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SPARSE CODING FOR SPEECH RECOGNITION

By

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PHILOSOPHIAE DOCTORAL

The brain is a complex organ that is computationally strong. Recent research in the field of neurobiology help scientists to better understand the working of the brain, especially how the brain represents or codes external signals. The research shows that the neural code is sparse. A sparse code is a code in which few neurons participate in the representation of a signal.

Neurons communicate with each other by sending pulses or spikes at certain times. The spikes send between several neurons over time is called a spike train. A spike train contains all the important information about the signal that it codes. This thesis shows how sparse coding can be used to do speech recognition. The recognition process consists of three parts. First the speech signal is transformed into a spectrogram. Thereafter a sparse code to represent the spectrogram is found. The spectrogram serves as the input to a linear generative model. The output of the model is a sparse code that can be interpreted as a spike train. Lastly a spike train model recognises the words that are encoded in the spike train.

The algorithms that search for sparse codes to represent signals require many computations. We therefore propose an algorithm that is more efficient than current algorithms. The algorithm makes it possible to find sparse codes in reasonable time if the spectrogram is fairly coarse.

The system achieves a word error rate of 19% with a coarse spectrogram, while a system based on Hidden Markov Models achieves a word error rate of 15% on the same spectrograms.

Key terms: sparse code, sparse code measurement, speech recognition, linear generative model, spike train, spike train classification, mathematical optimization, spectrogram, overcomplete dictionary, dictionary training.

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Die brein is ’n komplekse orgaan wat berekeningsgewys baie sterk is. Onlangse navorsing in die veld van neurobiologie help wetenskaplikes om die werking van die brein beter te verstaan, veral hoe die brein eksterne seine voorstel of kodeer. Die navorsing wys onder andere dat die neurale kode yl is. ’n Ylkode is ’n kode waar min breinselle deelneem om ’n spesifieke sein voor te stel.

’n Breinsel kommunikeer met ander selle deur pulse op spesifieke tye te stuur. Die pulse tussen verskeie breinselle oor tyd word ’n pulsreeks genoem. Die pulsreeks bevat al die belangrikke inligting oor die sein wat dit kodeer. Hierdie tesis wys hoe ylkodes gebruik kan word om spraakherkenning te doen. Die herkenningsproses bestaan uit drie dele. Eers word ’n spraaksein voorgestel as ’n spektrogram. Daarna word ’n ylkode gesoek wat weer die spektrogram voorstel. Die spektrogram dien as inset tot ’n lineêr–genererende model. Die model se uitset is ’n ylkode wat geinterpreteer kan word as ’n pulsreeks. Laastens herken ’n pulsreeksmodel die woorde wat voorgestel word in die pulsreeks.

Die algoritmes wat ylkodes soek om seine voor te stel verg baie berekeninge. Ons stel daarom ’n algoritme voor wat meer effektief is as huidige algoritmes. Die algoritme maak dit moontlik om ylkodes te vind binne ’n redelike tyd as die spektrogram taamlik grof is.

Met ’n growwe spektrogram behaal die herkenningsstelsel ’n woordfouttempo van 19%, terwyl ’n stelsel gebaseer op verskuilde Markovmodelle (“Hidden Markov Models”) ’n woordfouttempo van 15% behaal vir dieselfde voorstelling.

Sleutelterme: ylkode, ylkode maatstaf, spraakherkenning, lineêr–genererende model, pulsreekse, pulsreeksherkenning, wiskundige optimering, spektrogram, oorvolledige woordeboek, woordeboek afrigting.

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CHAPTER ONE

INTRODUCTION

The brain needs to form an internal representation of the outside world to interact with it and does so by representing or coding information in the spatio-temporal activities of neurons. Neurons are complex computational units. They communicate mainly with electrical pulses called spikes, but they also make use of chemical communication. A neuron can be viewed as a temporal binary element; it is either silent or it fires a spike. This is a rather simple view of a neuron (Aur, Connolly and Jog, 2006) but it captures the main means of neural communication (Rieke, Warland, de Ruyter van Steveninck and Bialek, 1996). Figure 1.1 shows the activity pattern of some neurons in the visual cortex; it reveals a pattern in the response of neurons to a repeating stimulus.

1.1 THE NEURAL CODE

The neural code is the “language” of the brain. It contains information about stimuli, it contains the commands the brain sends to muscles, it contains thought etc. Researchers have

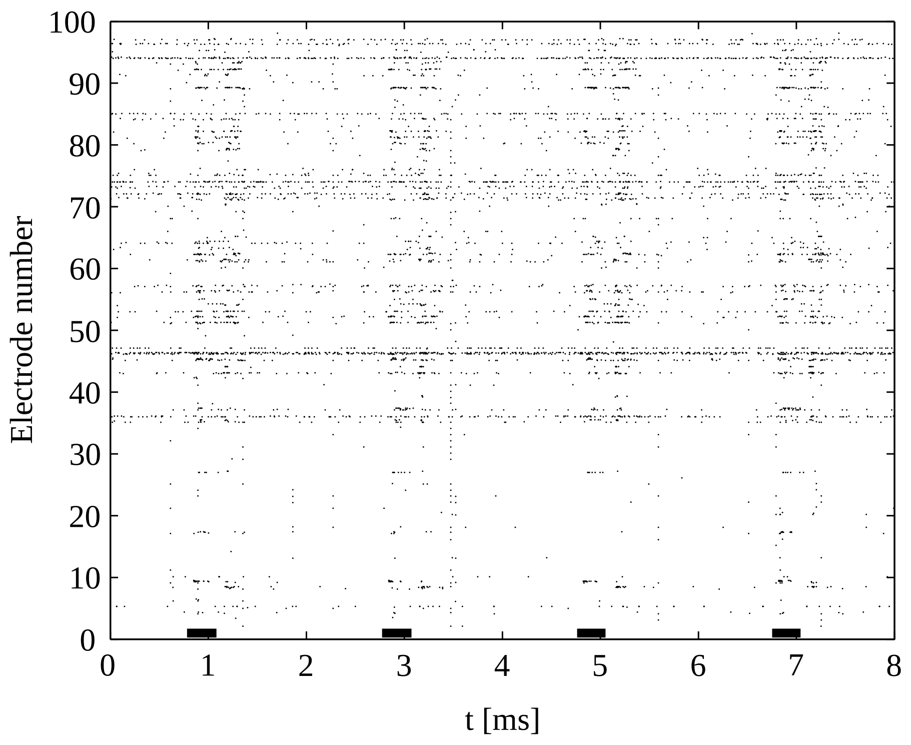


Figure 1.1: Here the activity of 100 neurons in the visual cortex are shown in response to a simple repeating black-white stimulus. Each dot signals a spike. The stimulus is white for 300ms, indicated by the dark bars at the bottom, and black for 700ms. (This data is part of a sample file distributed with the software program CORTIVIS).

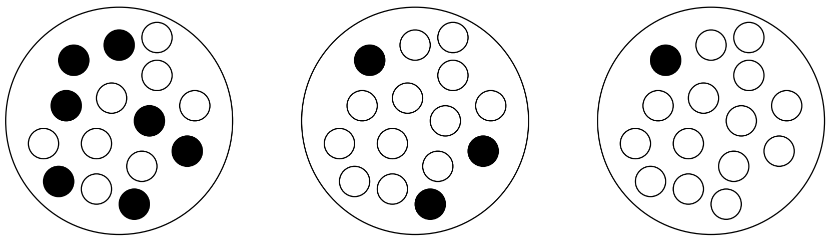

(a) Dense coding. (b) Sparse coding. (c) Local coding.

Figure 1.2: In dense coding (a) many of the available neurons are active in the representation of a stimulus, whereas in local coding (c) only one neuron is active in the representation. Sparse coding (b) is a scheme somewhere between dense coding and local coding.

not been able to read the neural code or understand exactly how the brain process information (Bair, 1999; Eggemont, 1998; Theunissen, 2003). This remains an active area of research and experimental work adds to our understanding of the neural code.

It is useful to take a simple view of a neuron in order to compare the neural code with common computational codes. Such a comparison helps us to understand certain aspects of the neural code. Figure 1.2 illustrates some coding schemes for binary elements. Information can be represented by a *dense code* (figure 1.2(a)). For such a code few neurons are required to code many different stimuli but the neurons will be very active. Many of the available neurons will participate in the representation. A particular neuron therefore does not convey that much information about the stimulus, instead the information is distributed among the neurons. A code can also be a *local code* (figure 1.2(c)). In this case there are many neurons available but only one participates in coding a specific stimulus. Both of these extremes are biologically implausible. The local code will require too many neurons to code all the possible stimuli. Even though there are billions of neurons in the brain, there are still too few so that each neuron can code one and only one stimulus. A dense code is also implausible as the neurons will have to be too active.

Generally the lower level sensory neurons are more active than higher level neurons. At the highest level neurons fire on average once every second. The brain adopts a coding scheme somewhere between local coding and dense coding, which is called *sparse coding*(figure 1.2(b)) (Földiák and Young, 1995; Vinje and Gallant, 2000). The optimal level of

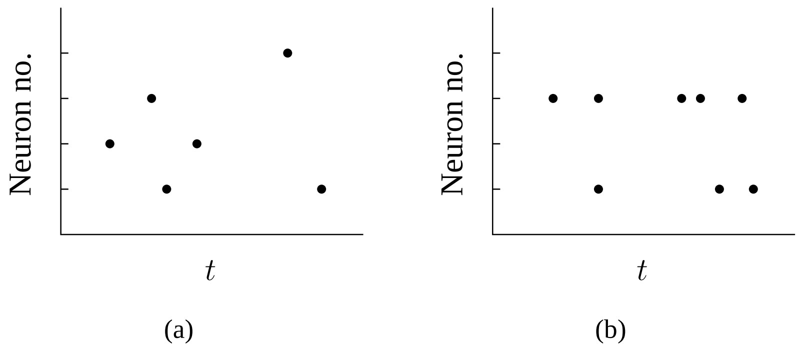


Figure 1.3: The figure shows two spike trains that code the same stimulus. Spike train (b) has a population activity that is more sparse than spike train (a) eventhough it has more spike. This is because population sparseness depends on the number of neurons that participate in a representation and not on the number of spike per representation.

sparseness may be different for various areas in the brain.

1.1.1 SPARSE CODING

Sparse coding is a general principle that is employed in most areas of the brain (Beloozerova, Sirota and Swadlow, 2003; DeWeese, Wehr and Zador, 2003; Laurent, 2002; Vinje and Gallant, 2000; Vinje and Gallant, 2002). There are two forms of sparseness namely, *population sparseness* and *lifetime sparseness*. Population sparseness refers to the fact that only a *small subset* of neurons is active in coding a stimulus, whereas *lifetime sparseness* refers to a particular neuron being seldom active over a long time period (Willmore and Tolhurst, 2001) (see figures 1.3 and 1.4). Both forms of sparseness are present in the brain and they are not identical. For example, each neuron in a group may have a good lifetime sparseness (not very active over long time spans), but the population sparseness will be poor if all the neurons are active at the same time. Whenever we refer to sparseness, it includes lifetime sparseness and population sparseness.

In a sparse code a small subset of neurons participate in coding a particular stimulus. Sparse coding has several advantages over other types of coding. A sparse code can also store more patterns in associative memory than either a dense code (Földiák, 1998; Rolls

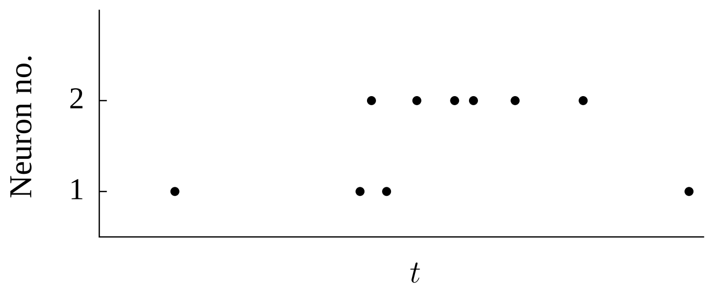


Figure 1.4: The activity of two neurons over a long time period in which several stimuli occured. The lifetime activity of neuron 1 is more sparse than neuron 2 because it fires fewer spikes.

and Treves, 1990) or a local code (1-of-N representation). Sparse coding makes structure in the input more explicit (Olshausen and Field, 2004), it is then easier for other areas in the cortex that receive the sparse code to perform computations on it. Furthermore, evidence suggests that the representation is efficient in an information sense (Vinje and Gallant, 2002), i.e. it contains as much information about the outside world as possible, given the physical constraints of the brain.

1.1.2 EVIDENCE FOR SPARSE CODING

Vinje and Gallant (2000) and Vinje and Gallant (2002) found that neurons in V1 (a large and important visual cortical area) respond sparsely to natural stimuli. Interestingly their responses are more dense for unnatural stimuli. This shows that the neural code is adapted to represent natural stimuli in a sparse manner. Sparse coding has also been observed in the auditory cortex (DeWeese *et al.*, 2003), the olfactory system (Laurent, 2002) and the motor cortex (Beloozerova *et al.*, 2003).

Computational studies have shown that sparse coding leads to receptive fields that resemble the actual receptive fields, again proving that the brain makes use of sparse coding. For most of the studies a *linear generative model* is used. The model linearly adds symbols or dictionary elements together in order to reconstruct a stimulus. The code gives the coefficients with which the dictionary elements have to be scaled before they are added to

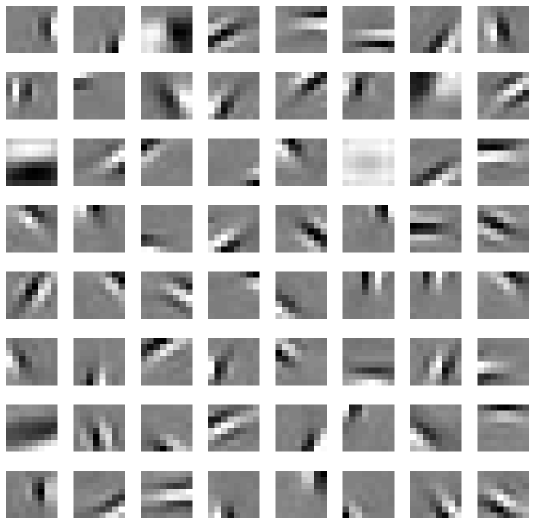


Figure 1.5: The dictionary elements as calculated with SPARSENET (Olshausen and Field, 1996b; Olshausen, 1996). The elements resemble the Gabor filter-like receptive fields of some neurons in V1.

reconstruct the signal. The dictionary is adapted to represent a dataset with sparse codes. Olshausen and Field (1996a) found that for image patches from natural scenes, the adapted dictionary elements are Gabor filter-like receptive fields (figure 1.5). Receptive fields that look similar are found in simple cells of V1. A simple cell responds to an edge that is orientated in the preferred direction of the cell, and is located in a specific area in the receptive field. Simple cells make up about a quarter of the cells in V1 while the remainder of the cells are complex. Complex cells also respond to specifically orientated edges, but the edge can be anywhere in the receptive field. Other complex cells respond to specifically orientated edges that move in a certain direction.

The receptive fields of more complex cells cannot yet be explained by the linear generative model. However a more complex model together with sparse coding shows some correlation between the receptive fields of complex cells and the dictionary elements (Hyvärinen and Hoyer, 2001). Sigman, Cecchi, Gilbert and Magnasco (2001) used a complex model to explain some properties of complex visual cells.

Less work has been done on senses other than sight. Lewicki (2002) found that when a linear generative model codes natural sounds in the time domain, the elements of the trained dictionary have similar tuning properties to auditory nerve fibres.

1.1.3 OVERCOMPLETE CODES

There are 3500 inner hair cells in the human cochlea. Roughly ten auditory nerve fibres connect to each inner hair cell so that 30 000 nerve fibres extend from the cochlea. Information gathered by the hair cells goes through several stages of processing. The last three stages are the inferior colliculus which has around 392 000 neurons, the medial geniculate body which has 570 000 neurons and the auditory cortex which has about 100 000 000 neurons. There is a great expansion of neurons from the senses to the cortex.

All the sound information about a stimulus is gathered by the hair cells. The amount of information about a stimulus cannot increase from one processing stage to the next. It is however possible that additional information, such as visual clues, may add information about a sound stimulus, but this only appears to happen from the auditory cortex and upwards (Kayser, Petkov, Augath and Logothetis, 2007). Given that the amount of information cannot increase, what advantage is there to having such an expanded representation in higher stages of processing?

Imagine a system similar to the brain that should be able to code an N dimensional *random* signal, but it can use an $N_d > N$ dimensional code. When the dimensionality of the code is more than the dimensionality of the signal the code is *overcomplete*. The code will always contain N active code elements, except if by chance the signal corresponds to a feature that a particular element code for. There is not much advantage in having an overcomplete representation when random signals are coded. However natural stimuli are not random, it has statistical structure. In fact we can only recognize stimuli if it is similar to stimuli we have experienced before.

Consider a code that has as many code elements as the fundamental features that can generate a structured signal. This code can be optimally sparse when each code element corresponds to one of the features. The representation will be sparse as it will have only as many active elements as there are fundamental features in the signal. In this case the dictionary has to be overcomplete if the number of features is more than the dimensionality of the signal.

The great expansion of the neural representation from lower to higher levels allows neurons to code for very specific features. For example, simple cells in the early visual cortex have Gabor filter-like receptive fields - their responses are broadly tuned. Higher up, in the primary visual cortex neurons respond to more complex features, such as edges, lines, or gratings. In even higher visual areas like V4, neurons respond to more complex features such as faces and hands (Földiák, 1998). There even exist "grandmother" cells (Quirago, Reddy, Kreiman, Koch and Fried, 2005). These neurons are so finely tuned that they only respond to very specific stimuli, such as the thought or sight of the proverbial grandmother. This shows that some neurons in the higher levels of the brain use the one extreme form of sparse coding, namely local coding.

1.2 SPARSE CODING FOR PATTERN RECOGNITION

Sparse overcomplete coding should perform well as a means to pattern recognition because it can capture the underlying structure in the signal. It is a powerful method to extract statistically significant features from a signal.

Most of the published work on sparse coding focuses on the properties of the adapted dictionary, algorithms that can find sparse codes or on models that are more complex than the linear generative model. There are some studies that have used sparse coding to create a feature set for sound or speech recognition tasks. The studies follow the same general approach: first a sound signal is encoded into a spike train, then the recognition task is performed by decoding the spike train. Cho and Choi (2005) perform sound classification with spikes. They classify a sound as belonging to one of ten classes, which include male speech, foot steps and flute sounds. Näger, Storck and Deco (2002) show that transitions between vowels can be classified by learning the delays between spikes. Kwon and Lee (2004) use independent component analysis (ICA) to extract features from speech in order to do phoneme recognition. ICA is an algorithm that provides a sparse representation of a signal. Some studies illustrate isolated digit recognition (Loiselle, Rouat, Pressnitzer and Thorpe, 2005; Mercier and Séguier, 2002; Verstraeten, Schrauwen, Stroobandt and Campenhout, 2005) while Holmberg, Gelbart, Ramacher and Hemmert (2005) demonstrate isolated letter recognition. All of these studies consider only isolated samples.

Very few studies have used sparse coding on practical image recognition problems. Sun, Zhou, Ma and Wang (2001) applied sparse coding to face recognition. Sparse coding achieved a recognition rate of 95% compared to 89% for Fisherface (a popular face recognition method).

All of the work mentioned above use inputs that are already segmented. In other words, it is not necessary to first divide the input into segments before it can be classified. However in this study we use sparse coding to do *continuous speech recognition*. The classification algorithm should be able to segment the signal.

The aims of the study are discussed in the last section of this chapter, but first we give an overview of the speech recognition problem and briefly discuss current speech recognition systems.

1.3 SPEECH RECOGNITION IN GENERAL

Automatic speech recognition (ASR) is a convenient interface between man and machine, because speech is such a natural way for people to convey information. It can also allow people that are unfamiliar with some technologies or some disabled people to make use of modern technology.

An ASR system can be categorized as speaker-dependent or speaker-independent; as an isolated word recognizer or a continuous speech recognizer; as working with a limited vocabulary or a large vocabulary. The most versatile ASR task would be to do speaker-independent, continuous speech recognition with a large vocabulary. This is also the most difficult ASR task, and the performance of computers on this task does not yet come close to that of humans. But other types of ASR systems have successfully been applied to everyday

ASR problems.

Persons with certain disabilities, such as quadriplegics, may find limited vocabulary, isolated word recognition systems very useful. With such a system they could give commands to a television set, a motorized bed etc. Some cars also use a similar system to allow the driver access to functions that are usually situated on the steering wheel, to operate the navigation system or the car stereo.

Speaker-independent continuous speech recognition with limited vocabulary is often used in switchboard applications, where a caller gives the name of the person he would like to speak to, and his call is automatically forwarded to that person. Other systems allow a caller to book movies, make hotel or flight reservations.

Dictation systems employ state-of-the-art technology continuous speech recognition. These systems can produce documents much faster than many people can type, once it is trained on a specific user, and if the user learns to speak a bit slower and clearer. A dictation software program is distributed as a speaker-independent system, but the accuracy of an off-the-shelf system is not good enough for the system to be useful. The user has to train the system for one or two hours before the recognition accuracy increases to a useful level. After the system has been trained that much, it is not really speaker-independent anymore.

Unfortunately the performance of a truly speaker-independent ASR system, are not yet good enough to be used in everyday applications. These systems are very susceptible to noise, even office background noises can cause their performance to degrade considerably.

There is therefore a need for ASR to move closer to human-like speech recognition.

1.4 CURRENT SPEECH RECOGNITION SYSTEMS

The most common ASR systems use across-frequency features to recognize speech. A signal is divided into small, usually 20ms, segments and some type of frequency analysis of each segment yields the feature set. The most widely used feature set is the Mel Frequency Cepstrum Coefficients (MFCC) and time derivatives thereof (Rabiner and Juang, 1993).

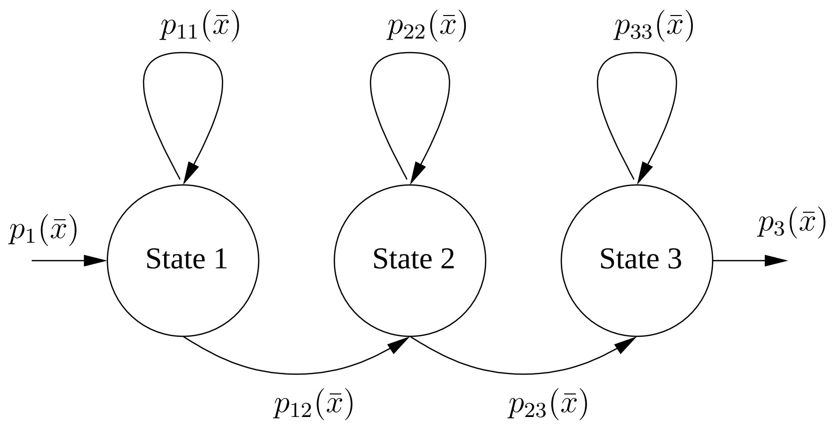


Figure 1.6: An acoustic unit is modeled with a three state Hidden Markov model. The probability of transition from one state to the next is a function of the feature vector $\bar{x}$.

The features serve as inputs to models of acoustic units, which can be words, phonemes, triphones etc. An acoustic unit model is usually a 3 or 5 state Hidden Markov Model (HMM) (see figure 1.6). The probability of transition from one state to another is determined by the feature vector for a particular segment. By concatenating the unit models, bigger units can be modelled, and eventually sentences. A language model ensures that the acoustic units are concatenated in a language consistent manner. A very basic language model would give the probability of a specific acoustic unit following another specific acoustic unit.

Continuous ASR aims to recognize sentences given a sequence of feature vectors. It is possible to find the most likely sequence of acoustic units, by finding the sequence of models that best fit the feature vectors and the language model. Once the sequence of acoustic units is determined, a post processor combines the units into sentences.

This approach to ASR does not contain much prior knowledge of speech, as information on the human communication process is limited. It rather makes use of a large amounts of data to self discover relevant patterns in speech within the framework of an HMM. This approach should work very well, provided there is enough information and provided the HMM framework can model speech decoding adequately. Later in this chapter we will show that there is an alternative speech decoding model that fit speech data better than the HMM approach.

1.4.1 SOME PROBLEMS WITH CURRENT SYSTEMS

There are a few reasons why the performance of current systems does not compare to that of humans. These reasons relate to the feature set, the acoustic units and the HMM framework.

Temporal dynamics play an important role in human speech recognition. It has been found that the relevant information for speech recognition lies in the modulation range of 1 to 15Hz (Hermansky, 1998). That is because the vocal tract cannot change faster than 15Hz, and information outside this range can therefore not contain anything relevant to speech recognition. Perceptual experiments (Allen, 1994; Kral, 2000; Moller, 1999; Shannon, Zeng, Kamath, Wygonski and Ekelid, 1995) have further showed that temporal information in speech is much more important than spectral information.

Current systems use an across-frequency feature set where each segment is assumed to be stationary and independent of neighbouring segments. This assumption does not take the temporal dynamics of speech into account at all. The problem is solved in part by augmenting the basic feature set with its time derivatives. Temporal dynamics are modelled with the HMM.

Across-frequency features in a short time segment is susceptible to noise, because it is difficult to separate noise from signal over a short time interval. This is one of the main reasons why current ASR systems fail in noisy environments, even when humans do not find it a difficult task.

Another problem with current ASR systems lies with the acoustic units. These units are very basic, and smaller units have been used more successfully that larger units. This is in contrast to human perception of speech, where the basic unit is probably on the syllable level (Nguyen and Hawkins, 2003).

A further shortcoming of current ASR systems is in the way the most likely sequence is found. Longer sounds will carry more weight than shorter sounds, because there is a cost associated with each time segment. The most likely sequence may end up not to be the best choice.

Finally the HMM framework may not be well suited to model speech. This notion is supported by the fact that huge amounts of data is required to train such a system, which even after training, performs poorly in conditions that are not very close to that under which the training set was gathered.

It's reasonable to assume that the best speech decoder is the auditory cortex. We should therefore look to include aspects of the cortex in ASR systems in order to improve them.

1.5 SPARSE CODING FOR SPEECH RECOGNITION

Psychological studies have showed that mammalian auditory cortical neurons respond to spectro-temporal sound patterns (deCharms, Blake and Merzenich, 1998; Shamma, 2001), that is patterns across-frequency *and* across-time.

There are some models that use across-time processing (Hermansky and Sharma, 1999; Hermansky and Morgan, 1994; Ikbal, Magimai.-Doss, Misra and Bourlard, 2004), and others that use spectro-temporal features (Kleinschmidt, 2002; Klein, König and Körding, 2003; Kwon and Lee, 2004). These models do not achieve the same recognition results as state-of-the-art ASR systems in relative noise free environments, but they have performed better than HMMs in noisy environments.

Sparse coding with an overcomplete dictionary can give a code whose features are meaningful spectro-temporal sound patterns. The linear generative model (LGM) easily fit into a sparse coding scheme. The model reconstructs a signal of a preset length using a dictionary

$$\bar{x} = \mathbf{\Phi}\bar{a} \tag{1.1}$$

The signal $\bar{x}$ is encoded by a set of coefficients $\bar{a}$, one for each dictionary element in $\mathbf{\Phi}$ (the columns in $\mathbf{\Phi}$ contain the dictionary elements).

Principle Component Analysis (PCA) can be seen as a generative model. PCA chooses a dictionary such that the elements are all orthogonal, and such that the elements point in the directions ordered from the largest to the lowest variance of a given set of signals. Another choice of dictionary is given by Independent Component Analysis (ICA). ICA tries to find dictionary elements that will result in a code whose coefficients are independent of each other for a given set of signals. Both PCA and ICA use a complete dictionary, i.e. for an N dimensional signal vector, $\mathbf{\Phi}$ is an $N \times N$ sized matrix. As already mentioned there are various advantages to having an overcomplete dictionary, where $\mathbf{\Phi}$ is an $N \times N_d$ sized matrix, and $N_d > N$.

