A decision support model for the cash replenishment process in South African retail banking

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Dedicated with love to

my father, Kris

(the father of Industrial Engineering in South Africa),

my mother, Riekie

and my sons, Herman and Kristian.

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ABSTRACT

Title: A decision support model for the cash replenishment process in South African retail banking

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Degree: *Philosophiae Doctor* in the Faculty of Engineering

The objective of the research was to establish a scientifically-based decision-making procedure for determining the amount of cash to be held at a cash point at any time without compromising the customer service level or incurring undue cost. To reach the objective, the problem was divided into the following subproblems:

- To determine the cost parameters describing the nature of the problem of cash provision in South Africa.
- To investigate the characteristics unique to South African retail banking.
- To determine the nature of the demand distribution for a cash point.
- To develop a forecasting method appropriate for retail banking, although it was clearly stated that the methods used were specific to the branch studied.
- To investigate the existing order policies used by retail banks, as well as alternative order policies, with the aim of improving the cash replenishment process.

As a result of the investigation a generic decision model was developed which

may be used to improve the process at branch level for retail banks in South Africa. Some suggestions were also made regarding the implementation and maintenance of the model.

To investigate the cash replenishment problem, the cooperation of one of the leading retail banks in South Africa was obtained. A typical branch was selected. The total withdrawal, deposit patterns and the withdrawal patterns at the automated teller machines (ATM's) for a three month period during 1998 were investigated. The cost parameters relevant to the cash replenishment process were quantified. The approach followed was based on the classical inventory theory where the total cost of carrying inventory comprised three cost categories, *i.e.* storage cost, supply cost and shortage cost. Since the banks do not quantify the shortage cost, assumptions regarding the scope of the shortage cost had to be made.

The next step was to determine the cost of the existing order policy followed by the branch. This figure was used as a benchmark once alternate policies were investigated. The investigation resulted in alternate policies which significantly reduced the daily cost involved in carrying inventory as well as reduced the average amount of cash carried at the branch.

It was also shown, that the branch should consider using an appropriate forecasting method, since once forecasting was combined with an appropriate order policy, it was possible to reduce the cost of carrying cash inventories even further.

In conclusion, the research report suggested an implementation plan to be followed at branch level pointing out that certain changes to information systems were required. In addition, training needs were identified to enable the branch operations manager to successfully use the decision support model.

A comparison was drawn between the existing approach followed at the branch (which is mainly experience-based and largely of a random nature) to the proposed method. It was shown that the daily cost of carrying cash inventory could be reduced by 13 per cent per day. This represented a daily bottom line cost reduction of R358. At the time that the research was carried out, this retail bank had 75 similar branches. Should the saving at this representative branch be extrapolated, it shows a potential saving of R8 000 000 per year at this category of branch. It was further shown that the average cash inventory at this branch could be reduced by 52 per cent using the proposed method.

The study was limited to an investigation at one particular branch of a leading South African retail bank. The figures used to describe cash movements at the branch were of an extremely sensitive nature and were fairly difficult to obtain due to the way in which transactions are reported. The accuracy of the data provided by the branch could not be verified, but had to be accepted at face value. Although a particular case was investigated, a concerted effort was made to point out how the methodology may be used in the generic situation.

During the period under review, the branch relocated to a complex across the street from its previous location in a busy shopping mall. This had a direct impact on the ATM withdrawal patterns at the two ATM's located at the branch. In addition, soon after the research was carried out, a number of other branches of the same retail bank were consolidated into this one particular branch. This would impact on the validity of the branch specific factors determined as part of the research.

The study proved the applicability of industrial engineering principles in a service environment, where the added value of having the optimum cash amount available when required would impact directly on the bottom line of the bank and thereby enhance share-holder value. In the changing environment confronting retail banks, enhanced share-holder value is of the utmost importance to increase competitiveness and long-term survival.

Key terms

Decision support model

Cash replenishment

Cash replenishment cost parameters

Retail banking

Characteristics of retail banking in South Africa

Forecasting techniques

Order policies for cash replenishment

Deposit and withdrawal patterns

Implementation of decision support model

Industrial engineering in services

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SAMEVATTING

Titel: ʻn Besluitsteunmodel vir die kontantaanvullingsproses in Suid-Afrikaanse handelsbanke

Outeur: Susanna Aletta Adendorff

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Universiteit van Pretoria

Graad: *Philosophiae Doctor* in die Fakulteit Ingenieurswese

Die doel van die navorsing was om ʻn wetenskaplik gefundeerde besluitprosedure daar te stel om te bepaal hoeveel kontant op enige tydstip by ʻn kontantvoorsieningspunt gehou moet word, sonder om die diensvlak aan kliënte te kompromitteer en sonder om onnodige koste aan te gaan. Om hierdie doelwit te bereik, is die probleem in die volgende subprobleme verdeel:

- Om die kosteparameters te bepaal wat die aard van die probleem van kontantvoorsiening in Suid-Afrika beskryf.
- Om die unieke eienskappe van die Suid-Afrikaanse handelsbankwese te ondersoek.
- Om die verdeling wat die vraag na kontant by ʻn voorsieningspunt beskryf te bepaal.
- Om ʻn vooruitskattingsmetode te ontwikkel wat geskik is vir handelsbankwese, alhoewel dit uitdruklik gestel is dat die metodes wat ondersoek is, spesifiek was aan die tak wat bestudeer is.
- Om die bestaande bestelbeleid wat deur handelsbanke gevolg word te ondersoek, asook om na alternatiewe te kyk met die doel om die kontantaanvullingsproses te verbeter.

Na aanleiding van die navorsing is 'n generiese besluitmodel ontwikkel, wat aangewend kan word om die proses op takvlak in Suid-Afrikaanse handelsbanke te verbeter. Enkele voorstelle is ook gemaak aangaande die implementering en instandhouding van die model.

Om die kontantaanvullingsprobleem te ondersoek, is die samewerking van een van die toonaangewende handelsbanke in Suid-Afrika verkry. 'n Tipiese tak is geselekteer. Die totale onttrekkings-, deposito- en geoutomatiseerde teller-masjienonttrekkingspatrone oor 'n periode van drie maande gedurende 1998 is ondersoek. Die kosteparameters relevant tot die kontantaanvullingsproses is gekwantifiseer. Die benadering wat gevolg is, was gebaseer op die klassieke voorraadteorie waar die totale koste van voorraadhouding uit drie koste-kategorieë bestaan, naamlik, houkoste, bestelkoste en tekortekoste. Aangesien banke versuim om die tekortekoste te kwantifiseer, was dit nodig om aannames te maak aangaande die tekortekoste.

Die volgende stap was om die koste van die bestaande bestelbeleid wat deur die tak gevolg word te bepaal. Hierdie syfer is as 'n baken gebruik tydens die ondersoek na alternatiewe tot die bestelbeleid. Die ondersoek het gelei tot alternatiewe bestelbeleide wat die daaglikse koste van die hou van voorraad betekenisvol verminder asook die gemiddelde hoeveelheid kontant wat deur die tak gehou word, verlaag.

Daar is ook aangetoon dat die tak oorweging moet skenk aan die gebruik van 'n geskikte vooruitskattingsmetode, aangesien die kombinasie van 'n gepaste vooruitskattingsmetode en 'n geskikte bestelbeleid tot verdere veminderings in die voorraadhoukoste gelei het.

Ten slotte het die verslag 'n implemeteringsplan voorgehou wat op takvlak gevolg kan word, wat uitgewys het dat bepaalde veranderings wat betref die inligting-stelsel nodig is. Verder is opleidingsbehoeftes geïdentifiseer wat die operasionele bestuurder van die tak in 'n posisie sou stel om die besluitsteunmodel suksesvol te kan aanwend.

‘n Vergelyking is getref tussen die bestaande benadering wat gevolg word by die tak (wat hoofsaaklik op ondervinding gebaseer en grootliks op toevalswyse uitgevoer word) en die voorgestelde metode. Daar is aangetoon dat die daaglikse koste van voorraadhouding met 13 persent per dag verminder kan word. Dit het ‘n daaglikse besparing van R358 bewerkstellig. Ten tye van die studie het hierdie handelsbank 75 soortgelyke takke gehad. Sou die besparing by hierdie verteenwoordigende tak geëkstrapoleer word, sou dit ‘n potensiële besparing van meer as R8 000 000 per jaar impliseer by hierdie kategorie takke. Daar is verder getoon dat die gemiddelde kontantvoorraad by hierdie tak met 52 persent verminder kon word deur die voorgestelde metode te gebruik.

Die studie was beperk tot ‘n ondersoek by een spesifieke tak van ‘n toon-aangewende Suid-Afrikaanse handelsbank. Die syfers wat gebruik is om die kontantbewegings by die tak te beskryf was van ‘n uiters sensitiewe aard en betreklik moeilik om te bekom vanweë die wyse waarop transaksies gerapporteer word. Alhoewel ‘n spesifieke geval ondersoek is, is ‘n doelgerigte poging aangewend om aan te dui hoe die metode ook in die generiese situasie toepassing vind.

Tydens die navorsingsperiode het die tak hervestig na ‘n gebou oorkant die straat van waar dit tevore in ‘n besige winkelsentrun geleë was. Dit het ‘n direkte impak gehad op die ontrekkings by die twee geoutomatiseerderde tellermasjiene (OTM’s) wat by die tak geleë is. Bykomend, pas nadat die navorsing voltooi is, het konsolidasie van verskeie takke binne dieselfde bankgroep plaasgevind. Dit sou ‘n impak hê op die geldigheid van die takspesifieke faktore wat tydens die studie bepaal is.

Die studie het die toepaslikheid van die beginsels van bedryfsingenieurswese in ‘n diensomgewing bewys, waar die bykomende waarde van die hou van die optimum kontanthoeveelheid soos benodig, ‘n direkte impak op die winsgewend-heid van die bank sal hê, en sodoende die waarde van die aandeelhouersbelang sou verhoog. In die hedendaagse, veranderende omgewing wat handelsbanke konfronteer, is verhoogde waarde van aandeelhouersbelang krities vir verbeterde mededingendheid en langtermynoorlewing.

Sleutelterme

Besluitsteunmodel

Kontantaanvulling

Kontantaanvulling kosteparameters

Handelsbankwese

Eienskappe van handelsbankwese in Suid-Afrika

Vooruitskattingstegnieke

Bestelbeleide vir kontantaanvulling

Deposito- en onttrekkingspatrone

Inplementering van besluitsteunmodel

Bedryfsingenieurswese in dienste

Preface

The application of industrial engineering in a service environment has always been of great interest to me. I have always been of the opinion that much scope exists for the expansion of the role of industrial engineers specifically in the service industries in South Africa. As a result of this belief and interest, I decided to undertake the research in the field of South African retail banking. My conviction was confirmed by the results of the research project and I trust that the results and conclusions of the research will stimulate further work in this field.

On completing a work such as this, it is appropriate to give thanks to the people who were directly involved or merely provided the crucial support to make the research possible. I am indebted to the following people:

- The decision makers at the retail bank who were prepared to give me access to the required information;
- the staff at the branch who were extremely cooperative and had enormous patience in answering all my questions;
- my supervisor, Prof Paul Kruger in the Department of Industrial and Systems Engineering at the University of Pretoria; and
- my parents and my sons, Herman and Kristian, who supported me throughout the endeavour.

I hereby declare that this is my own work.

Signed in Pretoria on 19 September 1999.

S A ADENDORFF

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Acronyms and terminology

ABA	American Bankers' Association
ABSA	Amalgamated Banks of South Africa
ATM	Automated teller machine
Banking Council	The Banking Council - South Africa
CIMC	Crime Information Management Centre
COSAB	Council of South African Banks
FIT	Forecast including a trend
MAD	Mean absolute deviation
MAPE	Mean absolute percent error
MSR	Moving seasonal relatives
SD	Standard deviation
SSR	Simple seasonal relatives
Reserve Bank	South African Reserve Bank
RSFE	Running sum of forecast errors
RSME	Root mean square error
SAPS	South African Police Service
SARB	South African Reserve Bank
South Africa	Republic of South Africa
TS	Tracking signal
UK	United Kingdom
USA	United States of America

NOTE: *Throughout this document, the decimal point is used to denote the decimal separator. This is done to comply with standard practice in the banking environment.*

Definition of variables

Variable	Definition	Unit
C	Total cost of cash handling	Rand/period
C_1	Storage cost	Rand/period
c_{11}	Cash float cost	Rand/unit period
c_{12}	Insurance cost	Rand/unit period
c_{13}	Labour cost element of storage	Rand/unit period
C_{13}	Labour cost involved in storage	Rand/period
C_2	Shortage cost	Rand/period
C_3	Supply cost	Rand/period
C_{31}	Order cost element	Rand/period
c_{31}	Unit replenishment cost	Rand/order
c_{311}	Internal order and processing cost for SBV visit	Rand/order
c_{312}	Internal order and processing cost for agency visit	Rand/order
C_{32}	Total cash processing cost	Rand/period
C_{33}	Transportation cost element	Rand/period
c_{331}	Normal unit transportation cost	Rand/order
c_{332}	Interim unit transportation cost	Rand/order
C_{34}	In-transit insurance cost	Rand/period
c_{34}	In-transit insurance cost	Rand/unit period
Q_O	Demand for current scheduling period	Units
$\sum Q_i$	Amount held in current scheduling period	Units
Q_D	Amount ordered from cash centre per delivery	Units
D	Total demand in planning period	Units
Q_A	Amount ordered/returned from agencies per delivery	Units
A	Total demand at agencies during planning period	Units
Q	Order quantity	Rand
SQ	Special order size	Rand
SS	Safety stock level	Rand
I_0	Initial inventory	Rand
UL	Upper limit	Rand
RA	Return amount	Rand
ROP	Reorder point	Rand

CHAPTER 1

The scope of the research

1.1 General background to the problem

1.1.1 The scope of the problem

Retail financial services have been evolving at a great pace in the recent past. Issues that have had to be addressed include (KPMG *s.a.*:6):

- Declining margins;
- consolidation;
- increasing shareholder demands;
- new regulatory pressures;
- growing complexities and costs of technology;
- new market entrants; and
- higher customer expectations.

In this changing global financial services market, the role of retail banks is continuously being redefined, specifically with reference to the provision of cash to a global society, which despite the move towards a cashless world, continues to prefer cash as a method of payment in concluding transactions. It is claimed that 50% of all payments world-wide are made with notes and coin. The question of cash remains central to how banks perceive their roles and strategies as well as how they are perceived by their customers. (De La Rue *s.a.*:2)

Given the fact that retail banks have to provide for the needs of their customers with regard to cash, the following illustrates the scope of the problem:

- Retail banks have to carry certain amounts of cash at branches, agencies and in automated teller machines (ATM's), but in doing so incur certain costs.
- The nature of the cost of providing cash facilities, has to a certain extent, been obscured due to a reluctance to regard cash as an inventory item. This unwillingness is most probably the legacy of accounting practice, which must be appreciated.
- The cost elements involved in providing cash facilities *inter alia* include holding cost, insurance cost, transportation cost, processing cost and shortage cost.
- Various other factors exacerbate the cash replenishment problem, for example, unpredictability in demand patterns and unreliability in supply lead time.

The result of the above is that retail banks tend to hold excessive cash at their various cash points (be it a branch, agency or ATM). If a holistic view of the problem is taken, identifying the true nature of all of the costs involved, without overemphasizing a single element, it would lead to a reduction in the amount of cash held at a cash point and it would minimise the unnecessary movement of cash, an activity that hardly adds value from the customer's perspective. It is claimed that *"cash frustrates bankers because the customer is reluctant or unwilling to pay for cash services"* (De La Rue *s.a.*:9).

Although the problem of providing cash in the correct quantities, and denominations, and at the right time, is common world-wide, a number of factors specific to the Republic of South Africa contribute to the extent and scope of the problem locally. Oosthuysen (1995:3) states that South African banks in general are faced with a number of challenges such as increased local and international competition, an increase in fraud and money laundering activities, bank robberies, customer resistance to excessive price increases as well as aggressive and innovative marketing initiatives. The specifics of the South African situation are discussed further in Chapter 3.

1.1.2 Quantifying the need for a solution

During an interview with representatives of De La Rue Cash Systems, UK, the following claims were made to illustrate the ignorance which exists globally with regard to the problem of optimising cash provision:

- 20% of all retail banks world-wide are aware of the problem of optimising cash provision and are attempting to address it.
- 30% of all retail banks world-wide are considering addressing the problem, but are not yet doing it.
- 50% of all retail banks are not even aware of the problem or of the benefits that may accrue if the problem is addressed.

1.1.3 Sources of profit in retail banking

The increasingly competitive and complex environment of retail financial services has compounded the focus on key profit drivers and has led to fundamental questions about the management and exploitation of distribution channels (KPMG *s.a.*:6). It is important to note the two major sources of profit for retail financial service institutions. The first source of revenue is loan activities, *i.e.* the difference between income on funds lent and the cost of deposits. The second source is commission and the fees recovered for financial services rendered (Falkena *et al.* 1995:68). From the above, it is obvious that the cost of deposits detracts from the first source of income. Therefore if anything can be done to reduce the cost of deposits, it will lead to an increase in profit.

In a thesis submitted to the Department of Applied Accountancy, UNISA, in 1995, Oosthuysen investigates the problems with current management information reporting in South African banks, which is not timeous and lacking in reliability. He states, *inter alia*, that information regarding the cost of a product and the profitability of a product is not readily available. The provision of cash is but one of the products (or services) provided by retail banks. As stated by De La Rue (*s.a.*:2): *"As banks attempt to unbundle costs to identify cross-subsidy across product lines, the cost of cash has come in for increasing scrutiny"*. Therefore any investigation into the cost of providing this service will be to the advantage of the industry.

1.2 Definitions

According to Falkena *et al.* (1995:68) a commercial bank is defined as *"... an institution carrying on a business of which a substantial part consists of the acceptance of deposits of money withdrawable by cheque, draft or order"*. This definition covers the most important activities carried out by commercial banks, although a wide variety of other functions are performed by these institutions. Most commercial banks are mainly involved in retail services, *i.e.* aimed at the individual client, although the emphasis is shifting to corporate or wholesale services. Due to the emphasis on individual banking services, this type of institution is often referred to as a retail bank. As various "non-banks" enter the financial services market (refer to paragraph 3.5.3 for further elucidation in this regard), it is perhaps cash handling that distinguishes and defines the retail bank (De La Rue *s.a.*:2).

1.3 Objectives of the research

1.3.1 Formulation of objectives

The main objective of the research is to establish a scientifically-based decision-making procedure for optimising the amount of cash to be held at a cash point (be it branch, agency or ATM) at any time without compromising the customer service level or incurring undue cost. In reaching the objective, the problem has been divided into a number of subproblems, each having its own objective. The subproblems are as follows:

- Determining the cost parameters describing the nature of the problem of cash provision in a South African context.
- Investigating the characteristics unique to the South African retail banking environment.
- Determining the nature of the demand distribution (a function of deposits and withdrawals) for a cash point.
- Developing a forecasting method appropriate for the retail banking environment in South Africa.
- Investigating the existing order policies used by retail banks, as well as alternative order policies, with the aim of improving the process of cash replenishment.

As a result of the investigation into these subproblems, a generic decision model is developed which may be used to optimise the cash replenishment process for retail banks. The decision model is used to evaluate the impact of changes in the nature of the cost parameters, changes in the demand distribution, as well as the impact of other factors, for example unpredictable lead time.

1.3.2 The use of management science in banking

Miller & Orr (1967:133) commented as follows on the use of operations research in the field of finance:

> *One stream of current research in finance involves the extension to the field of finance of the methods and approaches that have come to be called 'operations research' or 'management science'. Researchers working along these lines try to develop mathematical representations or 'models' of typical decision making problems in finance and, where they are given the opportunity to do so, to test and apply these models in actual decision settings.*

They continue to comment that this research is a mere trickle if compared to work done in other fields, but is expected to grow as the computer technology develops.

In 1978, Kalman (1978:16) commented as follows on the use of Management Science in the field of finance:

> *Recent advances in the development of quantitative models for finance and banking, along with the increasing utilization of electronic computers, have made it feasible to adopt a management science framework in approaching many types of financial problems.*

Concurrent with the previous comment, Fabozzi & Trovato (1978:24-29) reported on a study that was carried out, to establish the use of quantitative techniques (such as linear programming, queueing theory, simulation, game theory, statistical sampling and so forth) by commercial banks. The study included 92 banks, of which 13 were non-users of quantitative techniques. A correlation was established between the size of the bank (in terms of deposits) and the use of the techniques - smaller banks did not make use of the techniques. When asked why the techniques were not used, the main reason was a lack of understanding of these techniques, rather than the expected answer of high cost. The report on the study concluded by predicting that the use of these techniques would definitely increase especially as top management realised that such models do not replace decision makers, but assist them in making improved decisions.

Eilon & Fowkes (1972:1-24) provide a good overview on the use of Management Science in banking and finance, whereas Brunsen (1976:1-6) specifically provides an overview of the use of linear programming as a bank management tool.

1.3.3 The suitability of inventory models in approaching the problem

As early as 1952, Baumol published an article in the Quarterly Journal of Economics titled *The transactions demand for cash: An inventory theoretic approach* (Homonoff & Mullins 1975:3). In this model the transaction demand decisions were formulated as a deterministic inventory control problem. At the Graduate School of Business Stanford Finance Conference held in June 1966, Miller & Orr presented a paper on *An application of control-limit models to the management of corporate cash balances* (Miller & Orr 1967:133-151). Homonoff & Mullins (1975) expanded on the Miller-Orr model in a book titled *Cash Management: An inventory control limit approach.* None of these publications specifically investigated the position of a retail bank relating to cash management. However, the research certainly points to the suitability of the underlying theory to solve this class of problem.

1.4 Research methodology

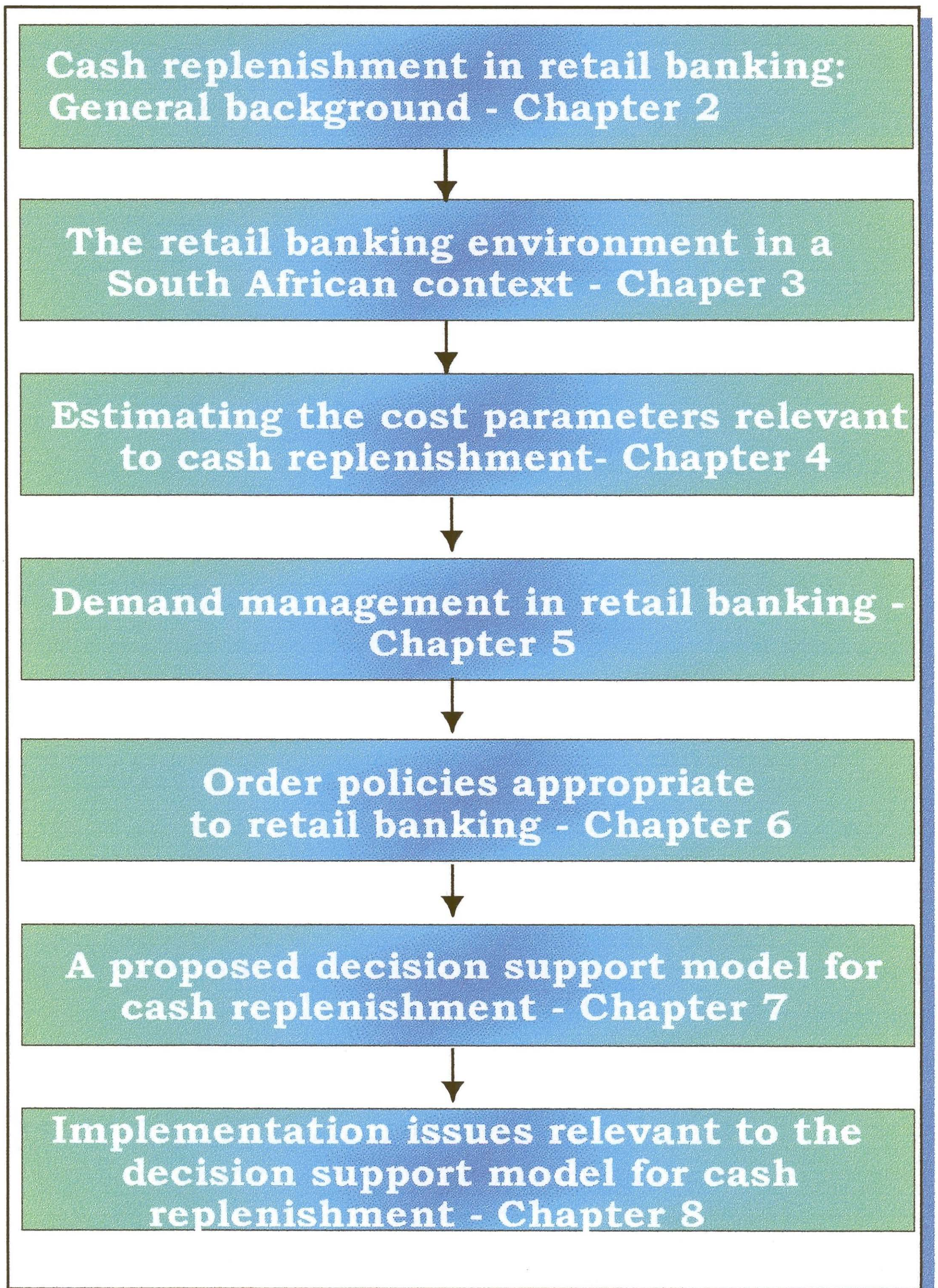
Figure 1.1 provides a graphic overview of the chapters of the research report.

The methodology used to investigate the use of inventory theory to model the cash management problem experienced by retail banks, was the following:

- The general problem was investigated as determined by the features of the specific situation in retail banking, culminating in the development of a conceptual mathematical model of the total cost of handling cash - reported on in Chapter 2 of the thesis.
- The features of the problem specific to the South African retail banking environment were investigated - reported on in Chapter 3 of the thesis.
- The cost parameters, as included in the conceptual model of the total cost of handling cash, were empirically determined by means of a case study - reported on in Chapter 4 of the thesis.

Figure 1.1

An overview of the research into the cash replenishment problem in retail banking



- Chapter 5 investigates the nature of demand and withdrawal patterns for the case study presented in Chapter 4. Since no formal demand forecasting takes place at this particular branch, various methods are proposed to forecast the demand.
- In Chapter 6, the existing order policy of the branch described in Chapter 4 is investigated so as to obtain clarity regarding the impact of the various parameters on the problem. Alternative policies are investigated and evaluated.
- Chapter 7 proposes the structure for a decision tool for use in a retail bank.
- Chapter 8 addresses implementation issues relevant to the successful use of the decision tool in cases other than the one under review.

1.5 Limitations of the study

The study was limited to an investigation at one particular branch of a leading South African retail bank. The figures used to describe cash movements at the branch obviously were of an extremely sensitive nature and were fairly difficult to obtain due to the way in which transactions are reported. The accuracy of the data provided by the branch could not be verified, but had to be accepted at face value. Although a particular case was investigated, a concerted effort was made to point out how the methodology may be used in the generic situation.

During the period under review, the branch relocated to an office complex across the street from its previous location in probably one of the busiest shopping malls in the city. This had a direct impact on the ATM withdrawal patterns at the two ATM's located at the branch. As the branch operations manager stated at the time, it took the customers almost a month "to find us again", before the withdrawal patterns started to normalise, although at a level much lower than before due to the lack of passing traffic. In addition, soon after the research was carried out, a number of other branches of the same retail bank were consolidated into this one particular branch. This would impact on the validity of the branch specific factors determined as part of the research.

In addition changes at SBV, the sole supplier in this particular case, as regards ownership may have a bearing on the supply cost structure. Rumours of a management buy-out or a take-over by the South African Reserve Bank in contrast to the current ownership may impact directly on how the cash is supplied. Some of the constraints adhered to by the branches at present, for example a single delivery per week is preferred, may then be challenged.

1.6 Contribution to knowledge base

Although limited to a particular branch, the study proved the applicability of industrial engineering principles in a service environment, where the added value of having the optimum cash amount available when required would impact directly on the bottom line of the bank and thereby achieve a cost reduction which can only enhance share-holder value. In the changing environment confronting retail banks in South Africa, enhanced share-holder value is of the utmost importance to increase competitiveness and long-term survival.

CHAPTER 2

The complexity of cash replenishment in retail banking

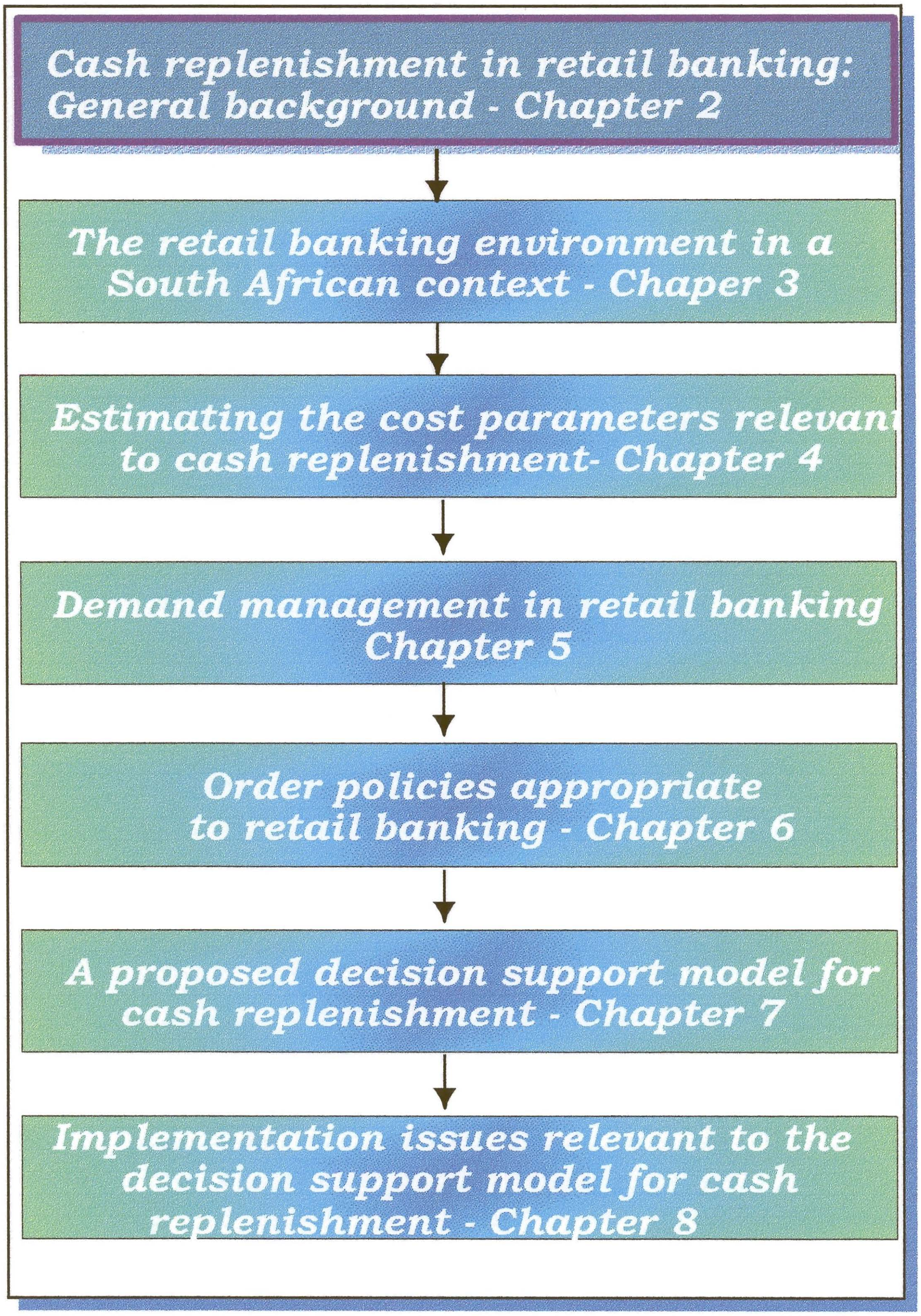
2.1 Introduction

Research commissioned by De La Rue (*s.a.*:2) indicates that any cash handling strategy must be built on individual national environments, bank business mixes together with an understanding of the industry (such as the role of monetary authorities and competition in the industry), an understanding of customer needs, an understanding of the costs involved in cash handling and taking a systems view of the problem.

In this chapter, the complexity of cash replenishment in retail banking is discussed at length. The characteristics of the problem which are discussed include the nature of the mix of cash withdrawals and deposits, factors influencing the demand for cash, the perspective of the customer in respect of good service with specific reference to cash availability, the specifics of cash handling from the point of view of a retail bank, the cost elements involved in cash handling and the effect of the planning or scheduling period. Figure 2.1 shows the relevance to this chapter as regards the subsequent chapter s included in this report.

Figure 2.1

The structure of the report indicating the relevance of Chapter 2



2.2 Problem description

2.2.1 Preamble

A part of the business conducted in retail banking concerns the provision and receipt of cash as required by the customers in turn to conduct their business, whether it is private or commercial in nature. A driving force in the continued high usage of cash despite the development of various other methods of payment is the immediacy and convenience of cash for the individual (De La Rue *s.a.*:5). In studying the cash replenishment problem in retail banking, the specific characteristics of the situation determine the nature of the problem.

In the South African banking system cash replenishment at branches, agencies and automated teller machines (ATM's) of retail banks, takes place on a daily basis within the following framework:

- Cash replenishment occurs by means of a single delivery per day at the normal reorder cost. An interim delivery is possible, but the order cost is significantly higher in such a case. (Refer to paragraph 2.3.2.3 in this regard.)

- Fourteen different denominations (coin and notes) represent the South African currency in circulation. (Refer to paragraph 3.3.1 for circulation figures.)

- A shortage situation is highly unacceptable at a branch or agency, due to the perception formed in the mind of the customer and the ripple effect this may have. At an ATM, a shortage may occur, and is tolerated by the customer, since it is accepted that an ATM may run out of cash from time to time (especially when the location is remote from a branch or agency). Often the customer is unaware of the exact reason for a transaction not being processed at an ATM and therefore accepts a shortage situation.

Cash balances held in branches, agencies and ATM's of a bank represent a sizable amount of unproductive capital and it is therefore in the interest of the bank to reduce such amounts to as low a level as is practical and possible. As stated by Derwa (1978:111):

> *The problem is part of the general class of stock problem. The question is one of determining the amounts to be delivered and the delivery dates that will minimise the total cost, which is the weighted sum of the costs of storage, supply and shortage.*

Johnson (1994:31-33) makes the following statement when describing the traditional bankers' approach to cash: *"Cash is certainly an expensive commodity, but most bankers contend that it is an inevitable expense in the current system."* Wagner (1969:786-787) provides an apt description of the insensitivity in business to the relevance of inventory management, which indeed is the case in retail banking. The employees in this environment in South Africa show an ignorance of the scientific approach to inventory management when cash balances are discussed. Not only is ignorance evident, an unwillingness exists to treat cash as an inventory item. As Miller & Orr (1967:133) state: *"It may be a little startling at first to think of your firm's cash balance as just another inventory – an inventory of dollars so to speak – but is it really so farfetched?"*

Some of the features of the problem require further elucidation.

2.2.2 Cash mix: deposits versus withdrawals

An important complication with regard to cash provision, is the mismatch both in terms of location and timing of cash needs (De La Rue *s.a.*:2). If deposits and withdrawals are compared, there is a significant difference in the mix in terms of denominations. The result of this difference may lead to a situation where on a particular day, the total amount of cash available may be sufficient to provide for the expected demand, but the mix is incorrect. A situation may therefore result where the amount of cash ordered equals the amount returned to the depot, but the composition differs, necessitating a denomination switch.

Johnson (1994:31-33) claims that most of the cash received by a bank is cash on deposit. The customer base of a particular branch will therefore have an effect on the nature of the deposits received. If the customer base is largely retail, huge amounts of cash will be deposited, whereas a predominantly household/private individual customer base will lead to a different deposit pattern.

In addition, some substitution exists between some of the denominations. For example, should a customer request a withdrawal in R100 notes, if unavailable, the customer would most probably accept either R50 notes or R200 notes. It may be assumed that a natural barrier exists with regard to substitution between R5 coin and R10 notes.

There are indeed limits with regard to legal currency in any transaction. The maximum amounts concerned are R50 when coins of denominations of R1 and higher are presented; R5 when coins of denominations 10c to 50c are presented and fifty cents when coins of denominations 5c or less are presented (Falkena *et al.* 1999:53). Substitution however implies a reduced service level, since the customer is not receiving exactly what is required. The issue of the customer's perspective with regard to service level is addressed in paragraph 2.2.4.

2.2.3 Factors influencing the demand for and supply of cash

Various studies have been conducted in the United States of America to determine customer preferences in bank selection. The factors quoted most frequently included the convenience of branch location in relation to both home and work location, the bank's reputation, the bank's financial strength, the completeness of service as well as the quality of service provided by the bank. As a result of these studies, it was concluded that banking to a large extent is seen as a *"convenience good"*, implying that retail customers largely believe

"...that all banks are alike" (Reich 1977:12-16). A criticism of many of these studies is that the research focussed mainly on demand deposit balances, and not the full line of services provided by the bank. In respect of the demand for and supply of cash, a number of other factors are relevant.

Cash demand patterns vary according to numerous factors, for example the occurrence of month ends, public holidays, school holidays, unusual occurrences (such as a special sporting occasion taking place in the vicinity of the branch, agency or ATM), the end of the financial year as well as any other random event which might play a role.

At the individual level, a factor such as socio-economic grouping also has an impact on cash usage (De La Rue *s.a.*:5). Demand patterns are therefore also dependent on the physical location of the branch, agency or ATM which will impact on the composition of the customer base. The demand patterns at a branch located in the country or a rural area will differ significantly from the demand patterns experienced at a city or urban branch. Inherent differences also occur between the demand patterns at either of the aforementioned and a suburban branch or a branch in an industrial area. As Derwa (1978:111) aptly states: *"There is no way of knowing with any degree of certainty how much in cash balances a branch will need."*

A concept which is relevant at this point, is the so-called service area. In determining the cash demand patterns, it is necessary to establish the service area for that particular branch, agency or ATM. Hansen as quoted in Reich (1977:24) uses the concept of trading area rather than service area. The trading area is defined as *"... a geographically delineated region, containing potential customers for whom there exists a probability greater than zero of their purchasing a given class of products or services offered for sale by a particular firm or by a particular agglomeration of firms."* This definition implies that competition in a particular trading area will have an impact on the volume of business conducted.

Reich (1977:51) further states that rather than categorise a branch according to its location (*i.e.* urban, city, commercial and so forth), this could be done using the volume of expected branch transactions and their distribution by transaction type (for example cashed cheques, cheque account deposits, savings account deposits, *etcetera*). Obviously the total demand at a particular cash point is of importance, but what is stressed by the above statement is the cost implication of having a cash point where numerous small transactions are performed in contrast to a branch with fewer transactions but greater amounts per transaction.

At the *Societe Generale de Banque* the following facts relating to cash demand patterns have been established (Derwa 1978:111):

- The cash requirements follow a symmetrical distribution. In conformity with statistical tests, fitting the normal curve is acceptable.
- The parameters of this distribution vary from one branch to another and for any individual branch, between one day of the week and another.

In the approach discussed above, the model applied was made more realistic by taking other factors such as the mix of denominations into account (Derwa 1978:111).

According to Naddor (1982:22) the probability distribution *P(x)* is used to designate the demand distribution. The following equation then holds:

$$\sum_{x=x_{min}}^{x_{max}} P(x)=1 \qquad (2\text{-}1)$$

where x_{max} denotes the maximum demand, and x_{min} the minimum demand.

For the system under review, the following holds for the demand *x*:

$$x=\sum_{i=i}^{14} Q_i \qquad (2\text{-}2)$$

where Q_i denotes the demand for the i^{th} denomination available in the South African economy.

2.2.4 The perspective of the customer

One of the greatest challenges in providing retail financial services is to determine how to operationally achieve such a detailed understanding of the customer to be able to target products and services in such a way to effectively meet real needs (De La Rue *s.a.*:5). Relationship banking is the phrase used in retail banking to describe strategies that bankers follow in an attempt to retain customers. Despite these attempts, studies have shown that banking scores *"second only to insurance as the industry perceived least responsive to customers"* (Violano & Van Collie 1992: 9).

A feature of the banking industry is a proliferation of new services provided to customers, while existing services which are of greater importance to customers and are used more frequently, have been neglected (Beatty & Gup 1989:15-16). As mentioned in an earlier paragraph, it is important to establish the customer's view of cash handling. In an environment where many customers perceive banks as being *"all alike"* (refer to paragraph 2.2.3), providing superior service is a way of distinguishing the service provided from that rendered by the next bank (Goodman *et al.* 1989:15). As Gray & Harvey (1992:61) quote: *"Quality service is one of the few ways a financial institution can differentiate itself sufficiently in the marketplace to achieve exceptional business growth and earnings performance."*

This statement is confirmed by an international research programme conducted by KPMG's Financial Sector Consulting Group. The research study confirmed that *"most banks identified quality of customer service as the probable key differentiator between retail financial service organisations in the future"* (KPMG *s.a.*:4). From work reported on by Clavert (1990:54), the following quote confirms the above: *"..... that service quality must be considered as a vital factor in* *performance improvement in order to meet the challenges of the changing financial service industry"*. The KPMG study however found that the relationship between the customer and the retail financial service organisation is currently typically defined by the product used, rather than the existence of a relationship (KPMG *s.a.*:4).

Quality in banking consists of three components: Internal excellence, effectiveness and efficiency; superior customer service; and an organisation structure that is designed explicitly to support the quality orientation (Gray & Harvey 1992:62). From this it is obvious that the cash handling problem could result from an internal problem (i.e. the incorrect amount and/or mix of cash ordered), leading to a reduction in customer service.

In the preamble to this chapter, reference was made to the necessity of avoiding a cash shortage at a branch or agency. The impact of word-of-mouth in this regard is significant. A customer who is dissatisfied with the way a bank has handled a request for assistance will tell an average of 16 people about the experience, while a satisfied customer will tell an average of eight people about the positive experience (Goodman *et al.* 1989:16). A study quoted in the ABA Banking Journal confirms this: *"Dissatisfied customers tell far more people about their experience than do routinely satisfied customers"* (Anonymous 1997:73-74).

First American Corporation (FAC), a regional bank holding company in Tennessee, USA, began a customer satisfaction survey of all its banks in 1989 to focus on retail customer perceptions of the banks' overall level of service provided. The specific objectives of the survey, which is repeated once a year, are the following (Calvert 1990:57):

- To measure overall customer satisfaction with FAC;
- to measure satisfaction with individual branches;
- to identify positive and negative customer experiences;
- to assess customers' willingness to recommend FAC; and

- to determine customer intention to increase business with FAC.

Obviously the way in which FAC manages its cash availability and denomination mix will impact on more than one of these objectives.

2.3 Managing cash balances in a retail bank

2.3.1 The cash handling process

Johnson (1994:31-33) provides an apt description of the cash handling process at a retail bank:

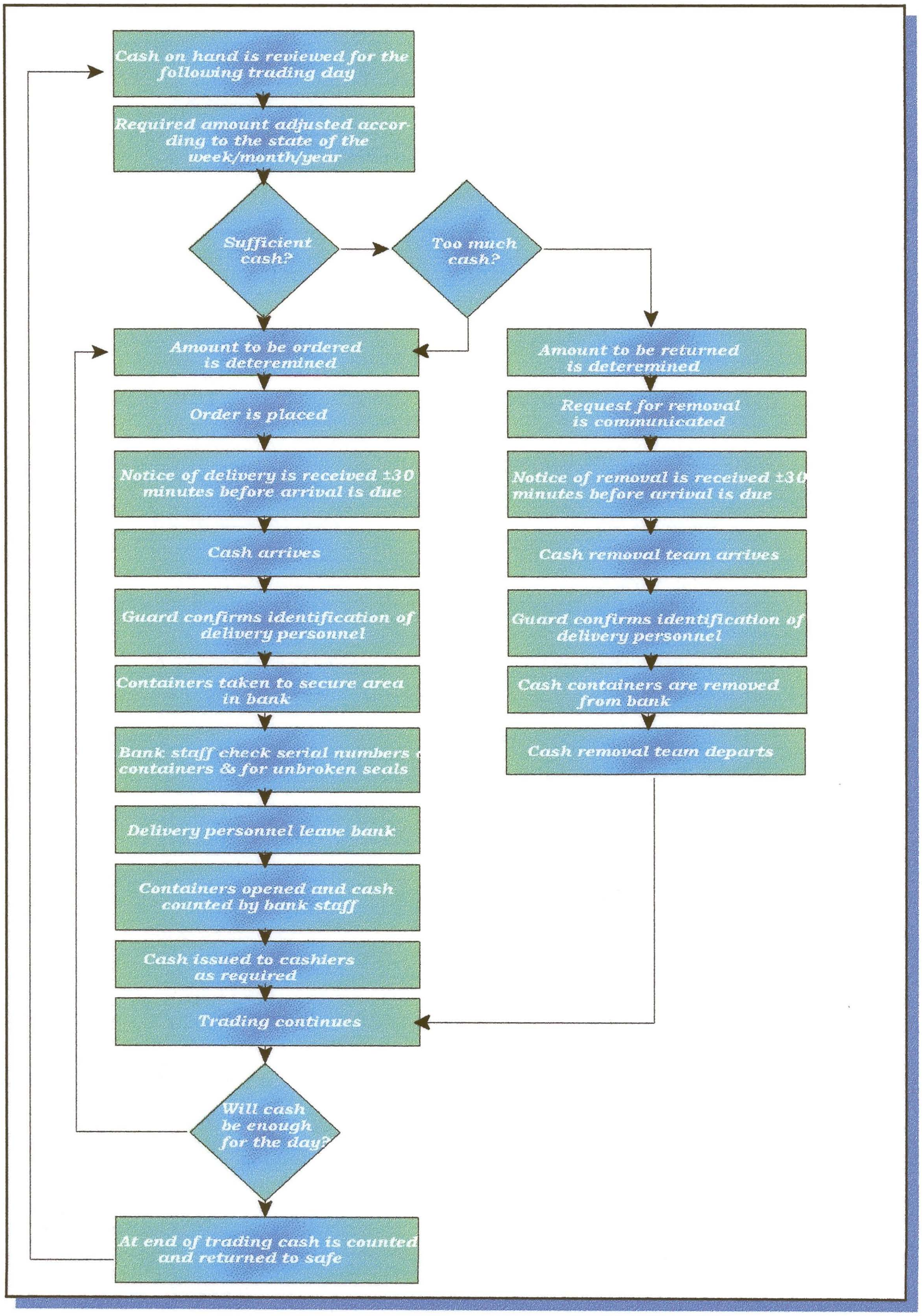
> *As any bank teller will tell you, cash can be a great deal of bother. Handling it is a menial and tedious job. It has to be sorted, counted, stacked into bundles in which all the bills face the same way. It's heavy, it gives you paper cuts, and it turns your hands black. When it's stored, it has to be locked into a vault or bolted into an impenetrable bank machine. When it's moved, it has to be accompanied by armed guards with shotguns and shipped across country in armoured trucks.*

From this description it is obvious that the handling costs are significant due to the equipment involved and the labour-intensive nature of the process.

