ADDENDUM A

CLOSED-LOOP SYSTEM FOR THE INTER-SAMPLING SIMULATION STUDY

The closed-loop system, which are used in the simulation study of the variance of models identified from inter-sampled closed loop data, is shown in Fig. A.1.

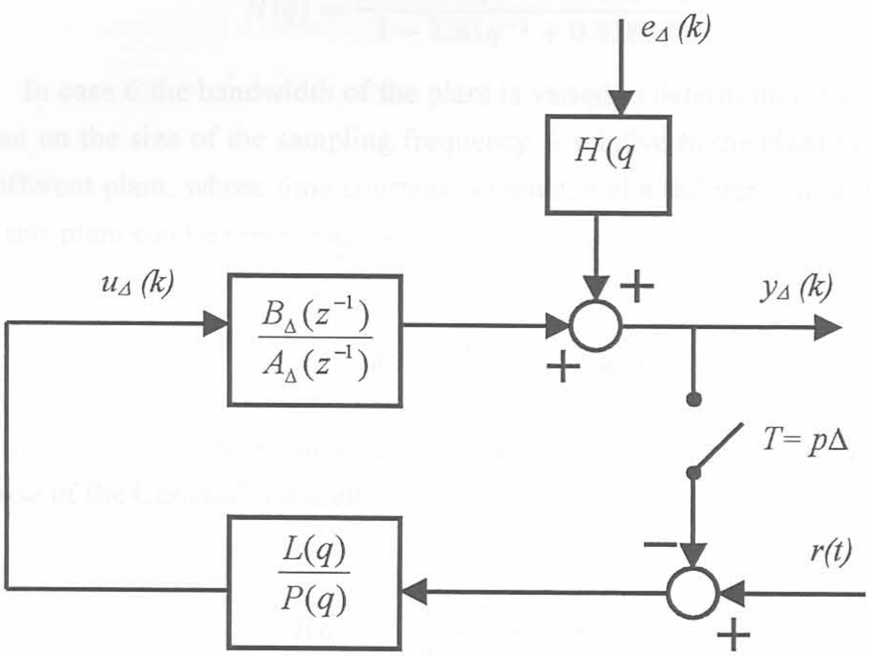

Figure A.1: The closed-loop system that was used in the simulation study.

A second order plant, which is directed by a feedback controller, is considered. In the Laplace form the true plant can be described with

$$G(s) = \frac{s+2}{s^2+2s+1}. \tag{A.1}$$

With the control interval T equal to $0.1s$, the discrete-time T-model, as described in Section 3.9.2, is

$$\frac{B(q)}{A(q)} = \frac{0.09984q^{-1} - 0.08173q^{-2}}{1 - 1.81q^{-1} + 0.8187q^{-2}}. \tag{A.2}$$

The plant is controlled by a second-order controller,

$$C(q) = \frac{L(q)}{P(q)} = \frac{4 - 7.323q^{-1} + 3.356q^{-2}}{1 - 1.67q^{-1} + 0.6703q^{-2}}. \tag{A.3}$$

The noise is generated by a white random process through a stable filter with the same poles as the plant and a zero at $s = 0$. A unit noise variance is used. In discrete-time form of this transfer function is

$$H(q) = \frac{0.004679q^{-1} + 0.004377q^{-2}}{1 - 1.81q^{-1} + 0.8187q^{-2}}. \tag{A.4}$$

Case 6: In case 6 the bandwidth of the plant is varied to determine if the influence of p is dependent on the size of the sampling frequency $\frac{1}{T}$ relative to the plant bandwidth. In this case a different plant, whose time constant is varied, and a different controller are used . In Laplace this plant can be represented as

$$G(s) = \frac{K/\tau}{1 + 1/\tau} e^{-\theta}, \text{ with} \tag{A.5}$$

$K = 1$ and $\theta = 1$. For each value of τ the parameters of the controller are determined by making use of the Cohen-Coon method [30]:

$$\begin{aligned} Kc &= \frac{1}{K} * \frac{\tau}{\theta} * (0.9 + \frac{\theta}{12\tau}), \\ \tau_i &= \frac{\theta * (30 + 3 * \theta)}{(9 + 20 * \frac{\theta}{\tau})}, \\ C(s) &= K_c(1 + \frac{1}{\tau_i s}). \end{aligned} \tag{A.6}$$

Here the noise is also generated by a white random process through a stable filter with the same poles as the plant and a zero at $s = 0$.

