Chapter 5

Single Parameter Equivalents of Radiometric Entities

5.1 Introduction

Radiometric entities such as radiance and atmospheric transmittance can be highly variable with wavelength. It is therefore not always accurate to replace such a spectrally variant entity with a single parameter that is required by OpenGL. The purpose of this chapter is to investigate the mapping of atmospheric transmittance, path radiance and sources with constant emissivity to single parameters that can be used in OpenGL.

The MODTRAN atmospheric transmittance model was used to generate atmospheric transmittance data at selected distances from 100m to 15km. The input parameters were selected to represent good, average and bad atmospheric transmittance conditions. The data was generated at a wavenumber increment of 2 (2 cm^{-1}). An example of atmospheric transmittance versus wavelength is shown in Figure 5.2.

5.2 Atmospheric transmittance

Effective atmospheric transmittance is determined by calculating the irradiance on the detector taking the atmospheric transmittance into account and dividing it with the irradiance calculated without the atmospheric transmittance, as shown in Equation (5.1). Equation (5.1) was adapted from Holst [21, p287].

$$\tau_{eff} = \frac{\int_{\lambda 1}^{\lambda 2} L(\lambda)\tau_{system}(\lambda)\tau_{atmosphere}(\lambda)d\lambda}{\int_{\lambda 1}^{\lambda 2} L(\lambda)\tau_{system}(\lambda)d\lambda}. \tag{5.1}$$

L_λ is the spectral radiance of the source, $\tau_{system}(\lambda)$ is the system's spectral response and $\tau_{atmosphere}(\lambda)$ is the spectral atmospheric transmittance.

The effective atmospheric transmittance as function of distance were determined for three different spectral filters, namely 3.9-4.1 μm, 3.9-5 μm and 3-5 μm. The effective transmittance and the position of the filter in the atmospheric window are shown in Figure 5.1 to Figure

5.6. The source was a blackbody at 920°C. The filter bandwidths were selected to represent transmittance in a single atmospheric window (3.9-4.1μm), transmittance in a window including a strong absorption band (3.9-5μm), and transmittance in a wide band that includes transmittance windows, an absorption band and line absorption bands (3-5μm).

The equivalent effective transmittance was determined by fitting a line to the log of the effective transmittance curve using:

$$\alpha = \frac{\sum_i x_i log(\tau_i)}{\sum_i x_i^2}. \tag{5.2}$$

The equivalent effective transmittance is a single parameter equation that can be used to calculate the atmospheric transmittance at a distance r by using

$$\tau = e^{\alpha r}, \tag{5.3}$$

with α the value determined in Equation (5.2). The technique was tested for the three filter bandwidths mentioned above. The results are shown in Figure 5.7 to Figure 5.12. The different figures are:

- Figure 5.7 shows a comparison between the effective transmittance that was calculated using Equation (5.1) and the single parameter equivalent that was calculated using Equation (5.2). Figure 5.8 shows the difference between the effective transmittance and the single parameter equivalent. The data was calculated for a 3.9-4.1 μm filter.
- Figure 5.9 shows a comparison between the effective transmittance that was calculated using Equation (5.1) and the single parameter equivalent that was calculated using Equation (5.2). Figure 5.10 shows the difference between the effective transmittance and the single parameter equivalent. The data was calculated for a 3.9-5 μm filter.
- Figure 5.11 shows a comparison between the effective transmittance that was calculated using Equation (5.1) and the single parameter equivalent that was calculated using Equation (5.2). Figure 5.12 shows the difference between the effective transmittance and the single parameter equivalent. The data was calculated for a 3-5 μm filter.

In Figure 5.7 to Figure 5.12 τ is used to indicate atmospheric transmittance. The atmospheric transmittance curves shown in Figures 5.2, 5.4 and 5.6 were calculated for a path length of 1km.

