 and 1.0. Probability is usually expressed as a cardinal value.

Table 5.2 and 5.3 shows description and definition of typical risk values for the High–Medium–Low categories. Use the same group of folks that helped develop the risk scales to determine the percentage that should be assigned to each category.

Table 5.2 Probability Scales with Risk Values

SCORE	DESCRIPTION	DEFINITION
High 0.8	Critical	Will occur frequently, has occurred on the past projects, and the present conditions exist for it to recur
Medium 0.5	Significant	Will occur sometimes, has happened a minimal number of times on past projects, and present conditions are somewhat likely for it to recur
Low 0.1	Negligible	Will not likely occur, has never occurred on past project, and present conditions do not exist for it to recur

Table 5.3 Risk Impact Scales with Risk Values

SCORE	DESCRIPTION	DEFINITION
High 0.8	Critical	A consequence that will cause loss, cause severe interruptions to the profit or project
Medium 0.5	Significant	A consequence that may cause loss, may cause annoying interruptions to the profit or project
Low 0.1	Negligible	A consequence that may cause minimal loss, cause minimal interruptions to the profit or project

5.4 Calculating risk scores

The risk, the probability, and the impact can be listed into a table as individual components. Table 5.4 shows the calculation of risk scores for each category.

Table 5.4 Calculation of Risk Score for Each Category

Category	Risk	Probability	Impact	Risk score
Technical Risk	Reserve	High–0.8	High–0.8	Probability × Impact
		Medium–0.5	Medium–0.5	
		Low–0.1	Low–0.1	
	Completion	High–0.8	High–0.8	Probability × Impact
		Medium–0.5	Medium–0.5	
		Low–0.1	Low–0.1	
	Production	High–0.8	High–0.8	Probability × Impact
		Medium–0.5	Medium–0.5	
		Low–0.1	Low–0.1	
Economic Risk	Price	High–0.8	High–0.8	Probability × Impact
		Medium–0.5	Medium–0.5	
		Low–0.1	Low–0.1	
	Demand	High–0.8	High–0.8	Probability × Impact
		Medium–0.5	Medium–0.5	
		Low–0.1	Low–0.1	
	Foreign Exchange	High–0.8	High–0.8	Probability × Impact
		Medium–0.5	Medium–0.5	
		Low–0.1	Low–0.1	
Political Risk	Currency Convertibility	High–0.8	High–0.8	Probability × Impact
		Medium–0.5	Medium–0.5	
		Low–0.1	Low–0.1	
	Environment	High–0.8	High–0.8	Probability × Impact
		Medium–0.5	Medium–0.5	
		Low–0.1	Low–0.1	
	Tax	High–0.8	High–0.8	Probability × Impact
		Medium–0.5	Medium–0.5	
		Low–0.1	Low–0.1	
	Nationalization	High–0.8	High–0.8	Probability × Impact
		Medium–0.5	Medium–0.5	
		Low–0.1	Low–0.1	

The total risk score is calculated by multiplying the probability by the impact. Using the reserve risk, for example, this risk has a low probability of occurring but a medium impact. Therefore, the risk score is calculated with 0.1× 0.5 for a final value, also known as an expected value, of 0.05. Total risk scores are calculated by summing up each risk score and converting risk premium.

5.5 Determining the discount rate

The rate of discount can be regarded in two ways. In the first case, if a company raises funds from external sources, the discount rate is regarded as the cost of the capital. It is the percentage rate of return that the firm must generate to compensate those outside investors, who supply funds to the company rather than to invest their money in another company or activity.

Secondly, if a company uses internal funds, the discount rate is regarded as the opportunity cost. This opportunity cost, therefore, is the rate of return the company could earn in the best use of its money.

Each project will have its own degree of risk, and so own discount rate. The greater the risk, the higher the discount rate should be, raising the discount rate reduces the NPV of a set of cash flows.

Determining the risk-adjusted discount rate is the most difficult aspect of cash-flow analysis where it is important to determine discount rate by the systematic method.

5.5.1 The risk premium

A risk-adjusted discount rate may be developed by using a risk-free rate of return, plus a subjectively determined risk premium, which is expected to compensate the investor for the extra risk involved. In practice the selection of a risk-free rate of return is relatively simple. In the majority of case, the yield on government bonds, under non-inflationary conditions, is adopted as the risk-free rate of return (Whitney, 1979).

The real problem relates to the selection of the risk premium which must be sufficient to compensate for the additional risks associated with the investment at hand. The risk premium is the premium which is required by the average investor to invest in a risky project

The risk premium is entirely dependent on the risks influencing the mineral development project; which are the result of analysis of:

- The macroeconomic circumstances
- The industry trends
- The project's strong points and weak points and key economic and financial variables
- All other information that significantly affects future cash flows of the project.

When determining an appropriate risk premium, all risks affecting the discount rate should be considered. This however is an extensive exercise and will encompass a greater number of risks, which makes the determination very difficult to work through and use. Furthermore, there are significant difficulties in structuring an involved analysis with many factors, for the obvious reason that it is complex and multi-faceted. In order to facilitate the implementation of the determination, there are usually focused on a definite number of key risks such as technical, economical and political risks.