Figure 2.2 provides a graphic representation of the cash replenishment process. The various elements of the process are described in greater detail in the later chapters of this thesis. In representing the steps involved in the process of cash replenishment, the generic process rather than the typical South African process is described. The process shown in Figure 2.2 does not allow for cash movement between branches. It only allows for replenishment from a central cash centre. This characteristic holds for the South African situation and may be different in other countries.

Figure 2.2

The cash replenishment process



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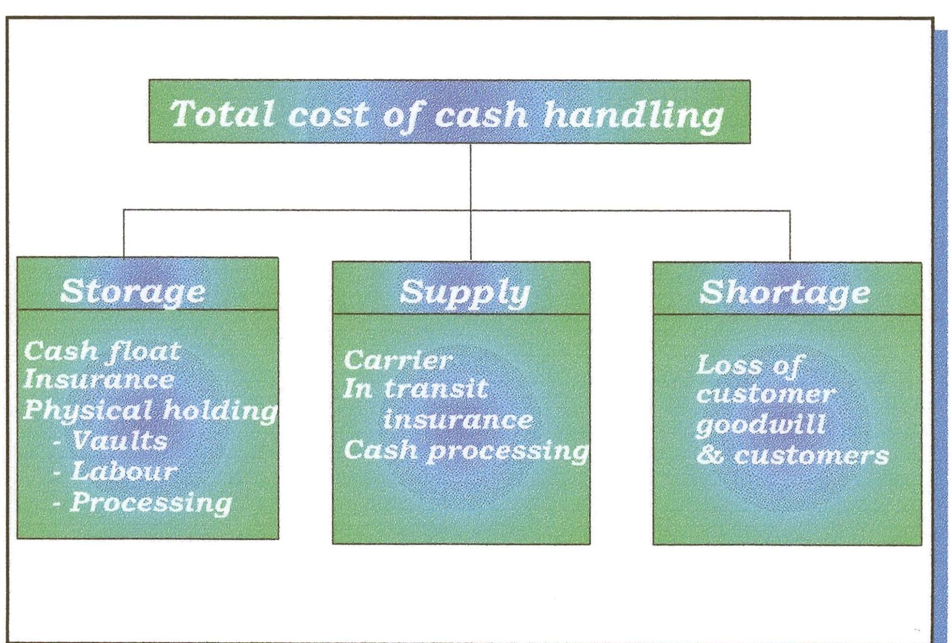
2.3.2 The cost elements involved in cash handling

According to a study commissioned by De La Rue Payment Systems Division, part of De La Rue PLC, the cost of handling cash is substantial. The estimated cost of handling cash reported by the study in 1995, was £3 billion per annum in the United Kingdom, Fr 55 billion per annum in France and DM 15 billion in Germany. These figures do not take interest into account (Anonymous 1995:10-11). The figures are staggering if the fact is considered that these are developed countries where the use of electronic payment methods is common.

As stated in paragraph 2.2.1, the aim in determining the amounts to be delivered or deposited, is to minimise the total cost, which is the weighted sum of the costs of storage, supply and shortage (Derwa 1978:111). Figure 2.3 illustrates the cost components involved in the handling of cash.

Figure 2.3

The cost components of cash handling



It may therefore be said that the following equation holds:

Total cost of cash handling (C) = Storage cost (C_1) + Shortage cost (C_2) + Supply cost (C_3)

The dimensions of each of these measures (C, C_1, C_2 and C_3) are [R]/[T].

The components of C_1, C_2 and C_3 are discussed in greater detail in the following paragraphs. In developing the mathematical formulation of the model representing the cost of cash handling, notation used by Naddor (1982) in describing the variables is adhered to as far as possible.

2.3.2.1 Storage cost (C_1)

In classical inventory theory, storage cost is referred to as inventory holding cost. This is in fact the principal factor which limits the order quantity when considering the number of physical items to keep in stock. As Wagner (1969:789) states: *"Keeping items in stock is costly because inventories tie up capital that might otherwise be profitably employed."* It is indeed no different when the inventory items are various denominations of cash, ready to be supplied to the customer in the correct amount and mix as required.

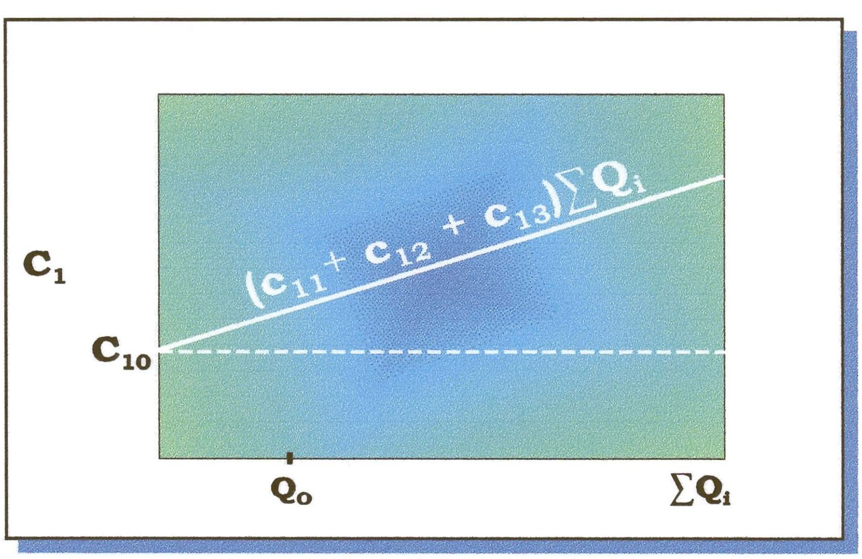
As indicated in Figure 2.3, the storage cost elements include cash float, insurance and the physical holding costs such as the provision of vaults, the labour required and the processing equipment. Figure 2.4 shows the typical behaviour of storage or holding cost as a function of the cash amount. The vertical axis represents the storage cost in Rand per period, whereas the horizontal axis shows the amount of cash held with the amount Q_O indicating the expected demand for the current scheduling period in units. Q_O would, for example, be the total amount of cash required from the bank per trading day (withdrawals), whereas ΣQ_i would represent the final amount carried on that day.

The offset from the origin on the vertical axis in the graph (C_{10}) in the figure above indicates the fixed component of holding the cash – vaults and processing equipment have to be in place. The other elements of holding cost, for example insurance and labour, have a linear relationship with the amount of cash held – for each additional Rand held, the cost is incurred. The cash float cost (c_{11}) is the opportunity cost or interest forgone for every Rand held at the branch, agency or ATM, and therefore also represents a linear relationship with the amount of cash held (De La Rue 1997:7). The insurance cost per unit is represented by c_{12}, while the labour cost per unit is represented by c_{13}. Therefore the total storage cost (C_1) for any amount of cash consisting of a mix of 14 denominations is:

$$C_1(Q_i) = C_{10} + (c_{11} + c_{12} + c_{13})(\sum_{i=1}^{14} Q_i) \qquad \text{(2-3)}$$

Figure 2.4

The storage cost element (C_1) of the total cost of handling cash



An aspect of the system under review which has not yet been addressed, concerns the limited storage space. Providing the appropriate storage space for a branch or agency is definitely a concern, and may become a constraint should large quantities of cash need to be carried. Clearly, limited storage space is a constraint when replenishing an ATM. In addition, the storage space of a branch or agency is not only limited, but is explicitly dedicated to the storage of cash.

In practice, a limit is set by head office on the amount of cash carried in a particular branch during any scheduling period. Although this does not represent a physical limit to the amount of cash carried, it does represent a constraint when optimising the amount of cash to be ordered for the next scheduling period.

2.3.2.2 Shortage cost (C_2)

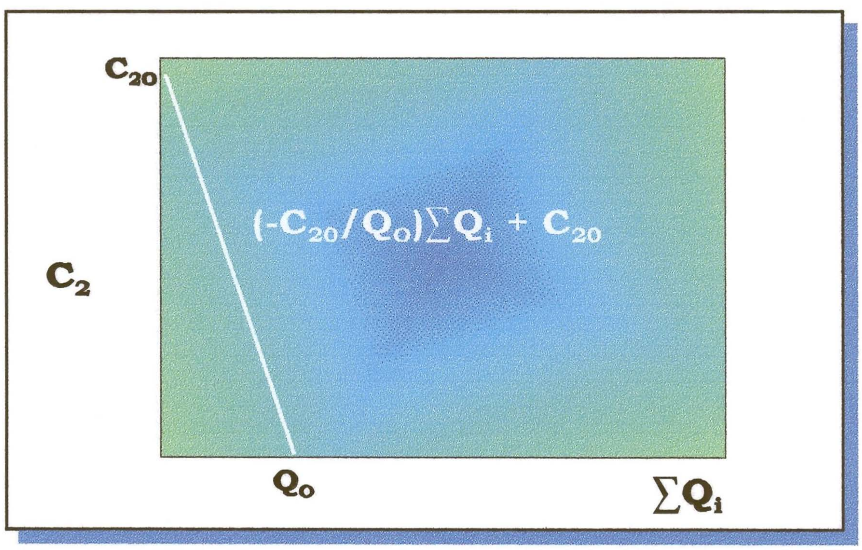
The shortage cost (C_2) of handling cash has two components, *i.e.* loss of customer goodwill and loss of customers. The impact of not fulfilling customer expectations and the effect of word-of-mouth communication amongst customers were touched upon in paragraph 2.2.4. However, the most important aspect in this regard is the effect that a cash shortage at a branch will have. It is said that the penalty for running out of cash is largely loss of goodwill, the perception of a poorly run and probably of an unstable financial institution (De La Rue 1997:7).

Shortage cost is indeed extremely difficult to measure, but cannot be ignored. The effect of a cash shortage at a branch, differs significantly from that at an ATM or agency. One element of shortage cost which is quantifiable, is the cost incurred by a bank when its ATM has run out of cash forcing a customer to make use of a competitor's ATM. This, however, represents only a small part of the total shortage cost.

Due to the difficulty in quantifying the shortage cost, a qualitative method such as a customer survey, would be a first step in determining the extent of the shortage cost. In addition, expert opinion may be solicited from sources internal to the bank to place a value on shortage cost. Figure 2.5 shows the behaviour of shortage cost in respect of the amount of cash carried. The variable C_{20} represents the intercept on the vertical axis and all variables are as defined before.

Figure 2.5

The shortage cost element (C_2) of the cost of handling cash



The function which describes the behaviour of shortage cost is also dependent on the amount of cash held.

For $\sum Q_i < Q_O$

$$C_2(Q_i) = -\frac{C_{20}}{Q_O}\sum_{i=1}^{14} Q_i + C_{20} \qquad \text{(2-4)}$$

For $\sum Q_i \geq Q_O$

$$C_2(Q_i) = 0 \qquad \text{(2-5)}$$

2.3.2.3 Supply cost (C_3)

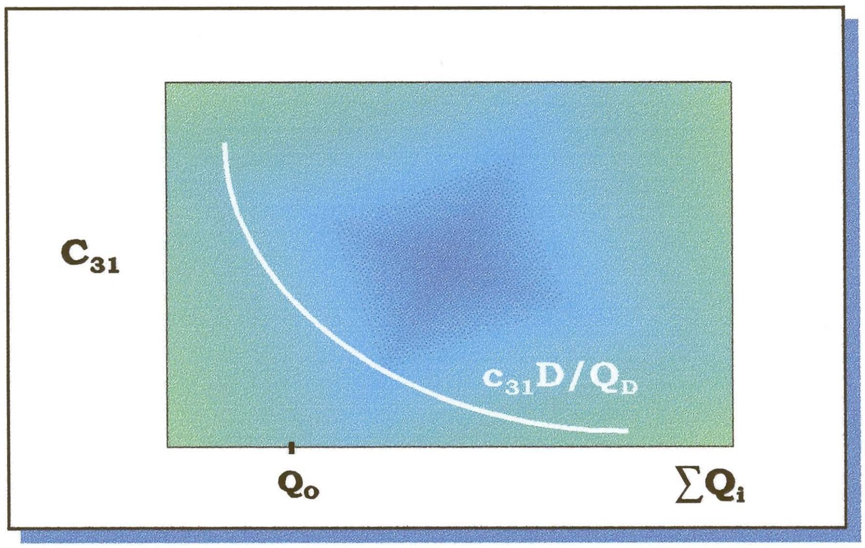
The third group of cost elements contributing to the total cost of handling cash (as shown in Figure 2.3) refers to the supply cost, or as according to the classical inventory theory, the replenishing cost. Naddor (1982:39) describes replenishing cost (C_3) for items that are sourced from an agency outside the organisation under review as *"...the costs associated with the unit cost of replenishment (and) may include clerical and administrative costs, transportation costs, unloading costs and other costs."* From the retail banking perspective this would include the carrier or transportation cost, the in-transit insurance as well as the processing cost (*i.e.* counting, packing, handling and administration performed by the carrier and bank). Figures 2.6 to 2.9 describe the four elements of supply cost relevant to retail banking.

Figure 2.6 illustrates the order cost per period (C_{31}) which represents the cost that would be incurred to replenish cash irrespective of the size of the order. It does not include the cost of cash processing, the carrier or transportation cost or in-transit insurance. It therefore represents all other preparation costs (administrative and clerical), particularly incurred internally by the bank necessary to activate a cash delivery.

Since the unit replenishment cost, c_{31}, is constant per order, the order cost per period, C_{31}, is dependent on the total demand (D) during the planning period, as well as the quantity ordered (Q_D). The latter is a function of the amount of cash held during the previous scheduling period, the demand during that period as well as the expected demand for the current scheduling period.

Figure 2.6

The order cost element (C_{31}) of the supply cost of handling cash



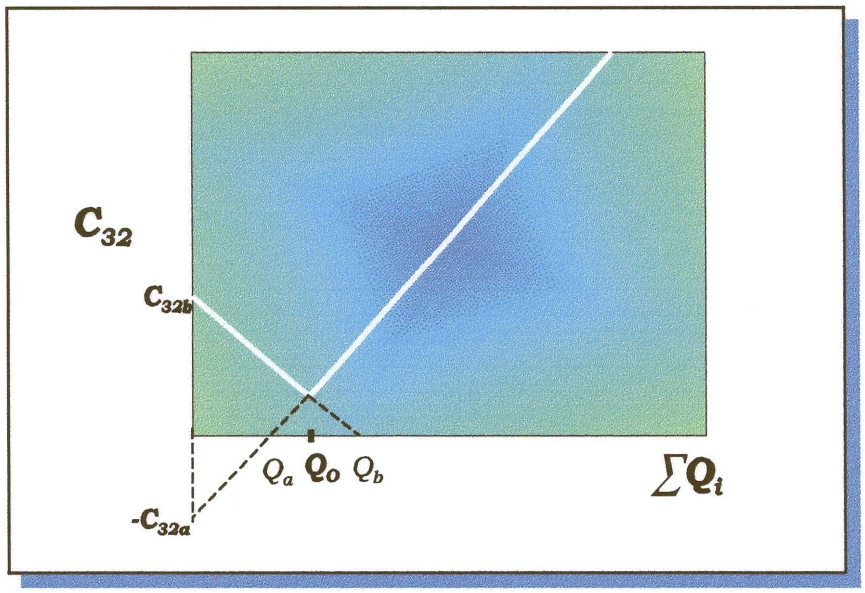
From the above, the equation for the order cost (C_{31}) is as follows:

$$C_{31}(Q_i) = c_{31}\frac{D}{Q_D} \quad \text{(2-6)}$$

Figure 2.7 shows the cash processing cost (C_{32}), the second element of the supply cost. If the actual demand for a particular day exceeds the expected demand for that day (Q_O), cash processing costs are incurred to obtain additional cash from the cash distribution centre, whereas if the actual demand is less than the expected demand (Q_O), cash processing costs are incurred when returning the surplus cash to the cash distribution centre. Even if a branch holds exactly the correct amount of cash for a particular day, the cost will not equal zero, since some counting and packaging will take place. The probability of having the exact amount and mix of denominations for the following planning period is remote, therefore it may be assumed that C_{32} will always be greater than 0. Once again this is a cost element internal to the branch.

Figure 2.7

The cash processing cost element (C_{32}) of the supply cost of handling cash



In the figure above, the cash processing cost is represented by two linear equations depending on whether the amount of cash held is less than or greater than the expected demand (Q_O). The following two equations describe the relationship:

For $\sum Q_i < Q_O$

$$C_{32}(Q_i) = -\frac{C_{32b}}{Q_b}(\sum_{i=1}^{14} Q_i) + C_{32b} \qquad \text{(2-7)}$$

For $\sum Q_i \geq Q_O$

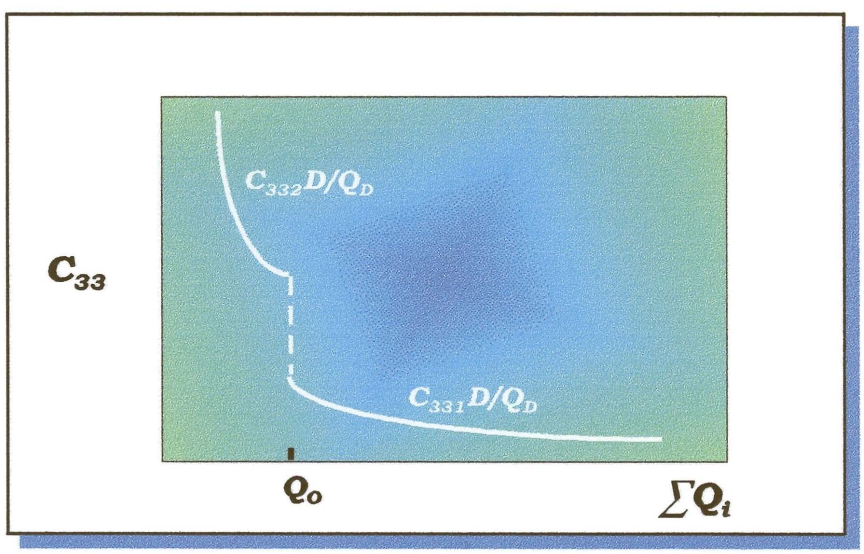
$$C_{32}(Q_i) = \frac{C_{32a}}{Q_a}(\sum_{i=1}^{14} Q_i) - C_{32a} \qquad \text{(2-8)}$$

All variables are as defined before, with C_{32b} and C_{32a} representing the offsets for the two straight line functions describing the cash processing cost below and above Q_O.

Figure 2.8 represents the carrier or transportation cost element (C_{33}) of the supply cost of handling cash. In the South African retail banking environment replenishment occurs on a daily basis at the normal cost. (Refer to paragraph 2.3.3 for further comments in this regard.) Should a branch require an interim delivery resulting from a projected shortage, the delivery cost is increased substantially. If this penalty cost did not apply, the transportation cost would remain constant per trip. As a result the transportation cost is described by a step function consisting of two hyperbolas. The derivation of the transportation cost per period (C_{33}) is analogous to the derivation of the order cost per period (C_{31}) shown earlier.

Figure 2.8

The transportation cost element (C_{33}) of the supply cost of handling cash



For $\sum Q_i < Q_O$

$$C_{33}(Q_i) = c_{332}\frac{D}{Q_D} \quad \text{.......................... (2-9)}$$

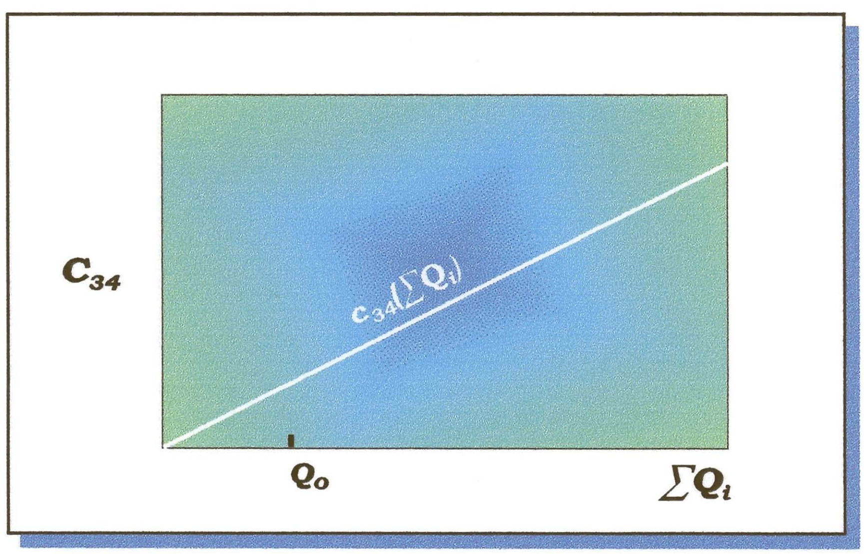
For $\sum Q_i \geq Q_O$

$$C_{33}(Q_i) = c_{331}\frac{D}{Q_D} \quad \text{.......................... (2-10)}$$

The last cost component of supply cost results form the in-transit insurance cost (C_{34}) required per trip. This is indeed extremely necessary in a South African context due to the high incidence of in-transit robberies. (Refer to paragraph 3.5.1 in this regard). The relationship in this instance is of a linear nature as shown in Figure 2.9.

Figure 2.9

The in-transit insurance cost element (C_{34}) of the supply cost of handling cash



The equation describing the in-transit insurance cost is as follows:

$$C_{34}(Q_i) = c_{34}(\sum_{i=1}^{14} Q_i) \quad \text{(2-11)}$$

The total supply cost is therefore

$$C_3(Q_i) = C_{31}(Q_i) + C_{32}(Q_i) + C_{33}(Q_i) + C_{34}(Q_i)$$

which is described by two mutually exclusive equations, depending on whether $\sum Q_i < Q_O$ or $Q_O \leq \sum Q_i$.

For $\sum Q_i < Q_O$, the following holds:

$$C_3(Q) = c_{31}\frac{D}{Q_D} - \frac{C_{32b}}{Q_b}\left(\sum_{i=1}^{14} Q\right) + C_{32b} + c_{332}\frac{D}{Q_D} + c_{34}\left(\sum_{i=1}^{14} Q\right) \qquad \text{.......... (2-12)}$$

For $Q_O \leq \sum Q_i$, the following holds:

$$C_3(Q) = c_{31}\frac{D}{Q_D} + \frac{C_{32a}}{Q_a}\left(\sum_{i=1}^{14} Q\right) - C_{32a} + c_{331}\frac{D}{Q_D} + c_{34}\left(\sum_{i=1}^{14} Q\right) \qquad \text{.......... (2-13)}$$

2.3.2.4 Conceptual mathematical model of the total cost of handling cash

Based on the deductions in the preceding paragraphs, it is possible to construct a conceptual mathematical model for the total cost of handling cash in a branch, agency or ATM of a retail bank. The model is dependent on whether the amount of cash held during the particular planning or scheduling period is greater than or less than the demand for cash which indeed materialises. The model is as follows:

For $\sum Q_i < Q_O$, the following holds:

$$C(Q) =$$

$$C_{10} + (c_{11} + c_{12} + c_{13})\sum_{i=1}^{14} Q_i - \frac{C_{20}}{Q_O}\left(\sum_{i=1}^{14} Q\right) + C_{20} + c_{31}\frac{D}{Q_D} - \frac{C_{32b}}{Q_b}\left(\sum_{i=1}^{14} Q\right) + C_{32b} + c_{332}\frac{D}{Q_D} + c_{34}\left(\sum_{i=1}^{14} Q\right)$$

$$\text{..} \qquad \text{(2-14)}$$

For $Q_O \leq \sum Q_i$, the following holds:

$$C\ (Q_i) = C_{10}+(c_{11}+c_{12}+c_{13})\sum_{i=1}^{14} Q_i+c_{31}\frac{D}{Q_D}+\frac{C_{32a}}{Q_a}(\sum_{i=1}^{14} Q_i)-C_{32a}+c_{331}\frac{D}{Q_D}+c_{34}(\sum_{i=1}^{14} Q_i) \quad (2\text{-}15)$$

It must be stressed that the above merely represents a conceptual model for which the various cost parameters need to be determined. Once the parameters are known, it will be possible to investigate the model in depth to establish the optimum of the problem to, for example, interim deliveries. At present in South African banking circles, it is assumed that a single daily delivery provides the optimum solution to the cash replenishment problems of a branch, agency or ATM.

2.3.2.5 Conclusion

Oosthuysen (1995:33) points out that the need for better costing information is critical for South African banks because of the increasingly competitive environment in which they operate. The result is a need for sound decision-making to ensure long-term survival based on accurate information. As Oosthuysen (1995: 36) aptly states: *"Banks are forced to shift the focus from growth in interest income to an increase in operating revenue and/or a reduction in operating expenses"*. This will only be possible if the true nature and extent of the cost of handling cash is established to facilitate the decision-making process.

2.3.3 The scheduling period and the effect of lead time

The scheduling period is the length of time between consecutive decisions with respect to replenishments and the lead time is the length of time between the scheduling of a replenishment and its actual addition to stock (Naddor 1982:26). In the South African retail banking industry the scheduling period, by popular conviction, is presumed fixed at one day, with the replenishment of cash or the removal of surplus cash taking place on a daily basis. The scheduling period in this inventory system is therefore constant. It is referred to as a prescribed scheduling period and is denoted by t_p (Naddor 1982:27).

The incentive for adhering to this scheduling period, is embodied in the transportation cost element (C_{33}) of the supply cost of handling cash as illustrated earlier in Figure 2.7. A problem arises however with regard to the lead time. Under the assumption and enforcement of a prescribed scheduling period, it is crucial that the lead time should also be prescribed and constant. However this is not the case. The lead time is denoted by L_j, where j = 1,2,...N and j denotes subsequent scheduling periods.. The adherence to a t_p = 1 trading day, is acceptable if $L_j \leq t_p$. However, as soon as $L_j > t_p$, a cash shortage becomes a possibility. It is precisely this occurrence in the present approach to cash replenishment in retail banking which leads to branches carrying more cash than is necessary as protection against a shortage.

In an attempt to optimise the amount of cash carried in a branch, agency or ATM, it would therefore be required to establish the lead time probability distribution. Once this has been established, the replenishment policy may be formulated to make provision for the demand during the lead time.

Initially, it may be hypothesized that the cash replenishment process is a Poisson process with a negative exponential lead time distribution. If λ = average arrival rate of the cash replenishment process and $1/\lambda$ = the average time between arrivals, then the lead time probability density function is given by the following equation:

$$f(L) = \lambda e^{-\lambda L} \qquad \text{(2-16)}$$

The cumulative distribution is given by:

$$F(L) = \int_0^L f(t)dt = 1 - e^{-\lambda L} \qquad \text{(2-17)}$$

The validity of the assumptions regarding the lead time distribution will be investigated thoroughly in a later chapter. It would also be prudent to propose that the adherence to a prescribed scheduling period be investigated.

2.3.4 Summary of inventory system characteristics

Having proposed a conceptual model to describe the inventory system used to manage cash holdings in retail banking services in South Africa, it is appropriate to summarise the characteristics of the system. The following has been postulated:

- The demand for (and supply of) cash is described by a probability distribution, which could be approximated by the normal curve, but the parameters of the distribution will vary between cash points (branches, agencies and ATM's). It is therefore an inventory system with probabilistic demand.

- The system is subject to two mutually exclusive conditions, *i.e.* cash held at the cash point for a specific scheduling period may be greater than or less than the demand which finally materialises during that period. The occurrence of each of these conditions will have an impact on the cost equation describing the total cost of handling cash. The cost parameters have yet to be determined.

- The scheduling period is prescribed, but the lead time varies according to a probability distribution, the specific nature thereof as yet unknown.

The system may therefore be described as a (z,q) system according to the notation used by Naddor (1982).

2.4 Conclusion

The aim of Chapter 2 was to highlight the attendant problems of cash replenishment in retail banking. It described the problems relating to the discrepancy in deposit and withdrawal mix and the factors which influence the demand for and supply of cash at a cash point (be it a branch, agency or ATM). The perspective of the customer on good service specifically related to the provision of cash was discussed. The bulk of the chapter was devoted to the development of a mathematical model to describe the costs involved in handling cash from the perspective of the branch, agency or ATM. In addition the impact of the scheduling period and the replenishment lead time were also discussed.

Chapter 3 focuses on the retail banking environment in South Africa, specifically those characteristics that impact on cash handling.

CHAPTER 3

The retail banking environment in a South African context

3.1 Introduction

This chapter provides a description of the retail banking environment in South Africa. The aim is to focus on aspects relevant to the research and not to provide an exhaustive description of retail banking in South Africa. The chapter discusses the legal position of retail banks in South Africa, the current environment in which retail banks have to operate, as well as the role of the South African Reserve Bank. It further provides an overview of the regulatory structure of the financial services industry and discusses the various role players in the industry relevant to the research. Finally, some exogenous factors impacting on retail banking services are discussed. Figure 3.1 shows the relevance of Chapter 3 with regard to the other chapters in this report.

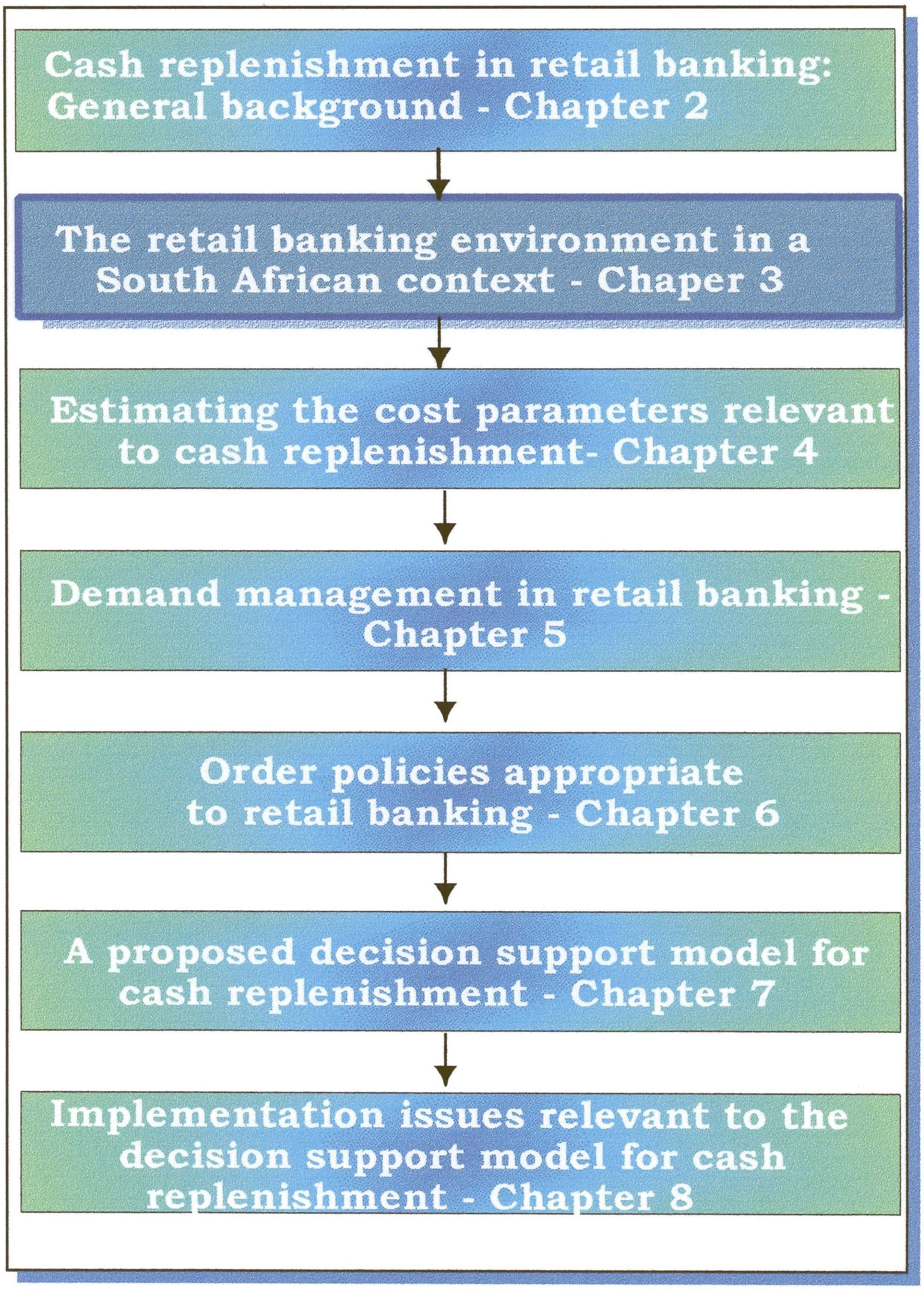
3.2 Retail banking

Retail or commercial banking is defined in Chapter 1. As pointed out earlier, commercial or retail banks are profit-seeking institutions, deriving profit from two major sources. In the process of realising profit, retail banks perform the following major functions (Falkena *et al.* 1995:69-70):

- Accept deposits, notably cheque deposits;
- provide credit, mainly in the form of overdrawn accounts;
- make payments and perform collections by means of a clearing system;
- render financial services;
- act as authorised currency dealers; and
- assist in the execution of monetary policy.

Figure 3.1

The structure of the report indicating the relevance of Chapter 3



With reference to the specific problem under review, the first main function mentioned above, i.e. accepting deposits, is of the greatest significance. As Falkena *et al.* (1995:69) state:

> *Commercial banks accept from the public (persons, business concerns, and other institutions) demand repayable deposits on current account that can be drawn by way of cheques or other electronic funds transfer means.*

The Banking Council – South Africa (1997:5-6) prefers the following description:

> *The commercial banks, now more commonly referred to as clearing banks, are involved in one way or another in almost every monetary transaction that takes place in the country, from cheque-processing and the provision of cash to electronic transmission of funds and the handling of credit and debit card transactions. A substantial distribution network of branches, agencies and ATM's handle these transactions.*

These banks therefore provide retail banking services.

3.2.1 Legal position of banks in South Africa

Falkena *et al.* (1995:75-76) state that all deposit-taking businesses are regulated by the Banks Act (Act 94 of 1990). This Act stipulates certain prudential requirements in respect of capital, cash reserves, liquid assets and large exposures. The cash reserve requirements may be used as an instrument of monetary policy (an increase in reserves would naturally be restrictive), while other requirements may be structural in nature. According to the Banks Amendment Act of 1994, capital requirements are the higher of R50 million or a specified percentage of the bank's risk-weighted assets and off-balance sheet activities.

Relevant at this point is to note that similar requirements which were in force in Canada have been lifted. Prior to this, Canadian banks had absolutely no incentive to reduce their cash holdings, but as a result of the lifting of the requirement, cash has become an important issue. Gammage of Toronto Dominion Bank is quoted as saying: *"All of a sudden, our inventory of cash becomes a non-earning asset"*. (Johnson 1994:31-33).

In addition to the capital requirements enforced upon South African banks, certain requirements pertaining to liquid assets and cash reserves (a subset of liquid assets) have to be adhered to. A bank must hold an average daily amount of liquid assets of not less than 5% of the average daily amount of its total liabilities to the public. Liquid assets would *inter alia* include any credit balance in a clearing account with the South African Reserve Bank and bank notes and coins in a bank's vaults and automated teller machines. (Falkena *et al.* 1995: 79).

It is interesting to note that methods have been developed to maintain a zero surplus on capital requirements in a federal reserve account. The application of a time series forecasting model to this situation is described by Balzano (1978: 99-105). A South African banking official, in an interview, remarked that South African retail banks do not pay much attention to this aspect and as a rule carry a huge surplus on the capital requirements as determined by the South African Reserve Bank. This is confirmed by Johnson (1994:31-33) as the attitude found in Canadian banks prior to the lifting of the capital reserve requirements regulation.

A further four acts have a bearing on banking institutions. These include the Companies Act, No. 61 of 1973, the Currency and Exchanges Act, 1933, the Usury Act, 1968 and the Credit Agreements Act, 1980 (Falkena *et al.* 1995:79-80).

3.2.2 Current environment of retail banking

At the end of 1996, more than 50 banks were operating in South Africa, providing banking services in more or less the same financial market. On May, 13, 1997, according to a list supplied by the South African Reserve Bank, 53 banks were finally and provisionally registered in South Africa (KPMG 1997:50-51).

Banking in South Africa is conducted by (Banking Council 1997:5):

- Four "major" banking groups with national distribution networks, each with assets in excess of R90 billion;
- five "medium-sized" banks with distribution networks, and/or assets in excess of R5 billion;
- twenty-four "small" South African banks with assets of less than R3 billion;
- ten subsidiaries of foreign banks;
- nine branches of foreign banks;
- four mutual banks; and
- fifty-nine registered representative offices of foreign banks.

According to the annual report of the Banking Council dated 31 December 1997, a total of 3 446 branches and agencies and 7 200 ATM's serve the South African retail banking industry. These far-flung distribution networks and large staff complements, especially of the country's major banking groups are expensive to maintain. Together with the fact that banks are not recovering the real cost of handling and moving cash, has resulted in high cost ratios. (Banking Council 1997:5)

Not only is the competition among banks extremely keen, but also between banks and other financial institutions. For example, some competition arises between banks' deposits and life assurers' saving schemes. It is frequently alleged that the South African financial sector is over banked (Fourie *et al.*

1999:75-76). The comment is often made that the market is unlikely to continue to support this number of banks and some consolidation in the financial services sector has recently occurred with retail banks merging with life assurers in an attempt to gain full advantage of utilising a common customer base. The consolidation in the financial services sector is far from over – refer to articles such as *"Southern en Momentum se belange smelt saam"* and *"Liberty oorweeg opsies met Standard-belang"* (Sake-Beeld 1998a:16).

It was recently stated that the increasingly competitive and complex nature of the retail financial services industry has compounded the focus on key profit drivers such as return on capital, gaining market share, maintaining/increasing margins and so forth. In this list, reducing the cost base is quoted as the fourth most important issue and one of the most critical due to the current pressure on margins (KPMG 1997:24).

A challenge facing banking institutions in South Africa is to expand their services to the third world component of the South African population. It is claimed that 60% of all South Africans do not make use of any banking services (Anonymous 1996a:1). With the growth in the informal sector of the economy in South Africa, the number of prospective customers in this segment is therefore significant, but numerous obstacles have to be overcome in the attempt to reach this market. These problems include the following (Falkena *et al.* 1995:83):

- Potential customers are not sufficiently interested in the services provided;
- typical deposits made and loans required are smaller than those of the existing customer base; and
- technologies which have proven successful in other environments may not be particularly suitable to this market segment.

Bankers in fact differ on the nature of the specific needs of low-income consumers of banking services. Basic banking, which is assumed to be what low-income consumers require, is defined as addressing three needs, *i.e.* a way to obtain cash, a way to make payments to third parties and a safe and accessible place to keep money (Scott 1988:32). Traditionally bankers have seen consumers in this category as high risk customers and have charged accordingly for services rendered. In addition to inaccessibility, the high cost of financial services rendered by banking institutions has deterred consumers from making use of these services.

Despite the problems in servicing this market segment, the major South African banks have made massive investments in technological infrastructure to extend banking services to low-income earners nation-wide. The international banking community indeed regards South African banks as world leaders in providing appropriate banking services for low-income communities (Banking Council 1997:6).

It is however the presence of this component of the market in South Africa which leads to an on-going need for cash handling in banks, rather than a move toward a cashless society.

3.3 The role of the South African Reserve Bank

The South African Reserve Bank in its role as the central bank of the Republic of South Africa has the following main functions (Falkena *et al.* 1995:55):

- The issuing of bank notes and coin;
- acting as banker to the government;
- acting as banker to other banks;
- providing facilities for the clearing and settlement of claims between banks;
- acting as custodian of the country's gold and foreign reserves;
- acting as "bank of rediscount" and "lender in the last resort";
- engaging in public debt management and open-market operations;

- supervising banks;
- collecting, processing and interpreting economic statistics and other information; and
- formulating and implementing monetary and exchange rate policies in co-operation with the Ministry of Finance.

Only some of these functions have particular reference to the research in question. These functions are discussed briefly in the following paragraphs.

3.3.1 Issuing of bank notes and coin

The Reserve Bank has the sole right to issue bank notes and coin in South Africa as stipulated by the Reserve Bank Act of 1944. All bank notes issued by the Bank are printed by the South African Bank Note Company (Pty) Ltd, a wholly owned subsidiary of the Bank. At present notes are issued in denominations of R10, R20, R50, R100 and R200. In 1989 the sole right to make, issue and destroy coins was transferred to the Bank by Act No. 49 of 1989. Another wholly owned subsidiary, the South African Mint Company (Pty) Ltd was established to manage these functions. The denominations, masses of the coins and the standard fineness of the relevant metals are set out in the Secondary Schedule to the South African Reserve Bank Act, 1989. It allows for the production of ordinary circulation coins in denominations of 1c, 2c, 5c, 10c, 20c, 50c, R1, R2 and R5. (Falkena *et al.* 1995:55-56). At the end of March 1997, the notes in circulation amounted to R19 249 million and the total amount of coins in circulation outside the banking sector amounted to R1 508 million. (Fourie *et al.* 1999:54)

3.3.2 Custodian of banks' cash reserves

The South African Reserve Bank acts as custodian of the cash balances of other banking institutions by virtue of the legal cash reserve requirements that have been in force at different times and in different forms, as determined by the South African Reserve Bank Act. Under normal circumstances, the banking institutions will endeavour not to hold reserve balances in excess of the minimum requirements with the Reserve Bank, although this is possible. The Reserve Bank is allowed to pay interest on deposit liabilities, but does not necessarily do so (Falkena *et al.* 1995:57). The existence of required reserves on which no interest return is received by the financial firm, reduces revenue (Hancock 1991:21).

Nearly all depository institutions must keep some minimum portion of assets in cash or otherwise liquid form. These reserve requirements affect the marginal prices of various financial services. (Hancock 1991:20-21).

3.3.3 Supervision of banks' activities

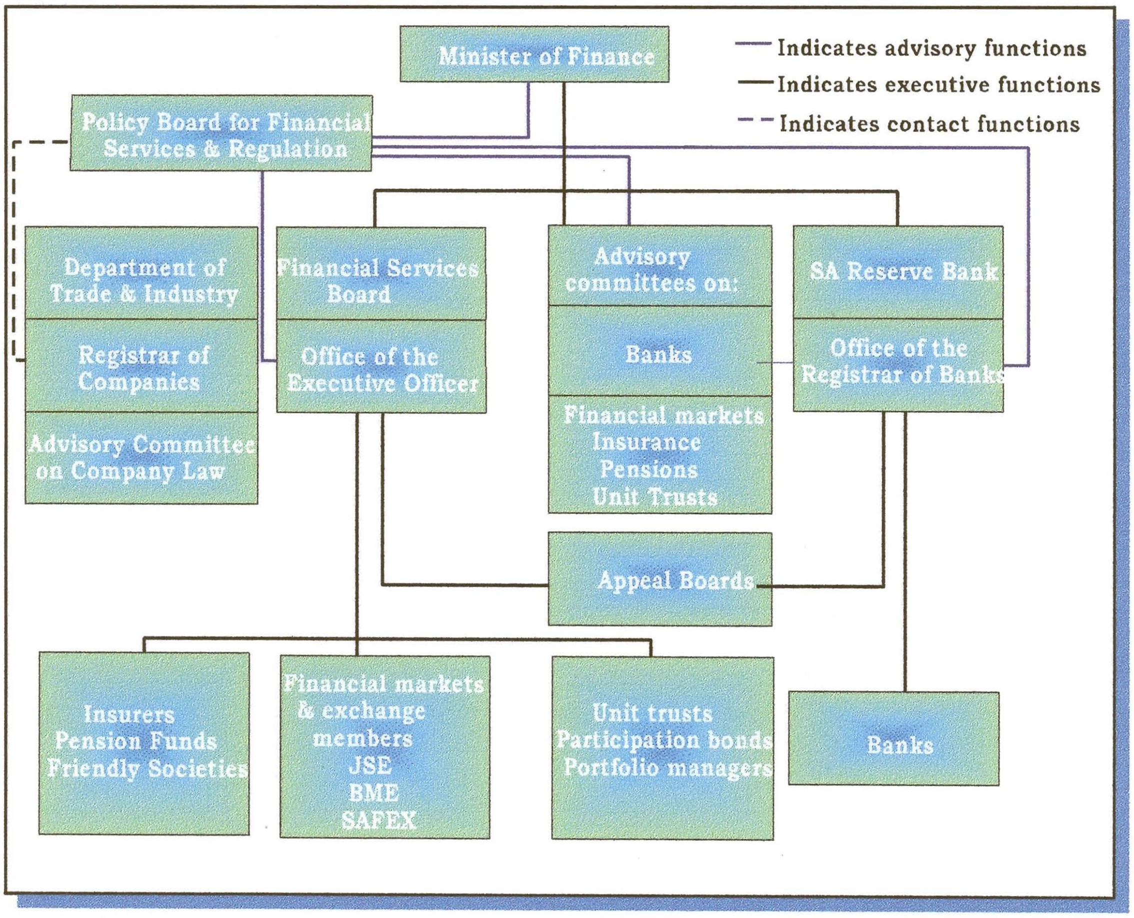
The main aim of bank supervision by the Reserve Bank is to create a legal and regulatory environment that will optimise the quality and effectiveness of risk management in banking institutions. The actions of the supervisory authorities are thus aimed at enhancing the proper management of risks, *i.e.* credit risk, liquidity risk, interest rate risk, market risk and currency risk, thereby ensuring a safer environment for depositors. The extent of supervision entails *inter alia* the establishment of certain capital and liquidity requirements and the continuous monitoring of the institutions' adherence to these legal requirements and other guidelines. In addition, the performance of an individual bank is measured on a continuous basis against developments in the relevant sector as a whole. (Falkena *et al.* 1995:61).