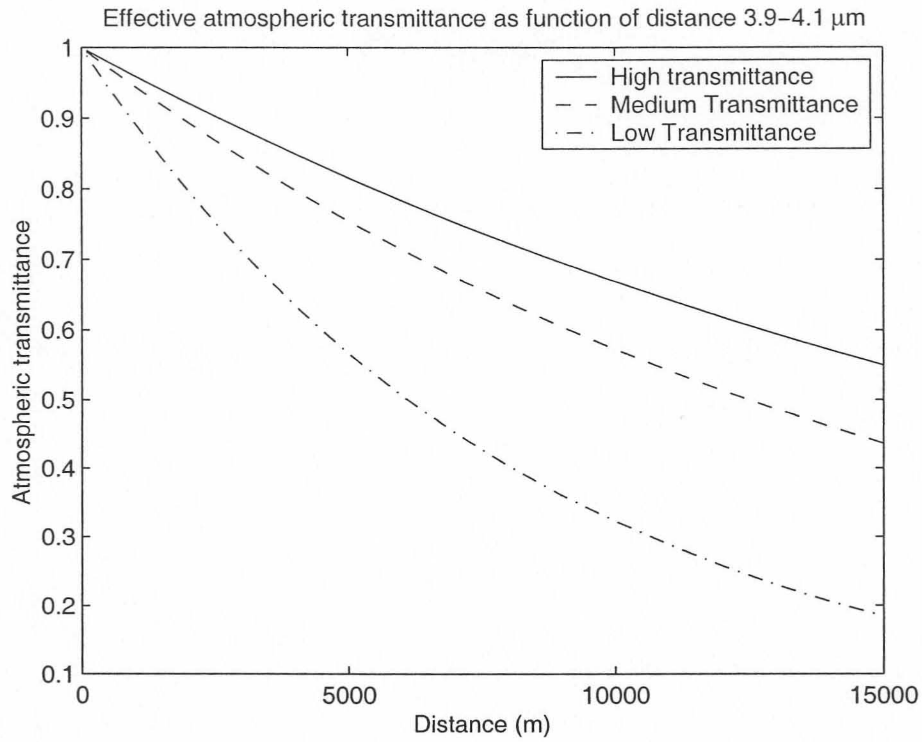

Figure 5.1: Effective atmospheric transmittance for a 3.9-4.1 μm system response

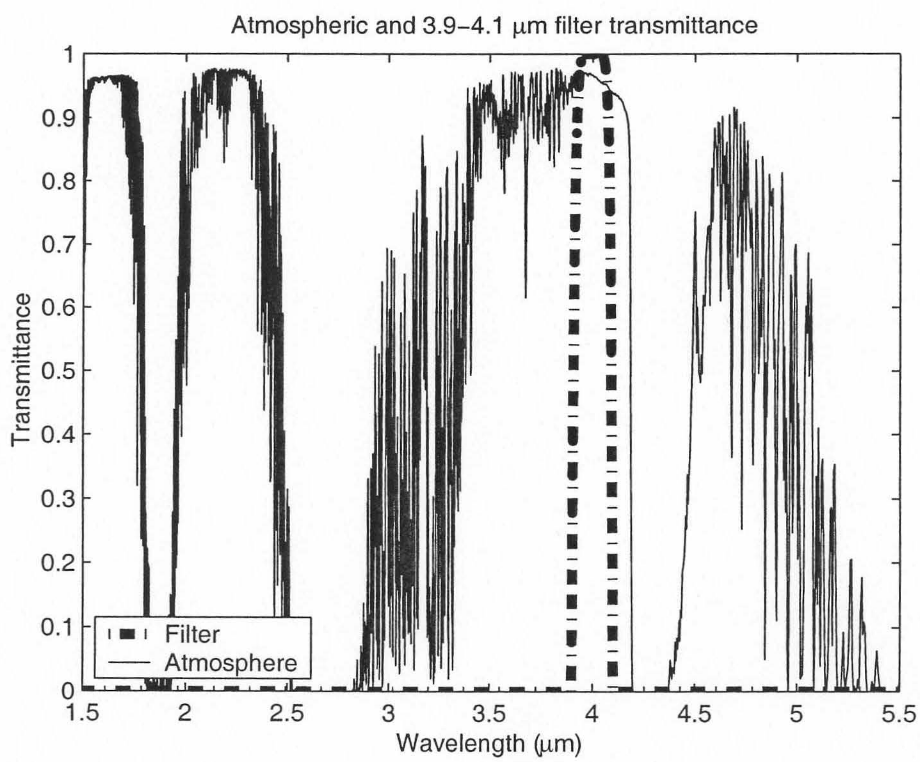

Figure 5.2: Position of the 3.9-4.1 μm bandpass filter

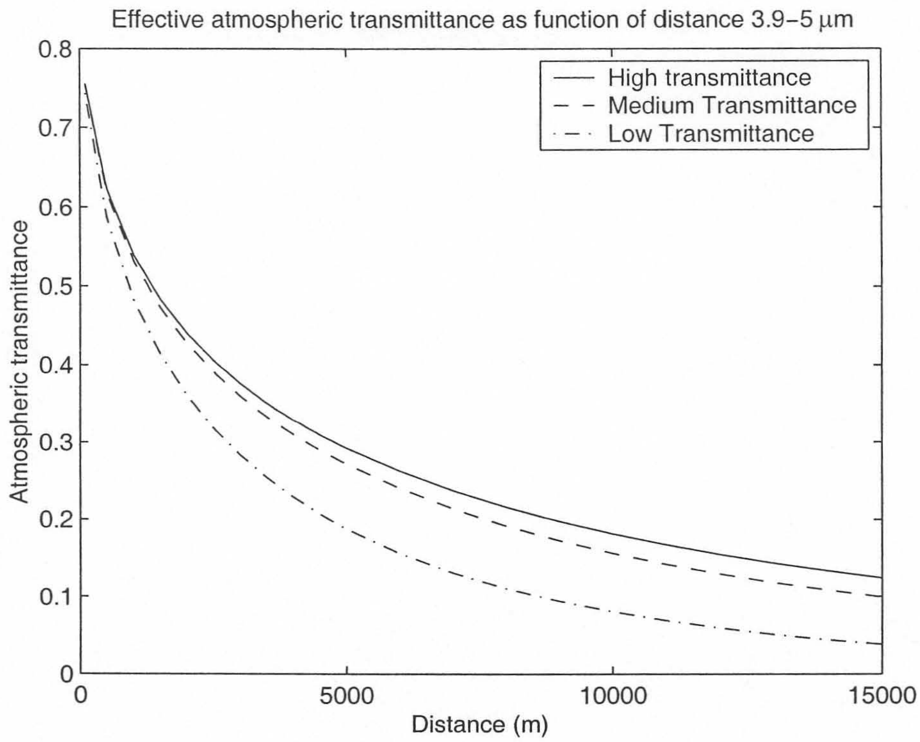

Figure 5.3: Effective atmospheric transmittance for a 3.9-5 μm system response

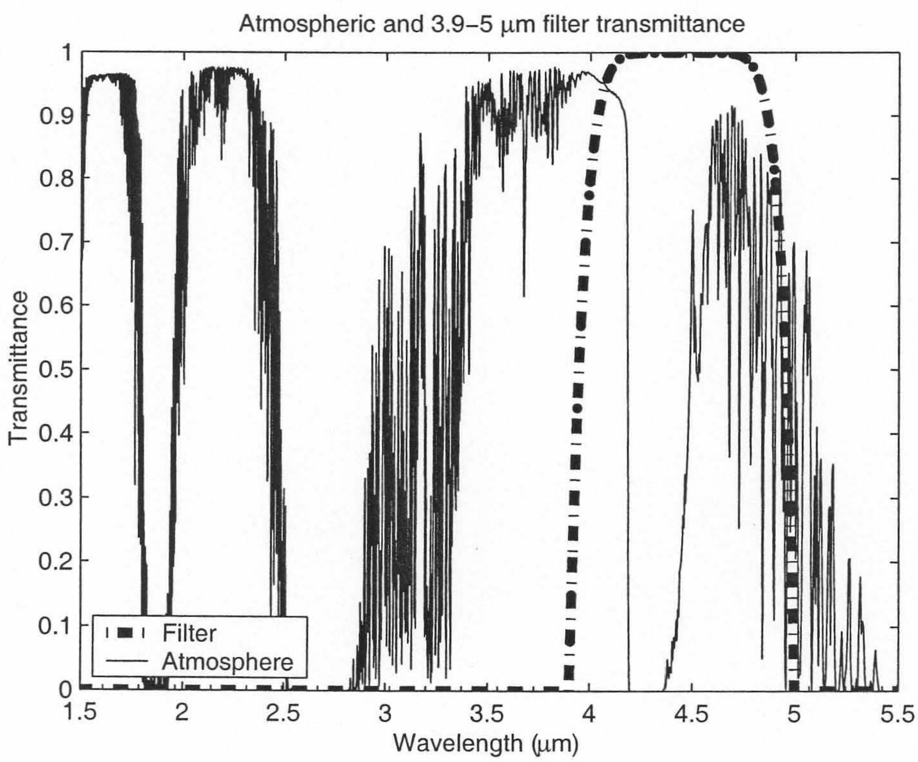

Figure 5.4: Position of the 3.9-5 μm bandpass filter

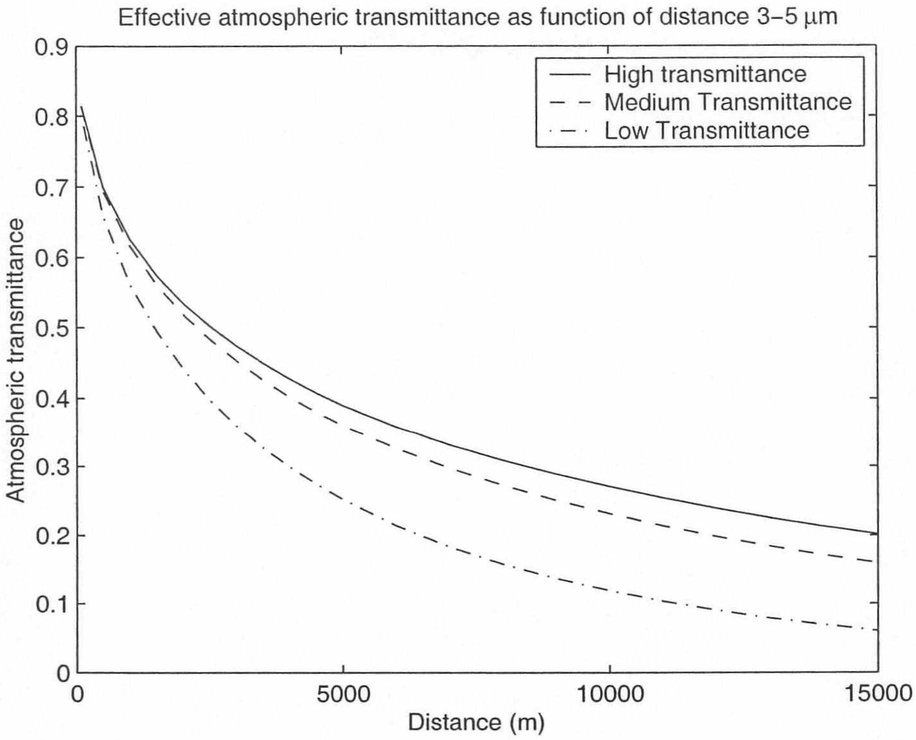

Figure 5.5: Effective atmospheric transmittance for a 3-5 μm system response

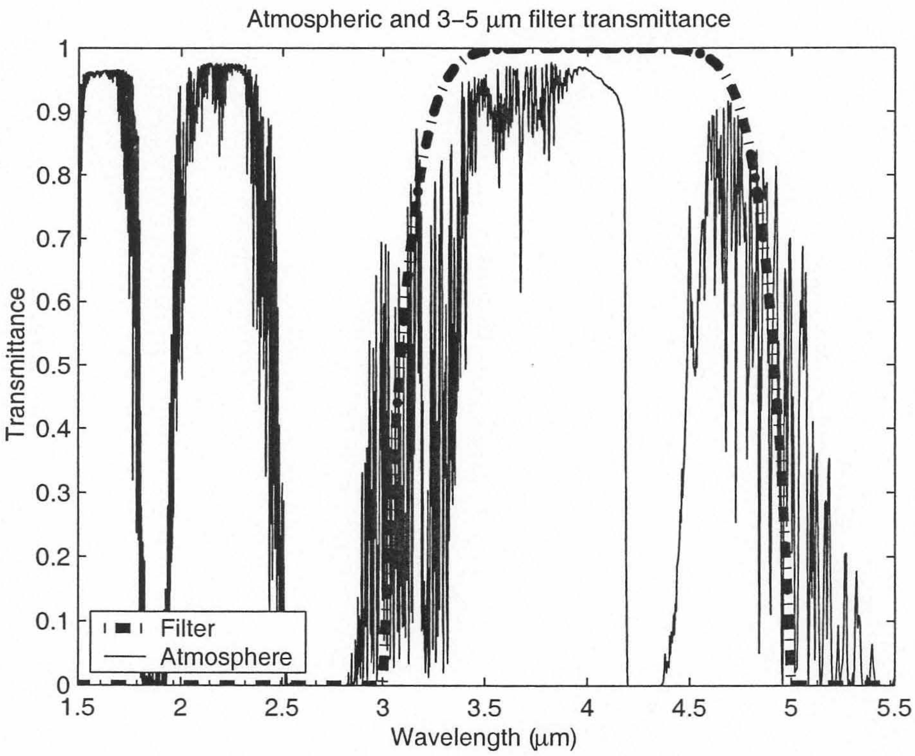

Figure 5.6: Position of the 3-5 μm bandpass filter

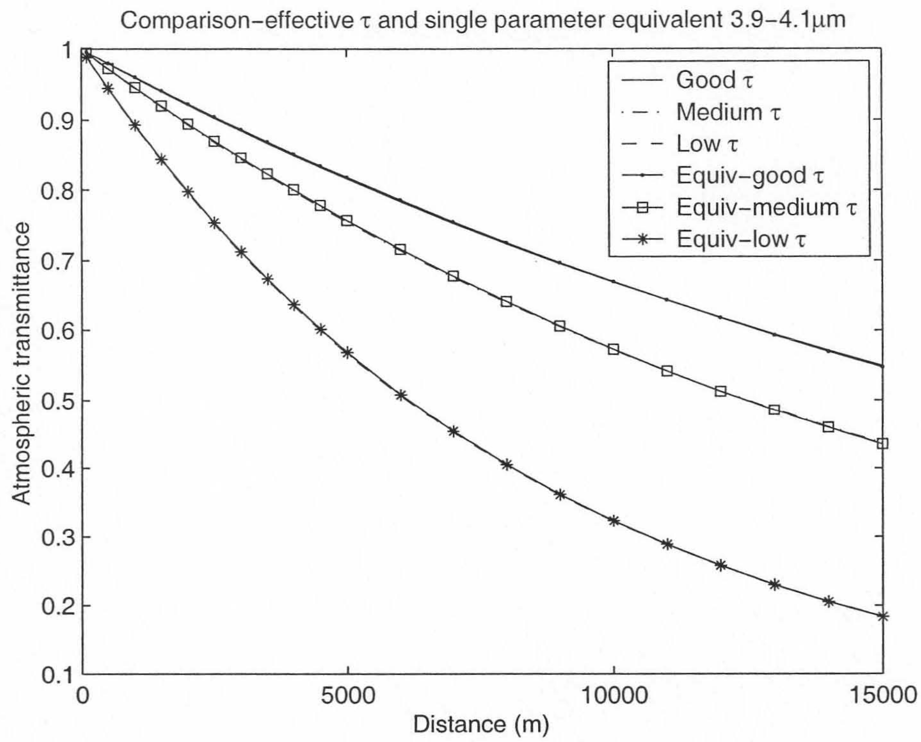