Thus, the key risks crucial for success of the mineral development project are as follows:
1) Technical risk– reserve, completion, production
2) Economical risk –price, demand, foreign exchange
3) Political risk – currency conversion, environment, tax, nationalization

Table 5.5 Key Risks Influencing Mineral Development

Category	Risk	Remark
Technical Risk	reserve	anticipated possibility to differ from initial estimates
	completion	anticipated possibility not to start production as expected
	production	anticipated possibility not to proceed production as expected
Economic Risk	price	variability of possible future mineral price
	demand	variability of possible future mineral demand
	foreign exchange	variability of possible future foreign exchange rate
Political Risk	currency convertibility	possibility of unforeseen government action to currency convertibility
	environment	public concern about environment
	Tax	possibility of unforeseen government action to tax
	nationalization	possibility of unforeseen government action to nationalization

To determine risk premium, an expected value (risk score) as calculated in the previous section (5.4) has to be converted to an overall value and risk premium.

An overall value and risk premium can be determined as shown in Table 5.6.

Table 5.6 Risk Premium Determination

Expected Value	Value	Risk Premium
4.46–6.40	High–High	20.0%
2.51–4.45	High	16.0%
1.51–2.50	Medium	12.0%
0.11–1.50	Low	9.0%
0 –0.10	Low–Low	6.0%

The determination of risk premium is incumbent on the impact of the factor and the potential possibility of its affecting success in the mineral development.

5.5.2 The risk–adjusted discount rate

Put simply but rather crudely, we can represent a risk–adjusted discount rate as follows:

Risk–Adjusted Discount Rate = Risk–free rate of return + Risk premium

- **The risk–free rate of return** – for mineral development projects, it is advisable use a 10–year bond that yields 1.2 percent
- **The risk premium –** use 6~20 percent as the generally accepted range.

The application of these numbers to the risk–adjusted discount rate formula yields the following risk–adjusted discount rate for mineral development projects.

Risk-Adjusted Discount Rate = Risk-free rate of return + Risk premium

= 1.2% + 6~20% = 7.2~21.2%

Thus, the risk-adjusted discount rate required by mining companies ranges between 7.2 and 21.2 percent.

6 Case study

This Case Study is based on developing a nickel mine in Madagascar, Ambatovy Project. This gives an example of risks involved in selecting a discount rate such as exploration, reserve calculation, construction phase, the operation and the sales associated with the product. Discount rate for the Ambatovy project was selected by using the quantitative methodology explained in section 5 to assess the economic viability of the project.

6.1 Introduction

- Name: Ambatovy Nickel Project in Madagascar
- Location: located in nation of Madagascar.
- Current status of licenses
  - Type: mining permit
  - Commodities: Ni, Co, Cu, Pt, Cr
- Mining technique: open pit
- Status: completed feasibility study & preparing for construction

6.2 Development plan

6.2.1 Mineral reserves

- 125 million tonnes @ 1.04% Ni, 0.10% Co (0.8% nickel cutoff)
  - Additional 39.4 million tonnes @ 0.69% Ni, 0.064% Co
  - Potential to increase reserves with additional drilling

6.2.2 Mining method

- 4 separate open pits
- Mine limonite and low magnesium saprolite "LMS" after stripping overburden of 3m from the surface
- Mine ore delivered by truck to ore preparation plant
- Ore then conveyed to scrubber where water is added to slurry the ore
- Slurry thickened and delivered to pipeline

6.2.3 Transportation of ore

- Ore transformed into a slurry form at the Ore Preparation Plant is transported through the pipeline buried 1.5m below the surface to the processing plant
- Pipeline is 220km long and 600mm in diameter
- Single pump station at mine site is installed to transport the slurry ore while using the gravity as a dragging force since the elevation difference is about 1,000m

6.2.4 Processing and refinery

- ㅇ Project to utilize only proven metallurgical processes, all process unit operations can be found elsewhere operating on a commercial scale
- ㅇ High Pressure Acid Leaching technique is used to produce nickel briquette and cobalt.
- ㅇ This process is separated into two parts where pressure leach is applied to produce mixed sulphides and the stage where the mixed sulphides are smeltered and refined.

6.3 Capital expenditure (Capex): U$2,500 millions

6.4 Operating expenditure (Opex)

- ㅇ Average Opex during 27 yrs of mine life: Ni- 1.99U$/lb (with credit, 0.97U$/lb)
- ㅇ 10 year average Opex after ramp-up period

: Ni- 1.75U$/lb (with Credit, 0.77U$/lb)

6.5 Market analysis and forecast

6.5.1 Nickel

<u>Overview</u>

Nickel, a principal product metal, had world sales in 2005 of $18.7billion on the average LME nickel price of $6.68/lb. The dominant end use of nickel is in stainless steel, accounting for two-thirds of nickel demand. For the Western World plus China, long-term nickel demand growth (1960 to 2005) has been 4% per year for nickel and 6% per year for stainless steel.