ADDENDUM B

APPROXIMATE REALIZATION OF STEP RESPONSE DATA

The function that was used to construct approximated state-space models from step response coefficients, makes use of the modified algorithm of Kung, described by Van Helmont, *et al.* [49], for step response data [51]:

- For a finite sequence of step response coefficients $\{s(t)\}_{t=1,\ldots N}$ a matrix $R_{n_r n_c}$, with $n_r + n_c = N$, is constructed as in equation (B.1)

$$R_{n_r n_c} = \left[\begin{array}{cccc} s(1)-s(0) & s(2)-s(0) & \cdots & s(n_c)-s(0) \\ s(2)-s(1) & s(3)-s(1) & \cdots & s(n_c)-s(1) \\ \vdots & \vdots & \vdots & \vdots \\ s(n_r)-s(n_r-1) & \cdots & \cdots & s(n_r+n_c-1)-s(n_r-1) \end{array} \right]. \tag{B.1}$$

- Singular Value Decomposition is then applied: $R_{n_r n_c} = U\Sigma V^T$.
- The decrease in the singular values with growing index is evaluated and a number n of significant singular values is chosen.
- The approximated rank n matrix is then constructed: $R(n) = U_n \Sigma_n V_n^T$ with

$$\begin{aligned} U_n &= U \left[\begin{array}{c} I_n \\ 0 \end{array} \right], \\ V_n &= V \left[\begin{array}{c} I_n \\ 0 \end{array} \right], \text{ and} \\ \Sigma_n &= \left[\begin{array}{cc} I_n & 0 \end{array} \right] \Sigma_n \left[\begin{array}{c} I_n \\ 0 \end{array} \right]. \end{aligned} \tag{B.2}$$

- The Ho-Kalman algorithm [50] is then applied to construct the state-space realization:

$$\begin{aligned} C &= \text{first } p \text{ (number of outputs) rows of } U_n \Sigma_n^{1/2}, \\ B &= \text{first } m \text{ (number of inputs) rows of } \Sigma_n^{1/2} V_n^T, \\ A &= \Sigma_n^{-1/2} U_n^T \cdot R^{\uparrow} \cdot V_n \Sigma_n^{-1/2}, \text{ and} \\ D &= s(1), \end{aligned} \tag{B.3}$$

where $R^{\uparrow}$ is $R(n)$ shifted over one block row upwards.

The continuous state-space matrices is computed with the MATLAB function *d2cm*, making use of the known sampling time and the ZOH method. The state-space model,

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t), ...x(0), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{B.4}$$

is then computed with the MATLAB function *ss*.

ADDENDUM C

RESIDUAL ANALYSIS OF EXPERIMENTAL DATA

In Figs. C.1, C.2, C.3, C.4, C.5 and C.6 the auto-correlation and cross-correlation functions of the residuals, for the model identified from the real closed-loop data are shown and in Figs. C.7, C.8, C.9, C.10, C.11 and C.12 these functions for the open-loop identified model are shown. The functions for the other combinations of inputs and outputs can be found in Section 6.3.2.2.

