Chapter 4

The Dynamic Range of Graphics Accelerators

4.1 Introduction

Commercial-off-the-shelf renderers such as the GeForce 2 card from Nvidia can render images at resolutions of up to 24 colour bits per pixel. The pixel is made up of the three basic colours, red, green and blue. The colours are rendered at a resolution of 8 bits each and the three colour channels are independent of each other. The resolution of the rendered infrared images is therefore limited to the resolution of a single colour channel. Olsen, *et al.* [6] and [20], described a technique whereby the resolution of a radiometric image can be increased by combining the outputs from the three colour channels.

The increased dynamic range is obtained by assigning parts of the input dynamic range to the different colour channels. This is illustrated in Figure 4.1, from Olsen *et al.* [20]. Figure 4.1 shows the assignment as it will be used on a personal computer, with a colour resolution of 8 bits per channel and a 14-bit output resolution. The 14-bit input word, which is related to the input radiance, is split into the least significant 8 bits which are assigned to the red channel, the next most significant 3 bits which are padded with 5 zeros are assigned to the green channel and finally the most significant 3 bits which are padded with 5 zeros and assigned to the blue channel.

The image is then rendered in normal fashion and recombined in a single 14-bit word. The method of recombination is shown in Figure 4.2.

4.2 Dynamic range of a thermal image

The dynamic range of the radiance values in a thermal image is dependent on the input temperatures in the scene. The radiance was calculated for input temperatures from 0°C to 1000°C and a spectral band of 3 to 5.5 μm using:

$$L(T) = \int_{3}^{5.5} \frac{M(\lambda, T)}{\pi} d\lambda, \tag{4.1}$$

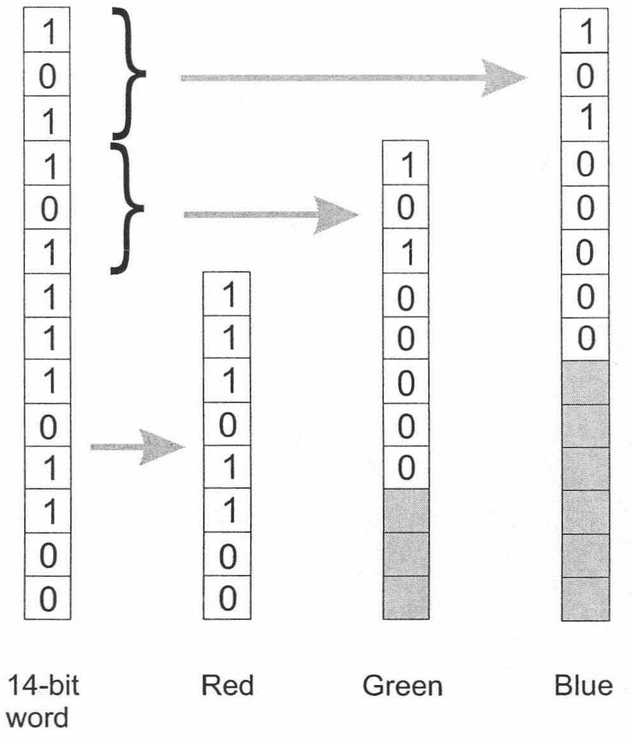

Figure 4.1: Assignment of radiance levels to RGB channels

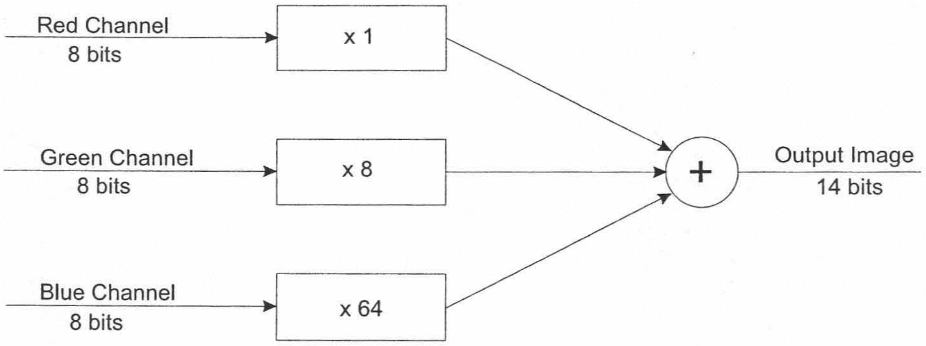

Figure 4.2: Recombination of RGB channels

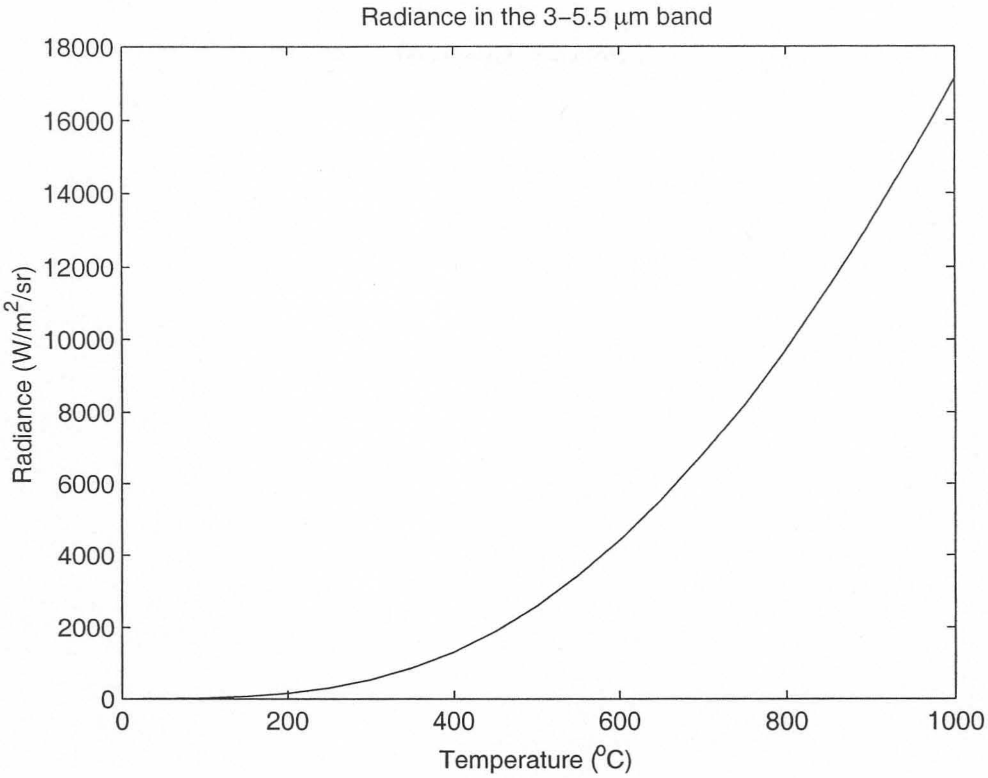

Figure 4.3: Radiance from sources at a range of temperatures

with the exitance as defined in Equation (2.1). The result is shown in Figure 4.3.