World nickel consumption in the 1980s grew on trend at 2.5% per year, peaking in 1989 at 854,000 tonnes. In the following four years, consumption declined 8.4% to reach 782,000 tonnes in 1993, down 72,000 tonnes, as demand in the former Soviet Union collapsed. From 1994, nickel consumption increased by 3.4% per year to reach 1,274,000 tonnes in 2005.

Forecasts from 2005 to 2012 indicate a 4.7% per year trend growth rate with consumption rising by 483,000 tonnes to 1,757,000 tonnes. This is equivalent to an average increase of 60,400 tonnes per years. All end uses are forecast to increase, however 80% of the growth will come from increasing demand for nickel in the dominant stainless steel sector.

The annual average price for LME nickel from 1980 to 2005, in dollars of the day, has fluctuated between a low of $1.76/lb (1986) and a high of $6.68/lb(2005) and has average $3.42/lb over this period. As nickel's demand in linked to global industrial activity, price volatility will continue in the future.

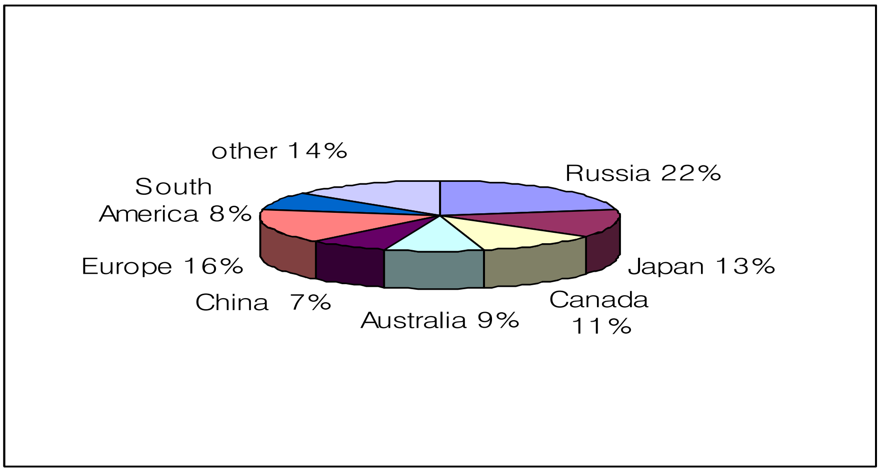
The annual average LME nickel price from 1980 to 2005 in constant 2005 US dollars was $4.52/lb. A project evaluation price of $4.00/lb in constant 2005 US dollars is recommended.

Supply/Demand/Price

World production of primary finished nickel by region is indicated in Figure 6.1 for 2005. Finished nickel includes refined metal, nickel oxide and ferronickel, products that are sold directly to nickel consumers. The world's largest nickel producer is Russia with 22%, followed by Japan at 13%, Canada at 11% and Australia at 9%. These four totals are 55%.

Figure 6.1 World Nickel Production by region 2005

2005 Production: 1,280,000 Tonnes



*Source :International Nickel Study Group, Brook Hunt,

Yates Mineral Consultants Inc.

The average 2003 LME nickel price was $4.37/lb, 42% higher than the 2002 average of $3.07/lb, due largely to strong demand, particularly from China and significant increase in global nickel supply.

Global nickel demand is forecast to advance by 4.7% per year from 1,724,000 tonnes in 2005 to 1,757,000 tonnes in 2012 for an increase of 483,000, equivalent to an average annual increase of 60,400 tonnes.

The increase in 2004 global demand of only 3.2% was capped by limited supplies of primary refined nickel production resulting in a genuine physical tightness in the global nickel market. The global market had a 28,000 tonnes and the LME average annual nickel price moved up strongly by 44% to $6.28/lb.

For 2005 global nickel demand was down fractionally by 0.6% due to stainless steel de-stocking. Stainless output dropped as a number of producers reduce production in the third and fourth quarters due to excess inventories of about 500,000 tonnes worldwide.

Lower stainless production to reduce excessive inventories in the second half of 2005 moved the nickel market into surplus. LME inventory moved up from 7,000 tonnes in mid 2005 to close the year at 36,000 tonnes.

The LME nickel price is FOB warehouse whereas nickel producers sell their finished production delivered customer in the US and delivered CIF major port in Europe and also in Asia. The premium over the LME price is charged to the customer to cover the delivery and sales charges. In today's situation where there is global shortage, premiums everywhere will tend to be higher than normal. Under normal conditions, premiums will not be more than a " Pimple" on the overall price received as delivery and sales charges account for most of the premium.

6.5.2 Cobalt

Overview

Cobalt, a by-product metal, had world sales in 2005 of $1.9 billion based on the average high grade (99.8%) cobalt price of $15.77/lb. Cobalt is used in a number of specialized applications and is critical for many diverse commercial, industrial and military applications.