For an overcomplete dictionary the code cannot be uniquely determined. It is then possible to search for a sparse code from all the possible codes. The sparse code of a LGM is a pulse like structure, similar to the spike structure by which biological neurons communicate. Patterns in this structure can be related to specific events in the signal, such as spoken words. Another benefit of encoding a signal with a pulse structure is that the decoding algorithms are not such a strong function of time as the decoding algorithm in current ASR systems is.

1.5.1 AIMS OF THIS STUDY

The *primary aim* of this study is to use overcomplete sparse coding for pattern recognition, in particular continuous speech recognition. This study will serve as an initial investigation into sparse coding for real-world pattern recognition problems. It will show what aspects of the implementation of sparse coding are important and also what the shortcomings of current methods are. It will further highlight the factors that limit the use of sparse coding for pattern recognition.

We use the LGM to select codes that can represent a signal. The dimensionality of the code is very high, especially as the code is overcomplete. The current algorithms that find sparse codes of such high dimensional problems are either computationally expensive or they yield a code that is not well suited for pattern recognition. We therefore need to develop an algorithm that satisfies both requirements.

A *secondary aim* is to see what improvements sparse coding can bring to current speech recognition systems. Overall sparse coding via a generative model seems to be a better model for speech recognition than MFCC coefficients and HMMs, because:

- it uses spectro-temporal features,
- it does not prescribe which acoustic units should be used, but the dictionary is free to choose any unit,
- it combines the basic units into sentences by using an algorithm that is not a linear function of time, and
- the properties of the model correlate well with that of the auditory cortex, making it biologically more plausible.

This study will show whether these benefits actually improve speech recognition performance, and whether there is merit in doing further investigation along these lines.

In chapter 2 we show how a signal can be represented by a sparse code. The sparse code resembles a spike train (the spatio-temporal activities of neurons). Chapter 3 discuss our method for decoding the sparse code; the chapter shows how to determine the words that are coded by the spike train.

CHAPTER TWO

SPARSE CODING

As mentioned earlier, we would like to use the idea of sparse coding in a speech recognition system. This requires that a stimulus or signal be transformed into a sparse code, and that patterns in the sparse code be associated with spoken words. In this chapter we will first look at ways to find a sparse code that will represent a signal. We then look at the implementation issues that arise when sparse coding is applied to speech recognition. Finally, we give the results of sparse coding for speech. The next chapter will discuss the pattern recognition problem.

2.1 THE LINEAR GENERATIVE MODEL

Signals are often represented as a linear sum of elementary signals; for example, with the Fourier transform a signal is represented as a linear combination of sinusoids. Such a representation may have favourable properties. It may, for example, highlight important elements or objects that are present in the signal, or it may distinguish between components that behave in characteristic fashion.

We want to transform a speech signal into a sparse representation. Thus far the most successful approach is to use a linear generative model (LGM) (Lewicki and Sejnowksi, 1999; Lewicki, 2002):

$$\overline{x} = \mathbf{\Phi}\overline{a} + \overline{\epsilon} \tag{2.1}$$

generally, $\overline{x}$ is the N dimensional signal, $\overline{a}$ the N_d dimensional representation or code, $\mathbf{\Phi}$ the $N \times N_d$ transformation matrix and $\overline{\epsilon}$ is an N dimensional additive noise term. $\bar{x}_R = \mathbf{\Phi}\overline{a}$ is the reconstructed signal. The elements a_i in the code reveal which objects (the columns in $\mathbf{\Phi}$) are present in the signal.

A representation of the above form is only helpful if the dictionary elements capture in some way the structure of the signal. A random dictionary would not be nearly as useful as a dictionary of sinusoids if one is looking for the frequency content of a signal.

The type of model in equation 2.1 is also successfully applied in blind source separation (Lee, Lewicki, Girlami and Sejnowski, 1999), in image denoising, image compression (Lewicki and Olshausen, 1999; Lewicki and Sejnowski, 2000; Kreutz-Delgado, Murray, Rao, Engan, Lee and Sejnowksi, 2003) and digital water marking (Bounkong, Toch, Saad and Lowe, 2003). The transform $\mathbf{\Phi}$ is named differently depending on the area of application. It can be called a basis matrix, mixing matrix, or dictionary. The columns in $\mathbf{\Phi}$ are called synthesis functions, basis functions, words or dictionary elements. We will use the term dictionary to refer to $\mathbf{\Phi}$ and dictionary elements to the refer to the columns in $\mathbf{\Phi}$.

As we are interested in speech recognition, we will use the generative model to do feature extraction. There is a lot of structure in speech, as in all natural stimuli. If the dictionary elements reflect this structure, then the code will reveal the basic elements that make up a particular signal.

The dictionary determines the features that will be extracted and also the properties of the code. Depending on the application, the best choice of a dictionary may not be a predetermined set such as sinusoids or wavelets, but may be data dependent. For example, in image compression, Fourier bases which are data independent achieve a 5.34 bits/pixel coding efficiency, while a data dependent dictionary such as ICA achieves a significantly better coding efficiency of 4.71 bits/pixel (Lewicki and Olshausen, 1999).

If the number of dictionary elements equals the signal dimension ($N = N_d$), we speak of a *complete* dictionary. An *overcomplete* code ($N_d > N$) has several advantages over a complete code. It is more stable in the presence of signal noise (Kreutz-Delgado *et al.*, 2003), small changed in the signal does not cause great disturbances in the code; it can give a sparse representation of the signal; and the code can be more efficient in an information sense (Kreutz-Delgado *et al.*, 2003; Lewicki and Olshausen, 1999).

However if the dictionary is overcomplete, then the code may not be uniquely determined, because there are many different codes that can represent the same signal. In this case a sparseness function can be used to choose the best code $\bar{a}^*$ from among the viable codes. The fact that the code cannot uniquely be determined from the signal signifies that the code is not linearly dependent on the signal. The sparse code for $\bar{x}_3 = \bar{x}_1 + \bar{x}_2$ does not have to be simply the sum of the sparse codes for $\bar{x}_1$ and $\bar{x}_2$.

Section 2.4 will look at the code selection problem in more detail. It can generally be stated as an optimization problem that is a function of the reconstruction error $R(\bar{a}) = ||\overline{x} - \mathbf{\Phi}\overline{a}||^2$ and the sparseness function $S(\bar{a})$ (S is small for sparse codes). For example the code $\overline{a}^*$ to represent a signal $\overline{x}$ is

$$\overline{a}^* = \min_{\overline{a}} S(\overline{a}) \quad \text{such that} \quad R(\bar{a}) \leq \epsilon \tag{2.2}$$

with ϵ the noise level. Another way of formulating the optimization problem is

$$\overline{a}^* = \min_{\overline{a}} R(\bar{a}) + \lambda S(\bar{a}) \tag{2.3}$$

where λ sets the balance between reconstruction and sparseness.

The sparseness function plays an important role in the code that will be selected to represent a signal. In section 2.3 we turn our attention to the sparseness function S.

2.1.1 NEURAL REPRESENTATION AND THE CODE

A nonzero code element corresponds to a spike in the neural code. There is a discrepancy between the neural code and the code of a LGM. A neural spike is a binary event whereas a code element can take on any real value. We can interpret positive real values as some number that is related to the number of neurons that fire spikes simultaneously. The brain appears to be redundant in this manner: more than one neuron can code the same information, which helps the brain to deal with physical damage. Negative real values are more difficult to interpret (although they can be viewed as inhibitory spikes). We choose to rather constrain the code elements to be nonnegative $a_i \geq 0$.

It has not yet been shown that a nonnegative sparse code is in some way better than a sparse code that includes negative terms, but it is easier to interpret physiologically, and, as we will show later, it provides a lower bound to a possible nonconvex minimization problem.

2.2 SPARSE CODING OF LETTERS

Here we give a simple application of sparse coding of letters to illustrate the LGM and sparse codes. The dataset appears in figure 2.1. It has 26 samples, one for each letter of the alphabet. Noise can be added to the samples to create a more realistic dataset, but for the purpose of illustration we consider only the noiseless case.

The dimensionality of the dataset is the number of pixels per letter, which is 15. We use an overcomplete dictionary of 20 elements to reconstruct the letters. The code will be maximally sparse if the dictionary has at least as many elements as there are letters. The elements of the optimal dictionary will then be exactly the dataset. However an overcomplete code will still be sparse even if it is not maximally sparse.

The dictionary is adapted to the data and appears figure 2.2. It contains dictionary elements that fit some letters almost exactly, such as elements 1, 5, 11 and 12; some elements are very similar to letters of the alphabet such as 2, 7, 8 and others; element 10 is not like any letter in the dataset. Inspection of the results reveals that element 10 is used, with very

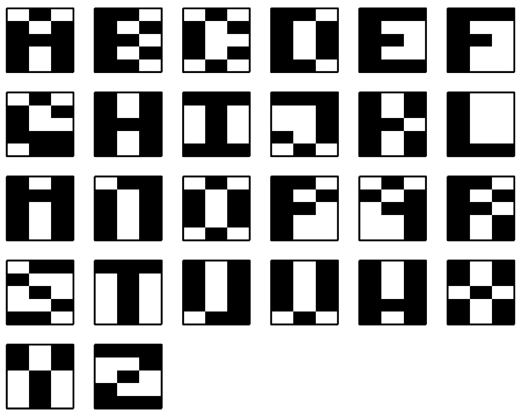


Figure 2.1: The letters that make up the dataset we use to illustrate sparse coding.

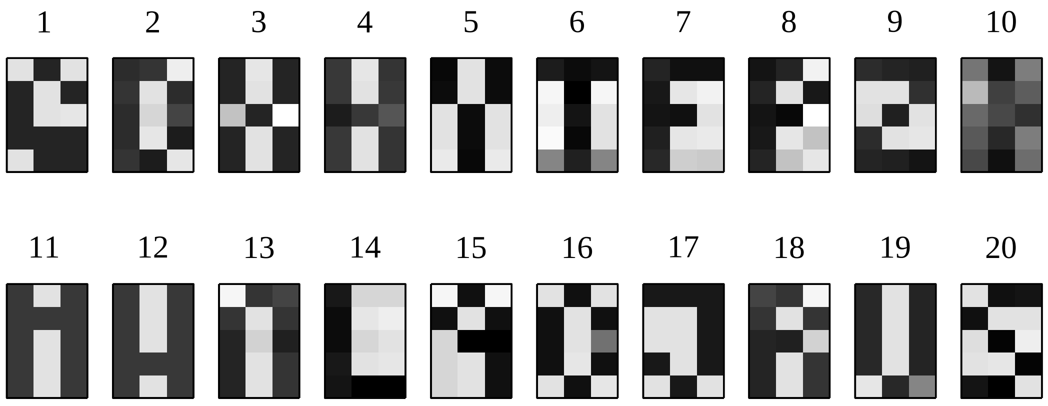
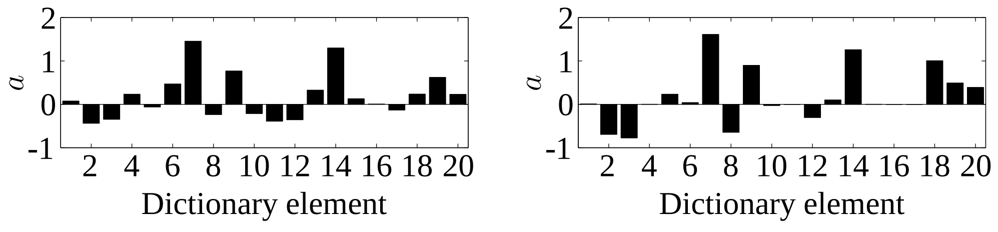


Figure 2.2: The overcomplete dictionary that is adapted to the data. Some dictionary elements have negative components (white represents a value of -0.05 and black represents 0.4).

small code values, in coding the letters “A” and “G”.

Figure 2.3 shows two different codes based on the dictionary in figure 2.2, that both reconstruct the letter “E” perfectly. The reconstruction is simply a sum of scaled dictionary elements, where the scaling factor for each element is given by the code. The codes make significant use of dictionary elements 7 and 14. These two elements are similar to the letters “F” and “L”; their combination in turn is similar to the letter “E”. The code in figure 2.3(b) is sparser than the code in figure 2.3(a) as it has fewer nonzero code elements. This illustrates the point that for an overcomplete dictionary there are many codes to represent a signal. In



(a) A dense code. (b) A sparse code.

Figure 2.3: The figure shows two codes that perfectly reconstruct the letter “E”. However, the code in (b) is sparser than the code in (a) because it has fewer nonzero code elements.

such a case a sparseness function can be used to select a sparse code.

2.3 MEASURES OF SPARSENESS

The neural code is sparse, it has both population sparseness and lifetime sparseness. We would like to measure and quantify sparseness, so that a single code can be selected to represent a signal with the LGM.

The neural code is also efficient in an information sense. This implies that the activities of two neurons cannot be correlated, otherwise there will be reduncies in the code. It follows then that the activities of any two neurons should be *independent*.

2.3.1 POPULATION SPARSENESS

At first we will address the measurement of population sparseness. Let the population sparseness function be

$$S_{pop}(n) = S(\overline{a}^{(n)}) \tag{2.4}$$

with $\overline{a}^{(n)}$ the code for stimulus n.

The values of the code elements have to be independent which implies that the sparseness function should be permutation invariant i.e. the sparseness value should not be dependent on the order of the individual components. A sufficient condition for a function to be permu-

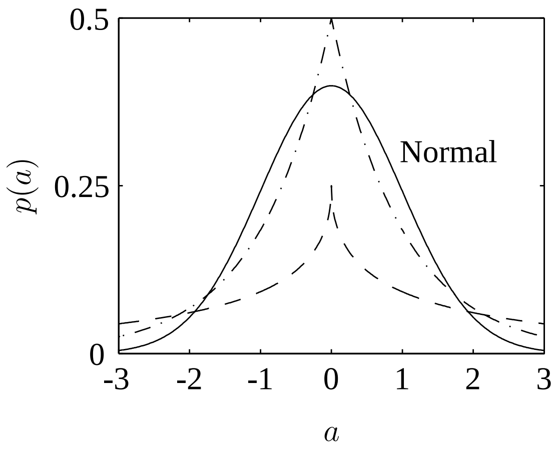


Figure 2.4: Code element values that follow a normal distribution are not considered sparse. Generally a code will be sparse if the probability that a code element value is close to zero is high. The chain ($p(a) = 0.5\exp(-|a|)$) and dashed ($p(a) = 0.25\exp(-|a|^{0.5})$) lines display probability distributions of code element values that are sparse.

tation invariant is separability

$$S(\overline{a}^{(n)}) = \sum_i S(a_i^{(n)}) \tag{2.5}$$

Suppose a neuron is represented by a code element a_i that only take on a binary value, then an appropriate sparseness function would be the l_0 norm, $S(\overline{a}) = \sum_i |a_i|^0$ where $0^0 = 0$. This norm simply counts the number of active or nonzero elements used to represent a stimulus. The l_0 norm will be small for if the code for that stimulus is sparse.

Sometimes a neuron is represented by a real-valued code element a_i, such as a perceptron or a code element in a LGM. Now the l_0 norm may not be appropriate anymore. A sparseness function for a real-valued code may not only consider the fact that a code element is nonzero, but also the exact value of that code element. For real-valued codes the probability distribution $p(\overline{a})$ of the values that code elements assume can indicate the sparseness of that code. The code will be sparse when the probability that an element value is zero or small is much larger than the probability that an element value is large. Section 2.3.3 lists the requirements for a sparseness function to ensure a sparse code. Figure 2.4 shows as an illustration a few probability distributions for which only the normal distribution is not consistent with a sparse code.

Interestingly in literature the l_0 norm is sometimes assumed to be the best measure for sparseness even though the code may be real valued (Field, 1994; Kreutz-Delgado *et al.*, 2003; Wipf and Rao, 2006).

We will show in section 2.4.2 how a sparseness function for a real-valued code can be derived from a probability distribution of code element values.

2.3.2 LIFETIME SPARSENESS

A neuron that is seldom active in the representation of a dataset has high lifetime sparseness. The lifetime sparseness function should not be dependent on the order in which stimuli are coded, which implies that the lifetime sparseness function is a permutation invariant function. For the sake of simplicity let the lifetime sparseness function and the population sparseness function use the same sparseness function. Now the lifetime sparseness function of code element i for all stimuli in the dataset is

$$S_{life}(i) = \sum_n S(a_i^{(n)}) \tag{2.6}$$

In this case the total population sparseness function for all stimuli in a dataset equals the total lifetime sparseness function of all code elements over the same dataset.

$$\sum_n S_{pop}(n) = \sum_n \left[\sum_i S(a_i^{(n)}) \right] = \sum_i \left[\sum_n S(a_i^{(n)}) \right] = \sum_i S_{life}(i) \tag{2.7}$$

There is an important property that arises when lifetime sparseness and population sparseness use the same sparseness function: by selecting the most sparse population code for a *single* given stimulus, we are also selecting the code that will yield the greatest lifetime sparseness over the *entire* dataset.

2.3.3 HOW TO ENSURE SPARSENESS

The sparseness function should have certain properties for it to produce a sparse code. It is generally believed that when the probability distribution of a code element value is peaked at zero and is heavy-tailed, then that code element is consistent with a sparse code (Olshausen and Field, 1997; Lewicki and Olshausen, 1999; Olshausen and Field, 2004). Probability distributions in the form of

$$p(\bar{a}) \propto \sum_i e^{-S(a_i)} \tag{2.8}$$

are often used to describe the distribution of sparse codes. Several sparseness functions have been proposed (Olshausen and Field, 1997; Lewicki and Olshausen, 1999; Lewicki, 2002): $S(a) = \beta \ln(1 + a^2)$, $S(a) = -\exp^{-a^2}$, $S(a) = |a|^q$ with $0 < q < 2$, and others.

Kreutz-Delgado *et al.* (2003) have derived sufficient conditions for functions to be valid measures of sparseness, i.e. functions that promote sparseness. The conditions are:

- The function should be permutation invariant. A function is permutation invariant if its value is independent of the order of its components.
- $S(|a| + \Delta a) < S(|a|) + S(\Delta a)$, i.e. the cost of increasing the amplitude of an existing spike by $\Delta a > 0$ should be less than the cost of adding a new spike with that same amplitude to the code.

This implies that $S(a) = |a|^q$ is a valid sparseness functions only for $0 < q \leq 1$. The second condition is not satisfied for $1 < q < 2$, therefore the general belief that all supergaussian functions ($0 < q < 2$) enforce sparseness is not well-grounded.

We use the sparseness function

$$S(a) = a - \frac{\beta}{2} a^2 \tag{2.9}$$

within the bounds $0 \leq a \leq 1/\beta$. The lower bound ensures that components are positive while the upper bound is needed to satisfy the second condition above. Figure 2.5 gives a contour plot of equation 2.9 for a two component code. It shows that the sparseness function is lower for points closer to the axes, i.e. sparser codes.

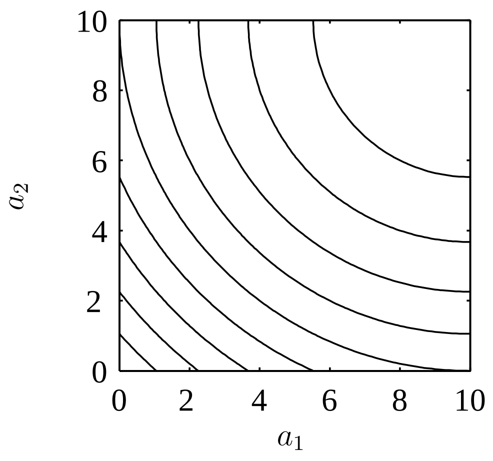


Figure 2.5: A countour plot of $S(\bar{a}) = \sum_i a_i - \frac{0.1}{2}{a_i}^2$, $S([0,0]) = 0$ and $S([10,10]) = 10$. It shows that the sparseness function is smaller for sparse codes. For example, $S([5,0]) = 3.75$ while $S([2.5,2.5]) = 4.375$.

2.4 FINDING A SPARSE CODE

A sparse code should be one that represents a signal well, but that is also sparse. Sparse codes are usually found in one of two ways: by viewing the LGM in a probabilistic framework and finding the most likely code, or by explicitly stating the problem as a mathematical optimization problem. The following sections expand on both approaches.

2.4.1 EXPLICITLY STATING THE MATHEMATICAL OPTIMIZATION PROBLEM

The sparse code $\overline{a}^*$ that represents a signal is given in equation 2.2

$$\overline{a}^* = \min_{\overline{a}} S(\overline{a}) \quad \text{such that} \quad R(\bar{a}) \leq \epsilon$$

where the reconstruction error is $R(\bar{a}) = \|\overline{x} - \mathbf{\Phi}\overline{a}\|^2$. It is useful to state the sparse coding problem in this way when the sparseness function is nondifferentiable.

Consider a sparse coding problem based on the l_0-norm. The problem of minimizing l_0 while ensuring that reconstruction error is small enough is NP-hard and is a combinatorial problem. A class of algorithms that attempt to solve this problem is termed *basis selection*. Of the many basis selection algorithms, *matching pursuit* (MP) is one of the most basic.

According to (Mallet and Zhang, 1993) matching pursuit

> ...decomposes any signal into a linear expansion of waveforms that are selected from a redundant dictionary of functions. These waveforms are chosen in order to best match the signal structures.

MP is an iterative algorithm that adds one element at a time to the code, until the reconstruction error falls below a predetermined threshold. The element that is added to the code during the kth iteration is

$$a_i^{(k)} = \max_i \frac{(\bar{x} - \bar{x}_R^{(k-1)})^T \cdot \Phi_i}{\|\Phi_i\|^2} \tag{2.10}$$

$\bar{x}_R^{(k-1)} = \mathbf{\Phi}\bar{a}^{(k-1)}$ is the reconstruction at iteration $k-1$. MP therefore adds during each iteration that single element to code that will most reduce the norm of the residue $\left\| \bar{x} - \bar{x}_R^{(k-1)} \right\|$. Even though MP does not explicitly consider sparseness, it still yields a sparse solution because the algorithm is stopped before it can perfectly reconstruct the signal.

If we view $R(\bar{a}) = \|\bar{x} - \mathbf{\Phi}\bar{a}\|^2$ as a cost function that needs to be minimized, then MP minimizes the cost function by moving the solution along one dimension at a time.

MP is generally faster than *gradient based* algorithms (Perrinet, 2004a). However we show in section 2.7.2.1 that codes from pure MP are not suitable for pattern recognition problems, as small changes in the signal cause big changes in the code.

2.4.2 A PROBABILISTIC APPROACH

We have mentioned that the sparseness of real-valued code elements can conveniently be measured according to the probability distribution of the code element values $p(\bar{a})$. In this section we view the LGM in a probabilistic framework in order to find a sparse code.

For the case of no noise ($\epsilon = 0$) and with a Laplacian prior for the code elements, there are methods that can find the code efficiently by maximizing sparseness while constraining the solution so that $\bar{x} = \mathbf{\Phi}\bar{a}$ (Chen, Donoho and Saunders, 1998). Approaches to finding the code of a model that includes nonzero noise ($\epsilon > 0$), are independent factor analysis (Attias, 1999), an expectation maximization algorithm (Girolami, 2001) and the FOCUSS algorithm (Kreutz-Delgado *et al.*, 2003). These approaches can only solve problems that can be cast into the form of equation 2.1. More general approaches that also apply to other forms of equation 2.1 (for example time-dependent generative models) are gradient based algorithms (Olshausen, 2002).

Barlow's hypothesis states that we should be looking for the most efficient codes to represent stimuli. An efficient code $\bar{a}$ is one that requires few bits to encode the source output $\bar{x}$. According to Shannon's source coding theorem, a source cannot be coded with fewer bits than its entropy H

$$L \geq H(X) = \int p_{true}(\bar{x}) \ln \frac{1}{p_{true}(\bar{x})} d\bar{x} \tag{2.11}$$

L is the average number of bits needed to encode the source, it is termed the average code length. $p_{true}(\bar{x})$ is the actual probability distribution of the data. We do not know the actual probability distribution, instead it is modelled with a distribution $p(\bar{x})$, where some parameters of the distribution can be changed to better fit the data. The more $p(\bar{x})$ deviates from $p_{true}(\bar{x})$, the less efficient the code becomes

$$\begin{aligned} L &\geq \int p_{true}(\bar{x}) \ln \frac{1}{p(\bar{x})} d\bar{x} && (2.12) \\ &\geq \int p_{true}(\bar{x}) \ln \frac{1}{p_{true}(\bar{x})} d\bar{x} + \int p_{true}(\bar{x}) \ln \frac{p_{true}(\bar{x})}{p(\bar{x})} d\bar{x} && (2.13) \end{aligned}$$

The second term on the right-hand side is the *Kullback-Leibler divergence* (*KL*). It measures how closely the probability distribution of signals from a model approximates the true distribution. Codes that minimize this term will be efficient.

Minimizing *KL* corresponds to maximizing $\langle \ln p(\bar{x}) \rangle$ since

$$\langle \ln p(\bar{x}) \rangle = \int p_{true}(\bar{x}) \ln p(\bar{x}) d\bar{x} \tag{2.14}$$

differs from *KL* by a quantity that does not depend on $p(\bar{x})$. $p(\bar{x})$ is found by marginalizing over $\bar{a}$:

$$p(\bar{x}) = \int p(\bar{x} \mid \bar{a}) p(\bar{a}) d\bar{a} \tag{2.15}$$

We need to specify the probability distributions of $p(\bar{x} \mid \bar{a})$ and $p(\bar{a})$. If we assume additive normally distributed noise on the signals, then the noise term in

$$\bar{x} = \mathbf{\Phi}\bar{a} + \bar{\epsilon} \tag{2.16}$$

is a random vector with entries that are sampled from $\mathcal{N}(0, \sigma_n^2)$. Now it is possible to specify:

$$p(\bar{x} \mid \bar{a}) \propto \exp(-\frac{\|\bar{x} - \mathbf{\Phi}\bar{a}\|^2}{2\sigma_n^2}) \tag{2.17}$$

The choice of $p(\bar{a})$ is limited by the fact that $\bar{a}$ needs to be sparse. We assume that the components of $\bar{a}$ are independent and that they follow the given distribution:

$$p(\bar{a}) = \prod_i p(a_i) \propto \prod_i \exp(-S(a_i)) \tag{2.18}$$

$S(a_i)$ should be a suitable sparseness function.

$p(\bar{x})$ is found by integrating over all possible $\bar{a}$ (see equation 2.15). It is generally intractable to evaluate this integral exactly. If there is no noise and if the dictionary is complete, the integral can be evaluated, which then leads to the ICA algorithm (Olshausen and Field, 1997; Cardoso, 1997). There are however several ways to approximate the integral. One is to sample a number of points from the posterior $p(\bar{a} \mid \bar{x}) \propto p(\bar{x} \mid \bar{a})p(\bar{a})$, and to approximate the integral by a sum at these points. Another is to approximate the terms inside the integral with a normal distribution centred on the maximum posterior mode $\bar{a}^*$ (Lewicki and Olshausen, 1999; Lewicki and Sejnowski, 2000):

$$\bar{a}^* = \max_{\bar{a}} p(\bar{a} \mid \bar{x}) \tag{2.19}$$

after which the integral in equation 2.15 becomes analytically solvable.

Alternatively, the integral can be approximated by just sampling at the maximum posterior mode (Olshausen and Field, 1997). This last approximation neglects the volume under the integral, and assumes that the term in the integral is a delta function. We will use this approximation for its simplicity although it has two significant drawbacks. Firstly, the actual problem we solve is only an approximation to the true problem we would like to solve. Secondly, it is shown in section 2.5 that the optimal dictionary will be one for which the dictionary elements have norms that approach infinity.