Wiese (as quoted in KPMG 1997:3) states that the Office of the Registrar of Banks, as part of the South African Reserve Bank, under the provisions of the Banks Act, 1990 and the Mutual Banks Act, 1993, is responsible not only for supervising the business of banks and mutual banks, but for the prevention of activities whereby deposits are solicited or accepted from the general public in contravention of these acts.

Figure 3.2 indicates the regulatory structure pertaining to the financial services industry (KPMG 1997:7).

Figure 3.2

Financial services regulatory structure



Source: KPMG. 1997. *Banking Survey Africa 1997.* South Africa: Financial Services Group of KPMG South Africa.

3.4 The South African cash cycle

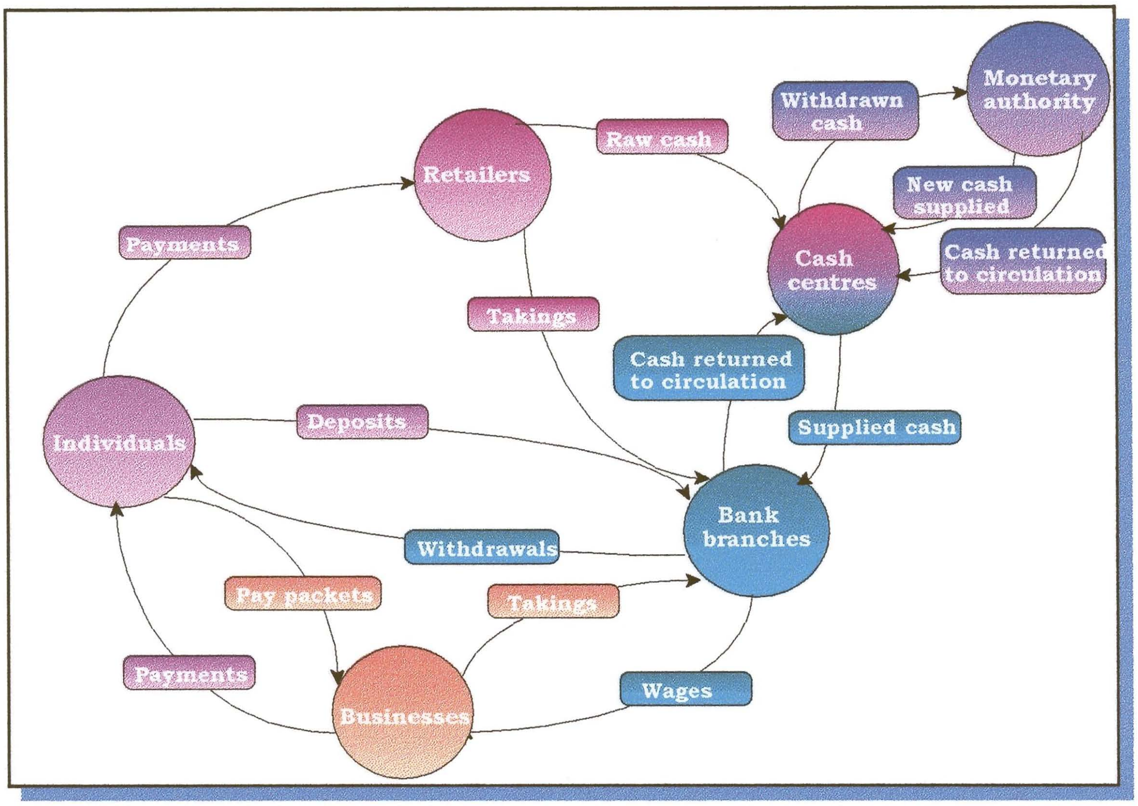
Figure 3.3 shows a typical currency cash cycle prevalent in every national economy (De La Rue *s.a.*:3). For purposes of this research, emphasis will be placed on the flows between the cash centres and bank branches (embodied by branches, agencies and ATM's) as well as certain factors influencing the flows downstream from the bank branches, *i.e.* deposits, withdrawals, takings and wages, which determine the cash demand patterns at a particular branch.

An interesting statistic provides insight into the scope of the South African cash cycle. According to the Banking Council there were 300 million cheque transactions to the value of R5 000 billion and 160 million ATM transactions to the value of R24 billion in South Africa during 1997 (Banking Council 1997:5).

In South Africa the function of providing cash to bank branches has been outsourced to SBV. This company was established in 1986 by three retail banks, Standard Bank of South Africa, Barclays (now First National Bank) and Volkskas (now part of ABSA), hence the name SBV. The company is owned by the banks and at present moves 95% of the bulk money in circulation in South Africa. It operates country-wide, performing three functions, *i.e.* transport, security and treasury. In addition to moving cash, it is also responsible for transporting gold coins, travellers cheques and foreign currency. It does not serve individuals customers, only banks. The value of notes handled on a daily basis by SBV is in excess of R1,5 billion. About 60 tons of coin are moved daily. This is achieved by daily having on average 300 vehicles transporting cash, travelling approximately 70 000 kilometres per day. Functions performed by SBV include sorting notes into three categories, *i.e.* ATM quality notes, fit notes and unfit or soiled notes. The latter are returned to the SARB to be destroyed. During the sorting procedure counterfeit notes are identified (approximately 30 notes per day) as well as notes that have been sorted incorrectly for example a R20 note amongst R200 notes. Of these sorting errors, 90% represent fraudulent transactions (Gregor 1998). It has been rumoured that a management buy-out is in the offing in the near future or that the SARB may take-over SBV.

Figure 3.3

The currency cash cycle



Source: De La Rue. *s.a. Cash handling strategy.* United Kingdom: De La Rue Systems.

3.5 Exogenous factors influencing the environment

It is of equal importance to take note of various exogenous factors that are at present influencing the retail banking environment in South Africa.

3.5.1 The crime situation

The most important exogenous factor impacting on the retail banking environment is the occurrence of bank related crime. According to the Crime Information Management Centre (CIMC) of the South African Police Service, 497 bank robberies were reported during 1997, while 230 robberies of cash in transit (not only bank related) occurred during the same period. The figures represent a decrease of 31% when compared to the 1996 figures (22,6% fewer bank robberies and 43,9% fewer robberies of cash in transit). If compared to the figures quoted by the Banking Council - South Africa (formerly known as the Council of South African Banks or COSAB), there is a discrepancy. According to the Banking Council 465 bank robberies occurred during 1997 against 408 in 1996 – an increase of 12,3% (Banking Council 1997:20). Of the 465 robberies reported in 1997, 103 were bank related robberies of cash in transit (SAPS 1998:10-12). According to the Banking Council figures, the bank-related robberies which occurred during 1997 involved R140 million, attacks on ATM's cost R2 million and burglaries cost R2 million (Banking Council 1997:20).

Various explanations are put forward for the discrepancy in the figures quoted by the CIMC and the Banking Council. These explanations are indeed irrelevant to the present study. However, the conclusion reached in the CIMC report is very relevant:

> *It logically follows from the latter facts that the conventional banks suffered an increase in the frequency of bank robberies, while robberies at other financial institutions decreased. At the same time, the amounts stolen from conventional banks and the degree of violence employed by the criminals involved also increased.*

This has also lead to the intense media interest in crimes targeting financial institutions. (SAPS 1998:10-12).

The banking sector has made on-going efforts in 1997 to counteract the increase in bank and cash-in-transit robberies at great cost. Extensive capital expenditure was undertaken on items such as video cameras, metal detectors, double doors and so forth (estimated at R100 million), guard services employed (estimated at R80 million) and protection of cash-in-transit (estimated at R200 million). (Banking Council 1997:20)

From the above, it is obvious that optimisation of the amount of cash to be held at any branch, agency or ATM, as well as the optimisation of deliveries (especially minimisation of interim deliveries), will be to the advantage of any retail bank. Not only will this reduce the amount of cash held unnecessarily at a cash point, it will also reduce the risk of cash in transit robberies – not only of occurring, but also minimising the loss should they take place.

3.5.2 The emergence of the cashless society

It is often claimed that in the not too distant future the need for cash to facilitate monetary transactions will disappear altogether. From the point of view of central banking, the impact of so-called e-cash will for example, be significant on monetary policy, whereas the impact on retail banking will be a marked effect on cash handling operations (Birch 1998a:1-2). In anticipation of this phenomenon conferences, such as *Preparing for a cashless society in the 21st century: Strategies for implementing an electronic payment system,* were arranged in 1996 by ICM Conferences, Inc. of Chicago.

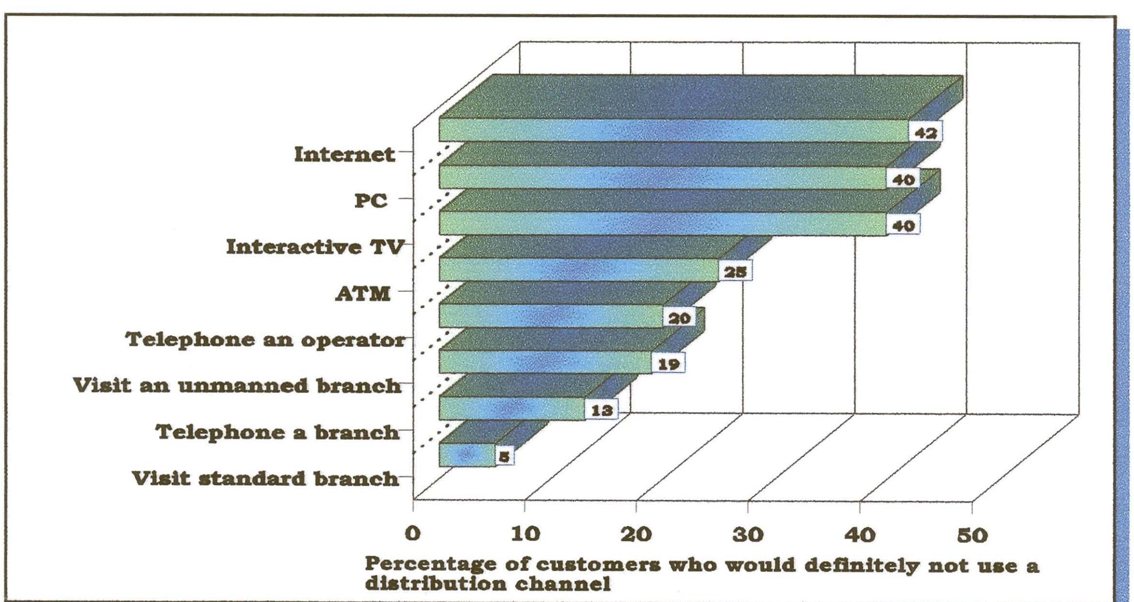
However, despite the expectations with regard to e-cash and the preparations being made to facilitate the use of e-cash (for example development of appropriate technology to prevent fraud), a claim made by senior members of the banking world at the conference on *Cash handling into the 21st century* held in Singapore during September 1997 was the following: *Cash is here to stay – the cashless society will not happen.* This claim was based on new industry studies which show that electronic forms of payment were substitutions for cheque rather than cash transactions. Recent research in the UK has shown that cash still accounts for 85% of all personal payments and 66% of all payments over £1 stressing the central position of cash even in a first-world economy (De La Rue *s.a.*:7).

It is claimed that the developing countries (for example in Eastern Europe) are just moving out of totally cash-based societies (De La Rue *s.a*:7). Despite the debate on a developing economy such as that of South Africa will ever become a cashless society – refer to articles such as *"Cashless world a long way off"* which appeared in the Financial Mail of 5 December 1997 (Bidoli 1997) – it is a known fact that the South African retail banking industry, specifically the so-called big four South African banks (ABSA, Standard, First National and Nedcor), is investing huge amounts in pioneering new methods of electronic banking (Anonymous 1998:72).

In a study quoted by KPMG(1998:26) customers were asked to indicate which channels they would definitely not use for obtaining various banking products. The results of this study seem to support the claim that the cashless society will not happen. The results of the study are shown in Figure 3.4. It is further claimed that customers maintain a strong preference for personal service, particularly for cash and that cash is either fully or partly responsible for about 75% of branch visits (De La Rue *s.a.*:9).

Figure 3.4

Percentage of customers who would definitely not use a distribution channel



Source: KPMG. 1997. *Banking Survey Africa 1997*. South Africa: Financial Services Group of KPMG South Africa.

3.5.3 Unconventional competitors in retail banking

The retail financial services industry is fiercely competitive. However, competition in this industry does not only emanate from traditional rivals, but recently other sources of competition have come to the fore. In the United States of America, NCR and 7-Eleven stores are the leaders in new era retail banking with the installation of full-service automated financial service centres in 7-Eleven stores in Austin, Texas. These centres provide cheque cashing, bill payment, money transfers, money orders and so forth (NCR 1998:1-2).

In South Africa, a national retailer has recently begun providing deposit-taking services *"in partnership"* with one of the local banks, and is in fact presenting itself to the public as a bank. Informal lenders and deposit-takers are also playing an increasingly important role in the provision of banking type services in South Africa, particularly at the lower end of the market (Banking Council 1997:27). In January 1996 the general manager of First National Bank stated (Anonymous 1996b): *"The new approach to retail banking is the result of increasing competition from traditional sources (including overseas banks) and non-traditional sources, including retailers."*

3.6 Conclusion

This chapter provided a description of the retail banking environment in South Africa. It discussed the legal position of retail banks in South Africa, the current environment in which retail banks have to operate, as well as the role of the South African Reserve Bank. It further provided an overview of the regulatory structure of the financial services industry and discussed the various role players in the industry relevant to the research. Finally, some exogenous factors impacting on retail banking services were discussed.

Chapters 4 and 5 investigate the behaviour of cost parameters and the demand patterns of a particular branch of a South African bank. The aim is to determine whether the model put forward in Chapter 2 applies to the South African situation. Although a particular case is investigated, the results will be generalised to reflect the general case in the South African retail banking industry, while distinguishing between branch-specific and generic factors which apply to the situation.

CHAPTER 4

Estimating the cost parameters relevant to cash replenishment

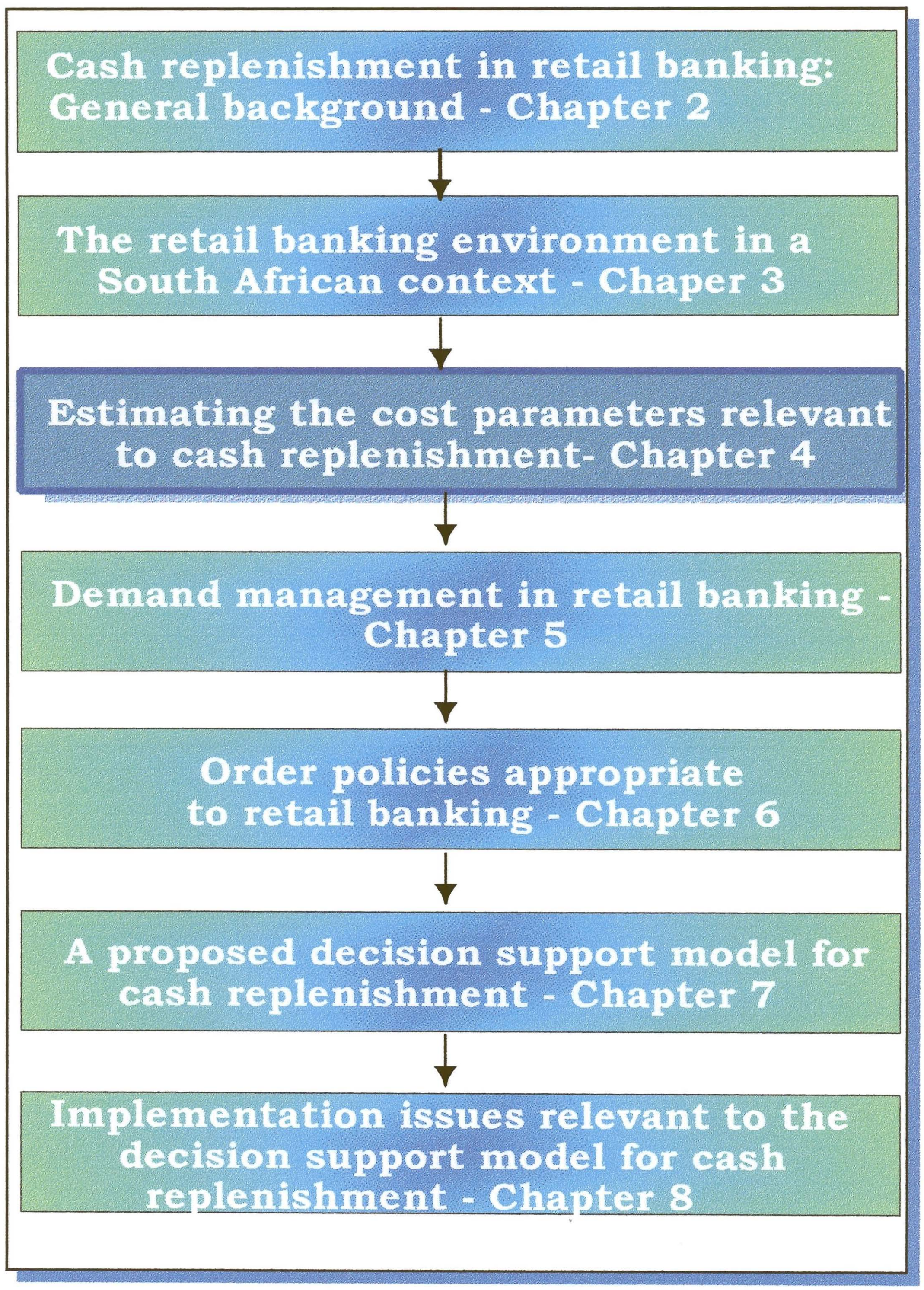
4.1 Introduction

Chapter 2 proposed a mathematical model to represent the cost elements involved in the replenishment of cash at a branch of a retail bank. To test the model, a typical branch of a leading South African retail bank was used. In the following paragraph, the specific characteristics of the situation at this particular branch are explained, where after the cost parameters discussed in Chapter 2 are quantified. Figure 4.1 shows the relevance of Chapter 4 to the research.

It would be appropriate at this point to comment briefly on the need for better cost information in the retail banking environment. It is a known fact that the operating expenses in retail banking are continuously increasing, resulting in a decline in profit margins. This situation is however not new. In a book published in 1970, the author John Walker in the foreword states the following (Walker 1970:iii): *"This book was written out of a conviction of mine that bank cost accounting procedures ... do not provide management with the type of information needed for decision making purposes."* In addition, a compounding factor has been the recent rapid increase in South African interest rates (and subsequent decline) as proposed by the previous president of the South African Reserve Bank, Dr Chris Stals in his last presidential address (Sake-Beeld 1998c:1). A rising inflation rate will also prevent interest rates from being reduced significantly in the near future (Sake-Beeld 1998b:1).

Figure 4.1

The structure of the report indicating the relevance of Chapter 4



From the above it is quite clear that decision-making will be facilitated if the true cost of holding cash is determined. Oosthuysen (1995:124-125) proposes a product costing methodology for a commercial bank, but although this methodology enhances decision-making from a head office perspective, it does not facilitate decision-making at the branch level, such as optimising the amount of cash carried by the branch.

4.2 Case background

Permission was obtained from one of the retail banks in South Africa to make use of the information pertaining to a branch, which would represent one of the more complex situations to be found at a typical branch in the country. For obvious reasons, the information is extremely sensitive and therefore the bank and branch will remain unnamed, as per prior arrangement with the bank.

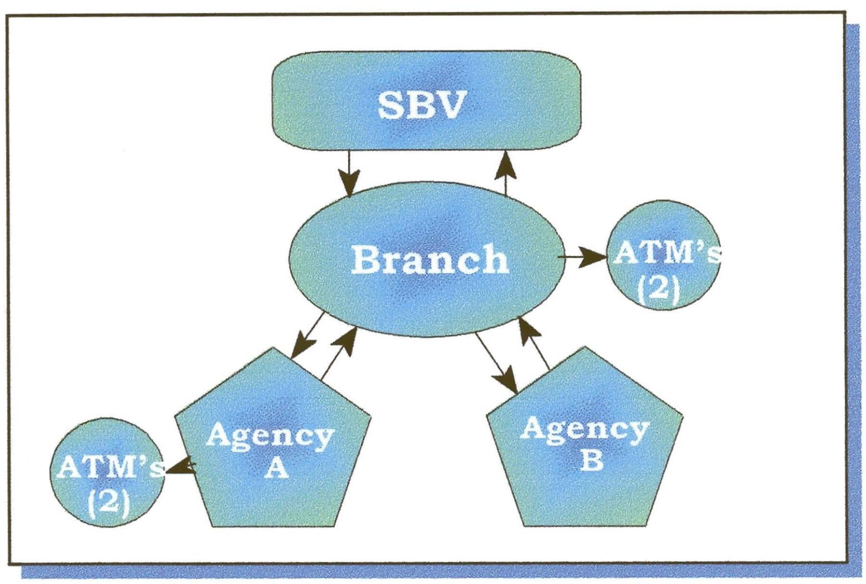
This specific branch serves both a commercial and a domestic customer base. The branch has two agencies and four automated teller machines which are dependent on it for the provision of cash. Two of these ATM's are located at the branch, while the other two are located at one of the agencies. The other agency does not have an ATM. Appendices A and B show the total deposit and withdrawal patterns at the branch and its two agencies for the period April to June 1998. The following information regarding savings and cheque accounts at the branch during the time of the investigation was obtained:

- Number of savings accounts: 7 530
- Average total monthly balance on savings accounts: R18 183
- Number of cheque accounts: 3 877
- Number of cheque accounts showing a credit balance: 2 768
- Number of cheque accounts showing a debit balance: 1 109
- Average total monthly credit balance on cheque accounts: R7 734 768
- Average total monthly debit balance on cheque accounts: R5 202 739

Figure 4.2 provides a graphic representation of cash movements to and from the branch. The process of supplying a branch with cash was detailed in Figure 2.2. This would represent the total picture where the branch does not have agencies. However, in a case where the branch is responsible for one or more agencies, the situation becomes more complex.

Figure 4.2

Cash movements to and from branch



Delivery from the cash centre, SBV, is possible once per trading day at the normal delivery cost. However, the supply lead time is two days. An order must be placed with SBV before 9:00 for delivery two days later. Delivery on the assigned day may take place any time between 7:45 and 20:00. Should the branch realise that more cash is needed for a specific day, a special order may be placed before 13:00 for delivery that same day. A special delivery is more expensive than a normal delivery. Special deliveries almost always arrive after 17:00. The minimum amount by special delivery is R500 000. A special delivery is an unusual occurrence – this branch has only had one special delivery during the three month period under review. A special delivery may only be arranged once it has been authorised by the regional head office – the operations manager at the branch has to obtain such authorisation.

When ordering cash from SBV, coin may only be ordered on a Friday. Notes may be ordered for any trading day. When ordering notes from SBV, the minimum order is set at 200 notes of any denomination, for example 200 R20 notes or 200 R200 notes. The branch, as a rule, attempts to order cash only once a week, but this is not always possible, particularly at the end of the month. Experience has shown that deliveries for the last Friday of the month invariably arrive too late on the day, therefore the branch has taken the decision to arrange for the cash to be delivered on the last Thursday of the month. Appendix F shows the schedule of SBV deliveries for the period April to June 1998. Obviously, since coin may only be ordered for delivery on a Friday, the decision to have notes delivered on the Thursday will lead to two deliveries that week, as for example at month end in June 1998.

As a rule the branch does not return notes to SBV, unless the notes are deemed to be unfit for further use or if a denomination switch is required. This is a highly unusual occurrence. During the period under review a denomination switch took place on only one occasion, *i.e.* on Friday, 22 June 1998 (see Appendix F).

Although money is moved from the branch to its agencies, no money may be transported between branches. Transportation between the branch and its agencies occurs twice a day – once early in the morning to deliver cash and once in the afternoon to return surplus cash. Whereas SBV is responsible for transporting cash to the branch, two other security companies are responsible for moving the cash between the branch and its agencies.

Agency B is quite small and serves a finite population of employees at a particular business. Agency B does not keep any notes overnight, but may keep coin. Therefore, notes left at the end of the trading day are returned to the branch. This agency is also closed on a Saturday. Agency A, in contrast, is open Monday to Saturday. It has a pattern of large deposits, therefore, the branch normally only sends an amount of approximately R26 000 in the mornings to the agency. After trading, the agency may not keep more than R360 000 overnight – it therefore returns any excess to the branch. Returns typically amount to approximately R200 000. Cash movements between the branch and its agencies from April to June 1998 are shown in Appendix F.

The ATM's are filled by staff members at the branch and the agency respectively. No cash is ever removed from an ATM for use in the branch or agency. Cash deposits at the ATM are removed and are then treated as ordinary deposits in the branch or agency at a cashier. Withdrawals from an ATM on a Sunday, are not treated separately, but added to the Monday withdrawals. Appendix C1 shows cash movements at the four ATM's which are the responsibility of the branch and Agency A.

4.3 Cash holding cost parameters

In determining the various cost parameters, Figures 2.2 and 4.2 serve as background throughout. The various steps in replenishing cash were described in these figures. Each of the three elements, C_1, C_2 and C_3, will be discussed separately, whereafter the proposed model of Chapter 2 is evaluated.

4.3.1 Storage cost (C_1)

As discussed in paragraph 2.3.2.1, the storage cost consists of a fixed component as well as a variable component. The fixed cost component (C_{10}) is discussed first.

4.3.1.1 The fixed component of storage cost

The fixed cost component (C_{10}) represents expenditures made directly to facilitate the cash holding process. These include factors such as expenditures for special security entrance doors (known as a mantrap door system) to the cashier area in the branch or agency, alarm system installations, a closed circuit television observation system, under-counter safes for each cashier, the main safes, the safety screen installations in front of the cashiers' desk, special protection of the ATM backrooms as well as security guards employed by the branch or agency. Since the branch under review has recently moved to new premises, the security arrangements at the branch are state of the art.

In addition to the security expenditures, the branch also incurs some other fixed elements attributable to the holding of cash. These include a counting machine as well as the rent payable for the space utilised by the cash in the branch or agency. Since the space provided is purpose-built for the holding of cash, it can not be used for other purposes.

To determine the scope of C_{10}, the safety expenditure figures shown in Table 4.1 were obtained from the branch. These figures include expenditures at the two agencies.

Table 4.1

Initial safety expenditures

Closed circuit television installation	R 75 000
Alarm system installations (2 each @ R22 500)	45 000
Installation of mantrap door (2 each @ R69 000)	138 000
Cashiers' safety screen (3 each @ R60 000)	180 000
Main safes (2 each @ R 12 500)	25 000
Cashiers' safes (10 each @ R 9 500)	95 000
Protection of ATM backroom (2 each @ R5 500)	11 000
Automated teller machines (4 each @ R180 000)	720 000
Counting machine (5 each @ R12 900)	64 500
TOTAL	**R1 353 500**

In addition to the initial expenditures attributable to the handling of cash, certain running expenses are incurred which may also be attributed to the holding of cash. The first is the cost of the security guards and the second is the rent payable for floor space dedicated to cash holding activities. The rent cost is considered to be a fixed monthly amount, since the space allocated to the holding of cash is dedicated to that activity and can not be used for anything else. It is therefore not dependent on the amount of cash carried by the branch. The figures obtained from the branch are shown in Table 4.2.

Table 4.2

Running monthly expenses

Security guards (3 each @ R 1 550 per month	4 650
Rent: Branch @ R 70/m^2 per month	
Cashiers' area behind safety screen (contains safes) -28m^2	1 960
ATM backroom -20m^2	1 400
Agencies @ R 28.44/m^2 per month	
Cashiers' area behind safety screen (contains safes) -39m^2	1 110
ATM backroom -9m^2	256
TOTAL	**R9 376**

The total value of the fixed component of storage cost is R1 353 500. The value of C_{10} as depicted in Figure 2.4 has to be expressed as a daily figure, since the cost on the vertical axis has the unit of Rand per day. Depreciation is done over a period of three years, therefore if straight line depreciation is assumed, the value of the fixed component of C_{10} may be rounded to R1 237 per day. The monthly component is R9 376, which expressed as a daily figure, based on 30 days per month, may be rounded to R313.

The value for C_{10} for this particular branch is therefore R1 550 per day.

From the above it is obvious that the total value of C_{10} is branch specific and should be established separately for each branch.

4.3.1.2 The variable component of storage cost

As pointed out earlier, the variable component of C_1 consists of three distinct elements. The first, c_{11}, is the cash float cost incurred by the branch. At present, the branch pays an interest rate of 15,5% per annum on every Rand carried by the branch overnight. This will include any amount kept overnight at the agencies or in the ATM's.

The second variable element of storage cost is insurance, c_{12}. The cash held by this branch is not insured, since the bank policy determines that amounts in excess of R5 000 000 only are insured. At no point for the period under review, did the branch carry an amount in excess of R5 000 000. Appendix D1 shows the total daily cash amounts on hand for the period April 1998 to June 1998. The cash amount peaked at R2.73 million during June. Therefore, c_{12} is 0.

The last element of storage cost pertains to the labour cost per unit, c_{13}. The activities represented in this component include removing cash from the safe for use by the cashiers and balancing by the cashiers at the end of the trading day. It does not include preparation of cash amounts to be sent to agencies or the replenishment of ATM's. These elements are included in the determination of C_{31} in a later paragraph. Each cashier is responsible for balancing his/her safe at the end of the trading day. In contrast to the behaviour of the labour cost as proposed in the model in Chapter 2, the labour cost involved is fairly insensitive to the amount of cash remaining at a cashier at the end of the trading day. This activity takes approximately 30 minutes to perform, irrespective of the amount involved. Since the branch and the agencies in total have nine cashiers, the total time spent on balancing is 4.5 hours at an average labour cost of R18 per hour. Therefore, the daily cost is R81. Since it is regarded as fixed rather than dependent on the amount of cash involved, it is denoted by C_{13} in the revised version of Equation 2-3, *i.e.* Equation 4-1.

4.3.1.3 The revised calculation of storage cost

Based on the preceding two paragraphs it is necessary to revise Equation 2-3 proposed in paragraph 2.3.2.1 to calculate the storage cost. The revised expression is as follows:

$$C_1(Q_i)=C_{10}+c_{11}(\sum_{i=1}^{14} Q_i)+C_{13} \qquad (4\text{-}1)$$

where C_{10} and c_{11} are as defined earlier, but instead of c_{13}, the labour cost is also regarded as a fixed component, denoted by C_{13}.

When the actual values determined in the preceding paragraphs are substituted into the expression it becomes:

$$C_1(Q_i)=1550+(0.155/365)c_{11}(\sum_{i=1}^{14} Q_i)+81$$

which reduces to

$$C_1(Q_i)=1631+4.247x10^{-4}c_{11}(\sum_{i=1}^{14} Q_i) \qquad (4\text{-}2)$$

Appendix E shows the calculation of the daily cash storage cost, as well as the monthly totals for the three months April to June 1998. The storage cost is in excess of R70 000 per month with the interest cost representing on average a third of the total storage cost.

4.3.2 Shortage cost (C_2)

The banks are hesitant to put a value on shortage cost, since it is extremely difficult to quantify. It will be ignored for the time being, but once the model has been evaluated, the impact of a shortage will be investigated. If the patterns reflecting amounts of cash carried by the branch are investigated (refer to Appendices A and B in this regard, and in particular to the graph in Appendix

B5), it is obvious that a shortage is avoided by carrying high levels of cash. This implies that the banks regard the shortage cost as significant. The impact of the shortage cost on order policy will be discussed further in Chapter 6.

4.3.3 Supply cost (C_3)

Although Chapter 2 identified four elements constituting supply cost, the actual situation is rather different. Firstly, it is necessary to separate the supply to and from the branch by SBV from the transportation of cash between the branch and the agencies, since different carrier companies are involved and the cost structures for the situations differ. Figure 4.2 shows the upstream and downstream movement of cash to and from the branch. Secondly, the order cost and the cash handling cost elements of supply cost are combined in determining the actual cost. This is discussed in paragraph 4.3.3.1. The transportation cost and in-transit cost elements are discussed in paragraphs 4.3.3.2 and 4.3.3.3.

4.3.3.1 The order and cash processing cost elements of the supply cost

Although the proposed model separates order cost and cash processing cost when determining the supply cost, the actual situation allows for these two elements to be combined. Since the time involved in processing cash is fairly short, the assumption is made that the time is constant irrespective of the amount involved, therefore the cost of cash processing is regarded as fixed per event.

However, to allow for the separation of movements upstream and downstream from the branch, Equation 2-6 for determining the order cost element (C_{31}) is revised as follows:

$$C_{31}(Q_i)=c_{311}\frac{D}{Q_D}+c_{312}\frac{A}{Q_A} \qquad \text{(4-3)}$$

where c_{311} represents the internal order and processing cost involved when a visit from SBV occurs, c_{312} is the internal order and processing cost regarding supply of cash to the two agencies, A represents the total demand for cash at the agencies during the planning period and Q_A is the cash amount involved in the trip between the agencies and the branch.

The internal activities involved to arrange for a visit from SBV to the branch are the responsibility of the operations manager at the branch, but may also be performed by the treasury custodian, although the responsibility remains that of the operations manager. This includes reviewing the amount of cash in the branch at the end of a trading day taking into consideration whatever is held overnight at the agencies, as well as the balance in the ATM's. Based on this amount a decision is made whether to place an order for the next day. Since the supply lead time is two days when trading on consecutive days, this implies considering the required amounts for trading two days hence. Obviously a disruption in trading and replenishing due to a Sunday or a public holiday will be considered.

The ordering procedure consists of a telephone call being made and a facsimile message being sent to SBV by the operations manager. In all the total duration of these activities is approximately 10 minutes. The labour cost involved is R45 per hour, therefore the labour cost is R7.50 per order. The cost of the facsimile is estimated at R1, whereas the telephone call of two minutes is estimated to cost R1.50. The total cost to place the order, therefore, is R10.

Internal proceedings when delivery or removal by SBV occurs, take approximately 20 minutes. Again this should be dependent on the cash amount involved in the delivery or removal, but since the period involved is relatively short, an average time is taken. The cost of the labour involved in the process is on average R37 per hour, therefore since two staff members are involved the total labour cost may be rounded to R25 per order.

The total internal order cost, c_{311}, when a delivery or removal by SBV occurs, is therefore R35.

It would be appropriate to comment on the existing order policy followed by the operations manager regarding the placement of orders to replenish, replace or reduce the amount of cash held at the branch. This policy is not formalised in any way, but is based on experience at this particular branch. As a rule of thumb, the amount of cash in the main safe is used as a benchmark. The reasoning behind the policy is depicted in Figure 4.3. Appendix I compares this experience-based policy to actual orders placed, showing that the policy is not adhered to strictly. The policy is investigated in greater detail in Chapter 6.

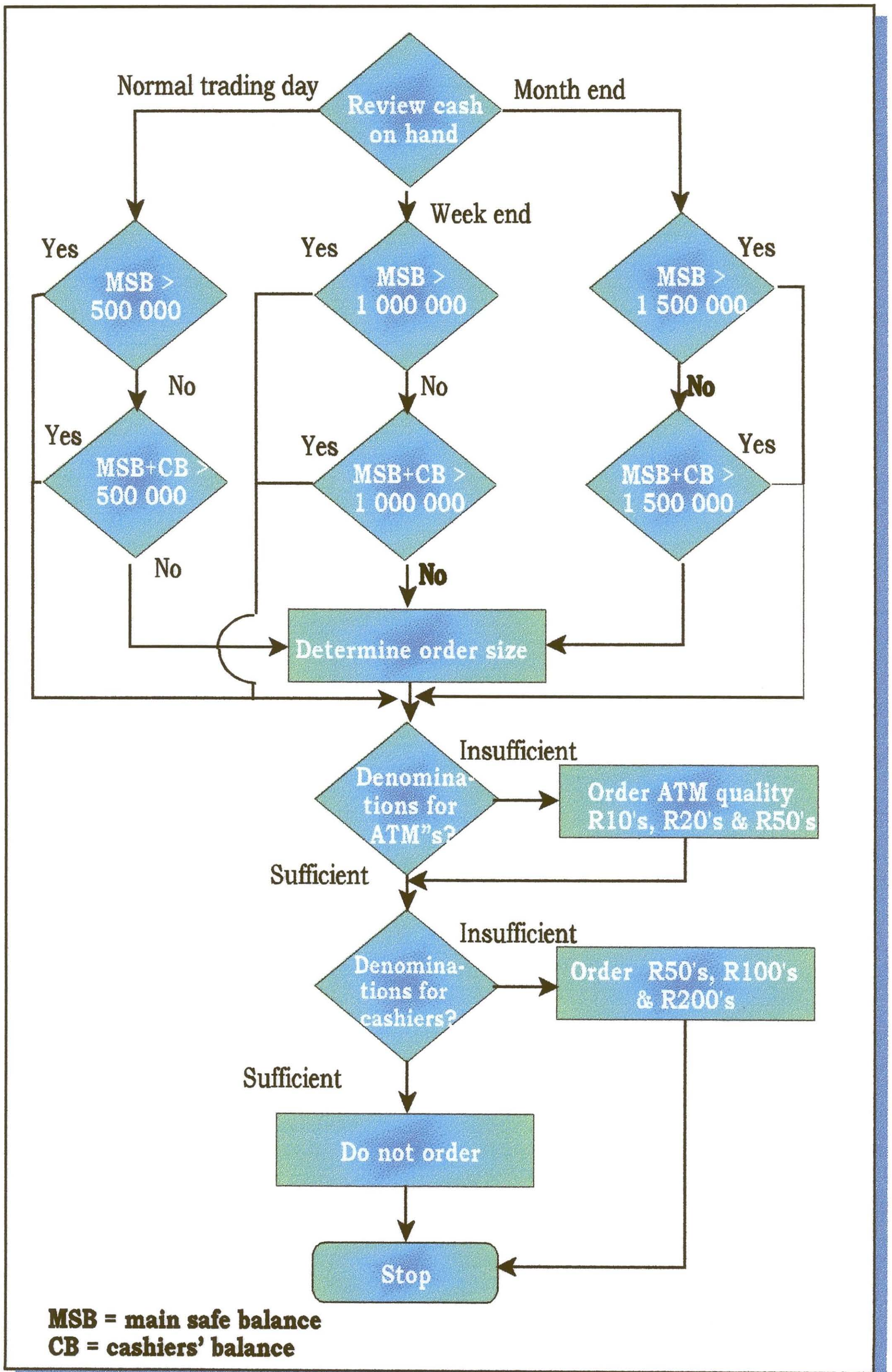
With regard to supplying the agencies with cash, transportation occurs twice a day – to each agency each morning with a return from each agency every afternoon after trading. The proceedings at the branch and the agency attributable to the movement of cash are mirror images – what happens at the branch in the morning and in the afternoon occurs in reversed order at each agency. It takes approximately 20 minutes for the preparation of each order to be sent to the agencies and 20 minutes when the cash is returned in the afternoon. Once the cash arrives at each agency and when it is returned after trading, the duration is the same. This implies that the total time involved in these activities is 60 minutes per trip, where two staff members are involved at an average labour cost of R37 each. Therefore the value of c_{312} is R74 per trip. These movements of cash occur twice a day according a fixed schedule.

If the values for c_{311} and c_{312} are substituted into Equation 4-3, it becomes:

$$C_{31}(Q_i)=35\frac{D}{Q_D}+74\frac{A}{Q_A} \qquad \text{(4-4)}$$

Figure 4.3

Order policy followed at branch



4.3.3.2 The transportation cost element of the supply cost

The transportation cost consists of two elements. The first reflects the cost of delivery or removal by SBV to and from the branch. The second pertains to the supply of cash to the agencies. In the model proposed in Chapter 2, no distinction was made between these two elements. The cost of transportation was shown to be a step function, the distinction between the two curves emanating from the difference in transportation cost for normal and interim (or special) deliveries by SBV to the branch.

The cost of a normal delivery by SBV (c_{331}) is R500, whereas the cost of a interim delivery (c_{332}) is R1 000. For the period under review, the branch received only one interim delivery due to special circumstances. The interim delivery occurred on April, 29, and was a direct result of the branch having moved premises during the previous days. SBV visited the branch on 18 occasions during the review period of three months. Of these visits, 14 were for the purpose of delivering cash, whereas four visits were for the removal of cash. On one occasion delivery and removal of notes occurred simultaneously, on one occasion coin and notes were delivered on the same day and on one occasion notes were delivered and coin removed on the same day. When delivery and removal occur simultaneously (as for example in the case of a denomination switch), it does not affect the transportation cost (*i.e.* it remains R500 for the trip).

The timing of the deliveries has been a problem for the branch, since the delivery time on a particular day is only communicated to the branch 30 minutes before the delivery occurs. This is done for obvious security reasons. A contributing factor is that all banks experience the same peaks in their demand patterns, therefore SBV just has so much more to deliver when the demand peaks occur (for example at month end). The impact of the supply lead time will be investigated in greater detail in a subsequent chapter. Suffice it to say that the branch has taken a decision to order cash for the month end peak for the previous day, due to the uncertainty regarding the delivery time.

The values for c_{331} and c_{332} are substituted into Equations 2-9 and 2-10. Instead of using C_{33} to denote the transportation cost, C_{33B} is used to indicate transportation to and from the branch by SBV.

For a special delivery

$$C_{33B}(Q_i) = 1000\frac{D}{Q_D} \qquad \text{(4-5)}$$

For a normal delivery

$$C_{33B}(Q_i) = 500\frac{D}{Q_D} \qquad \text{(4-6)}$$

As stated above, the model does not provide for deliveries from the branch to the agencies. Obviously, if a branch has no agencies, this factor may be omitted. However, the branch under review does have two agencies and whereas visits from SBV are the exception rather than the rule, cash is moved to and from the agencies on a daily basis. This implies ten trips per week to each agency. If required a special trip may be requested on a Saturday to Agency A, but this is not provided for in the normal contract amount. Although the trip needs to be specially arranged, the cost remains the same as for a normal trip. The fixed amount of R6 053 for transportation of cash between the branch and the agencies is based on an average of 20 days on which trips are undertaken, in other words 40 trips per month to each agency or a total of 80 trips per month. Therefore the cost per trip is R76. A special trip to Agency A on a Saturday would cost the same.

Shortages at the agencies are not catered for. Should a customer arrive with a request for a particularly large withdrawal without prior arrangement, the customer is referred to the branch. The customer would then have to travel to the branch which is located approximately five kilometres from Agency A and three kilometres from Agency B.

Equation 4-7 is used to determine the transportation cost between the branch and the agencies:

$$C_{33A}(Q_i)=76\frac{A}{Q_A} \qquad \text{(4-7)}$$

where C $_{33A}$ represents the transportation cost between the branch and the agencies.

The total transportation cost C $_{33}$ will be equal to the sum of C $_{33B}$ and C $_{33A}$.

4.3.3.3 The in-transit insurance cost element of the supply cost

The in-transit insurance cost element is not treated as a separate component of the supply cost. It is included in the transportation cost, C $_{33}$, which remains constant irrespective of the size of the order. This implies that a small order delivered during the same trip as a large order, subsidises the in-transit insurance of the large order. The value of C $_{34}$ is 0 for this particular case and for all other South African retail banks that make use of the services of SBV. The same applies to the two carrier companies that are used to transport cash between the branch and the agencies. In-transit insurance is included in the amount charged per trip.

4.3.3.4 The total supply cost

The total monthly supply cost for the period April to June 1998 is summarised in Appendix F. As mentioned before, the branch follows a policy of avoiding special deliveries from SBV and limiting normal deliveries to a single occasion per week. The resulting supply cost is low in comparison to the total monthly storage cost over the same period calculated in Appendix E.

4.3.4 The total cost of cash holding

Appendix G summarises the total daily cost of cash holding at the branch and its two agencies for the period April to June 1998. The next step is to investigate the effect of changes to the above-mentioned policies on the total cost of holding cash with the aim of reducing the total daily cost of holding cash without compromising customer service and without running due risk of a shortage situation.

4.4 Conclusion

Chapter 4 investigated a case representative of the situation found at South African retail banks with the aim of illustrating how the cost parameters of holding cash may be estimated. Subsequently, the cost of the current order policy was determined for a period of three months. In Chapter 5 the demand patterns are investigated. Once this has been concluded, the effect of changes to the existing order policy will be investigated in subsequent chapters.

CHAPTER 5

The impact of demand management issues in retail banking

5.1 Introduction

Demand management may be defined as the process whereby all factors influencing the demand for a particular item are recognised and identified, the aim being to control the demand for that item as far as possible. Demand management encompasses demand forecasting, order entry, the order promise, determination of demand at subsidiary plants as well as the determination of the need for replacement parts or items. As Melnyk & Denzler (1996:444-445) state:

> *Operations managers first seek to discover product characteristics that drive customer demand. With this knowledge they can explore ways to influence the timing pattern of demand by certain customers to benefit both them and the producing firm.*

In fact, the relationship between forecasting and inventory management appears to have first been recognised by R.G. Brown in 1959 in a book titled *Statistical forecasting for Inventory Control* (Nahmias 1993:91).