If it is assumed that the maximum render resolution is 14 bits (16384 levels) and that the mapping from radiance to gray levels is linear, the maximum temperature range that can be accommodated in 14 bits is from 0°C to 981°C. The temperature values were obtained by inspection of the data used to generate Figure 4.3. The requirement for a linear mapping is added to eliminate an additional processing step on the image, increasing the rendering speed. On an 8-bit rendering system radiance values of 0 to 255 $\mathrm{Wm^{-2}sr^{-1}}$ would be assigned to the red channel, 256 to 2048 $\mathrm{Wm^{-2}sr^{-1}}$ to the red and green channel and 2048 to 16384 $\mathrm{Wm^{-2}sr^{-1}}$ to the red, green and blue channel. The examples in Table 4.1 should clarify the technique.

4.3 Errors due to artificially extending the render resolution

In order to investigate the errors caused by artificially extending the rendering resolution, a test object was defined as shown in Figure 4.4. The test object was selected so that its radiometric properties could be calculated exactly, as shown in Equations (4.2) and (4.5). The calculated properties could then be compared with the object that was rendered using OpenGL. The triangle was rendered in OpenGL using:

```
    glBegin(GL_TRIANGLES);
```

Table 4.1: Examples of mapping radiance values to colours

Input Value	Input value in binary	Dynamic Range to be applied	Red Value (8-bits)	Green Value (8-bits)	Blue Value (8-bits)	Comment
200	11001000	10-bit	11001000	00000000	00000000	
812	1100101100	10-bit	00101100	10000000	10000000	
812	1100101100	12-bit	00101100	11000000	00000000	
812	1100101100	14-bit	00101100	01100000	00000000	
812	1100101100	16-bit	00101100	00110000	00000000	
1487	10111001111	10-bit				> 10-bit
1487	10111001111	12-bit	11001111	01000000	10000000	
1487	10111001111	14-bit	11001111	10100000	00000000	
6869	1101011010101	10-bit				> 10-bit
6869	1101011010101	12-bit				> 12-bit
6869	1101011010101	14-bit	11010101	01000000	01100000	

```
    glColor3f(red1, green1, blue1);
    glVertex3f(-1.0, -1.0, 0.0);
    glColor3f(red2, green2, blue2);
    glVertex3f(1.0, -1.0, 0.0);
    glColor3f(red3, green3, blue3);
    glVertex3f(0.0, 1.0, 0.0);
glEnd();
```

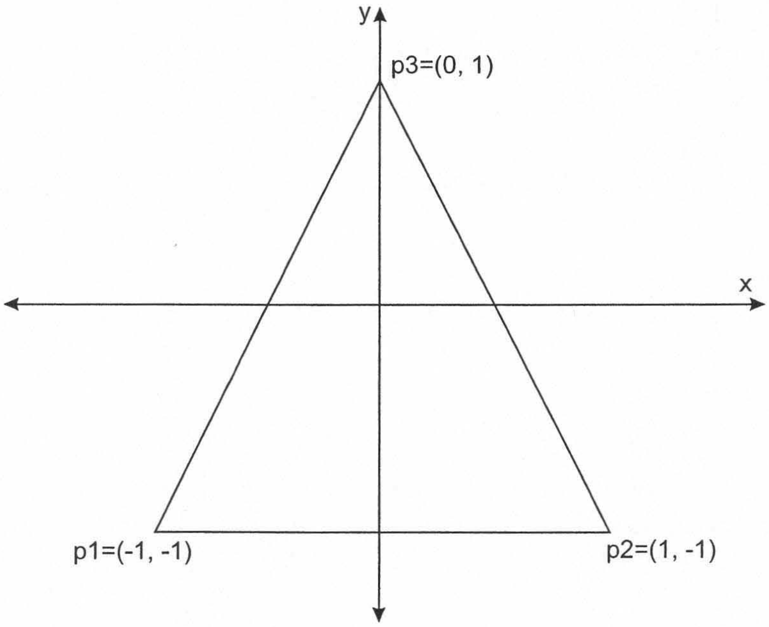

Figure 4.4: Definition of the test object

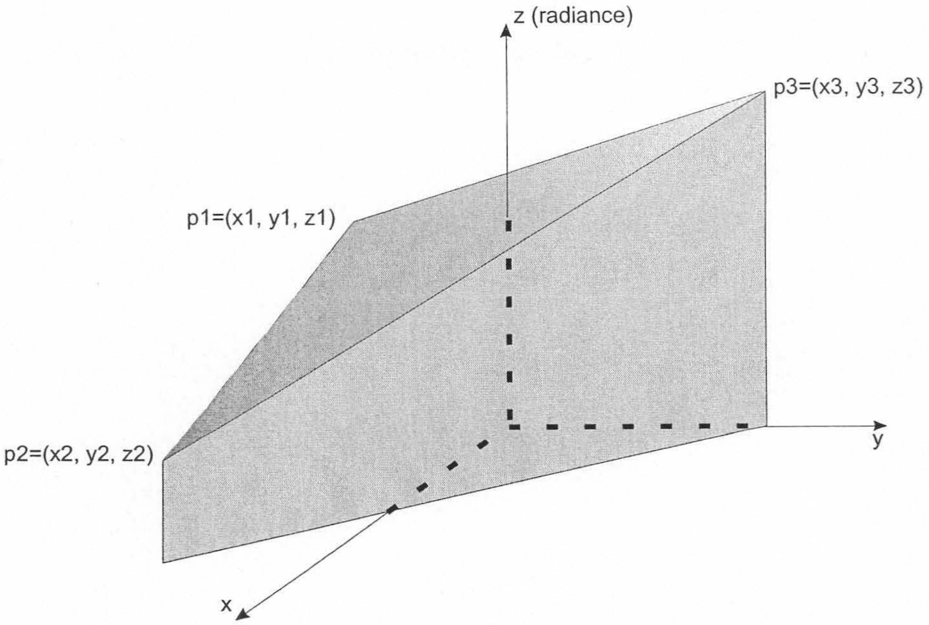

Figure 4.5: Test object with radiance values assigned to the z-axis

By assigning a colour related to radiance to each vertex, it was possible to calculate the exact radiance value of an x,y-point within the triangle, or to calculate the exact intensity of the triangle. An example of the triangle, with radiance values assigned to the z-axis, is shown in Figure 4.5.

The equation of the surface connecting points p1, p2 and p3 in Figure 4.5 is given by:

$$ax + by + cz = d. \tag{4.2}$$

It is therefore possible to find the radiance at any point (x, y) in the triangle by using Equation (4.2). The intensity of the test object is given by:

$$\begin{aligned} I &= LA, && (4.3)\\ &= \int_{-1}^{1}\int_{\frac{y-1}{2}}^{\frac{1-y}{2}} LA \quad dxdy, && (4.4)\\ &= \frac{6d+2b}{3c}. && (4.5)\end{aligned}$$

L is the radiance of the target, A the projected area and b, c and d the constants defining the surface in Equation (4.2).