Figure 5.7: Atmospheric transmittance and equivalent transmittance - 3.9-4.1μm

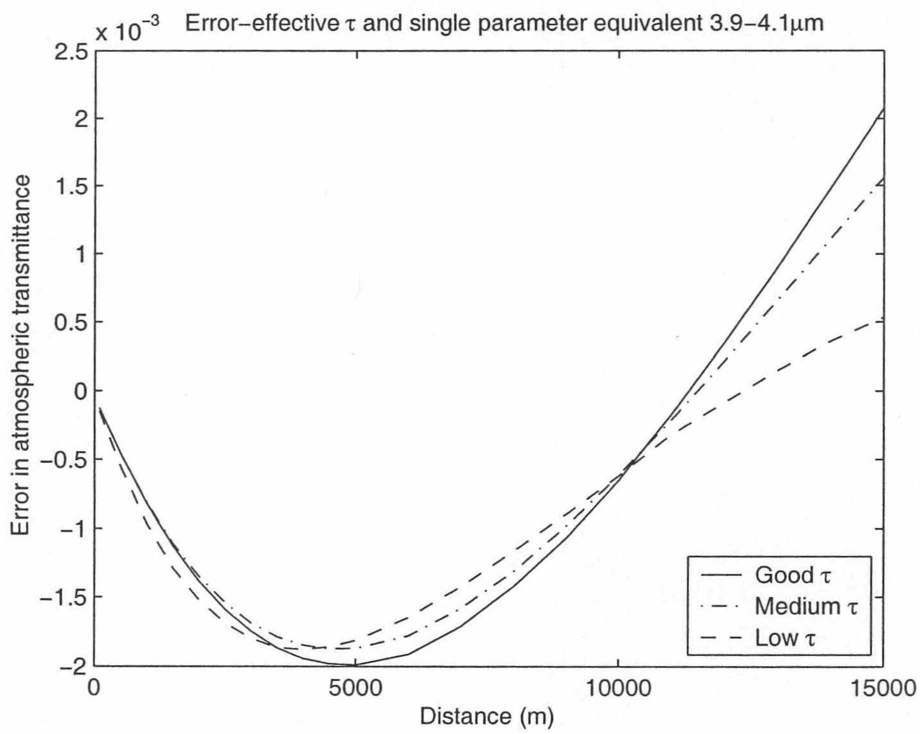

Figure 5.8: Error between actual and equivalent atmospheric transmittance - 3.9-4.1μm

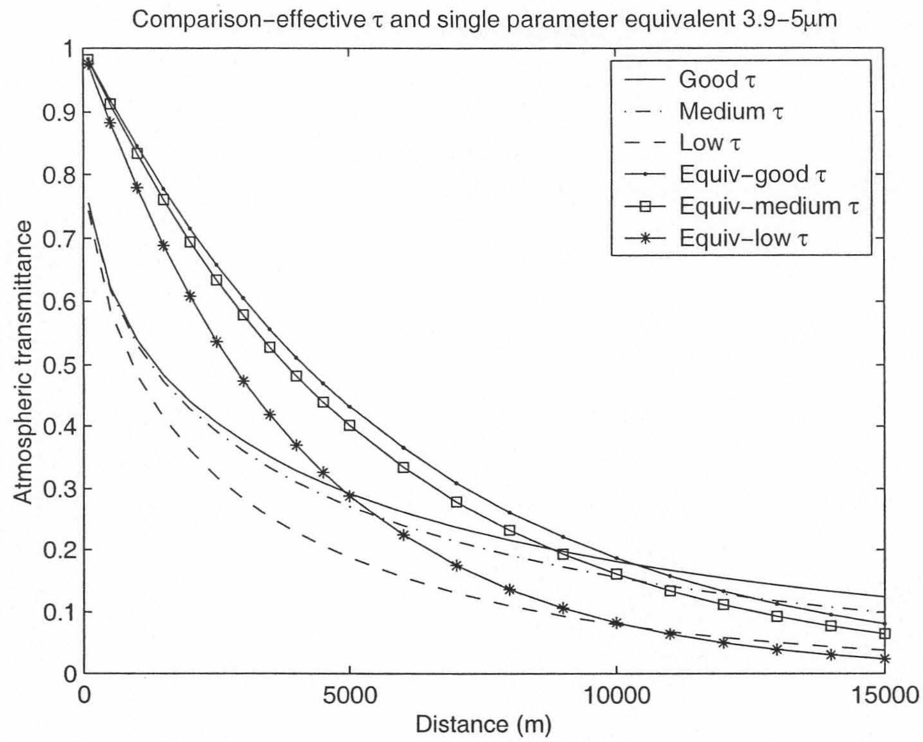

Figure 5.9: Atmospheric transmittance and equivalent transmittance - 3.9-5μm

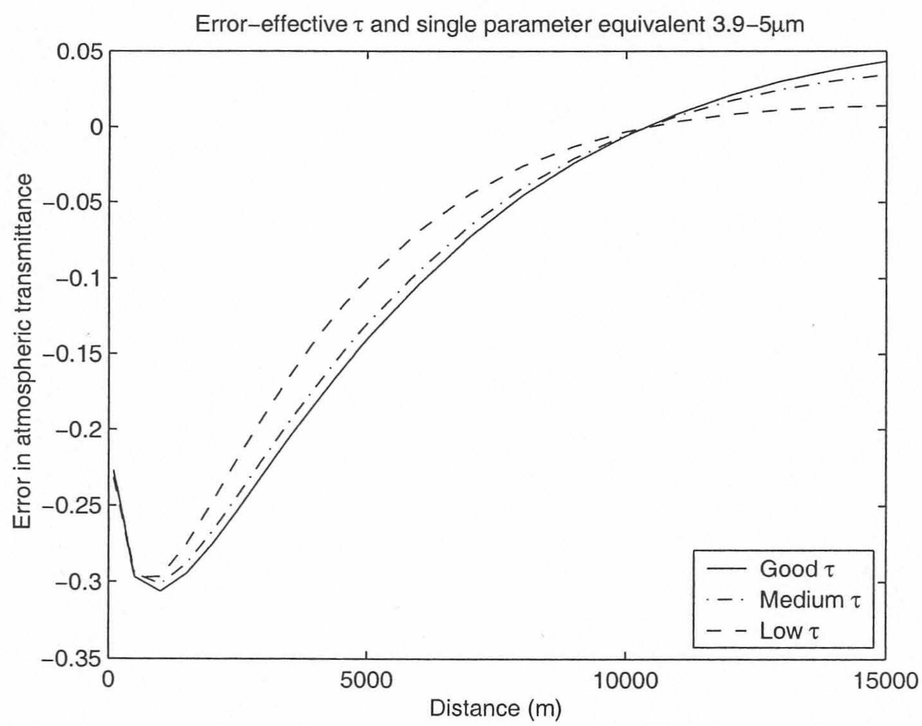

Figure 5.10: Error between actual and equivalent atmospheric transmittance - 3.9-5μm

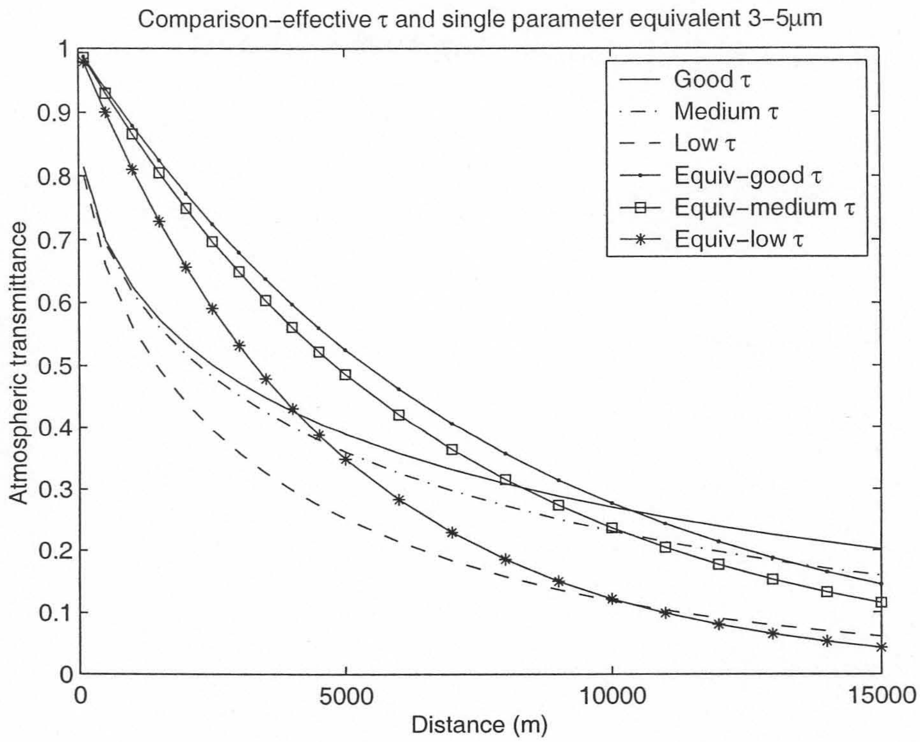

Figure 5.11: Atmospheric transmittance and equivalent transmittance - 3-5μm

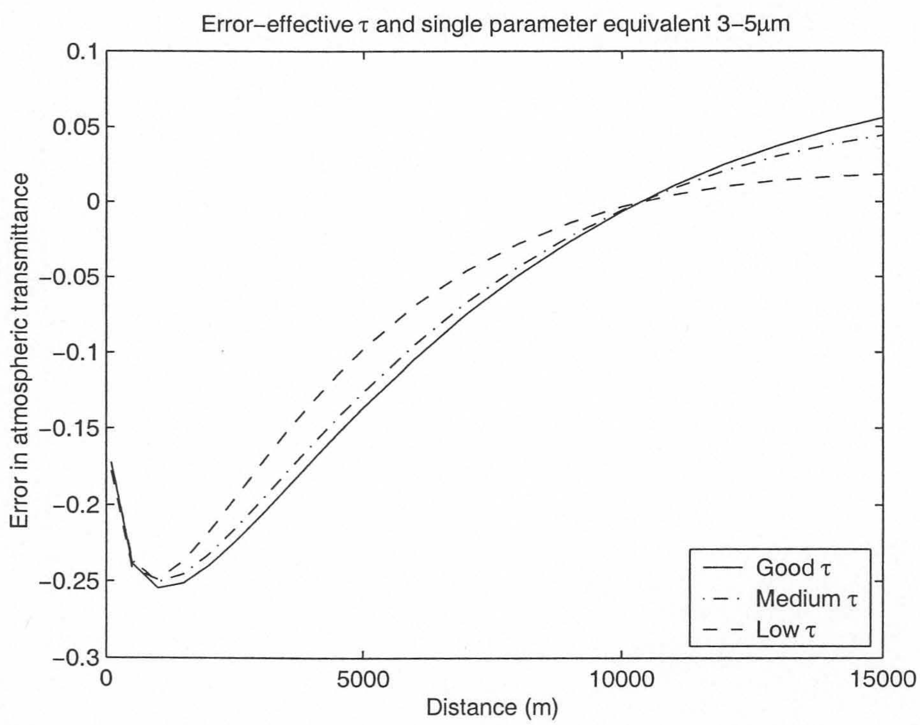

Figure 5.12: Error between actual and equivalent atmospheric transmittance - 3-5μm