The optimization problem in equation 2.19 can be simplified. By using Bayes' theorem together with the fact that $p(\bar{x})$ is not a function of $\bar{a}$ we have

$$\begin{aligned} \bar{a}^* &= \max_{\bar{a}} \ln p(\bar{a} \mid \bar{x}) && (2.20) \\ &= \max_{\bar{a}} \ln \frac{p(\bar{x} \mid \bar{a})p(\bar{a})}{p(\bar{x})} && (2.21) \\ &= \max_{\bar{a}} \ln p(\bar{x} \mid \bar{a})p(\bar{a}) && (2.22) \end{aligned}$$

Maximizing $p(\bar{a} \mid \bar{x})$ is equivalent to maximizing $\ln p(\bar{a} \mid \bar{x})$ since the logarithm is a monotonic function.

The sparse code $\bar{a}^*$ is the solution to the minimization problem

$$E(\bar{a} \mid \bar{x}) = \frac{1}{2\sigma_n^2} \|\bar{x} - \mathbf{\Phi}\bar{a}\|^2 + \sum_i g(|a_i|) \tag{2.23}$$

where we have made use of equations 2.17 and 2.18. E can also be written as

$$E(\bar{a} \mid \bar{x}) = \|\bar{x} - \mathbf{\Phi}\bar{a}\|^2 + \lambda S(\bar{a}) \tag{2.24}$$

with $\lambda = 2\sigma_n^2$.

The cost function is a combination of a reconstruction term and a penalty term. The penalty term ensures that a sparse solution will be selected, and the reconstruction term ensures that the code will give a good representation of the signal. These two terms are opposing forces: a very sparse code cannot usually give a good representation, and vice versa. λ is a critical parameter that sets the balance between sparseness and reconstruction error.

The minimum of equation 2.24 can be found with a *gradient based* approach. The gradient of the error function with respect to the code is

$$\frac{\partial E}{\partial \bar{a}} = -2\mathbf{\Phi}^T (\bar{x} - \mathbf{\Phi}\bar{a}) + \lambda S'(\bar{a}) \tag{2.25}$$

MP and the gradient based approach are related; both approaches minimize the same cost function when $\lambda = 0$. However MP will find a sparse solution as it only adds one element at a time to the code, whereas the gradient based approach will not find a sparse solution as it can employ every code element in order to minimize the cost function. The penalty term $\lambda S(\bar{a})$ is therefore crucial to ensure a sparse solution for the gradient based approach.

2.4.2.1 SETTING THE RECONSTRUCTION ERROR-SPARSENESS BALANCE

λ is related to the noise present in the model. This noise is general, it incorporates any noise source: either noise in the signal and/or noise in the representation. There are various ways to choose the optimal value of λ and thereby set the noise level (Rao, Engan, Cotter, Palmer and Kreutz-Delgado, 2003). In a signal representation problem, λ is chosen so as to ensure a minimum signal-to-noise ratio (SNR). This approach is termed *quality-of-fit*; λ is simply set according to a prescribed SNR.

In a compression problem, the number of nonzero entries in the code is predetermined. For a given signal λ is then set such that the code has the preset number of nonzero entries. With this method λ has to be determined iteratively for each signal.

The *L-curve* method is another approach. Here λ is to set to some value that gives the best trade-off between the reconstruction error and sparseness. A plot of the reconstruction error versus the sparseness term for different values of λ is shaped like an "L". The L-curve theory states that the best choice for λ is the one that corresponds to the corner of the "L". The graph does not always have a definite corner, so it is proposed that the point of maximum curvature corresponds to the corner. This approach requires the model to be evaluated at various values of λ, making it impractical in most cases.

Ideally we would like to select that value for λ which will give the best performance of the entire system, in our case the best classification performance. This would require the entire system to be trained with several values of λ, which is not viable as the training of the entire system takes very long. Instead we use a different approach: λ should be chosen so that the reconstructed signal captures speech features that are important for speech recognition. Making λ too small will require the code to capture features that are irrelevant to speech recognition, while not all the important speech features will be captured when λ is too big. A proper value for λ can be estimated by visual inspection of several reconstructed signals at various values of λ.

2.5 FINDING THE OPTIMAL DICTIONARY

The dictionary has to capture the underlying statistical structure of the data for the code to be efficient. The optimal dictionary for a given dataset is difficult to find, but it is possible to find a good dictionary by using an iterative approach. This process is called dictionary *training*.

The following two sections show how to train a dictionary. The first section does so for codes that are selected by means of basis selection, the following section for codes that are selected in a probabilistic framework.

2.5.1 TRAINING A DICTIONARY FOR CODES FROM BASIS SELECTION

Here we derive a procedure that iteratively adapts the dictionary to make the codes that represent a set of stimuli more and more sparse.

The best code to represent a stimulus is the sparsest code that yields a reconstruction error less than a prescribed noise level. Generally, the sparser a code is the worse it reconstructs a signal. This means that the optimal code from equation 2.2 will have a reconstruction error that equals the prescribed noise level i.e. the constraint will be active.

Suppose the dictionary is adapted for a given signal and given code so that the reconstruction error is reduced. The adapted dictionary will allow a sparser code to be selected than the initial one, because the reconstruction error is smaller than the prescribed noise level; the code will have room to become sparser until the reconstruction error again equals the prescribed noise level.

This suggests a *sequential* procedure to adapt the dictionary. A change in the dictionary $\Delta\mathbf{\Phi}^{(n)}$ is determined for each signal $\bar{x}^{(n)}$ and associated code $\bar{a}^{(n)}$, one at a time. The dictionary is adapted with the term $\Delta\mathbf{\Phi}^{(n)}$ before the algorithm moves on to the next signal $\bar{x}^{(n+1)}$ in the stimulus set

$$\mathbf{\Phi}^{(n+1)} = \mathbf{\Phi}^{(n)} + \Delta\mathbf{\Phi}^{(n)} \tag{2.26}$$

The change in the dictionary $\Delta\mathbf{\Phi}^{(n)}$ should reduce the reconstruction error $R = \|\bar{x}^{(n)} - \mathbf{\Phi}^{(n)}\bar{a}^{(n)}\|^2$. The direction in which to change the dictionary is then

$$\Delta\mathbf{\Phi}^{(n)} = -\eta\frac{\partial R}{\partial\mathbf{\Phi}} \tag{2.27}$$

with η a learning parameter. The update formula is now

$$\Delta\mathbf{\Phi}^{(n)} = -\eta\left[\bar{x}^{(n)} - \mathbf{\Phi}^{(n)}\bar{a}^{(n)}\right]\bar{a}^{(n)T} \tag{2.28}$$

where $\bar{a}^{(n)T}$ is the transpose of $\bar{a}^{(n)}$.

The sequential update rule is difficult to perform in parallel, as a single $\bar{a}^{(n)}$ is determined for an update of the dictionary. It would be easier to parallelize a *batch* version of the update equation. In the batch version all the codes for the stimulus set is first determined for a given dictionary, only then is the dictionary updated.

For very small values of η, the batch version of the update formula approximates the sequential version. The batch update version can compactly be written as

$$\Delta\mathbf{\Phi} = -2\eta(\boldsymbol{X} - \mathbf{\Phi}\boldsymbol{A})\boldsymbol{A}^T \tag{2.29}$$

where the nth stimulus is the nth column in matrix $\boldsymbol{X}$, and the code that represents stimulus n is the nth column of matrix $\boldsymbol{A}$. This update formula is also used by Perrinet (2004b) to train dictionaries on codes from MP. Perrinet (2004b) notes that it is necessary to introduce a mechanism to ensure that the choice of any one dictionary element is not favoured above any other element. One way to achieve this requirement is to set the norms of all dictionary elements equal to a preset value.

2.5.2 TRAINING A DICTIONARY IN A PROBABILISTIC FRAMEWORK

The same arguments used in the previous section to find the optimal code can be used to find the optimal dictionary. In the previous section we were only interested in finding the optimal code, accordingly we use the notation $p(\bar{x})$. However, the probability distribution of signals arising from the model is also function of the dictionary. It follows that the Kullback-Leibler divergence is a function of the dictionary

$$KL(\mathbf{\Phi}) = \int_{\bar{x}} p_{true}(\bar{x}) \ln\frac{p_{true}(\bar{x})}{p(\bar{x} \mid \mathbf{\Phi})} d\bar{x} \tag{2.30}$$

The optimal dictionary is then

$$\mathbf{\Phi}^* = \max_{\mathbf{\Phi}} \langle \ln p(\bar{x} \mid \mathbf{\Phi}) \rangle \tag{2.31}$$

We approximate $\ln p(\bar{x} \mid \mathbf{\Phi})$ by sampling at its maximum posterior (see the discussion leading to equation 2.19)

$$\mathbf{\Phi}^* = \min_{\mathbf{\Phi}} \langle \min_{\bar{a}} E(\bar{a} \mid \bar{x}, \mathbf{\Phi}) \rangle \tag{2.32}$$

The optimal dictionary is found by integrating over all possible signals. If the distribution of the data set reflects the true distribution we try to model, then

$$\langle \min_{\bar{a}} E(\bar{a} \mid \bar{x}, \mathbf{\Phi}) \rangle \approx \sum_n \min_{\bar{a}} E(\bar{a}^{(n)} \mid \bar{x}^{(n)}, \mathbf{\Phi}) \tag{2.33}$$

Training of the dictionary is an iterative process that involves two steps: firstly the optimal codes for the stimulus set is found (solve $\min_{\bar{a}} E(\bar{a} \mid \bar{x}, \mathbf{\Phi})$), then the dictionary is adapted. Training can stop when there is not much change in the dictionary from one iteration to the next.

The dictionary can be adapted by a gradient based method. The cost function is $E_T = \sum_n E(\bar{a}^{*(n)} \mid \bar{x}^{(n)}, \mathbf{\Phi})$, for which the gradient with respect to the dictionary is

$$\frac{\partial E_T}{\partial \mathbf{\Phi}} = \sum_n \frac{\partial E(\bar{a}^{*(n)} \mid \bar{x}^{(n)}, \mathbf{\Phi})}{\partial \mathbf{\Phi}} \tag{2.34}$$

$$= \sum_n 2(\bar{x}^{(n)} - \mathbf{\Phi}\bar{a}^{*(n)})\bar{a}^{*(n)T} \tag{2.35}$$

This can be compactly written as

$$\frac{\partial E_T}{\partial \mathbf{\Phi}} = 2(\boldsymbol{X} - \mathbf{\Phi}\boldsymbol{A})\boldsymbol{A}^T \tag{2.36}$$

which is in the same form as equation 2.29.

We mentioned previously that this approximation leads to an optimal dictionary that has dictionary elements whose norm approach infinity. To see this consider that the optimal dictionary will be one that maximize $\langle \ln p(\bar{x} \mid \mathbf{\Phi}) \rangle$. The term $p(\bar{x} \mid \mathbf{\Phi})$ is approximated by the peak $\bar{a}^*$ of $p(\bar{x} \mid \bar{a}, \mathbf{\Phi})p(\bar{a})$. The maximum possible value that $p(\bar{x} \mid \bar{a}, \mathbf{\Phi})p(\bar{a})$ can reach will be a product of the maximum value of $p(\bar{x} \mid \bar{a}, \mathbf{\Phi})$ and the maximum value of $p(\bar{a})$. If the dictionary elements have norms that approach infinity, then the $\bar{a}$ that maximize $p(\bar{x} \mid \bar{a}, \mathbf{\Phi})$

will be close to the all zero vector, but the prior $p(\bar{a})$ also has its peak at the all zero vector. therefore the optimal dictionary is the one for which the dictionary elements have norms that approach infinity.

The trivial solution to the optimal dictionary does not exist if the norm of the dictionary elements is constrained. The constraint can be enforced strictly by fixing the norm of each dictionary element to a specific value (Kreutz-Delgado *et al.*, 2003), or it can be a soft constraint by adapting the norm of each dictionary element so the code element associated with that dictionary element has a prescribed variance.

Olshausen and Field (1997) use the soft constraint. They scale the norm of each dictionary element l_i after each iteration according to $l_{i,new} = l_{i,old}\left[\frac{\langle a_i^2 \rangle}{\sigma_{goal}^2}\right]^{\alpha}$ with $0 < \alpha < 1$. When α is set too small, it will take a long time for the dictionary norms to converge. On the other hand if α is set too large, some dictionary elements may never learn useful structure. This will happen because the training algorithm only adapts elements to the extend that they are used by the code. If it so happens that a particular element is not used often early in the training process, it will initially have a small variance. This in turn will reduce its norm. An element with a small norm requires a relative large code value to make a significant contribution to reconstructing the signal. An element with a small norm is therefore less likely to be selected for a code since the sparseness function for large code values will penalize it. As it is not selected often its variance reduces even further. This leads to a point where the element is never selected as it did not learn any structure and it has a very small norm. We therefore choose to use the hard constraint as it avoids the problem of setting α.

The hard- and soft constraint approaches to bounding the dictionary norms will yield suboptimal dictionaries. A dictionary that is closer to the optimal dictionary can be found when the term $p(\bar{x} \mid \mathbf{\Phi})$ is not approximated by the peak $\bar{a}^*$ of $p(\bar{x} \mid \bar{a}, \mathbf{\Phi})p(\bar{a})$, but when the volume of the integral around the peak is estimated (Lewicki and Olshausen, 1999; Lewicki and Sejnowski, 2000). The volume estimation is unfortunately expensive to compute.

2.6 SPARSE CODING FOR SPEECH RECOGNITION

Speech recognition systems, which includes the brain, forms multichannel representations of raw speech waveforms. They can also handle signals of variable length. The sparse coding model discussed so far finds a sparse code only for a single channel signal of fixed length. In this section the simple LGM model is expanded to take multichannel signals of variable length as input.

2.6.1 SPEECH REPRESENTATION

We use the TIDIGITS (Leonard and Doddington, 1993) data set of continuous spoken digits by male and female speakers. The utterances are of variable length, consisting of a variable number of random digits. There are 11 different spoken digits, one for each number from "one" to "nine", a "zero" and an "oh". We choose this data set because it is a rather simple set: it has a limited vocabulary and simple language model (since the probability that a certain word follows any other word is approximately equal for all the words).

The label data supplied with TIDIGITS is limited to orthographic transcriptions. That is, the digit sequence of each utterance is given, but the start- and endpoints of each digit in the waveform are not specified. We estimated these points by doing forced alignment with the *HVite* tool that is part of the *Hidden Markov Model Toolkit* (HTK) (University of Cambridge, 2006). During forced alignment, HVite aligns a transcription with a waveform. The automatic forced alignment is accurate as HTK is able to model the data well: it is able to recognize almost all words in the test set correctly (WER[1]=3%).

It is time consuming to train sparse coding models. We therefore use a reduced data set so that the system can be trained in reasonable time. (With the reduced set, the entire system is trained in a week on sixteen Pentium 4 PCs working in parallel). The training set suggested for the TIDIGITS data set has 8623 utterances and contains 28329 words. Our training set is made up of every second sample of the suggested data set. The reduced set contains at least 2000 samples of every word, and is therefore sufficient for this initial investigation.

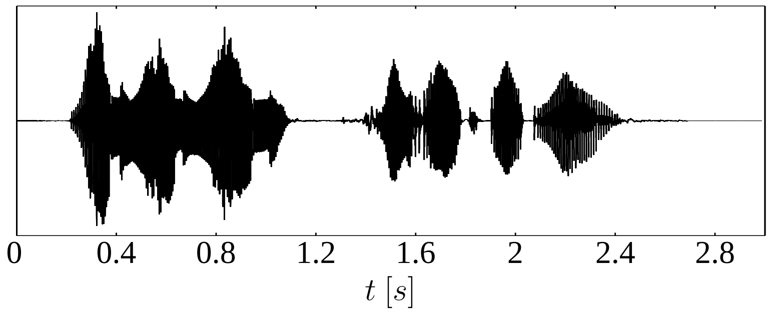
[1]Word Error Rate (WER) $= \frac{\#\text{deletions}+\#\text{insertions}+\#\text{replacements}}{\text{number of words}} 100\%$

The simplest way to represent a raw speech signal is as a time waveform. However, time domain representations of speech are not as natural as frequency domain representations, as is evidenced by the fact that the auditory pathway uses a frequency domain representation. Even so, sparse coding of the time waveform of speech signals has shown some correspondence with physiological data (Lewicki, 2002). The auditory pathway forms a complex nonlinear representation of auditory stimuli. The representation produces phenomena like forward masking and two-tone suppression. There are models that provide frequency domain representations that to some extent agree with physiological measurements along the auditory pathway (Hermansky and Morgan, 1994; Wang and Shamma, 1995); however, for this study we choose to use a simpler spectrogram representation. The spectrogram we use is constructed by first dividing the raw signal into segments. The segments span 25.6ms (512 points with a sampling frequency of 20kHz). A new segment starts every 20ms or every 400 points, which gives an overlap between adjacent segments of 5.6ms. Each segment is then windowed with a Hanning window before the log magnitude of the Fast Fourier Transform for that segment is calculated. The magnitude is chosen because the human ear is insensitive to phase information; the log of the magnitude is used because our perception of sound intensity is often approximated by the logarithm although we actually perceive sound intensity on a cube-root scale (Hermansky, 1990).

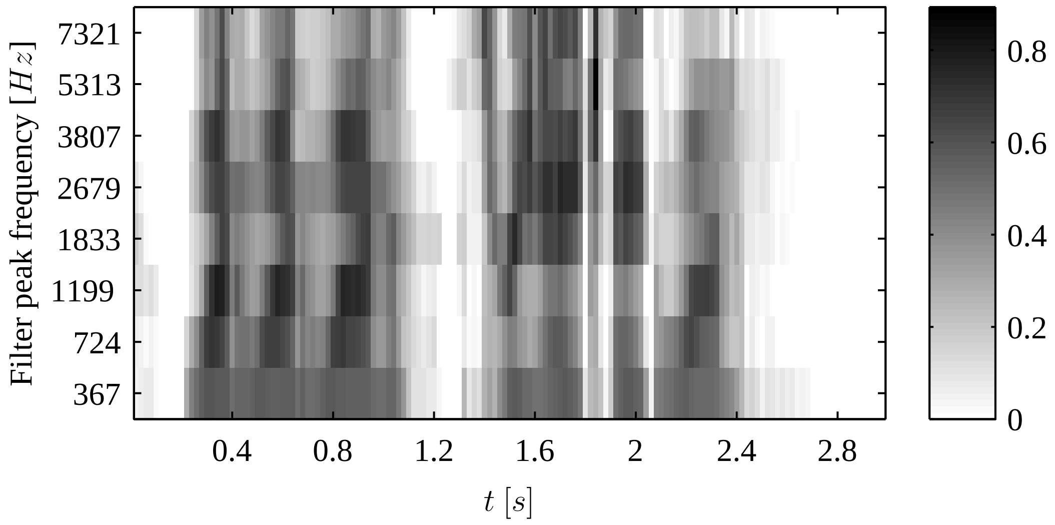
Humans do not perceive the content of a signal on a linear frequency scale. We therefore use a Mel-spaced filter-bank to incorporate this nonlinearity. Each filter is a triangular bandpass filter whose centre frequencies are linearly spaced on a Mel-scale. Current automatic speech recognition systems use up to 32 filters, but we use only 8 filters for computational reasons. A small number of filters is preferred because too many filters put significant computational strain on the algorithms, and accurate temporal information is apparently more important than accurate spectral information for speech recognition (Allen, 1994; Kral, 2000; Moller, 1999; Shannon *et al.*, 1995). The outputs of the filters (with centre frequencies as given in figure 2.6) are then scaled so that the variance of each filter output over the data set is one.

We use a linear generative model to find the sparse code of a spectrogram. This model works best if important speech features have large values in the representation and if silences have values equal to zero. From inspection it seems for this particular data set and the spectrogram transform described above, that values below $15dB$ do not carry important speech features. We therefore subtract $15dB$ from the spectrogram after filtering and set any negative value equal to zero.

Figure 2.6 gives the signal representation for the utterance "one one one three eight eight one". The spectrogram representation $\boldsymbol{x}$ is an $N_c \times N_t$ matrix with N_c the number of channels or filters and N_t the number of segments. It serves as input to the linear generative model.



(a)



(b)

Figure 2.6: (a) The speech waveform for the utterance "one one one three eight eight one", and (b) its spectrogram representation. The gray scale indicates the signal amplitude $[dB]$.

2.6.2 TEMPORAL LINEAR GENERATIVE MODEL

The linear generative model applies to signals of a fixed length. Speech signals of variable length can be modelled with a LGM if the signal is segmented into fixed length segments. The code for each segment is then solved independently of other segments and the code for the complete signal is constructed by concatenating the codes from various segments to yield a temporal code. Such a code will have redundancies because the correlation between the code elements of neighbouring segments will be high, making the code less efficient.

A better way to model signals of variable length is to explicitly model the temporal code (Olshausen, 2002; Smith and Lewicki, 2005). The LGM now becomes a temporal linear generative model (TLGM). The reconstructed signal (in our case a spectrogram) $\boldsymbol{x}_R$ is a convolution of the code $\boldsymbol{a}$ with the dictionary $\boldsymbol{\Phi}$. An element at time t in channel c of the reconstructed spectrogram is found with:

$$x_{Rc,t} = \sum_{d=1}^{N_d} \sum_{T=1}^{N_t} \Phi_{c,\Delta t}^{\{d\}} a_{d,T} \tag{2.37}$$

For convenience we use the integer index t to correspond to a segment in the spectrogram. The index t is related to actual time by multiplying t with $20ms$. N_t is the number of segments in the spectrogram that is being reconstructed. d is an index to a dictionary element; there are $N_d = 32 \times 7 = 224$ dictionary elements in our dictionary (we will explain why this number is chosen below). $\boldsymbol{\Phi}^{\{d\}}$ is an $N_c \times \Delta\Phi$ matrix representing the dth dictionary element; $\Phi_{c,\Delta t}^{\{d\}}$ refers to the element in $\boldsymbol{\Phi}^{\{d\}}$ located in channel c at time Δt. N_c is the number of channels in the signal, for the spectrogram used here $N_c = 8$. $\Delta t = \Delta\Phi - T + t$ where $\Delta\Phi$ is the temporal width of a dictionary element. The width is taken as $260ms$ (or 13 segments), as evidence points to auditory memory being around $250ms$ in duration (Hermansky, 1998; Huggins, 1975; Massaro, 1972). When T is smaller than the temporal width of a dictionary element, the first $\Delta\Phi - T$ columns of the dictionary element are truncated. $\boldsymbol{a}$ is therefore an $N_d \times N_t$ matrix. We will refer to $a_{i,j}$ as a code element.

We now also extend the sparseness function to take a matrix $\boldsymbol{a}$ as an argument

$$S(\boldsymbol{a}) = \sum_{d} \sum_{T} S(a_{d,T}) \tag{2.38}$$

This sparseness function assumes that the activity of any two code elements is independent. It assumes that the activity of a channel is independent over time, and that the activity of different channels is independent. The assumptions fit the requirements of an efficient code.

2.6.3 FINDING THE SPARSE CODE OF A TLGM

For most practical problems the dimensionality of an LGM is small enough so that a sparse code can be found quickly. However the dimensionality of a TLGM is too large to find a sparse code in reasonable time. For example, a code of 40 dictionary elements that represents a $1s$ spectrogram of $20ms$ segments would yield a $40 \times 50 = 2000$ dimensional problem. A pure gradient based algorithm would take too long to converge to a sparse code. We need a more efficient way to find the sparse code.

The error function is a quadratic function of the code when the sparseness function is a quadratic function of the code. We can choose the sparseness function to be quadratic

$$S(a) = a - \frac{\beta}{2}a^2 \tag{2.39}$$

with $\beta > 0$. This is a valid sparseness function for $0 \leq a \leq 1/\beta$. The lower bound is consistent with a neural code that cannot have negative spikes; it is also required for this particular sparseness function to be valid (the sparseness function is negative for negative code elements). The upper bound is the point where a maximizes the sparseness function. The sparseness function decreases for values beyond this point, which does not fit a sparseness function. We found in our application that the upper constraint is never active for $\beta = 0.1$ and therefore does not play an important role in the code selection. We use a bound constrained quadratic programming algorithm called MINQ (Neumaier, 1998) to minimize the error function; it is much more efficient than a pure gradient based algorithm.

Consider again the 2000 dimensional problem. The Hessian matrix that is used in the quadratic programming algorithm would be a 2000×2000 matrix which is too big for efficient solution. Here follows an algorithm that does not require the entire Hessian.

We expect very few nonzero components in the solution, because we are looking for a sparse code. It is therefore not necessary to include all the components in a search for the solution. In fact, it is only necessary to include those components that are likely to be nonzero. If it is possible to know these components beforehand, it will greatly reduce the dimensionality of the optimization problem.

However it is difficult to know which components will be nonzero, so we make use of an iterative process that chooses a small subset of components (size N_{set}) to be optimized in each iteration. The subset is determined by selecting the N_{set} most "promising" components from a set of candidate components. A candidate component is either nonzero in the current iteration, or it has a negative gradient with respect to the error function E. This criterion does not select *all* the components that could reduce the error function, because the error function is nonconvex, but it does select a few of them. The components that are included in the subset are the N_{set} candidate components with the largest gradient magnitude. The process of selecting components and optimizing them iterates until the reduction in the error function falls below a preset threshold. We used a threshold of 10^{-4} and select subset of size $N_{set} = 200$ during each iteration. We refer to this algorithm as subset selection and quadratic programming (SSQP). Its pseudo code appears in algorithm 1.

Fan, Chen and Lin (2005) describe a similar approach for support vector machines. Their algorithm selects a subset of two elements at a time, however the subset may be larger than two elements (Liao, Lin and Lin, 2002). Blumensath and Davies (2006) have also used a subset selection approach to find the sparse code of a high dimensional problem. An important difference between SSQP and their work is that SSQP performs several iterations of subset selection, whereas their approach selects the subset only once. We could not use such a selection criterion, as a reasonable subset would be too large to quickly calculate the Hessian matrix (size $N_{set} \times N_{set}$) required by quadratic programming.

SSQP is a gradient based algorithm which attempts to find the minimum of the quadratic error function $E(\boldsymbol{a})$. E is possibly a nonconvex function. This implies that there may be several local minima. We expect that most of the code elements in the optimal solution will be zero as we are looking for a sparse code. It is therefore sensible to start the optimization with an initial guess where all the code elements are equal to zero instead of a random guess, since such a starting point is expected to be close to the optimal solution. Other large scale

Algorithm 1 SSQP: Subset selection and quadratic programming

```
Input: $\boldsymbol{a}_{start}$ and $\boldsymbol{\Phi}$
Output: $\boldsymbol{a}_{final}$

$q \leftarrow 0$
$\boldsymbol{\alpha}_q \leftarrow \boldsymbol{a}_{start}$
repeat
    $s_{active} \leftarrow \left\{[i,j] \mid \alpha_{q[i,j]} > 0\right\}$
    $s_{neg} \leftarrow \left\{[i,j] \mid \frac{\partial E}{\partial \alpha_{q[i,j]}} < 0\right\}$
    $s_{candidate} \leftarrow s_{active} \cup s_{neg}$
    sort $s_{candidate}$ in descending order of $\left|\frac{\partial E}{\partial \alpha_{q[i,j]}}\right|$
    $s_{use} \leftarrow \{s_{candidate\,[k]} \mid k = 1, 2, 3, \ldots N_{set}\}$
    $\overline{\alpha}_{use} \leftarrow \{\alpha_{q[i,j]} \mid [i,j] \in s_{use}\}$
    $\overline{\alpha}_{new} \leftarrow \min_{\overline{\alpha}_{use}} E(\boldsymbol{\alpha}_q, \boldsymbol{\Phi})$ {using quadratic programming}
    $\boldsymbol{\alpha}_{q+1} \leftarrow \boldsymbol{\alpha}_q$ but with the $s_{use}$ components replaced by $\overline{\alpha}_{new}$
    $q \leftarrow q + 1$
until $E(\boldsymbol{\alpha}_q, \boldsymbol{\Phi})$-$E(\boldsymbol{\alpha}_{q-1}, \boldsymbol{\Phi}) < 10^{-4}$
$\boldsymbol{a}_{final} \leftarrow \boldsymbol{\alpha}_q$
```

optimization techniques may also be used to find sparse codes (a typical problem has around 30 000 variables).