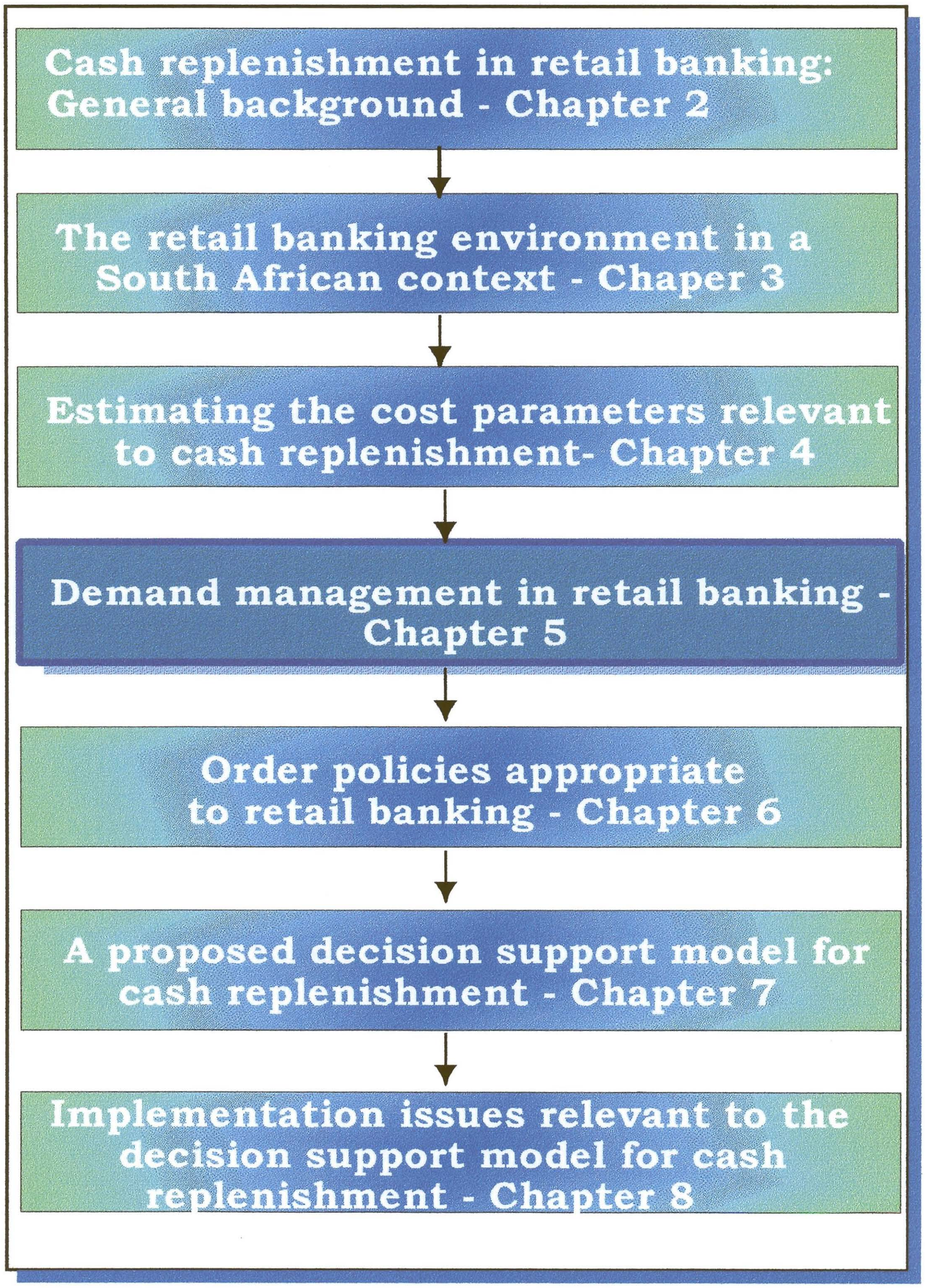
If the above definition of demand management is applied to the retail banking environment, it implies that the operations manager at the branch should investigate demand patterns in an attempt to discover the how, when and why of these patterns. This information should subsequently be used to the advantage of the customer as well as the service provider, *i.e.* the bank. The definition also refers to subsidiary plants, which in the case under investigation, would refer to the demand patterns which occur at the two agencies of the branch (refer to Figure 4.2 in this regard). The order promise in the case of a bank is an implicit rather than an explicit statement. The customer assumes and expects that, when requested, the bank will always be in a position to provide the amount of cash required by the customer. Providing replacement parts in a banking environment is not as important an issue as in the case of a manufacturing concern, although the bank is responsible for removing unfit notes from circulation and returning those to SBV (refer to paragraph 3.4 in this regard). Order entry in the case of the bank refers to the record keeping of transactions, *i.e.* cash flowing into and out of the bank. This particular issue will be addressed in Chapter 6.

Melnyk & Denzler (1996:445) state further that demand management seeks to control demand in two ways, *i.e.* by influencing the pattern of customer orders and by reducing the uncertainty of its demand pattern. It is exactly this last point which leads to businesses holding too much inventory. Chapter 4 attempted to quantify the impact of the current inventory levels on the cost structure of the branch. However, before any suggestions regarding a reduction in the amount of cash held at the branch and therefore a concomitant reduction in the cost of holding inventory may be achieved - as will be shown in Chapter 6 - the demand patterns need to be investigated. It is also prudent at this point to investigate the demand management activities which are in place at the branch.

Figure 5.1 shows the relevance of demand management to the other chapters included in the research report.

Figure 5.1

The structure of the report indicating the relevance of Chapter 5



5.2 Existing demand forecasting practices

As indicated in Chapter 4, no formal procedure is followed to forecast deposits, face-to-face withdrawals or ATM withdrawals at the branch. With the exception of the rough estimate of the required cash on hand per day to fulfill expected total withdrawals as indicated in Figure 4.3, the branch has no other demand forecasting system. In addition, there is also no tracking system providing regular feedback on actual face-to-face deposits or withdrawals per day. Feedback on ATM withdrawals may be obtained without much trouble.

The approach followed at the branch is an example of an intuitive forecasting method based on experience, particularly that of the branch operations manager and some estimate of seasonality in the withdrawal patterns based on the time of week or month. An investigation into the rationale followed by the branch according to Figure 4.3 based on the actual figures is appropriate at this point.

5.2.1 Withdrawals and deposits quantified

Appendices A, B and C show the real cash deposits, total withdrawals and ATM withdrawals for the period April to June 1998. ATM withdrawals form part of the total withdrawals shown in Appendix B. The appendices show both the actual runs, as well as a histogram of the total daily deposits and withdrawals. Appendix B5 compares the total withdrawals to the deposits. In Appendices A2, B2 and C2 various averages are calculated. Table 5.1 compares the averages for the six trading days of a week as well as the averages for normal trading days, Fridays (*i.e.* week end) and the five trading days at the end of the month.

The operations manager at the branch stated that week ends (*i.e.* Fridays) require double the amount cash on hand for withdrawals, whereas the five days at the end of a calendar month require three times the amount of cash when compared to the required amount on a normal trading day. The rule of thumb applied by the branch, uses R500 000 as the minimum amount of cash on hand for withdrawal purposes on a normal trading day. Friday would therefore require R1 000 000, whereas the five days at the end of a calendar month would require R1 500 000.

ATM replenishment over weekends requires some clarification. Although the ATM's are filled with sufficient cash on a Saturday to provide for expected demand until Monday, the ATM withdrawals are monitored centrally. Should a shortage occur before trading reopens on a Monday, a branch staff member is on standby to meet cash delivery staff at the ATM to replenish the machine where the shortage has occurred.

Table 5.1

A comparison of deposit, total withdrawal and ATM withdrawal averages

Day	Deposit average	Total withdrawal average	ATM withdrawal average
Monday	796 202	733 464	196 793
Tuesday	683 714	796 072	179 839
Wednesday	683 168	830 725	168 736
Thursday	508 489	800 817	182 243
Friday	559 616	1 029 475	207 615
Saturday	281 805	447 764	166 518
Normal trading day	550 261	622 531	172 480
Month end	704 882	1 124 768	212 115
Overall average	579 090	766 797	182 658

From Table 5.1, it is obvious that with the exception of Monday, the average total withdrawal is always greater than the average deposit. On Mondays, the two amounts are of the same scope. The deposit average on a Monday may be explained if deposit patterns on business accounts are investigated, particularly at this branch which serves a number of retail businesses which are contractually bound to do business until five o' clock on a Saturday and have to open from 9am until 2pm on a Sunday. Appendix B5 shows the differences on a daily basis between deposits and total withdrawals. On 13 of the 73 trading days, deposits exceeded total withdrawals. On seven out of thirteen occasions, this occurred on a Monday. Based on the figures in Table 5.1, this amount used for a Friday is fairly accurate, although the amount used for a month end seems rather high. The amounts withdrawn at the ATM's are subject to a maximum daily ceiling of R1 000 per account. In addition, the move of the branch from the old location (in a busy shopping mall) had an impact on the withdrawals, since withdrawals at an ATM are often done by a person who does not necessarily have an account with that particular bank. Face-to-face transactions tend to be carried out at the client's specific bank - often the specific branch at which the account is held. The move to an office complex across the street, removed the presence of passing traffic, since the bank is at present one of just a few businesses located in the new complex. It is quite obvious that these patterns are very much branch-specific.

5.2.2 Implied seasonality factors used by the branch

The implied seasonality factors used by the branch for withdrawals on normal trading days, Fridays and the five days at the end of the month are summarised in Table 5.2. These factors are based on a month consisting of a total of 26 trading days. The factor used varies slightly according to the day of the week on which a calendar month commences. The weight for a normal trading day will, for example, be 0.67 for months when the first day occurs on a Saturday, Monday or Tuesday, but will be 0.65 when the first day is on a Wednesday, Thursday or Friday.

Table 5.2

Seasonality factors for total withdrawals used by the branch

DAY	Total withdrawal factor
Normal trading day	0.65 or 0.67
Friday	1.30 or 1.33
Five days at the end of the month	1.95 or 2.00

However, the seasonality factors of deposits and ATM withdrawals are not quantified in the policy followed at the branch, although the operations manager does admit that the branch receives substantial deposits on a Monday. No guideline in this regard was available from the branch for ATM withdrawals. The validity of these factors will be investigated in a later section of this chapter.

5.3 Appropriate forecasting methods

5.3.1 Introduction

It is appropriate at this point to consider the characteristics of a good forecast before discussing various forecasting methods suited to the particular situation. Nahmias (1993:50-51) lists the following characteristics of forecasts:

- They are normally wrong, therefore the planning system should be sufficiently robust to be able to react to anticipated forecast errors.
- A good forecast is more than a single number; it should include some measure of anticipated forecast error.
- Aggregate forecasts are more accurate.
- The longer the forecast horizon, the less accurate the forecast will be.
- Forecasts should not be used to the exclusion of known information; cognisance should be taken of factors influencing future demand not represented in the historical data.

If these requirements are related to the branch of a retail bank, it has specific implications for the forecasting technique used. Firstly, the fact that the forecast will not be correct, implies the use of some safety stock to cover for expected errors. Secondly, the safety stock calculation should be based on the anticipated forecast error. In the third place, forecasting total daily withdrawals, deposits and ATM withdrawals would represent aggregate forecasts, which would result in a smaller error than forecasting individual transaction values or even demand for specific denominations. In the case under review the forecast horizon is relatively short, *i.e.* two days for normal orders and one day for special orders. Inclusion of known information would imply that knowledge of a public holiday and the impact that may have on the demand at the branch should be considered, or that the impact of the December school holidays on activity levels at the branch should be taken into account.

Stevenson (1999:90) states further that a properly prepared forecast should fulfill certain requirements:

- The forecast should be timely.
- The forecast should be accurate and the degree of accuracy should be stated.
- The forecast should be reliable and work consistently.
- The forecast should be expressed in meaningful units.
- The forecast should be in writing to ensure that all parties involved use the same information and to permit an objective basis for evaluating the forecast once actual results are available.
- The forecasting technique should be simple to understand and to use.

The implications of the above for the branch are the following: The forecast should be available in time for the operations manager to use when placing the cash order with SBV. Monitoring the forecast to prove or disprove its accuracy and reliability, and therefore usefulness would be necessary. Finally, the importance of having a simple system which is easy to use cannot be overemphasised.

5.3.2 The nature of the demand patterns

5.3.2.1 Seasonality present in the demand patterns

From the previous paragraph it is obvious that some seasonality is present in the total withdrawal and demand patterns. It may be assumed that the ATM withdrawal patterns will exhibit similar fluctuations over time. It may also be assumed that as a result of inflation, some trend should be evident in the demand patterns, particularly over the longer term. The suitability of the implied factors used by the branch require investigation.

Given the above description, a method for deseasonalising the data is required. Seasonal relatives are calculated for each day of the week or month depending on the season selected. This is done by dividing the demand for that day by the average demand during the season. In using the deseasonalisation method, the repetitive patterns over six, 26, 24 and 30 days are investigated. *HOM Operations Management Software for Windows®* (Moses *et al.* 1999) was used to calculate the seasonal factors. Since a number of public holidays occurred during the period under review, an assumption regarding the amounts on these days was necessary. To facilitate calculation of the seasonal averages, it was assumed that the reported amount on the day after the public holiday could be divided by two to provide a number appropriate for the public holiday.

Table 5.3 provides a summary of the seasonal factors for each day of the week for both deposits, total withdrawals and ATM withdrawals. In this case the assumption is that the cyclical pattern repeats itself every six days.

Table 5.3

Seasonality factors for each day of the week based on a six day cycle

DAY	Deposit factor	Total withdrawal factor	ATM withdrawal factor
Monday	1.36	0.98	0.96
Tuesday	1.10	0.93	1.07
Wednesday	1.21	1.13	1.13
Thursday	0.94	1.12	0.87
Friday	0.93	1.28	1.16
Saturday	0.48	0.56	0.81

The seasonal factors to an extent, confirm some of the patterns identified by the operations manager at the branch, showing the withdrawal peak on a Friday and the deposit peak on a Monday. These factors also confirm the timing mismatch between withdrawal and deposit patterns.

However, implied in the reasoning of the operations manager at the branch, the pattern does not repeat itself every six days, but over and above the weekly cycle repeating itself over the six trading days per week, an additional cycle of 26 days exists. Concomitant to peaks occurring weekly, the last five trading days of every calendar month exhibit additional increases. This implies two cycles superimposed on one another, the shorter however repeating itself 4.3 times for every one of the longer cycles.

Should the same method for calculating seasonal factors again be employed using *HOM Operations Management Software for Windows®* (Moses *et al.* 1999), the values for the 26 days of the month are as shown in Table 5.4. This probably provides a more accurate picture of the true seasonality present in the total withdrawals, deposits and ATM withdrawals, which should be used when taking cash inventory decisions.

The calculations in Table 5.4 are based on the first month commencing on a Wednesday. This resulted in the month having four Fridays, Saturdays, Mondays and Tuesdays, but five Wednesdays and Thursdays. The second month of the review period commenced on a Friday, resulting in five Fridays and Saturdays during the month, but having only four Mondays, Tuesdays, Wednesdays and Thursdays. The third month consisted of four Wednesdays, Thursdays, Fridays and Saturdays, but had five Mondays and Tuesdays. This occurrence may have an impact on the factors calculated. The impact will be discussed later when various forecasting methods are discussed.

In an attempt to evaluate the validity of the estimates used by the branch for total withdrawals, a comparison is made between seasonal factors for a 26 period cycle determined using the deseasonalisation method in Table 5.4 and those used by the branch in Table 5.2, and in addition, a comparison is made between actual total withdrawals per day based in part on Appendix B2 and the estimate used by the branch as indicated in Figure 4.3. These comparisons are shown in Table 5.5.

Table 5.4

Seasonality factors for each day of the month based on a 26 day cycle

DAY	Deposit factor	Total withdrawal factor	ATM withdrawal factor
First Wednesday	1.02	1.05	1.12
First Thursday	0.86	1.87	1.06
First Friday	1.27	1.32	1.18
First Saturday	0.81	0.76	1.11
First Monday	1.39	1.04	1.05
First Tuesday	0.61	0.55	0.83
Second Wednesday	1.12	1.05	1.15
Second Thursday	0.73	0.89	1.09
Second Friday	0.96	0.71	0.65
Second Saturday	0.81	0.60	0.54
Second Monday	0.64	0.76	1.20
Second Tuesday	0.46	0.44	0.82
Third Wednesday	0.95	0.98	1.29
Third Thursday	0.61	0.43	0.57
Third Friday	1.31	1.12	1.14
Third Saturday	0.84	0.67	0.92
Third Monday	1.47	1.13	1.02
Third Tuesday	0.84	1.02	0.87
Fourth Wednesday	1.01	0.78	0.73
Fourth Thursday	0.79	0.90	0.88
Fourth Friday	1.24	1.51	1.12
Fourth Saturday	1.20	1.08	0.58
Fourth Monday	0.84	1.18	1.23
Fourth Tuesday	0.79	1.11	1.13
Fifth Wednesday	1.90	2.08	1.25
Fifth Thursday	1.53	2.03	1.50

Table 5.5

Comparison of withdrawal factors and amounts used and calculated

DAY	Total with-drawal sea-sonal factor	Branch withdrawal factor	Average amount withdrawn	Branch allowance
First Wednesday	1.05	0.65	790 420	500 000
First Thursday	1.87	0.65	447 585	500 000
First Friday	1.32	1.30	763 677	1 000 000
First Saturday	0.76	0.65	364 343	500 000
First Monday	1.04	0.65	886 848	500 000
First Tuesday	0.55	0.65	702 459	500 000
Second Wednesday	1.05	0.65	714 661	500 000
Second Thursday	0.89	0.65	695 540	500 000
Second Friday	0.71	1.30	690 822	1 000 000
Second Saturday	0.60	0.65	305 374	500 000
Second Monday	0.76	0.65	402 336	500 000
Second Tuesday	0.44	0.65	388 119	500 000
Third Wednesday	0.98	0.65	410 624	500 000
Third Thursday	0.43	0.65	613 592	500 000
Third Friday	1.12	1.30	1 083 932	1 000 000
Third Saturday	0.67	0.65	381 246	500 000
Third Monday	1.13	0.65	825 985	500 000
Third Tuesday	1.02	0.65	530 435	500 000
Fourth Wednesday	0.78	0.65	986 272	500 000
Fourth Thursday	0.90	0.65	980 241	500 000
Fourth Friday	1.51	1.30	926 591	1 000 000
Fourth Saturday	1.08	1.95	458 679	1 500 000
Fourth Monday	1.18	1.95	579 355	1 500 000
Fourth Tuesday	1.11	1.95	778 957	1 500 000
Fifth Wednesday	2.08	1.95	1 902 638	1 500 000
Fifth Thursday	2.03	1.95	2 199 751	1 500 000

Again the day of the week on which the month commences will have an impact on the calculation of the seasonal factor as well as the average amount withdrawn on that particular day. The comparison in Table 5.5 was based on the three month period under review, where the first of the three months started on a Wednesday. The differences between the figures used by the branch and calculated on the basis of a 26 day season are quite significant in some cases.

In an attempt to find a longer cycle (monthly) corresponding to the shorter cycle (weekly) as identified by the branch, two alternatives are proposed. The first is to use a longer cycle of 24 days (monthly), or four weeks consisting of six trading days each. The second proposal is to use a 30 day longer cycle (monthly) under the assumption that the month end is not as clear-cut as assumed by the branch, *i.e.* month end could have an effect on the first few trading days at the start of a new month. The second proposal suggests a season consisting of five weeks consisting of six trading days each. The factors for these two proposed alternatives a summarised in Table 5.6.

The suitability of the various periods as well as the validity of the seasonal factors will be discussed in detail in paragraph 5.3.3.

Table 5.6
Seasonality factors based on a 24 and 30 day cycle

DAY	Deposit factor		Total withdrawal factor		ATM factor	
	24 day	30 day	24 day	30 day	24 day	30 day
First Wednesday	1.66	1.24	1.69	1.20	1.20	1.35
First Thursday	1.06	0.56	1.87	0.79	1.75	1.13
First Friday	1.04	0.97	1.38	1.42	1.23	1.23
First Saturday	0.50	0.41	0.72	0.63	1.19	1.34
First Monday	1.81	1.42	1.31	0.59	1.16	1.05
First Tuesday	1.35	0.80	1.04	0.47	0.97	0.66
Second Wednesday	1.10	0.86	1.06	0.92	0.99	1.38
Second Thursday	0.78	0.80	0.84	1.01	1.01	1.15
Second Friday	0.84	0.61	1.15	0.87	0.94	1.06
Second Saturday	0.44	0.46	0.52	0.47	0.82	0.54
Second Monday	1.06	1.43	0.59	1.11	0.99	1.33
Second Tuesday	0.81	0.88	0.57	0.57	0.68	0.87
Third Wednesday	1.13	0.68	1.11	0.74	1.15	1.03
Third Thursday	0.89	0.91	0.75	0.95	0.83	0.89
Third Friday	0.88	1.17	1.29	1.27	1.28	1.12
Third Saturday	0.36	0.47	0.49	0.66	0.83	1.19
Third Monday	2.00	1.57	1.22	1.13	1.42	1.50
Third Tuesday	1.02	1.31	0.78	1.27	0.67	0.79
Fourth Wednesday	0.59	0.76	0.51	0.85	0.70	0.83
Fourth Thursday	0.97	0.74	1.10	1.26	0.81	1.09
Fourth Friday	1.10	0.95	1.47	1.60	0.92	1.26
Fourth Saturday	0.54	0.69	0.54	0.67	0.63	0.60
Fourth Monday	0.87	1.30	0.86	1.27	1.04	0.87
Fourth Tuesday	1.18	1.44	1.15	1.13	0.79	0.94
Fifth Wednesday	-	2.02	-	1.82	-	0.70
Fifth Thursday	-	1.40	-	1.71	-	1.30
Fifth Friday	-	0.78	-	0.91	-	0.64
Fifth Saturday	-	0.44	-	0.43	-	0.52
Fifth Monday	-	1.77	-	1.21	-	1.00
Fifth Tuesday	-	1.19	-	1.04	-	0.63

5.3.2.2 Trends evident in the demand patterns

Again using the *HOM Operations Management Software for Windows®* (Moses *et al.* 1999), an analysis was performed on the available data to investigate the trends present in the total withdrawals, deposits and ATM withdrawals. In all three cases trends were evident. In both total withdrawals and deposits the trend showed an increase, whereas the ATM withdrawals exhibited a decreasing trend. The latter may be explained by the change of location of the branch from a busy shopping mall to an office complex across the street from the mall with no passing pedestrian traffic. The decrease is obvious from Appendix C3, where the change in level of ATM withdrawals at the end of April is quite visible. If the data points describing ATM withdrawals prior to the move are ignored, the remaining data points again show an increasing trend.

The level change, so obviously present in the ATM withdrawals, does not manifest in the deposit pattern (Appendix A3) or total withdrawal pattern (Appendix B3), although the total withdrawals include the ATM withdrawals.

The presence of a trend component in the three demand patterns will be taken into consideration when selecting a forecasting method. This component of the demand pattern is ignored in the reasoning of the operations manager at the branch in applying the weights summarised in Table 5.2.

5.3.3 Selection of a forecasting method

Stevenson (1999:122) states unequivocally that no single technique works best in every situation and in selecting a technique for a given situation, the two most important factors to consider are accuracy and cost. Other factors to consider include the availability of historical data, the availability of computers, the ability of decision-makers to utilise the technique, as well as the time needed to gather and analyse data and to prepare the forecast. Chase *et al.* (1999:529) state that in selecting a forecasting method, a measure of forecast error, such as mean absolute deviation (MAD), together with a tracking signal based the relationship between the mean absolute deviation and the running sum of forecast errors be used to evaluate various approaches. This would support the claim by Stevenson (1999:122) that accuracy is very important.

Eppen *et al.* (1993:801) propose the following features that distinguish one forecasting situation from the next:

- The importance of the decision.
- The availability of relevant data.
- The time horizon for the forecast.
- The cost of preparing the forecast.
- The time until the forecast is needed.
- The number of times such a forecast will be needed.
- The stability of the environment.

In addition the components of the demand patterns discussed in paragraph 5.3.2 should be considered when selecting an appropriate method.

5.3.4 Methods investigated

Based on the reasoning put forward in the previous two paragraphs, sophisticated methods, such as the Box-Jenkins model, were deemed to be unsuitable. Given the particulars of the situation, a simplistic, user-friendly approach providing a rapid response was regarded as the most appropriate method.

Winter's method for forecasting demand, where the demand patterns exhibit some form of seasonality, was regarded as a possibility, since in addition to compensating for seasonal behaviour, it also provides for a trend component (Montgomery & Johnson 1976:99-108). In addition, Holt's method which combines exponential smoothing with a trend, was considered (Winston 1991:1169-1171). Finally, in an attempt to consider as many options as possible, the forecasting module of *HOM Operations Management Software for*

Windows® (Moses *et al.* 1999:176-199) was used to evaluate 16 different approaches. The methods investigated included simple exponential smoothing, FIT smoothing (or double exponential smoothing also known as Holt's method), trend regressed exponential smoothing, simple average, moving average and Winter's method. Since seasonality and a trend component were present in all three data series, two approaches to treating the seasonality and two methods for incorporating the trend component were investigated respectively.

Two techniques were used to calculate seasonality for each forecasting method investigated, *i.e.* simple seasonal relatives (SSR) and moving seasonal relatives (MSR). In the second case, the seasonal relative is an average based on, for example, the preceding and following seasonal weights, whereas the first approach merely determines the weight for that particular season (Stevenson 1999:106-109). Both approaches were investigated and are reported on in the following result summaries.

Two approaches to calculating the initial value of the trend component were used for each forecasting method. The first (default) approach used an initialisation value of zero, whereas the second approach used a regressed value for initialisation purposes. The regression was carried out over the starting and ending periods of the data (Moses *et al.* 1999:195).

The software has the capability to find the best option from five methods, *i.e.* exponential smoothing, FIT smoothing, exponential smoothing with a regressed trend, simple average and moving average. The selection of the best of the methods is based on the forecast error. In addition, the software is capable of optimising the values of the smoothing constants, where applicable (Moses *et al.* 1999:176-199). This is achieved by minimising the root mean square error (RMSE). The full results for the techniques investigated may be found in Appendix H.

Table 5.7 summarises the best results for total withdrawals based on seasons of differing lengths using all available data points and Table 5.8 that for total withdrawals using the most recent 56 data points, *i.e.* after the move of the branch to the new location.

The measures of forecast error reported on in Tables 5.7 to 5.11 are the root mean square error (RMSE), the mean absolute percent error (MAPE) and the mean absolute deviation (MAD). The measures of forecast error will be discussed in more detail in paragraph 5.3.5.

Table 5.7

Comparison of forecasting methods for total withdrawals for differing seasons using all available data points

Season	Forecasting method	Seaso-nality	Measures of forecast error		
			RMSE	MAPE	MAD
6 days	Moving average	SSR	384 732	50.20%	258 837
	Winter's method with regressed trend	SSR	370 683	55.04%	270 157
24 days	FIT smoothing with default trend	SSR	301 406	38.54%	220 001
	Simple average	SSR	298 723	39.97%	218 769
	Simple average	MSR	316 419	40.88%	218 248
26 days	Simple average	SSR	291 437	44.49%	226 927
30 days	FIT smoothing with default trend	SSR	338 206	39.90%	209 357
	Simple average	SSR	333 285	43.83%	223 189

Table 5.8

Comparison of forecasting methods for total withdrawals for differing seasons using 56 most recent data points

Season	Forecasting method	Seasonality	Measures of forecast error		
			RMSE	MAPE	MAD
6 days	Moving average	MSR	321 613	44.08%	221 423
24 days	Moving average	SSR	286 826	30.98%	190 958
	Moving average	MSR	305 407	31.45%	183 103
26 days	Simple average	SSR	318 145	47.66%	253 211
30 days	FIT smoothing with default trend	SSR	447 901	46.52%	196 393
	Winter's method with regressed trend	SSR	439 816	49.51%	211 868

From Tables 5.7 and 5.8, it may be construed that fitting a forecasting technique to the most recent 56 data points will provide a better result than using all the available points. Table 5.8 proves that irrespective of the measure of forecast error used to select an appropriate forecasting method, the best result will be obtained if a cycle of 24 days is used and a moving average forecasting method is applied. If all the data points are used, as shown in Table 5.7, the most suitable cycle is not as easily identifiable. Each measure of forecast error points to a different cycle, with the exclusion of a six day cycle, which proves to be unsuitable in all instances.

Table 5.9 shows the summary of results for forecasting techniques applied to deposits using all available data points. This was done under the assumption that the move did not have such an impact on deposits as it had on withdrawals. From Table 5.9 the obvious conclusion is that a cycle of 24 days provides the best results when fitting a forecasting method. Irrespective of the measure of forecast error used to select an appropriate method, the cycle in all three instances will be 24 days. However, the measure of forecast error used to select an appropriate method will determine which approach is applied.

Table 5.9

Comparison of forecasting methods for deposits for differing seasons using all available data points

Season	Forecasting method	Seaso-nality	Measures of forecast error		
			RMSE	MAPE	MAD
6 days	Moving average	SSR	299 207	47.91%	194 297
	Winter's method with regressed trend	SSR	281 227	52.10%	205 376
24 days	Simple exponential smoothing	MSR	246 084	41.02%	171 956
	FIT smoothing with regressed trend	MSR	245 944	41.60%	168 784
	Simple average	SSR	239 617	47.38%	174 917
26 days	Simple average	SSR	268 642	58.08%	197 049
	Winter's method with regressed trend	SSR	270 903	56.22%	195 660
	Winter's method with regressed trend	MSR	300 479	54.46%	204 932
30 days	Simple exponential smoothing	SSR	284 331	43.81%	193 144
	Simple average	SSR	271 597	49.18%	193 199
	Winter's method with regressed trend	SSR	274 551	45.35%	192 724

Table 5.10 shows the results for ATM withdrawals using all the data points whereas Table 5.11 shows the results for ATM withdrawals using the 56 data points after the move of the branch.

Again the analysis of forecasting methods applied to ATM withdrawals is inconclusive, since the two of the measures of forecast error point to a 26 day cycle, but the mean absolute deviation points to a 24 day cycle.

Table 5.10

Comparison of forecasting methods for ATM withdrawals for differing seasons using all available data points

Season	Forecasting method	Seasonality	Measures of forecast error		
			RMSE	MAPE	MAD
6 days	FIT smoothing with regressed trend	MSR	72 653	43.81%	53 282
	Moving average	SSR	64 835	44.81%	47 724
	Moving average	MSR	65 544	44.06%	47 204
24 days	Moving average	SSR	58 723	41.48%	44 564
	Moving average	MSR	66 208	39.34%	48 372
26 days	FIT smoothing with regressed trend	SSR	56 779	34.35%	42 937
	Moving average	SSR	54 825	35.42%	40 642
30 days	FIT smoothing with regressed trend	SSR	65 462	42.80%	48 739
	Winter's method with regressed trend	SSR	65 698	40.85%	47 530

Table 5.11

Comparison of forecasting methods for ATM withdrawals for differing seasons using the 56 most recent data points

Season	Forecasting method	Seasonality	Measures of forecast error		
			RMSE	MAPE	MAD
6 days	Moving average	SSR	54 159	40.20%	39 428
	Moving average	MSR	54 526	39.99%	39 911
24 days	Moving average	MSR	49 108	31.67%	30 153
26 days	Simple average	SSR	46 900	30.43%	33 194
	Moving average	MSR	59 056	32.74%	31 036
30 days	Simple exponential smoothing	SSR	73 596	34.38%	35 025
	Winter's method with regressed trend	SSR	66 367	31.79%	35 682

5.3.5 Measures of forecast error

Chase *et al.* (1998:513) discuss the various sources of forecast error and categorise errors as being either of a bias or random nature. Bias errors are said to occur when a consistent mistake is made for example employing an incorrect trend line or mistakenly shifting the seasonal demand from where it normally occurs. Random errors can be defined as those errors that cannot be explained by the forecast model being used.

A forecast is generally deemed to perform adequately when the errors exhibit only random variations. The key to judging when to reexamine the validity of a particular forecasting technique is whether forecast errors are random. If the errors are not random, an investigation needs to be carried out to determine which other sources of error (*i.e.* bias errors) are present and how to correct the problem. (Stevenson 1999:117). An indication of bias present in a forecasting method would be the running sum of the forecast errors (RSFE) which should not deviate too far from zero (Nahmias 1993:59).

Various methods for measuring forecast error are used, for example, mean absolute deviation (MAD), mean absolute percent error (MAPE) and mean square error (MSE). As illustrated in the previous paragraph, these measures of forecast error when applied to a range of forecasting techniques, do not necessarily judge the same method to be the most suitable to the particular situation.

It was decided to use the mean absolute deviation (MAD) as a measure of forecast error. The decision was based on the simplicity and usefulness of the mean absolute deviation in obtaining tracking signals (Chase *et al.* 1998:513). The use of a tracking signal to identify unusually large values of forecast error is an option when monitoring the success of a forecasting method.

However, an approach similar to the use of statistical control charts, is suggested as part of the proposed method for forecasting withdrawal and deposit patterns. The control chart approach involves setting upper and lower limits for individual forecast errors (rather than cumulative errors as in the case of the tracking signal). The limits are multiples of the square root of the mean square error. This method assumes that forecast errors are randomly distributed around a mean of zero and that the distribution of the errors is normal. The square root of the mean square error is used as an estimate of the standard deviation of the distribution errors. (Stevenson 1999:118). Winston (1991:1175) suggests that the mean absolute deviation (MAD) may also be used to estimate the standard deviation under these conditions and that the standard deviation will equal 1.25MAD.

This will be illustrated in Chapter 7 when a decision support methodology for the branch is suggested.

5.4 Availability of data

It is prudent at this stage to comment on the availability of data. The sensitivity of such data, accurately describing the deposit and withdrawal patterns at the branch, is obvious. Should this data become available to the forces of evil actively at work in South Africa - as shown in Chapter 3 - the branch and thus the bank could suffer heavy losses - financial and other. However, despite the sensitivity of the information, it is of the utmost importance for the operations manager at the branch to take an informed decision on the amount of cash required for the next planning period. At present, this decision is very much experience-based rather than based on accurate information. To obtain the data from the branch to perform the calculations reflected in Chapters 4, 5 and 6, tremendous effort was required from the branch operations manager to siphon off fictitious transactions and systems error (delayed transactions because of lines being down and so forth), before reasonably accurate figures could be obtained. Often, the odd-looking number was identified by the operations manager with a comment such as *"we definitely did not have that amount of cash in the branch on that day"*. To the untrained eye this would have been impossible. A real-time, on-line information system providing the required visibility with regard to cash inventory movement would facilitate decision-making and open up opportunities for cost reduction at the branch by matching supply and demand. Chapter 6 investigates the specific opportunities for cost reduction in detail.

5.5 Factors influencing deposit and withdrawal patterns

Although an attempt was made in this chapter to show that a more sophisticated quantitative method of forecasting deposit and withdrawal patterns at the branch may prove useful, it is crucial to point out that qualitative factors influencing the patterns experienced by the branch should not be ignored. Such an issue (change of branch location) had a direct bearing on the validity of the withdrawal data made available by the branch. Subsequent to the period under review, an amalgamation of branches in the vicinity of the branch investigated took place, resulting in the branch size doubling. Obviously, such a factor should be considered when forecasting demand for the branch. A decision, for example, to change one of the existing agencies into a fully fledged branch will have a definite impact on the demand patterns. The influence of such changes will have to be evaluated in a qualitative way initially, since the relevance of historical data will be limited.

5.6 Conclusion

The importance of finding a valid and accurate method for forecasting the deposit and withdrawal patterns at the branch will become clear at the conclusion of Chapter 6. Chapter 7 will therefore describe a suitably simplistic method to be used in conjunction with a forecasting method to ensure that the inventory replenishment decision at the branch is indeed an informed one.

CHAPTER 6

Order policies appropriate to retail banking

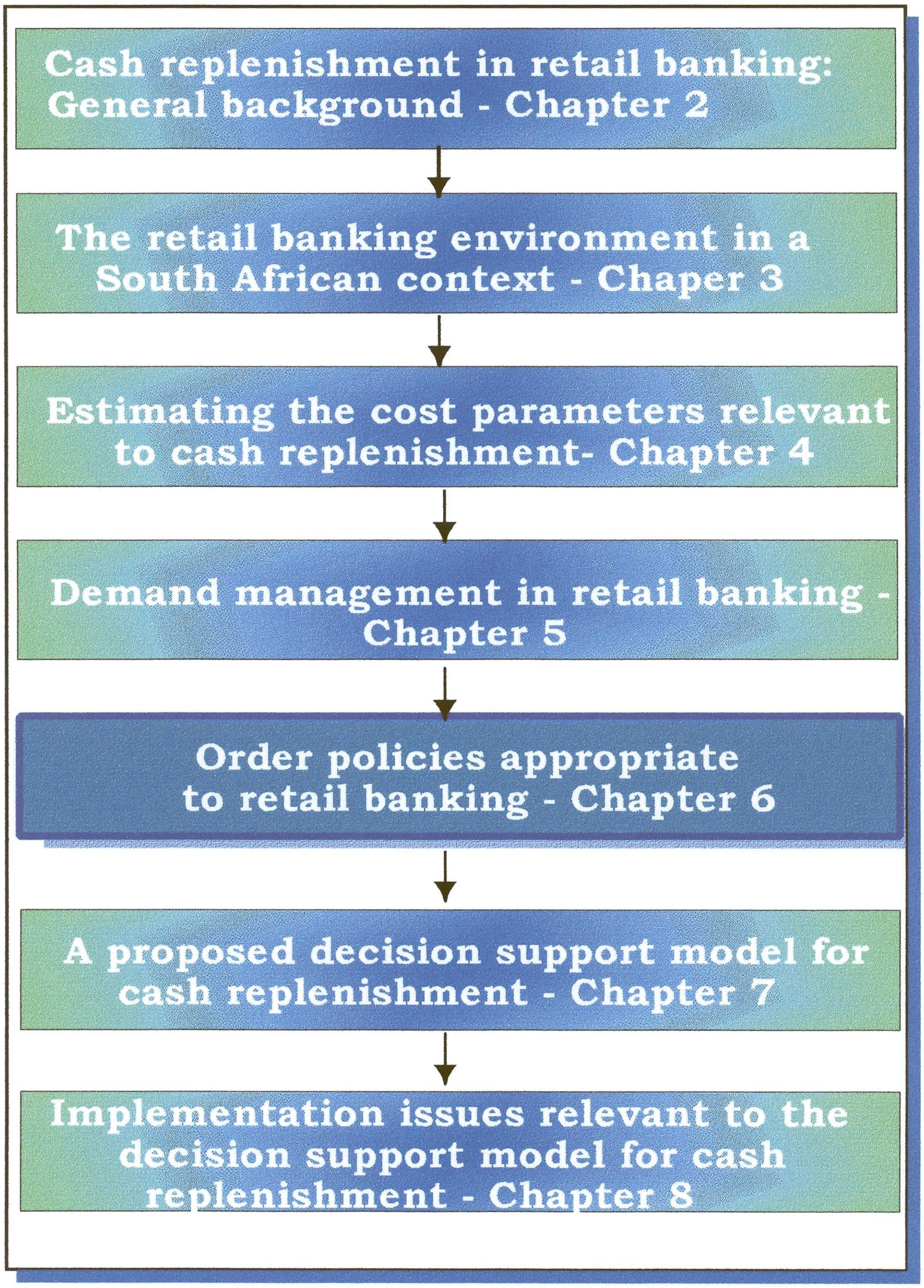
6.1 Introduction

In this chapter, proof will be provided that alternative order policies to the policy currently in use at the branch may be employed without compromising customer service (embodied by a shortage at the cash point) and at the same time, reducing the cost of holding cash inventories. The current policy is investigated as well as the adherence to this policy. The daily cost of holding inventory under the current policy is estimated and compared to the cost of other policies challenging some of the constraints embedded in the current policy. Furthermore, the effect of changes to some of the cost parameters is also investigated. Finally, in Chapter 7, the information provided by this and the previous chapter will be used to propose a decision support model - the details of which will be discussed in the next chapter.

Figure 6.1 shows the relevance of appropriate order policies to the other chapters included in the thesis.

Figure 6.1

The structure of the report indicating the relevance of Chapter 6



6.2 Fitting distributions to the demand and withdrawal patterns

In developing alternative order policies for the branch, simulation runs were performed to evaluate the cost of the proposed policies. Before this could be done, a series of random numbers representing the total demand and total withdrawal patterns at the branch were generated based on appropriate probability distributions.

Using the *Input Analyzer* function of *Arena®* (Kelton *et al.* 1998), various distributions were fitted to the available data describing demand and withdrawal patterns. *Input Analyzer* fits a number of different probability distributions, providing an indication of the goodness of fit. Since the Weibull distribution provided an acceptable fit to both the deposits (squared error = 0.022) and withdrawals (squared error = 0.0125), it was selected to represent the distributions for both deposits and withdrawals. Kelton *et al.* (1998:514) state that the Weibull distribution is particularly useful to represent non-negative values that are skewed to the left. (Refer to appendices A4, B4 and C4 for confirmation of the shape of the deposit, total withdrawal and ATM withdrawal histograms respectively.)

As a first step to simulating conditions at the branch, 960 data points describing deposits and total withdrawals per day were generated from the fitted probability distributions. The differences between the daily deposits and total withdrawals generated, were calculated. The normal distribution was fitted to these differences, providing a square error of 0.0033 - indicating that the differences were normally distributed as would be expected. This data string was used as an initial test to investigate the sensitivity of the total cost of holding cash inventory to changes in certain parameters, for example safety stock, reorder point, order quantity and also the impact of an estimate of the shortage cost.

The second step in the investigation was to test the best alternatives from each simulation run against the actual total withdrawal and deposit patterns obtained from the branch, and to compare the cost of the proposed policies to the actual cost of the existing policy.

6.3 Existing order policy

6.3.1 Formulation

The branch does not have an explicit policy on cash replenishment although various unwritten rules do exist within a broad framework. As pointed out in Chapter 4, the guidelines followed at the branch *inter alia* include:

- As far as possible a single order is placed per week;
- an order is placed if the main safe balance is below R500 000, implying the use of a safety stock level of R500 000;
- special deliveries (minimum order quantity of R500 000), with a lead time of one day, are only possible if authorised by the regional office, and are to be avoided;
- a lead time of two days is used, unless the order is placed for a Friday, when a lead time of three days is used; and
- only in exceptional cases, for example where notes are unfit for use by the cashiers, is cash returned to SBV.

From the above, the branch-specific nature of some of these rules is obvious in contrast to some rules that are generic in character.

6.3.2 Application

Appendix I provides a comparison of the application of the order policy detailed in Figure 4.2 as well as in the previous paragraph, to the actual activities which occurred at the branch during the period under review. From the appendix, it is quite clear that many of the rules incorporated in the policy are not strictly adhered to. The only rule which seems to be followed fairly consistently, is the avoidance of special orders. This is obvious from Appendix F, since only one special order was placed at the end of April. This occurrence was the direct result of the move of the branch to its new location.

6.3.3 Cost of existing policy

Appendices A to D show the calculations required to determine the actual amount of cash on hand in the branch at the end of each trading day during the three month period under review. Once these actual on hand amounts had been calculated, it was possible to determine the cost of holding the cash. The cost calculations were based on the cost parameters determined in Chapter 4. In Appendix E the daily cash storage cost is determined, while the daily cash supply cost is calculated in Appendix F. The total daily cost of holding cash is finally summarised in Appendix G.

The total cost over the three month period is as follows:

April	R 79 915
May	R 82 952
June	R 85 463
TOTAL	**R 248 330**

On a daily basis this is equal to R 2 728.90.

The next paragraph is dedicated to finding alternative policies which will reduce the daily cost.

6.4 Alternative order policies

6.4.1 Method of investigation

The method used to investigate alternative order policies which could produce a reduction in the total cost of holding cash inventories, to a certain extent represented a random process of sub optimisation. As a first step, random numbers were generated to represent the total withdrawal and deposit patterns as described earlier in paragraph 6.2. The next step was to decide on the scope of the shortage cost involved, since the bank refrained from placing a value on that component of the total cost of holding cash. An assumption was made that a relationship exists between storage cost and shortage cost, since in this particular case, if the branch cannot provide the required cash amount, the situation may be seen as one where the branch is borrowing that amount from the customer. It was decided to use various values for shortage cost, for example shortage cost equal to twice the storage cost, then ten times the storage cost and finally 50 times the storage cost.

The branch essentially sets safety stock and the reorder point at the same level, *i.e.* R500 000. For purposes of the simulation runs, the safety stock was not assumed to be equal to the reorder point. Therefore the reorder point would trigger a normal order having a lead time of two days, whereas the safety stock level would trigger a special order having a lead time of one day. The safety stock was set at three different levels, initially R200 000, then R500 000, and finally R1 000 000. In paragraph 6.4.2.2 further levels of safety stock were investigated. At the same time, the special order size was set at R500 000. In paragraph 6.4.2.3 other special order sizes were investigated. The branch policy precludes returning excess amounts of cash to SBV - only coin and notes unfit for circulation may be returned. In paragraph 6.4.2.4 the option of returning excess cash is investigated.

Finally, the branch operations manager mentioned that delivery of normal orders often occurred only after 13:00 on the designated day, rendering the true lead time two and a half days. A special order was almost always delivered after trading had ended, *i.e.* after 15:30, which implies a lead time of two days rather than one day. One way of overcoming this uncertainty is to assume the respective lead times being three and two days, rather than two and one day. The effect of this assumption is investigated in paragraph 6.4.2.5, since it could have a bearing on the safety stock level.

As was explained earlier, the initial runs were carried out on 960 simulated data points generated from the fitted distributions. These distributions did not allow for the seasonality present in the deposit and withdrawal patterns described in the previous chapter. Therefore when investigating a specific policy, it was deemed viable only if it lead to a situation where shortages occurred on fewer than 48 out of 960 days, *i.e.* 5% of the days. Subsequently, when testing a policy against the actual figures, no shortages were allowed. Therefore situations arose where the best policy based on the initial run lead to a shortage in the second stage. The policy would then be adjusted until the next best option was found avoiding shortages. The investigation did not limit the number of order placed per week and no explicit avoidance of special orders was built into the simulation.