The values in Equation (4.5) were determined for

$$p1 = (-1, \quad -1, \quad z_1), \tag{4.6}$$
$$p2 = (1, \quad -1, \quad z_2) \quad \text{and} \tag{4.7}$$
$$p3 = (0, \quad 1, \quad z_3), \tag{4.8}$$

as shown in Figure 4.4. The triangle can be rendered in OpenGL and the resulting values compared with the theoretically calculated values. The following two experiments were carried out to determine the errors caused by increasing the dynamic range to more than 8-bits.

Test object at a fixed spatial position. Random radiance values are determined for each vertex of the test object. The radiance values are assigned to the three colours according to the technique described in Section 4.2. The image is rendered and the radiance value at a random pixel is determined. The radiance value at the same position is calculated using Equation (4.2). The difference between the two values is recorded.

Test object at an increasing distance from the sensor. The vertices of the test object are assigned three different radiance values. The radiance values are assigned to the three colours according to the technique described in Section 4.2. The test object is moved from a distance of 100m from the camera to a distance of 2100m with a step size of 10m. The image is rendered at each distance and the intensity is compared with the intensity calculated from Equation (4.5). The rendered intensity is recorded at each distance.

4.4 Results

4.4.1 Determining difference between rendered and calculated radiance

In this experiment the radiance of the three vertexes of the test object were assigned random radiance values. The radiance value on a random pixel were rendered and theoretically calculated and the results compared. The experiment was repeated 10000 times. The position of the pixel on the triangle and the radiance levels of the three vertexes were varied in each case. The difference between the calculated and the rendered radiance values were determined at resolutions of 8-bit, 10-bit, 12-bit, 14-bit and 16-bit. The differences between the theoretical and rendered radiance values and the error histograms for the 5 cases are plotted in Figure 4.7 to Figure 4.16. The figures are:

- **Figures 4.7 and 4.8** The difference between the rendered and calculated radiance is shown as absolute error and percentage error as function of radiance value in Figure 4.7. The histogram of the absolute error values is shown in Figure 4.8. The radiance resolution was 8-bit or 256 radiance levels.
- **Figures 4.9 and 4.10** The data was determined for a radiance resolution of 10-bit or 1024 gray levels and is presented in the same format as Figures 4.7 and 4.8.
- **Figures 4.11 and 4.12** The data was determined for a radiance resolution of 12-bit or 4096 gray levels and is presented in the same format as Figures 4.7 and 4.8.

- **Figures 4.13 and 4.14** The data was determined for a radiance resolution of 14-bit or 16384 gray levels and is presented in the same format as Figures 4.7 and 4.8.
- **Figures 4.15 and 4.16** The data was determined for a radiance resolution of 16-bit or 65536 gray levels and is presented in the same format as Figures 4.7 and 4.8.
- **Figure 4.6** shows the error histograms of all the dynamic range cases on one graph.

The errors can be ascribed to the following:

- **Rounding Errors** The radiance value of the pixels are calculated using integers. The radiance value is therefore immediately rounded to the nearest integer, leading to an error of ± 0.5 in the 8-bit case, which is red only. This is multiplied by the scaling value for the green and blue pixels in the cases where more than 8-bits are used. The maximum rounding error would then be: $\pm(0.5 + 0.5 * greenscalefactor + 0.5 * bluescalefactor)$. This is ±3.5, ±10.5, ±36.5 and ±136.5 for 10-bit, 12-bit, 14-bit and 16-bit respectively.
- **Value for pixel compared to value calculated for a point** The theoretical radiance value is calculated for a point, whereas the radiance of a pixel is determined by the radiance values at the four corners of the pixel. The amplitude of this error is determined by the equivalent position of the point in the pixel and the radiance values at the four corners of the pixel.

The maximum error percentage in all the cases is less than 20%. The error percentage drops to below 5% when the rendered radiance value is more than 20% of the maximum rendered value. It can therefore be concluded that it is possible to render images with a less than 5% error in more than 80% of the dynamic range of the card, even in cases where the dynamic range is artificially extended. The histograms of the errors of all the dynamic range cases are shown for comparison purposes in Figure 4.6.