2.6.4 THE DICTIONARY

We stated earlier that there are certain benefits to using an overcomplete dictionary. How many dictionary elements constitute a complete dictionary of the TLGM? This question is equivalent to finding the minimum number of dictionary elements required to reconstruct a signal perfectly. Each dictionary element spans across all the channels, and the code can select dictionary elements to be used at any time. Therefore a signal $\boldsymbol{x}$ of size $N_c \times N_t$ can be perfectly reconstructed with a minimum of N_c dictionary elements. However we constrain the code elements to be nonnegative, so the minimum dictionary size to reconstruct any signal perfectly is $2 \times N_c$.

The dictionary elements of a trained dictionary could model phonemes, which are the smallest meaningful acoustic units, or it can model syllables, or even complete words. The dictionary training process is unsupervised, which means that a particular dictionary element may learn any unit of speech. In fact it would be interesting to see what type of features the trained dictionary elements model. There are 20 phonemes in the speech that make up the data set, 11 syllables and 11 words. It is difficult to find the best dictionary size. The dictionary should at least be complete so that the code can successfully represent a signal, and the size of the dictionary may be related to the basic speech units that make up the data set. We use a two times overcomplete dictionary, so there are 32 dictionary elements. This dictionary is large enough to capture at least all the phonemes present in the data set.

In speech there are certain features which can be stretched over time without changing the meaning of the speech. To reflect this in the model, time scaled versions of the basic dictionary elements are constructed. We construct six scaled versions of each basic dictionary element and append them to the dictionary. The dictionary now contains $N_d = 32 \times 7 = 224$ elements.

A dictionary element $\boldsymbol{\Phi}^{\{d\}}$ is scaled over time with a scaling matrix $\boldsymbol{M}_s$

$$\boldsymbol{\Phi}^{\{d\}}_{\{s\}} = \boldsymbol{\Phi}^{\{d\}}\boldsymbol{M}_s \tag{2.40}$$

where $\boldsymbol{\Phi}^{\{d\}}_{\{s\}}$ is the scaled version of basic dictionary element d. $s \in 0, 1, 2, \ldots 6$ where $s = 0$ represents the unscaled dictionary element that spans $260ms$, $s = 1$ corresponds to the dictionary element that is scaled to span $280ms$, etc. $\boldsymbol{M}_s$ is determined in such a way that that the jth column of $\boldsymbol{\Phi}^{\{d\}}_{\{s\}}$ (which spans $20ms$) is

$$\left[\boldsymbol{\Phi}^{\{d\}}_{\{s\}}\right]^{\langle j\rangle} = \frac{1}{0.02}\int_{0.02\cdot(j-1)}^{0.02\cdot j} \boldsymbol{\Phi}^{\{d\}}_{\{0\}}(\tau\frac{0.26}{0.26+0.02s})\mathrm{d}\tau \tag{2.41}$$

$\boldsymbol{\Phi}^{\{d\}}_{\{s\}}$ is a matrix where each column is used to reconstruct a $20ms$ segment of a spectrogram. We cast $\boldsymbol{\Phi}^{\{d\}}_{\{s\}}$ into the continuous time domain by letting $\boldsymbol{\Phi}^{\{d\}}_{\{s\}}(\tau) = \left[\boldsymbol{\Phi}^{\{d\}}_{\{s\}}\right]^{\langle i\rangle} \quad \forall \quad \tau \in [0.02(i-1), 0.02i]$. Figure 2.7 illustrates how a dictionary element is stretched.

The dictionary that includes the scaled elements is

$$\boldsymbol{\Phi} = \left[\boldsymbol{\Phi}_{\{0\}}, \boldsymbol{\Phi}_{\{1\}}, \boldsymbol{\Phi}_{\{2\}}, \ldots, \boldsymbol{\Phi}_{\{6\}}\right] \tag{2.42}$$

$\boldsymbol{\Phi}_{\{0\}}$ is the basic dictionary which has 32 elements; $\boldsymbol{\Phi}$ therefore has $32 \times 7 = 224$ elements.

The dictionary may be scaled in many other ways, for example linear interpolation of the basic dictionary should also work.

2.6.5 TRAINING THE DICTIONARY

The code will be more efficient if the dictionary is trained to reflect the regularities of the data. We use a gradient based training approach. First the sparse codes are determined for a given dictionary, then the dictionary is optimized for the given codes. The optimization is done by means of gradient descent with a line search. The derivative of the total error function E_T with respect to the dictionary is used as a search direction. A golden section method then finds the minimum of the error function along the search direction. The pseudo code for the training of dictionary elements appears in algorithm 2.

During the training process the norm of the dictionary elements tend to grow without bounds. We address this by forcing all dictionary elements to have a norm of one. Every

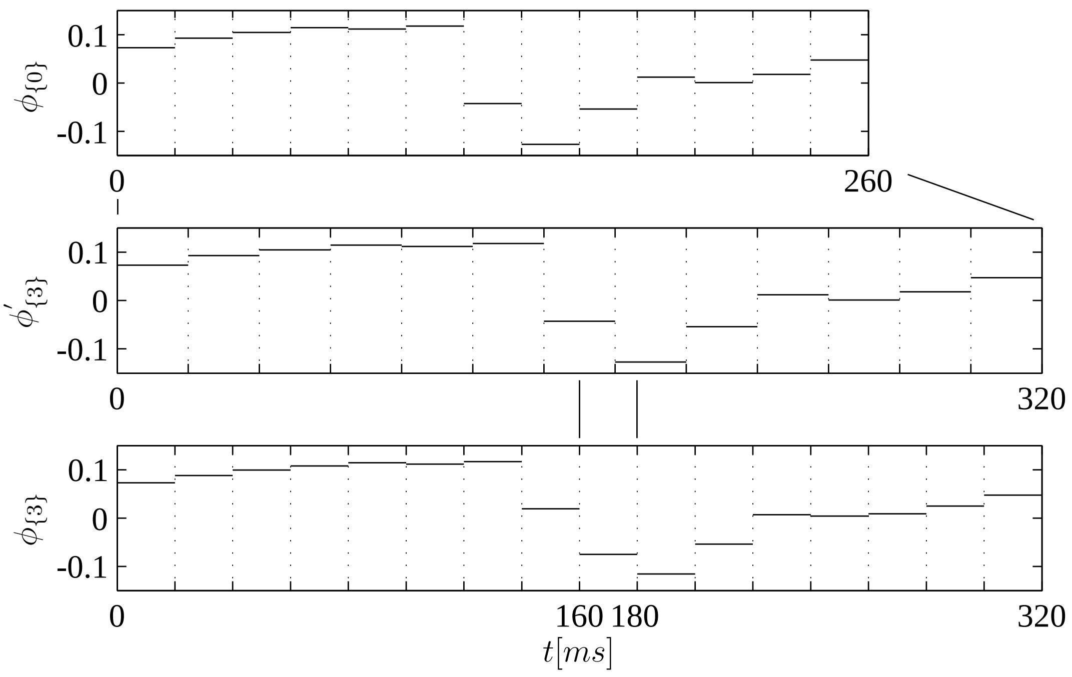


Figure 2.7: An unscaled dictionary element $\mathbf{\Phi}^{\{d\}}_{\{0\}}$ is a $N_c \times \Delta\Phi$ matrix. A row in the dictionary element corresponds to a channel in the spectrogram. This figure illustrates how a row of an unscaled dictionary element is scaled to $320ms$. $\phi_{\{s\}}$ refers to a row of the dth dictionary element $\mathbf{\Phi}^{\{d\}}_{\{s\}}$. The *top* plot shows a row of an unscaled dictionary element. The *middle* plot shows an intermediate step where the row is stretched to span $320ms$; $\phi'_{\{3\}}$ still has only 13 entries. The *bottom* plot shows that $\phi_{\{3\}}$ is created by resampling from $\phi'_{\{3\}}$. For example, the ninth entry which represents the interval $160ms$ to $180ms$, is the time averaged integral of $\phi'_{\{3\}}$ over that same interval. Every row of a dictionary element is stretched in the same way.

dictionary element of the initial dictionary is randomized. $\mathbf{\Phi}_{\{s\}}^{\{d\}}$ is a random $N_c \times \Delta\Phi$ matrix with entries sampled from a standard uniform distribution $U(0,1)$. We choose to use a distribution that will not give negative values. The motivation is that the dictionary elements are used to build up a representation that does not have any negative terms. Initially the elements does not have any structure so it will be unlikely to find a reasonable signal reconstruction by using elements that also have negative terms.

Algorithm 2 Training the dictionary

```
$k \leftarrow 0$
initialize $\mathbf{\Phi}_0$ as a random dictionary
normalize every dictionary element of $\mathbf{\Phi}_0$
**repeat**
    **for all** $n$ **do**
        $\boldsymbol{a}_{k+1}^{(n)} \leftarrow SSQP(\boldsymbol{a}_k^{(n)}, \mathbf{\Phi}_k)$
    **end for**
    $\boldsymbol{A}_{k+1} \leftarrow \left\{ a_{k+1}^{(n)} \mid n = 1, 2, 3, \ldots N_s \right\}$
    $\delta_{k+1} \leftarrow \min_\delta E_T(\boldsymbol{A}_{k+1}, \mathbf{\Phi}_k + \delta \frac{\partial E_T}{\partial \mathbf{\Phi}_k})$ {using a golden section search}
    $\mathbf{\Phi}_{k+1} \leftarrow \mathbf{\Phi}_k + \delta_{k+1} \frac{\partial E_T}{\partial \mathbf{\Phi}_k}$
    normalize every dictionary element of $\mathbf{\Phi}_{k+1}$
    $k \leftarrow k + 1$
**until** $\mathbf{\Phi}$ has converged
```

It is computationally very expensive to train the dictionary. The reasons are that it is necessary to find the sparse code of every utterance at each iteration. The SSQP algorithm can take a long time to converge, especially if the starting point is far from the final solution. For example, when we want to find the sparse code to the utterance "one one one three eight eight one", we use a trained dictionary and set the starting point as the all-zero code $\boldsymbol{a}_{start} \leftarrow \mathbf{0}$ in the SSQP algorithm. This utterance is $2.6s$ long; it takes 121 iterations of SSQP to find the sparse code which has 172 nonzero elements. On an Intel Core2 1.86GHz PC it takes $88s$ for SSQP to converge.

To speed up the training process, we set the starting point of each training iteration as the solution to the previous training iteration $\boldsymbol{a}_{start} \leftarrow \boldsymbol{a}^k$. In this case it usually takes less than five SSQP iterations to find the sparse code. It is only for the first iteration that we set the starting point as $\boldsymbol{a}_{start} \leftarrow \mathbf{0}$.

2.7 RESULTS

2.7.1 TRAINING THE DICTIONARY

It is computationally expensive to train the dictionary. The most expensive step is to find the sparse code of each utterance. We performed the coding step in parallel on sixteen Pentium 4 PCs. The code for each utterance can be found independently of the other utterances which makes it easy to parallelize the coding step.

To further speed up the training process, we start with a bigger value of λ than the one we would like to use; the bigger λ, the fewer nonzero elements are in the code and the quicker it is solved.

Initially the dictionary does not have any structure that corresponds to the data set. During the first few iterations the dictionary only has to be train on a few samples from the data set in order to learn some structure.

For the first 50 iterations we train the dictionary with $\lambda = 0.4$ and only on 1-in-5 samples of the data set. Thereafter we use $\lambda = 0.3$. For iterations 51 to 180 we train the dictionary on 1-in-5 samples. For iterations 181 to 245 we train it on 1-in-2 samples and from iteration 246 onwards on the entire data set (remember that our data set is not the entire TIDIGITS data set).

Figure 2.8 shows the performance of the training process on the *entire* data set, even though at times it is trained on a smaller data set. The error function is still decreasing at iteration 490 when the training is stopped. Further training should reduce the error function even more, but it appears if the improvement would not be significant. We also checked training progress against a validation set, and the cost function of the validation set is also still decreasing at iteration 490 (the validation results are not shown).

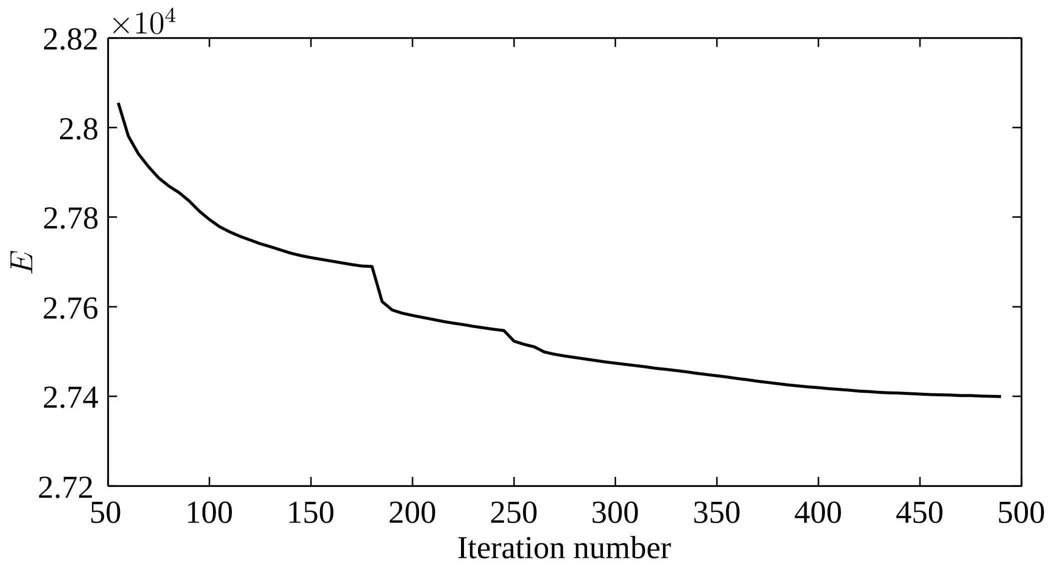


Figure 2.8: The history of the error function for the *entire* data set is shown in this figure. Refer to the text for more information on the training process.

2.7.2 PERFORMANCE OF CODE SELECTION ALGORITHMS

We tested the performance of three code selection algorithms using the trained dictionary: MP, SSQP and L-BFGS. L-BFGS is a limited memory BFGS algorithm (Byrd, Lu, Nocedal and Zhu, 1995). It is considered to be an efficient optimization algorithm for large-scale problems. The results are summarized in table 2.1.

MP is stopped when the value of the code element that is added to the code is less than 0.009. SSQP is stopped when the reduction in error function from one iteration to the next is less than 10^{-4} and L-BFGS stops when the norm of the gradient is less than 10^{-5}.

Of all the algorithms MP converges in the shortest time, but the solution is not nearly optimal. SSQP converges much faster than L-BFGS and the code it finds has the same error function value.

Notice that the value of the sparseness function for MP is more than that of SSQP, even though the MP code has fewer nonzero elements. This is a result of the sparseness function that we use; it is only a function of the code element value and not of the fact that an element

Table 2.1: The performance of three different sparse code selection algorithms. The input signal is "one one one three eight eight one"; the dictionary is trained with SSQP.

Algorithm[a]	$E(\boldsymbol{a})$	$R(\boldsymbol{a})$	$S(\boldsymbol{a})$	No. of nonzero elements	Time[b]
Matching pursuit (MP)[c]	25.399	13.907	38.305	80	17.7s
SSQP	12.268	1.890	34.594	162	34.4s
L-BFGS	12.268	1.860	34.700	164	320.9s

[a]All the codes are MATLAB 7 scripts.

[b]CPU time on an Intel Core2 1.86GHz PC.

[c]The MP algorithm does not make use of the sparseness function. The results for $E(\boldsymbol{a})$ and $S(\boldsymbol{a})$ are only given for comparison with other algorithms.

is zero or not.

The performance of an algorithm depends on the dictionary. The performance of MP will improve significantly when a MP-trained dictionary is used. However the results of L-BFGS should not change even if the dictionary is trained with L-BFGS. The reason is that L-BFGS finds very similar solution to SSQP as both algorithms are gradient based.

Table 2.2 gives the results to the same experiment as in table 2.1, but now the dictionary is trained on codes from MP. MP in the first row of the table is stopped when the code value that is added to the code is less than 0.008. This value is far too small, it allows the code to add elements that does not contribute significantly to the representation of the signal. It is better to stop MP when the SNR of the reconstructed signal falls below a predetermined threshold. MP$*$ is stopped as soon as the reconstruction error falls below that of the SSQP algorithm. We see that MP$*$ finds a code that reconstructs the signal just as well as SSQP, but it requires about half the number of nonzero code elements and finds the code in a fraction of the time it takes SSQP to converge.

The performance of L-BFGS for the MP-trained dictionary is again similar to the performance of SSQP. This emphasize the fact that these two algorithms are alike in their solutions, the only important difference is in the time it takes the algorithms to converge.

A comparison of tables 2.1 and 2.2 makes it clear that an algorithm performs much better with a dictionary trained by itself. We cannot use these results to determine which class of algorithm is better: basis selection or gradient based. One reason is that there are better basis selection algorithms than MP, such as *orthogonal matching pursuit* (Davis, Mallat and Avellaneda, 1997) and *optimized orthogonal matching pursuit* (Rebollo-Neira and Lowe, 2002). Another reason is that the application of the code is important. The code may be used as a means of signal compression or as a feature for pattern recognition.

Table 2.2: The same experiments as in table 2.1 but here the dictionary is trained on codes from MP.

Algorithm	$E(\boldsymbol{a})$	$R(\boldsymbol{a})$	$S(\boldsymbol{a})$	No. of nonzero elements	Time
MP[a]	24.086	1.767	74.395	400	44.4s
MP$*$	18.925	3.948	49.926	69	5.6s
SSQP	15.3082	3.988	37.735	108	42.2s
L-BFGS	15.0248	4.003	36.739	116	442.3s

[a]The MP algorithm does not make use of the sparseness function. The results for $E(\boldsymbol{a})$ and $S(\boldsymbol{a})$ are only given for comparison with other algorithms.

2.7.2.1 MP FOR PATTERN RECOGNITION

Matching pursuit is not a suitable algorithm to use when the sparse code it finds is a feature for pattern recognition. Robust pattern recognition requires that the feature representation does not change much if the input does not change much. Figure 2.9 shows the spectrograms of two utterances of “one six five seven”. There is not much difference between the spectrograms. The figure also shows the sparse code that MP finds for each utterance using the MP-dictionary; as well as the SSQP codes for each utterance using the SSQP-dictionary. The MP codes for the two similar utterances are dissimilar, while the two SSQP codes are similar. We found that this property of the MP codes also applies when MP codes are based on the SSQP-dictionary. When MP makes a poor selection of a code element in the first few iterations, it cannot correct that mistake. This may be the primary reason that MP does not select robust codes.

2.7.3 PERFORMANCE OF SPARSENESS FUNCTIONS

Here we compare the performance of four sparseness functions for three different code selection algorithms. The sparseness functions[2] are l_0, $l_{0.5}$, l_1 and $S_{quad}(a) = a - 0.05a^2$. We base the performance on the signal-to-noise ratio (SNR) of the reconstructed signal and on the sparseness of the code.

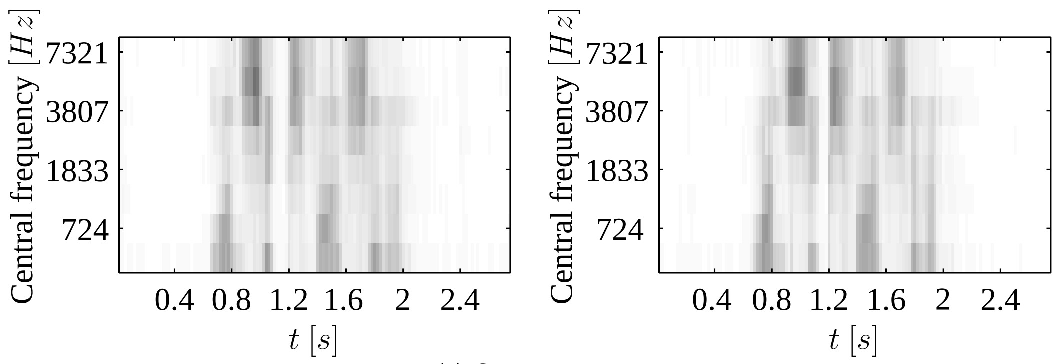
The code selection algorithms we use are MP, MP$^+$ and SSQP. MP$^+$ is the same as MP, except that it adds a final optimization step. A simple optimization can be performed on *any* given code that will not increase the l_0 norm while it may decrease the reconstruction error, or in the worst case leave the reconstruction error unchanged. It is the optimization of the values of the nonzero code elements. The values are changed so as to minimize the reconstruction error. MP$^+$ uses this final optimization step.

We train a dictionary for each of the algorithms. MP and MP$^+$ are implicitly trained on the l_0 norm as a sparseness function, SSQP is trained with S_{quad} as a sparseness function. The results appear in figure 2.10. Different points in the graphs are obtained by varying λ in the case of SSQP and the stopping threshold in the case of MP and MP$^+$. The points for which the dictionaries are trained are shown in each plot; SSQP is trained with $\lambda = 0.4$; MP and MP$^+$ with a threshold of 0.1. It is important to note that the SSQP dictionary we use to obtain figure 2.10 is not the dictionary we finally use to do speech recognition. Our final dictionary is trained with $\lambda = 0.3$. The exact value of λ is not important for the results of figure 2.10 but rather the trends in the figure.

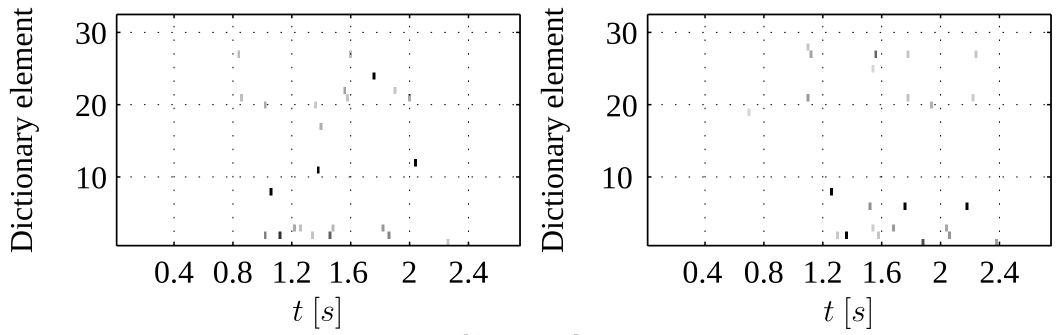
MP$^+$ performs best when the algorithms are compared using the l_0 norm, as can be expected. On the other hand, SSQP with S_{quad} performs best when the comparison is based on S_{quad}.

There is little difference between figures 2.10(c) and (d). The reason is that most of the

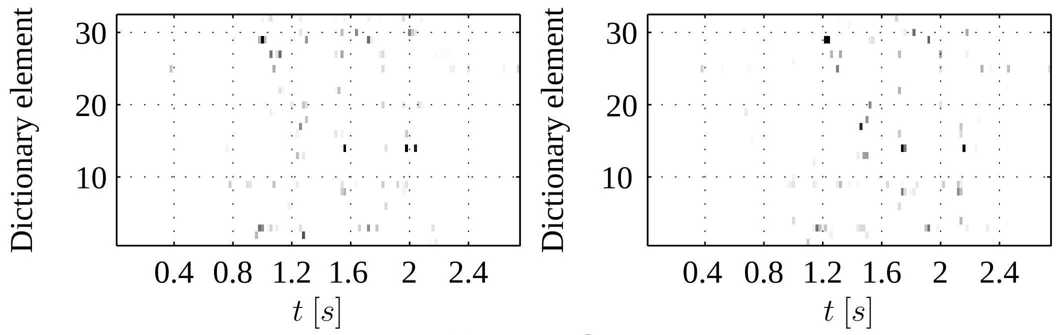
[2] $l_p(a) = |a|^p$. l_p is a valid sparseness function for $0 \leq p \leq 1$.



(a) Spectrograms.



(b) MP codes.



(c) SSQP codes.

Figure 2.9: (a) The spectrograms of two utterances of “one six five seven”. Black represents $1dB$. (b) The compressed MP code for each utterance using the MP-trained dictionary. A code is compressed by adding the code elements associated with a particular dictionary element together irrespective of the scaling of that code element. The codes are compressed for better visualization. Black represents a code element value of 4.5. (c) The compressed SSQP code for each utterance using the SSQP-trained dictionary. Black represents a value of 1.2.

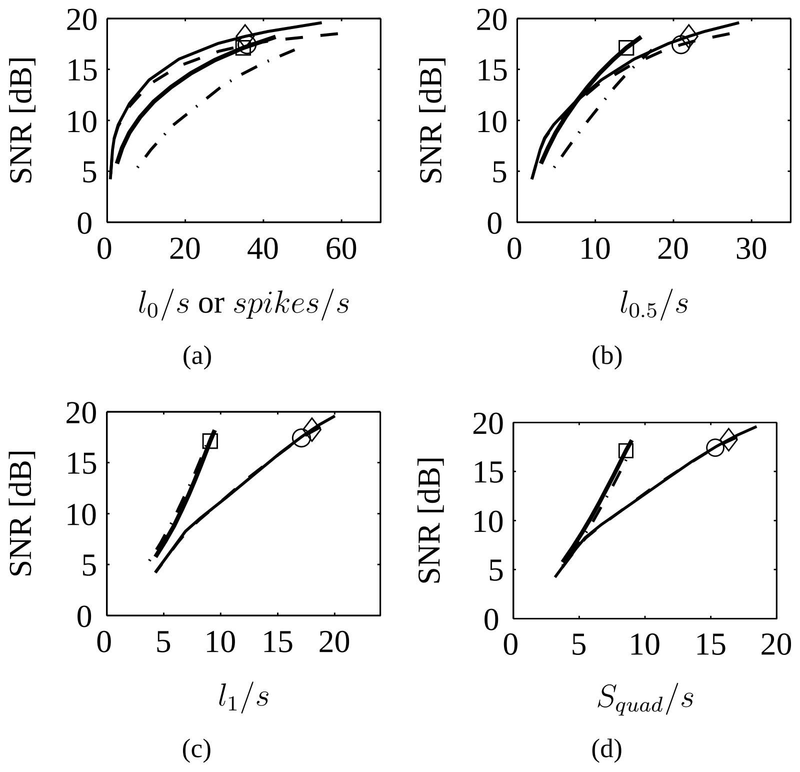


Figure 2.10: The four plots show the results for MP codes (thin dashed line), MP$^+$ codes (thin continuous line), SSQP codes obtained with S_{quad} as sparseness function (thick continuous line) and SSQP codes obtained with l_1 as sparseness function (thin chain line). The SSQP results are obtained with a dictionary trained on SSQP codes with the S_{quad} sparseness function, while MP and MP$^+$ use dictionaries optimized for each respective algorithm. The lines for MP and MP$^+$ in plots (c) and (d) are almost coincident. The x-axis is an average sparseness value per second, the y-axis is the SNR of the reconstructed signal for the dataset. A marker indicate that point on the plot where the stopping threshold or λ is the same as was used during training of the dictionary (circle for MP, diamond for MP+, square for SSQP).

code element values for MP and SSQP lie in that range where S_{quad} and l_1 are very close together. S_{quad} and l_1 have almost identical functional plots in this range ($a < 2$). It would seem at first that there is not any benefit to using the computationally more expensive S_{quad} when l_1 yields almost the same results. However from figure 2.10(a) we see that codes calculated with S_{quad} has far fewer nonzero components than codes calculated with l_0 for the same SNR. This property of the S_{quad} sparseness function is very desirable. Remember that ideally we want to use l_0 as a sparseness function, but cannot as MP yields codes that are not suited to pattern recognition. S_{quad} is a better sparseness function than l_1 in this respect.