World cobalt demand grew in the 1990's at close to 3% per year. From 1997 to 2005 growth accelerated to a rate of 7.9% per year to reach 54,000 tonnes in 2005. Forecasts indicate a growth rate of 6.3% per year with demand rising to 83,900 tonnes in 2012, equivalent to an average increase of 3,600 tonnes per years. All and uses are expected to increase, but especially cobalt used in rechargeable batteries for electronic devices, driven by Asia and particularly China.

Annual average prices for high grade cobalt (99.8%) from 1980 to 2005 in constant 2005 US dollars indicate a 26 year average of $19.93/lb. The recommended project evaluation price is $10.0/lb in constant 2005 US dollars.

Price fluctuations are expected to continue as cobalt's end uses are dominated, high technology applications. In addition, price manipulation by traders does contribute to price volatility, particularly for by-product metals like cobalt, which lack market transparency.

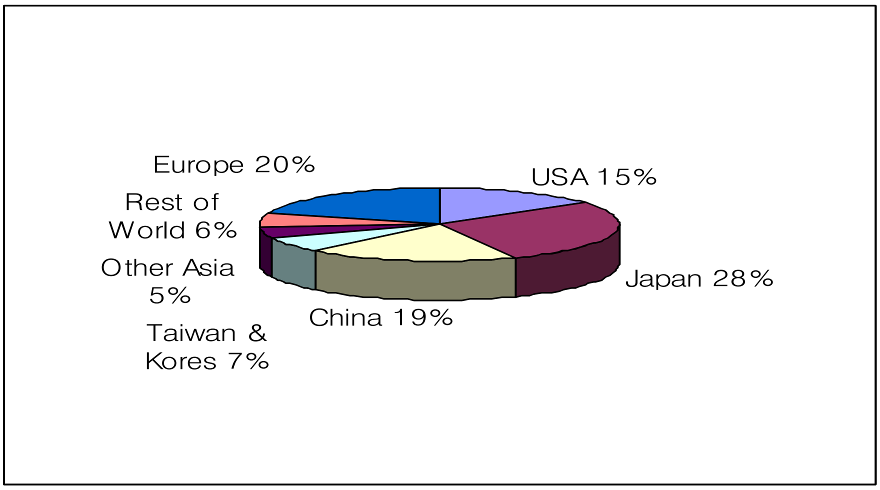
Supply/Demand/Price

Cobalt is produced as a by-product of nickel and copper and as a primary product from ore produced in the DRC. Prior to 1978, world cobalt supply came predominantly from the Africa Copper Belt comprising two countries- the Democratic Republic of Congo (DRC) and Zambia, with the DRC being the larger. DRC has 45% of the world's reserves of cobalt, the largest of any country.

Figure 6.2 shows World cobalt demand by region in 2004. The three largest consuming countries are Japan at 28%, China at 19% and the US at 15%. Europe is at 20%. The largest consuming region is Asia at 59%. In 2002 Asia accounted for only 44%. This massive jump up in demand from Asia, particularly in China and Japan, was led by rapidly advancing production of rechargeable batteries.

Figure 6.2 World Cobalt Demand by Region 2004

2004 Refined Demand: 51,400 Tonnes



*Sources: Cobalt Development Institute, Yates Mineral Consultants Inc.

Global cobalt demand is forecast to advance at 6.3% per year from 54,500 tonnes in 2005 to 89,000 tonnes in 2012 for an increase of 29,400 tonnes, equivalent to an annual average increase of 3,675 tonnes.

Following the bursting of the high tech bubble in late 2000, the global cobalt market by 2002 was in surplus and the annual average price hit bottom at $6.91/lb. Significant deficits were recorded in 2003 and 2004 followed by a minor surplus in 2005

6.6 Determining the discount rate for Ambatovy Project

The following is a procedure using a quantitative risk analysis in order to select the discount rate for the Ambatovy Project

①Identifying Risk

②Developing Rating Scales

③Determining Risk Values

④Calculating Risk Scores

⑤Determining Discount Rate

6.6.1 Identifying risk for Ambatovy Project

Potential risks associated with the project are ①Technical risk(reserve, completion and production risk) ②Economic risk(price, demand and foreign-exchange risk ③Political risk(currency convertibility, environment, tax and nationalization)

Table 6.1 Key Risks in Ambatovy Project

category	Risk	Remark
Technical Risk	reserve	anticipated possibility to differ from initial estimates
	completion	anticipated possibility not to start production as expected
	production	anticipated possibility not to proceed production as expected
Economic Risk	price	variability of possible future mineral price
	demand	variability of possible future mineral demand
	foreign exchange	variability of possible future foreign exchange rate
Political Risk	currency convertibility	possibility of unforeseen government action to currency convertibility
	environment	public concern about environment
	tax	possibility of unforeseen government action to tax
	nationalization	possibility of unforeseen government action to nationalization

6.6.2 Developing rating scales for Ambatovy Project

Effects of possible technical, economical and political risks on the project's schedule, budget, resources, deliverables, costs and quality are evaluated by the High-Medium-Low rating scales.