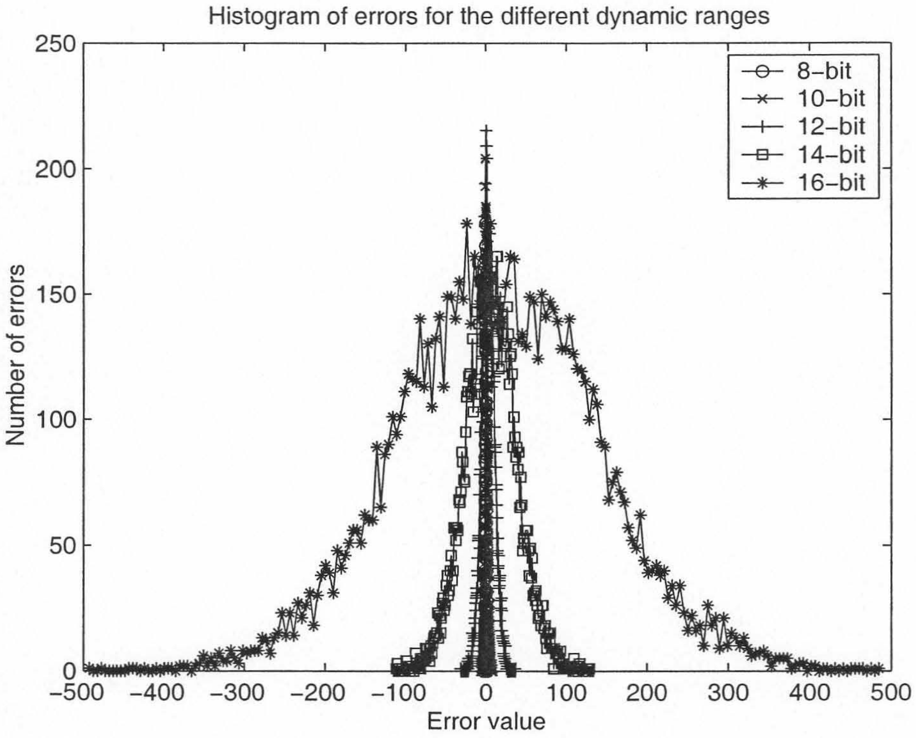

Figure 4.6: Histogram of the errors for different dynamic ranges

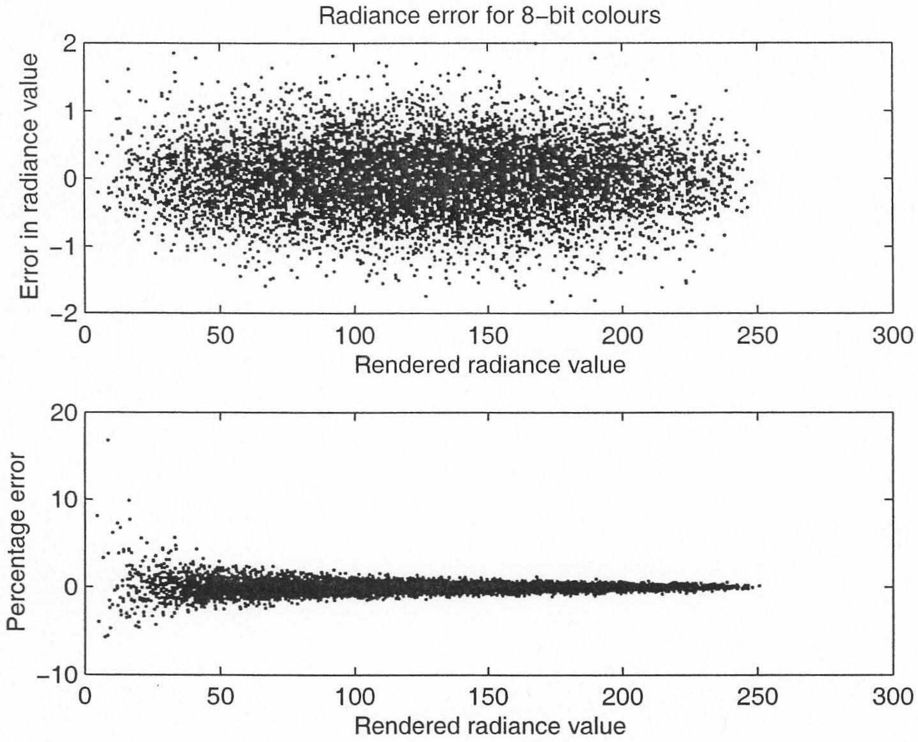

Figure 4.7: Difference between calculated and rendered radiance values - 8bit

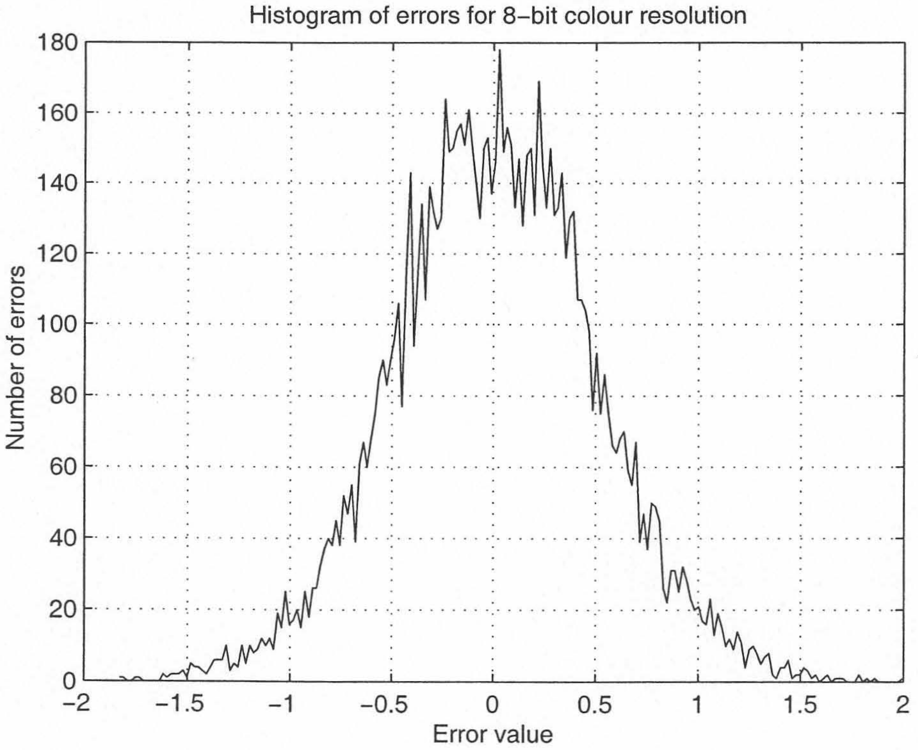

Figure 4.8: Histogram of difference between calculated and rendered radiance values - 8bit