2.7.4 THE TRAINED DICTIONARY

The basic dictionaries $\mathbf{\Phi}_{\{0\}}$ that are trained with MP and SSQP are shown in figures 2.12 and 2.11. Each dictionary has dictionary elements whose structure reveal complex features that span across frequency and time to varying degrees. It is clear that some elements are strongly localized in frequency and others more localized in time; several seem to encode specific transitions of frequency content as a function of time.

We see that the SSQP dictionary has more dictionary elements that code complex features. For example elements 12 to 16 and others of the MP dictionary are similar and are not as complex as element 2 for example. This is in contrast to the image dictionaries published by Perrinet (2004a) who used MP and Olshausen and Field (1996b) who use a gradient based approach. Comparing the MP image dictionary with the gradient based image dictionary shows that the MP dictionary has elements that are less localized in space and higher portion of elements that are high-frequency Gabors. We found that the gradient based approach has more localized elements than MP. The difference may be due to the different type of dataset we use.

We can use histograms of phoneme occurrences to form an idea of the sounds coded by the elements. A histogram is created for each phoneme p and each dictionary element d. The bins of the histogram span over time where each bin is $20ms$ wide. We associate a time period t_b with each bin; it is a period that precedes a spike associated with dictionary element d. The number of entries in bin b of histogram (p, d) is the number of times phoneme

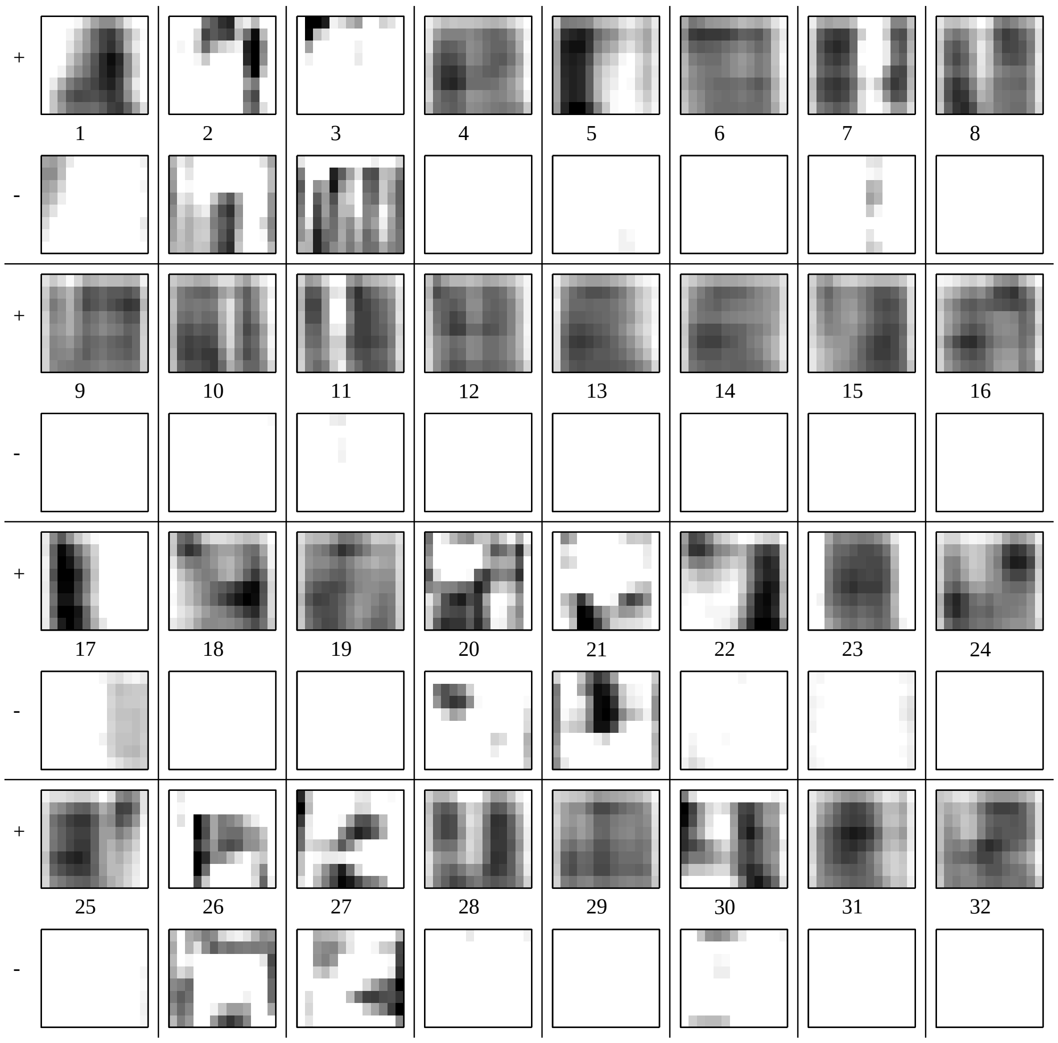


Figure 2.11: This figure shows the dictionary elements of the basic dictionary $\Phi_{\{0\}}$ trained with MP. Each element is split into positive and negative parts for better visualization. A column of a dictionary element spans $20ms$; a complete dictionary element therefore spans $260ms$. Each row in a dictionary element correlates with a channel in the spectrogram (see figure 2.6). Thus, the bottom row of a dictionary element represents signal content at $367Hz$ and the top row $7321Hz$. The dictionary elements have a norm of 1. White represents values of 0 and black 0.2. Elements are numbered from left-to-right and from top-to-bottom.

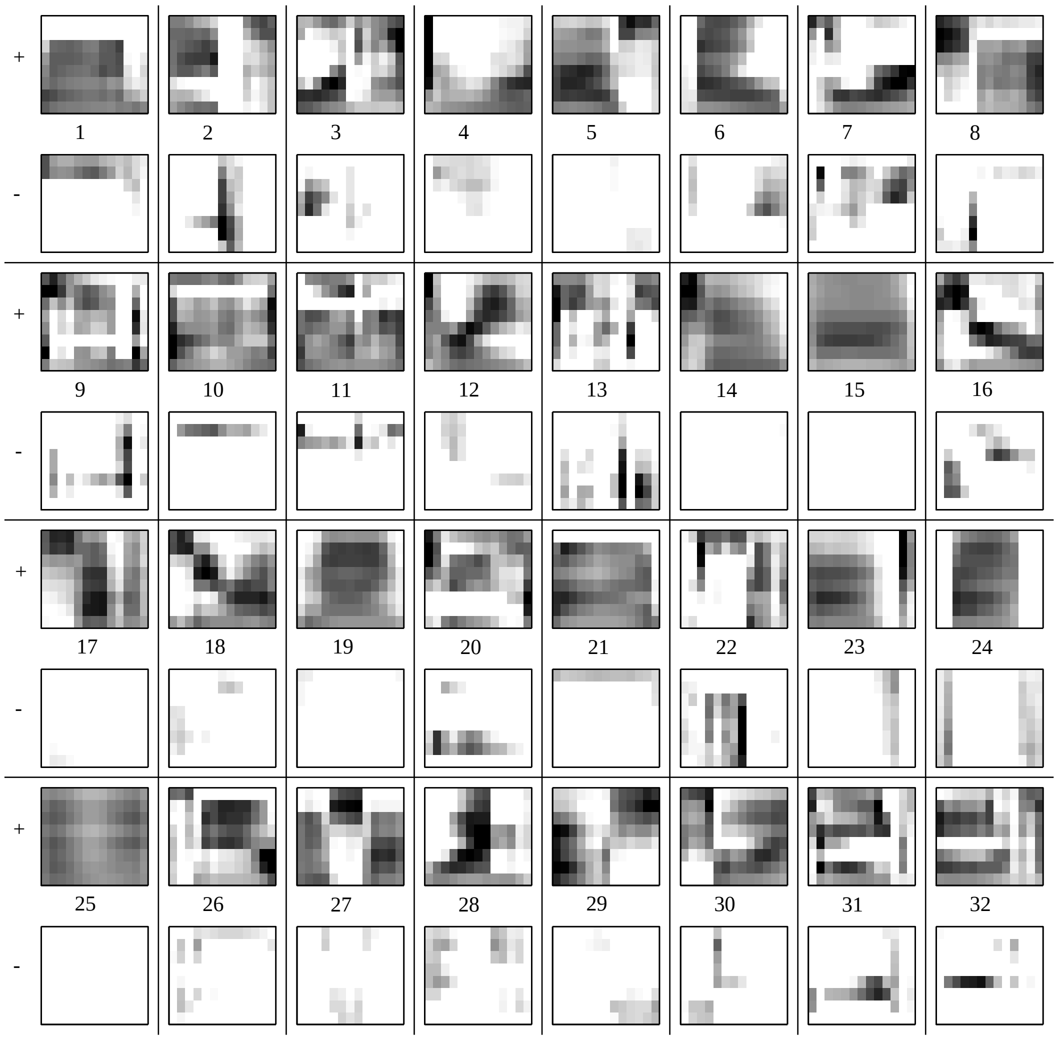


Figure 2.12: This figure shows the 32 dictionary elements of the basic dictionary $\mathbf{\Phi}_{\{0\}}$ trained with SSQP. Each element is split into positive and negative parts for better visualization. A column of a dictionary element spans $20ms$; a complete dictionary element therefore spans $260ms$. Each row in a dictionary element correlates with a channel in the spectrogram (see figure 2.6). Thus, the bottom row of a dictionary element represents signal content at $367Hz$ and the top row $7321Hz$. The dictionary elements have a norm of 1. White represents values of 0 and black 0.2. Elements are numbered from left-to-right and from top-to-bottom.

p precedes a spike of dictionary element d during t_b for the entire data set. In other words, the histogram shows how often and at what times a phoneme occurs before a spike. In the TLGM a spike in the code means that a dictionary element is used to reconstruct a part of the spectrogram that *precedes* the spike. (Note that we ignore the fact that time-scaled versions of a dictionary element may in fact have been used; all the scaled versions are grouped together and are viewed as a single dictionary element).

Figure 2.13 shows the histograms of phoneme occurrences for four dictionary elements. From figure 2.13(a) is appears that dictionary element $d = 2$ is used in coding the “ih-k-s” sound in “six”. It also seems that it is used to code “ey-t” which occurs in “eight”. It is not possible to conclude from this histogram alone that element $d = 2$ actually codes the “ih” sound. The reason is that in our limited dictionary the “ih” sound always precedes the “k-s” sound. If $d = 2$ codes only the “k-s” sound the histogram would still show that the “ih” sound often precedes a spike associated with $d = 2$. However, by taking a closer look at the dictionary element itself, we see that it has a period of silence between $80ms$ and $140ms$ before the spike occurs. This corresponds to the closure between the “ih” and “k” sounds. therefore it does seem that element $d = 2$ codes the “ih” sound or at least a part of it. There are certain implications to $d = 2$ having a large negative part. Our spectrogram transform has only positive parts, which means that $d = 2$ cannot be used on its own. It has to be used with other dictionary elements.

Figure 2.13(b) shows a dictionary element that codes a sound with a rising frequency. We see from the histogram that the entire word “three” often precedes a spike from $d = 12$. The plot of the dictionary element itself shows that it contains information to code the word “three”. The “th” fits the left part of the dictionary element; it shows a broad distribution of energy across most of the frequencies. The right part of the dictionary element fits the rising centre-of-mass (in frequency) of the sound in “iy”.

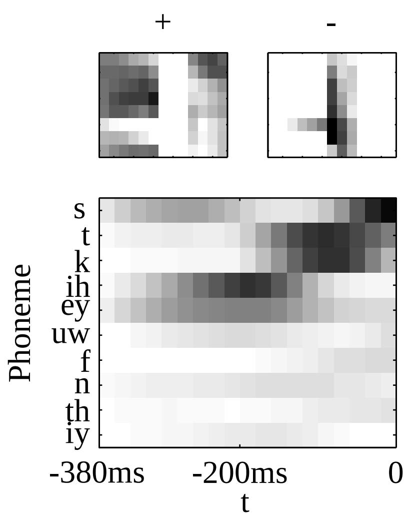
Dictionary element $d = 25$ (figure 2.13(c)) does not code a particular sound, but is rather used as a constant offset. Lastly figure 2.13(d) shows that $d = 31$ is a dictionary element with a very complex structure. The histogram shows that it is mostly used during the coding of “ey” in “eight”.

There is good reason to believe that the basic unit of speech is on the syllable level (Nguyen and Hawkins, 2003). The dictionary elements are in good agreement with this, at least some of them (for example $d = 2$) are on syllable level. Dictionary elements are not used in isolation, they are used in conjunction with other elements. It is complicated to say which acoustic unit a particular dictionary element codes as it depends on the other dictionary elements that are used in with it. We see for example from figure 2.13(a) that $d = 2$ is used in coding both the words "six" and "eight" although the words does not share a phoneme.

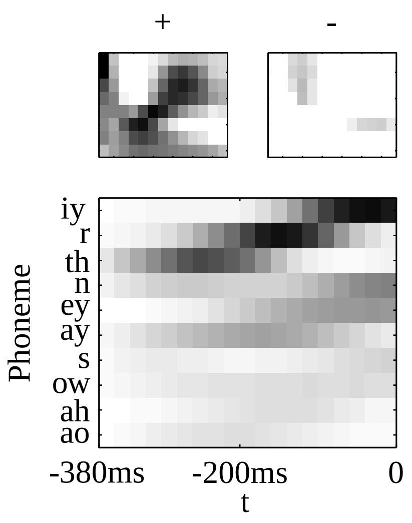
2.7.5 THE CODE ELEMENTS

Figure 2.14 gives the histogram of all the spikes for the entire dataset. It does not show the code elements that did not spike. Only 0.0042% of the code elements are active. The distribution agrees with that of a sparse code where most of the code elements are zero. A great percentage of the code elements have values close to zero. This is unwanted feature is caused by the prescribed prior probability that has a peak for values close to zero. The figure shows the shape of the prescribed prior probability $p(a) \propto e^{-(a-0.05a^2)}\ \forall\ 0 \leq a \leq 10$. The shape of the histogram does not follow that of the prior. Two important reasons for this are that the dictionary elements are constrained to have a norm of one and that the underlying distribution of the data may not fit the prescribed prior. The shape of the histrogram shows that the computed codes are more sparse than the prior prescribes. This is not a problem as we are looking for sparse codes.

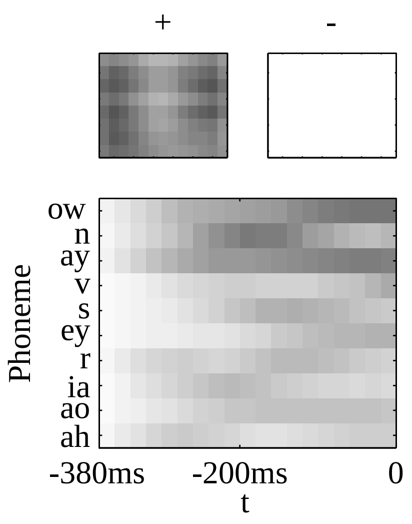
Actual data agrees much better with the prior when the optimization problem is formulated so that a constraint on the dictionary elements is not necessary (Girolami, 2001). Such a formulation unfortunately makes it computationally even more expensive to find a sparse code. Another reason for the difference between the data and the prior is that the prior only applies to a dictionary where every dictionary element is trained. In our dictionary only the basic unscaled dictionary elements are trained, not the scaled versions. The sparseness function is not chosen for its shape but the fact that it makes the error function quadratic, therefore the exact shape of the spike histogram is not that important to our application.



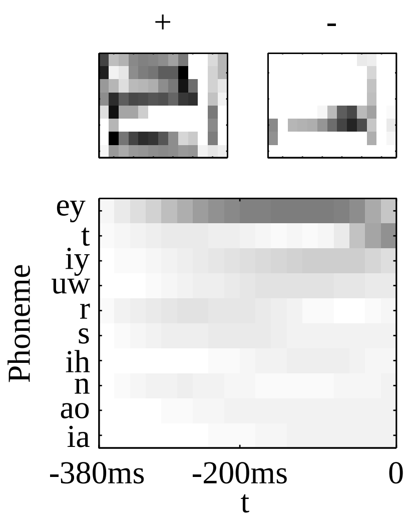
(a) d=2



(b) d=12



(c) d=25



(d) d=31

Figure 2.13: The top row of each plot gives the positive and negative parts of the dictionary element. The bottom row of each plot depicts the phonemes' correlation with the given elements: it is a histogram plot that shows which phonemes preceded the spike associated with that particular model, and at what times the phoneme occurred. We considered only spikes with an amplitude of more than 0.124; smaller spikes are not that significant. The histogram plot shows only the ten phonemes that occurred most for each dictionary element. The histograms are scaled so that black represents 2000 entries in a bin.

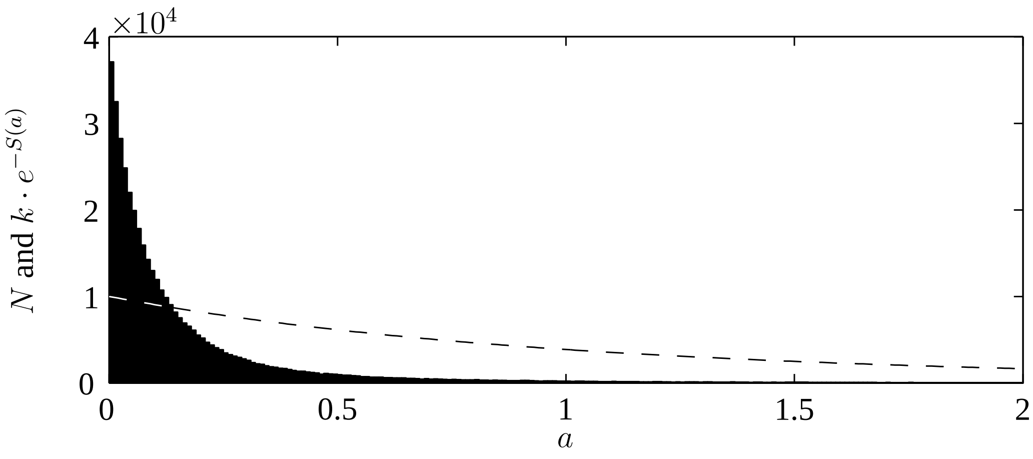
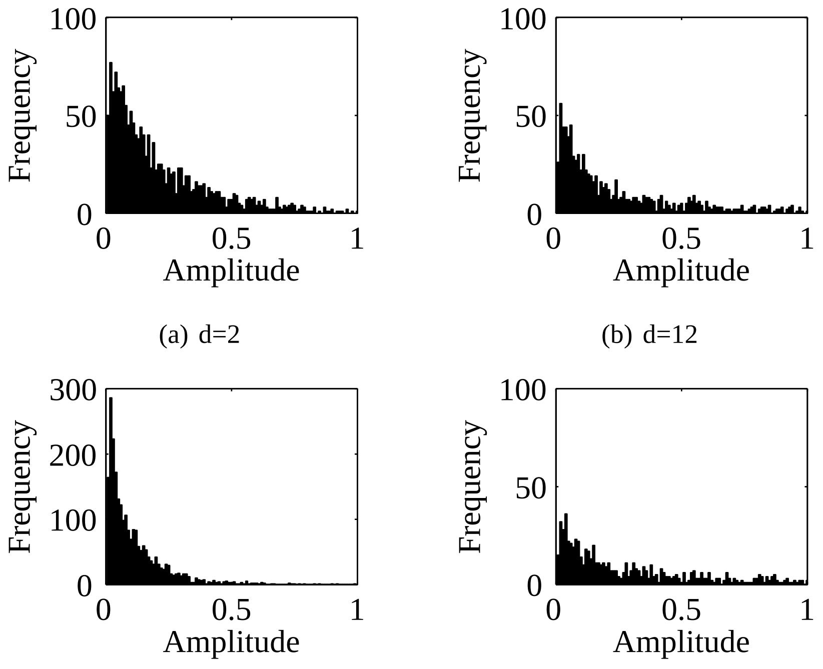


Figure 2.14: The histogram of spike values for the training set. It is gathered from the codes for the complete dictionary (includes all the scaled dictionary elements). The dashed line gives the shape of the probability density function $p(a) \propto e^{-S(a)}$. The histogram should ideally follow the shape of $p(a)$.

It is interesting to look at the amplitudes that spikes assume for a particular dictionary element. Figure 2.15 shows histograms of the spike amplitudes for various dictionary elements. We see that the dictionary elements are not used equally often; $d = 31$ is used less often than any of the other dictionary elements that are shown. This may be due to the complex and irregular structure of $d = 31$, we do not expect such a structure in speech. $d = 25$ is used often but has a small average spike value. The spike value indicates the contribution that a dictionary element makes to reconstruct a signal. The fact that $d = 25$ has a smaller average spike value than the other dictionary elements suggests that it too does not capture significant structure in the speech.

Figure 2.16 shows the frequency with which a given dictionary element is used, as well as the frequency with which a given scale is used. Dictionary element $d = 15$ is seldom used. This element seems to serve a similar purpose as $d = 25$ which is to provide an offset. However $d = 15$ has significant low frequency content which may be the reason its usage is different from $d = 25$. The figure also shows that the maximum scaling ($s = 6$) of dictionary elements are used more often than any other scaling. This is because the sparseness function promotes sparse codes; it is "cheaper" to increase the amplitude of a



(a) d=2 (b) d=12

(c) d=25 (d) d=31

Figure 2.15: The histograms of spike amplitudes for four of the dictionary elements are given. These are compiled from the sparse codes of the training set. Only nonzero spikes are included in the histograms. The histograms show that most of the spikes have very small amplitudes. We explain in the text that such spikes are not desirable. It also shows that the dictionary elements are not used equally often; $d = 25$ is used more often than any of the other three dictionary elements.

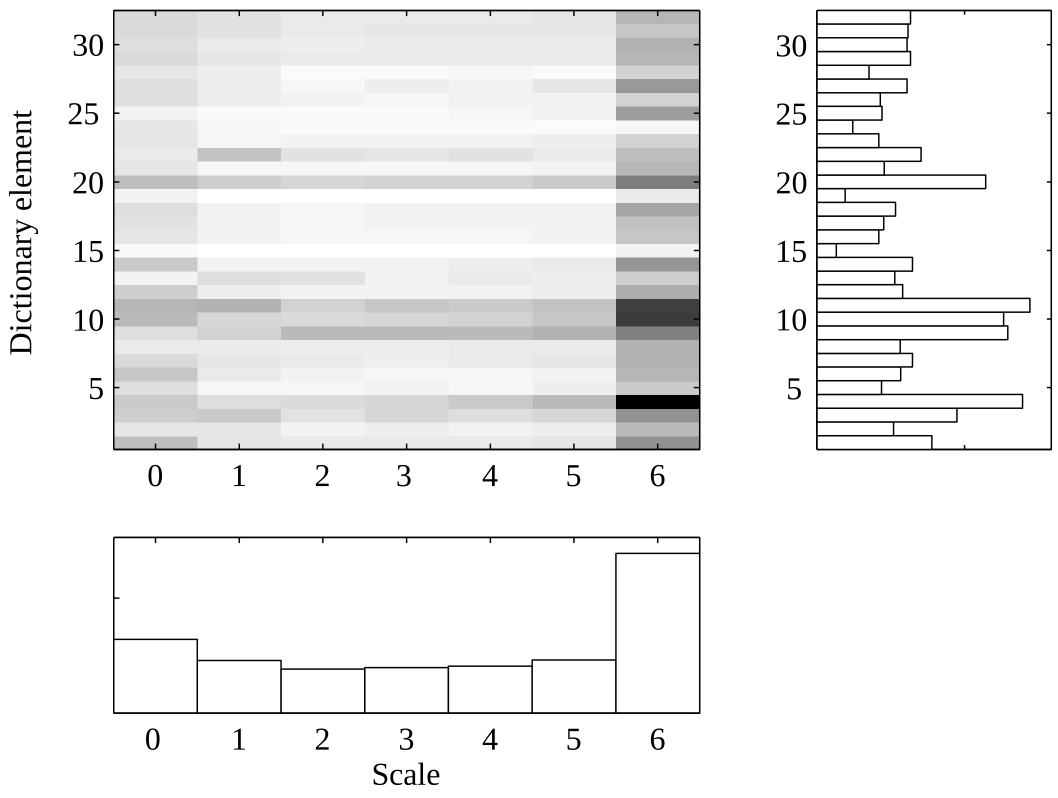


Figure 2.16: The *top left* plot consist of 32 histograms, each histogram is a row in the plot. A histogram shows how often a particular scale is used for the given unscaled dictionary element. The *bottom left* histogram shows the frequency with which a particular scale is used, independent of the unscaled dictionary element. The *top right* histogram gives the frequency with which a particular unscaled dictionary element is used, independent of scale.

spike in order to reconstruct a code than to use an additional spike. A code can consist of fewer nonzero entries if the dictionary elements span over long time periods than if the dictionary elements are temporally short.

Not all dictionary elements are most often used at the maximum scale. For example $d = 12$ is also often used unscaled. The reason for this is explained by the fact that $d = 12$ codes the entire word “three” (see figure 2.13(b). 43% of the occurences of “three” in the dataset are shorter than $280ms$ (the time span of an unscaled dictionary element). It makes sense that the unscaled version of $d = 12$ is used so often.

A code is efficient when the code elements are independent. Figure 2.17 shows the

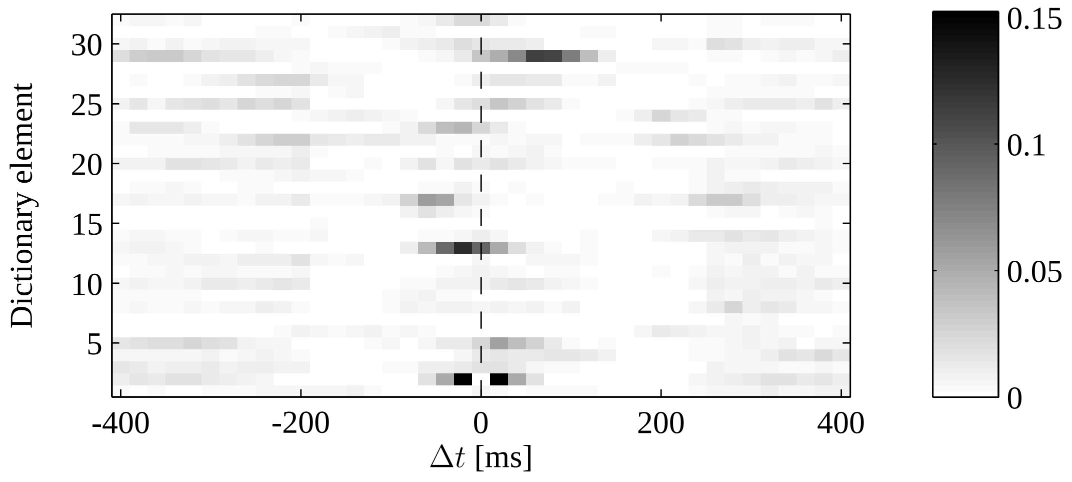


Figure 2.17: The probability that a spike at time t_i precedes or follows a spike of dictionary element $d = 2$ that occurs at t_2. The x-axis is the time relative to the $d = 2$ spike, $\Delta t = t_i - t_2$. The entry at $(d = 2, \Delta t = 0)$ is 1, but is removed to give a better range to the shades in the plot. The plot is constructed for spikes with an amplitude larger than 0.124 and *only* for spikes from the $s = 6$ scale (dictionary elements that span $380ms$).

correlation between spikes from $d = 2$ with other spikes. There is overall little correlation between spikes. A weak correlation exist between $d = 2$ and itself around $\Delta t = \pm 20ms$. This shows that one spike sometimes immediately follows another. One would not expect this to occur as often as it does. We closely inspected some of the codes that have two consecutive $d = 2$, $s = 6$ spikes, but could not find a solution for those particular codes that does not have the two consecutive spikes. Different starting points and different gradient based algorithms all yielded codes that have the two spikes. It therefore seems that the occurrence of two consecutive spikes is not an artefact of the optimization but rather a feature of the code.