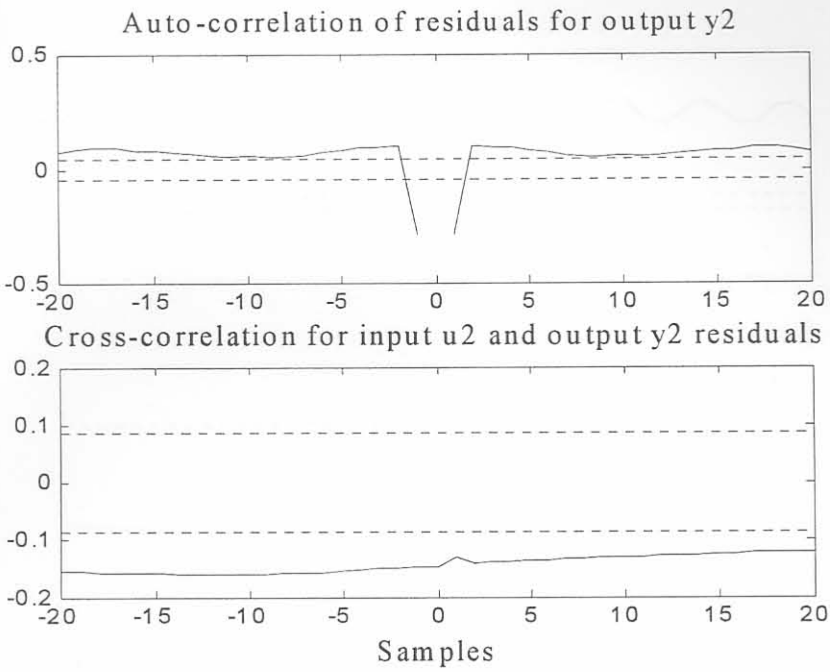

Figure C.1: The cross-correlation and auto-correlation of the residuals for y_2 and u_2 of the closed-loop identified model.

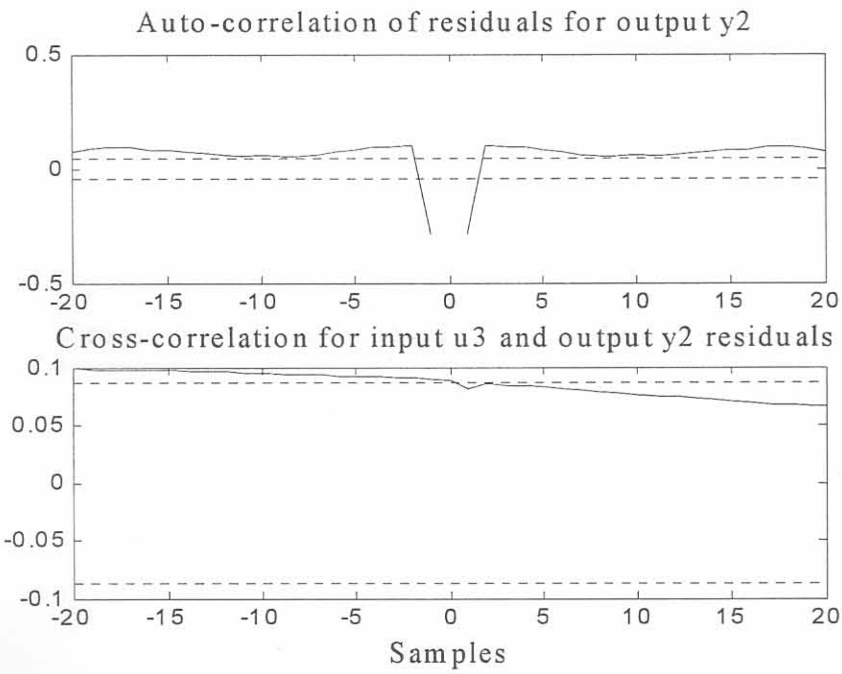

Figure C.2: The cross-correlation and auto-correlation of the residuals for y_2 and u_3 of the closed-loop identified model.