Table 6.2 Probability Scales for Ambatovy Project

Score	Definition
High	Will occur frequently, has occurred on the past projects, and condition exist for it to recur
Medium	Will occur sometimes, has happened a minimal number of times on past projects, and conditions are somewhat conclusive for it to recur
Low	Is not likely to occur, has never occurred on past projects, and conditions do not exist for it to recur

The effects of potential risks on cost, revenue, time or scope were also evaluated using High-Medium-Low rating scale.

Table 6.3 Risk Impact Scales for Ambatovy Project

Constraint	Risk	Low	Medium	High
Cost	Environmental Tax	Less than 5% increase	6-10% increase	Greater than 10% increase
Revenues	Price Foreign exchange	Less than 5% decrease	6-10% decrease	Greater than 10% decrease
Time	Completion	Less than 5% increase	6-20% increase	Greater than 20% increase

Table 6.3 Continued: Risk Impact Scales for Ambatovy Project

Constraint	Risk	Low	Medium	High
Scope	Reserve Production Demand Currency Convertibility Nationalization	Insignificant change	Change to major deliverable	Change to critical path task

Probability and impact scales of the Ambatovy Project categorized in the Table 6.3 above are shown in Table 6.4.

Table 6.4 Rating Scales for Ambatovy Project

category	Risk	Probability	Impact	Remarks
Technical Risk	Reserve	Low	Low	Drilled: 1,282holes, 54,888M
	Completion	Medium	Medium	Period of construction: 36months
	Production	Low	Medium	Utilizing globally proven technology
Economic Risk	Price	Medium	High	Changes in the price range is large
	Demand	Low	Medium	Shortage of supply
	Foreign Exchange	Low	Medium	Stable currency market forecasted

Table 6.4 Continued: Rating Scales for Ambatovy Project

category	Risk	Probability	Impact	Remarks
Political Risk	Currency Convertibility	Low	Medium	Low Currency Convertibility risk due to specialized law
	Environment	Low	High	EA approved by the government
	Tax	Low	Medium	Tax incentives due to specialized law
	Nationalization	Low	High	Low risk in nationalization

6.6.3 Determining risk values for Ambatovy Project

Numeric values need to be applied in the probability and impact explained in the previous section 6.6.2 in order to calculate risk score of the Project. However, this process is very hard to carry out especially for calculating a value that represents a possible risk in the future.

Therefore, the quantitative risk analysis method as previously explained is used to decide the risk value of 0 and 1.0 for the probability and impact.

Scale	High	Medium	Low
Value	0.8	0.5	0.1

6.6.4 Calculating risk scores for Ambatovy Project

As explained before the risk, the probability, and the impact can be listed into a table as individual components. The risk score was calculated by multiplying the probability of the risk by the impact as shown in Table 6.5.

Table 6.5 Calculation of Risk Score for Ambatovy Project

category	Risk	Probability	Impact	Remarks
Technical Risk	Reserve	Low –0.1	Low–0.1	0.1×0.1 = 0.01
	Completion	Medium–0.5	Medium–0.5	0.5×0.5 = 0.25
	Production	Medium–0.5	Medium–0.5	0.5×0.5 = 0.25
Economic Risk	Price	Medium–0.5	High–0.8	0.5×0.8 = 0.40
	Demand	Low–0.1	Medium–0.5	0.1×0.5 = 0.05
	Foreign Exchange	Medium–0.5	Medium–0.5	0.5×0.5 = 0.25
Political Risk	Currency Convertibility	Low–0.1	Medium–0.5	0.1×0.5 = 0.05
	Environment	Low–0.1	Medium–0.5	0.1×0.5 = 0.05
	Tax	Low–0.1	Medium–0.5	0.1×0.5 = 0.05
	Nationalization	Low–0.1	High–0.8	0.1×0.8 = 0.08
Total				1.44

Total risk scores are 1.44 which is calculated by summing up each risk score.

6.6.5 Determining the Discount Rate for Ambatovy Project

The risk premium

To determine risk premium, an expected value (risk score) as calculated in the previous section (6.6.4) have to be converted to an overall value and risk premium.

An overall value and risk premium for Ambatovy Project can be determined as shown in Table 6.6.

Table 6.6 Expected Value and Risk Premium for Amatovy Project

Expected Value	Value	Risk Premium
4.46–6.40	High–High	20.0%
2.51–4.45	High	16.0%
1.51–2.50	Medium	12.0%
0.11–1.50	Low	9.0%
0–0.10	Low–Low	6.0%

Risk premium calculated in accordance with the table above is 9.0% since the risk score of the Project calculated by the previous section (6.6.4) is 1.44.