Figure 2.17 also shows a weak correlation between $d = 2$, $d = 13$ and $d = 29$. This correlation exist because the combination of these dictionary elements are part of the reconstruction of the word "six". This pattern can in fact be used to recognize the word "six" from a code.

2.7.6 THE IMPORTANCE OF SCALING DICTIONARY ELEMENTS

We use scaled versions of dictionary elements because in speech certain features may be stretched over time without changing its meaning. Therefore we do not expect useful information for speech recognition to be contained in the particular scaled version of a dictionary element that is used, i.e. we expect a spike associated with $\mathbf{\Phi}^{\{d\}}_{\{s\}}$ to signal the same event as a spike associated with $\mathbf{\Phi}^{\{d\}}_{\{s' \neq s\}}$.

Figures 2.18 and 2.19 shows whether there are any information useful for speech recognition contained in the scaling itself. These figures are constructed similarly to the histograms of phoneme occurrences in figure 2.13; they show how often a word precedes a spike. The difference is that here we correlate activity with words and not phonemes, we also create a different plot for each scale.

Figure 2.18 confirms that $d = 2$ is used most often to code “six” and “eight”. We see for a particular scale that there is very little difference between the histograms of “six” and “eight”. This means that scaling cannot be used to distinguish whether the spike signals a “six” or an “eight”. The plot for $s = 6$ does show that scaling can be used to distinguish “two” from either “six” or “eight”. We see that “two” is mostly used at the $s = 6$ scale. A classification system that uses the scaling information will classify “two” more accurately as it will use the information that a spike associated with $d = 2$ and $s = 0, 1, 2, \ldots 5$ probably does not signal a “two”. It appears that in most cases scaling does not contain useful information, although scaling may be useful in other cases. Figure 2.19 leads us to a similar conclusion. It shows that a spike associated with $d = 12$ most probably signals a “three”, independent of the scaling. However for $s = 6$ the spike may also signal an “eight”.

Figures 2.18 and 2.19 suggest that scaling can be a good feature to use in speech recognition, although scaling adds very little information about the spikes.

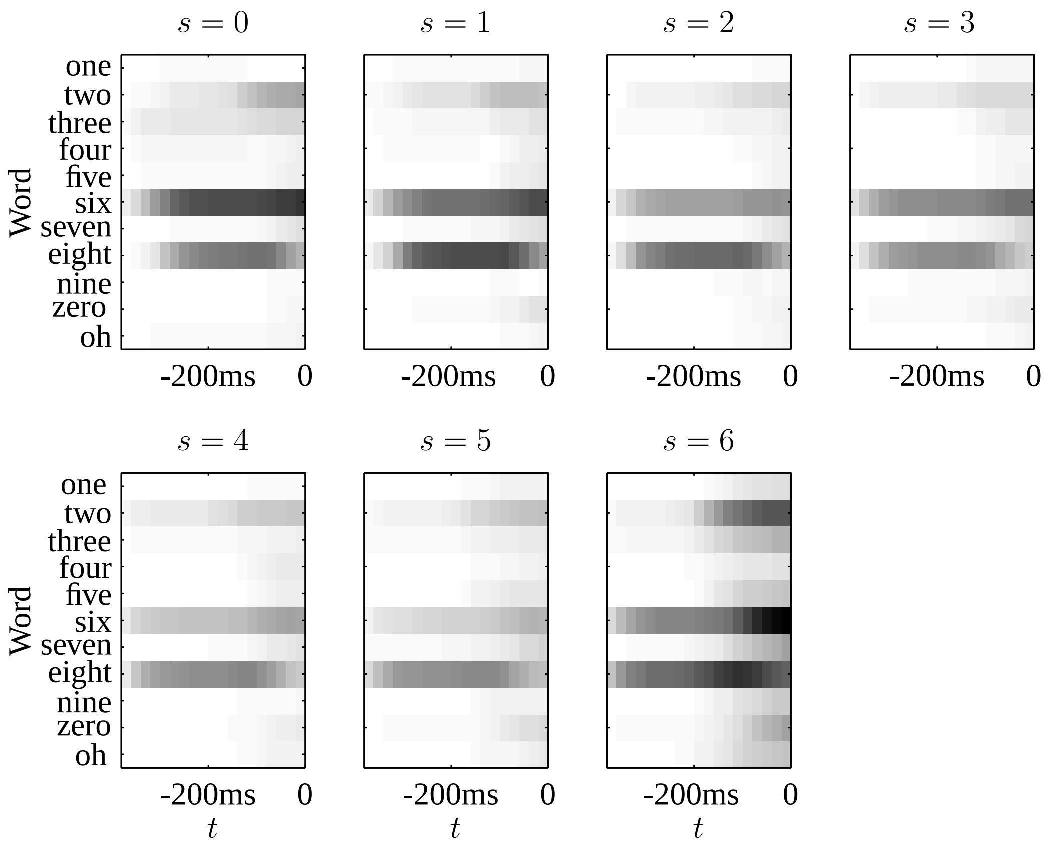


Figure 2.18: The correlation between words and spikes of $d = 2$. The intensity of an entry in the plot shows how often a particular word precedes a spike at a particular time. Each plot is constructed for a different scale. We only considered significant spikes (spikes with an amplitude larger than 0.124).

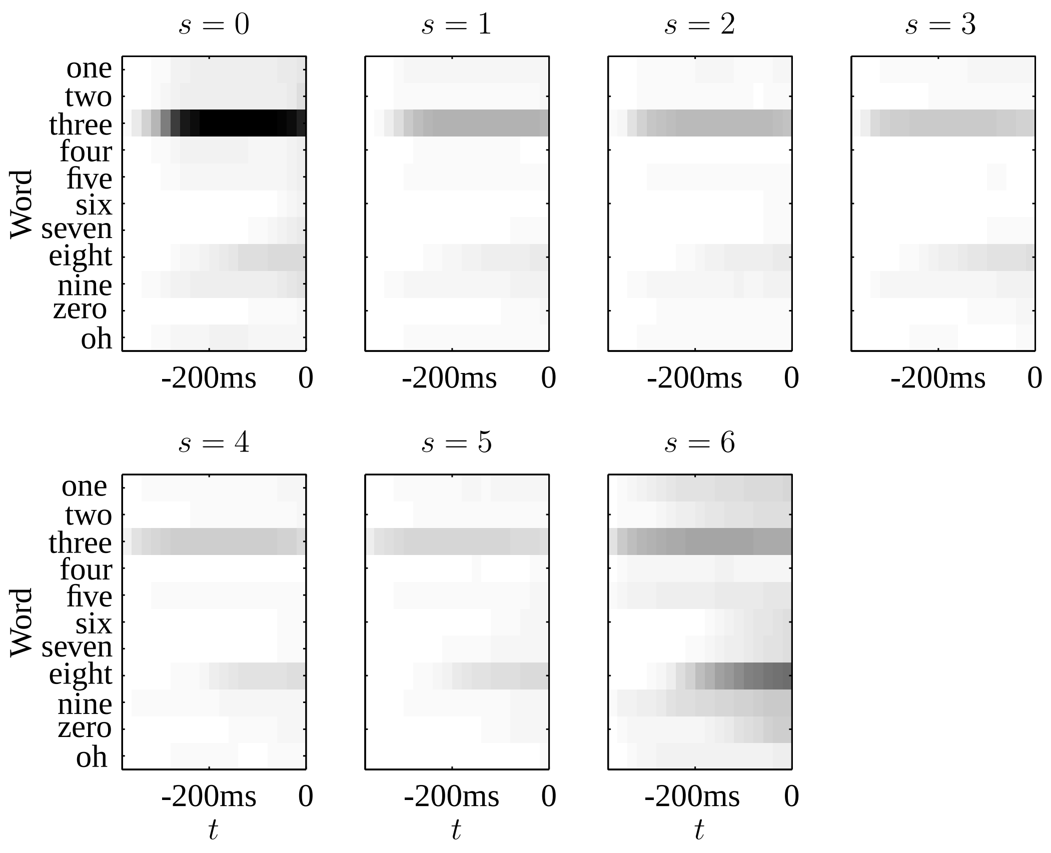


Figure 2.19: The same as figure 2.18, but for $d = 12$.

Chapter Three

Spike train classification

The previous chapter shows how a speech signal can be transformed into a multichannel or multineural spike train. In this chapter we will address the problem of decoding the spike train, i.e. how to infer or classify the spoken words encoded by the spike train.

A spike train can be decoded when the way in which spikes carry information is fully understood. For example, does the specific time at which a spike occur carry any meaning, or is the information contained in the average firing rate of a neuron? We first look at a few of the coding schemes proposed for spike trains. After that we present a probabilistic model for spike trains and we show how to use this model to infer the words that are encoded by a spike train.

3.1 CODING SCHEMES

In order to decode a spike train, it is necessary to understand how information is encoded by the spike train. We do not understand the neural code completely. However recent advances in the technology of neuroscience have made it possible to form a better picture of the neural code.

The neural code represents information about the external world. Neurons very close to the senses carry the most information about stimuli. Subsequent stages in the neural system transform the information and discard irrelevant information. The purpose of processing sensory information is to extract features in the stimulus relevant to behaviour.

Originally it was thought that information is encoded only in the *firing rate* of neurons. This led to the development of the very successful multi-layer perceptron. The numerical value of a perceptron represents the firing rate of a neuron; the sigmoid activation function corresponds to the limited range of firing rates that a real neuron can achieve. Spike train models that are based on the firing rate view often make use of a Poisson model for which the rate parameter is stimulus dependent.

Another view of the neural code, the *temporal coding* view, is that the precise timing of a spike also carries information in addition to the firing rate (Dayan and Abbott, 2001). It is difficult to verify experimentally which coding scheme fits the neural code best. Figure 3.1 shows two time-varying stimuli and the response of a neuron to those stimuli. The neuron is a stochastic model with the probability that a spike occur proportional to the firing rate. In figure 3.1(a) there is no temporal pattern in the spikes that code the slowly varying time stimulus. The information is coded in the firing rate of a neuron measured over a rather long time frame. In figure 3.1(b) it appears as if there is a pattern in the spike train that may contain additional information. However this is not the case, the patterns are the result of a quickly varying stimulus. The firing rate of a quickly varying stimulus should be decoded from the spike train by taking the average number of spikes in a short time frame. We see that the difference between rate coding and temporal coding is more than a matter of time scales. A code cannot be called a temporal code only because there are patterns in the spike train that occur often.

Oram, Wiener, Lestienne and Richmond (1999) has proposed a *spike count-matched model*. The model is based on the *firing rate* profile (firing rate as a function of time) and the *spike count* (number of spikes in response to a stimulus). This model fits actual spike train

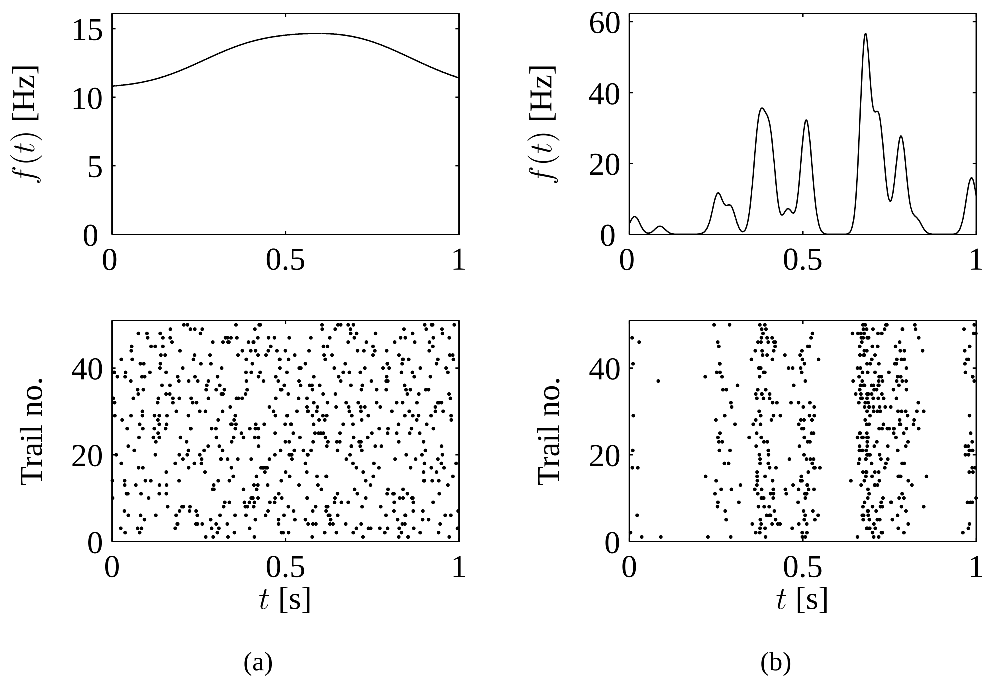


Figure 3.1: The response of a stochastic neuron to a time-varying stimulus is shown in each figure. The experiment is repeated for 50 trials. (a) There is no pattern in the spike train of the neuron in response to a slowly varying stimulus. (b) There is an obvious pattern in the spike train of a neuron in response to the quickly varying stimulus, but this is not enough reason to assume that it is a temporal code.

data from the lateral geniculate nucleus, the primary visual cortex (V1), and primary motor cortex of a monkey. They show that the precise patterns in spike trains are directly related to the firing rate modulation and the spike count distribution. It is therefore not a temporal coding model. The precise timing of spikes does not carry information beyond the firing rate in the three mentioned areas of the monkey brain.

There are also other coding schemes that fit neural data. The *time-to-first-spike* scheme considers only the time from stimulus onset to the first spike. This scheme fits real data from certain tasks very well; tasks that require a quick response allow only time for neurons in the different processing stages to fire a single spike. For example, faces are recognized by humans within 150ms (Thorpe, Fize and Marlot, 1996), just enough time for each stage to fire a single spike. This scheme is just a special case of the spike count-matched model. Another coding scheme is *synchrony*. The fact that certain neurons spike at almost the same time carries information. Again this is a special case of the spike count-matched model that is extended to include multiple neurons.

The spike count-matched model fits *single neuron* data from three areas of the monkey brain. The model should be expanded and be tested on multineural spike trains. The discussion between neuroscientists on the actual neural coding scheme is not yet over. More data needs to be analyzed to find the true coding scheme. It seems as if different areas of the brain may use different coding schemes.

3.2 CLASSIFICATION OF THE SPIKE TRAIN

Accurate classification of spike trains requires that patterns in the spike trains correlate well with the words in the utterance. For example, if two different people say the same words, we would ideally like the spike trains that code those utterances to be identical. This is seldom possible because people speak at different rates, at a different pitch etc. Even if spike the trains that correspond to the same word are not identical, it is still possible to classify spike trains accurately. This is possible when the spike trains of the same word but different utterances are similar enough, and when the spike trains of different words are dissimilar enough.

A spike train can be classified by matching patterns in a spike train with templates. The templates could correspond to a specific part of a word, for example a phoneme, a triphone, a syllable or even a complete word. There are a few models that can find temporal patterns in multichannel spike trains (Chi, Rauske and Margoliash, 2003; Kass and Ventura, 2001; Gat and Tishby, 2001); they do not give the probability that a spike train fits a model.

Current speech recognition systems use a powerful probabilistic approach to classification: the most likely sequence of words given the features is determined. There are well established methods for finding the model parameters that will lead to good classification for the probabilistic approach. We use the same classification approach as current automatic speech recognition systems, but we have a different feature set.

The spike count-matched model with order statistics (Wiener and Richmond, 2003) is well suited to this problem. With this model the probability $p(\zeta_i \mid m)$ that segment i from the spike train $\boldsymbol{\zeta}$ fits a model m can be calculated and is used to find the most likely sequence of models. The model can easily be extended to include multichannel spike trains; this extended model is the spike train model we use. We use supervised training of the models and accordingly force them to correspond to words. This choice is discussed below in more detail.

3.2.1 THE SPIKE TRAIN MODEL

A spike train is completely described by the times $\bar{t}$ at which spikes occur, the amplitudes $\bar{a}$ of those spikes and the channels $\bar{c}$ in which they occur. The ith spike is described by its firing time t_i, amplitude a_i and channel c_i.

The probability that spike train $\boldsymbol{\zeta}$ fits model m is

$$p(\boldsymbol{\zeta} \mid m) = p(\bar{t}, \bar{a}, \bar{c} \mid m). \tag{3.1}$$

In section 2.6.2 we showed that the sparseness function assumes the activity of different code elements to be independent. Therefor

$$p(\bar{t}, \bar{a}, \bar{c} \mid m) = \prod_c p(\bar{t}_c, \bar{a}_c, n_c \mid m, c) \tag{3.2}$$

with $\bar{t}_c$ and $\bar{a}_c$ the respective subsets of $\bar{t}$ and $\bar{a}$ that contain only the spikes in channel c. n_c is the spike count of channel c.

In order to reduce the complexity of the model, we assume that the spike times and spike amplitudes are independent of each other. It is reasonable to assume that the spike times are independent. Figure 2.17 shows that there is a weak correlation between spike times, but the correlation exists because it codes the same word. So

$$p(\bar{t}_c, \bar{a}_c, n_c \mid m, c) = p_t(\bar{t}_c \mid m, c, n_c) p_a(\bar{a}_c \mid m, c) P_n(n_c \mid m, c) \tag{3.3}$$

Here p_t is the spike time probability density function. It gives the probability that the times at which spikes occur in the spike train fit the firing rate profile. p_a is the spike amplitude probability density function. P_n is the normalized spike count distribution. It gives the probability than n_c spikes will occur in a given channel.

The spike time probability density function is a function of the order statistics and the firing rate profile (Wiener and Richmond, 2003)

$$p_t(\bar{t}_c \mid m, c, n_c) = \prod_h p_t(t_{c,h} \mid m, c, n_c, h) \tag{3.4}$$

$$p_t(t_{c,h} \mid m, c, n_c, h) = \frac{n_c!}{(h-1)!(n_c-h)!} F_t(t_{c,h})^{h-1} f_t(t_{c,h}) [1 - F_t(t_{c,h})]^{n_c - h} \tag{3.5}$$

with $f_t(t)$ the normalized spike density function and $F_t(t)$ is its cumulative probability density. $p_t(t_{c,h} \mid m, c, n_c, h)$ is the probability that for channel c of model m, the hth spike in a spike train occur at time $t_{c,h}$.

The amplitudes of the spikes within a channel are also assumed to be independent so that

$$p(\boldsymbol{\zeta} \mid m) = \prod_c \left[P_n(n_c \mid m, c) \prod_h p_t(t_{c,h} \mid m, c, n_c, h) \prod_h p_a(a_{c,h} \mid m, c, h) \right] \tag{3.6}$$

p_a and $f_t(t)$ are modelled with Gaussian mixture models (GMMs). p_a has two components and $f_t(t)$ three. P_n is modelled with a discrete GMM having two components.

3.2.2 AN EXAMPLE OF SPIKE TRAIN MODELS

Figure 3.2(a)-(c) shows three simplified spike train models. The models do not use spike amplitude information. One gets a good idea of how spike train models are able to classify a spike train by comparing a few models. Consider for example models $m = 1$ and $m = 3$. Their spike count distributions are similar except for channel 4. The probability that model $m = 3$ does not contain a spike in channel 4 is much lower than the probability that model $m = 1$ does not contain a spike in that channel. By just counting the number of spikes in channel 4 of an unclassified spike train, we have a good indication whether that spike train was generated by model $m = 1$ or model $m = 3$.

The number of spikes in channel 4 would not be a good way to classify spike trains generated by models $m = 2$ and $m = 3$. The spike count distributions of these two models are very similar for channel 4. In this case it will be better to use the spike times of spikes in channel 3. By inspecting all three models we see that if a random spike train is generated with equal probability $P(m) = 1/3$ by any of the models, then we will be able to predict very accurately which model generated that spike train.

Figure 3.2(d) shows a spike train that is generated by model $m = 1$. The spike train will be correctly classified as it fits model $m = 1$ the best.

3.2.3 FINDING THE MOST LIKELY SEQUENCE OF MODELS

Here we show how to determine the most likely sequence of models given an unknown spike train. From the sequence of models we can easily determine the sequence of words as each model corresponds to a word. The spike train models can be trained in an unsupervised manner, in which case a model can represent any sound or sequence of sounds. Once the models are trained, it is necessary to determine the correspondence between sounds and models. However, supervised training is easier to conceptualize than unsupervised training. For this study we force the models to correspond to words. Unfortunately this choice may limit the classification performance.

Part of the problem of finding the most likely sequence is segmenting the spike train. The classification problem is now to find the most likely sequence of models $\overline{m}^*$ and segment sizes $\overline{\Delta t}^*$ given a spike train, i.e.

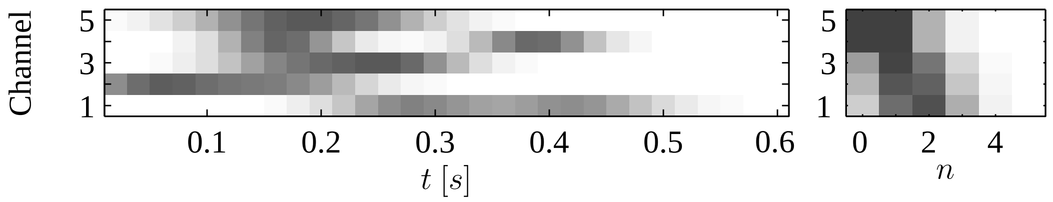
$$\overline{m}^*, \overline{\Delta t}^* = \max_{\overline{m}, \overline{\Delta t}} p(\overline{m}, \overline{\Delta t} \mid \boldsymbol{\zeta}) \tag{3.7}$$

We assume that each segment is independent of all others, so that using Bayes' theorem we have

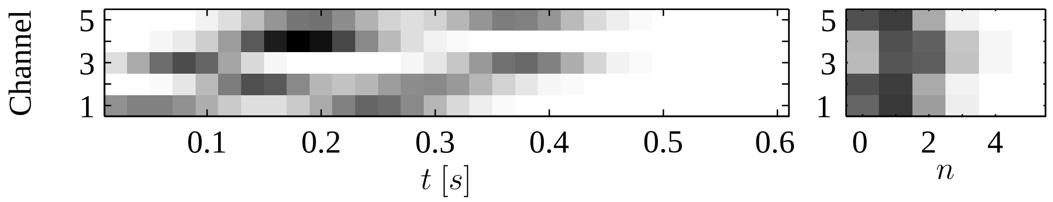
$$p(\boldsymbol{\zeta} \mid \overline{m}, \overline{\Delta t}) = \prod_i p(\zeta_i \mid m_i, \Delta t_i) \tag{3.8}$$

where $p(\zeta_i \mid m_i, \Delta t_i)$ is the probability that the ith spike segment ζ_i is Δt_i long and fits model m_i (equation 3.1). The joint probability is

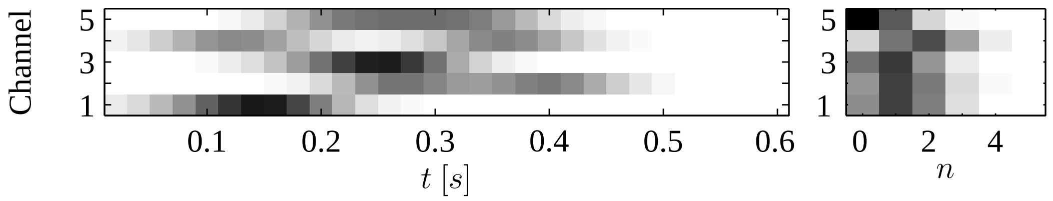
$$p(\overline{m}, \overline{\Delta t}) = p_{\Delta t}(\overline{\Delta t} \mid \overline{m}) P(\overline{m}) \tag{3.9}$$



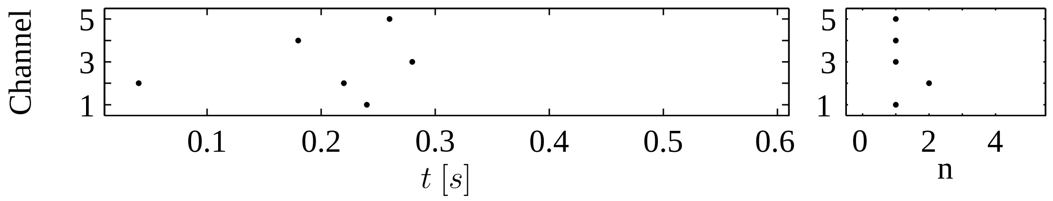
(a) m=1



(b) m=2



(c) m=3



(d)

Figure 3.2: The left column of (a) to (c) gives the normalized spike density function f_t, while the right column gives the spike count distribution P_n. (d) shows a spike train that best fits model $m = 1$. In fact $\ln(p(\boldsymbol{\zeta} \mid m))$ equals for models (a), (b) and (c) respectively -22.3, -29.6 and -36.9.

with

$$p_{\Delta t}(\overline{\Delta t} \mid \overline{m}) = \prod_i p_{\Delta t}(\Delta t_i \mid m_i) \tag{3.10}$$

again because we assume the segments to be independent. $p_{\Delta t}(\Delta t_i \mid m_i)$ is the probability that a spike train will have a length of Δt_i given model m_i. It is modelled with a two-component continuous GMM. $P(\overline{m})$ is the language model, which for a data set consisting of random digits and models corresponding to words is

$$P(\overline{m}) = \prod_i P(m_i) \tag{3.11}$$

If the models do not correspond to words, the language model would be slightly more complex. Accurate language models can be designed for a specific data set (Rabiner and Juang, 1993). Finally, the most likely model sequence and segment sizes are

$$\overline{m}^*, \overline{\Delta t}^* = \max_{\overline{m}, \overline{\Delta t}} \prod_i p(\zeta_i \mid m_i, \Delta t_i) p_{\Delta t}(\Delta t_i \mid m_i) P(m_i) \tag{3.12}$$

or

$$\overline{m}^*, \overline{\Delta t}^* = \min_{\overline{m}, \overline{\Delta t}} \sum_i [-\ln p(\zeta_i \mid m_i, \Delta t_i) - \ln p_{\Delta t}(\Delta t_i \mid m_i) - \ln P(m_i)] \tag{3.13}$$

The above equation can be solved with dynamic programming (see for example (Cormen, Leiserson, Rivest and Stein, 2001)), which is an efficient way to find the shortest path through a lattice.

Our set of spike train models also includes a silence model m_{sil}. This model is fixed. It has a spike count distribution

$$P_n(n_c) = \begin{cases} 1 & \text{if } n_c = 0 \\ 0 & \text{otherwise} \end{cases}$$

for every channel. The time duration of the model is set as a uniform distribution $p(\Delta t \mid m = m_{sil}) = U(140ms, 1s)$. A silence period shorter than 140ms is probably part of a word or a transition between words. It can therefore not be labelled “silence”. We select $1s$ as the upper bound as there is no silence longer than that in our data set. $p(\Delta t \mid m = m_{sil})$ could also be trained instead of being preset but it would still be necessary to limit the shortest duration that a silence can be.

Table 3.1: A summary of the probability density functions (pdf) used in spike train classification. There are three spike train models for each word and there is one silence model.

Pdf	Type	No. of components
$p(\zeta \mid m)$	Spike train model	34
For each channel in a spike train model we have:		
P_n	Discrete GMM	2
$f_t(t)$	Continuous GMM	3
p_a	Continuous GMM	2
Associated with each spike train model there is:		
$p_{\Delta t}$	Continuous GMM	2

A set of three models is assigned to correspond to each word in the dictionary before training starts. We use more than one model per word because the spike trains for a given word may not all be similar. It is not easy to determine the optimal number of models per word class to use. Table 3.1 gives a summary of the probability density functions we use.