6.4.2 Results

6.4.2.1 Effect of shortage cost

As explained earlier, the bank does not quantify shortage cost - a shortage is something to be avoided at all cost. In an attempt to gauge the impact that a shortage would have on the total cost, a value had to be placed on a shortage should it occur. Three different values were investigated based on the assumption that a relationship exists between shortage cost and inventory holding cost. Initially, shortage cost was assumed to be twice the inventory holding cost, then ten times the inventory holding cost and finally 50 times the inventory holding cost. A summary of the results of this investigation is shown in Table 6.1. In each of the three cases investigated an initial inventory of R1 000 000 was assumed, safety stock was set at three different levels (*i.e.* R200 000, R500 000 and R1 000 000), the order size was first set at R500 000, then at R750 000 and finally at R1 000 000, and the reorder point was increased in increments of R100 000, from R300 000 to R2 500 000. The full results of this investigation are reported in Appendix J. The abbreviations used in Tables 6.1 to 6.9 are SS for safety stock, SQ for special order size, Q for the reorder quantity, ROP for the reorder point and UL for upper limit for the amount of cash held at the branch.

Table 6.1

Summary of the investigation into the scope of shortage cost

(Based on Appendix J)

Safety stock	Order quantity	Optimum reorder point	Daily cost
Shortage cost = 2 x Inventory holding cost ($c_2 = 2c_1$)			
SS = 200 000	Q = 500 000	ROP = 1 100 000	R2 800
SQ = 500 000	Q = 750 000	ROP = 900 000	R2 782
(Appendix J-1)	Q = 1 000 000	ROP = 900 000	R2 814
SS = 500 000	Q = 500 000	ROP = 1 100 000	R2 852
SQ = 500 000	Q = 750 000	ROP = 700 000	R2 805
(Appendix J-2)	Q = 1 000 000	ROP = 600 000	R2 860
SS = 1 000 000	Q = 500 000	ROP = 600 000	R2 873
SQ = 500 000	Q = 750 000	ROP = 300 000	R2 861
(Appendix J-3)	Q = 1 000 000	ROP = 300 000	R2 888
Shortage cost = 10 x Inventory holding cost ($c_2 = 10c_1$)			
SS = 200 000	Q = 500 000	ROP = 1 100 000	R2 869
SQ = 500 000	Q = 750 000	ROP = 900 000	R2 831
(Appendix J-4)	Q = 1 000 000	ROP = 900 000	R2 872
SS = 500 000	Q = 500 000	ROP = 1 100 000	R2 894
SQ = 500 000	Q = 750 000	ROP = 800 000	R2 863
(Appendix J-5)	Q = 1 000 000	ROP = 900 000	R2 896
SS = 1 000 000	Q = 500 000	ROP = 600 000	R2 924
SQ = 500 000	Q = 750 000	ROP = 300 000	R2 911
(Appendix J-6)	Q = 1 000 000	ROP = 600 000	R2 939
Shortage cost = 50 x Inventory holding cost ($c_2 = 50c_1$)			
SS = 200 000	Q = 500 000	ROP = 1 500 000	R3 136
SQ = 500 000	Q = 750 000	ROP = 900 000	R3 078
(Appendix J-7)	Q = 1 000 000	ROP = 1 400 000	R3 107
SS = 500 000	Q = 500 000	ROP = 1 500 000	R3 093
SQ = 500 000	Q = 750 000	ROP = 1 400 000	R3 038
(Appendix J-8)	Q = 1 000 000	ROP = 1 000 000	R3 054
SS = 1 000 000	Q = 500 000	ROP = 1 000 000	R3 082
SQ = 500 000	Q = 750 000	ROP = 800 000	R3 068
(Appendix J-9)	Q = 1 000 000	ROP = 800 000	R3 064

From the table it is obvious that the policy which consistently provides the lowest cost option under the first two shortage cost assumptions, is a safety stock level of R200 000 and an order quantity of R900 000. However, when shortage cost is assumed to be 50 times the storage cost, the best option is a safety stock level of R500 000 and an order quantity of R1 400 000. These "best" options correspond to expectations in as much as a higher shortage cost would lead to a higher safety stock level and reorder point to avoid incurring the shortage cost.

If these policies are tested against the actual withdrawals and deposits over the three month period, the cost of holding cash may be significantly reduced compared to the cost of the current policy (discussed in paragraph 6.3.3). Table 6.2 shows the cost of these policies for the actual withdrawal and deposit amounts.

Table 6.2

Proposed policies applied to actual amounts

(Special order size SQ = R500 000)

Policy	Cost		Number of orders		Number of shortages
	Total	Daily	Normal	Special	
$c_2 = 2\ c_1$ Q = 750 000 SS = 200 000 ROP = 900 000	R 224 435	R2 466	16	2	0
$c_2 = 10\ c_1$ Q = 750 000 SS = 200 000 ROP = 900 000	R 224 435	R2 466	16	2	0
$c_2 = 50\ c_1$ Q = 750 000 SS = 500 000 ROP = 1 400 000	R 246 791	R2 712	18	0	0

Compared to the actual cost for the period under review, a substantial reduction would have been possible, had any one of the above policies been followed. A reduction of almost 10% could have been achieved by following a policy where the reorder quantity equals R750 000, safety stock is set at R200 000 and a reorder point of R900 000 is used. This was under the assumption that shortage cost is equal to twice (or ten times) the storage cost.

The impact of the safety stock level has been included to an extent in the analysis performed above, but deserves further probing.

6.4.2.2 Effect of safety stock

To investigate the impact of the safety stock level, the order quantity was fixed at R750 000, which seemed to provide good results based on the conclusions drawn in paragraph 6.4.2.1. The shortage cost was assumed to be equal to ten times the inventory holding cost (*i.e.* $c_2 = 10c_1$). Safety stock levels of zero, R200 000, R300 000, R500 000, R750 000 and R1 000 000 were investigated. The full results are shown in Appendix K, although some of the results presented in Appendices J-4-2, J-5-2 and J-6-2 are also relevant. A summary of the best results at each safety stock level is given in Table 6.3. All calculations were based on the 960 simulated data points.

Table 6.3

Summary of the investigation into the impact of safety stock levels

(Simulated figures with special order size SQ = R500 000 and $c_2 = 10c_1$)

Safety stock	Order quantity	Optimum reorder point	Daily cost
SS = 0 (Appendix K-1)	Q = 750 000	ROP = 1 000 000	R2 863
SS = 200 000 (Appendix J-4-2)	Q = 750 000	ROP = 900 000	R2 831
SS = 300 000 (Appendix K-2)	Q = 750 000	ROP = 900 000	R2 847
SS = 500 000 (Appendix J-5-2)	Q = 750 000	ROP = 800 000	R2 863
SS = 750 000 (Appendix K-3)	Q = 750 000	ROP = 600 000	R2 880
SS = 1 000 000 (Appendix J-6-2)	Q = 750 000	ROP = 300 000	R2 911

From Table 6.3 it may seem that the safety stock level does not have much impact on the daily cost. The "best" policy seems to be a safety stock level of R200 000 combined with a reorder point of R900 000. To confirm the findings, the safety stock levels are tested against the actual withdrawals and deposits. The results of this investigation are shown in Table 6.4.

Table 6.4

Proposed safety stock levels applied to actual amounts

(c_2 = 10 c_1, Q = 750 000 and special order size = R500 000)

Policy	Cost		Number of orders		Number of shortages
	Total	Daily	Normal	Special	
SS = 0 ROP = 1 000 000	R 231 822	R 2 547	18	0	0
SS = 200 000 ROP = 900 000	R 224 435	R 2 466	16	2	0
SS = 300 000 ROP = 900 000	R 224 435	R 2 466	16	2	0
SS = 500 000 ROP = 800 000	R 233 858	R 2 570	13	7	1[1]
SS = 500 000 ROP = 900 000	R 231 672	R 2 546	14	5	0
SS = 750 000 ROP = 600 000	R 235 578	R 2 589	9	13	0
SS = 1 000 000 ROP = 300 000	R 236 419	R 2 598	1	24	0

1. Since shortages are not acceptable, the next reorder point level for a safety stock level of R500 000 is found, which will prevent shortages of occurring. The reorder point level which does not lead to shortages, is R900 000.

As would be expected, the reorder point increases as the safety stock level decreases as seen in Table 6.4. Noticeable though, is the impact of no safety stock. Even if the safety stock level is set at 0, a reorder point of R1 000 000, will prevent shortages.

As before, all policies tested would have lead to a reduction in cost. The lowest daily cost is obtained where safety stock is equal to R200 000 or R300 000 and the reorder point is R900 000.

6.4.2.3 Special order size

Throughout the analyses performed to this point, the special order size has been set at a fixed amount of R500 000, which also is the minimum amount that may be ordered by special order. The next step in the analysis was to investigate other possible special order sizes, for example R750 000 and R1 000 000. Again, the relationship between shortage cost and inventory holding cost was assumed to be $c_2=10c_1$. The safety stock levels were set at R200 000, R500 000 and R1 000 000 respectively. The full results are shown in Appendix L, although the results shown in Appendices J-4-2, J-5-2 and J-6-2 also apply. The results are summarised in Table 6.5.

From Table 6.5, the special order size does not seem to have a significant impact on the daily cost based on the simulated figures. However, a safety stock level of R200 000 consistently provides the lowest cost alternative irrespective of the special order size. As before, the policies investigated are tested against the actual figures supplied by the branch. The results are summarised in Table 6.6.

Table 6.5

Summary of the investigation into the impact of special order sizes

(Simulated figures where $c_2 = 10\ c_1$ and Q = 750 000)

Safety stock	Special order size	Order quantity	Optimum reorder point	Daily cost
SS = 200 000 (Appendix J-4-2)	SQ = 500 000	Q = 750 000	ROP = 900 000	R2 831
SS = 500 000 (Appendix J-5-2)	SQ = 500 000	Q = 750 000	ROP = 800 000	R2 863
SS = 1 000 000 (Appendix J-6-2)	SQ = 500 000	Q = 750 000	ROP = 300 000	R2 911
SS = 200 000 (Appendix L-1-1)	SQ = 750 000	Q = 750 000	ROP = 700 000	R2 820
SS = 500 000 (Appendix L-1-2)	SQ = 750 000	Q = 750 000	ROP = 500 000	R2 830
SS = 1 000 000 (Appendix L-1-3)	SQ = 750 000	Q = 750 000	ROP = 300 000	R2 884
SS = 200 000 (Appendix L-2-1)	SQ = 1 000 000	Q = 750 000	ROP = 700 000	R2 815
SS = 500 000 (Appendix L-2-2)	SQ = 1000 000	Q = 750 000	ROP = 500 000	R2 843
SS = 1 000 000 (Appendix L-2-3)	SQ = 1 000 000	Q = 750 000	ROP = 300 000	R2 892

Table 6.6

Proposed special order sizes applied to actual amounts

(c_2 = 10 c_1 and Q = 750 000)

Policy	Cost		Number of orders		Number of shortages
	Total	Daily	Normal	Special	
SQ = 500 000					
SS = 200 000 ROP = 900 000	R 224 435	R2 466	16	2	0
SS = 500 000 ROP = 800 000	R 233 858	R2 570	13	7	1[2]
SS = 500 000 ROP = 900 000	R 231 672	R 2 546	14	5	0
SS = 1 000 000 ROP = 300 000	R 236 419	R 2 598	1	24	0
SQ = 750 000					
SS = 200 000 ROP = 700 000	R 231 153	R 2 540	13	5	2[2]
SS = 200 000 ROP = 900 000	R 231 866	R 2 548	16	2	0
SS = 500 000 ROP = 500 000	R 228 798	R 2 514	9	9	1[3]
SS = 500 000 ROP = 700 000	R 225 506	R 2 478	12	5	0
SS = 1 000 000 ROP = 300 000	R 237 774	R 2 613	1	17	0
SQ = 1 000 000					
SS = 200 000 ROP = 700 000	R 228 403	R 2 510	13	4	0
SS = 500 000 ROP = 1 000 000	R 242 067	R 2 660	12	5	0
SS = 1 000 000 ROP = 300 000	R 239 685	R 2 634	1	13	0

2. Since shortages are not acceptable, the next reorder point level which prevents shortages is found. This reorder point is R900 000.
3. Since shortages are not acceptable, the next reorder point level which prevents shortages is found. This reorder point level is R700 000.

Based on the results of Table 6.6, no further reduction beyond the 10% achieved by an order policy using a safety stock level of R200 000, a reorder point of R900 000, a special order size of R500 000 and an order quantity of R750 00 was possible. These calculations were based throughout on the assumption that the shortage cost is ten times the storage cost.

6.4.2.4 Returning excessive cash amounts to SBV

The branch does not return excess cash amounts to SBV. Instead, the cash is carried at the branch until required. Since the average total withdrawals at this particular branch exceeds the average deposits, this situation is tenable. Over time the excess cash will be employed.

The question which arises, however, concerns the storage cost associated with this approach. It implies that large amounts of cash will be carried at the branch for extended periods. If Appendix D1 is studied, the observation may be made that the amount of cash on hand peaks at R2 700 000 on a number of occasions. On only three days during the three month period under review, was the amount of cash on hand below R1 000 000.

Using the best policy identified thus far, the impact of returning excess amounts of cash to SBV is investigated. The policy which has provided the lowest cost option in the preceding paragraphs, prescribes an order quantity of R750 000, a safety stock level of R200 000 and special order size of R500 000. This policy was selected under the assumption that the shortage cost (c_2) is equal to ten times the storage cost (c_1). In addition, a safety stock level of R500 000 was investigated.

During the investigation, the reorder point was varied between R300 000 and R2 000 000 for an upper limit of R2 000 000 and R2 500 000. Trial and error proved that only a return amount of R500 000 seemed at all feasible, therefore only this option was investigated fully as shown in Appendix M. The lead time for returns was assumed to be two days. The results of the investigation shown in Table 6.7, are based on the simulated figures as before. The abbreviation RA is used to denote return amount and UL to indicate the upper limit.

Table 6.7

Summary of the investigation into the effect of returning excess cash

(Simulated figures where $c_2 = 10c_1$, Q = 750 000, SQ = 500 000 and RA = 500 000)

Upper limit	Optimum reorder point	Daily cost
SS = R200 000		
R 2 000 000 (Appendix M-1-1)	ROP = 1 600 000	R3 136
R 2 500 000 (Appendix M-1-2)	ROP = 1 100 000	R2 856
SS = 500 000		
R 2 000 000 (Appendix M-2-1)	ROP = 1 400 000	R3 072
R 2 500 000 (Appendix M-2-2)	ROP = 900 000 ROP = 1 100 000	R2 877 R2 893

From Table 6.7 it is obvious that an upper limit of R2 500 000 combined with a safety stock level of R200 000 and a reorder point of R1 100 000 provides the best alternative for a special order size of R500 000, a R500 000 return amount and a reorder quantity of R750 000. It is important to note the effect that returning excess cash amounts has on the number of orders as shown in Appendix M. The number of normal and special orders, as well as the shortages increased markedly. As before this approach is tested against the actual figures. The results are shown in Table 6.8.

Table 6.8 shows that a policy prescribing an upper limit of R2 500 000 combined with a safety stock level of R500 000 leads to a situation where no returns are made. An upper limit of R2 000 000 in both cases (safety stock equal to R200 000 or R500 000) would lead to many returns, compared to the number of orders placed.

Based on Table 6.8, the cost could be reduced significantly from current levels at the branch, but no further reductions were possible beyond the 10% reduction achieved earlier in the chapter where returns were not considered. These results, combined with the theoretically confirmed characteristic of the total cost curve in analysing inventory costs, of flatness in the optimum region, lead to the decision to perform a full analysis on the actual figures, similar to the analyses performed thus far on the simulated figures.

Table 6.8

Proposed return amounts and upper limits applied to actual amounts

(c_2 = 10 c_1, Q = 750 000 and SQ = 500 000)

Policy	Cost		Number of orders		Number of shortages	Number of returns
	Total	Daily	Normal	Special		
SS = 200 000						
UL = 2 000 000 ROP = 1 600 000	R 258 853	R2 845	34	0	0	25
UL = 2 500 000 ROP = 1 100 000	R 233 533	R2 566	19	0	0	2
SS = 500 000						
UL = 2 000 000 ROP = 1 400 000	R 256 695	R 2 821	30	4	0	21
UL = 2 500 000 ROP = 900 000	R 231 672	R 2 546	14	5	0	0
UL = 2 500 000 ROP = 1 100 000	R 233 596	R 2 567	17	1	0	0

6.4.2.5 The "best" order policy

As explained before, various combinations of the parameters are applied to the actual figures in an attempt to verify the conclusions reached in this chapter and to identify a "best" policy for the branch which will minimise the cost of holding inventory without compromising customer service. The full results of the investigation are shown in Appendix N. A summary of the results appear in Table 6.9. The analysis was based on an order quantity of R750 000, a special order size of R500 000 and a return amount of R500 000.

Appendices N1 to N4 show that the investigation based on the simulated figures (reported in Table 6.7) did not point out the true minimum cost alternative when applied to the actual amounts. A full investigation as shown in Appendix N suggests lower reorder point levels. This is reflected in Table 6.9.

Table 6.9

Summary of "best" policies

(c_2 = 10 c_1, Q = 750 000, SQ = 500 000, RA = 500 000)

Policy	Cost		Number of orders		Number of shortages	Number of returns
	Total	Daily	Normal	Special		
SS = 200 000						
UL = 2 000 000 ROP = 900 000 (Appendix N-1)	R231 186	R2 541	22	2	0	9
UL = 2 500 000 ROP = 900 000 (Appendix N-2)	R224 435	R2 466	16	2	0	0
SS = 500 000						
UL = 2 000 000 ROP = 500 000	R228 096	R 2 507	12	12	0	4
UL = 2 500 000 ROP = 900 000	R231 672	R 2 546	14	5	0	0

From Table 6.9 the "best" policy corresponds to the policy identified in Table 6.4, *i.e.* an order quantity of R750 000, a special order size of R500 000, a safety stock level of R200 000 and a reorder point of R900 000. Since this policy, when combined with the possibility of returning excess cash amounts, did in fact not lead to any returns. However, this does not imply that excess cash should not be returned. In Table 6.4, a second "best" policy provided the same daily cost, however, using a reorder point of R300 000. If this reorder level is combined with a return amount of R500 000 and an upper limit of R2 500 000 (with all other parameters as before), it provides the same results as the policy using a safety stock level of R200 000.

6.4.2.6 Lead time

During discussions with the operations manager at the branch, it was pointed out that the deliveries from SBV often arrived after or very close to the end of a trading day, thereby lengthening the lead time for normal orders to three days and that for special orders to two days. In Appendix O the three "best" policies identified in the preceding paragraphs were tested for further cost reductions should the extended lead times be used. From Appendix O it is obvious that none of these policies would have been feasible since at all reorder levels shortages would have occurred. In addition, all policies lead to an increase in the daily cost of holding cash inventory. The policies identified in the previous paragraph were therefore accepted as being the "best" based on the analyses performed.

6.5 Conclusion

In this chapter it was proven that a cost reduction could be achieved by altering the existing policy of ordering cash at the branch. However, in ordering the cash amounts, cognisance was not taken of the seasonality of the data. In the next chapter, the results of Chapter 5 will be combined with the results of Chapter 6 to propose a decision support system to optimise the order policy employed at the branch.

CHAPTER 7

A proposed decision support model for cash replenishment

7.1 Introduction

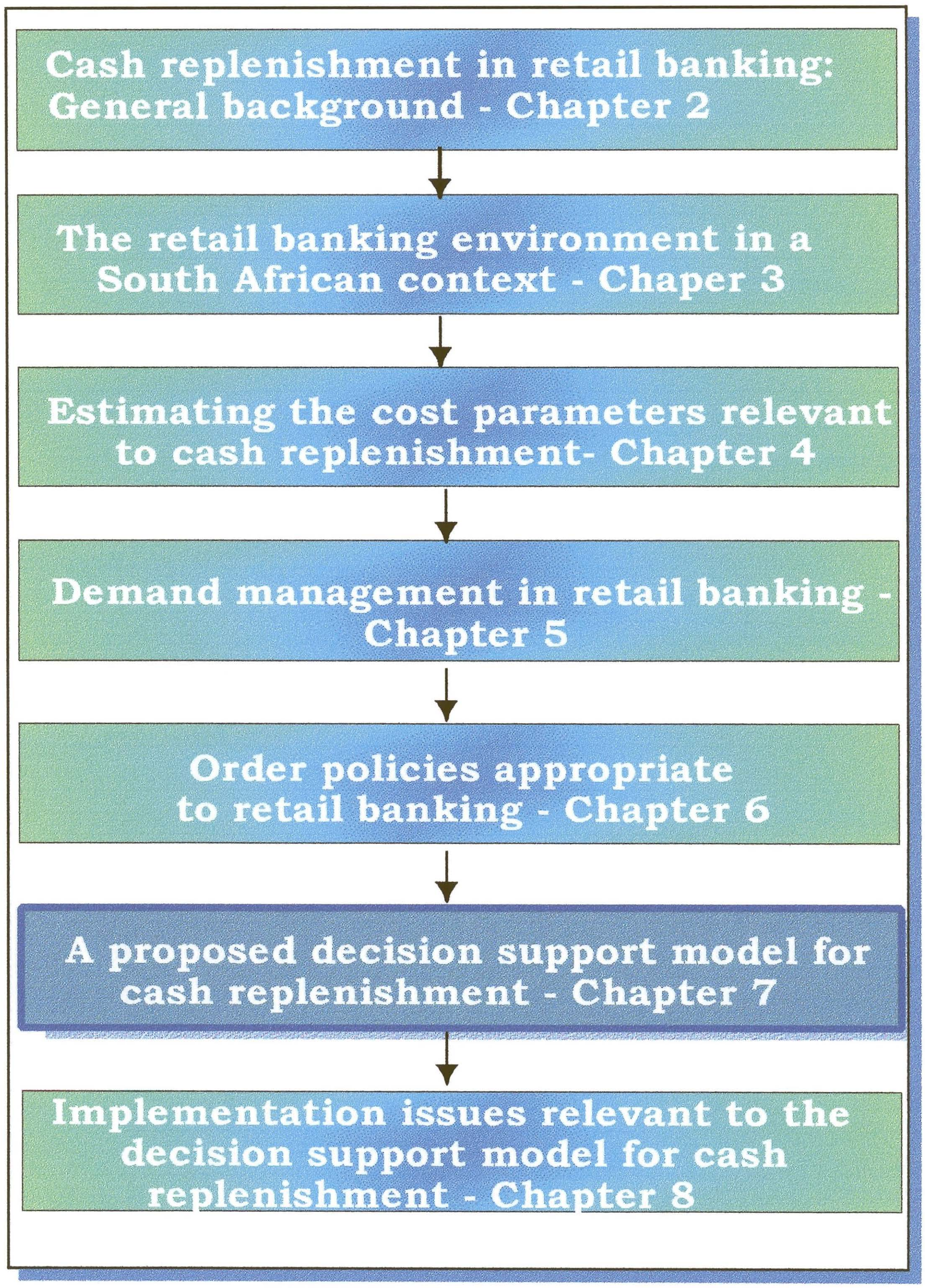
In Chapter 5 various demand management issues were discussed which were deemed to be relevant to the cash replenishment situation in retail banking. The most important conclusion arrived at in this chapter, was that the weights used by the branch in determining the scope of deposits and withdrawals may be refined to improve the estimates of the amounts. In Chapter 6, the implicit order policy adhered to by the branch was challenged. In this chapter it was shown that alternate order policies, which do not even consider the seasonality present in the deposit and withdrawal patterns, will reduce the total cost of holding inventory at the branch.

In Chapter 7, a decision support model is proposed which combines the forecasting methods investigated in Chapter 5 and the order policies suggested in Chapter 6. The conceptual model is subsequently evaluated using the real data patterns provided by the branch, in an attempt to show that a combination of the forecasting techniques and order policies will lead to an even greater cost reduction than that achieved in Chapter 6 without compromising the customer service level provided at the branch.

Figure 7.1 shows the relevance of this chapter with regard to the research project.

Figure 7.1

The structure of the report indicating the relevance of Chapter 7

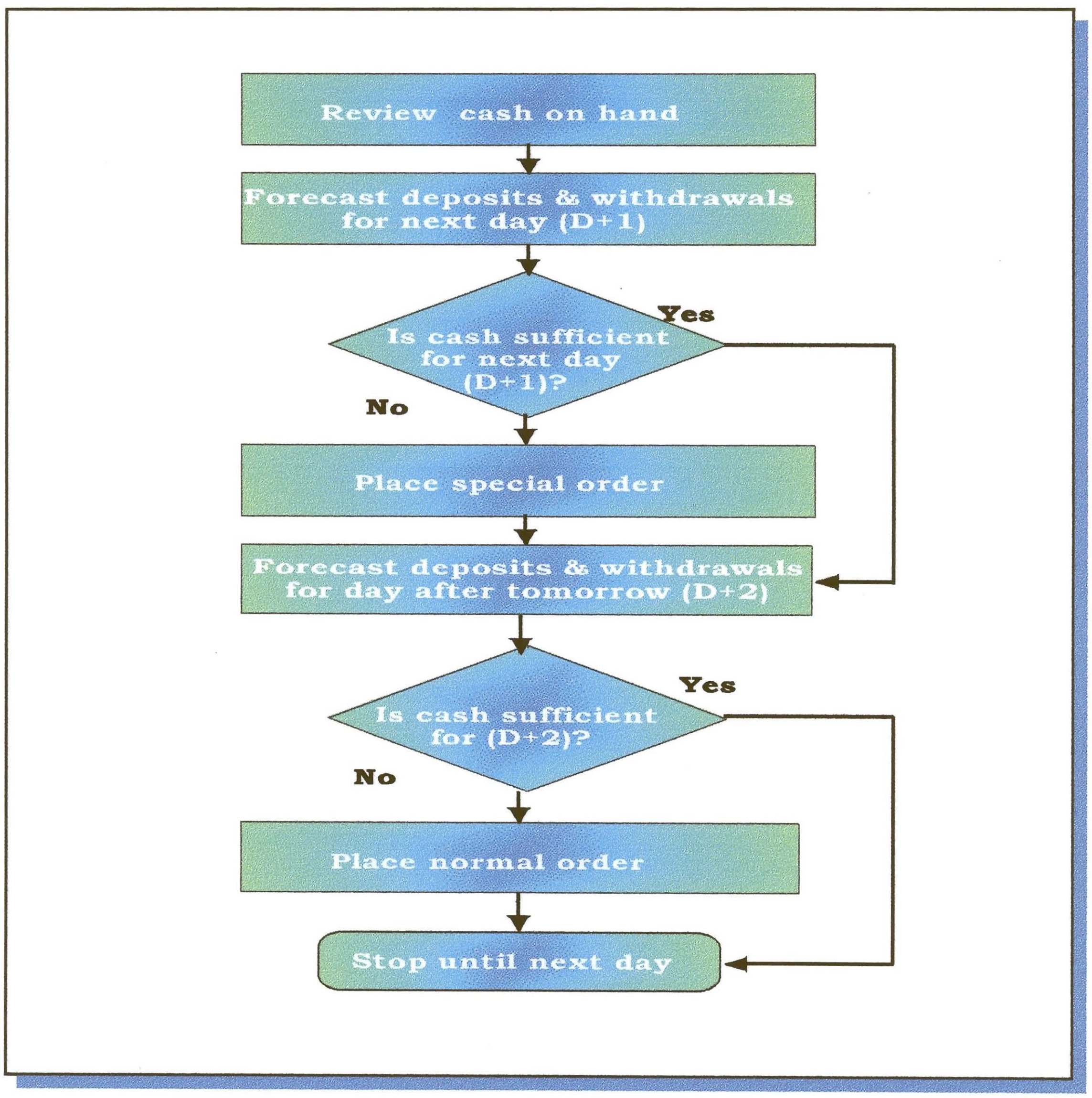


7.2 A decision support model for cash replenishment

Figure 7.2 shows a diagrammatic representation of the proposed decision support model for the branch which combines the order policy with a forecast based on seasonality.

Figure 7.2

A decision support model for cash replenishment at branch level



7.3 Application of the proposed decision support model

In Chapter 5, four different cycles were suggested as possibilities in forecasting deposit and withdrawal patterns. Various methods were applied and for each cycle a "best" method was suggested. In this paragraph, the assumption is made that the seasonality of deposit and withdrawal patterns correspond. Since three measures of forecast error were used in Chapter 5, and since these error measures do not necessarily confer with regard to the "best" method, all possibilities were explored. Table 7.1 provides a summary of the methods investigated for the four different cycles. The methods represented in the table are based on the results reported earlier in Chapter 5 in Tables 5.7 and 5.9. The full results of the use of the methods combined with the "best" order policy as determined in Chapter 6, are shown in Appendix P.

The suitability of each of the cycles, combined with the forecasting method and the order policy, is discussed in the following paragraphs. In each case the criteria for suitability reflect the avoidance of shortages as well as the cost involved. The benchmark for cost is the cost of the "best" order policy as determined in Chapter 6, *i.e.* an order quantity of R750 000, a special order size of R500 000, a safety stock level of R200 000 and a reorder point of R900 000. The cost of holding cash inventories in this instance was R2 466 per day.

However, as before, the reorder point is not fixed at the amount of R900 000, but varied from R300 000 to R1 000 000 to study the effect on the total cost of holding inventory. In addition, when necessary, the rule of simplicity with regard to use and understanding is used to select a preferred forecasting method.

Table 7.1

Combinations investigated assuming that the seasonality cycles of withdrawal and deposit patterns correspond

Seasonality	Withdrawal forecast method	Deposit forecast method	Result summary
6 days	Moving average with simple seasonal relatives	Moving average with simple seasonal relatives	Appendix P1-1
		Winter's method with regressed trend & simple seasonal relatives	Appendix P1-2
	Winter's method with regressed trend & simple seasonal relatives	Moving average with simple seasonal relatives	Appendix P1-3
		Winter's method with regressed trend & simple seasonal relatives	Appendix P1-4
24 days	FIT smoothing with default trend & simple seasonal relatives	Simple exponential smoothing with moving seasonal relatives	Appendix P2-1
		FIT smoothing with regressed trend & moving seasonal relatives	Appendix P2-2
		Simple average with simple seasonal relatives	Appendix P2-3
	Simple average with simple seasonal relatives	Simple exponential smoothing with moving seasonal relatives	Appendix P2-4
		FIT smoothing with regressed trend & moving seasonal relatives	Appendix P2-5
		Simple average with simple seasonal relatives	Appendix P2-6
	Simple average with moving seasonal relatives	Simple exponential smoothing with moving seasonal relatives	Appendix P2-7
		FIT smoothing with regressed trend & moving seasonal relatives	Appendix P2-8
		Simple average with simple seasonal relatives	Appendix P2-9

Table 7.1 (Continued)
Combinations investigated assuming that the seasonality cycles of withdrawal and deposit patterns correspond

Seasonality	Withdrawal forecast method	Deposit forecast method	Result summary
26 days	Simple average with simple seasonal relatives	Simple average with simple seasonal relatives	Appendix P3-1
		Winter's method with regressed trend & simple seasonal relatives	Appendix P3-2
		Winter's method with regressed trend & moving seasonal relatives	Appendix P3-3
30 days	FIT smoothing with default trend & simple seasonal relatives	Simple exponential smoothing with simple seasonal relatives	Appendix P4-1
		Simple average with simple seasonal relatives	Appendix P4-2
		Winter's method with regressed trend & simple seasonal relatives	Appendix P4-3
	Simple average with simple seasonal relatives	Simple exponential smoothing with simple seasonal relatives	Appendix P4-4
		Simple average with simple seasonal relatives	Appendix P4-5
		Winter's method with regressed trend & simple seasonal relatives	Appendix P4-6

7.3.1 Seasonality based on a six day cycle

Two methods were considered to forecast withdrawal patterns if the seasonality is based on a six day cycle, *i.e.* a moving average combined with simple seasonal relatives (based on mean absolute percent error and mean absolute deviation) and Winter's method with a regressed trend and simple seasonal relatives (based on the root mean square error). The deposit patterns were forecast using the same two methods. The results are shown in Appendices P1-1, P1-2, P1-3 and P1-4.

From Appendices P1-1 and P1-2 the combination of withdrawals and deposits both forecast using moving averages and simple seasonal relatives and the combination of withdrawals forecast using moving averages and simple seasonal relatives and deposits forecast using Winter's method with a regressed trend and simple seasonal relatives, create situations where no shortages occur irrespective of the reorder point. In both cases the reorder point of R300 000 provides the lowest cost per day - in the first case R2 398 and in the second case R2 392. Both of these represent an improvement on the previous lowest cost per day of R2 466, achieved in Chapter 6.

The results of the combinations represented in Appendices P1-3 and P1-4 are disqualified based on the erratic nature in which shortages occur as the reorder point is varied from R300 000 to R1 000 000.

7.3.2 Seasonality based on a 24 day cycle

Appendix P2 summarises the results for the order policy explained before combined with a seasonal cycle based on 24 days. Based on the root mean square error, both withdrawals and deposits were forecast using a simple average combined with simple seasonal relatives. Using the mean absolute percent error, withdrawals were forecast using FIT smoothing with a default trend combined with simple seasonal relatives, whereas deposits were forecast using simple exponential smoothing combined with moving seasonal relatives. If the mean absolute deviation is used as a measure of forecast error, withdrawals were forecast using a simple average combined with moving seasonal relatives, whereas deposits were forecast using FIT smoothing with a regressed trend together with moving seasonal relatives.

In studying the results of these combinations, only two do not display erratic shortages patterns. In the first case withdrawals were forecast using FIT smoothing with a default trend combined with simple seasonal relatives, while deposits were also forecast using FIT smoothing, but a regressed trend is used in combination with moving seasonal relatives (results in Appendix P2-2). In the second case withdrawals were forecast using a simple average with moving seasonal relatives, while deposits were forecast using simple exponential smoothing with moving seasonal relatives (results in Appendix P2-9). In the first case, the suggested reorder point is R400 000 which results in a daily cost of R2 471, whereas the second case suggests a reorder point of R500 000 resulting in a daily cost of R2 470. None of these represent an improvement on the benchmark of R2 466 or an improvement on the daily cost achieved in paragraph 7.3.1.

7.3.3 Seasonality based on a 26 day cycle

When using a 26 day cycle, one forecasting method consistently provided the "best" results for withdrawals based on the three methods of forecast error used to evaluate the methods investigated, *i.e.* a simple average combined with simple seasonal relatives. The methods used to forecast the deposit patterns were a simple average with simple seasonal relatives (based on root mean square error), Winter's method with a regressed trend and moving seasonal relatives (based on mean absolute percent error) and Winter's method with a regressed trend and simple seasonal relatives (based on mean absolute deviation).

The three possible combinations all provide feasible alternatives having a daily cost lower than the benchmark cost of R2 466. The results are shown in Appendices P3-1, P3-2 and P3-3. The results of the 26 day cycle are however not as good as the results based on the six day cycle reported in paragraph 7.3.1.

7.3.4 Seasonality based on a 30 day cycle

Two methods were proposed for forecasting withdrawal patterns for a 30 day cycle, *i.e.* FIT smoothing using a default trend and simple seasonal relatives (based mean absolute percent error and mean absolute deviation) and a simple average with simple seasonal relatives (based on root mean square error). To forecast deposit patterns, three methods were suggested, *i.e.* a simple average with simple seasonal relatives (based on root mean square error), simple exponential smoothing combined with simple seasonal relatives (based on mean absolute percent error) and Winter's method with a regressed trend and simple seasonal relatives (based on mean absolute deviation).

The six possible combinations all provided daily costs improving on the benchmark cost of R2 466. Although some of the combinations did create shortages at lower reorder points, the shortage patterns were not erratic as shown for the 24 day cycle before. The results are shown in Appendices P4-1, P4-2, P4-3, P4-4, P4-5 and P4-6. The best results were obtained in Appendices P4-1 and P4-3. In both cases the withdrawals were forecast using FIT smoothing with a default trend and simple seasonal relatives, whereas the deposits were first forecast using simple exponential smoothing combined with simple seasonal relatives and, subsequently, using Winter's method with a regressed trend together with moving seasonal relatives. In both cases the reorder point was R300 000 and the daily cost was equal to R2 371. This not only substantially improves on the benchmark cost, but provides the "best" alternative achieved thus far.

7.3.5 Conclusion

Table 7.2 summarises the results of the investigation reported on in the preceding paragraphs.

The results show that under the assumption that deposits and withdrawals display the same seasonal pattern, the "best" approach based on lowest daily cost of holding cash inventory would be to use a 30 day cycle. The lowest cost alternative was achieved using a FIT smoothing forecasting technique with a default trend combined with simple seasonal relatives to forecast withdrawals. The deposits could be forecast using either simple exponential smoothing combined with simple seasonal relatives or Winter's method with a regressed trend with moving seasonal relatives. In both cases the daily cost is R2 371 compared to the current cost of R2 466 (determined in Chapter 6). Based on the criteria of simplicity and ease of use, exponential smoothing combined with simple seasonal relatives would be the preferred method for forecasting deposit patterns over a 30 day cycle.

The other notable result of combining the forecasting method with the order policy, is that the reorder point has dropped form R900 000 to R300 000 implying that the cash inventories held at the branch have reduced significantly leading to a reduced risk should a bank robbery occur.

7.4 An investigation into different cycles for withdrawals and deposits

In contrast to the assumption made before, where withdrawal and deposit patterns were assumed to have corresponding cyclical behaviour, this assumption is now revoked. The selection of the withdrawal and deposit forecasting method is merely based on the minimisation of the forecast error, irrespective of the cycle involved. In the case of deposit patterns, the three methods of forecast error were all minimised where deposits were assumed to have a 24 day cycle. Refer to Table 5.9 for confirmation of the results. However, in the case of the withdrawal patterns, the minimisation of forecast error does not indicate a "most suitable" cycle, since the three measures of forecast error each indicate a different period to be appropriate. The root mean square error indicated a cycle of 26 days, the mean absolute percent error pointed to a 24 day cycle, whereas the mean absolute deviation was minimised over a 30 day cycle. The various combinations of the "best" methods are summarised in Table 7.3.

Table 7.3

Combinations of forecasting methods based on minimisation of forecast error

Withdrawal seasonality	**Withdrawal forecast method**	**Deposit forecast method (using a 24 day cycle)**	**Result summary**
24 days (based on the minimisation of the mean absolute percent error)	FIT smoothing with default trend & simple seasonal relatives	Simple exponential smoothing with moving seasonal relatives	Appendix P2-1
		Simple average with simple seasonal relatives	Appendix P2-3
		FIT smoothing with regressed trend & moving seasonal relatives	Appendix P2-2
26 days (based on the minimisation of the root mean square error)	Simple average with simple seasonal relatives	Simple exponential smoothing with moving seasonal relatives	Appendix Q1-1
		Simple average with simple seasonal relatives	Appendix Q1-2
		FIT smoothing with regressed trend & moving seasonal relatives	Appendix Q1-3
30 days (based on the minimisation of the mean absolute deviation)	FIT smoothing with default trend & simple seasonal relatives	Simple exponential smoothing with moving seasonal relatives	Appendix Q2-1
		Simple average with simple seasonal relatives	Appendix Q2-2
		FIT smoothing with regressed trend & moving seasonal relatives	Appendix Q2-3

However, if the results from Appendices P2, Q1 and Q2 are studied, none of the combinations provide a solution which reduces the daily cost further than the figure of R2 371 achieved in the previous paragraph. The assumption therefore has to be made that at this particular branch, the deposit and withdrawal patterns correspond with regard to cyclical behaviour.

7.5 The proposed model compared to the reality

In conclusion, the proposed method for replenishment of cash may be compared to the reality at the branch during the three month period under review. The comparison is shown in Table 7.4. Figure 7.3 compares the daily amount of cash on hand as it really occurred and what would have been the case had the proposed method been used during the three month period under review.

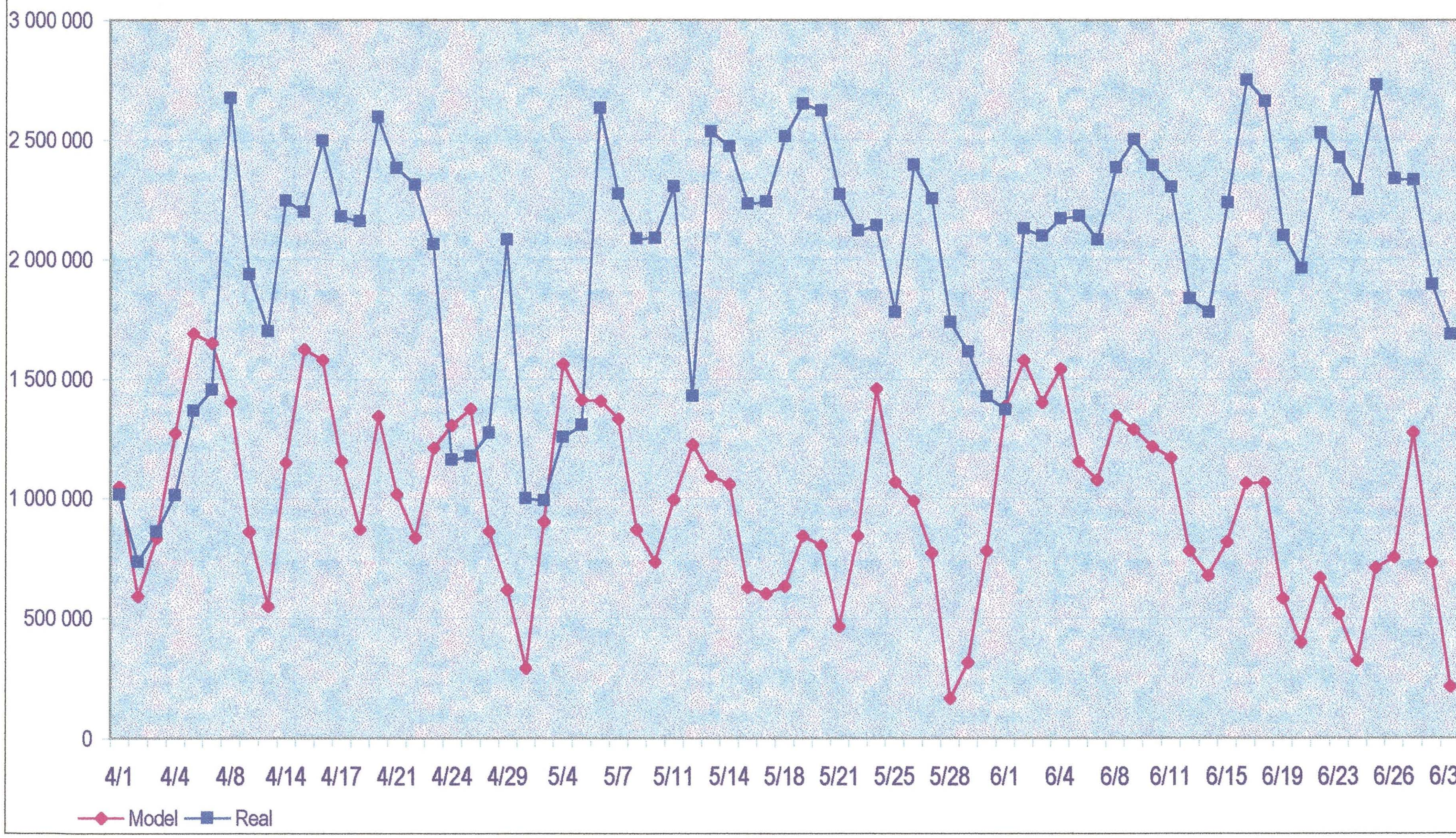
Table 7.4

The proposed model compared to the reality at the branch

Feature	Method used at branch	Proposed method
Average cash on hand	R2 009 264	R970 858
Minimum cash on hand	R736 043	R215 343
Maximum cash on hand	R2 751 331	R1 690 575
Reorder point	From R500 000 to R1 500 000	R300 000
Cash holding cost/day	R2 729	R2 371
Reorder quantity	From R250 000 to R1 300 000	R750 000
Safety stock	R500 000	R200 000
Special order size	R500 000 (minimum)	R500 000
Number of normal orders	16[1]	16
Number of special orders	1	1
Number of shortages	0	0

1 On six occasions these orders concerned coin rather than notes

Figure 7.3: Daily cash on hand from April 1998 to June 1998



7.6 Conclusion

From the comparison in Table 7.4, the advantages of using the proposed method rather than the existing method are obvious. The improvement achieved represents a 13 per cent bottom line cost reduction which equates to R358 per day. If it assumed that a year comprises 300 trading days, the annual saving at the branch is equal to R107 400. Should similar savings be achieved at all branches, the impact will be even more significant. At the time that the research was carried out, 75 branches of a similar size existed within the portfolio of this particular retail bank. If the saving achieved at this branch is extrapolated, it implies a potential annual saving of over R8 000 000. Obviously, the scope of such savings at branches that exhibit other deposit and withdrawal patterns will have to be determined. As stated earlier in the research, the cost calculations were based on an assumption regarding shortage cost, *i.e.* that the shortage cost is equal to ten times the storage cost.

The final chapter of this report discusses issues regarding the implementation of a decision support model of this nature.

CHAPTER 8

Implementation issues relevant to the decision support model for cash replenishment

8.1 Introduction

The aim of Chapter 8 is to discuss various issues relevant to the implementation of an integrated inventory management system in a retail banking environment specifically aimed at improving the cash replenishment process. Figure 8.1 shows the relevance of this chapter with regard to the research project.

In addition to discussing general issues that need to be considered, the chapter will also address specific issues that need to be improved.