The risk–adjusted discount rate

From the above extrapolation the risk–adjusted discount rate can be determined from the risk–adjusted discount rate formula as follows:

Risk–Adjusted Discount Rate = Risk–free rate of return + Risk premium

– The risk–free rate of return – a 10–year bond that yields 1.2 percent

– The risk premium – 9.0%

Thus, the risk–adjusted discount rate required for Ambatovy Project is **10.2 percent.**

7 Conclusions

It is concluded that the quantitative methodology for discount rate should be a process of identifying the factors influencing mineral development projects. Therefore, a method is required whereby the key factors crucial for success of the mineral development projects are used as a fundamental base.

These key factors can be group into three categories of mineral-development risk according to the cause of the risk

- Technical risk: reserve, completion, production
- Economic risk: price, demand, foreign exchange
- Political risk: currency convertibility, environment, tax, nationalization

A procedure for identifying the key factors relevant to discount rate is then required. In this way, existing knowledge should be used in an optimal way to determine discount rate. Hence, it is proposed the quantitative methodology for discount rate is a process considering the following steps:

1. Identifying risks
2. Developing rating scales
3. Developing risk values
4. Calculating risk scores
5. Determining the discount rate

Put simply but rather crudely, we can represent a risk-adjusted discount as follows:

Risk-Adjusted Discount Rate = Risk-free rate of return + Risk premium

- **The risk-free rate of return**: for mineral development projects, it is advisable use a 10-year bond that yields 1.2 percent
- **The risk premium** : use 6~20 percent as determined by the researcher

Thus, the risk-adjusted discount rate required by mining companies ranges between **7.2 and 21.2 percent.**

It is obvious that the risk-adjusted discount rate approach is useful to the decision-maker in that it produces decision tool in the form of a risk-adjusted rate of return. The inherent disadvantage of this approach is that the selection of the risk premium which is subjective and hence the reliability of the method is often suspect.

The risk-adjusted discount rate is not the end-all and be-all criterion for the decision to invest in a mineral development project under consideration, although it is generally one of the motivating factors considered by the firm's management.

The attitude of investors to risk taking is entirely subjective and very difficult to express in quantitative terms. Investors who are not particularly averse to risk, tend to choose the low level of discount rate, whereas the more cautious and risk-averse investors will usually tend to select the medium level of discount rate. The decidedly risk-averse investors will usually opt for a high level of discount rate.

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Appendix A Terminology

Accelerated depreciation

Is depreciation methods that write-off the cost of an asset at a faster rate than the straight line methods.

Acceptance

A risk response strategy that accepts the consequences of a risk event should it occur

Amortization

The periodic charge for intangible assets.

Amortize

To liquidate on an installment basis; an amortized loan is one on which the principal amount of the loan is paid off in installments during the life of the loan.

Avoidance

A risk response strategy that attempts to avoid or eliminate risk events and their impacts

Assets

Resources owned by the company.

Business risk

The basic risk inherent in a firm's operation. The uncertainty or variability of expected pre-tax returns on the firms assets.

Capital

Anything of value having earning power that is owned by the company.

Capital asset

An asset with a life of more than one year that is not bought and sold in the ordinary course of business.

Capital budget

The process of planning expenditures on assets whose returns are expected to extend beyond one year.

Capital expenditure

Benefits future fiscal periods as well as the current one, increases the carrying value of an asset, usually fixed asset, on the balance sheet.

Capitalized investment

Is an investment with an expected life-time and usefulness exceeding one year.

Cardinal scale values

Values expressed as numbers between 0 and 1.0 and referenced in the Qualitative Risk Analysis process

Co-efficient of variation

Standard deviation divided by the mean.

Compound interest

An interest rate that is application when interest in succeeding periods is earned not only on the initial principal, but also on the accumulated interest of prior periods.

Constraints

Actions or decisions that either restrict or dictate the actions of the project team.

Contingency planning

A risk response strategy that involves planning alternatives to deal with the risks should they occur.

Contingency reserves

Contingency reserves hold project funds, time, or resources in reserve to offset any unavoidable threats that might occur to project scope, schedule, cost overruns, or quality.

Cost of capital

The discount rate that should be used in the capital budgeting process.

Cut-off grade

Is the minimum ore grade to be economically extracted from a particular deposit, when blended with higher grade ore.

Decision tree analysis

A diagramming method that shows the sequence of interrelated decisions and the expected results of choosing one alternative over another. It is usually used for risk events that impact time or cost.

Degree of risk

Probability of not achieving or of exceeding the anticipated resources.

Depreciation

is a non cash charge. The periodic charge for tangible assets other than natural resources.

Development

Work carried out for the purpose of opening up a mineral deposit and making the actual ore extraction possible.

Discounted Cash Flow (DCF) Techniques

Methods of ranking investment proposals. Including: (1) internal rate of return (IRR), (2) Net Present Value (NPV)

EBIT

Earnings before interest and taxes.

Equity

Stockholders equity is the excess of assets over liabilities.

Expected value

An overall risk score derived by multiplying the risk impact value by the risk probability value.

Expense

Cost incurred in the normal course of business.

Expensed investment

is that annual investment that can be legally deducted as an expense at the time-period it occurred. It involves intangible assets and costs of unsuccessful exploration efforts.