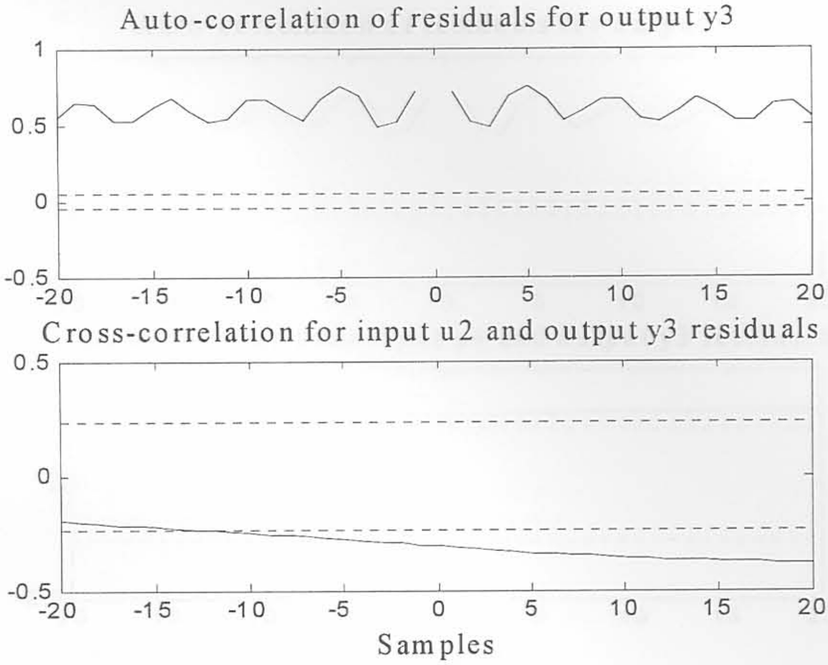

Figure C.3: The cross-correlation and auto-correlation of the residuals for y_3 and u_2 of the closed-loop identified model.

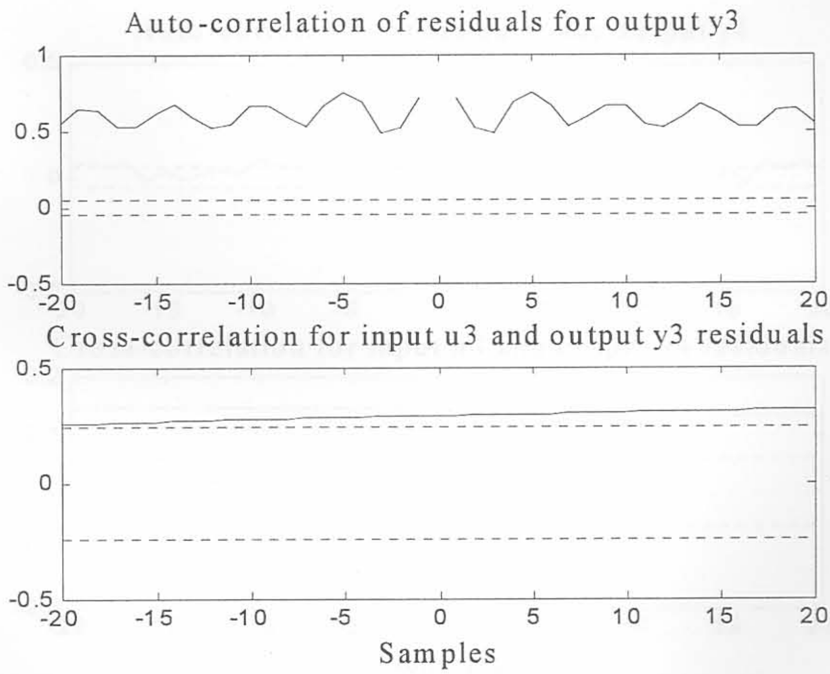

Figure C.4: The cross-correlation and auto-correlation of the residuals for y_3 and u_3 of the closed-loop identified model.