8.2 Inventory management across the supply chain in retail banking

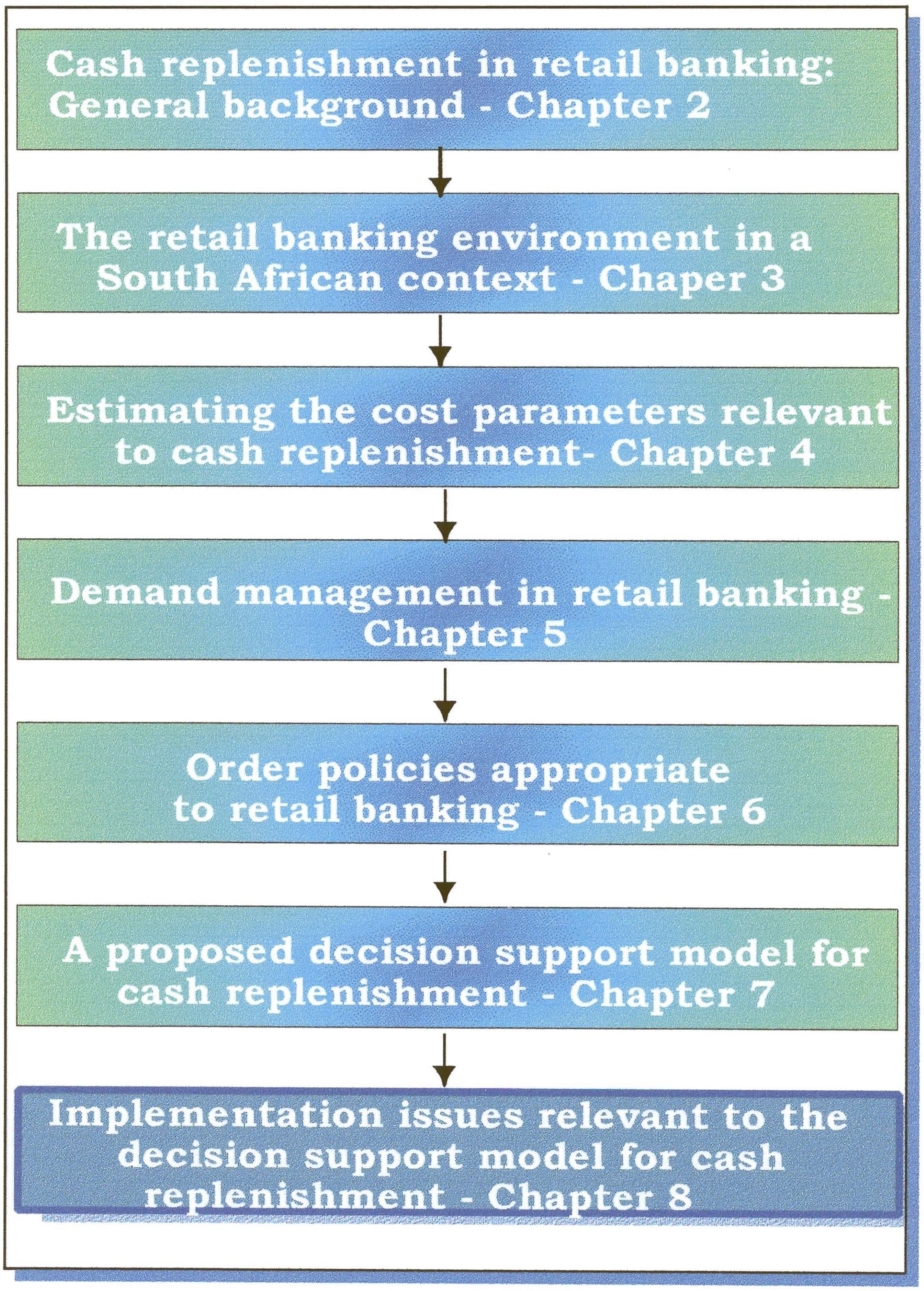
Handfield & Nichols (1999:8) discuss the changes that have come about in global markets resulting in an increasingly competitive environment. The impact of these changes is expressed as follows:

> *Managers throughout the supply chain are feeling the full effect of these changes. Cutbacks in staffing are forcing managers to handle a greater number of channels with fewer people, while cost pressures require that they do so with less inventory.*

From the above quote as well as the quote in paragraph 2.3.2.5, the critical importance of reducing inventories (cash or otherwise) is evident.

Figure 8.1

The structure of the report indicating the relevance of Chapter 8



Stevenson (1999:561) describes management as having two basic functions concerning inventory. The first is to establish a system of keeping track of items in inventory and the second is to take decisions on how much and when to order. The decision support model developed in this thesis sets out to facilitate this process at the branch level in retail banking, with the additional aim of reducing inventory levels without compromising the service level.

Stevenson (1999:561-567) expands on the requirements for effective inventory management as follows:

- A system to keep track of the inventory on hand and on order.
- A reliable forecast of demand that includes an indication of possible forecast error.
- Knowledge of lead times and lead time variability.
- Reasonable estimates of inventory holding costs, ordering costs and shortage costs.
- A classification system for inventory items.

With the exception of the last requirement, all others in the above list are relevant even when the inventory item concerns cash.

Pienaar (1999:3-7) states that optimum throughput through the supply chain should be achieved by design and not by accident and should *inter alia* include a careful plan of supply chain activities, close control of the execution of the planned supply chain activities and continuous reporting of the results of the supply chain activities. He continues that any business process is dependent on accurate data records, the best possible available demand information as well as the best possible supply information.

It is obvious from the discussion that the proposed model attempts to set up an effective system to support inventory management issues across the supply chain. However, some issues relevant to the successful implementation of the model at branch level will have to be addressed to create an environment conducive to the use of such a model.

8.3 Implementation issues at branch level

The first step in implementing the proposed decision support model would be to investigate the factors influencing the cash replenishment problem . A distinction between generic factors (factors characteristic of South African retail banking) and branch-specific factors (those elements unique to a particular branch) will have to be made.

The second step concerns support issues necessary as enablers in this particular situation. These enablers include the necessary information systems as well as the training required by the branch operations manager and his/her support staff to use the model effectively.

Finally, an implementation plan at branch level is proposed as well as the steps required to maintain the decision support model post-implementation.

8.3.1 Generic versus branch-specific factors

As has been stated on a number of occasions in this thesis, specifically in Chapters 2 and 4, certain parameters relevant to the decision support model are of a generic nature. The supply cost, for example, is generic irrespective of the branch location. To determine the scope of these generic factors, it is proposed that a study similar to this, is carried out at an additional five to ten branches to verify that the factors described and quantified in Chapter 2 and 4 are indeed generic and have been quantified correctly. At present, the cost of holding cash inventories at the branch is not seen as something which should be managed at branch level. In fact, the branch does not even consider this as an issue which if reduced, could result in a bottom line saving.

Other elements, particularly those discussed in Chapter 5, are branch-specific. It is obvious that each branch will have unique withdrawal and deposit patterns which have to be determined and tracked to ensure that changes are monitored and catered for.

Stevenson (1999:117) proposes monitoring forecast error by means of comparing forecast errors to predetermined values or action limits. Errors that fall within the limits are judged to be acceptable, whereas errors outside of either limit signal that corrective action is needed.

The control chart approach proposed by Stevenson (1999:118) involves setting upper and lower limits for individual values rather than cumulative errors (as is the case with a tracking signal). The limits are multiples of the square root of the mean square error.

At present, no forecasting is done at the branch level as was described in earlier chapters. Obviously, if withdrawal and deposit patterns are not monitored on a continuous basis, the branch will not be alert to changes (gradual level changes or even cyclical behaviour). As was shown in Chapter 7, by combining forecasting methods with a more flexible approach to replenishing cash, a significant reduction in the average amount of cash held at the branch was achieved.

8.3.2 Support factors

From the discussion in paragraph 8.3.1 it is obvious that the branch operations manager will need to have accurate information continuously to take the "best" possible decisions regarding the supply chain activities at the branch. At present there is no transparency with regard to the amount of cash held at the branch throughout the day. The branch operations manager is able to judge the amount of cash present in the branch at any time by physically going to the safe and observing the amount on hand. There is a limit to the number of times it would be possible for that person to physically perform that function. It is proposed that an information system is developed for the branch which will provide the branch operations manager with on-line information as to the amount of cash in the branch at any time during the day. In addition, it is proposed that deposit and withdrawal totals are monitored throughout the day, creating the necessary visibility if for example withdrawals are unexpectedly higher than forecast.

From the above it is clear that certain changes will be required from a systems point of view. However, the effect will be that although, as in the case of the branch investigated, cash inventory levels may be significantly reduced, the service level will not be impaired. By redesigning the information system, the branch operations manager will be in a position to manage and control cash inventory levels much better. Obviously this could then become a key performance area for the operations manager which should be evaluated on a continuous basis.

It will be crucial to provide the branch operations manager as well as his/her support staff with the necessary training regarding the decision support model. Issues relevant to the decision support model will include some basic inventory management theory and forecasting theory as well as training on how to use the redesigned information system.

8.3.3 A proposed implementation plan

The proposed decision support model was developed for a particular branch of a retail bank. Should the model be used for other branches, the following implementation plan is proposed:

Step 1

Test the validity of the cost parameters determined in Chapters 2 and 4 of the report for the branch where the model is being implemented. If the parameters are not valid, establish the scope of the cost parameter relevant to that branch.

Step 2

Establish historical withdrawal and deposit patterns for the branch as demonstrated in Chapter 5 of the report.

Step 3

Based on judgement, decide how much of the historical data patterns are representative of the situation and consider changes to the branch location, size, number of agencies, opening/closure of new branches/agencies, level of competition in the trading area (refer to paragraph 2.2.3) and so forth that in the near future may have a bearing on the data patterns at the branch.

Step 4

Investigate various forecasting models suited to the data patterns at the branch. As illustrated in Chapter 5, select a suitable method subject to the requirements of simplicity and ease of use from the perspective of the branch operations manager and his/her support staff.

Step 5

Use the forecasting model combined with various order policies to find the "best" policy for that branch as illustrated in Chapter 7.

Step 6

Monitor the forecast by means of the control chart approach described earlier. Take action should the monitoring of the forecast error indicate the necessity. This would include revisiting the "best" order policy since the data patterns have changed, *i.e.* return to Step 5.

Step 7

Monitor changes that have an impact on the cost parameters determined in earlier Step 1. Should changes occur, return to Step 5.

The proposed seven step plan will not only lead to successful implementation, but will guarantee post-implementation maintenance of the decision support model ensuring that accurate information populates the model.

Implementation at branch level will require the involvement of the branch operations manager and his/her support staff from the very inception of the project, and will require assistance from specialists who are familiar with inventory and forecasting theory. A task team comprising branch staff members and specialists should take joint responsibility for the implementation. Post-implementation, support should still be available from the specialist staff, but the responsibility and accountability for maintaining the model should be that of the branch operations manager. A suitable performance measure should be put in place to monitor the use of the model by the branch operations manager.

8.4 Conclusion

As quoted earlier in this chapter, supply chain activities should not occur by accident but by design. The system in use at the branch at present is very much an experience-based random effort with absolutely no theoretical foundation with regard to optimising the cash replenishment process. No performance measures are in place highlighting exceptional performance or even investigating the cost involved in providing this particular service element of branch operations. The proposed decision support model goes a long way to providing a means whereby supply chain activities will occur by design rather than by accident - a situation every manager irrespective of the environment in which he/she finds themselves, should feel exceedingly comfortable about.

CHAPTER 9

Conclusion

9.1 Research objectives revisited

As stated in Chapter 1, the main objective of the research was to establish a scientifically-based decision-making procedure for optimising the amount of cash to be held at a cash point (be it branch, agency or ATM) at any time without compromising the customer service level or incurring undue cost. In reaching the objective, the problem was divided into a number of subproblems, each having its own objective. The subproblems were as follows:

- To determine the cost parameters describing the nature of the problem of cash provision in a South African context - this was reported on in Chapters 2 and 4 of the research report.
- To investigate the characteristics unique to the South African retail banking environment - this was reported on in Chapter 3 of the research report.
- To determine the nature of the demand distribution (a function of deposits and withdrawals) for a cash point - the investigation was reported on in Chapter 5 .
- To develop a forecasting method appropriate for the retail banking environment in South Africa - also reported on in Chapter 5, although it was clearly stated that the methods used were specific to the branch under investigation.
- To investigate the existing order policies used by retail banks, as well as alternative order policies, with the aim of improving the process of cash

replenishment, as represented by typical branch of a South African retail bank.

As a result of the investigation into these subproblems, a generic decision model was developed which may be used to improve the cash replenishment process at branch level for retail banks in South Africa. Finally, some suggestions were made regarding the implementation and maintenance of the decision support model.

9.2 Research methodology

To investigate the cash replenishment problem the cooperation of one of the leading retail banks in South Africa was obtained. A typical branch was selected. The total withdrawal and deposit patterns as well as the ATM withdrawal patterns for a three month period during 1998 were investigated. The cost parameters relevant to the cash replenishment process were quantified. The approach followed was based on the classical inventory theory where the total cost of carrying inventory comprised three cost categories, *i.e.* storage cost, supply cost and shortage cost. Since the banks do not quantify the shortage cost, various assumptions regarding the scope of the shortage cost had to be made.

The next step was to determine the cost of the existing order policy followed by the branch. This figure was used as a benchmark once alternate policies were investigated. The investigation resulted in alternate policies which significantly reduced the daily cost involved in carrying inventory as well as reduced the average amount of cash carried at the branch. By reducing the average cash inventory level, the risk factor related to bank robberies was significantly reduced.

It was also shown, that the branch should consider using an appropriate forecasting method, since once forecasting was combined with an appropriate order policy, it was possible to reduce the cost of carrying cash inventories even further.

In conclusion, the research report suggested an implementation plan to be followed at branch level pointing out that certain changes to information systems were required. In addition, training needs were identified to enable the branch operations manager together with his/her support staff to successfully use the decision support model.

9.3 Results achieved

Table 7.4 compared the existing approach followed at the branch (which is mainly experience-based and largely of a random nature) to the proposed method based on the research at that particular branch. It was shown that the daily cost of carrying cash inventory could be reduced from R2 729 per day to R2 371 per day. This represented a 13 per cent bottom line cost reduction at the branch or R358 per day. As mentioned in Chapter 7, if this result is extrapolated for similar branches within the portfolio of this particular retail bank, the potential annual saving exceeds R8 000 000. Figure 7.3 showed the reduction in average inventory achieved following the proposed method as compared to the existing approach followed at the branch.

It was pointed out in Chapter 8 that some elements of the research would apply at any branch of this bank and for that matter, probably at any retail bank branch in South Africa. However, those elements that are branch-specific were also pointed out.

9.4 Contribution to the knowledge base

As pointed out in Chapter 1, the research was limited to a particular branch of a South African retail bank, the study proved the applicability of industrial engineering principles in a service environment, where the added value of having the optimum cash amount available when required would impact directly on the bottom line of the bank and thereby achieve a cost reduction which can only enhance share-holder value. In the changing environment confronting retail banks in South Africa, enhanced share-holder value is of the utmost importance to increase competitiveness and long-term survival.

9.5 Future research

It would be prudent at this point to indicate possible further research opportunities emanating from the research. It is quite clear that more work needs to be done to distinguish between branch-specific and generic factors impacting on the cash replenishment problem. In addition some elements which were varied on an *ad hoc* basis in this instance may be quantified in a scientific way. Examples that come to mind include safety stock levels, service levels, the determination of the trading area and many more. This was but a first attempt to show the advantages of approaching the problem in a more scientific way than is at present the case.

Finally, the role of industrial engineering in the service environment may no longer be challenged and the particular role in the retail banking environment should be a *fait accompli.*

Appendices

Appendix A

Total real cash deposits

April to June 1998

Explanatory notes

The figures provided by the branch do not reflect real cash deposits, but include transfers, interteller transactions, deposits received from the main safe as well as cash moved to the two agencies, although the cash movements to the agencies are not treated in the same way by the branch. Movements to Agency B are similar to interteller transactions whereas amounts returned from Agency A are shown as a deposit at the branch. In addition, certain fictitious withdrawal transactions are done at the end of a trading day to theoretically empty a cashier's safe of money. Therefore, to determine real cash deposits for a particular day, the transfer amounts for that day first have to be subtracted from the daily deposits, whereafter the cashiers' daily balance at the end of the previous trading day also has to be subtracted. The result reflects the real cash deposits taken by the branch and agencies that day.

Appendix A1: Total real cash deposits - April 1998

Date	Days (holidays)	Total deposits Deposits	Transfers	To safe	Interteller	A	B	Net deposits	Cashiers' daily balance	Real deposits
31-Mar-98	Tuesday								244 889	
1-Apr-98	Wednesday	2 935 082	1 600 000	70 000	267 000	130 000	45 480	822 602	214 321	577 713
2	Thursday	1 084 823	330 000	135 000	188 200	26 000	37 220	368 403	241 078	154 082
3	Friday	1 461 779	265 000	155 640	202 500	72 000	29 271	737 368	260 170	496 291
4	Saturday	513 231	59 750	4 000	19 000			430 481	341 481	170 311
5	Sunday									
6	Monday	1 981 311	419 000	8 000	307 400		20 000	1 226 911	329 220	885 430
7	Tuesday	1 229 657	370 000		192 500		35 284	631 873	217 858	302 653
8	Wednesday	1 487 353	443 750	126 000	193 040	7 000	24 013	693 550	189 248	475 692
9	Thursday	1 489 930	286 450	226 000	233 700	108 000	22 700	613 080	294 696	423 832
10	Good Friday									
11	Saturday	690 269	17 000	1 040	46 000			626 229	379 737	331 533
12	Sunday									
13	Family day									
14	Tuesday	2 140 966	540 200		742 420		25 620	832 726	185 665	452 989
15	Wednesday	6 498 088	5 517 000	100 075	84 000	26 000	36 895	734 118	253 593	548 453
16	Thursday	1 940 800	1 003 600		269 000	26 000	36 140	606 060	195 262	352 467
17	Friday	955 292	7 500	73 000	54 000	80 000	20 400	720 392	285 454	525 130
18	Saturday	802 563	200 000	6 000	149 000			447 563	365 591	162 109
19	Sunday									
20	Monday	3 400 723	966 198	22 040	706 500		35 107	1 670 878	179 439	1 305 287
21	Tuesday	1 479 033	250 000	270 000	145 000	26 000	32 338	755 695	145 763	576 256
22	Wednesday	883 762	1 100	110 000	115 000	28 000	43 360	586 302	212 390	440 539
23	Thursday	1 274 256	250 000	252 000	100 600	26 000	20 000	625 656	142 530	413 266
24	Friday	1 446 405	305 000	372 000	304 000	90 000	152 000	223 405	193 994	80 875
25	Saturday	690 794	100 000	6 000	63 000			521 794	222 406	327 800
26	Sunday									
27	Freedom day									
28	Tuesday	1 939 732	510 000	165 000	25 000			1 239 732	371 010	1 017 326
29	Wednesday	8 078 593	5 868 000	135 640	46 000			2 028 953	224 676	1 657 943
30	Thursday	3 807 833	1 620 000	490 440	68 000	230 000	50 000	1 349 393	303 859	1 124 717

Appendix A1: Total real cash deposits - May 1998

Date	Days	Total deposits						Net	Cashiers'	Real
	(holidays)	Deposits	Transfers	To safe	Interteller	A	B	deposits	daily balance	deposits
1-May-98	Workers' day									
2	Saturday	1 462 481	700 000	200	44 000			718 281	246 444	414 422
3	Sunday									
4	Monday	6 460 337	4 824 327		292 400		21 620	1 321 990	378 590	1 075 546
5	Tuesday	1 761 208	300 000	50 040	173 000	49 000	30 090	1 159 078	383 759	780 488
6	Wednesday	1 540 374	147 500	40 200	166 500	26 000	30 200	1 129 974	435 105	746 215
7	Thursday	1 241 277	134 000	90 200	55 000	26 000	63 000	873 077	282 705	437 972
8	Friday	3 049 437	1 762 000	147 000	135 000	163 000	24 994	817 443	551 500	534 738
9	Saturday	1 832 219	900 000		112 000			820 219	555 739	268 719
10	Sunday									
11	Monday	1 838 937	318 000	20 000	287 000		32 000	1 181 937	357 135	626 198
12	Tuesday	15 919 721	14 684 253	117 000	141 000	27 000	47 604	902 864	350 348	545 729
13	Wednesday	940 349	1 500	20 000	65 000	26 000	30 717	797 132	338 214	446 784
14	Thursday	948 847	15 000	20 000	79 000	26 000	38 053	770 794	309 699	432 580
15	Friday	2 486 175	1 357 500	120 040	95 600	102 000	20 000	791 035	498 160	481 336
16	Saturday	1 419 632	523 500		77 000			819 132	447 284	320 972
17	Sunday									
18	Monday	2 623 865	667 000	1 510	185 500		26 480	1 743 375	463 393	1 296 091
19	Tuesday	2 095 761	660 000	50 000	142 000	26 000	47 382	1 170 379	412 546	706 986
20	Wednesday	1 032 938	162 500	50 000	167 000	26 000	40 000	587 438	426 158	174 892
21	Thursday	2 123 682	618 000	200 000	218 000	26 000	20 000	1 041 682	334 170	615 524
22	Friday	1 931 430	267 000	209 000	226 000	128 000	40 000	1 061 430	556 737	727 260
23	Saturday	964 079	54 822		8 000			901 257	622 860	344 520
24	Sunday									
25	Monday	2 895 631	1 201 000	191 000	370 000		134 710	998 921	512 644	376 061
26	Tuesday	1 679 835	17 500	30 000	218 600	56 000	28 170	1 329 565	228 622	816 921
27	Wednesday	1 263 312	250 000	146 000	156 000	68 000	45 270	598 042	309 803	369 420
28	Thursday	2 138 598	731 000	345 000	286 000	68 000	20 000	688 598	319 798	378 795
29	Friday	2 532 266	617 500	212 000	290 000	144 000	20 000	1 248 766	349 490	928 968
30	Saturday	881 736	5 000	50 000	10 000	60 000		756 736	406 616	407 246
31	Sunday									

Appendix A1: Total real cash deposits - June 1998

Date	Days	Total deposits						Net	Cashiers'	Real
	(holidays)	Deposits	Transfers	To safe	Interteller	A	B	deposits	daily balance	deposits
1-Jun-98	Monday	1 867 128	265 350	22 000	263 000		31 310	1 285 468	492 282	878 852
2	Tuesday	2 054 619	225 000		263 000		45 342	1 521 277	331 943	1 028 995
3	Wednesday	1 136 033	11 150	20 000	211 000	27 000	40 200	826 683	484 720	494 740
4	Thursday	1 113 951	35 000		230 000			848 951	414 880	364 231
5	Friday	1 433 520	72 700	117 000	74 500	114 000	20 000	1 035 320	575 445	620 440
6	Saturday	923 852	0		91 000			832 852	595 171	257 407
7	Sunday									
8	Monday	2 121 413	89 500	90 000	505 000		32 964	1 403 949	371 165	808 778
9	Tuesday	1 204 838	180 000	20 000	72 000	26 000	46 260	860 578	430 245	489 413
10	Wednesday	1 386 428	6 000	41 000	89 280	26 000	25 000	1 199 148	329 038	768 903
11	Thursday	1 109 423	1 000	20 000	97 040	26 000	25 580	939 803	285 273	610 765
12	Friday	1 277 117	132 000	190 000	128 800	154 000	22 800	649 517	467 873	364 244
13	Saturday	971 382	200 000	20 040	196 800			554 542	532 398	86 669
14	Sunday									
15	Monday	2 095 482	668 000		332 000		41 470	1 054 012	296 004	521 614
16	Youth day									
17	Wednesday	1 839 538	302 000	20 000	546 000	26 000	24 000	921 538	337 831	625 534
18	Thursday	1 067 058	27 000	50 000	88 440	26 000	41 240	834 378	246 847	496 547
19	Friday	2 204 341	503 310	234 000	161 200	114 000	37 780	1 154 051	544 716	907 204
20	Saturday	874 577	35 000		89 000	40 000		710 577	548 892	165 861
21	Sunday									
22	Monday	1 432 346	150 000	77 000	138 900		40 270	1 026 176	271 740	477 284
23	Tuesday	11 075 922	10 012 000	70 400	133 000	26 000	36 170	798 352	301 787	526 612
24	Wednesday	2 338 618	117 000	140 000	174 020	26 000	25 460	1 856 138	246 854	1 554 351
25	Thursday	1 804 539	47 000	320 000	176 000	56 000	153 110	1 052 429	330 445	805 575
26	Friday	3 082 736	1 687 000	240 000	172 000	144 000	20 000	819 736	392 402	489 291
27	Saturday	24 950 296	24 028 500	50 000	73 500			798 296	294 888	405 894
28	Sunday									
29	Monday	1 642 178	22 500	364 200	415 000		38 510	801 968	387 836	507 080
30	Tuesday	17 815 786	15 800 000	217 000	313 200	101 000	36 550	1 348 036	473 335	960 200

Appendix A2

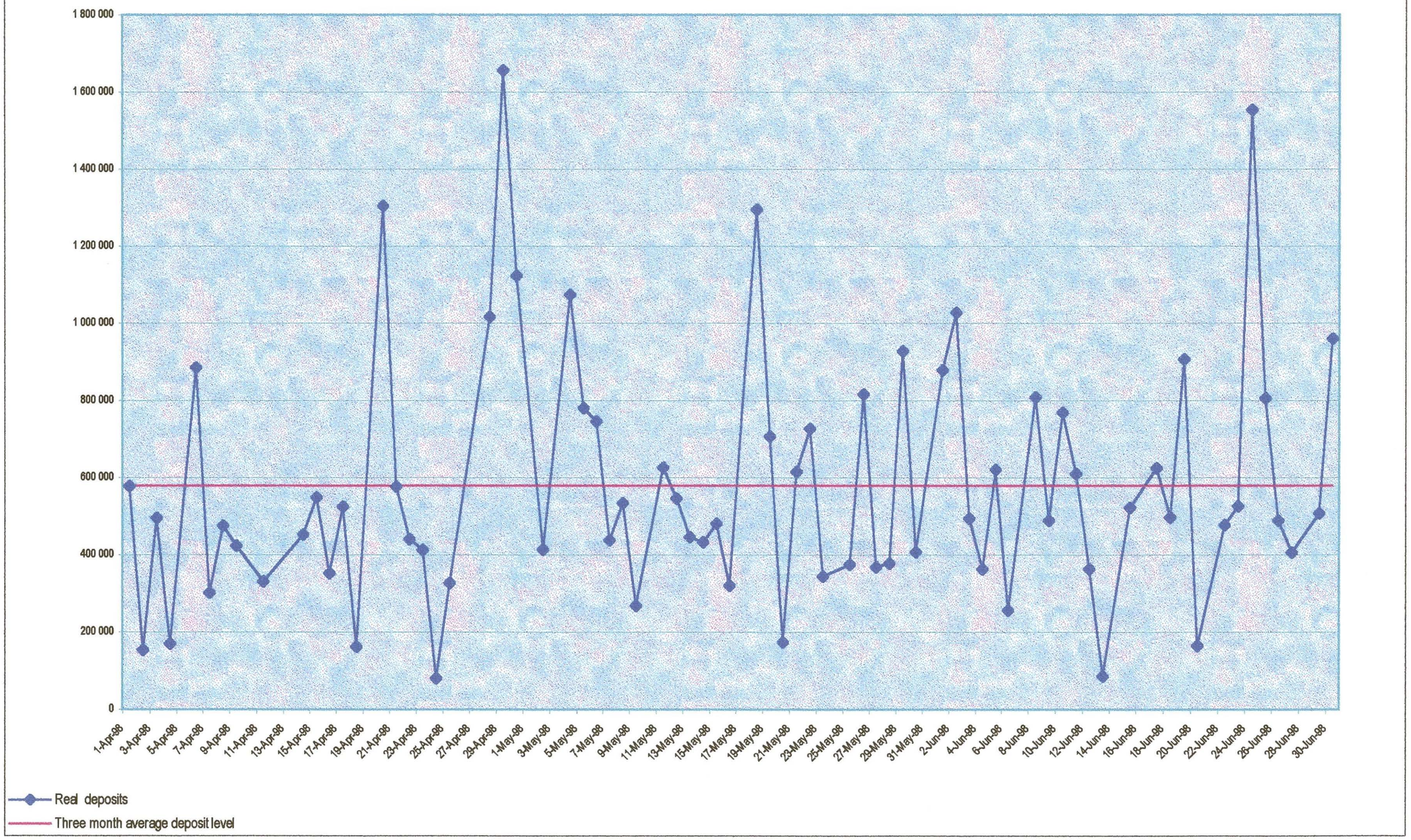
Calculation of various averages for deposits from April to June 1998

Cash deposits per day: April to June 1998

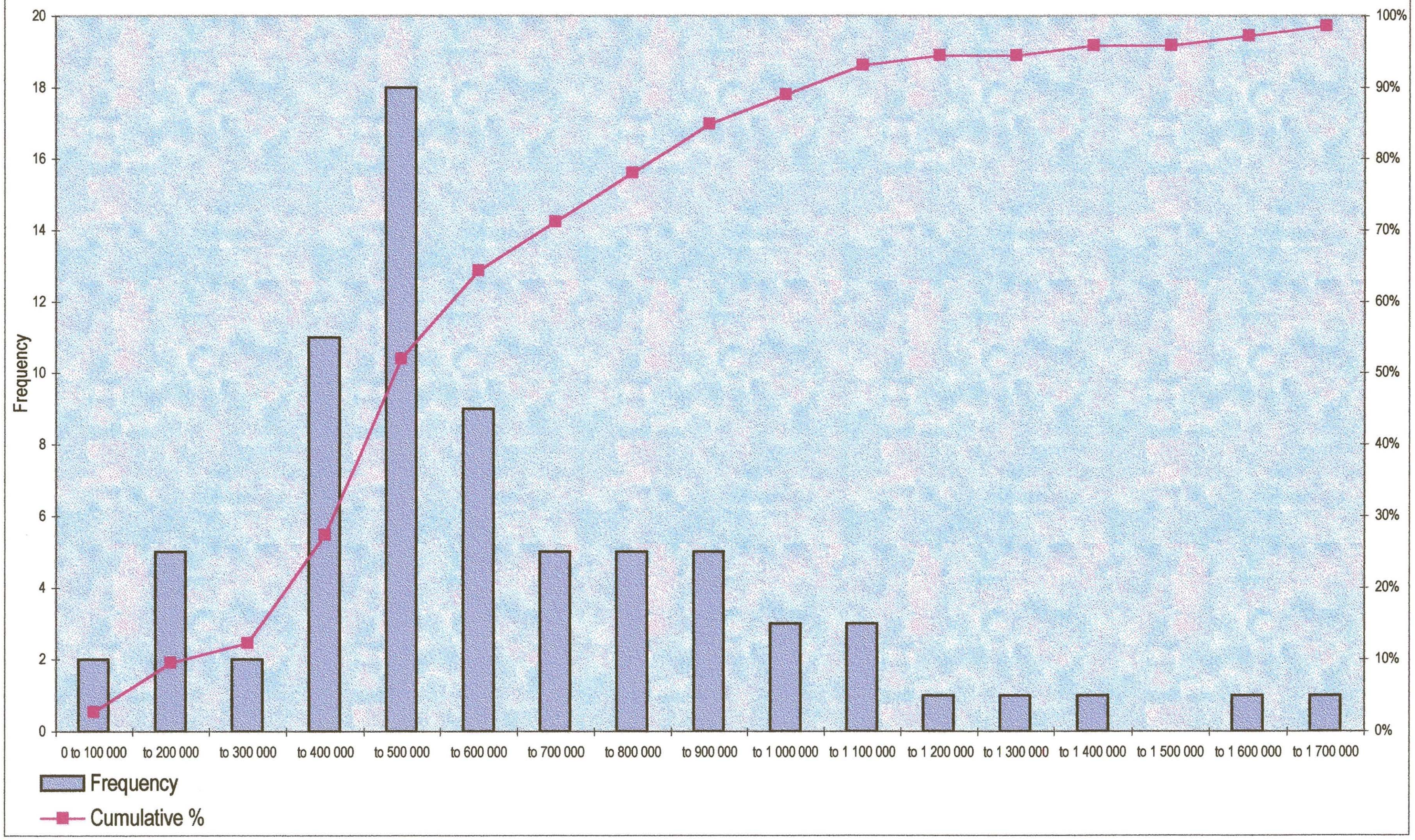
WEEK	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Weekly average
1 - 4 April			577 713	154 082	496 291	170 311	349 599
6 - 11 April	885 430	302 653	475 692	423 832		331 533	483 828
13 - 18 April		452 989	548 453	352 467	525 130	162 109	408 230
20 - 25 April	1 305 287	576 256	440 539	413 266	80 875	327 800	524 004
27 Apr - 2 May		1 017 326	1 657 943	1 124 717		414 422	1 053 602
4 - 9 May	1 075 546	780 488	746 215	437 972	534 738	268 719	640 613
11 - 16 May	626 198	545 729	446 784	432 580	481 336	320 972	475 600
18 - 23 May	1 296 091	706 986	174 892	615 524	727 260	344 520	644 212
25 - 30 May	376 061	816 921	369 420	378 795	928 968	407 246	546 235
1 - 6 June	878 852	1 028 995	494 740	364 231	620 440	257 407	607 444
8 - 13 June	808 778	489 413	768 903	610 765	364 244	86 669	521 462
15 - 20 June	521 614		625 534	496 547	907 204	165 861	543 352
22 - 27 June	477 284	526 612	1 554 351	805 575	489 291	405 894	709 835
29 - 30 June	507 080	960 200					733 640
Daily average	796 202	683 714	683 168	508 489	559 616	281 805	

Month end average	704 882
Normal trading day average	550 261
Average for three month period	579 090

Appendix A3: Real cash deposits per trading day April to June 1998



Appendix A4: Histogram of amounts deposited daily from April to June 1998



Appendix B

Total real cash withdrawals

April to June 1998

Explanatory notes

The figures provided by the branch do not reflect real cash withdrawals, but include transfers, interteller transactions, deposits received from the main safe as well as cash moved to the two agencies, although the cash movements to the agencies are not treated in the same way by the branch. Movements to Agency B are similar to interteller transactions whereas amounts returned from Agency A are shown as a deposit at the branch. In addition, certain fictitious withdrawal transactions are done at the end of a trading day to theoretically empty a cashier's safe of money. Therefore, to determine real cash withdrawals for a particular day, the transfer amounts for that day first have to be subtracted from the daily withdrawals, whereafter the cashiers' daily balance at the end of that trading day also has to be subtracted. The result reflects the real cash withdrawals taken by the branch and agencies that day.

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Appendix B1: Total real cash withdrawals - April 1998

Date	Days (holidays)	Total withdrawals: Withdrawals	Transfers	From safe	Interteller	A	B	Face to face withdrawals	ATM withdrawals	Cashiers' daily balance	Real cash withdrawals
31-Mar-98	Tuesday									244 889	
1-Apr-98	Wednesday	2 902 390	1 600 000		267 000	205 000	45 480	784 910	376 560	214 321	947 149
2	Thursday	1 108 781	330 000	1 200	188 200	23 000	37 220	529 161	320 740	241 078	608 823
3	Friday	1 479 901	265 000		202 500		29 271	983 130	282 390	260 170	1 005 351
4	Saturday	589 296	59 750	30 000	19 000			480 546	341 470	341 481	480 535
5	Sunday										
6	Monday	1 965 107	419 000	160 000	307 400	537 000	20 000	521 707	276 860	329 220	469 348
7	Tuesday	1 110 759	370 000	83 000	192 500	95 000	35 284	334 975	224 690	217 858	341 807
8	Wednesday	1 465 953	443 750	34 000	193 040	112 100	24 013	659 050	252 550	189 248	722 352
9	Thursday	1 593 365	286 450	100 000	233 700		22 700	950 515	310 530	294 696	966 349
10	Good Friday										
11	Saturday	769 126	17 000		46 000			706 126	316 970	379 737	643 359
12	Sunday										
13	Family day										
14	Tuesday	1 949 414	540 200	190 000	742 420	112 000	25 620	339 174	448 570	185 665	602 079
15	Wednesday	6 563 743	5 517 000	96 000	84 000	8 400	36 895	821 448	258 440	253 593	826 296
16	Thursday	1 879 925	1 003 600		269 000	187 040	36 140	384 145	205 750	195 262	394 633
17	Friday	1 043 276	7 500		54 000		20 400	961 376	274 330	285 454	950 252
18	Saturday	873 446	200 000		149 000			524 446	287 020	365 591	445 875
19	Sunday										
20	Monday	3 218 612	966 198	313 120	706 500	459 000	35 107	738 687	273 660	179 439	832 908
21	Tuesday	1 441 648	250 000	102 000	145 000	57 000	32 338	855 311	193 380	145 763	902 927
22	Wednesday	947 880	1 100		115 000	138 000	43 360	650 420	183 150	212 390	621 180
23	Thursday	1 201 908	250 000		100 600	93 000	20 000	738 308	193 720	142 530	789 498
24	Friday	1 495 791	305 000		304 000		152 000	734 791	194 340	193 994	735 137
25	Saturday	678 502	100 000	70 000	63 000			445 502	36 770	222 406	259 866
26	Sunday										
27	Freedom day										
28	Tuesday	2 126 848	510 000		25 000	75 000		1 516 848	382 800	371 010	1 528 638
29	Wednesday	7 931 574	5 868 000		46 000	2 640		2 014 934	112 380	224 676	1 902 638
30	Thursday	3 885 030	1 620 000		68 000		50 000	2 147 030	356 580	303 859	2 199 751

Appendix B1: Total real cash withdrawals - May 1998

Date	Days (holidays)	Total withdrawals: Withdrawals	Transfers	From safe	Interteller	A	B	Face to face withdrawals	ATM withdrawals	Cashiers' daily balance	Real cash withdrawals
1-May-98	Workers' day										
2	Saturday	1 396 321	700 000		44 000			652 321	149 710	246 444	555 586
3	Sunday										
4	Monday	6 592 226	4 824 327		292 400	93 060	21 620	1 360 819	183 190	378 590	1 165 419
5	Tuesday	1 761 208	300 000		173 000	61 200	30 090	1 196 918	116 440	383 759	929 599
6	Wednesday	1 583 078	147 500	30 200	166 500	128 000	30 200	1 080 678	106 450	435 105	752 023
7	Thursday	1 080 053	134 000		55 000	119 000	63 000	709 053	85 260	282 705	511 608
8	Friday	3 311 547	1 762 000		135 000		24 994	1 389 553	159 560	551 500	997 613
9	Saturday	1 832 219	900 000		112 000			820 219	139 320	555 739	403 800
10	Sunday										
11	Monday	1 636 450	318 000	80 000	287 000	295 000	32 000	624 450	98 230	357 135	365 545
12	Tuesday	15 911 323	14 684 253	125 000	141 000	260 040	47 604	653 426	13 510	350 348	316 588
13	Wednesday	926 059	1 500	40 310	65 000	112 000	30 717	676 532	240 810	338 214	579 128
14	Thursday	918 904	15 000		79 000	114 000	38 053	672 851	102 050	309 699	465 202
15	Friday	2 663 560	1 357 500	1 200	95 600		20 000	1 189 260	220 260	498 160	911 360
16	Saturday	1 361 430	523 500		77 000			760 930	34 310	447 284	347 956
17	Sunday										
18	Monday	2 641 898	667 000		185 500	288 000	26 480	1 474 918	254 240	463 393	1 265 766
19	Tuesday	2 039 225	660 000	198 000	142 000	168 000	47 382	823 843	86 160	412 546	497 457
20	Wednesday	1 045 881	162 500	60 000	167 000	87 000	40 000	529 381	111 360	426 158	214 583
21	Thursday	2 029 263	618 000		218 000		20 000	1 173 263	113 570	334 170	952 663
22	Friday	2 151 330	267 000	340 000	226 000		40 000	1 278 330	128 070	556 737	849 663
23	Saturday	1 024 301	54 822		8 000			961 479	141 600	622 860	480 220
24	Sunday										
25	Monday	2 783 927	1 201 000		370 000	65 000	134 710	1 013 217	265 610	512 644	766 183
26	Tuesday	1 393 729	17 500		218 600	94 600	28 170	1 034 859	89 860	228 622	896 097
27	Wednesday	1 340 699	250 000		156 000	108 000	45 270	781 429	114 990	309 803	586 615
28	Thursday	2 147 072	731 000	4 480	286 000		20 000	1 105 592	198 180	319 798	983 974
29	Friday	2 557 649	617 500	10 000	290 000		20 000	1 620 149	257 970	349 490	1 528 628
30	Saturday	934 773	5 000		10 000			919 773	179 380	406 616	692 537
31	Sunday										

Appendix B1: Total real cash withdrawals - June 1998

Date	Days (holidays)	Total withdrawals: Withdrawals	Transfers	From safe	Interteller	A	B	Face to face withdrawals	ATM withdrawals	Cashiers' daily balance	Real cash withdrawals
1-Jun-98	Monday	1 955 680	265 350		263 000		31 310	1 396 020	122 040	492 282	1 025 778
2	Tuesday	1 891 375	225 000		263 000	337 000	45 342	1 021 033	146 880	331 943	835 970
3	Wednesday	1 281 078	11 150		211 000		40 200	1 018 728	138 080	484 720	672 088
4	Thursday	1 045 064	35 000	16 000	230 000	236 040		528 024	109 180	414 880	222 324
5	Friday	1 597 162	72 700		74 500		20 000	1 429 962	153 370	575 445	1 007 887
6	Saturday	907 431	0		91 000			816 431	113 440	595 171	334 700
7	Sunday										
8	Monday	1 888 192	89 500	70 000	505 000	455 400	32 964	735 328	176 260	371 165	540 424
9	Tuesday	1 263 774	180 000	96 400	72 000	1 280	46 260	867 834	109 140	430 245	546 729
10	Wednesday	1 283 140	6 000	21 000	89 280	49 080	25 000	1 092 780	78 760	329 038	842 502
11	Thursday	1 064 232	1 000		97 040	106 800	25 580	833 812	106 530	285 273	655 069
12	Friday	1 456 688	132 000	100 000	128 800		22 800	1 073 088	147 960	467 873	753 174
13	Saturday	1 024 821	200 000		196 800			628 021	95 020	532 398	190 643
14	Sunday										
15	Monday	1 856 225	668 000	130 000	332 000	192 000	41 470	492 755	182 530	296 004	379 281
16	Youth day										
17	Wednesday	1 879 545	302 000		546 000	396 000	24 000	611 545	108 130	337 831	381 844
18	Thursday	974 536	27 000	5 400	88 440	170 040	41 240	642 416	97 910	246 847	493 479
19	Friday	2 496 709	503 310		161 200		37 780	1 794 419	140 480	544 716	1 390 183
20	Saturday	885 178	35 000		89 000			761 178	137 620	548 892	349 906
21	Sunday										
22	Monday	1 157 992	150 000	58 160	138 900	355 060	40 270	415 602	63 700	271 740	207 562
23	Tuesday	11 103 722	10 012 000	1 220	133 000	57 220	36 170	864 112	114 130	301 787	676 455
24	Wednesday	2 280 685	117 000	28 000	174 020	50 240	25 460	1 885 965	111 910	246 854	1 751 021
25	Thursday	1 879 666	47 000	120 000	176 000	55 020	153 110	1 328 536	169 160	330 445	1 167 250
26	Friday	3 141 346	1 687 000		172 000		20 000	1 262 346	325 030	392 402	1 194 974
27	Saturday	24 840 738	24 028 500		73 500			738 738	192 100	294 888	635 950
28	Sunday										
29	Monday	1 739 736	22 500		415 000	94 400	38 510	1 169 326	268 400	387 836	1 049 890
30	Tuesday	17 899 558	15 800 000	30 000	313 200	460	36 550	1 719 348	232 510	473 335	1 478 523

Appendix B2

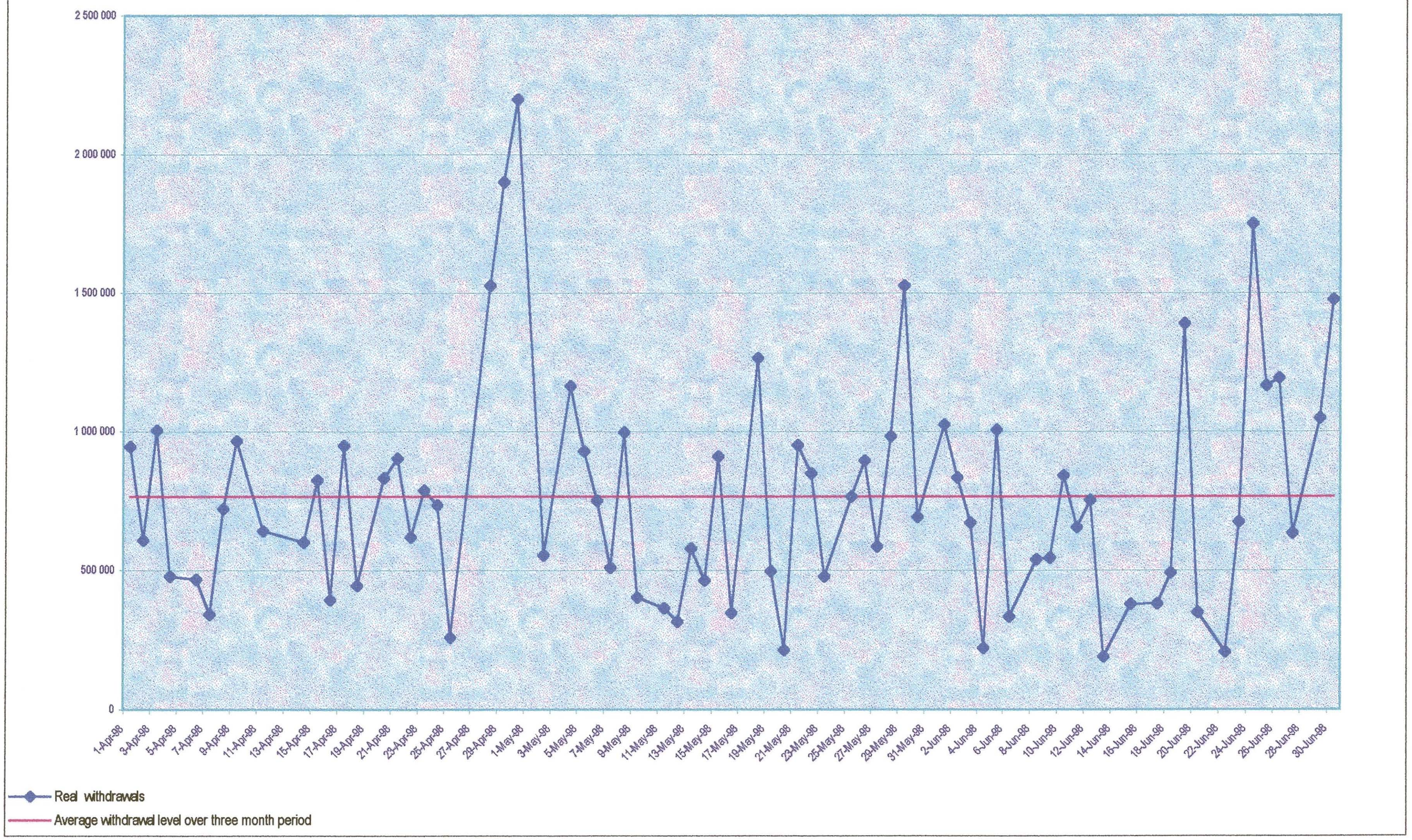
Calculation of various averages for withdrawals from April to June 1998