5.3 Polynomial model of the effective atmospheric transmittance

It is possible to model the transmittance in the narrow band case, shown in Figure 5.7, with a low error, as shown in Figure 5.8, using an equation of the form $e^{-\alpha r}$, with r the distance and α the extinction coefficient. The two wider band cases do not model well with a single exponential parameter. A higher order polynomial model was investigated to map the transmittance in the wide band cases. The higher order model cannot be used directly in OpenGL, because OpenGL uses a single input value for the extinction coefficient, but is useful in cases where the atmospheric transmittance is modeled as a second step in the image generation. The results shown in Figure 5.13 to Figure 5.16 were modeled with an equation of the form

$$\tau = e^{-\alpha_1 r^2 - \alpha_2 r - \alpha_3}, \tag{5.4}$$

with r the distance, and α_1, α_2 and α_3 the coefficients of a polynomial fitted to the logarithm of the transmittance data. The advantage of using a higher order polynomial instead of just a single parameter is shown in Table 5.1.

Table 5.1: Reduction in error after using a higher order polynomial

Spectral band of the system	Atmospheric Transmittance	Maximum Error - single parameter (%)	Average of the absolute value of the error - single parameter (%)	Maximum Error - polynomial (%)	Average of the absolute value of the error - polynomial (%)
3.9-5 μm	Good	-31.27	14.62	14.52	1.45
3.9-5 μm	Medium	-30.79	13.93	14.48	1.41
3.9-5 μm	Low	-30.46	12.19	15.42	1.40
3-5 μm	Good	-23.95	12.24	14.55	1.52
3-5 μm	Medium	-23.58	11.58	14.53	1.48
3-5 μm	Low	-23.90	10.20	15.91	1.48

The use of the polynomial model led to up to a 53% reduction in the error when calculating the equivalent atmospheric transmittance. This result is useful in cases where the atmospheric transmittance is calculated as a separate step in a simulation. It is not possible to map the polynomial approximation to the fog density parameter in OpenGL. The calculation of the atmospheric transmittance in an OpenGL-based simulation would require careful optimisation of the atmospheric model at the range of distances of interest in the simulation.

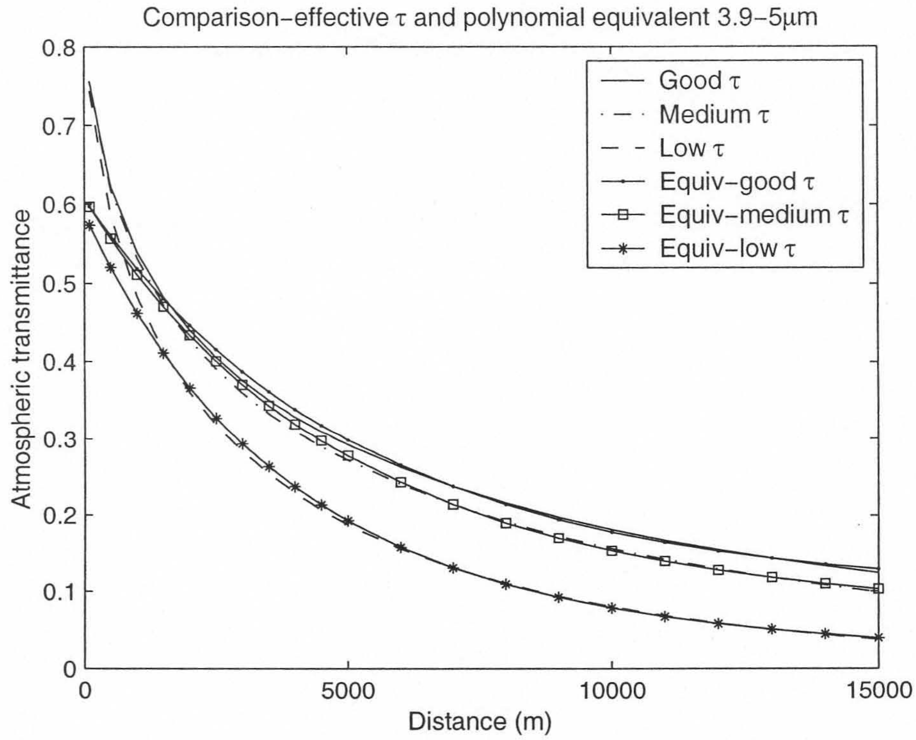

Figure 5.13: Atmospheric transmittance and equivalent transmittance - 3.9-5μm - polynomial fit

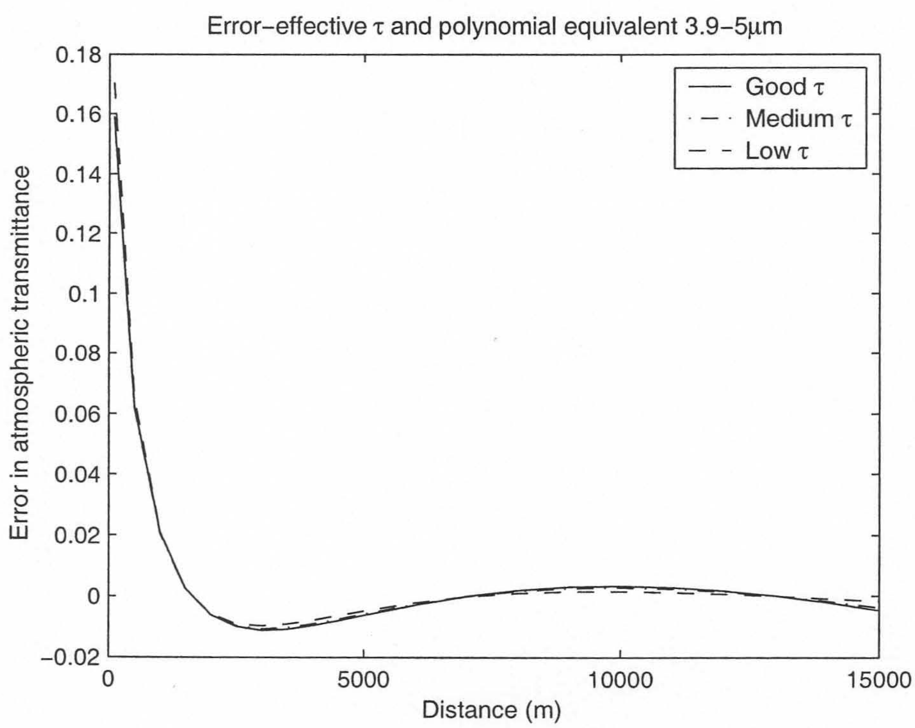

Figure 5.14: Error between actual and equivalent atmospheric transmittance - 3.9-5μm

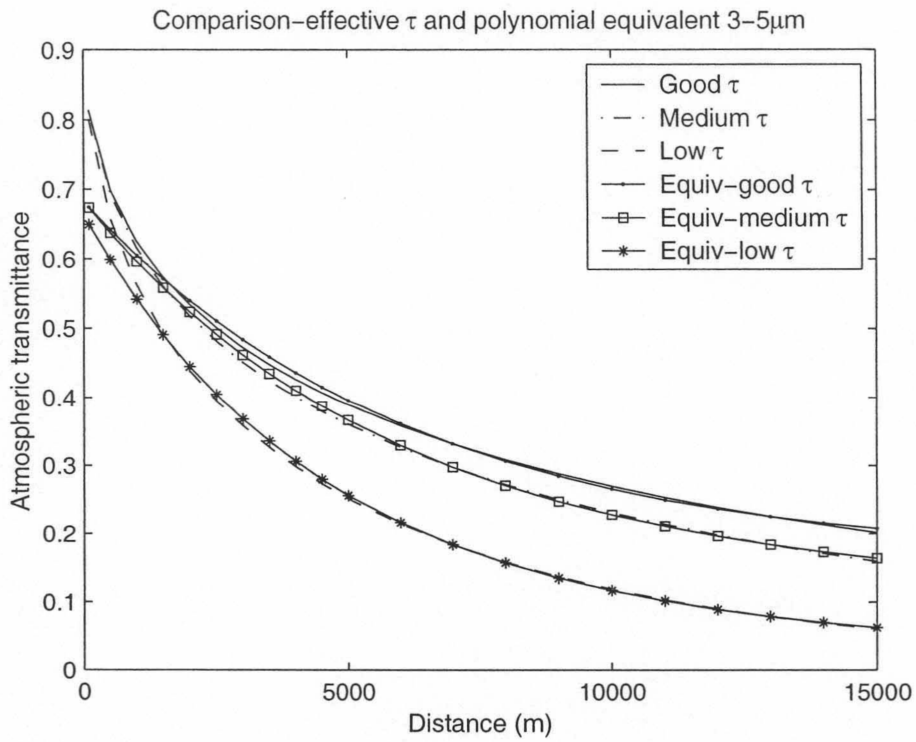

Figure 5.15: Atmospheric transmittance and equivalent transmittance - 3-5μm - polynomial fit