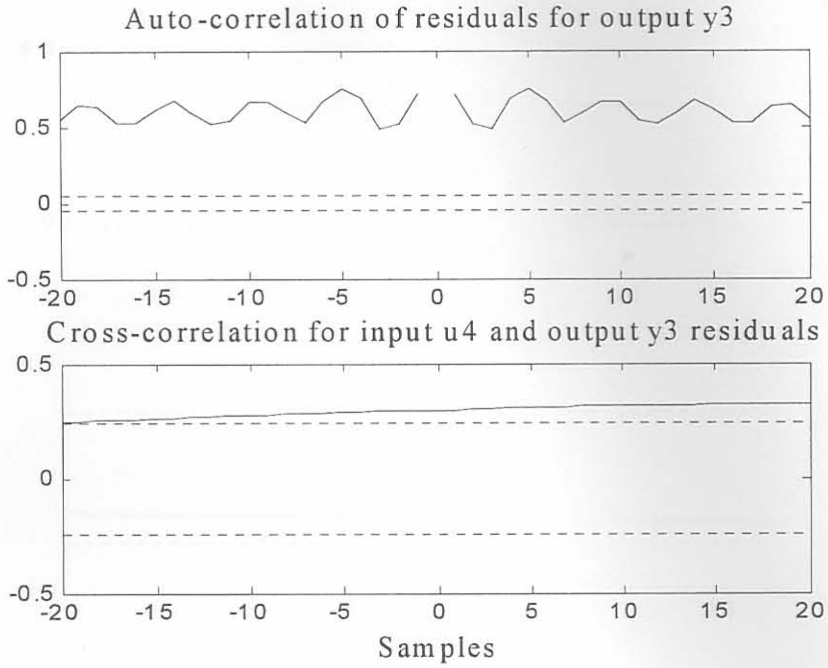

Figure C.5: The cross-correlation and auto-correlation of the residuals for y_3 and u_4 of the closed-loop identified model.

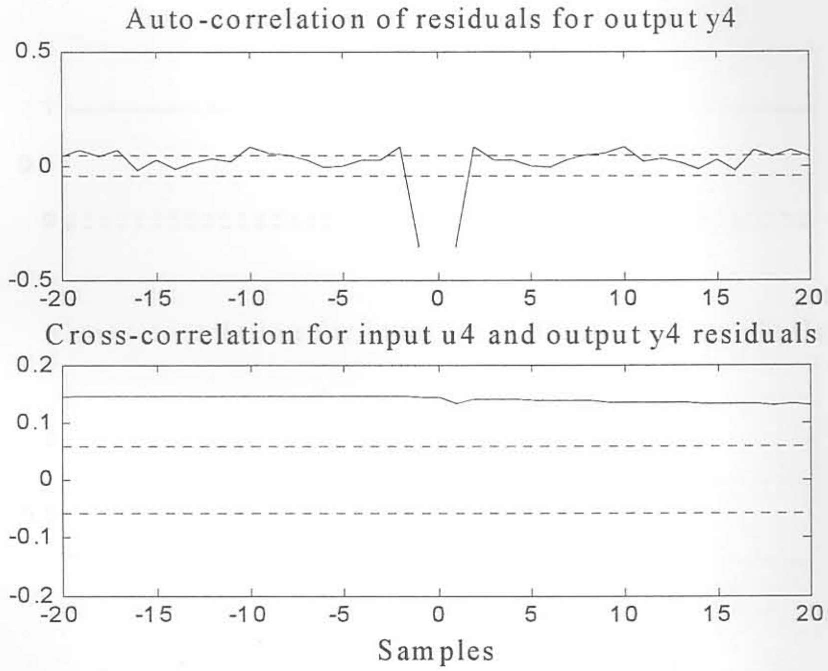

Figure C.6: The cross-correlation and auto-correlation of the residuals for y_4 and u_4 of the closed-loop identified model.