3.2.4 AN EXAMPLE OF SPIKE TRAIN CLASSIFICATION

In order to illustrate spike train segmentation and classification, we use the three models given in figure 3.2 as well as a silence model. The illustration is simplified by not taking the amplitude of spikes into account. Also for all three models $p(\Delta t \mid m)$ are set as uniform distributions, equal for all models; the prior probability of selecting the silence model and the prior probabilities of each of three other models are equal, so that $p(m) = 0.25$ with $m \in [1, 2, 3, m_{sil}]$.

Figure 3.3(a) shows a generated spike train that has to be classified. It is generated by sampling two spike trains from model $m = 1$, one spike train from $m = 2$ and one from $m = 3$. The spike trains are then concatenated in the order $m = [1, 3, 2, 1]$. The spike train is correctly classified; a period of silence is classified at the start of the spike train.

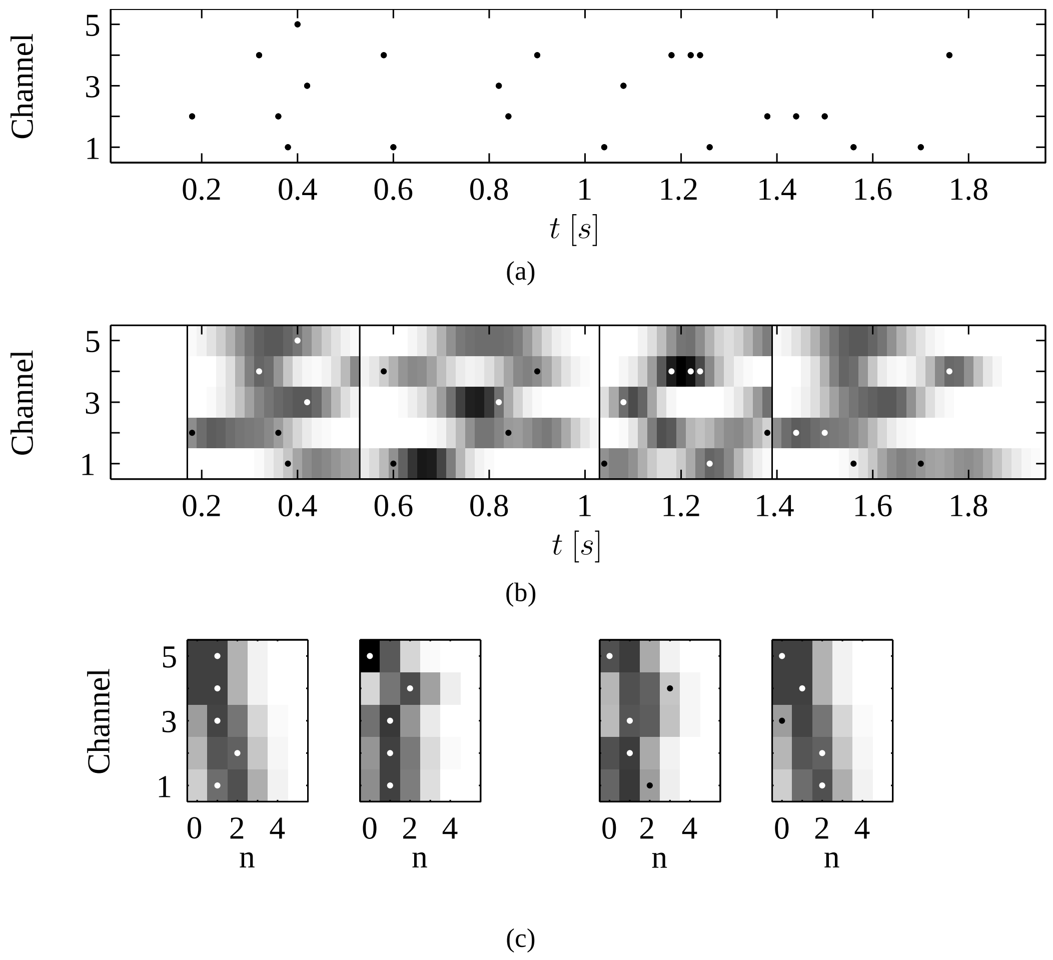


Figure 3.3: (a) shows a spike train that has to be segmented and classified. The spike train is correctly classified as $\overline{m} = [1, 3, 2, 1]$. The first part ($t = [1, 8]$) of the spike train is classified as "silence". (b) shows the segmentation as well as the normalized spike density function of each selected model. (c) gives the spike count distribution of each selected model.

3.3 TRAINING

In the training process the parameters of the spike train models are adapted to fit the data. We use expectation maximization (EM) to train the model. Firstly the expectation step finds the most likely sequence of models $\overline{m}^*$ and segment sizes $\overline{\Delta t}^*$ for the entire data set. The previous section on the classification of spike trains addressed exactly this issue. As the utterances are independent of each other, $\overline{m}_n^*$ and $\overline{\Delta t}_n^*$ can be determined for the nth utterance independently of all the other utterances in the data set.

The maximization step then adapts the model parameters to increase the likelihood of $\overline{m}^*$ and $\overline{\Delta t}^*$, or it maximizes the likelihood function $\mathcal{L}(\overline{\theta})$ with

$$\mathcal{L}(\overline{\theta}) = \prod_n p(\overline{m}_n^*, \overline{\Delta t}_n^* \mid \boldsymbol{\zeta}_n, \overline{\theta}) \tag{3.14}$$

where the subscript n refers to the nth utterance in the data set and $\overline{\theta}$ refers to the model parameters.

The problem is more readily solved by minimizing the negative log-likelihood function

$$\overline{\theta}^* = \min_{\overline{\theta}} - \sum_n \ln p(\overline{m}_n^*, \overline{\Delta t}_n^* \mid \boldsymbol{\zeta}_n, \overline{\theta}) \tag{3.15}$$

This can efficiently be done by taking the derivative of the negative log-likelihood function and finding the parameters for which the derivative is equal to zero. These derivatives are available in most text books that discuss GMMs.

The expectation step and the maximization step are repeated one after the other until the model parameters have converged.

3.3.1 UNSUPERVISED TRAINING

The data sets that are used for the training of speech recognition systems may be labelled. Labeled data sets are usually expensive to gather because each utterance has to be labelled. Unlabeled data sets on the other hand can very easily be gathered by recording speech from radio, television, phone conversations etc.

When models are trained on unlabeled data or when the label information is not used, the training procedure is unsupervised. Spike train models can be trained in an unsupervised fashion. There is one conceptual difficulty when using unsupervised training. Each EM iteration changes the data set because it changes $\overline{\Delta t}^*$. If EM converges to the global minimum, the fact that the data set changes each iteration would not influence the final solution. However, EM only finds a local minimum that is dependent on the data set, and the changing data set means that the minimum may not be a good enough solution to get the desired performance from the model.

We use supervised training so that we circumvent the problems of training with EM, and it also allows us to force spike train models to correspond to words. However unsupervised training of spike train models has one big advantage over supervised training: it would show which speech units are statistically significant. It would be an important result if the speech units that spike trains code compare well with speech units measured in the auditory cortex. A spike train classifier trained in an unsupervised manner may perform better than one trained in a supervised manner as it is free to choose its the speech units.

3.3.2 SUPERVISED TRAINING

A set of three models is assigned to correspond to each word in the dictionary before training starts. We use more than one model per word because the spike trains for a given word may not all be similar – it may be that the spike trains of a certain word class are grouped into several clusters of similar spike trains. If this is the case, then the classification performance will depend on the number of clusters that exist, and also whether these clusters are discovered during training. It is not easy to determine the optimal number of models per word class to use.

The expectation step uses a modified version of the standard dynamic programming algorithm used during the Viterbi search (Ostendorf, Digilakis and Kimball, 1996). The modification is that our Viterbi lattice is not two dimensional; it has a third dimension, called "number of words". A three dimensional Viterbi lattice is set up for each spike train. The three dimensions are "time", "model number" and the "number of words" selected from start of the sequence (see figure 3.4). Paths through the lattice always point in the increasing "time" dimension. When the transition is toward a word model, it has to increase one level along the "number of words" dimension. On the other hand, if the transition is toward the silence model it does not change its "number of words" dimension.

The supervised algorithm makes some of the vertices and transitions invalid in a manner consistent with the target word sequence and the target word boundary times. Figure 3.5 shows how the number of words that should be classified at specific times is bounded. From the bounds the valid vertices in the lattice are determined. At a specific time nodes along

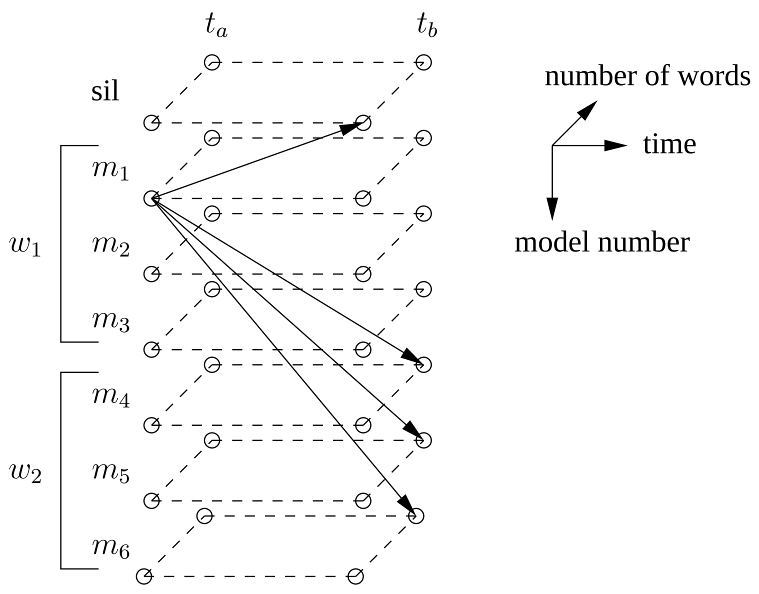


Figure 3.4: From a vertex there are only two types of valid transitions to a given time instant. There is the transition to the silence model, for which the number of words since the start of the sequence do not increase; and there is the transition to all the models in the set that correspond to the next word in the sequence.

the “number of words” dimension that fall outside the bounds are invalid. The target word sequence on the other hand determines which transitions are valid: transitions from models associated with word w_i to models associated with the following word w_{i+1} are valid, as are all transition to the silence model m_{sil}.

3.3.3 INITIALIZATION OF THE SPIKE TRAIN MODELS

The likelihood function has many local minima and EM finds one of them. The starting point for EM has a great influence on the quality of the solution. The solution that EM finds is much better when the starting point is in a region of a good solution, than when a random starting point is used.

The very first iteration of EM uses the starting point $\bar{\theta}_{start}$ to determine the most likely model sequence $\overline{m}^*$ and segment sizes $\overline{\Delta t}^*$. It then adapts the model parameters to better fit $\overline{m}^*$ and $\overline{\Delta t}^*$. We do not know what a good starting point should be, so we skip the first step

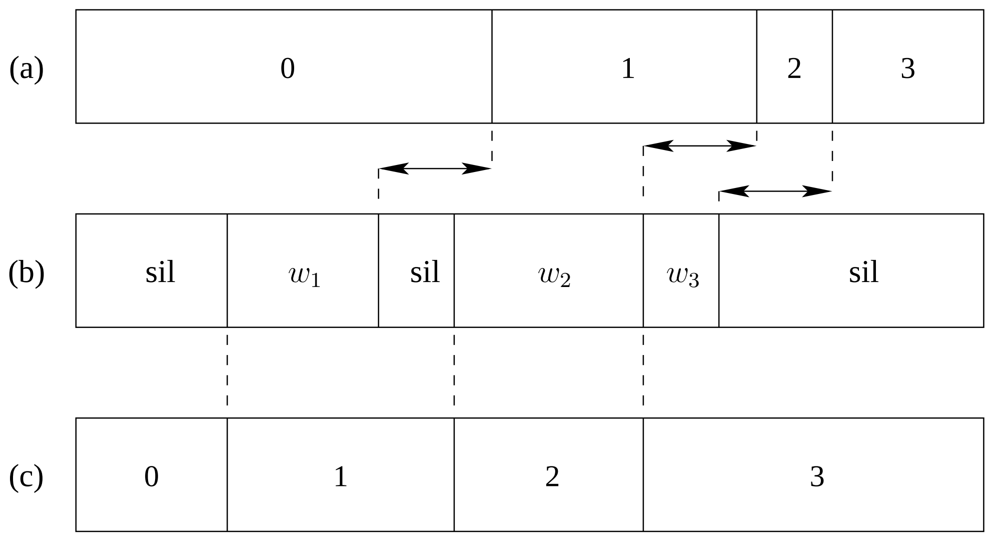


Figure 3.5: This figure shows how the number of words since the start of the sequence is bounded. (b) The target sequence of words. (a) The minimum number of words that has to be in the sequence at a specific time. This is determined by the word-end boundaries. A word has to occur before the time of the last spike that can reconstruct part of that word. This time cannot be later than the word-end boundary plus the temporal length of the longest dictionary element (the elements of $\Phi_{\{6\}}$ spans $380ms$). (c) The maximum number of words that can be in the sequence at a specific time is determined by the word-start boundary. A word cannot occur before its word-start boundary.

of the first iteration of EM and manually set $\overline{m}^*$ and $\overline{\Delta t}^*.\overline{\Delta t}^*$ is set according to the data set labels. $\overline{m}^*$ is set by dividing all the samples in a word class evenly between the models of that class. For example, the first occurrence of "three" in the data set is labelled as $m = 7$, the second occurrence as $m = 8$, the third as $m = 9$, the fourth again as $m = 7$ etc. This initialization works well, as all the models are forced to be in a region of good solutions.

3.4 IMPLEMENTATION DETAILS

There are two important details we implement for the classification of spike trains. Firstly we force the spike train models to correspond to words in order to keep the classification simple. Secondly we process the spike train before it is classified.

As mentioned earlier, the sparse code has many elements whose amplitudes are so small that they contribute little to the reconstruction of the signal. We remove them by setting their amplitudes equal to zero. Only those spikes that have amplitudes less than a preset threshold are removed. We choose the threshold at 0.124 so that 40% of the spikes remain. We tested the classification performance (see figure 3.6) against different threshold values and two values of λ. We use $\lambda = 0.3$ and choose the threshold as 0.124 so that only 40% of the spikes remain. The removal of small valued spikes could also be done during the process of dictionary training. We did not do that as we did not know beforehand which threshold would be best.

This step resembles *sparse code shrinkage* (SCS) (Hyvärinen, 1999b). SCS is a post processing step that applies to sparse codes calculated with ICA. ICA does not explicitly model noise, the code elements therefore also reconstruct the noise. Very small code values probably reconstruct noise but the shrinkage function removes these elements. A typical shrinkage function is

$$h(a) = \text{sign}(a) \max\left(0, |a| - \sqrt{2}\sigma^2\right) \tag{3.16}$$

where σ^2 is the variance of noise. Other shrinkage function can be derived based on assumptions about the noise and the prior probability $p(a)$. SCS is a rigorous method to denoise an ICA code.

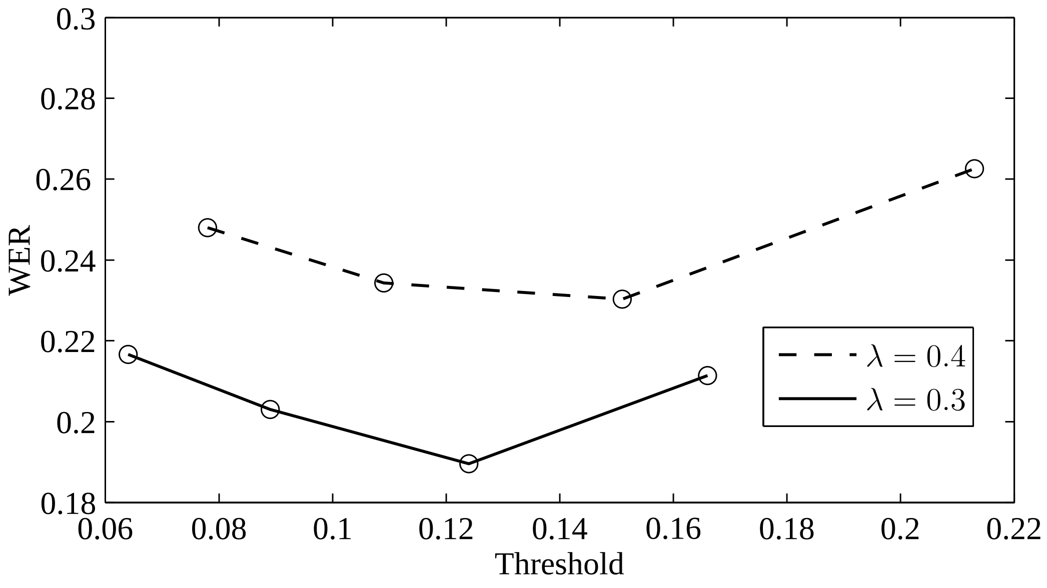


Figure 3.6: The figure shows the classification performance (Word Error Rate) of the system as a function of the threshold. Spikes with an amplitude less than the threshold are removed from the code. Each circle on a line corresponds to certain percentage of spikes that remain in the code. From the left most circle to the right the percentages are for each line: 60%, 50%, 40% and 30% respectively. The two lines are for different values of λ.

The sparse coding framework we use model noise explicitly, it is contained in $\lambda = 2\sigma^2$. Although our post processing step and the shrinkage function of SCS appear to be similar, they serve a different purpose. Our post processing step is performed because the sparseness function is not ideal, whereas the shrinkage function removes noise.

The spike train includes information about the temporal scaling of dictionary elements. Our motivation for using scaled dictionary elements is that some sounds may be scaled without changing the meaning of the sound. The scaling information is therefore not that important to a spike train classifier of words; what are important are the sounds that appear in the signal and the order in which they appear. We keep the spike train classifier simple by not including scaling information. It is still useful to have scaled versions of the dictionary elements, as this produces a sparse code that allows us to identify a particular sound irrespective of its duration.

The scaling information is removed by grouping spikes of the same sound together. The dictionary consists of all the scaled versions of the unscaled dictionary $\boldsymbol{\Phi} = \left[\boldsymbol{\Phi}_{\{0\}}, \boldsymbol{\Phi}_{\{1\}}, \boldsymbol{\Phi}_{\{2\}}, \ldots, \boldsymbol{\Phi}_{\{6\}}\right]$. We can split the code $\boldsymbol{a}$ in the same manner; $\boldsymbol{a} = \left[\boldsymbol{a}_{\{0\}}, \boldsymbol{a}_{\{1\}}, \boldsymbol{a}_{\{2\}}, \ldots, \boldsymbol{a}_{\{6\}}\right]$ where $\boldsymbol{a}_{\{s\}}$ refers to the sparse code associated with $\boldsymbol{\Phi}_{\{s\}}$. The compressed code is then

$$\hat{\boldsymbol{a}} = \sum_{s=0}^{6} \boldsymbol{a}_{\{s\}} \tag{3.17}$$

3.5 RESULTS RELATED TO SPIKE TRAIN CLASSIFICATION

In the EM training process the spike train models converged to a stable solution in fewer than ten iterations. The performance on the test set (1000 utterances) is a word error rate (WER) of 19%. Out of the 3222 words in the test set, there are 65 deletions, 178 insertions and 368 replacements. Table 3.2 shows the confusion matrix of the replacements. The majority of confusions are predictable from phonetic similarities – for example, “one” and “nine” are often confused because of their common terminal nasals and confusible initial phonemes.

Figure 3.7 shows the components p_t, p_a and P_n for the three models that code the word “six” (they are $m = 16$, $m = 17$ and $m = 18$). From the plot we see that $P_n(0)$ for

Table 3.2: The confusion matrix of classification of the test set. The diagonal entries are words that are correctly classified; the off-diagonal entries are words that have been replaced by incorrect words during classification.

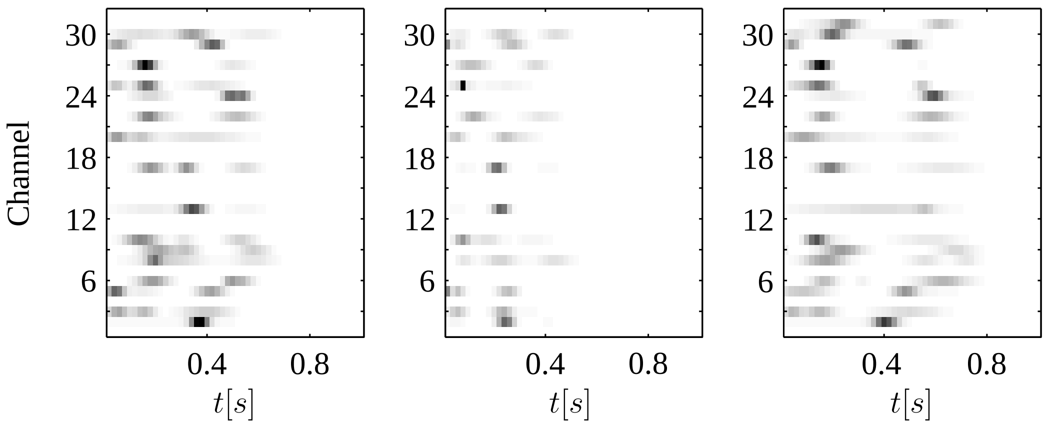
		True class										
		one	two	three	four	five	six	seven	eight	nine	zero	oh
Recognized class	one	240	2	2	14	4	0	0	9	12	1	8
	two	0	266	5	2	0	1	5	7	3	3	1
	three	2	9	283	1	0	0	0	13	0	1	0
	four	7	4	1	260	5	1	0	0	0	0	3
	five	2	3	2	11	230	1	3	1	8	2	19
	six	1	3	1	0	1	288	2	0	0	0	0
	seven	0	10	2	2	1	8	259	1	0	12	1
	eight	4	5	9	0	3	3	0	251	5	1	2
	nine	19	2	5	2	4	0	0	7	211	4	14
	zero	0	5	2	1	1	0	1	0	1	280	1
	oh	4	15	1	7	8	0	0	2	6	1	221

channel 2 of every model is very small. This shows that dictionary element $d = 2$ is used often in the reconstruction of "six". $m = 16$ has a very similar p_t to $m = 18$. There is an observable difference between the P_n of these two models, but the difference is not significant. It appears that the EM-algorithm was trapped in a local minimum where these two models are close together.

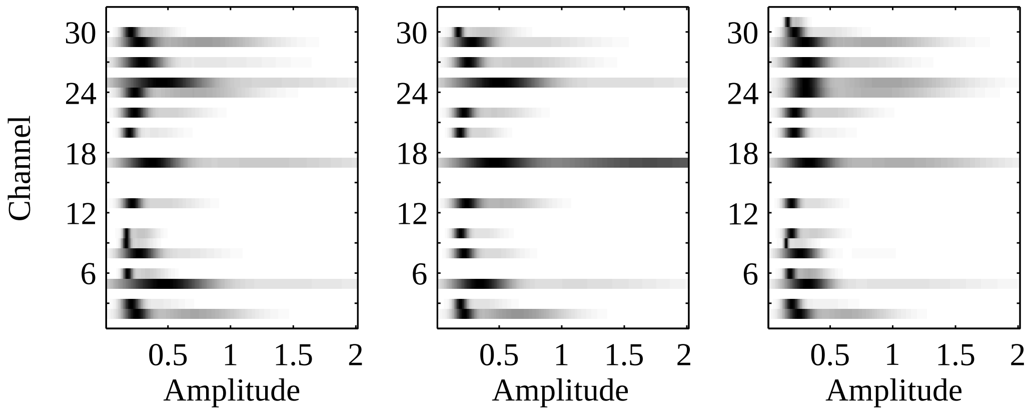
Figure 3.8 shows how often each model is used ($P(m)$) as well as the lengths of spike trains that are coded by each model ($p_{\Delta t}(\Delta t \mid m)$). We see from $p(\Delta t \mid 16)$ and $p(\Delta t \mid 18)$ that the models will mostly be selected when the time span Δt is rather long. The time span is in fact much longer than it takes to say the word "six". Inspection of the results reveal that this is because these models are mostly selected when "six" is the last word in an utterance, or when "six" is followed by a period of silence. If the models had been better trained, $p_{\Delta t}(\Delta t \mid m)$ would have had a higher probability that the lengths correspond to the time it actually takes to say "six". The expectation step in the EM algorithm would then use the silence model m_{sil} to take up the periods of silence following "six". This suggests that the EM algorithm became stuck in a non-optimal local extremum. We see from $P(16)$ and $P(18)$ in figure 3.8 that these models are not used often, which agrees with the fact that they are mostly used at the end of utterances or when long periods of silence follows "six".

A comparison between the performance of this sparse coding system and other systems for continuous speech recognition gives an indication of the effectiveness of the sparse coding approach. We compared the performance of the current system to that of Hidden Markov Model (HMM)-based speech recognition on the same spectrogram representations we use. HMMs are typically used with features such as Mel-frequency cepstral coefficients, which are approximately independent. Employing HMMs with features directly extracted from spectrograms requires some care, since the features will tend to be correlated to a significant degree. We were able to deal with this issue by using components with full covariance matrices. Recognition accuracy was not affected to a significant degree when we used more than one mixture component. The comparison could also be made with methods such as Dynamic Time Warping with full covariance matrices, but the complexities of developing such a system for speaker-independent recognition were not considered justified for our rather preliminary comparison.

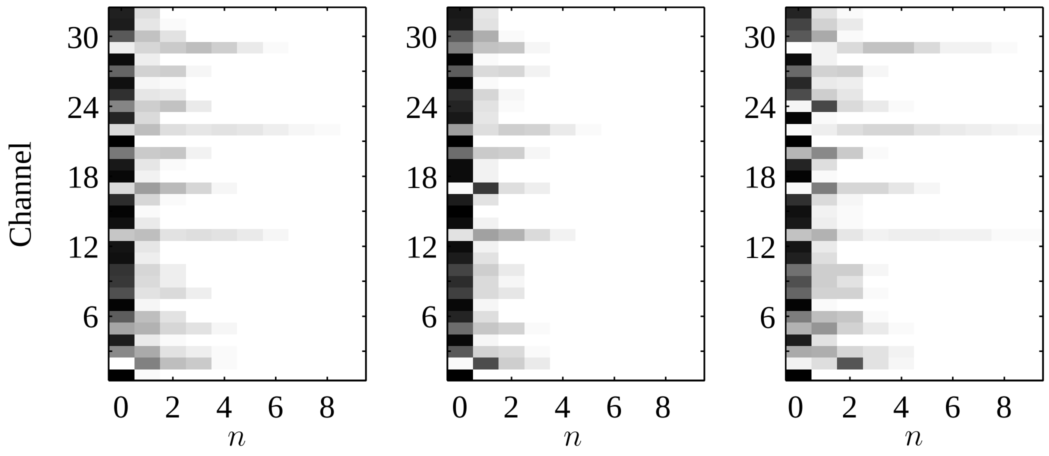
The HMM system achieved a WER of 15% for models based on monophones. The models were single mixture components with full covariance matrices. Straightforward HMM-based speech recognition with triphones achieves a 3% WER on the TIDIGITS dataset when Mel Frequency Cepstral Coefficients (MFCCs) are used instead of the coarse spectrograms we use. The WER of 15% for the HMM system is slightly better than the 19% WER that sparse coding achieves. We return to these performance differences in the next chapter.