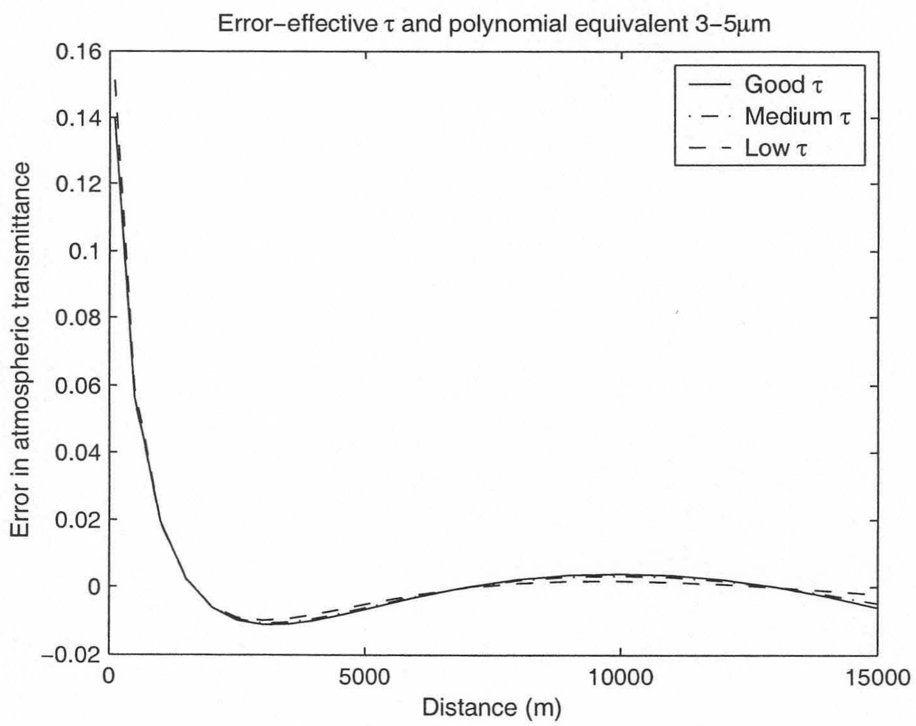

Figure 5.16: Error between actual and equivalent atmospheric transmittance - 3-5μm

5.4 The equivalent temperature of an object

The temperature of the source is an important parameter in determining the equivalent atmospheric transmittance. An additional complication arises when the object consists of a number of areas, each at a different temperature. In this case the different areas and temperatures must be combined into a projected area at an equivalent temperature. The average atmospheric transmittance for an object at a given temperature T is given by Holst [21, p287] as:

$$\tau_{ave} = \frac{\int_{\lambda_1}^{\lambda_2} \Delta M_e(\lambda,T)[\tau_{atm}(\lambda)]^R \tau_{optics}(\lambda) R_d(\lambda) d\lambda}{\int_{\lambda_1}^{\lambda_2} \Delta M_e(\lambda,T) \tau_{optics}(\lambda) R_d(\lambda) d\lambda}, \tag{5.5}$$

with $\Delta M_e(\lambda,T)$ the difference in spectral radiant exitance between the object and the background, τ_{atm} is the spectral atmospheric transmittance at distance R, $R_d(\lambda)$ is the detector's spectral response and $\tau_{optics}(\lambda)$ is the optical system's spectral transmittance. Equation (5.1) is a simplified form of Equation (5.5). The average temperature for an object consisting of a number of areas at different temperatures is given by Lauber *et al.*[22] as:

$$T_{ave} = \frac{\sum_{i=1}^{N} A_i T_i}{\sum_{i=1}^{N} A_i}, \tag{5.6}$$

where A_i is the area of each part, T_i is its temperature and the object consists of N areas at constant temperature.

The effective transmittance is influenced by the source temperature. This is illustrated in Figure 5.17. The data in the figure were generated using medium atmospheric transmittance, a system response of 3-5μm and the source temperatures as indicated in the figure.

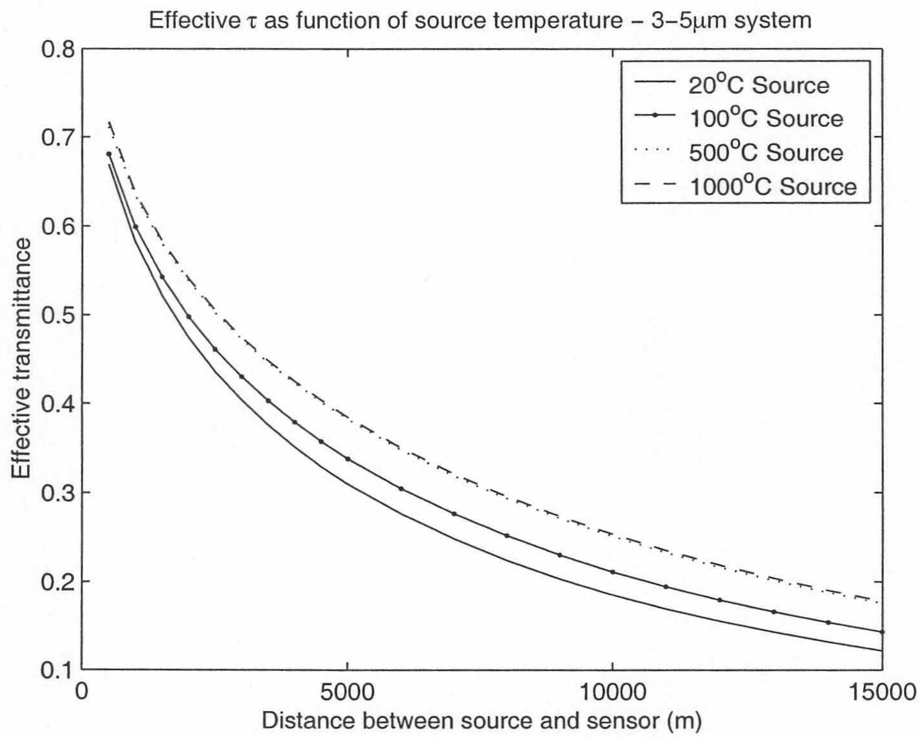

Figure 5.17: Effect of source temperature on the effective atmospheric transmittance

5.5 Path radiance

The purpose of this section is to investigate the accuracy of mapping path radiance to a single parameter. The path radiance can be be simulated in OpenGL by assigning a colour to the fog, as was discussed in Section 3.11.2.

The path radiance between the source and detector for a point source is given in Equation (2.10) as:

$$E_{path} = \int_{\lambda_0}^{\lambda_1} L(\lambda)_{path\ sensor\ to\ source} S_\lambda \Omega_{source}(1 - \tau_{\lambda,source})d\lambda. \tag{5.7}$$

Equation (5.7) can be re-written for a point-source after integration of spectral quantities as:

$$E_{path} = \frac{L_{path} A_{source}}{r^2} \varepsilon, \tag{5.8}$$

with r the distance between the source and sensor, L the radiance of a blackbody at the temperature of the path, A_{source} the area of the source and ε the equivalent emissivity. The area of the source is used because the path radiance is from the same area as that of the source. The path radiance can therefore be represented as a source at the temperature of the atmosphere with an emissivity equal to $(1-\tau_{\lambda,source})$, with $\tau_{\lambda,source}$ the atmospheric transmittance of the path between the sensor and the source. The path radiance is directly dependent on the atmospheric transmittance between the detector and the source.

The mapping of path radiance to an equation of the form:

$$E_{path} = ke^{\sigma r}, \tag{5.9}$$

was investigated for the three filter bandwidths and the three atmospheric transmittance conditions used previously in this chapter. The results are shown in Figure 5.18 to Figure 5.23. The figures are:

- Figure 5.18 shows the calculated and equivalent path radiance for the 3.9-4.1 μm filter. Figure 5.19 shows the error in the equivalent path radiance. A single parameter model was used to calculate σ in Equation (5.9).
- Figure 5.20 shows the calculated and equivalent path radiance for the 3.9-5 μm filter. Figure 5.21 shows the error in the equivalent path radiance. A single parameter model was used to calculate σ in Equation (5.9).
- Figure 5.22 shows the calculated and equivalent path radiance for the 3-5 μm filter. Figure 5.23 shows the error in the equivalent path radiance. A single parameter model was used to calculate σ in Equation (5.9).

The values for k and σ in Equation (5.9) were calculated for a point source and is not valid for an extended source. The path radiance for an extended source is given by:

$$E_{path} = L_{path}\Omega_{sensor}\varepsilon, \tag{5.10}$$

with Ω the sensor's field-of-view. The values of k and σ must be determined again for an extended source.

The mapping of the path radiance to Equation (5.9) led to large errors in the cases investigated. The narrow-band system had errors of up to 100%, whereas the two wide-band systems had errors of more than 250%. The path radiances were re-calculated using a second order polynomial model for the extinction coefficient. The polynomial model is of the form:

$$\sigma = \alpha_1 r^2 + \alpha_2 r + \alpha 3. \tag{5.11}$$

The results are shown in Figure 5.24 to Figure 5.29. The figures are:

- Figure 5.24 shows the calculated and equivalent path radiance for the 3.9-4.1 μm system. Figure 5.25 shows the error in the equivalent path radiance. Equation (5.11) was used to calculate σr in Equation (5.9).
- Figure 5.26 shows the calculated and equivalent path radiance for the 3.9-5 μm system. Figure 5.27 shows the error in the equivalent path radiance. Equation (5.11) was used to calculate σr in Equation (5.9).
- Figure 5.28 shows the calculated and equivalent path radiance for the 3-5 μm system. Figure 5.29 shows the error in the equivalent path radiance. Equation (5.11) was used to calculate σr in Equation (5.9).