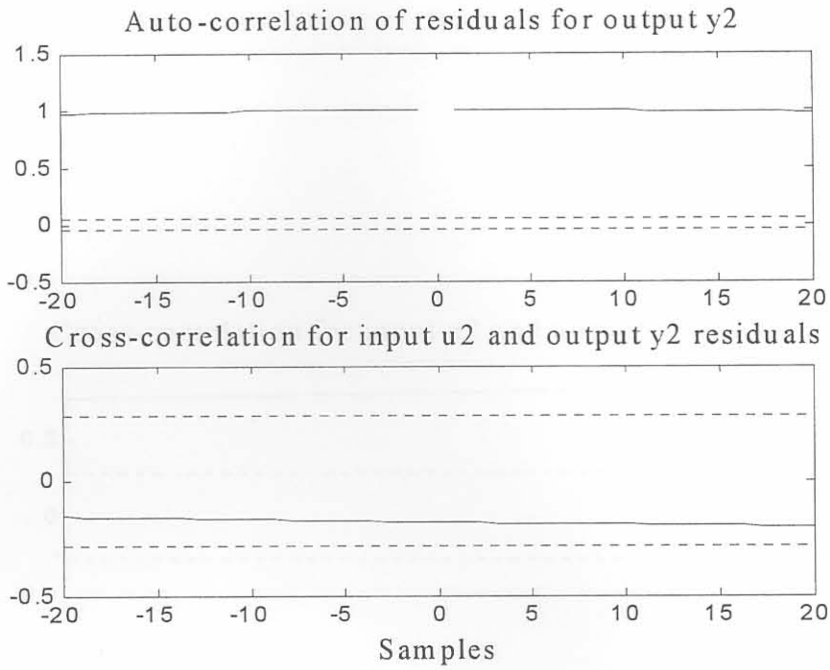

Figure C.7: The cross-correlation and auto-correlation of the residuals for y_2 and u_2 of the open-loop identified model.

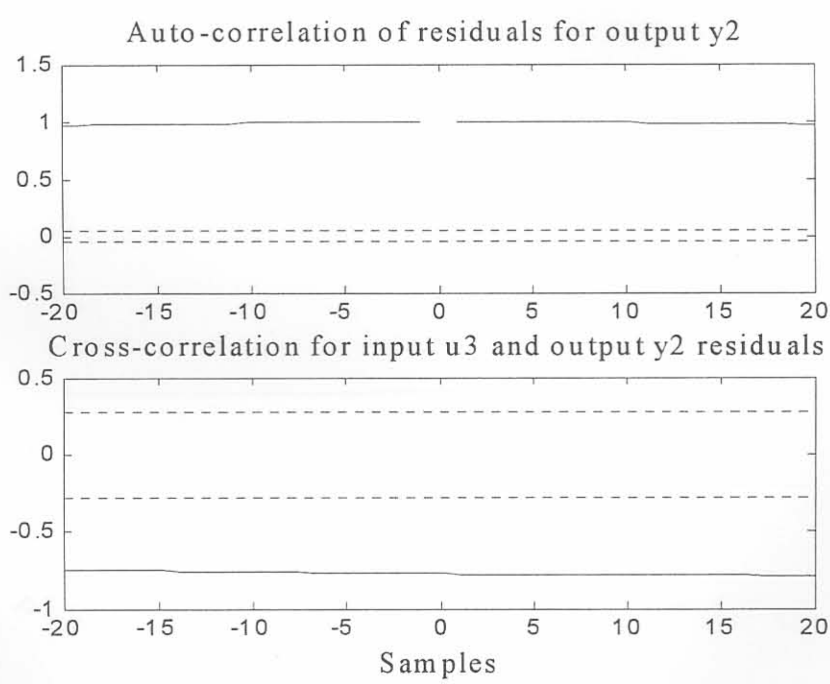

Figure C.8: The cross-correlation and auto-correlation of the residuals for y_2 and u_3 of the open-loop identified model.