Cash withdrawals per day: April to June 1998

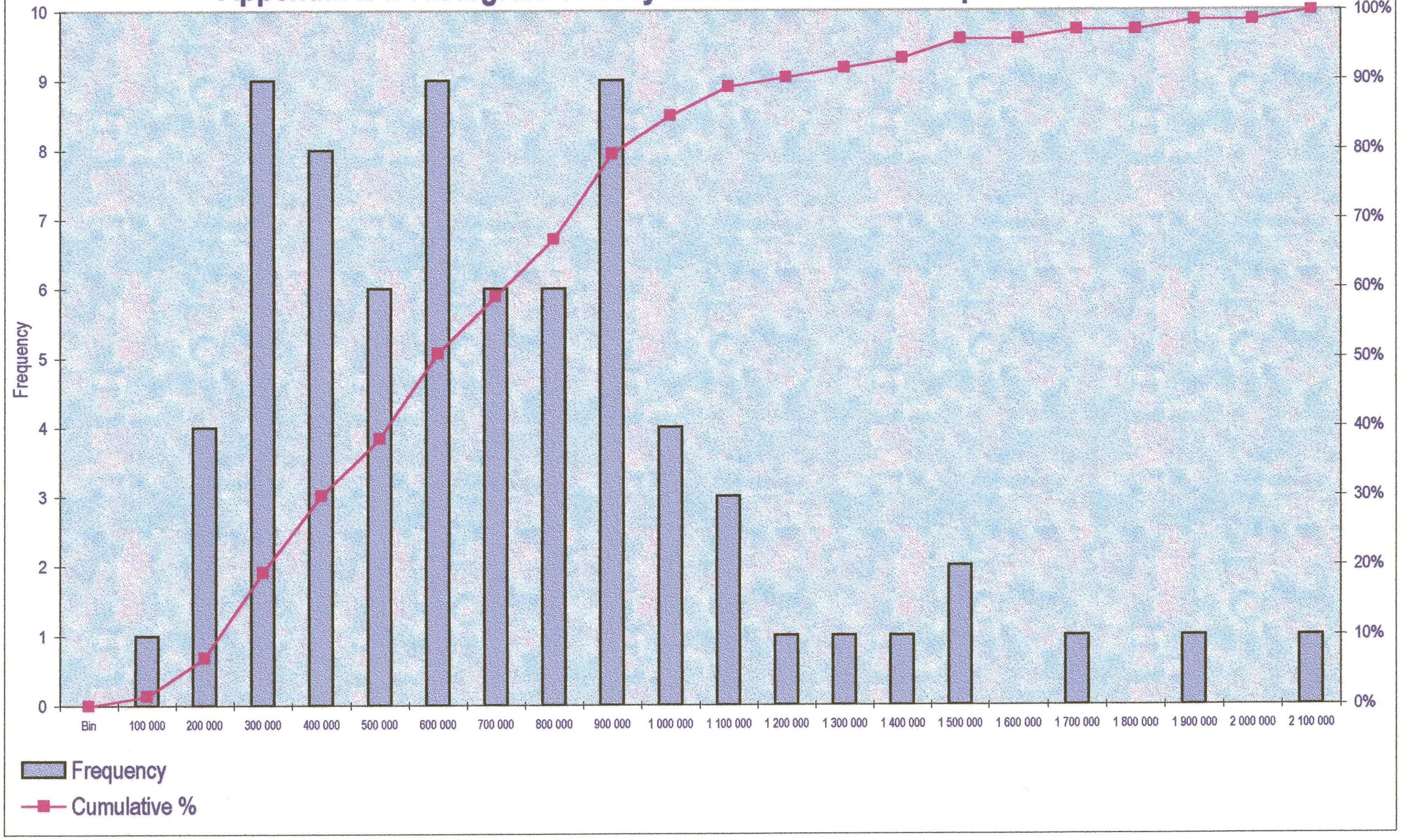
WEEK	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Weekly average
1 - 4 April			947 149	608 823	1 005 351	480 535	760 465
6 - 11 April	469 348	341 807	722 352	966 349		643 359	628 643
13 - 18 April		602 079	826 296	394 633	950 252	445 875	643 827
20 - 25 April	832 908	902 927	621 180	789 498	735 137	259 866	690 253
27 Apr - 2 May		1 528 638	1 902 638	2 199 751		555 586	1 546 653
4 - 9 May	1 165 419	929 599	752 023	511 608	997 613	403 800	793 344
11 - 16 May	365 545	316 588	579 128	465 202	911 360	347 956	497 630
18 - 23 May	1 265 766	497 457	214 583	952 663	849 663	480 220	710 059
25 - 30 May	766 183	896 097	586 615	983 974	1 528 628	692 537	909 006
1 - 6 June	1 025 778	835 970	672 088	222 324	1 007 887	334 700	683 125
8 - 13 June	540 424	546 729	842 502	655 069	753 174	190 643	588 090
15 - 20 June	379 281		381 844	493 479	1 390 183	349 906	598 939
22 - 27 June	207 562	676 455	1 751 021	1 167 250	1 194 974	635 950	938 869
29 - 30 June	1 049 890	1 478 523					1 264 207
Daily average	733 464	796 072	830 725	800 817	1 029 475	447 764	

Month end average	1 124 768
Normal trading day average	622 531
Average for three month period	766 797

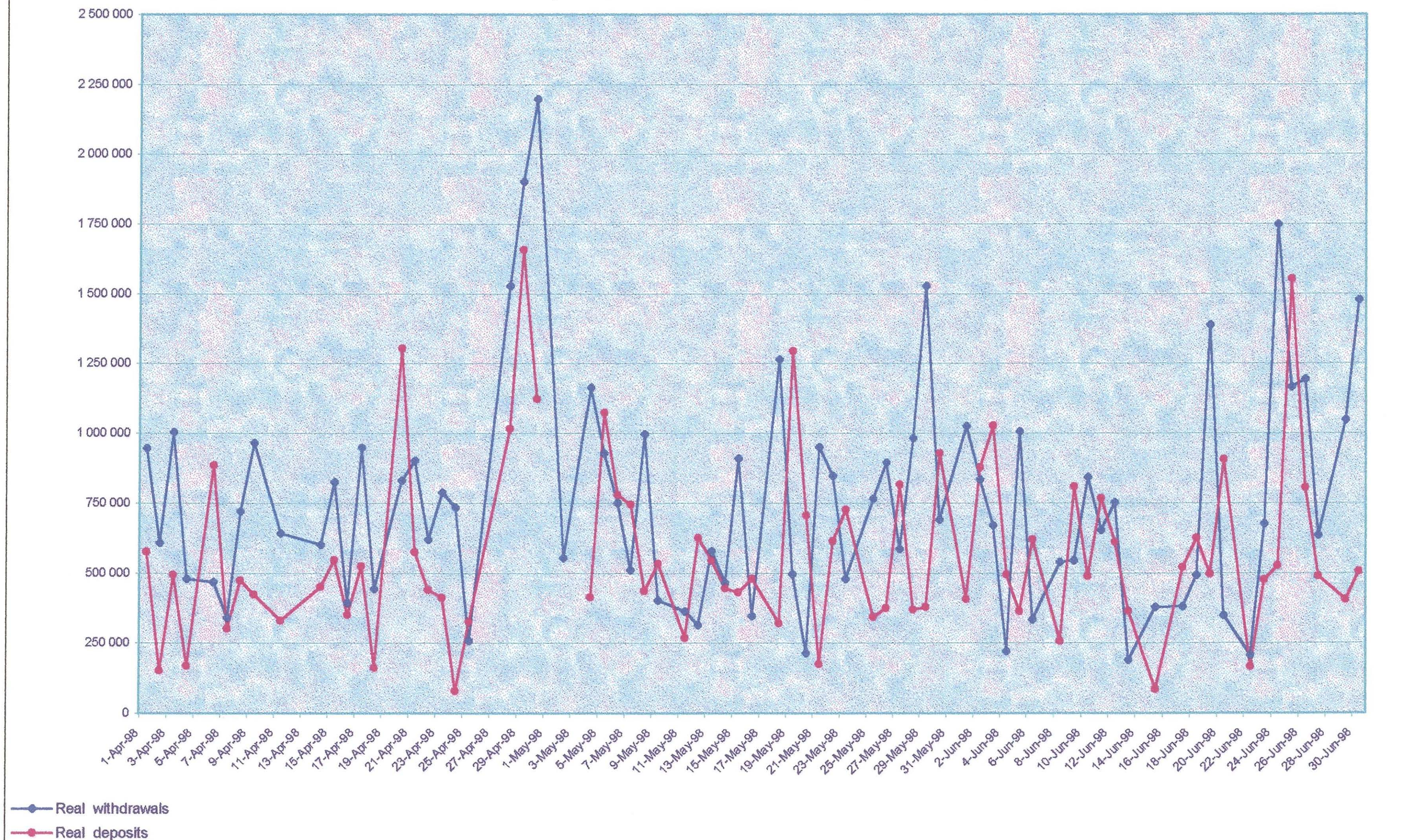
Appendix B3: Real cash withdrawals per trading day April to June 1998



Appendix B4: Histogram of daily amounts withdrawn April to June 1998



Appendix B5: Real withdrawals compared to real deposits per trading day April to June 1998



Appendix C

Cash movements at ATM's

April to June 1998

Explanatory notes

Appendix C1 shows the movement of cash over a period of three months at the four ATM's which are the responsibility of the branch. As explained in Chapter 4, two of the ATM's are located at the branch whereas the other two are located at Agency A.

Since the branch moved to a new location on April, 25, the normal patterns of replenishment and withdrawals at the two ATM's at the branch were disturbed quite severely. The ATM's were not operational from April, 25, until May, 5. During this period a reconciliation showed that one of the ATM's was R10 short. This was corrected by a replenishment of R10 on April, 30.

In addition to the above disruption, the demand patterns at the branch ATM's were significantly altered by the move. At the previous location, the ATM's were located in a very busy shopping mall, whereas the new location does not have passing pedestrian traffic. As a result the demand levels dropped to a much lower level on the reopening of the ATM's. The ATM's at the agency however show undisturbed replenishment and withdrawal patterns throughout the three month period.

Appendix C2 calculates the average withdrawals for various times of the month and week, whereas Appendices C3 to C5 show various graphs illustrating ATM withdrawal patterns.

Appendix C1: Cash movements at ATM's - April 1998

Date	Days	ATM's filled			ATM withdrawals			ATM
	(holidays)	Branch	Agency A	TOTAL	Branch	Agency A	TOTAL	balance
31-Mar-98	Tuesday							307 130
1-Apr-98	Wednesday	80 000	260 000	340 000	197 140	179 420	376 560	270 570
2	Thursday	160 000	151 000	311 000	178 110	142 630	320 740	260 830
3	Friday	80 000		80 000	151 930	130 460	282 390	58 440
4	Saturday	260 000	540 000	800 000	213 160	128 310	341 470	516 970
5	Sunday							
6	Monday	210 000		210 000	124 810	152 050	276 860	450 110
7	Tuesday		100 000	100 000	138 610	86 080	224 690	325 420
8	Wednesday	250 000	200 000	450 000	145 240	107 310	252 550	522 870
9	Thursday	260 000	200 000	460 000	170 870	139 660	310 530	672 340
10	Good Friday							
11	Saturday	400 000		400 000	197 820	119 150	316 970	755 370
12	Sunday							
13	Family day							
14	Tuesday		100 000	100 000	267 570	181 000	448 570	605 900
15	Wednesday	160 000	165 000	325 000	123 800	134 640	258 440	672 460
16	Thursday	160 000	60 000	220 000	114 840	90 910	205 750	686 710
17	Friday	80 000	200 000	280 000	152 920	121 410	274 330	692 380
18	Saturday	160 000	200 000	360 000	188 330	98 690	287 020	765 360
19	Sunday							
20	Monday	260 000	140 000	400 000	149 580	124 080	273 660	891 700
21	Tuesday		150 000	150 000	113 750	79 630	193 380	848 320
22	Wednesday		45 000	45 000	108 100	75 050	183 150	710 170
23	Thursday	80 000	200 000	280 000	89 420	104 300	193 720	796 450
24	Friday	10 000	100 000	110 000	2 610	191 730	194 340	712 110
25	Saturday			0	0	36 770	36 770	675 340
26	Sunday							
27	Freedom day							
28	Tuesday		400 000	400 000	0	382 800	382 800	673 400
29	Wednesday		200 000	200 000	0	112 380	112 380	761 020
30	Thursday	10	200 000	200 010	0	356 580	356 580	604 450

Appendix C1: Cash movements at ATM's - May 1998

Date	Days (holidays)	ATM's filled Branch	ATM's filled Agency A	ATM's filled TOTAL	ATM withdrawals Branch	ATM withdrawals Agency A	ATM withdrawals TOTAL	ATM balance
1-May-98	Workers' day							
2	Saturday		200 000	200 000	0	149 710	149 710	654 740
3	Sunday							
4	Monday		221 000	221 000	0	183 190	183 190	692 550
5	Tuesday		200 000	200 000	0	116 440	116 440	776 110
6	Wednesday	150 000	160 000	310 000	8 810	97 640	106 450	979 660
7	Thursday		15 000	15 000	7 680	77 580	85 260	909 400
8	Friday		20 000	20 000	11 550	148 010	159 560	769 840
9	Saturday		140 000	140 000	12 980	126 340	139 320	770 520
10	Sunday							
11	Monday	140 000	155 000	295 000	2 430	95 800	98 230	967 290
12	Tuesday			0	610	12 900	13 510	953 780
13	Wednesday		128 000	128 000	26 210	214 600	240 810	840 970
14	Thursday			0	10 060	91 990	102 050	738 920
15	Friday		320 000	320 000	27 130	193 130	220 260	838 660
16	Saturday		100 000	100 000	1 970	32 340	34 310	904 350
17	Sunday							
18	Monday		220 000	220 000	21 360	232 880	254 240	870 110
19	Tuesday		150 000	150 000	12 140	74 020	86 160	933 950
20	Wednesday			0	13 130	98 230	111 360	822 590
21	Thursday		80 000	80 000	9 530	104 040	113 570	789 020
22	Friday			0	12 900	115 170	128 070	660 950
23	Saturday		100 000	100 000	9 630	131 970	141 600	619 350
24	Sunday							
25	Monday		200 000	200 000	23 260	242 350	265 610	553 740
26	Tuesday	50 000	368 000	418 000	14 150	75 710	89 860	881 880
27	Wednesday			0	17 740	97 250	114 990	766 890
28	Thursday	100 000	78 000	178 000	25 360	172 820	198 180	746 710
29	Friday		260 000	260 000	54 020	203 950	257 970	748 740
30	Saturday	100 000	200 000	300 000	21 760	157 620	179 380	869 360
31	Sunday							

Appendix C1: Cash movements at ATM's - June 1998

Date	Days	ATM's filled			ATM withdrawals			ATM
	(holidays)	Branch	Agency A	TOTAL	Branch	Agency A	TOTAL	balance
1-Jun-98	Monday			0	25 040	97 000	122 040	747 320
2	Tuesday		228 000	228 000	21 570	125 310	146 880	828 440
3	Wednesday			0	18 320	119 760	138 080	690 360
4	Thursday			0	15 400	93 780	109 180	581 180
5	Friday	100 000	496 000	596 000	24 490	128 880	153 370	1 023 810
6	Saturday			0	15 760	97 680	113 440	910 370
7	Sunday							
8	Monday		260 000	260 000	21 860	154 400	176 260	994 110
9	Tuesday		115 000	115 000	13 670	95 470	109 140	999 970
10	Wednesday			0	9 010	69 750	78 760	921 210
11	Thursday			0	8 870	97 660	106 530	814 680
12	Friday			0	28 210	119 750	147 960	666 720
13	Saturday	100 000		100 000	12 000	83 020	95 020	671 700
14	Sunday							
15	Monday		550 000	550 000	19 260	163 270	182 530	1 039 170
16	Youth day							
17	Wednesday		230 000	230 000	11 290	96 840	108 130	1 161 040
18	Thursday			0	8 520	89 390	97 910	1 063 130
19	Friday			0	23 610	116 870	140 480	922 650
20	Saturday			0	12 370	125 250	137 620	785 030
21	Sunday							
22	Monday	50 000	510 000	560 000	9 350	54 350	63 700	1 281 330
23	Tuesday		18 000	18 000	15 620	98 510	114 130	1 185 200
24	Wednesday	130 000	120 000	250 000	18 720	93 190	111 910	1 323 290
25	Thursday		76 000	76 000	20 720	148 440	169 160	1 230 130
26	Friday		260 000	260 000	51 610	273 420	325 030	1 165 100
27	Saturday		343 000	343 000	18 190	173 910	192 100	1 316 000
28	Sunday							
29	Monday			0	28 530	239 870	268 400	1 047 600
30	Tuesday		157 000	157 000	32 630	199 880	232 510	972 090

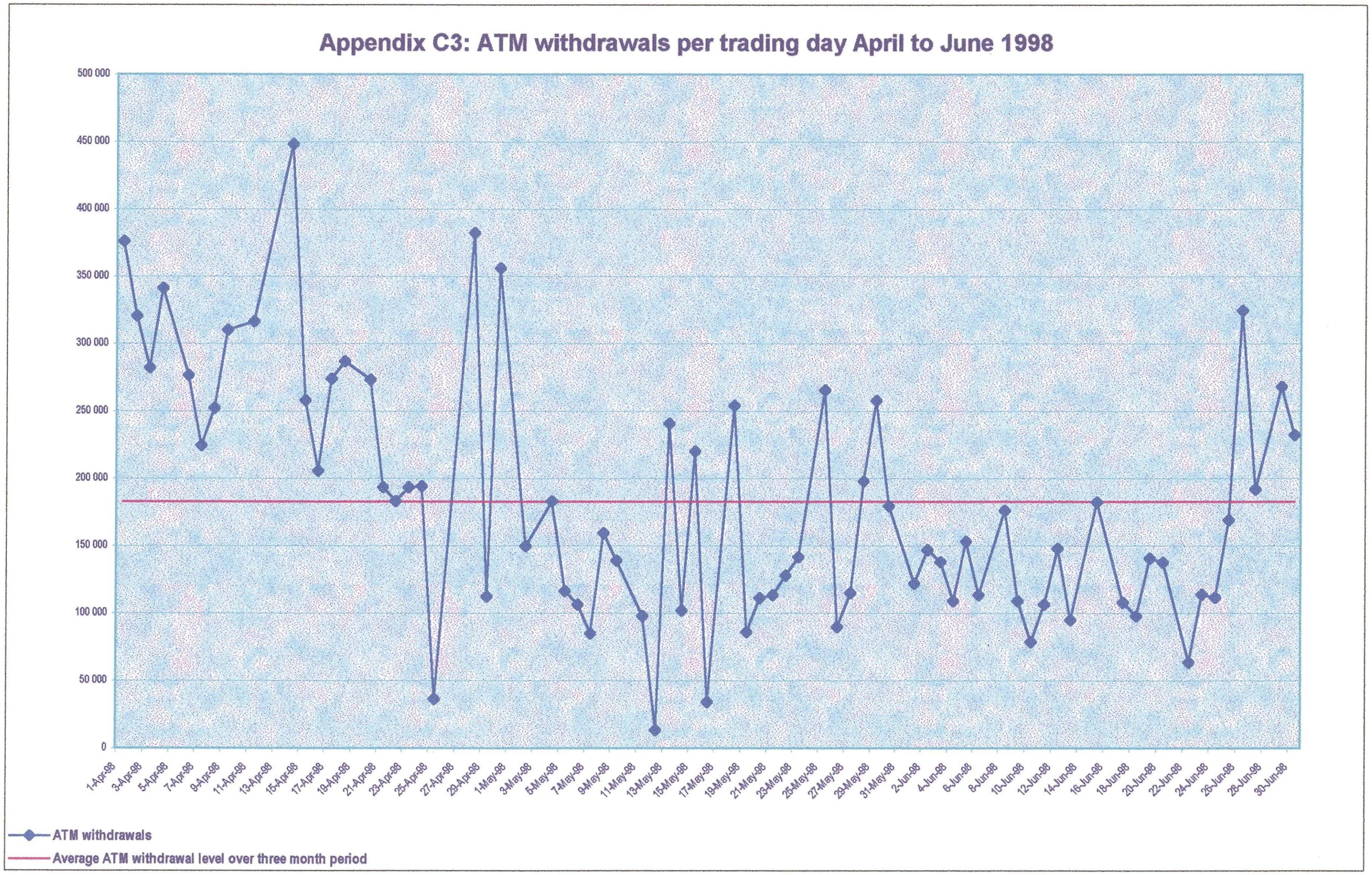
Appendix C2

Calculation of various averages for ATM withdrawals from April to June 1998

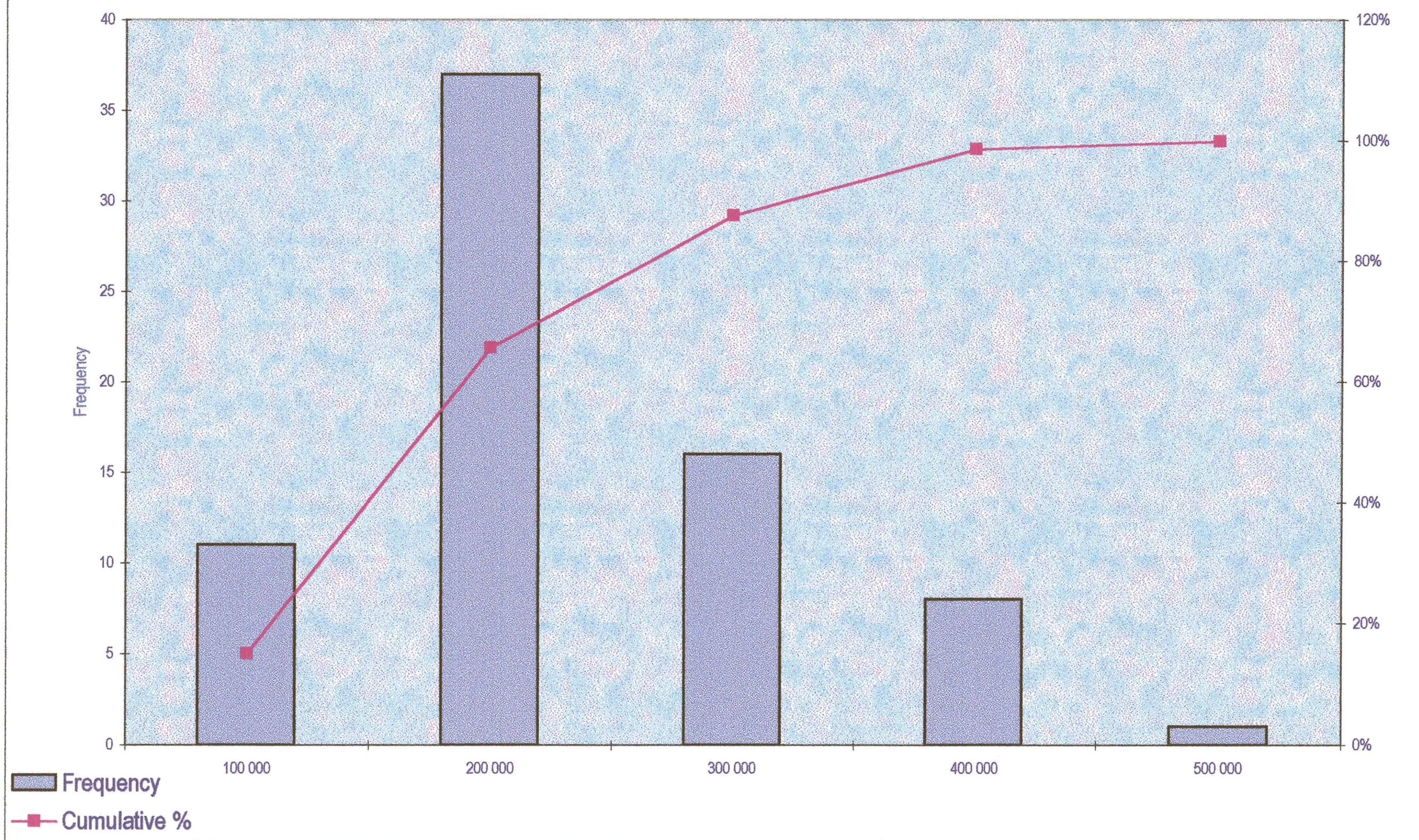
ATM cash withdrawals per day: April to June 1998

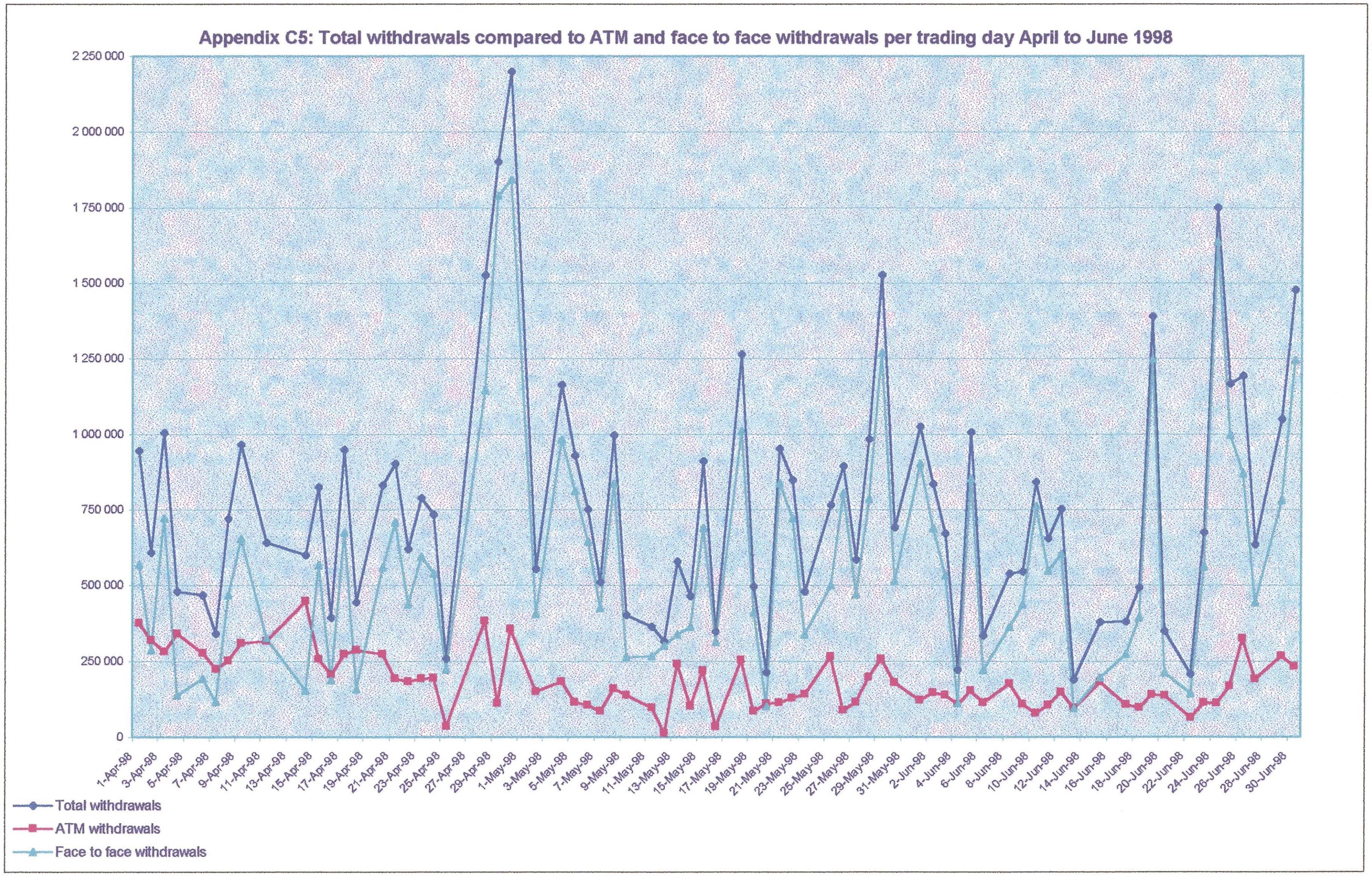
WEEK	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Weekly average
1 - 4 April			376 560	320 740	282 390	341 470	330 290
6 - 11 April	276 860	224 690	252 550	310 530		316 970	276 320
13 - 18 April		448 570	258 440	205 750	274 330	287 020	294 822
20 - 25 April	273 660	193 380	183 150	193 720	194 340	36 770	179 170
27 Apr - 2 May		382 800	112 380	356 580		149 710	250 368
4 - 9 May	183 190	116 440	106 450	85 260	159 560	139 320	131 703
11 - 16 May	98 230	13 510	240 810	102 050	220 260	34 310	118 195
18 - 23 May	254 240	86 160	111 360	113 570	128 070	141 600	139 167
25 - 30 May	265 610	89 860	114 990	198 180	257 970	179 380	184 332
1 - 6 June	122 040	146 880	138 080	109 180	153 370	113 440	130 498
8 - 13 June	176 260	109 140	78 760	106 530	147 960	95 020	118 945
15 - 20 June	182 530		108 130	97 910	140 480	137 620	133 334
22 - 27 June	63 700	114 130	111 910	169 160	325 030	192 100	162 672
29 - 30 June	268 400	232 510					250 455
Daily average	196 793	179 839	168 736	182 243	207 615	166 518	

Month end average	212 115
Normal trading day average	172 480
Average for three month period	182 658


Appendix C3: ATM withdrawals per trading day April to June 1998
500 000
450 000
400 000
350 000
300 000
250 000
200 000
150 000
100 000
50 000
0
1-Apr-98
30-Jun-98
ATM withdrawals
Average ATM withdrawal level over three month period

Appendix C4: Histogram of ATM amounts withdrawn daily April to June 1998




Appendix C5: Total withdrawals compared to ATM and face to face withdrawals per trading day April to June 1998
2 250 000
2 000 000
1 750 000
1 500 000
1 250 000
1 000 000
750 000
500 000
250 000
0
1-Apr-98
30-Jun-98
Total withdrawals
ATM withdrawals
Face to face withdrawals

Appendix D

Total daily amount of cash on hand

April to June 1998

Explanatory notes

Appendix D1 determines the total amount of cash held by the branch and its agencies on a daily basis. This is required to determine the cost of carrying these particular levels of inventory. To determine the total daily amount of cash on hand, the main safe overnight balance is added to the cashiers' total daily balance (both sets of figures provided by the branch) as well as the ATM balance (calculated in Appendix C). A complicating factor is that although ATM withdrawals occur on Sundays and public holidays, the withdrawals on those days are added to the withdrawal totals of the next trading day. The assumption is therefore made that the full amount held at the end of a trading day before a non-trading day is carried throughout that period. Refer also to Appendix E where the storage cost is calculated to view the impact of this assumption.

Appendix D2 shows the components constituting the total amount of cash on hand on a daily basis.

Appendix D1:
Total daily amount of cash on hand: April 1998

Date	Days (holidays)	Main safe balance overnight	Cashiers' total daily balance	ATM balance	Total daily cash on hand
31-Mar-98	Tuesday	865 935	244 889.04	307 130	1 417 954.04
1-Apr-98	Wednesday	530 935	214 321.02	270 570	1 015 826.02
2	Thursday	234 135	241 077.56	260 830	736 042.56
3	Friday	542 535	260 169.83	58 440	861 144.83
4	Saturday	158 535	338 713.69	516 970	1 014 218.69
5	Sunday				
6	Monday	587 535	329 219.64	450 110	1 366 864.64
7	Tuesday	910 535	217 857.58	325 420	1 453 812.58
8	Wednesday	1 963 635	189 247.56	522 870	2 675 752.56
9	Thursday	969 635	294 696.08	672 340	1 936 671.08
10	Good Friday				
11	Saturday	568 555	375 736.90	755 370	1 699 661.90
12	Sunday				
13	Family day				
14	Tuesday	1 455 555	185 664.82	605 900	2 247 119.82
15	Wednesday	1 273 880	253 592.60	672 460	2 199 932.60
16	Thursday	1 614 920	195 261.94	686 710	2 496 891.94
17	Friday	1 202 320	285 454.05	692 380	2 180 154.05
18	Saturday	1 036 320	358 985.83	765 360	2 160 665.83
19	Sunday				
20	Monday	1 526 400	179 439.18	891 700	2 597 539.18
21	Tuesday	1 389 400	145 763.43	848 320	2 383 483.43
22	Wednesday	1 389 400	212 390.24	710 170	2 311 960.24
23	Thursday	1 124 400	142 529.58	796 450	2 063 379.58
24	Friday	255 640	193 993.98	712 110	1 161 743.98
25	Saturday	319 640	182 286.21	675 340	1 177 266.21
26	Sunday				
27	Freedom day				
28	Tuesday	229 640	371 009.80	673 400	1 274 049.80
29	Wednesday	1 096 640	224 675.53	761 020	2 082 335.53
30	Thursday	93 200	303 858.82	604 450	1 001 508.82

Appendix D1:
Total daily amount of cash on hand: May 1998

Date	Days (holidays)	Main safe balance overnight	Cashiers' total daily balance	ATM balance	Total daily cash on hand
1-May-98	Workers' day				
2	Saturday	93 000	244 051.75	654 740	991 791.75
3	Sunday				
4	Monday	186 060	378 590.28	692 550	1 257 200.28
5	Tuesday	148 220	383 758.69	776 110	1 308 088.69
6	Wednesday	1 220 220	435 104.94	979 660	2 634 984.94
7	Thursday	1 083 020	282 704.69	909 400	2 275 124.69
8	Friday	767 020	551 499.76	769 840	2 088 359.76
9	Saturday	767 020	552 679.56	770 520	2 090 219.56
10	Sunday				
11	Monday	982 020	357 134.91	967 290	2 306 444.91
12	Tuesday	125 060	350 347.89	953 780	1 429 187.89
13	Wednesday	1 356 370	338 213.64	840 970	2 535 553.64
14	Thursday	1 424 370	309 698.95	738 920	2 472 988.95
15	Friday	895 930	498 159.54	838 660	2 232 749.54
16	Saturday	895 930	442 554.20	904 350	2 242 834.20
17	Sunday				
18	Monday	1 182 420	463 392.54	870 110	2 515 922.54
19	Tuesday	1 304 420	412 546.12	933 950	2 650 916.12
20	Wednesday	1 375 420	426 158.46	822 590	2 624 168.46
21	Thursday	1 149 420	334 170.27	789 020	2 272 610.27
22	Friday	902 420	556 737.06	660 950	2 120 107.06
23	Saturday	902 420	619 859.55	619 350	2 141 629.55
24	Sunday				
25	Monday	711 420	512 644.08	553 740	1 777 804.08
26	Tuesday	1 285 020	228 622.12	881 880	2 395 522.12
27	Wednesday	1 179 020	309 803.45	766 890	2 255 713.45
28	Thursday	670 500	319 798.17	746 710	1 737 008.17
29	Friday	514 500	349 490.36	748 740	1 612 730.36
30	Saturday	154 500	402 942.21	869 360	1 426 802.21
31	Sunday				

Appendix D1:
Total daily amount of cash on hand: June 1998

Date	Days (holidays)	Main safe balance overnight	Cashiers' total daily balance	ATM balance	Total daily cash on hand
1-Jun-98	Monday	132 500	492 282.25	747 320	1 372 102.25
2	Tuesday	969 500	331 943.34	828 440	2 129 883.34
3	Wednesday	922 500	484 720.05	690 360	2 097 580.05
4	Thursday	1 174 540	414 879.69	581 180	2 170 599.69
5	Friday	583 540	575 445.10	1 023 810	2 182 795.10
6	Saturday	583 540	588 892.89	910 370	2 082 802.89
7	Sunday				
8	Monday	1 018 940	371 164.72	994 110	2 384 214.72
9	Tuesday	1 070 620	430 245.02	999 970	2 500 835.02
10	Wednesday	1 143 700	329 037.97	921 210	2 393 947.97
11	Thursday	1 204 500	285 272.97	814 680	2 304 452.97
12	Friday	700 500	467 873.28	666 720	1 835 093.28
13	Saturday	580 460	526 568.25	671 700	1 778 728.25
14	Sunday				
15	Monday	902 460	296 004.40	1 039 170	2 237 634.40
16	Youth day				
17	Wednesday	1 252 460	337 831.10	1 161 040	2 751 331.10
18	Thursday	1 351 900	246 846.80	1 063 130	2 661 876.80
19	Friday	633 800	544 715.68	922 650	2 101 165.68
20	Saturday	633 800	544 878.16	785 030	1 963 708.16
21	Sunday				
22	Monday	977 020	271 739.97	1 281 330	2 530 089.97
23	Tuesday	939 060	301 786.51	1 185 200	2 426 046.51
24	Wednesday	721 300	246 853.96	1 323 290	2 291 443.96
25	Thursday	1 170 320	330 445.09	1 230 130	2 730 895.09
26	Friday	780 900	392 401.52	1 165 100	2 338 401.52
27	Saturday	730 900	286 718.91	1 316 000	2 333 618.91
28	Sunday				
29	Monday	461 100	387 835.98	1 047 600	1 896 535.98
30	Tuesday	240 560	473 335.20	972 090	1 685 985.20

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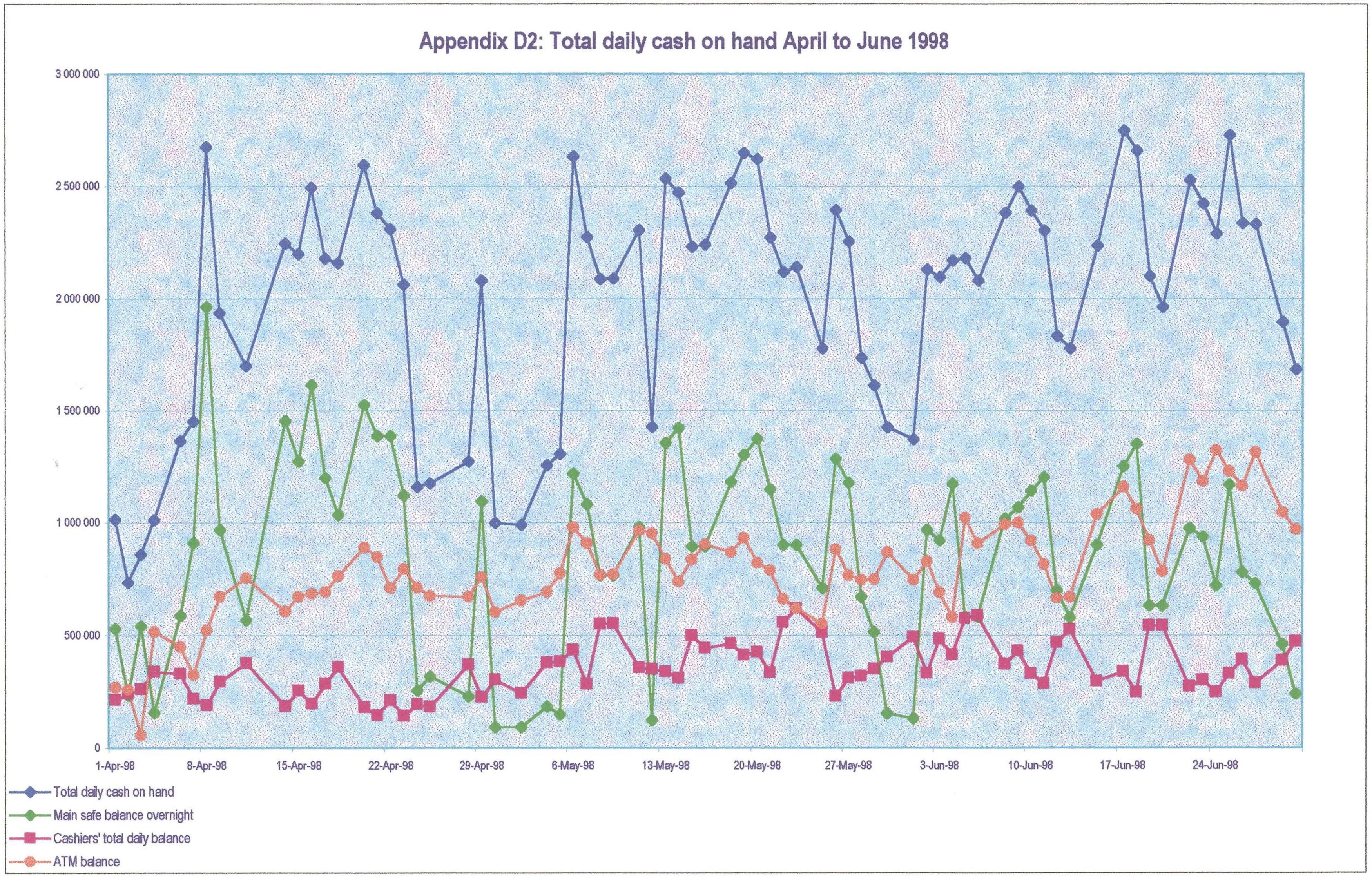


D5

Appendix E

Daily cash storage cost

April to June 1998

Explanatory notes

Appendix E determines the total daily storage cost based on the amount of cash held by the branch and its agencies on a daily basis. The calculations to determine the values of C_{10} and C_{13} were shown in paragraph 4.3.1. The value of c_{11} was provided by the branch. The value of Q_i - the daily amount of cash on hand was determined in Appendix D. Refer to the explanatory note on Appendix D for the assumption regarding amount on hand on non-trading days.

Appendix E:
Daily cash storage cost: April 1998

Date	Days (holidays)	Total daily cash on hand	Storage cost C10+C13	Storage cost c11(Qi)	Storage cost Total
31-Mar-98	Tuesday	1 417 954.04			
1-Apr-98	Wednesday	1 015 826.02	1 631	431.38	2 062
2	Thursday	736 042.56	1 631	312.57	1 944
3	Friday	861 144.83	1 631	365.69	1 997
4	Saturday	1 014 218.69	1 631	430.70	2 062
5	Sunday	1 014 218.69	1 550	430.70	1 981
6	Monday	1 366 864.64	1 631	580.45	2 211
7	Tuesday	1 453 812.58	1 631	617.37	2 248
8	Wednesday	2 675 752.56	1 631	1 136.28	2 767
9	Thursday	1 936 671.08	1 631	822.42	2 453
10	Good Friday	1 936 671.08	1 550	822.42	2 372
11	Saturday	1 699 661.90	1 631	721.77	2 353
12	Sunday	1 699 661.90	1 550	721.77	2 272
13	Family day	1 699 661.90	1 550	721.77	2 272
14	Tuesday	2 247 119.82	1 631	954.26	2 585
15	Wednesday	2 199 932.60	1 631	934.22	2 565
16	Thursday	2 496 891.94	1 631	1 060.32	2 691
17	Friday	2 180 154.05	1 631	925.82	2 557
18	Saturday	2 160 665.83	1 631	917.54	2 549
19	Sunday	2 160 665.83	1 550	917.54	2 468
20	Monday	2 597 539.18	1 631	1 103.06	2 734
21	Tuesday	2 383 483.43	1 631	1 012.16	2 643
22	Wednesday	2 311 960.24	1 631	981.79	2 613
23	Thursday	2 063 379.58	1 631	876.23	2 507
24	Friday	1 161 743.98	1 631	493.34	2 124
25	Saturday	1 177 266.21	1 631	499.93	2 131
26	Sunday	1 177 266.21	1 550	499.93	2 050
27	Freedom day	1 177 266.21	1 550	499.93	2 050
28	Tuesday	1 274 049.80	1 631	541.03	2 172
29	Wednesday	2 082 335.53	1 631	884.28	2 515
30	Thursday	1 001 508.82	1 631	425.30	2 056
TOTAL			48 363	21 642	70 005

Appendix E:
Daily cash storage cost: May 1998

Date	Days	Total daily cash	Storage cost		
	(holidays)	on hand	C10+C13	c11(Qi)	Total
1-May-98	Workers' day	1 001 508.82	1 550	425.30	1 975
2	Saturday	991 791.75	1 631	421.17	2 052
3	Sunday	991 791.75	1 550	421.17	1 971
4	Monday	1 257 200.28	1 631	533.88	2 165
5	Tuesday	1 308 088.69	1 631	555.49	2 186
6	Wednesday	2 634 984.94	1 631	1 118.97	2 750
7	Thursday	2 275 124.69	1 631	966.15	2 597
8	Friday	2 088 359.76	1 631	886.84	2 518
9	Saturday	2 090 219.56	1 631	887.63	2 519
10	Sunday	2 090 219.56	1 550	887.63	2 438
11	Monday	2 306 444.91	1 631	979.45	2 610
12	Tuesday	1 429 187.89	1 631	606.92	2 238
13	Wednesday	2 535 553.64	1 631	1 076.74	2 708
14	Thursday	2 472 988.95	1 631	1 050.17	2 681
15	Friday	2 232 749.54	1 631	948.15	2 579
16	Saturday	2 242 834.20	1 631	952.44	2 583
17	Sunday	2 242 834.20	1 550	952.44	2 502
18	Monday	2 515 922.54	1 631	1 068.41	2 699
19	Tuesday	2 650 916.12	1 631	1 125.73	2 757
20	Wednesday	2 624 168.46	1 631	1 114.37	2 745
21	Thursday	2 272 610.27	1 631	965.08	2 596
22	Friday	2 120 107.06	1 631	900.32	2 531
23	Saturday	2 141 629.55	1 631	909.46	2 540
24	Sunday	2 141 629.55	1 550	909.46	2 459
25	Monday	1 777 804.08	1 631	754.96	2 386
26	Tuesday	2 395 522.12	1 631	1 017.28	2 648
27	Wednesday	2 255 713.45	1 631	957.91	2 589
28	Thursday	1 737 008.17	1 631	737.63	2 369
29	Friday	1 612 730.36	1 631	684.86	2 316
30	Saturday	1 426 802.21	1 631	605.90	2 237
31	Sunday	1 426 802.21	1 550	605.90	2 156
TOTAL			48 525	25 602	74 127

Appendix E:
Daily cash storage cost: June 1998

Date	Days	Total daily cash	Storage cost		
	(holidays)	on hand	C10 + C13	c11(Qi)	Total
1-Jun-98	Monday	1 372 102.25	1 631	582.67	2 214
2	Tuesday	2 129 883.34	1 631	904.47	2 535
3	Wednesday	2 097 580.05	1 631	890.75	2 522
4	Thursday	2 170 599.69	1 631	921.76	2 553
5	Friday	2 182 795.10	1 631	926.94	2 558
6	Saturday	2 082 802.89	1 631	884.48	2 515
7	Sunday	2 082 802.89	1 550	884.48	2 434
8	Monday	2 384 214.72	1 631	1 012.47	2 643
9	Tuesday	2 500 835.02	1 631	1 062.00	2 693
10	Wednesday	2 393 947.97	1 631	1 016.61	2 648
11	Thursday	2 304 452.97	1 631	978.60	2 610
12	Friday	1 835 093.28	1 631	779.29	2 410
13	Saturday	1 778 728.25	1 631	755.35	2 386
14	Sunday	1 778 728.25	1 550	755.35	2 305
15	Monday	2 237 634.40	1 631	950.23	2 581
16	Youth day	2 237 634.40	1 550	950.23	2 500
17	Wednesday	2 751 331.10	1 631	1 168.37	2 799
18	Thursday	2 661 876.80	1 631	1 130.39	2 761
19	Friday	2 101 165.68	1 631	892.28	2 523
20	Saturday	1 963 708.16	1 631	833.90	2 465
21	Sunday	1 963 708.16	1 550	833.90	2 384
22	Monday	2 530 089.97	1 631	1 074.42	2 705
23	Tuesday	2 426 046.51	1 631	1 030.24	2 661
24	Wednesday	2 291 443.96	1 631	973.08	2 604
25	Thursday	2 730 895.09	1 631	1 159.70	2 791
26	Friday	2 338 401.52	1 631	993.02	2 624
27	Saturday	2 333 618.91	1 631	990.99	2 622
28	Sunday	2 333 618.91	1 550	990.99	2 541
29	Monday	1 896 535.98	1 631	805.38	2 436
30	Tuesday	1 685 985.20	1 631	715.97	2 347
TOTAL			48 525	27 848	76 373

Appendix F

Daily cash supply cost

April to June 1998

Explanatory notes

Appendix F determines the total daily supply cost based on the amount of cash delivered to the branch from SBV and sent from the branch to its agencies on a daily basis. The calculations to determine the cost parameters were shown in paragraph 4.3.3. The value of Q_i - the daily amount of cash on hand was determined in Appendix D. Order type is denoted by N for a delivery at the normal cost placed two days before delivery, and S for a special delivery, authorised and arranged for delivery on that day but at the increased cost.