The polynomial model reduced the errors in the path radiance compared to the single parameter model. The comparison is shown in Table 5.2.

Table 5.2: Comparison in path radiance error when using a single parameter and a polynomial model

Spectral band of the system	Maximum Error in single parameter model (%)	Maximum error in polynomial model (%)
3.9-4.1 μm	-100	30
3.9-5 μm	-250	50
3-5 μm	-250	50

When a simulation is implemented in OpenGL, a potential source of errors is the change in effective transmittance with a change in temperature. The path radiance is calculated using the effective transmittance of the source, because it is only possible to specify a single value for fog density, but the effective transmittance required to calculate the path radiance might differ a lot due to large temperature differences between the source and the atmosphere.

The large errors generated by the single parameter model would again imply that care must be taken in setting up a simulation where the transmittance, and therefore the path radiance, is represented by a single parameter.

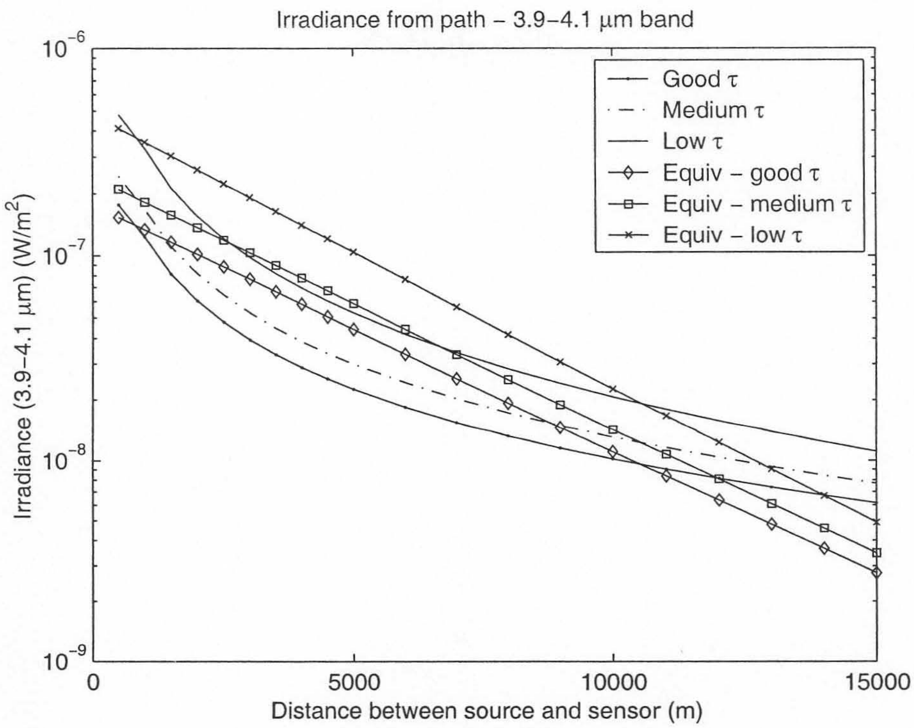

Figure 5.18: Irradiance due to path radiance for various atmospheric conditions, 3.9-4.1 μm system

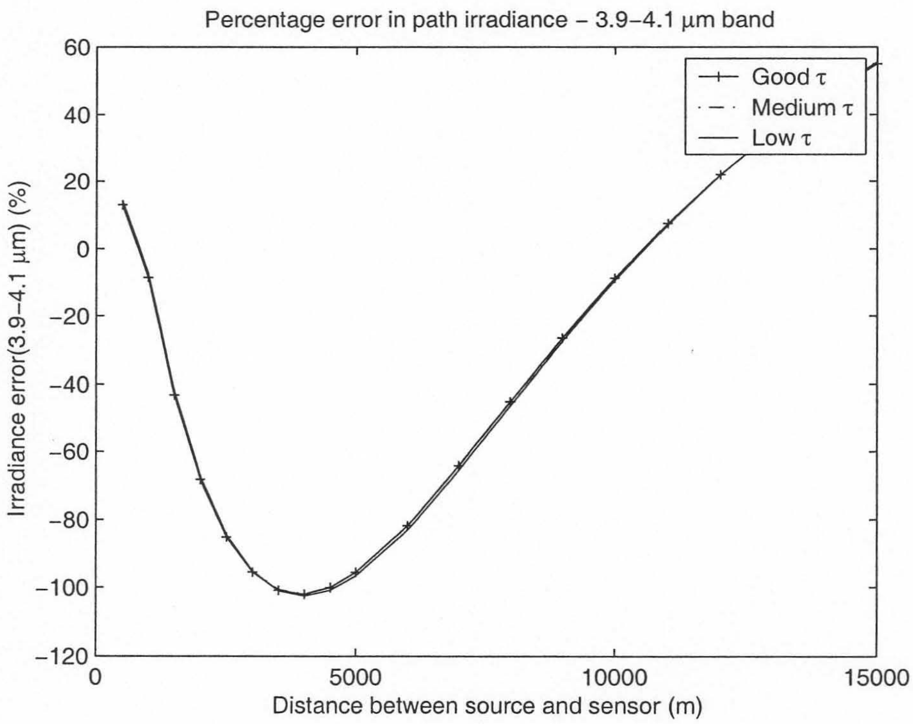

Figure 5.19: Error in irradiance due to path radiance for various atmospheric conditions, 3.9-4.1 μm system

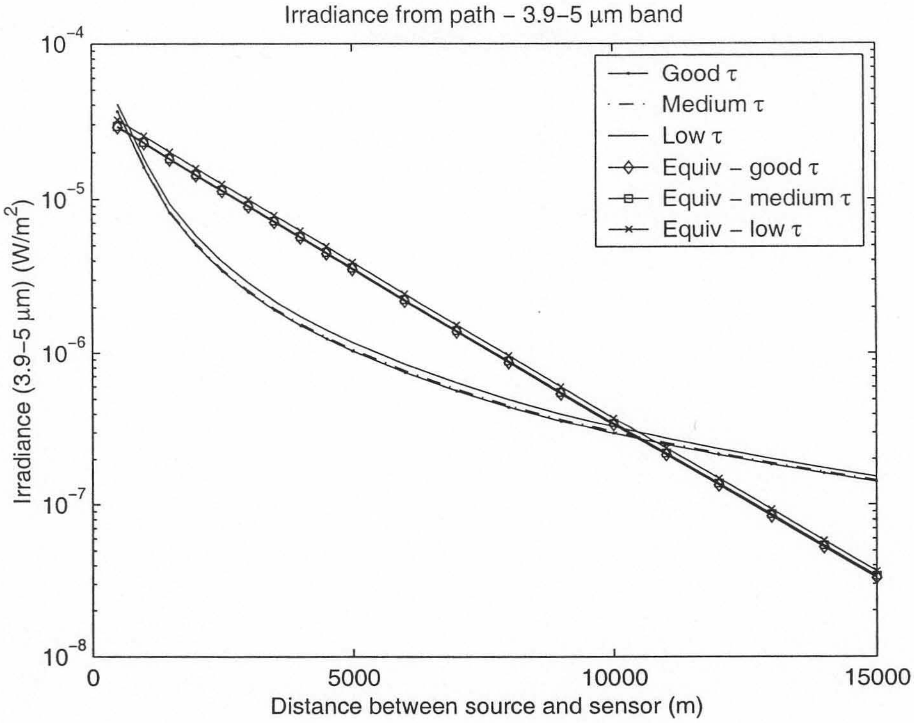

Figure 5.20: Irradiance due to path radiance for various atmospheric conditions, 3.9-5 μm system

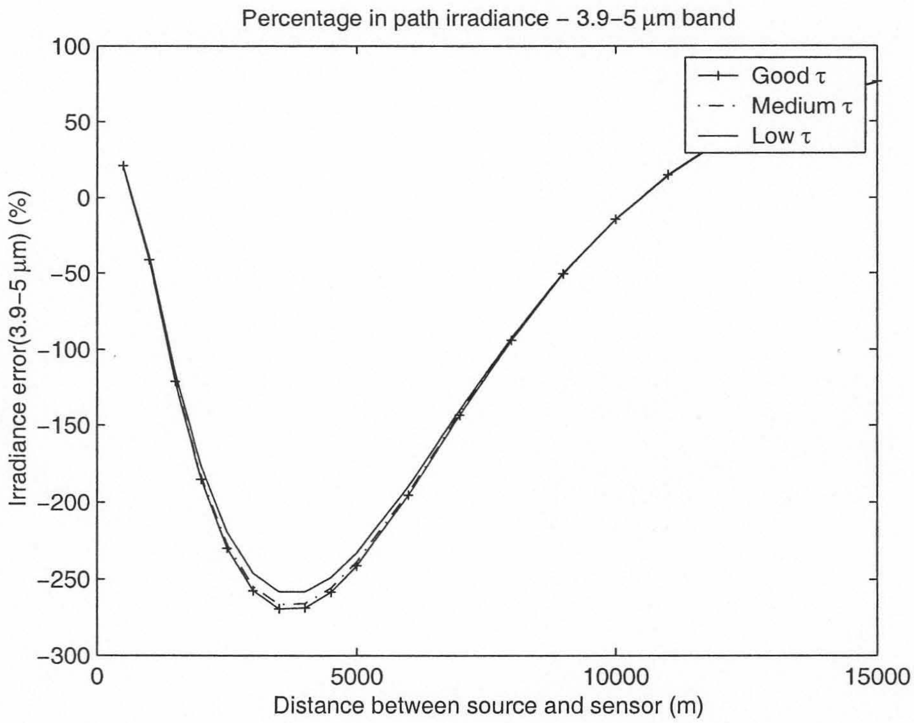

Figure 5.21: Error in irradiance due to path radiance for various atmospheric conditions, 3.9-5 μm system