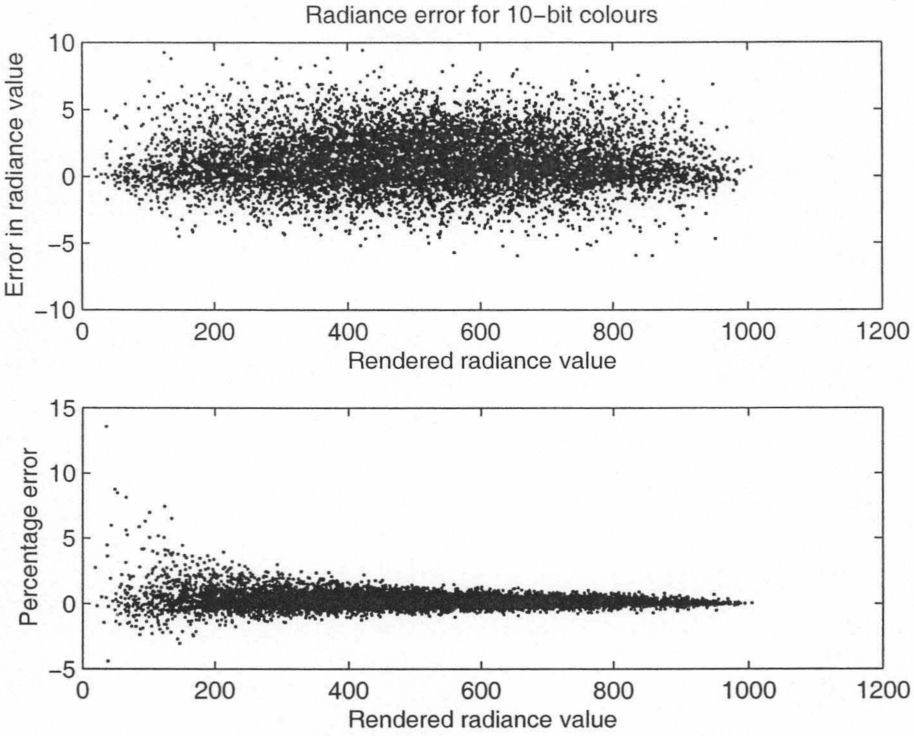

Figure 4.9: Difference between calculated and rendered radiance values - 10bit

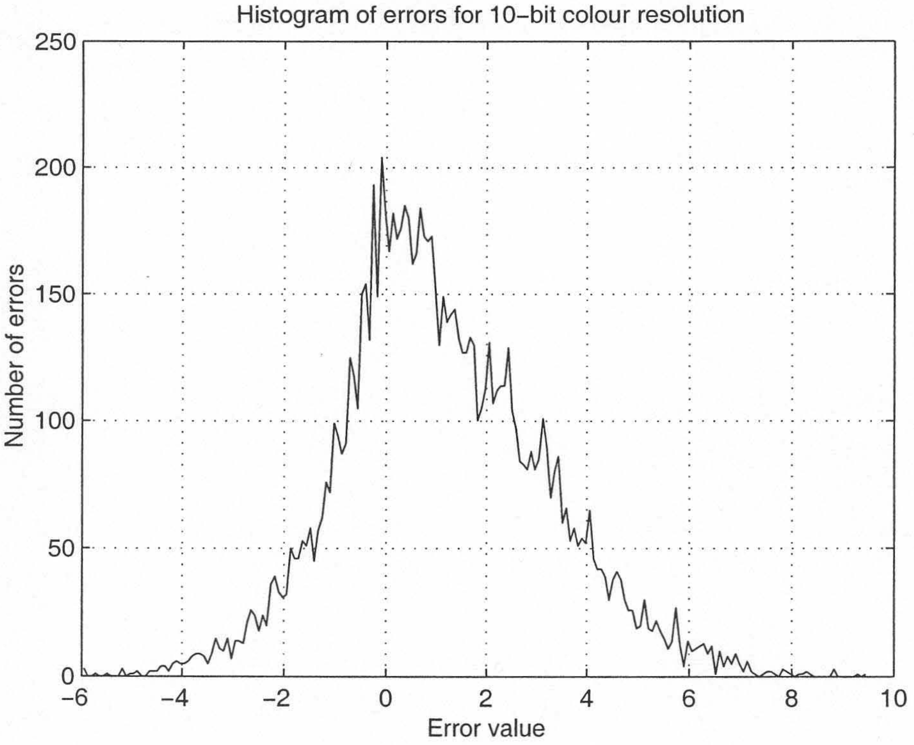

Figure 4.10: Histogram of difference between calculated and rendered radiance values - 10bit

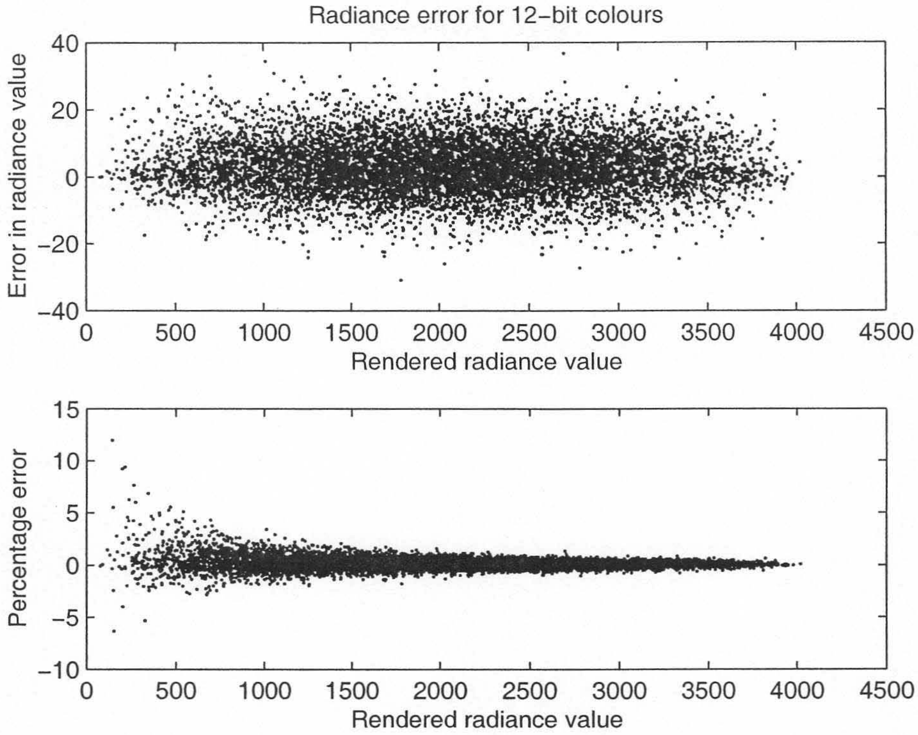

Figure 4.11: Difference between calculated and rendered radiance values - 12bit

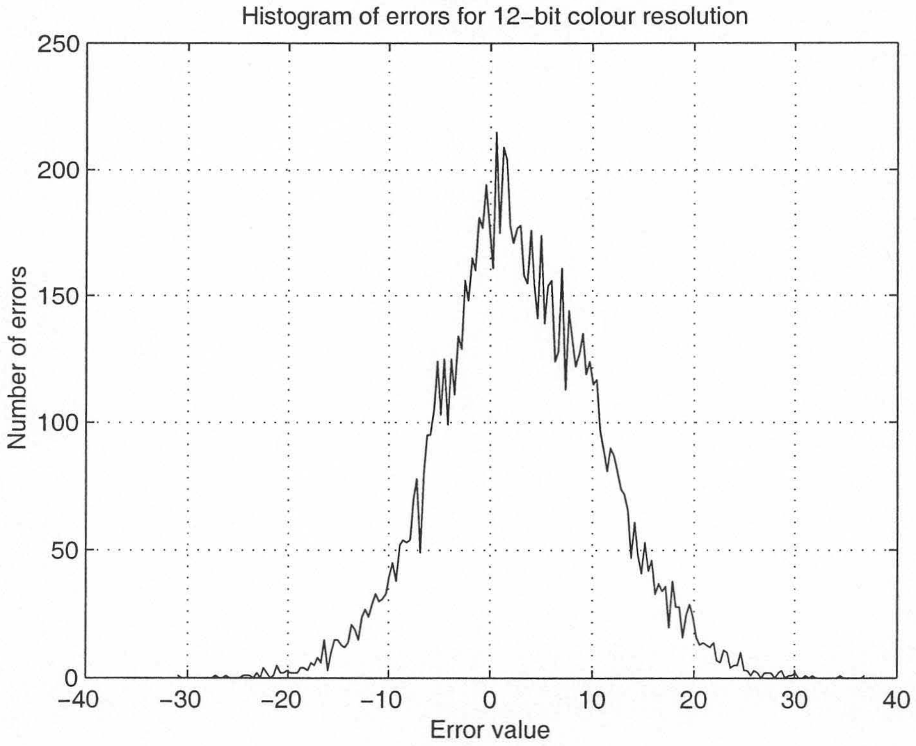

Figure 4.12: Histogram of difference between calculated and rendered radiance values - 12bit