(a) p_t



(b) p_a



(c) P_n

Figure 3.7: The plots shows the components of the three models that are associated with the word ‘six’. From the left column to the right the models are $m = 16$, $m = 17$ and $m = 18$ respectively. Plots (a), (b) and (c) show p_t, p_a and P_n for each respective model. For (a) and (b) we show only those channels that are highly active, i.e. only the channels for which $P_n(0) \leq 0.2$

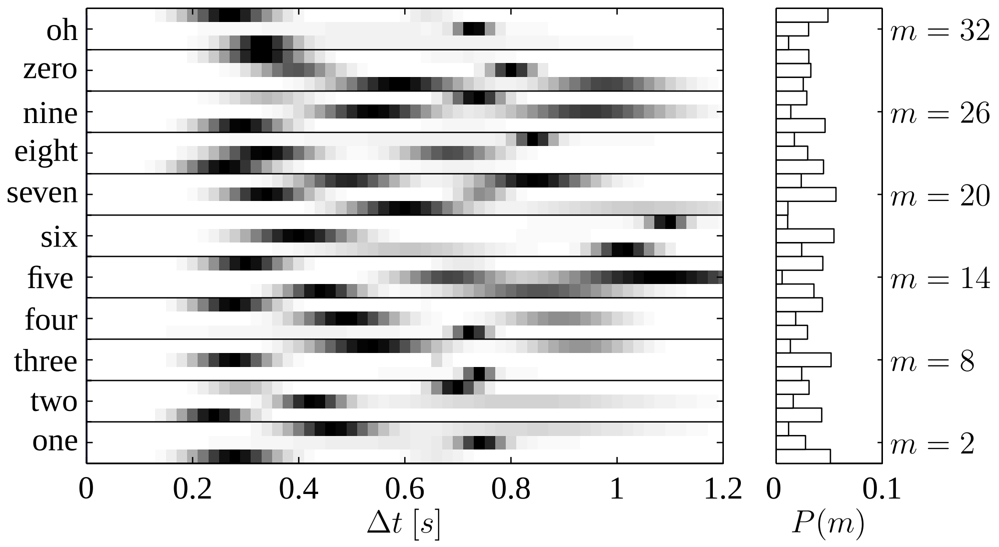


Figure 3.8: The *left* figure gives $p_{\Delta t}(\Delta t \mid m)$; the *right* figure gives $P(m)$. There are 33 rows in $p_{\Delta t}(\Delta t \mid m)$ and in $P(m)$, each row corresponds to a model. For example $p_{\Delta t}(\Delta t \mid 1)$ and $P(1)$ appear in the bottom row of each plot while models $m = 7$ to $m = 9$ are associated with the word "three".

CHAPTER FOUR

DISCUSSION

We have shown how sparse coding can be used for speech recognition. The steps to classify an unknown speech waveform are as follows:

1. The raw speech waveform is transformed in a frequency domain representation,
2. A sparse code that represents the signal is found by minimizing a cost function. The cost function consists of two terms: a reconstruction error and a sparseness function. The code that gives a low reconstruction error but that is also sparse is the code we select to represent a signal.
3. The sparse code is classified by searching for the most likely sequence of spike train models that fit the sparse code. Each spike train model is associated with a word from the dataset, thereby giving us the classification of the sparse code.

We found that for a relative simple signal representation and simple dataset, the system has a WER of 19%. This compares well with the performance of a HMM-based system which achieves a WER of 15% on the same dataset using the same coarse signal representation.

4.1 SPARSE CODING

4.1.1 THE DICTIONARY

Neuroscientists would like to understand the neural code better. Research are being conducted on several fronts, sparse coding being one of them. Thus far sparse coding is successful at explaining the receptive fields of some neurons. Dictionaries for image patches compare well with the receptive fields of simple cells in the visual cortex. It would be very interesting to see whether dictionaries for spectrograms are also related to properties of cells in the auditory pathway. However the dataset should then be natural sounds and not spoken digits.

The trained dictionary has some elements that are on syllable level. This is encouraging as the basic unit of speech is also believed to be on that level. It shows that there are at least some agreement between the dictionary and the receptive fields of neurons in the auditory pathway. It is necessary to investigate whether this property holds for more complex datasets. The digit dataset we use has very short words, most of which are single syllable words. We cannot conclusively say that sparse coding with a LGM yields a dictionary on the syllable level.

Sparse coding with a LGM can only explain the receptive fields of a quarter of cells in V1. The remaining cells have more complex receptive fields. The activity of complex cells is also sparse, but the LGM is not complex enough to model these cells. We showed in figure 2.17 that the activity of code elements is not independent as we would like them to be. This demonstrates that the LGM is limited in the order of statistical structures it can capture. A more complex model could produce receptive fields of complex cells and thereby giving a more powerful sparse code.

The size of the dictionary plays an important role in the properties of the dictionary elements and of the code. The code will be maximally sparse if the dictionary has as many elements as there are basic features in the dataset. Normally we do not know how many features there are in a dataset, this makes it difficult to determine the optimal dictionary size. If the dictionary size is increased while λ remains constant, the dictionary elements will code more complex features. For a dictionary that has too many elements, the features will not be general enough to capture the underlying structure in the signal. There will be very few samples where each dictionary element is used, which in turn will reduce the classification performance of the system.

We did not study the effect the size of the dictionary has on the type of features that the dictionary elements code or on the classification performance. Here more work needs to be done.

The dictionary training is stopped prematurely because the training takes a long time. There is a small improvement in the error function from one iteration to the next when it is stopped. Further training will reduce the error function more which should improve the classification performance. This too needs to be investigated.

The elements are linearly scaled to account for the fact that some sounds may be streched in time without changing the meaning of the sound. Phonemes however do not scale linearly. The classification performance may improve if the scaling mechanism better resembles the nonlinear scaling of actual phonemes. A more complex scaling mechanism, such as dynamic time warping, would increase the computational cost of finding a sparse code even more. It is necessary to study the effect of some nonlinear scaling mechanisms.

4.1.2 THE BALANCE BETWEEN RECONSTRUCTION ERROR AND SPARSENESS

We need a method to tell us how big a dictionary should be given the dataset and the value of λ. The number of phonemes and syllables that occur in a dataset should play an important role in determining the size of the dictionary. It is reasonable to expect that a dictionary which codes spoken digits will have to be smaller than a dictionary which codes conversational

English. The value of λ determines how well a signal will be reconstructed. If λ has a low value, fine detail in the signal is reconstructed. In this case the elements of a too small dictionary will be overused, that is they will have to be used in coding almost every signal, which will not allow them to capture the underlying structure.

The effect of λ on the performance also needs to be investigated. We found that for a 32 element dictionary, the performance decreases for $\lambda > 0.3$, but we did not check the performance for $\lambda < 0.3$. There should be a turning point for which a reduction in λ decrease the classification performance.

4.1.3 CONSTRAINING THE DICTIONARY NORMS

The fact that we chose to constrain the norms of dictionary elements plays an important role in the features that the dictionary code. Dictionary elements have to be constrained because in the problem formulation the integral in $p(\bar{x}) = \int p(\bar{x} \mid \bar{a})p(\bar{a})d\bar{a}$ (equation 2.15) is estimated by a delta function. When the integral can be more accurately evaluated the dictionary norms will not have to be constrained. The dictionary elements will then adapt in such a way that the code elements follow the prescribed prior probability $p(\bar{a})$. We have a good idea of what the prior probability should be for a true neural code where the code elements are binary. It can be estimated from the average firing rate of the neuron. However it is not clear what the prior probability for real valued code elements should be. It may even be different for different neurons.

The problem of constraining the norms of dictionary elements is not unique to gradient based algorithms. Codes that are determined with basis selection methods also suffer from this problem. MP has to have some mechanism that controls the norms of dictionary elements so that they stay competitive (Perrinet, 2004b). x

4.1.4 FINDING A SPARSE CODE

The problem of finding a sparse code is not yet satisfactorily solved. We use an iterative optimization approach that is computationally expensive. Although this algorithm is substantially faster than other gradient based algorithms, there is still a need for an even faster algorithm. In order to find sparse codes in reasonable time, we use a sparseness function that deviates significantly from the ideal sparseness function. Unfortunately with our sparseness function the sparse code has many spikes with very small amplitudes. We found that by removing 60% of the spikes the performance increases significantly. The spikes that are finally removed put unnecessary strain on the algorithm while it searches for a sparse code. These spike are included in every iteration even though they do not contribute to the solution. The algorithm will execute much faster if there are 60% less spikes in the code.

It will be better if the sparseness function has a form that not only includes the code element amplitudes but also the activity of code elements. This sparseness function will produce a code that does not require the removal of many spikes to give good performance. Such sparseness functions exist, but they cause the optimization problem to become non-quadratic and computationally much more expensive. Future work should focus on how to solve sparse codes efficiently, especially sparse codes of sparseness functions that include the activity of code elements.

We used a final optimization step for MP which we termed MP+. This step simply changes the values of the nonzero code elements to minimize the reconstruction error. It is a step that can reduce the reconstruction error of any code without increasing its l_0 norm. The classification performance of the system may improve if this step is also performed on the SSQP codes. We did not do any investigations along this line.

Basis selection algorithms such as MP are generally computationally cheap. They also use the l_0 sparseness function which better fits the neural code than the sparseness function used by gradient based algorithms. Be that as it may we found that MP does not give robust codes. A small change in the signal leads to a big change in the code. Other basis selection algorithms should be evaluated based on the requirement of robust codes and computational efficiency.

The quadratic sparseness function S_{quad} is a much better function to use than the simpler l_1 norm. S_{quad} codes have fewer nonzero code entries than l_1 norm codes for a given reconstruction error.

4.2 SPIKE TRAIN CLASSIFICATION

4.2.1 THE SPIKE TRAIN MODEL

The spike train models are able to decode a spike train successfully. We have forced the spike train models to correspond to words, while most modern HMM-based speech recognition systems model monophones or triphones. When it comes to discovering the basic units of speech, unsupervised training of the spike train models can be very powerful. With unsupervised training it is possible to discover those units of speech that are statistically significant. Hopefully they will correlate with the basic units that the brain uses. Another advantage of unsupervised training is that the number of models per class does not have to be specified; only the number of spike train models. At some stage it is necessary to use supervised training of the recognition part of the system. It may merely be to determine the sounds that each model codes.

The spike train model actually finds higher-order correlations among code elements. It uses these correlations to decode the spike train. A coding model that is more complex than the LGM may be able to model these higher-order correlations. If this is the case, then the spike train becomes very simple to decode; the activity of certain neurons may signal when a certain word is present in the utterance. A spike train can then still be decoded with the spike train model, but the model will be much simpler, and the result may be more accurate.

The classification of spike trains is done on trains that do not include the scaling of dictionary elements in any way. We showed earlier that the performance could improve when scaling information is used. This can be done by simply adding another dimension to a spike train, so that a given spike is completely described by its scale s, firing time t, amplitude a and channel c.

4.2.2 EXPECTATION MAXIMIZATION

The EM algorithm finds a solution for the spike train models that is a local minimum. For example the results show that two of the three models that correspond to the word “six” are very similar and both have unreasonable long time spans. These models could be used better, but the EM algorithm is stuck. The classification performance can potentially increase if the EM algorithm is guided more in its search for a minimum. The guidance can be user input or some automatic process.

The problem that EM gets stuck in local minima is a property of the algorithm and not of the dataset. Any improvements of the EM algorithm will also be useful here.

4.3 GENERAL REMARKS

For the input features used here, the performance of speech recognition based on sparse coding is comparable to HMM-based speech recognition. These results are, however, not competitive with those achievable using state-of-the-art features; incorporating such features into our system would not have been feasible for computational reasons.

We did not check the performance of the system with noise maskers. There is good reason to believe that sparse coding will perform well on noisy signals, as sparse coding can also be used as a means to remove noise from a signal (Hyvarinen, 1999a; Shang, Huang, Zheng and Sun, 2006). In other words, sparse coding captures the underlying structure of the signal and thereby is not so susceptible to noise.

There are many areas of the sparse coding approach that need to be investigated in order to determine whether these methods can be competitive with current state-of-the-art systems. Most obviously, other feature sets such as spectrograms with a greater number of channels, or even MFCCs should be investigated. It should be noted that we do not require the MFCC coefficients to be sparse - as long as they are relatively stable in a particular acoustic context, our coding algorithm will extract a sparse set of events that describe the non-sparse MFCC coefficients. The training process however requires significantly more efficient training algorithms when the size of the input representation increases.

The spike train model of Oram *et al.* (1999) with order statistics (Wiener and Richmond, 2003) appears to be a useful model to use for spike train classification. The system's performance is probably capped by the sparse coding process and not by the spike train classifier.

It will be interesting to use the same approach for image recognition as we use here for speech recognition. We have a more intuitive understanding of vision than we have of speech. It will be easier to interpret the dictionary elements as well as the spike train models.

The results obtained in our investigation suggest that the further study of these issues will be a worthwhile endeavor.

4.4 SUMMARY

This thesis shows that sparse coding can be used to do continuous speech recognition. It highlights some factors that currently limit the use of sparse coding for pattern recognition. One such factor is the computational effort required to find a sparse code. We propose the SSQP algorithm which it is much faster than popular gradient based algorithms. The thesis further showed that useful spectro-temporal features can be extracted from speech when the LGM dictionary is trained. This is an important point as most other speech recognition systems use across-frequency features. We have also showed that the spike train model with order statistics can successfully decode spike trains of variable length. This initial investigation emphasizes the areas that need further study before sparse coding will be more widely used.

REFERENCES

Allen, J. B. (1994). How do humans process and recognize speech?, *IEEE Transactions on Speech and Audio Processing* **2**(4): 567–577.

Attias, H. (1999). Independent factor analysis, *Neural Computation* **11**(4): 803–851.

Aur, D., Connolly, C. I. and Jog, M. S. (2006). Computing information in neuronal spikes, *Neural Processing Letters* **23**: 183–199.

Bair, W. (1999). Spike timing in the mammalian visual system, *Current Opinion in Neurobiology* **9**: 447–453.

Beloozerova, I. N., Sirota, M. G. and Swadlow, H. A. (2003). Activity of different classes of neurons of the motor cortex during locomotion, *Journal of Neuroscience* **23**: 1087–1097.

Blumensath, T. and Davies, M. (2006). Sparse and shift-invariant representation of music, *IEEE Transactions on Audio, Speech, and Language Processing* **14**(1): 50–57.

Bounkong, S., Toch, B., Saad, D. and Lowe, D. (2003). ICA for watermarking digital images, *Journal of Machine Learning Research* **4**: 1471–1498.

Byrd, R. H., Lu, P., Nocedal, J. and Zhu, C. (1995). A limited memory algorithm for bound constrained optimization, *SIAM Journal on Scientific and Statistical Computing* **16**(5): 1190–1208.

Cardoso, J. F. (1997). Infomax and maximum likelihood for blind source seperation, **4**: 109–111.

Chen, S. S., Donoho, D. L. and Saunders, M. A. (1998). Atomic decomposition by basis pursuit, *SIAM Journal on Scientific Computing* **20**: 33–61.

Chi, Z., Rauske, P. L. and Margoliash, D. (2003). Detection of spike patterns using pattern filtering, with applications to sleep replay, *Neurocomputing* **52–54**: 19–24.

Cho, Y. C. and Choi, S. (2005). Nonnegative features of spectro-temporal sounds for classification, *Pattern recognition letters* **26**: 1327–1336.

Cormen, T. H., Leiserson, C. E., Rivest, R. L. and Stein, C. (2001). *Introduction to Algorithms*, 2nd edn, MIT Press and McGraw-Hill, Cambridge, Mass.

Davis, G., Mallat, S. and Avellaneda, M. (1997). Adaptive greedy approximations, *Journal of Constructive Approximation* **13**: 57–98.

Dayan, P. and Abbott, L. F. (2001). *Theoretical Neuroscience: computational and mathematical modeling of neural systems*, MIT Press, Cambridge, Mass., chapter 1.5 - The Neural Code.

deCharms, R. C., Blake, D. T. and Merzenich, M. M. (1998). Optimizing sound features for cortical neurons, *Science* **280**: 1439–1444.

DeWeese, M., Wehr, M. and Zador, A. (2003). Binary spiking in auditory cortex, *Journal of Neuroscience* **23**: 7940–7949.

Eggemont, J. J. (1998). Is there a neural code?, *Neuroscience & Biobehavioral Reviews* **22**(2): 355–370.

Fan, R. E., Chen, P. H. and Lin, C. J. (2005). Working set selection using second order information for training support vector machines, *Journal of Machine Learning Research* **6**: 1889–1918.

Field, D. J. (1994). What is the goal of sensory coding?, *Neural Computation* **6**: 559–601.

Földiák, P. (1998). *The handbook of brain theory and neural networks*, 1st mit press paperback edn, MIT Press, Cambridge, Mass., chapter Sparse coding in the primate cortex, pp. 895–898.

Földiák, P. and Young, M. (1995). *The handbook of brain theory and neural networks*, MIT Press, Cambridge, Mass., chapter Sparse coding in the primate cortex, pp. 895–898.

Gat, I. and Tishby, N. (2001). Spotting neural spike patterns using an adversary background model, *Neural Computation* **13**: 2681–2708.

Girolami, M. (2001). A variational method for learning sparse and overcomplete representations, *Neural Computation* **13**: 2517–2532.

Hermansky, H. (1990). Perceptual linear predictive(PLP) analysis of speech, *Journal of the Acoustical Society of America* **87**: 1738–1752.

Hermansky, H. (1998). Should recognizers have ears?, *Speech Communication* **25**: 3–27.

Hermansky, H. and Morgan, N. (1994). Rasta processing of speech, *IEEE Transactions on Speech and Audio Processing* **2**(4): 578–589.

Hermansky, H. and Sharma, S. (1999). Temporal patterns (TRAPs) in ASR of noisy speech, *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, Phoenix, AZ.

Holmberg, M., Gelbart, D., Ramacher, U. and Hemmert, W. (2005). Automatic speech recognition with neural spike trains, *Interspeech 2005 - Eurospeech, 9th European Conference on Speech Communication and Technology*.

Huggins, A. W. F. (1975). Temporally segmented speech, *Perception and Psychophysics* **18**(2): 149–157.

Hyvarinen, A. (1999a). Sparse code shrinkage: Denoising of nongaussian data by maximum likelihood estimation, *Neural Computation* **11**(7): 1739–1768.

Hyvärinen, A. (1999b). Sparse code shrinkage: denoising of nonhaussion data by maximum likelihood estimation, *Neural computation* **11**: 1739–1768.

Hyvärinen, A. and Hoyer, P. O. (2001). A two-layer sparse coding model learns simple and complex cell receptive fields and topography from natural images, *Vision Research* **41**(18): 2413–2423.

Ikbal, S., Magimai.-Doss, M., Misra, H. and Bourlard, H. (2004). Spectro-temporal activity pattern (STAP) features for noise robust ASR, *Proceedings of the INTERSPEECH-International Conference on Spoken Language Processing*, Jeju Island, Korea.

Kass, R. E. and Ventura, V. (2001). A spike-train probability model, *Neural Computation* **13**: 1713–1720.

Kayser, C., Petkov, C. I., Augath, M. and Logothetis, N. K. (2007). Functional imaging reveals visual modulation of specific fields in auditory cortex, *Journal of Neuroscience* **27**(8): 1824–1835.

Klein, D. J., König, P. and Körding, K. P. (2003). Sparse spectrotemporal coding of souds, *EURASIP Journal on Applied Signal Processing* **7**: 659–667.

Kleinschmidt, M. (2002). Mehtods for capturing spectro-temporal modulations in automatic speech recognition, *Acustica* **88**: 416–422.

Kral, A. (2000). Temporal code and speech, *Acta Otolaryngol* **120**: 529–530.

Kreutz-Delgado, K., Murray, J. F., Rao, B. D., Engan, K., Lee, T. W. and Sejnowksi, T. J. (2003). Dictionary learning algorithms for sparse representation, *Neural Computation* **15**: 349–396.

Kwon, O. W. and Lee, T. W. (2004). Phoneme recognition using ICA-based feature extraction and transformation, *Signal Processing* **84**(6): 1005–1019.

Laurent, G. (2002). Olfactory network dynamics and the coding of multidimensional signals, *Nature Reviews Neuroscience* **3**: 884–895.

Lee, T. W., Lewicki, M. S., Girlami, M. and Sejnowski, T. J. (1999). Blind source seperation of more sources than mixtures using overcomplete representations, *IEEE Signal Processing Letters* **6**(4): 87–90.

Leonard, R. G. and Doddington, G. (1993). TIDIGITS, Philadelphia.

Lewicki, M. S. (2002). Efficient coding of natural sounds, *Nature Neuroscience* **5**(4): 356–363.

Lewicki, M. S. and Olshausen, B. A. (1999). Probabilistic framework for the adaptation and comparison of image codes, *Journal of the Optical Society of America* **16**(7): 1587–1601.

Lewicki, M. S. and Sejnowksi, T. J. (1999). Coding time-varying signals using sparse, shift-invariant representations, *in* M. S. Kearns, S. Solla and D. Cohn (eds), *Advances in neural information processing systems, 11*, MIT Press, Cambridge, Mass., London, pp. 730–736.

Lewicki, M. S. and Sejnowski, T. J. (2000). Learning overcomplete representations, *Neural Computation* **12**: 337–365.

Liao, S. P., Lin, H. T. and Lin, C. J. (2002). A note on the decomposition methods for support vector regression, *Neural Computation* **14**: 1267–1281.

Loiselle, S., Rouat, J., Pressnitzer, D. and Thorpe, S. (2005). Exploration of rank order coding with spiking neural networks for speech recognition, *International Joint Conference on Neural Networks*, Montreal, Canada.

Mallet, S. and Zhang, Z. (1993). Matching pursuits with time-frequency dictionaries, *IEEE Transactions on Signal Processing* **41**(12): 3397–3415.

Massaro, D. W. (1972). Preperceptual images, processing time and perceptual units in auditory perception, *Psychological Review* **79**(2): 124–145.

Mercier, D. and Séguier, R. (2002). Spiking neurons (stanns) in speech recognition, *3rd WSES International Conference on Neural Networks and Applications*, Interlaken.

Moller, A. (1999). Review of the roles of temporal and place coding of frequency in speech discrimination, *Acta Otolaryngol* **119**: 424–430.

Näger, C., Storck, J. and Deco, G. (2002). Speech recognition with spiking neurons and dynamic synapses: a model motivated by the human auditory pathway, *Neurocomputing* **44–46**: 937–942.

Neumaier, A. MINQ - general definite and bound constrained indefinite quadratic programming [online]. 1998. Available from: `http://www.mat.univie.ac.at/~neum/software/minq/` [cited December 1, 2007].

Nguyen, N. and Hawkins, S. (2003). Temporal integration in the perception of speech: introduction, *Journal of Phonetics* pp. 279–287.

Olshausen, B. Sparsenet [online]. 1996. Available from: `http://redwood.berkeley.edu/bruno/sparsenet/` [cited November 26, 2007].

Olshausen, B. A. (2002). Sparse codes and spikes, *in* R. P. N. Rao, B. A. Olshausen and M. S. Lewicki (eds), *Probabilistic models of the brain: perception and neural function*, MIT Press, pp. 257–272.

Olshausen, B. A. and Field, D. J. (1996a). Emergence of simple-cell receptive-field properties by learning a sparse code for natural images, *Nature* **381**: 607–609.

Olshausen, B. A. and Field, D. J. (1996b). Wavelet-like receptive fields emerge from a network that learns sparse codes for natural images, *Nature* **381**: 607–609.

Olshausen, B. A. and Field, D. J. (1997). Sparse coding with an overcomplete basis set: A strategy employed by V1?, *Vision Research* **37**: 3311–3325.

Olshausen, B. A. and Field, D. J. (2004). Sparse coding of sensory inputs, *Current Opinion in Neurobiology* **14**: 481–487.

Oram, M. W., Wiener, M. C., Lestienne, R. and Richmond, B. J. (1999). Stochastic nature of precisely timed spike patterns in the visual system neuronal responses, *Journal of Neurophysiology* **81**: 3021–3033.

Ostendorf, M., Digilakis, V. V. and Kimball, O. A. (1996). From HMMs to segment models: A unified view of stochastic modeling for speech recognition, *IEEE Transactions on Speech and Audio Processing* **4**: 360–378.

Perrinet, L. (2004a). Feature detection using spikes: the greedy approach, *Journal of Physiology-Paris* **98**: 530–539.

Perrinet, L. (2004b). Finding independent components using spikes: a natural result of Hebbian learning in a sparse spike coding scheme, *Natural Computing* **3**(2): 159–175.

Quirago, R. Q., Reddy, L., Kreiman, G., Koch, C. and Fried, I. (2005). Invariant visual representation by single neurons in the human brain, *Nature* **435**(7045): 1102–1107.

Rabiner, L. and Juang, B. H. (1993). *Fundamentals of Speech Recognition*, Prentice Hall, Englewood Cliffs, N.J.

Rao, B. D., Engan, K., Cotter, S. F., Palmer, J. and Kreutz-Delgado, K. (2003). Subset selection in noise based on diversity measure minimization, *IEEE Transactions on Signal Processing* **51**(3): 760–770.

Rebollo-Neira, L. and Lowe, D. (2002). Optimised orthogonal matching pursuit, *IEEE Signal Processing Letters* **9**(4): 137–140.

Rieke, F., Warland, D., de Ruyter van Steveninck, R. and Bialek, W. (1996). *Spikes: Exploring the Neural Code*, MIT Press, Cambridge, Mass.

Rolls, E. T. and Treves, A. (1990). The relative advantages of sparse versus distributed encoding for associative neuronal networks in the brain, *Network: Computation in Neural Systems* **1**: 407–421.

Shamma, S. A. (2001). On the role of space and time in auditory processing, *Trends in Cognitive Sciences* **5**(8): 340–348.

Shang, L., Huang, D. E., Zheng, C. H. and Sun, Z. L. (2006). Noise removal using a novel non-negative sparse coding shrinkage technique, *Neurocomputing* **69**: 874–877.

Shannon, R. V., Zeng, F. G., Kamath, V., Wygonski, J. and Ekelid, M. (1995). Speech recognition with primarily temporal cues, *Science* **270**(5234): 303–304.

Sigman, M., Cecchi, G. A., Gilbert, C. D. and Magnasco, M. O. (2001). On a common circle: Natural scenes and gestalt rules, *PNAS* **98**(4): 1935–1940.

Smith, E. and Lewicki, M. S. (2005). Efficient coding of time-relative structure using spikes, *Neural Computation* **17**: 19–45.

Sun, J., Zhou, Q., Ma, C. and Wang, W. (2001). Sparse image coding with clustering property and its application to face recognition, *Pattern recognition* **34**(9): 1883–1884.

Theunissen, F. E. (2003). From synchrony to sparseness, *Trends in Neuroscience* **26**(2): 61–64.

Thorpe, S., Fize, D. and Marlot, C. (1996). Speed of processing in the human visual system, *Nature* **381**(6582): 520–522.

University of Cambridge. Hidden Markov Model Toolkit ver 3.4 [online]. 2006. Available from: `http://htk.eng.cam.ac.uk/` [cited March 17, 2008].

Verstraeten, D., Schrauwen, B., Stroobandt, D. and Campenhout, J. V. (2005). Isolated word recognition with the liquid state machine: a case study, *Information Processing Letters* **95**: 521–528.

Vinje, W. E. and Gallant, J. L. (2000). Sparse coding and decorrelation in primary visual cortex during natural vision, *Science* **287**: 1273–1276.

Vinje, W. E. and Gallant, J. L. (2002). Natural stimulation of the nonclassical receptive field increases information transmission efficiency in V1, *Journal of Neuroscience* **22**(7): 2904–2915.

Wang, K. and Shamma, S. A. (1995). Spectral shape analysis in the central auditory system, *IEEE Transactions on Speech and Audio Processing* **3**(5): 382–395.

Wiener, M. C. and Richmond, B. J. (2003). Decoding spike trains instant by instant using order statistics and the mixture-of-Poissons model, *Journal of Neuroscience* **23**(6): 2394–2406.

Willmore, B. and Tolhurst, D. J. (2001). Characterizing the sparseness of neural codes, *Network* **12**: 255–270.

Wipf, D. P. and Rao, B. D. (2006). l_0-norm minimization for basis selection, *Advances in Neural Information Processing Systems 18*, MIT Press, Cambridge, Mass., London.