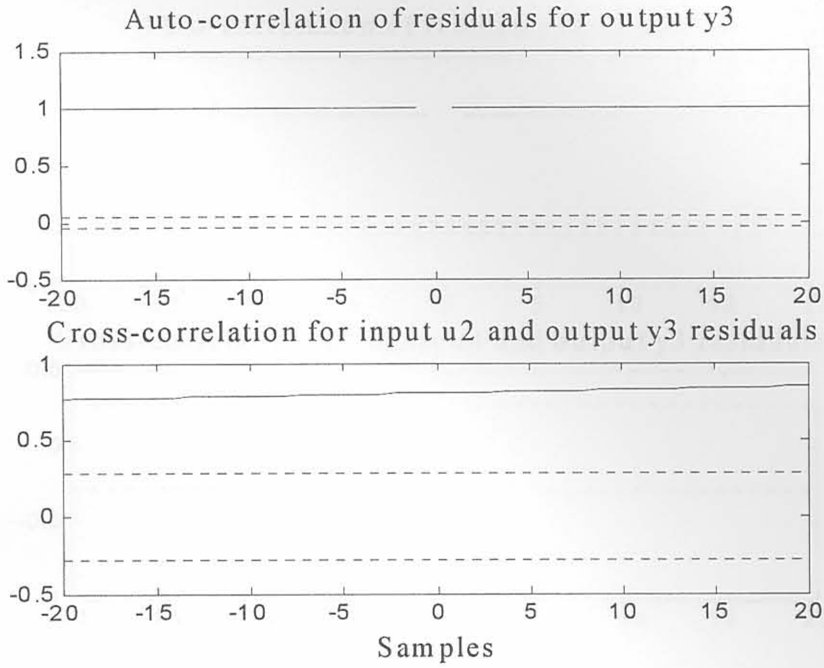

Figure C.9: The cross-correlation and auto-correlation of the residuals for y_3 and u_2 of the open-loop identified model.

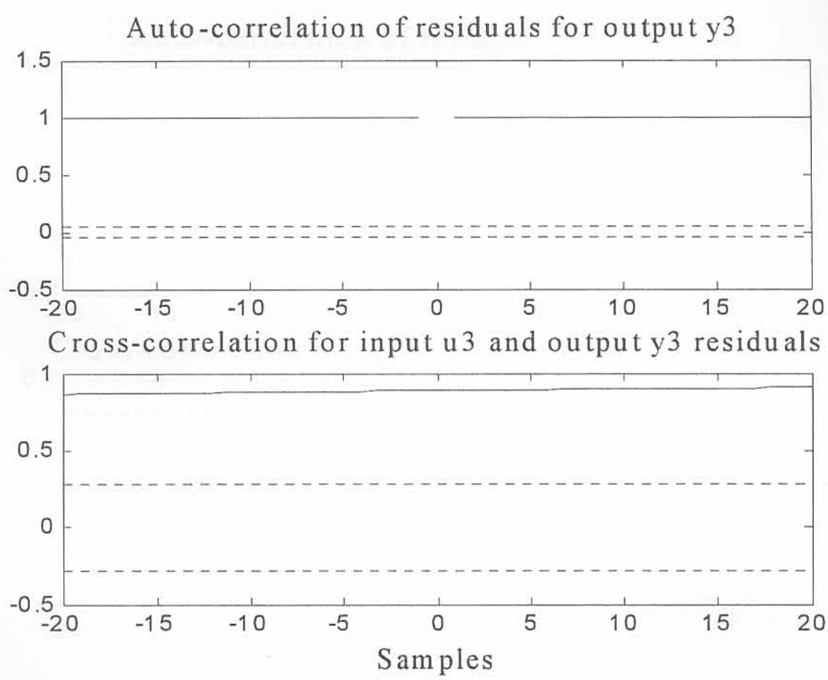

Figure C.10: The cross-correlation and auto-correlation of the residuals for y_3 and u_3 of the open-loop identified model.