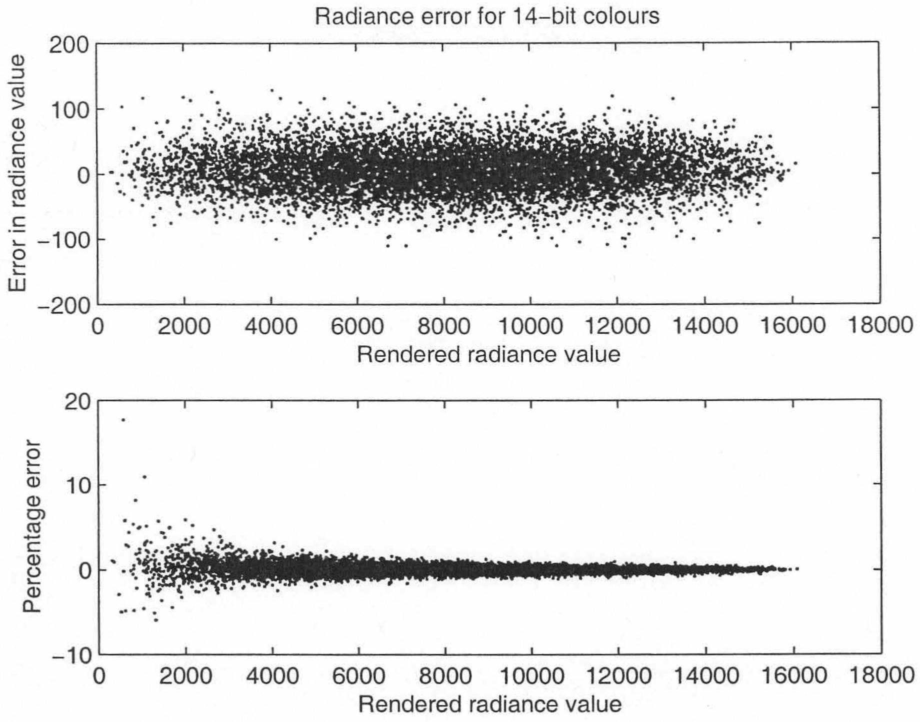

Figure 4.13: Difference between calculated and rendered radiance values - 14bit

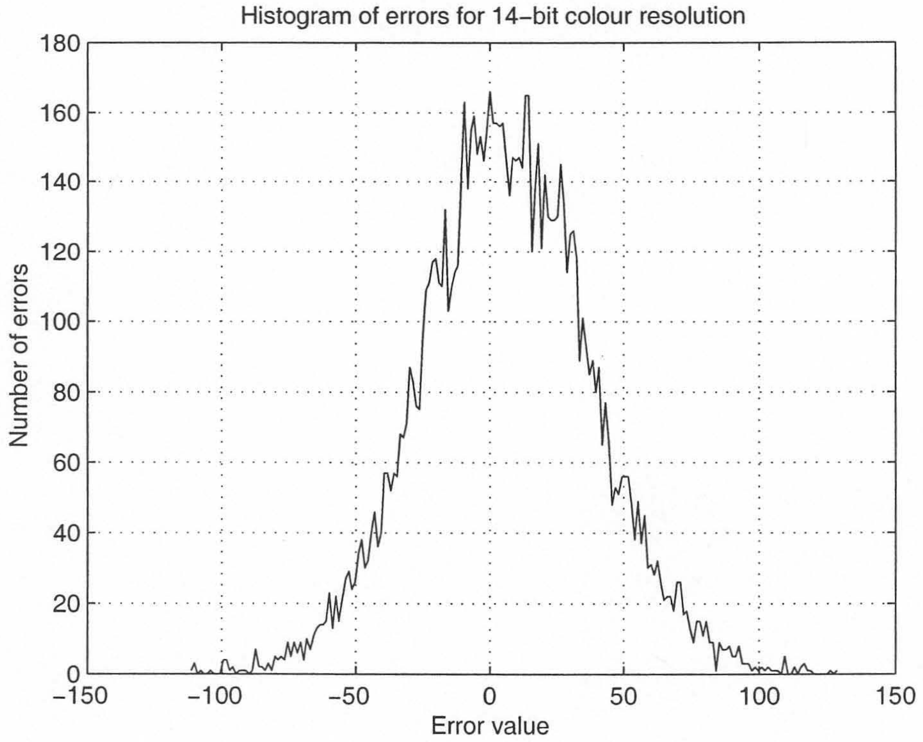

Figure 4.14: Histogram of difference between calculated and rendered radiance values - 14bit

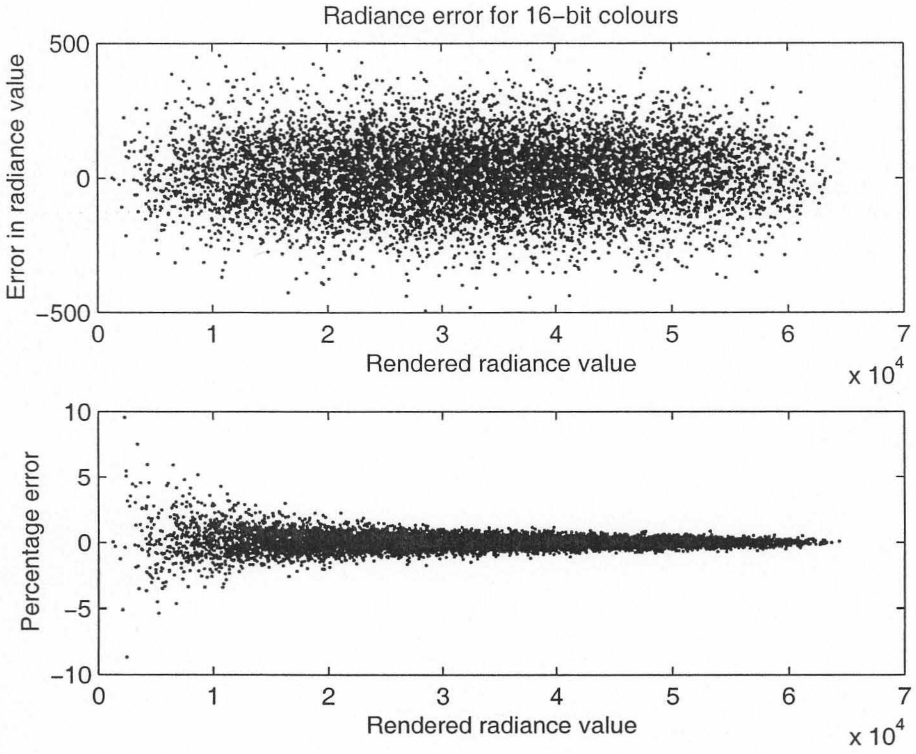

Figure 4.15: Difference between calculated and rendered radiance values - 16bit

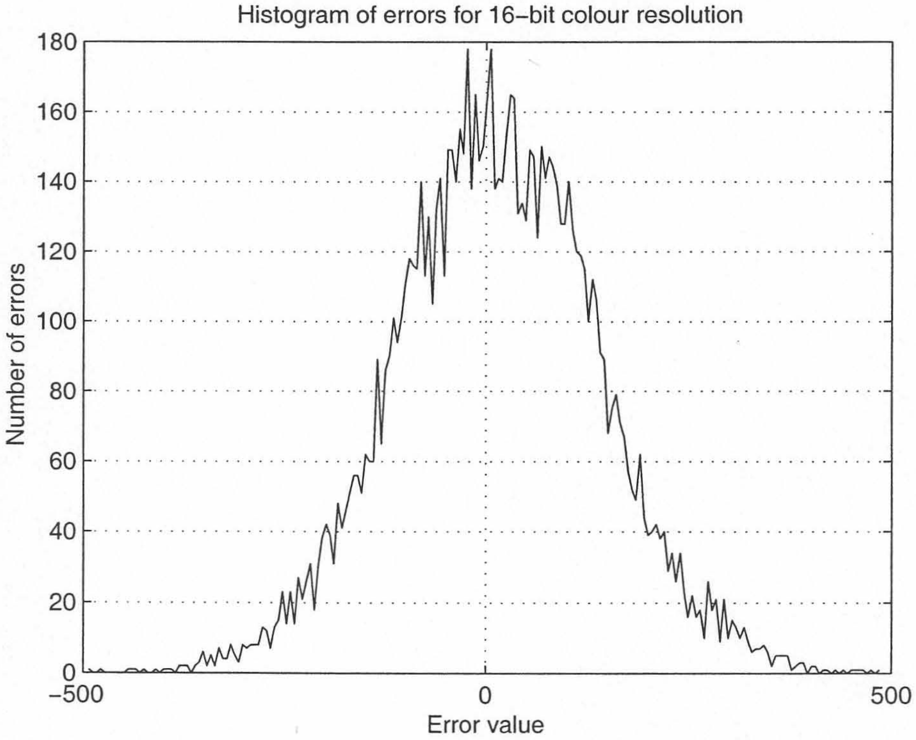

Figure 4.16: Histogram of difference between calculated and rendered radiance values - 16bit