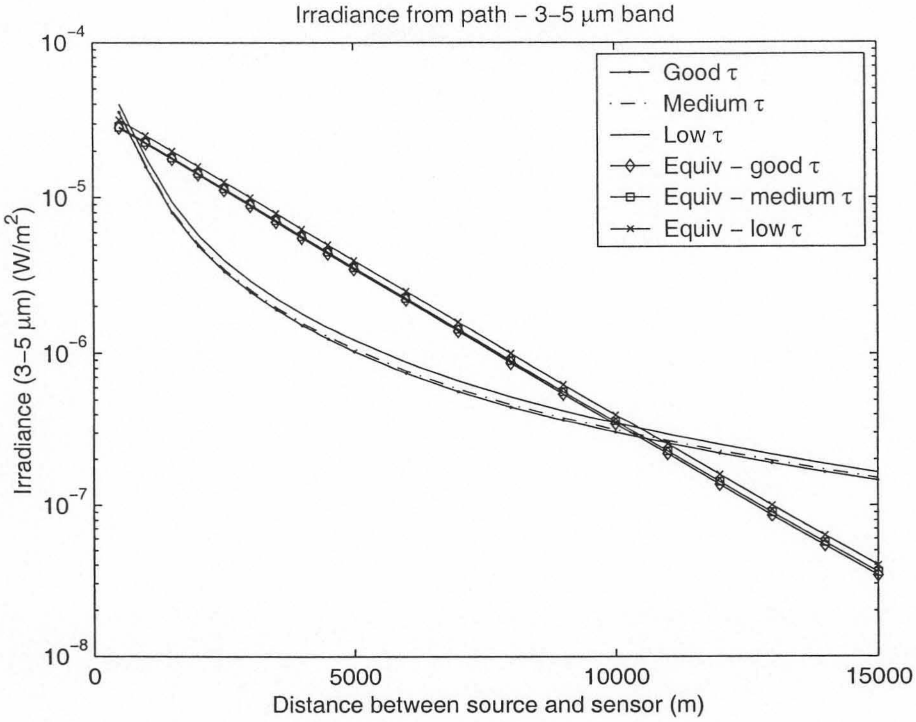

Figure 5.22: Irradiance due to path radiance for various atmospheric conditions, 3-5 μm system

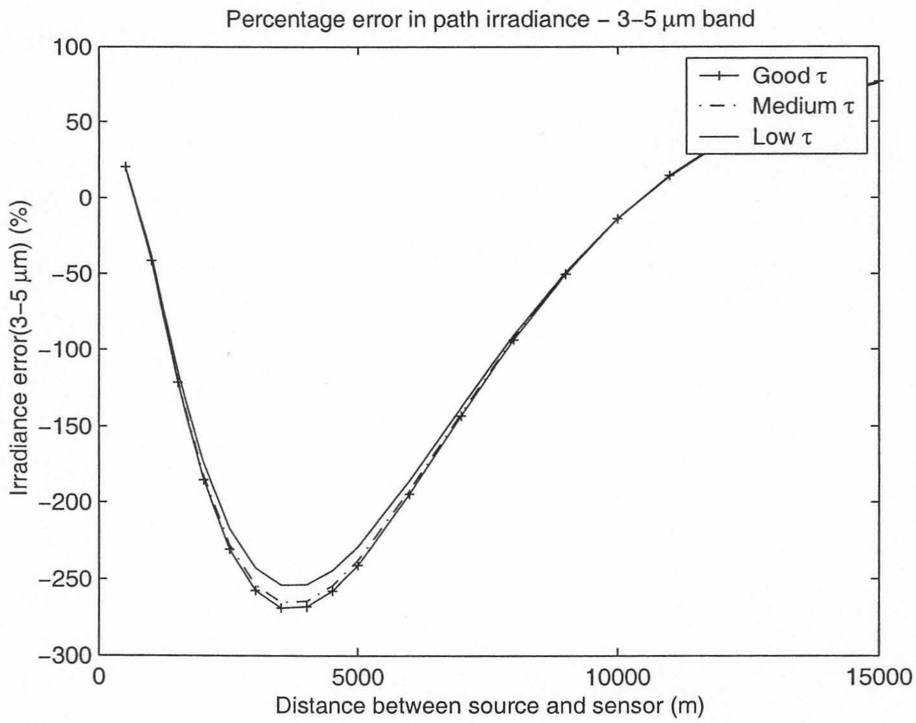

Figure 5.23: Error in irradiance due to path radiance for various atmospheric conditions, 3-5 μm system

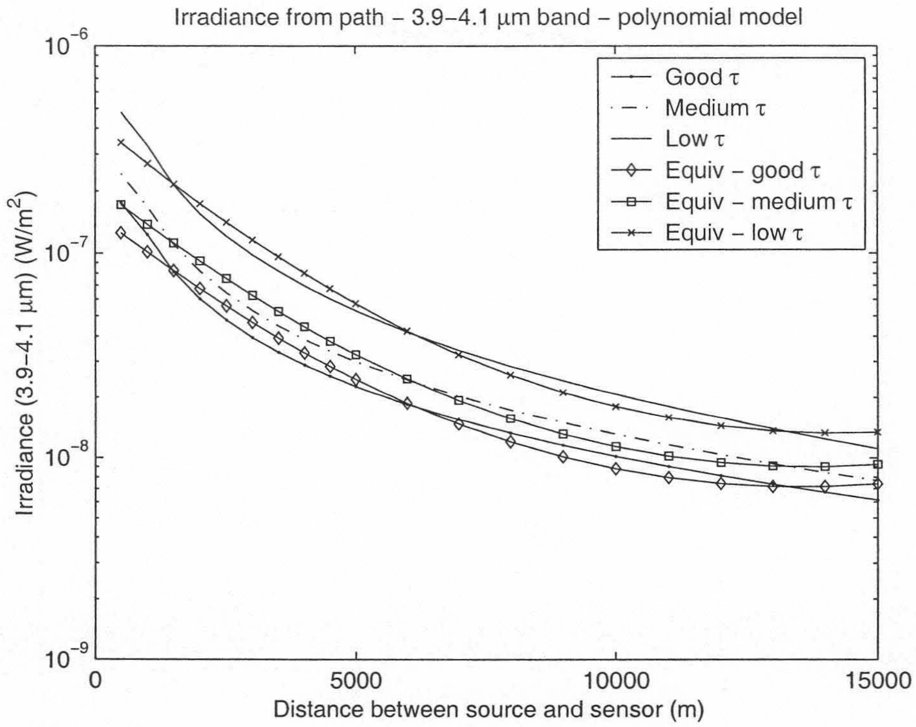

Figure 5.24: Irradiance due to path radiance - polynomial model for α, 3.9-4.1 μm system

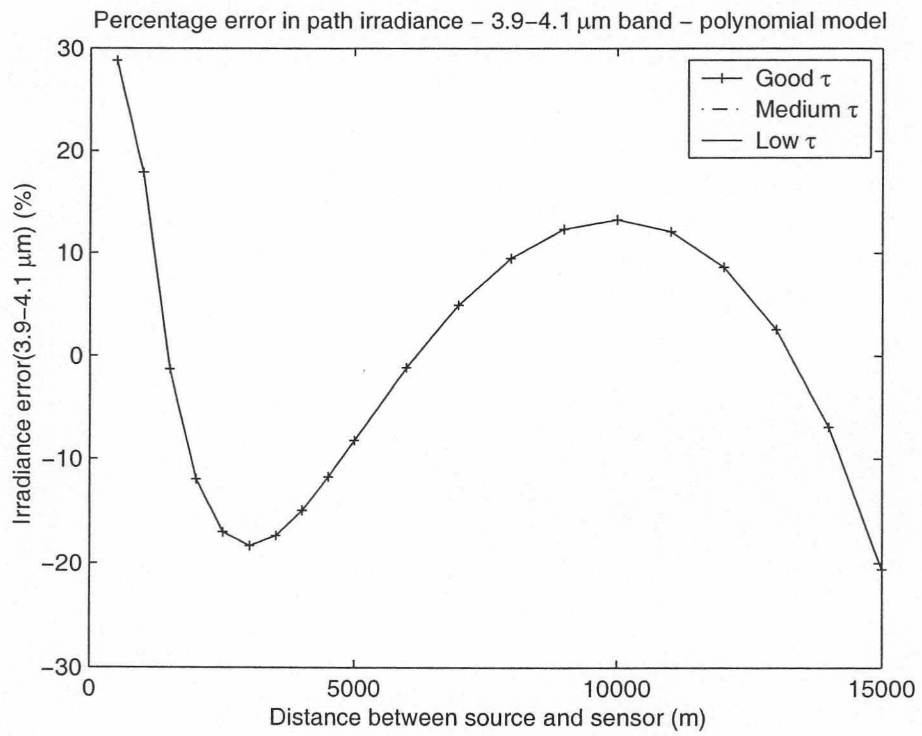

Figure 5.25: Error in irradiance due to path radiance - polynomial model for α, 3.9-4.1 μm system