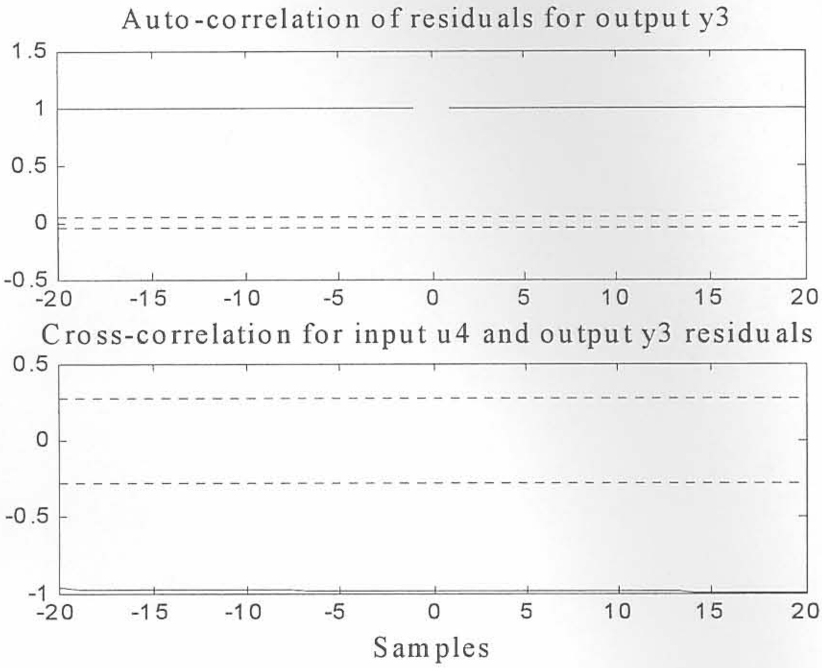

Figure C.11: The cross-correlation and auto-correlation of the residuals for y_3 and u_4 of the open-loop identified model.

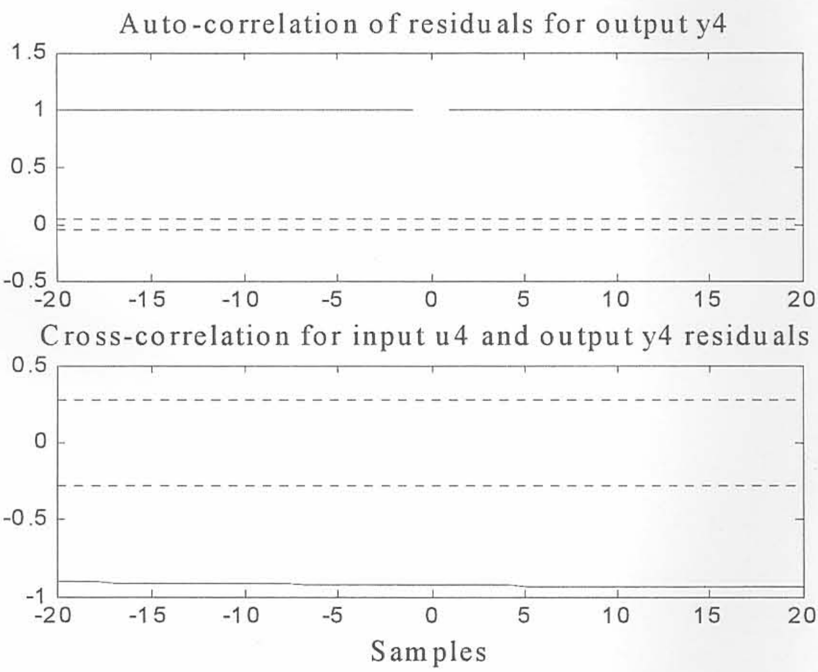

Figure C.12: The cross-correlation and auto-correlation of the residuals for y_4 and u_4 of the open-loop identified model.