4.4.2 Determining the error in intensity at increasing distances

The object was assigned the input radiance values shown in Table 4.2 for the second experiment. The intensity of the object was calculated using Equation (4.5) and the intensity was determined from the rendered image for different distances between the object and the sensor.

Table 4.2: Input values for the calculation of intensity

Resolution (bits)	Radiance of vertex 1 ($Wm^{-2}sr^{-1}$)	Radiance of vertex 2 ($Wm^{-2}sr^{-1}$)	Radiance of vertex 3 ($Wm^{-2}sr^{-1}$)	Calculated Intensity (Wsr^{-1})
8	105	250	96	300.667
10	105	250	968	882
12	1050	3250	368	3112
14	1050	250	12048	8898.667
16	1050	5250	36840	28760

Figures 4.17 to 4.21 show the rendered intensity as a function of the distance between the source and the sensor. The calculated intensity is also shown in each of the graphs. A summary of the errors is shown in Table 4.3.

The object was rendered every 10m, from 100m to 2100m. The field of view used in the experiment was 5°and the screen resolution was 512×512 pixels. The data shows a similar sawtooth trend in all the cases. There is therefore a range of distances, even at large distances, where the rendered intensity is within 1% of the actual intensity. It is therefore not possible to conclude that the intensity calculation can only be used up to a specific distance. The intensity at short distances is within 2% of the actual value in all the cases. The technique can be used to determine the intensity, with less than a 2% error, of objects at distances up to 200m from the sensor for the set of input parameters used in this experiment. A smaller pixel field of view would increase this distance and a larger pixel field of view would reduce it. The saw-tooth shape of the data might be due to a sampling effect, where the number of pixels filled with the test object varies as the distance between the observer is increased.

Table 4.3: Errors in calculating the intensity at different distances

Resolution (bits)	Maximum error (Wsr^{-1})	Maximum error (%)
8	46.6	15.5
10	132.54	15
12	519.7	16.7
14	1251	14.1
16	4051	14.1