Exploration

Work involved in searching for ore

Financial leverage

Ratio of the total debt to total assets.

Financial risk

Risk over and above the basic business risk that result from using financial leverage.

Intangible fixed assets

Those which have no physical substance. Examples include patents, copyrights, trademarks, franchises, and rights to the holder.

Internal Rate of Return (IRR)

The discount rate that equates the present value of future cash flows to the cost of the investment.

Liabilities

Debts owed by the company, claims of creditors.

Investment tax credit

Is an allowance given by the government as a percent of the investment undertaken through the year, to be deducted from the tax liability

Marginal cost (MC)

Is the cost of increasing output by one additional unit.

Mitigation

A proactive risk response strategy that reduce the probability of an event and its impacts to an acceptable level.

Monte Carlo Simulation Technique

A statistical technique that uses simulation to calculate a distribution of probable result. This method was developed during the Second World War for a secret project, where a roulette device was used to simulate the probable behaviour of neutron diffusion.

Net Present Value (NPV)

The present value of future returns, discounted at the marginal cost of capital, minus the present value of the cost of the investment.

Opportunity cost

The rate of return on the best alternative investment that is available.

Ordinal values

Rank-ordered values such as High, Medium, and Low. They are used during the Qualitative Risk Analysis process to assign risk scores to identified risks.

Ore

A mixture of valuable minerals and gangue from which at least one of the minerals can be extracted at a profit.

Payback period

The length of time required for the net revenues of an investment to return the cost of the investment.

Present value (PV)

The value of a future payment, or stream of payments, discounted at the appropriate discount rate.

Probability

The likelihood that a risk event will occur

Profit

Net earnings in a given period after all costs have been deducted from gross revenues.

Qualitative Risk Analysis

This process determines what impact the identified risks will have on the project and the probability they will occur. Rank orders risks in priority order according to their effect on the project objectives.

Quantitative Risk Analysis

This process assigns numeric probabilities to each identified risk and examines its potential impact on the project objectives.

Required rate of return

The rate of return that stockholders expect to receive on common stock investment.

Reserve (ore)

The quantity of ore of which the grade and tonnage have been established with reasonable assurance by drilling and other earns.

Residual risk

A risk that remains or is the result of implementing a risk response strategy.

Risk

An event that poses a threat or an opportunity to the project.

Risk audit

A method for examining the effectiveness of the risk management plan and risk processes.

Risk identification

This process identifies the potential project risks and documents their characteristics.

Risk management

Applying skills, knowledge, and risk management tools and techniques to the project in order to reduce threats to an acceptable level while maximizing opportunities.

Risk management plan

Details how the risk management processes will be implemented, monitored, and controlled throughout the life of the project.

Risk monitoring

An activity that includes gathering information, documenting the findings, and reporting the findings.

Risk Monitoring and Control

This process responds to risks as they occur, tracks and monitors identified risks, evaluates risk response plans for effectiveness, identifies new risks, and ensures that proper risk management procedures are followed as defined in the risk management plan.

Risk originator

Team members, stakeholders, or others who identify a potential risk and inform the project manager.

Risk owner

The team member responsible for managing an identified risk by tracking risk activities, monitoring the project and other events for risk triggers, carrying out the risk response plan and monitoring the effectiveness of the response plan.

Risk response plan

The risk response plan details how risk response strategies will be implemented when the risk event occurs.

Risk response planning

This process defines what steps to take to reduce threats and take advantage of opportunities, assigns departments or individuals the responsibility of carrying out risk response plans.

Risk tolerance

The level of risk an organization or individual I swilling to accept.

Risk trigger

Symptoms of a risk that imply a risk event is about to occur

Risk premium

The difference between the required rate of return on a particular risky asset and the rate of return on a riskless asset with the same expected value.

Salvage value

Amount that can be realized from sale of an asset after its useful life has ended.

Sensitivity analysis

A modeling technique that determines which risk event has the greatest potential for impact. It examines the extent to which the uncertainty of a risk event affects the project objectives.

Simulation

Analysis techniques that use various assumptions to calculate a distribution of probable result.

Standard deviation

A statistical term that measures the variability of a set of observations from the mean of the distribution.

Time value of money

The concept of time value of money deals with the varying values of equal sums of money over different time-periods—a future inflow of money is worth less right now at the present.

Transference

A risk response strategy that transfers the consequences of a risk to a third party. Insurance is an example of transference.

Trust

An equitable or beneficial right or title to land or other property, held for the beneficiary by another person, in whom resides the legal title or ownership, recognized and enforced by the courts of chancery.

Workaround

An unplanned response to an unknown, unidentified risk or a previously accepted risk.

Yield

The rate of return on an investment; the internal rate of return.