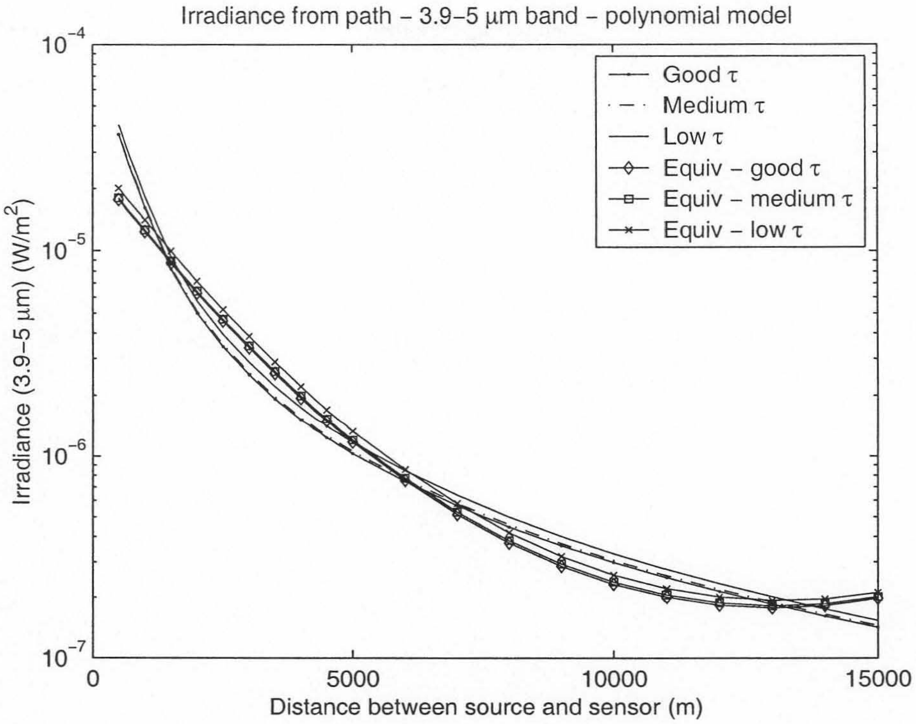

Figure 5.26: Irradiance due to path radiance - polynomial model for α, 3.9-5 μm system

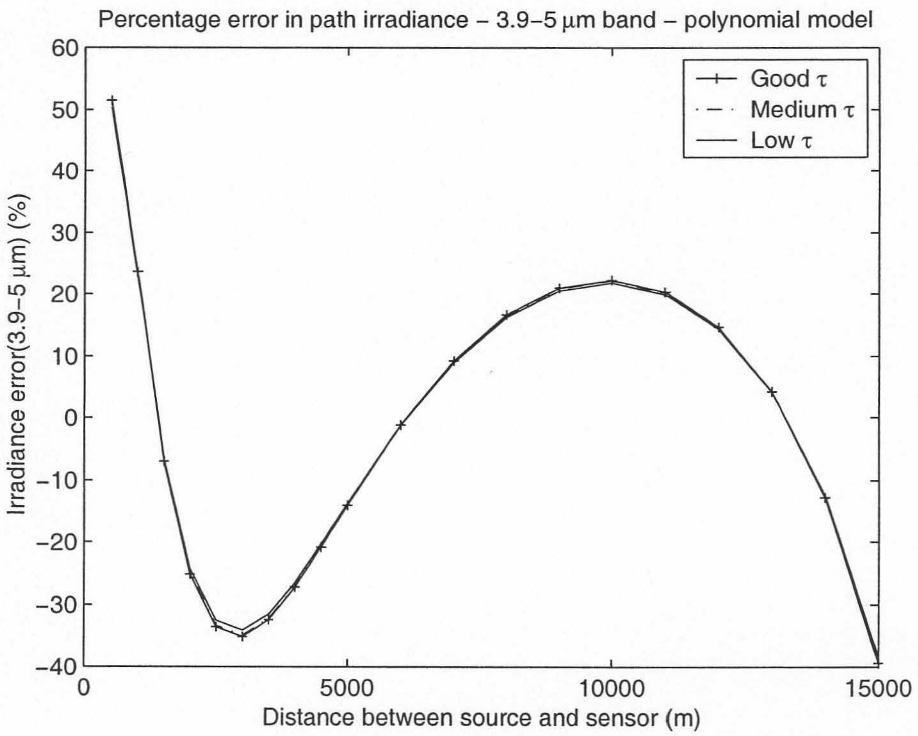

Figure 5.27: Error in irradiance due to path radiance - polynomial model for α, 3.9-5 μm system

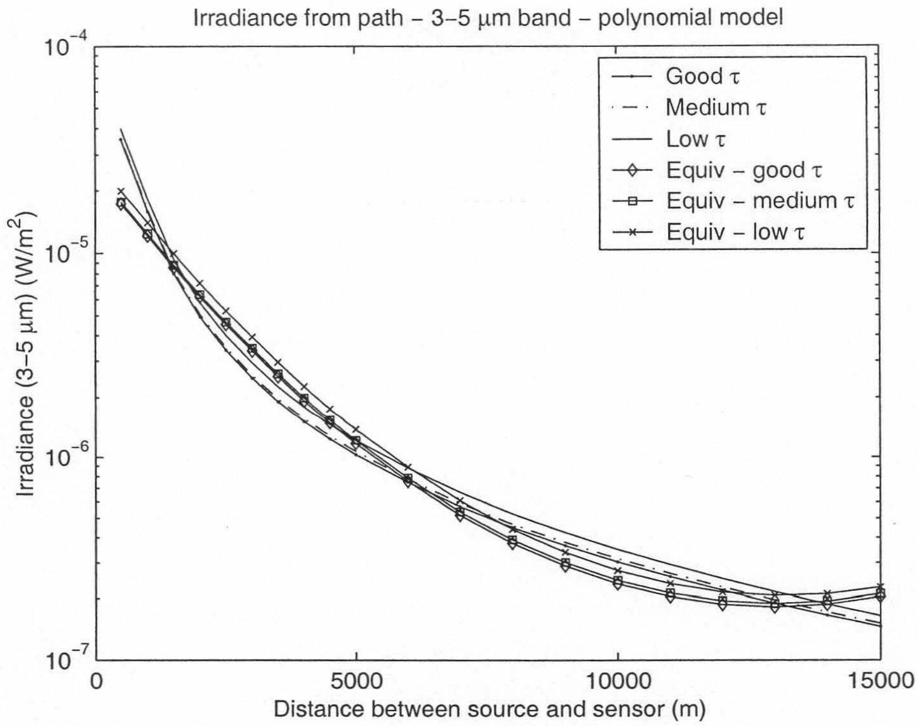

Figure 5.28: Irradiance due to path radiance - polynomial model for α, 3-5 μm system

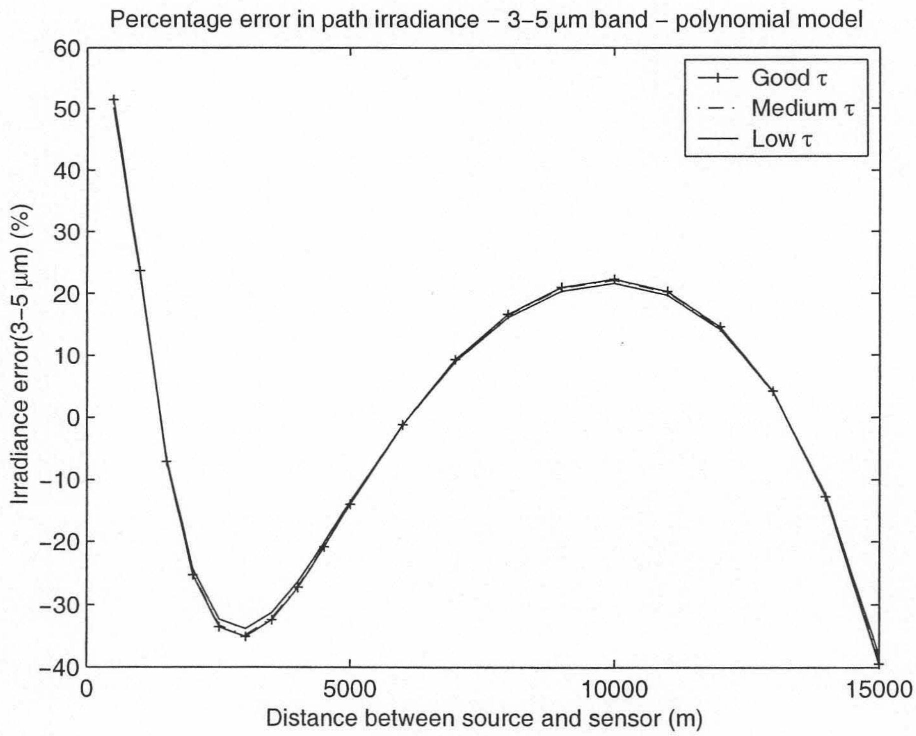

Figure 5.29: Error in irradiance due to path radiance - polynomial model for α, 3-5 μm system

5.6 Conclusion

5.6.1 Mapping the atmospheric transmittance to a single parameter equivalent

The mapping of atmospheric transmittance to a single parameter is only successful in cases where the spectral band of interest is within an atmospheric transmittance window. This is illustrated in Figure 5.7 and Figure 5.8. The effect of absorption bands leads to errors in the cases where a transmission band and an absorption band are included in the spectral band of interest. This is mainly due to the $e^{-\alpha r}$ function that approaches 1 when the distance r approaches 100 m. The 100 m distance was the shortest distance used in this evaluation. In cases where an absorption band is included, the transmittance at shorter distances, in this case 100 m, does not approach 1.

5.6.2 Mapping the atmospheric transmittance to a polynomial equivalent model

The mapping of the atmospheric transmittance to a polynomial model for the absorption coefficient, leads to the lower errors shown in Figure 5.13 to Figure 5.16. The reduction in errors is mainly due to the polynomial model that does not approach 1 when the distance approaches 100 m. The use of the polynomial model is not directly applicable to the mapping of atmospheric transmittance to OpenGL, but can be useful in simulations where the atmospheric transmittance is required as a function of distance and it is not possible to re-run the MODTRAN model.

5.6.3 Modeling the path radiance

There are a number of potential causes for errors in the modelling of the path radiance. The most important two are:

- **The effective transmittance.** The effective transmittance of the source and the atmospheric path can be substantially different due the the difference in temperature between the source and the path. This was illustrated in Section 5.5. The path radiance is calculated using the effective transmittance of the source, potentially leading to large errors.

- **The mapping of the atmospheric transmittance.** The path radiance is calculated from the effective atmospheric transmittance. The inaccuracy in calculating the atmospheric transmittance is reflected in the errors calculating the path radiance shown in Figure 5.18 to Figure 5.23. The use of a polynomial model for atmospheric transmittance leads to a reduction in the error, as illustrated in Figure 5.24 to Figure 5.29.