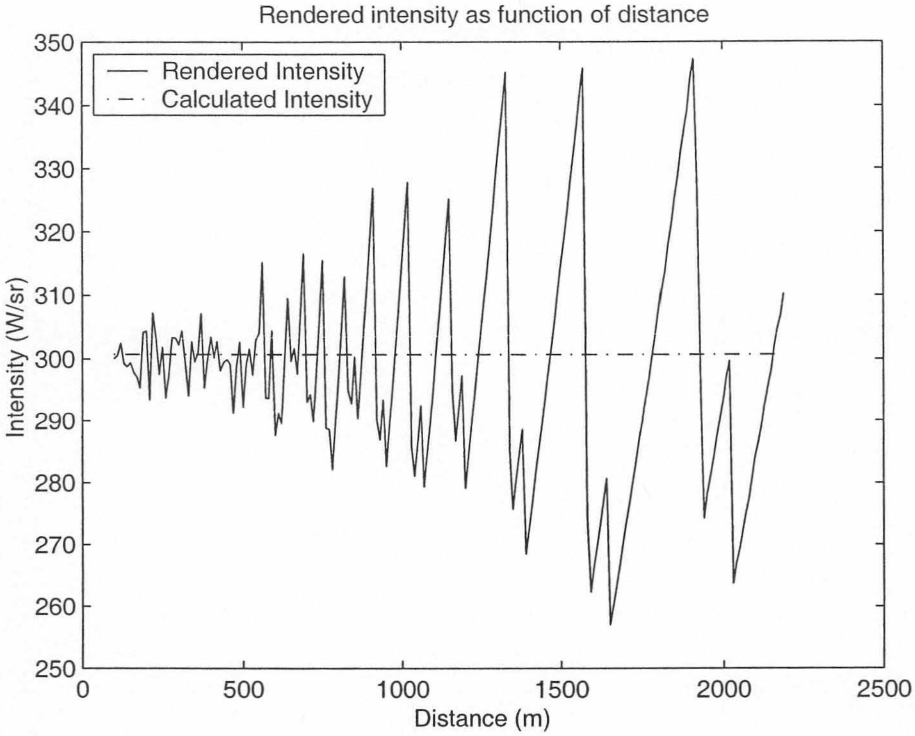

Figure 4.17: Rendered intensity values as function of distance - 8bit

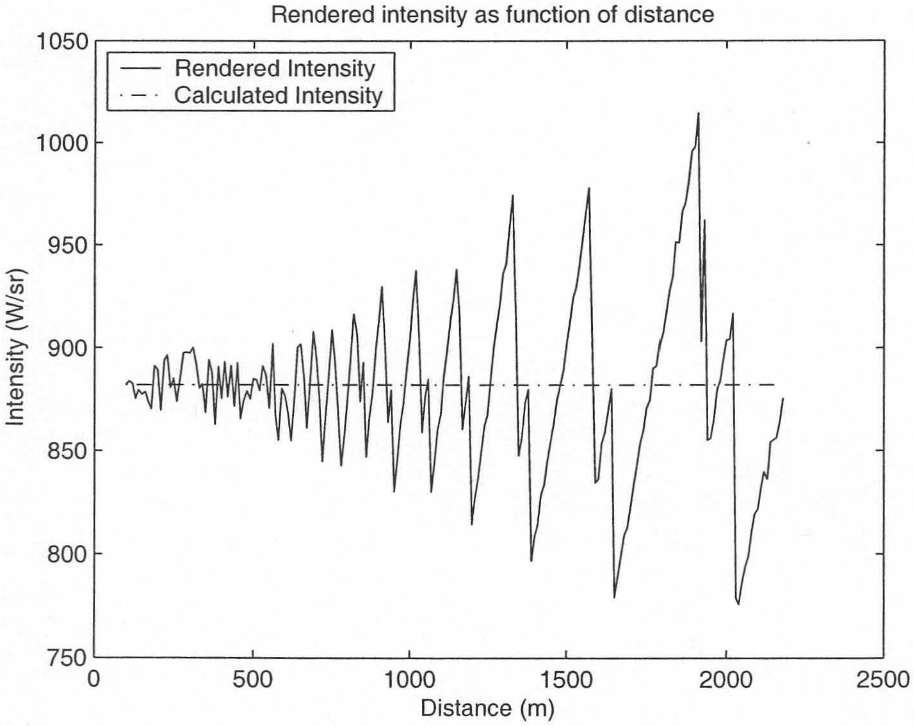

Figure 4.18: Rendered intensity values as function of distance - 10bit

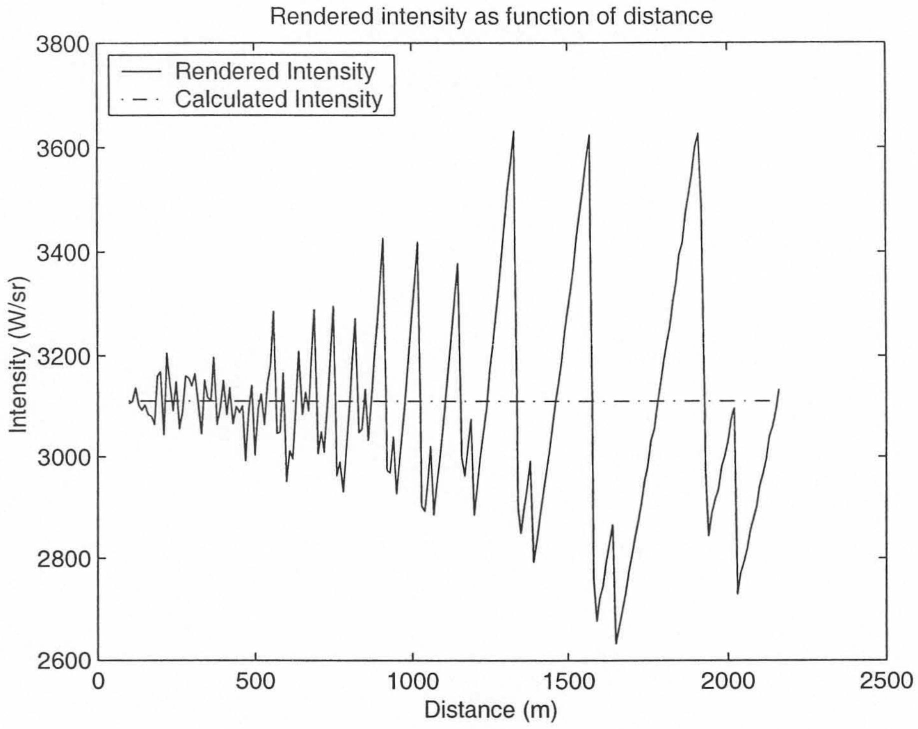

Figure 4.19: Rendered intensity values as function of distance - 12bit

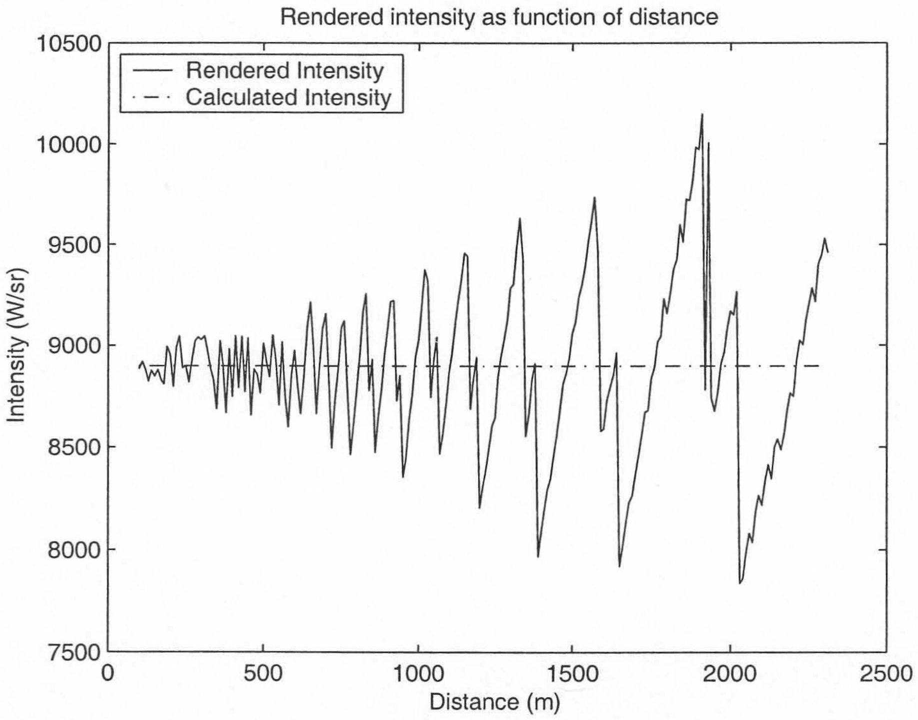

Figure 4.20: Rendered intensity values as function of distance - 14bit

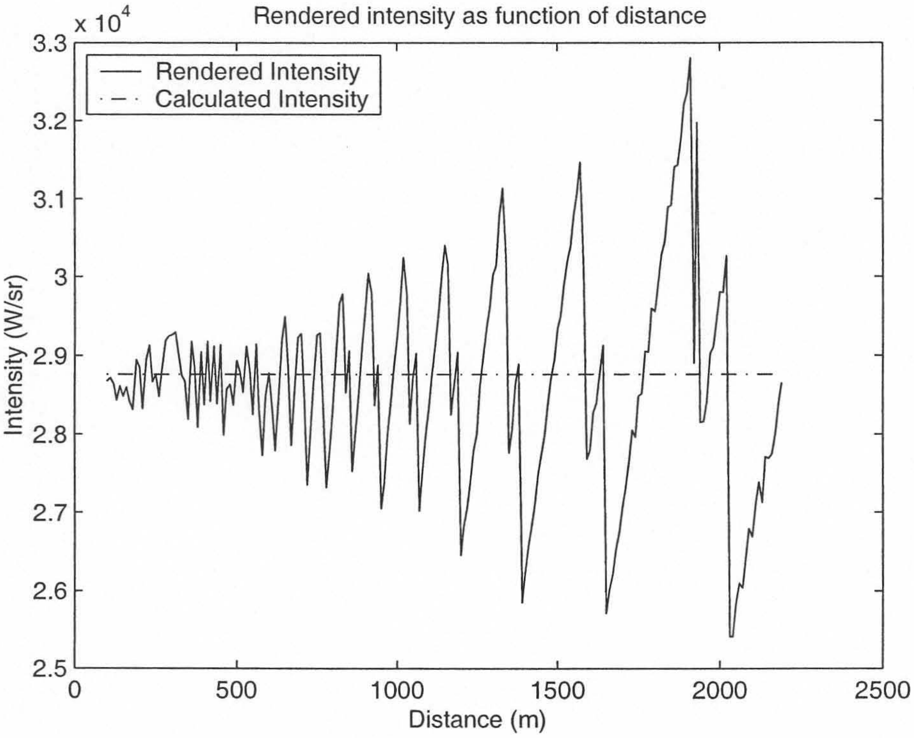

Figure 4.21: Rendered intensity values as function of distance - 16bit