Appendix B List of formulas

- Investment Evaluation
- Depreciation Methods
- Coefficient of variation
- Standard deviation

■ Investment Evaluation

- DISCOUNTING PRESENT VALUE

$$PV = \frac{FV}{(1+DISC)^n}$$

Where : PV = Present Value

FV = Future Value

DISC = Discount Rate

n = Year in The Future

- NET PRESENT VALUE

$$NPV = \sum_{i=0}^{n} \frac{CFi}{(1+DISC)^I}$$

Where : NPV = Net Present Value

i = Each Year in The Project Life

n = Last Year in The Project Life

CF_i = Net Cash Flow in Year i

DISC = Discount Rate

- INTERNAL RATE OF RATURN

$$0 = \sum_{i=0}^{n} \frac{CFi}{(1+IRR)^{i}}$$

Where :

n = Total Life of The Project Including Investment Period and Operation Period

I = Each Year in The Life of The Project

CF_i = Cash Flow for The *i* th Year in The Project Life

IRR = Internal Rate of Return for The Project.

- ANNUAL STRAIGT LINE DEPRECCIATION

$$D_n = \frac{(CB - SV)}{L}$$

Where : D_n = Depreciation Allowance in Year n

CB = Initial Cost Basis of Depreciable Asset

SV = Estimated Salvage Value of Depreciable Asset

L = Depreciable Life of Asset in Years

- ANNUAL LIITED DECLINING BALANCE DEPRECCIATION

$$D_n = (\frac{1.5}{L}) \times (CB - \sum_{i=1}^{n-1} Di)$$

Where : D_n = Depreciation Allowance for Year n
L = Depreciation Life of The Asset in Years
CB = Initial Cost Basis of The Depreciable Asset
i = Years Prior to Year n
Di = Depreciation Allowance in Years Prior to Year n

Subject to the limitation :

$$D_n \leq CB - SV - \sum_{i=1}^{n-1} Di)$$

Where : SV = Estimated Salvage Value of the Asset

- DOUBLE DECLINING BALANCE DEPRECCIATION

$$D_n = (\frac{2}{L}) \times (CB - \sum_{i=1}^{n-1} Di)$$

Where : D_n = Depreciation Allowance for Year n
L = Estimate Depreciation Life of The Asset
CB = Initial Cost Basis for The Asset
i = Years Prior to Year n
D_i = Depreciation Allowance for Year i
n = Current Year in The Asset Life

Subject to the follow limitation:

$$D_n \leq CB - SV - \sum_{i=1}^{n-1} Di)$$

Where : SV = Estimated Salvage Value for the Asset

- SUM OF THE YEAR'S DIGITS DEPRECIATION

$$D_n = \frac{(2)\times(L-n+1)}{L^2+L} \times (CB - SV)$$

Where : D_n = Depreciation Allowance for Year n

L = Estimated Depreciable Life of The Asset

n = Current Year in The Asset Life

CB = Initial Cost Basis for The Asset

SV = Estimated Salvage Value for The Asset

- UNITS OF PRODUCTION DEPRECCIATION

$$D_n = \left(\frac{Pn}{\Pr}\right) \times \left(CB - SV - \sum_{i=1}^{n-1} Di\right)$$

Where : D_n = Depreciation Allowance for Year n

P_n = Production during the Present Year

Pr = Production Remaining at the Beginning of The Year

CB = Initial Cost Basis for the Asset

SV = Estimated Salvage Value for the Asset

I = Years Prior to Year n

Di = Depreciation Allowance for Years Prior to Year n

■ DEPLETION METHODS

● COST DEPLETION

$$CD_n = (CB - \sum_{i=0}^{n-1} De \times \frac{(Un)}{Un + Ur}$$

Where : CD_n = Cost Depletion Allowance in Year n

CB = Original Cost Basis of the Property

$\sum_{i=0}^{n-1} De$ = Accumulated Value of Depletion taken in Preceding Years (Both Cost and Percentage)

Un = Units of Ore Mined During Year

Ur = Units of Ore Remaining at Year End

● PERCENTAGE DEPLETION

(1) PD_n = (MPDA)(GIM_n – RP_n)

(2) If PD 〉 (0.5)(TiBD) then PD_n = (0.5)(TiBD)

Where : PD_n = Allowance Percentage Depletion for the mineral Sold During The Year

MPDA = Mineral's Depletion Allowance

GIM_n = Gross Income from Mineral Sales in Year n

RP_n = Royalty Payments

$TiBD_n$ = Taxable Income Before Depletion

■ COEFFICIENT OF VARIARION

$$C = \frac{S}{\bar{X}}$$

Where ; C = Coefficient of Variation

S = Standard Deviation

$\bar{X}$ = Mean of All Occurrence

■ STANDARD DEVIATION OF VARIARION

(SQUARE-ROOT THE VARIANCE)

$$S = \sqrt{S^2} \; : \; S^2 = \left(\frac{1}{N-1}\right) \sum_{i=1}^{N} (Xi - \bar{X})^2 \qquad \bar{X} = \frac{1}{N} \sum_{i=1}^{N} Xi$$

Where : S = Standard Deviation

S^2 =Variance

N = Number of Occurrences

i = Individual Occurrence

Xi = Value for The ith Occurrence

$\bar{X}$ = Mean Value of All Occurrence