Appendix F:
Daily cash supply cost: April 1998

Date	Days (holidays)	Cash movements between branch and SBV: SBV in Notes	SBV in Coin	SBV out Notes	SBV out Coin	Arrival time & order type	Cost	Cost of cash movement: branch & agencies — Cash moved to and from agencies: To A	From A	To B	From B	Cost	Total supply cost
31-Mar-98	Tuesday												
1-Apr-98	Wednesday							130 000	205 000	20 000	25 480	300	300
2	Thursday							26 000	23 000	20 000	17 220	300	300
3	Friday	850 000	16 000			13:45 N	1 035	72 000		20 000	9 271	300	1 335
4	Saturday												0
5	Sunday												0
6	Monday								537 000	20 000		300	300
7	Tuesday								240 000	20 000	15 284	300	300
8	Wednesday	1 300 000				? N	535	7 000	112 100	20 013	4 000	300	835
9	Thursday							108 000		19 000	3 700	300	300
10	Good Friday												0
11	Saturday												0
12	Sunday												0
13	Family day												0
14	Tuesday								697 000	15 000	10 620	300	300
15	Wednesday							26 000	8 400	20 000	16 895	300	300
16	Thursday	600 000				12:05 N	535	26 000	187 040	20 000	16 140	300	835
17	Friday		20 400			11:00 N	535	80 000		20 400		300	835
18	Saturday												0
19	Sunday												0
20	Monday								459 000	20 000	15 107	300	300
21	Tuesday							26 000	57 000	20 000	12 338	300	300
22	Wednesday							28 000	138 000	20 000	23 360	300	300
23	Thursday							26 000	93 000	20 000		300	300
24	Friday				3 760	? N	535	90 000		152 000		300	835
25	Saturday												0
26	Sunday												0
27	Freedom day												0
28	Tuesday								75 000			300	300
29	Wednesday	1 000 000				15:10 S	1 035		2 640			300	1 335
30	Thursday							230 000		50 000		300	300
TOTAL							4 210					5 700	9 910

Appendix F:
Daily cash supply cost: May 1998

Date	Days (holidays)	Cash movements between branch and SBV						Cost of cash movement: branch & agencies					Total supply cost
		SBV in Notes	SBV in Coin	SBV out Notes	SBV out Coin	Arrival time & order type	Cost	Cash moved to and from agencies: To A	From A	To B	From B	Cost	
1-May-98	Workers' day												0
2	Saturday												0
3	Sunday												0
4	Monday								93 060	13 000	8 620	300	300
5	Tuesday							49 000	61 200	11 000	19 090	300	300
6	Wednesday	1 000 000				12:30 N	535	26 000	128 000	10 000	20 200	300	835
7	Thursday							26 000	119 000	50 200	12 800	300	300
8	Friday							163 000		20 000	4 994	300	300
9	Saturday												0
10	Sunday												0
11	Monday								295 000	20 000	12 000	300	300
12	Tuesday							27 000	260 040	30 000	17 604	300	300
13	Wednesday							26 000	112 000	20 000	10 717	300	300
14	Thursday							26 000	114 000	20 000	18 053	300	300
15	Friday		12 400			12:45 N	535	102 000		20 000		300	835
16	Saturday												0
17	Sunday												0
18	Monday								288 000	20 000	6 480	300	300
19	Tuesday							26 000	168 000	22 000	25 382	300	300
20	Wednesday							26 000	87 000	20 000	20 000	300	300
21	Thursday							26 000		20 000		300	300
22	Friday	250 000		250 000		11:40 N	535	128 000		20 000	20 000	300	835
23	Saturday												0
24	Sunday												0
25	Monday								65 000	132 000	2 710	300	300
26	Tuesday	500 000				11:40 N	535	56 000	94 600	20 000	8 170	300	835
27	Wednesday							68 000	108 000	20 000	25 270	300	300
28	Thursday							68 000		20 000		300	300
29	Friday	500 000				? N	535	144 000		20 000		300	835
30	Saturday							60 000				150	150
31	Sunday												
TOTAL							2 675					6 150	8 825

Appendix F:
Daily cash supply cost: June 1998

Date	Days	Cash movements between branch and SBV						Cost of cash movement: branch & agencies					Total
		SBV in	SBV in	SBV out	SBV out	Arrival time	Cost	Cash moved to and from agencies				Cost	supply
	(holidays)	Notes	Coin	Notes	Coin	& order type		To A	From A	To B	From B		cost
1-Jun-98	Monday									20 400	10 910	300	300
2	Tuesday	500 000				9:40 N	535		337 000	20 000	25 342	300	835
3	Wednesday							27 000		20 200	20 000	300	300
4	Thursday								236 040			300	300
5	Friday							114 000		20 000		300	300
6	Saturday												0
7	Sunday												0
8	Monday								455 400	20 000	12 964	300	300
9	Tuesday							26 000	1 280	20 000	26 260	300	300
10	Wednesday							26 000	49 080	20 000	5 000	300	300
11	Thursday							26 000	106 800	20 000	5 580	300	300
12	Friday							154 000		20 000	2 800	300	300
13	Saturday												0
14	Sunday												0
15	Monday								192 000	20 000	21 470	300	300
16	Youth day												0
17	Wednesday							26 000	396 000	20 000	4 000	300	300
18	Thursday							26 000	170 040	20 000	21 240	300	300
19	Friday				20 100	11:50 N	535	114 000		20 000	17 780	300	835
20	Saturday							40 000				150	150
21	Sunday												0
22	Monday								355 060	20 000	20 270	300	300
23	Tuesday							26 000	57 220	20 400	15 770	300	300
24	Wednesday							26 000	50 240	20 000	5 460	300	300
25	Thursday	650 000				14:00 N	535	56 000	55 020	130 000	23 110	300	835
26	Friday	300 000			45 420	12:35/13:45 N	1 035	144 000		20 000		300	1 335
27	Saturday												0
28	Sunday												0
29	Monday								94 400	30 200	8 310	300	300
30	Tuesday							101 000	460	30 000	6 550	300	300
TOTAL							2 640					6 450	9 090

Appendix G

Total daily cost of holding cash

April to June 1998

Explanatory note

Appendix G1 summarises the cost information determined in Appendices E and F for the period April to June 1998. Appendix G2 provides a graph of the total daily cost of holding cash as well as its constituent parts.

Appendix G1:
Total daily cost of holding cash: April 1998

Date	Days (holidays)	Total daily storage cost	Total daily supply cost	Total daily cost of holding cash
31-Mar-98	Tuesday			
1-Apr-98	Wednesday	2 062	300	2 362
2	Thursday	1 944	300	2 244
3	Friday	1 997	1 335	3 332
4	Saturday	2 062	0	2 062
5	Sunday	1 981	0	1 981
6	Monday	2 211	300	2 511
7	Tuesday	2 248	300	2 548
8	Wednesday	2 767	835	3 602
9	Thursday	2 453	300	2 753
10	Good Friday	2 372	0	2 372
11	Saturday	2 353	0	2 353
12	Sunday	2 272	0	2 272
13	Family day	2 272	0	2 272
14	Tuesday	2 585	300	2 885
15	Wednesday	2 565	300	2 865
16	Thursday	2 691	835	3 526
17	Friday	2 557	835	3 392
18	Saturday	2 549	0	2 549
19	Sunday	2 468	0	2 468
20	Monday	2 734	300	3 034
21	Tuesday	2 643	300	2 943
22	Wednesday	2 613	300	2 913
23	Thursday	2 507	300	2 807
24	Friday	2 124	835	2 959
25	Saturday	2 131	0	2 131
26	Sunday	2 050	0	2 050
27	Freedom day	2 050	0	2 050
28	Tuesday	2 172	300	2 472
29	Wednesday	2 515	1 335	3 850
30	Thursday	2 056	300	2 356
TOTAL		70 005	9 910	79 915

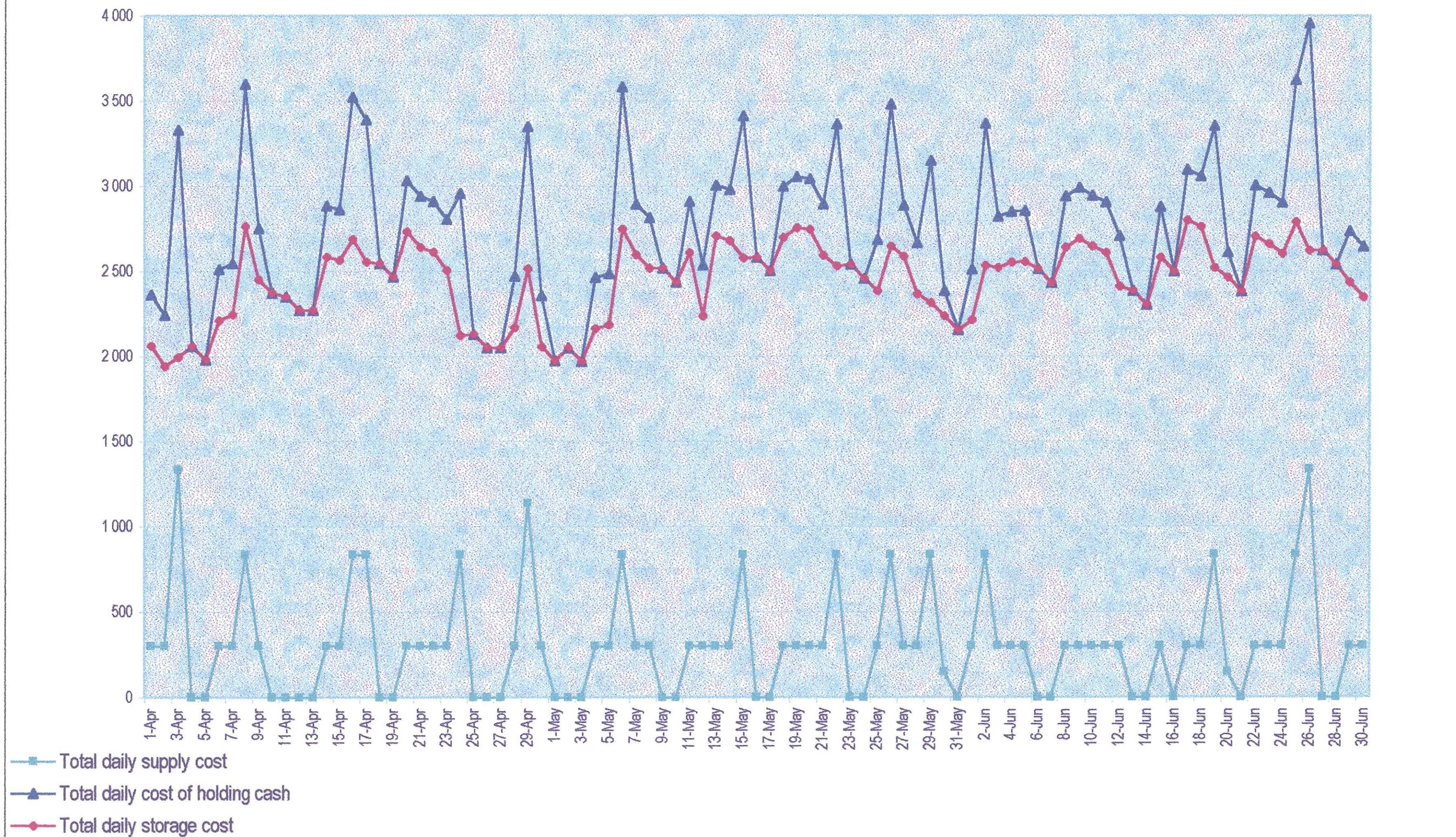
Appendix G1:
Total daily cost of holding cash: May 1998

Date	Days (holidays)	Total daily storage cost	Total daily supply cost	Total daily cost of holding cost
1-May-98	Workers' day	1 975	0	1 975
2	Saturday	2 052	0	2 052
3	Sunday	1 971	0	1 971
4	Monday	2 165	300	2 465
5	Tuesday	2 186	300	2 486
6	Wednesday	2 750	835	3 585
7	Thursday	2 597	300	2 897
8	Friday	2 518	300	2 818
9	Saturday	2 519	0	2 519
10	Sunday	2 438	0	2 438
11	Monday	2 610	300	2 910
12	Tuesday	2 238	300	2 538
13	Wednesday	2 708	300	3 008
14	Thursday	2 681	300	2 981
15	Friday	2 579	835	3 414
16	Saturday	2 583	0	2 583
17	Sunday	2 502	0	2 502
18	Monday	2 699	300	2 999
19	Tuesday	2 757	300	3 057
20	Wednesday	2 745	300	3 045
21	Thursday	2 596	300	2 896
22	Friday	2 531	835	3 366
23	Saturday	2 540	0	2 540
24	Sunday	2 459	0	2 459
25	Monday	2 386	300	2 686
26	Tuesday	2 648	835	3 483
27	Wednesday	2 589	300	2 889
28	Thursday	2 369	300	2 669
29	Friday	2 316	835	3 151
30	Saturday	2 237	150	2 387
31	Sunday	2 156		2 156
TOTAL		74 127	8 825	82 952

Appendix G1:
Total daily cost of holding cash: June 1998

Date	Days (holidays)	Total daily storage cost	Total daily supply cost	Total daily cost of holding cash
1-Jun-98	Monday	2 214	300	2 514
2	Tuesday	2 535	835	3 370
3	Wednesday	2 522	300	2 822
4	Thursday	2 553	300	2 853
5	Friday	2 558	300	2 858
6	Saturday	2 515	0	2 515
7	Sunday	2 434	0	2 434
8	Monday	2 643	300	2 943
9	Tuesday	2 693	300	2 993
10	Wednesday	2 648	300	2 948
11	Thursday	2 610	300	2 910
12	Friday	2 410	300	2 710
13	Saturday	2 386	0	2 386
14	Sunday	2 305	0	2 305
15	Monday	2 581	300	2 881
16	Youth day	2 500	0	2 500
17	Wednesday	2 799	300	3 099
18	Thursday	2 761	300	3 061
19	Friday	2 523	835	3 358
20	Saturday	2 465	150	2 615
21	Sunday	2 384	0	2 384
22	Monday	2 705	300	3 005
23	Tuesday	2 661	300	2 961
24	Wednesday	2 604	300	2 904
25	Thursday	2 791	835	3 626
26	Friday	2 624	1 335	3 959
27	Saturday	2 622	0	2 622
28	Sunday	2 541	0	2 541
29	Monday	2 436	300	2 736
30	Tuesday	2 347	300	2 647
TOTAL		76 373	9 090	85 463

Appendix G2: Total daily cost of holding cash: April to June 1998



Appendix J

An initial search for feasible alternative order policies with initial cash inventory at R1 000 000 and the special order size R500 000

Explanatory notes

Appendix K provides the results of the investigation into alternate values for safety stock. Appendix K-1 investigates a safety stock level of R0, K-2 investigates a safety stock level of R300 000 and K-3 shows the results for a safety stock level of R750 000.

Appendix J-1-1

Summary for Q = 500 000, I0 = 1 000 000, c2=2c1, SS = 200 000, SQ = 500 000

ROP	Q=500 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	500 000	3 008 716	2 686	216	190	133
400 000	500 000	3 014 512	2 692	229	176	130
500 000	500 000	3 019 573	2 696	242	163	117
600 000	500 000	3 036 253	2 711	266	140	99
700 000	500 000	3 054 977	2 728	277	129	83
800 000	500 000	3 063 904	2 736	289	117	76
900 000	500 000	3 103 018	2 771	305	101	68
1 000 000	500 000	3 118 448	2 784	312	94	65
1 100 000	500 000	3 136 257	2 800	334	74	43
1 200 000	500 000	3 173 173	2 833	338	70	40
1 300 000	500 000	3 193 841	2 852	348	60	34
1 400 000	500 000	3 221 381	2 876	357	51	27
1 500 000	500 000	3 247 440	2 899	361	47	25
1 600 000	500 000	3 292 521	2 940	369	40	24
1 700 000	500 000	3 349 821	2 991	374	35	18
1 800 000	500 000	3 372 513	3 011	379	30	16
1 900 000	500 000	3 414 116	3 048	386	23	12
2 000 000	500 000	3 434 207	3 066	388	21	11
2 100 000	500 000	3 481 941	3 109	391	19	11
2 200 000	500 000	3 542 366	3 163	395	15	11
2 300 000	500 000	3 562 994	3 181	396	14	9
2 400 000	500 000	3 601 517	3 216	401	9	8
2 500 000	500 000	3 635 264	3 246	401	9	7

Appendix J-1-2

Summary for Q = 750 000, C2=2C1, I0 = 1 000 000, SS = 200 000, SQ= 500 000

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	3 051 980	2 725	172	149	99
400 000	750 000	3 038 603	2 713	183	133	85
500 000	750 000	3 038 804	2 713	194	115	72
600 000	750 000	3 047 944	2 721	198	109	74
700 000	750 000	3 072 133	2 743	207	96	66
800 000	750 000	3 111 615	2 778	217	81	54
900 000	750 000	3 115 434	2 782	230	62	39
1 000 000	750 000	3 163 752	2 825	234	56	35
1 100 000	750 000	3 194 124	2 852	240	47	31
1 200 000	750 000	3 217 612	2 873	244	41	29
1 300 000	750 000	3 262 387	2 913	251	31	24
1 400 000	750 000	3 281 444	2 930	254	26	16
1 500 000	750 000	3 309 532	2 955	258	20	15
1 600 000	750 000	3 369 157	3 008	262	17	14
1 700 000	750 000	3 422 857	3 056	263	13	10
1 800 000	750 000	3 464 872	3 094	262	16	11
1 900 000	750 000	3 517 281	3 140	264	12	10
2 000 000	750 000	3 540 155	3 161	266	9	7
2 100 000	750 000	3 568 040	3 186	268	8	6
2 200 000	750 000	3 631 236	3 242	271	4	4
2 300 000	750 000	3 670 047	3 277	271	3	4
2 400 000	750 000	3 718 179	3 320	270	4	2
2 500 000	750 000	3 764 880	3 361	272	3	2

Appendix J-1-3

Summary for Q = 1 000 000, C2=2C1, I0 = 1 000 000, SS = 200 000, SQ = 500 000

ROP	Q=1 000 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	1 000 000	3 076 742	2 747	143	122	83
400 000	1 000 000	3 054 835	2 728	148	111	81
500 000	1 000 000	3 083 156	2 753	155	98	72
600 000	1 000 000	3 145 570	2 809	161	86	58
700 000	1 000 000	3 170 890	2 831	167	74	48
800 000	1 000 000	3 178 097	2 838	170	68	47
900 000	1 000 000	3 151 412	2 814	176	56	38
1 000 000	1 000 000	3 209 286	2 865	180	48	30
1 100 000	1 000 000	3 223 494	2 878	187	34	23
1 200 000	1 000 000	3 261 060	2 912	190	28	22
1 300 000	1 000 000	3 315 122	2 960	189	30	20
1 400 000	1 000 000	3 355 373	2 996	194	20	14
1 500 000	1 000 000	3 388 610	3 026	194	20	13
1 600 000	1 000 000	3 426 009	3 059	196	16	10
1 700 000	1 000 000	3 466 132	3 095	199	11	9
1 800 000	1 000 000	3 531 790	3 153	200	9	8
1 900 000	1 000 000	3 565 047	3 183	200	9	7
2 000 000	1 000 000	3 614 263	3 227	202	5	5
2 100 000	1 000 000	3 657 050	3 265	202	6	6
2 200 000	1 000 000	3 712 973	3 315	202	4	4
2 300 000	1 000 000	3 747 657	3 346	202	5	3
2 400 000	1 000 000	3 784 274	3 379	204	3	2
2 500 000	1 000 000	3 833 074	3 422	204	3	3

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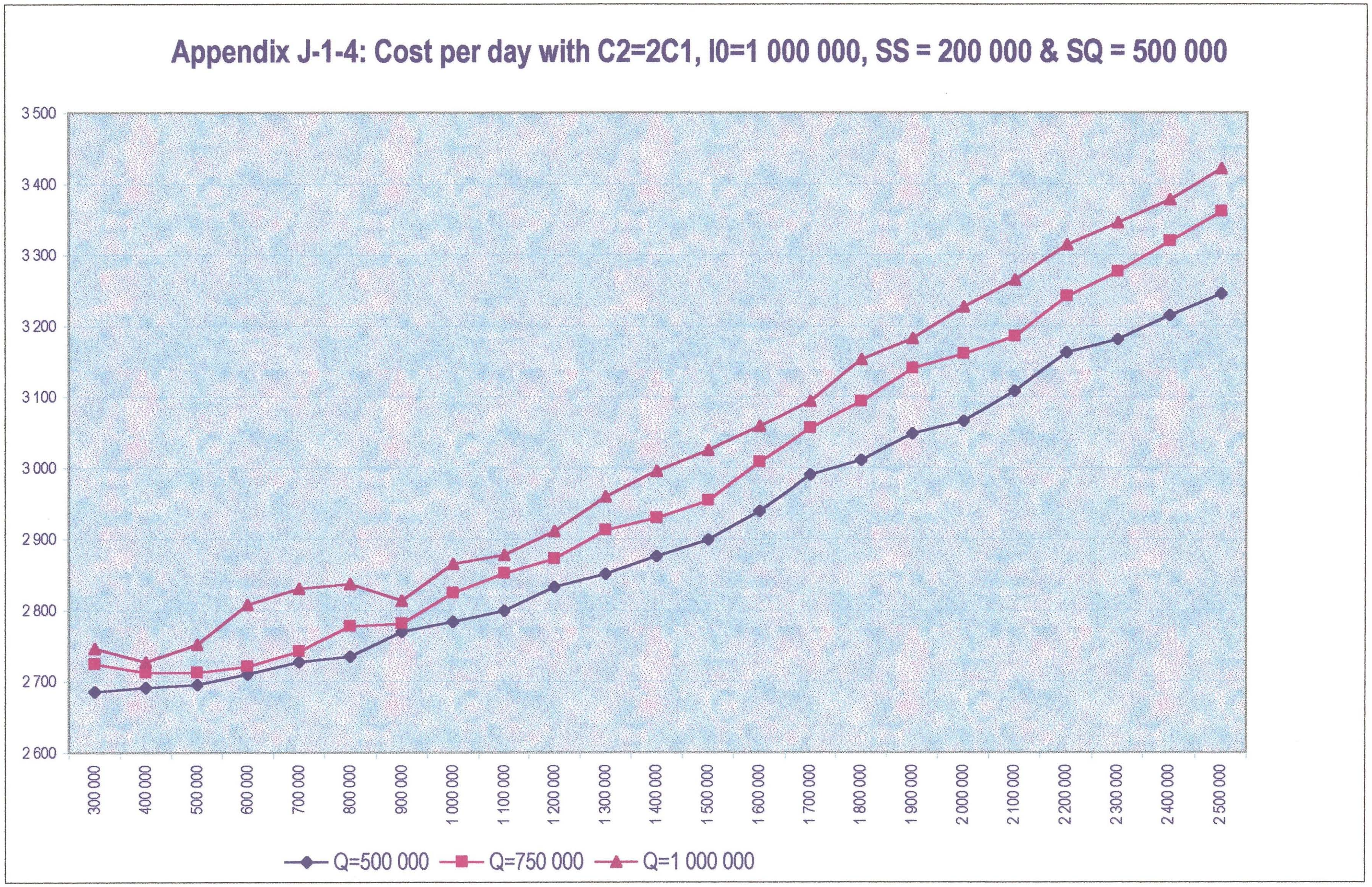


Appendix J-2-1

Summary for Q = 500 000, I0 = 1 000 000, c2=2c1, SS = 500 000, SQ = 500 000

ROP	Q=500 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	500 000	3 058 046	2 730	170	235	99
400 000	500 000	3 065 479	2 737	187	218	99
500 000	500 000	3 100 958	2 769	203	203	89
600 000	500 000	3 133 872	2 798	220	186	70
700 000	500 000	3 135 663	2 800	234	172	67
800 000	500 000	3 122 184	2 788	248	158	63
900 000	500 000	3 159 255	2 821	260	146	53
1 000 000	500 000	3 167 550	2 828	269	137	49
1 100 000	500 000	3 194 632	2 852	289	119	34
1 200 000	500 000	3 227 990	2 882	304	103	24
1 300 000	500 000	3 243 037	2 896	316	91	23
1 400 000	500 000	3 265 106	2 915	327	80	20
1 500 000	500 000	3 286 123	2 934	334	73	18
1 600 000	500 000	3 317 264	2 962	352	57	16
1 700 000	500 000	3 370 197	3 009	361	48	13
1 800 000	500 000	3 391 743	3 028	366	43	13
1 900 000	500 000	3 428 470	3 061	376	33	10
2 000 000	500 000	3 449 961	3 080	379	30	11
2 100 000	500 000	3 496 643	3 122	383	27	10
2 200 000	500 000	3 548 900	3 169	390	20	11
2 300 000	500 000	3 571 439	3 189	391	19	9
2 400 000	500 000	3 616 257	3 229	397	13	8
2 500 000	500 000	3 645 814	3 255	397	13	6

Appendix J-2-2

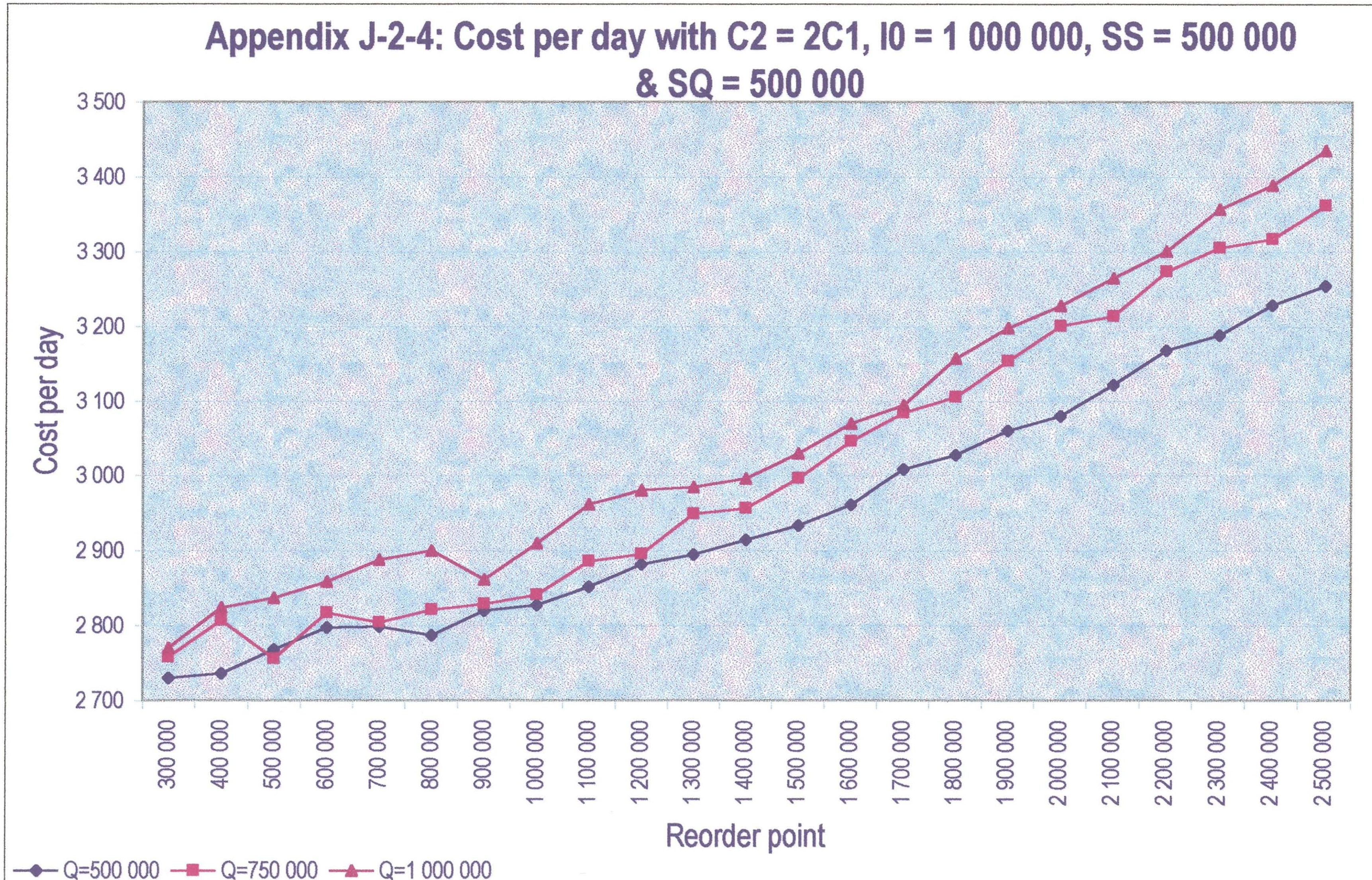
Summary for Q = 750 000, I0 = 1 000 000, c2=2c1, SS = 500 000, SQ = 500 000

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	3 089 622	2 759	144	191	76
400 000	750 000	3 144 413	2 808	154	175	68
500 000	750 000	3 087 607	2 757	163	163	54
600 000	750 000	3 155 971	2 818	172	150	53
700 000	750 000	3 141 336	2 805	184	132	44
800 000	750 000	3 160 408	2 822	199	108	34
900 000	750 000	3 168 780	2 829	202	105	32
1 000 000	750 000	3 182 511	2 842	210	92	39
1 100 000	750 000	3 232 653	2 886	216	84	27
1 200 000	750 000	3 243 910	2 896	224	72	19
1 300 000	750 000	3 303 967	2 950	239	49	15
1 400 000	750 000	3 312 343	2 957	238	51	11
1 500 000	750 000	3 356 151	2 997	246	39	12
1 600 000	750 000	3 412 368	3 047	252	30	14
1 700 000	750 000	3 454 826	3 085	252	30	12
1 800 000	750 000	3 478 281	3 106	256	25	10
1 900 000	750 000	3 532 571	3 154	260	18	5
2 000 000	750 000	3 584 892	3 201	262	15	5
2 100 000	750 000	3 600 006	3 214	264	15	6
2 200 000	750 000	3 666 758	3 274	265	11	5
2 300 000	750 000	3 702 327	3 306	266	11	5
2 400 000	750 000	3 715 304	3 317	266	10	3
2 500 000	750 000	3 765 874	3 362	268	9	3

Appendix J-2-3

Summary for Q = 1 000 000, I0 = 1 000 000, c2=2c1, SS = 500 000, SQ = 500 000

ROP	Q=1 000 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	1 000 000	3 103 577	2 771	119	169	62
400 000	1 000 000	3 163 716	2 825	129	149	57
500 000	1 000 000	3 177 996	2 837	136	136	56
600 000	1 000 000	3 202 755	2 860	147	114	46
700 000	1 000 000	3 235 335	2 889	153	103	37
800 000	1 000 000	3 248 579	2 901	155	99	33
900 000	1 000 000	3 205 545	2 862	166	76	30
1 000 000	1 000 000	3 259 419	2 910	167	74	22
1 100 000	1 000 000	3 318 468	2 963	170	68	20
1 200 000	1 000 000	3 339 095	2 981	178	53	14
1 300 000	1 000 000	3 343 457	2 985	182	45	16
1 400 000	1 000 000	3 356 787	2 997	184	41	13
1 500 000	1 000 000	3 393 699	3 030	187	35	11
1 600 000	1 000 000	3 438 671	3 070	190	29	12
1 700 000	1 000 000	3 466 357	3 095	193	23	9
1 800 000	1 000 000	3 536 363	3 157	194	21	8
1 900 000	1 000 000	3 581 967	3 198	197	15	7
2 000 000	1 000 000	3 615 519	3 228	198	13	5
2 100 000	1 000 000	3 657 148	3 265	200	10	6
2 200 000	1 000 000	3 697 570	3 301	201	8	4
2 300 000	1 000 000	3 760 843	3 358	201	8	3
2 400 000	1 000 000	3 796 337	3 390	201	8	2
2 500 000	1 000 000	3 848 473	3 436	203	4	2


Appendix J-2-4: Cost per day with C2 = 2C1, I0 = 1 000 000, SS = 500 000 & SQ = 500 000
Cost per day
3 500
3 400
3 300
3 200
3 100
3 000
2 900
2 800
2 700
300 000
400 000
500 000
600 000
700 000
800 000
900 000
1 000 000
1 100 000
1 200 000
1 300 000
1 400 000
1 500 000
1 600 000
1 700 000
1 800 000
1 900 000
2 000 000
2 100 000
2 200 000
2 300 000
2 400 000
2 500 000
Reorder point
Q=500 000
Q=750 000
Q=1 000 000

J9

Appendix J-3-1

Summary for Q = 500 000, I0 = 1 000 000, c2=2c1, SS = 1 000 000, SQ = 500 000

ROP	Q=500 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	500 000	3 195 393	2 853	94	312	55
400 000	500 000	3 205 318	2 862	110	296	55
500 000	500 000	3 209 718	2 866	124	282	51
600 000	500 000	3 217 842	2 873	141	265	41
700 000	500 000	3 232 093	2 886	159	247	34
800 000	500 000	3 254 084	2 905	170	236	32
900 000	500 000	3 262 650	2 913	188	218	34
1 000 000	500 000	3 281 940	2 930	203	203	26
1 100 000	500 000	3 340 780	2 983	221	186	22
1 200 000	500 000	3 346 690	2 988	235	172	19
1 300 000	500 000	3 335 376	2 978	249	158	20
1 400 000	500 000	3 376 375	3 015	261	146	15
1 500 000	500 000	3 386 738	3 024	270	137	13
1 600 000	500 000	3 419 107	3 053	290	119	10
1 700 000	500 000	3 454 736	3 085	305	103	10
1 800 000	500 000	3 470 266	3 098	317	91	9
1 900 000	500 000	3 492 506	3 118	329	79	9
2 000 000	500 000	3 515 456	3 139	336	72	7
2 100 000	500 000	3 546 746	3 167	354	56	7
2 200 000	500 000	3 601 061	3 215	363	47	7
2 300 000	500 000	3 622 147	3 234	367	43	8
2 400 000	500 000	3 662 025	3 270	377	33	5
2 500 000	500 000	3 682 342	3 288	380	30	6

Appendix J-3-2

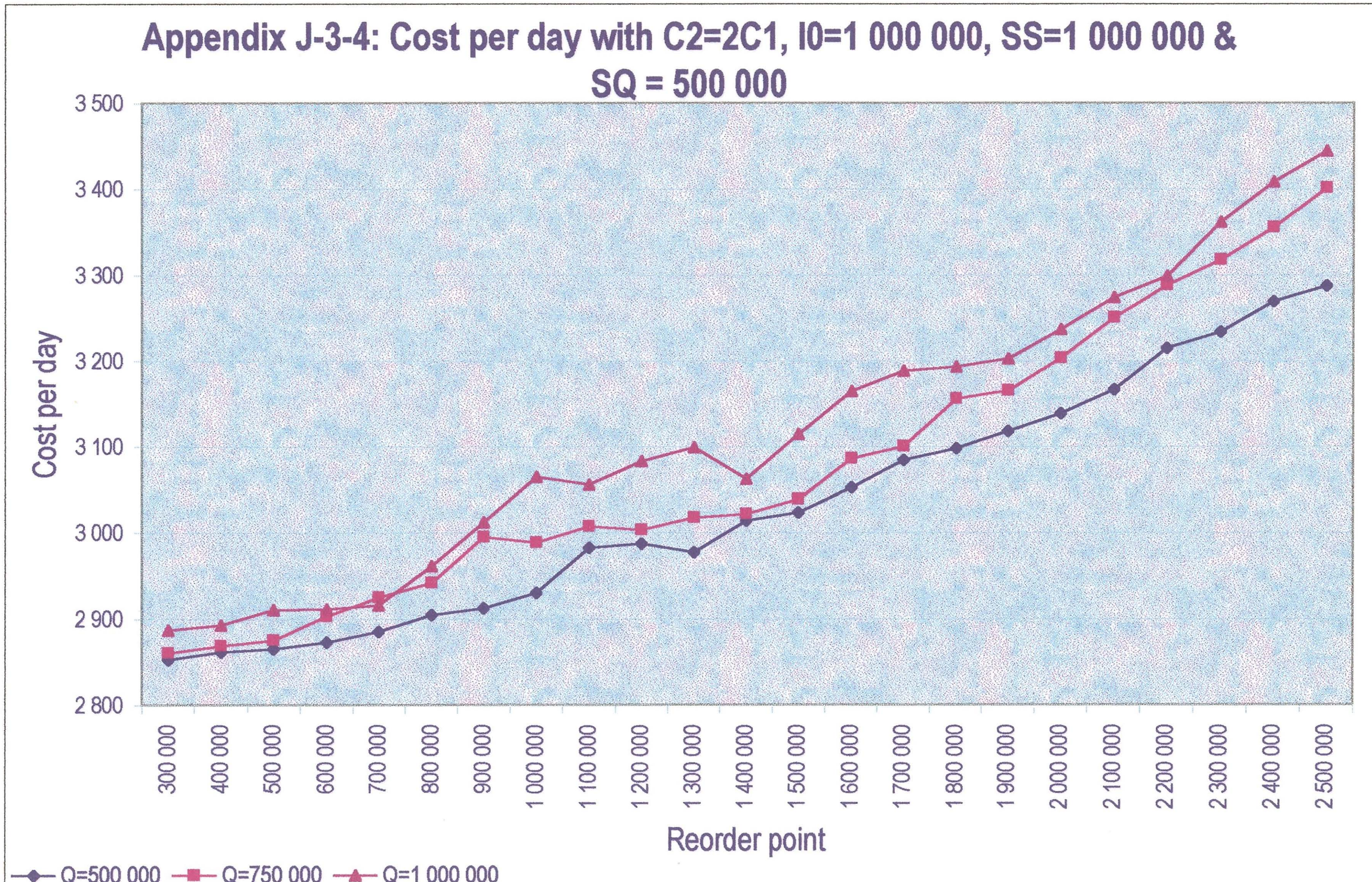
Summary for Q = 750 000, I0 = 1 000 000, c2=2c1, SS = 1 000 000, SQ = 500 000

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	3 204 112	2 861	87	277	36
400 000	750 000	3 213 311	2 869	95	265	43
500 000	750 000	3 220 833	2 876	104	251	41
600 000	750 000	3 251 876	2 903	116	233	34
700 000	750 000	3 276 080	2 925	131	211	25
800 000	750 000	3 295 452	2 942	144	192	18
900 000	750 000	3 355 280	2 996	154	176	22
1 000 000	750 000	3 347 979	2 989	163	163	19
1 100 000	750 000	3 368 601	3 008	174	148	19
1 200 000	750 000	3 364 202	3 004	184	133	16
1 300 000	750 000	3 379 808	3 018	201	106	10
1 400 000	750 000	3 384 491	3 022	204	103	10
1 500 000	750 000	3 404 048	3 039	212	90	8
1 600 000	750 000	3 456 874	3 086	216	85	9
1 700 000	750 000	3 472 645	3 101	226	70	6
1 800 000	750 000	3 534 780	3 156	239	50	5
1 900 000	750 000	3 545 267	3 165	238	52	3
2 000 000	750 000	3 587 661	3 203	246	40	3
2 100 000	750 000	3 641 051	3 251	252	31	4
2 200 000	750 000	3 683 402	3 289	252	31	4
2 300 000	750 000	3 716 323	3 318	256	26	3
2 400 000	750 000	3 758 603	3 356	260	19	2
2 500 000	750 000	3 809 862	3 402	262	16	2

Appendix J-3-3

Summary for Q = 1 000 000, I0 = 1 000 000, c2=2c1, SS = 1 000 000, SQ = 500 000

ROP	Q=1 000 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	1 000 000	3 234 136	2 888	70	268	42
400 000	1 000 000	3 240 322	2 893	78	252	39
500 000	1 000 000	3 260 512	2 911	85	238	35
600 000	1 000 000	3 260 980	2 912	99	210	23
700 000	1 000 000	3 266 524	2 917	109	188	23
800 000	1 000 000	3 317 556	2 962	120	168	18
900 000	1 000 000	3 374 063	3 013	129	150	13
1 000 000	1 000 000	3 433 308	3 065	136	136	15
1 100 000	1 000 000	3 423 426	3 057	147	115	12
1 200 000	1 000 000	3 453 344	3 083	153	104	14
1 300 000	1 000 000	3 471 519	3 100	156	98	8
1 400 000	1 000 000	3 430 278	3 063	167	75	7
1 500 000	1 000 000	3 488 546	3 115	168	73	6
1 600 000	1 000 000	3 544 945	3 165	171	67	9
1 700 000	1 000 000	3 571 639	3 189	179	52	4
1 800 000	1 000 000	3 576 362	3 193	183	44	5
1 900 000	1 000 000	3 587 378	3 203	185	40	3
2 000 000	1 000 000	3 625 344	3 237	188	34	5
2 100 000	1 000 000	3 667 417	3 274	191	28	4
2 200 000	1 000 000	3 695 163	3 299	194	22	4
2 300 000	1 000 000	3 765 806	3 362	195	20	3
2 400 000	1 000 000	3 818 021	3 409	197	16	1
2 500 000	1 000 000	3 858 478	3 445	199	12	1


Appendix J-3-4: Cost per day with C2=2C1, I0=1 000 000, SS=1 000 000 & SQ = 500 000
Cost per day
3 500
3 400
3 300
3 200
3 100
3 000
2 900
2 800
300 000
400 000
500 000
600 000
700 000
800 000
900 000
1 000 000
1 100 000
1 200 000
1 300 000
1 400 000
1 500 000
1 600 000
1 700 000
1 800 000
1 900 000
2 000 000
2 100 000
2 200 000
2 300 000
2 400 000
2 500 000
Reorder point
Q=500 000
Q=750 000
Q=1 000 000

Appendix J-4-1

Summary for Q = 500 000, I0 = 1 000 000, c2=10c1, SS = 200 000, SQ = 500 000

ROP	Q=500 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	500 000	3 266 805	2 917	216	190	133
400 000	500 000	3 233 979	2 887	229	176	130
500 000	500 000	3 217 776	2 873	242	163	117
600 000	500 000	3 192 105	2 850	266	140	99
700 000	500 000	3 176 905	2 837	277	129	83
800 000	500 000	3 171 279	2 831	289	117	76
900 000	500 000	3 206 936	2 863	305	101	68
1 000 000	500 000	3 221 886	2 877	312	94	65
1 100 000	500 000	3 213 611	2 869	334	74	43
1 200 000	500 000	3 242 193	2 895	338	70	40
1 300 000	500 000	3 250 909	2 903	348	60	34
1 400 000	500 000	3 271 595	2 921	357	51	27
1 500 000	500 000	3 291 511	2 939	361	47	25
1 600 000	500 000	3 336 750	2 979	369	40	24
1 700 000	500 000	3 389 318	3 026	374	35	18
1 800 000	500 000	3 408 978	3 044	379	30	16
1 900 000	500 000	3 435 532	3 067	386	23	12
2 000 000	500 000	3 457 510	3 087	388	21	11
2 100 000	500 000	3 503 545	3 128	391	19	11
2 200 000	500 000	3 563 969	3 182	395	15	11
2 300 000	500 000	3 580 922	3 197	396	14	9
2 400 000	500 000	3 613 332	3 226	401	9	8
2 500 000	500 000	3 647 609	3 257	401	9	7

Appendix J-4-2

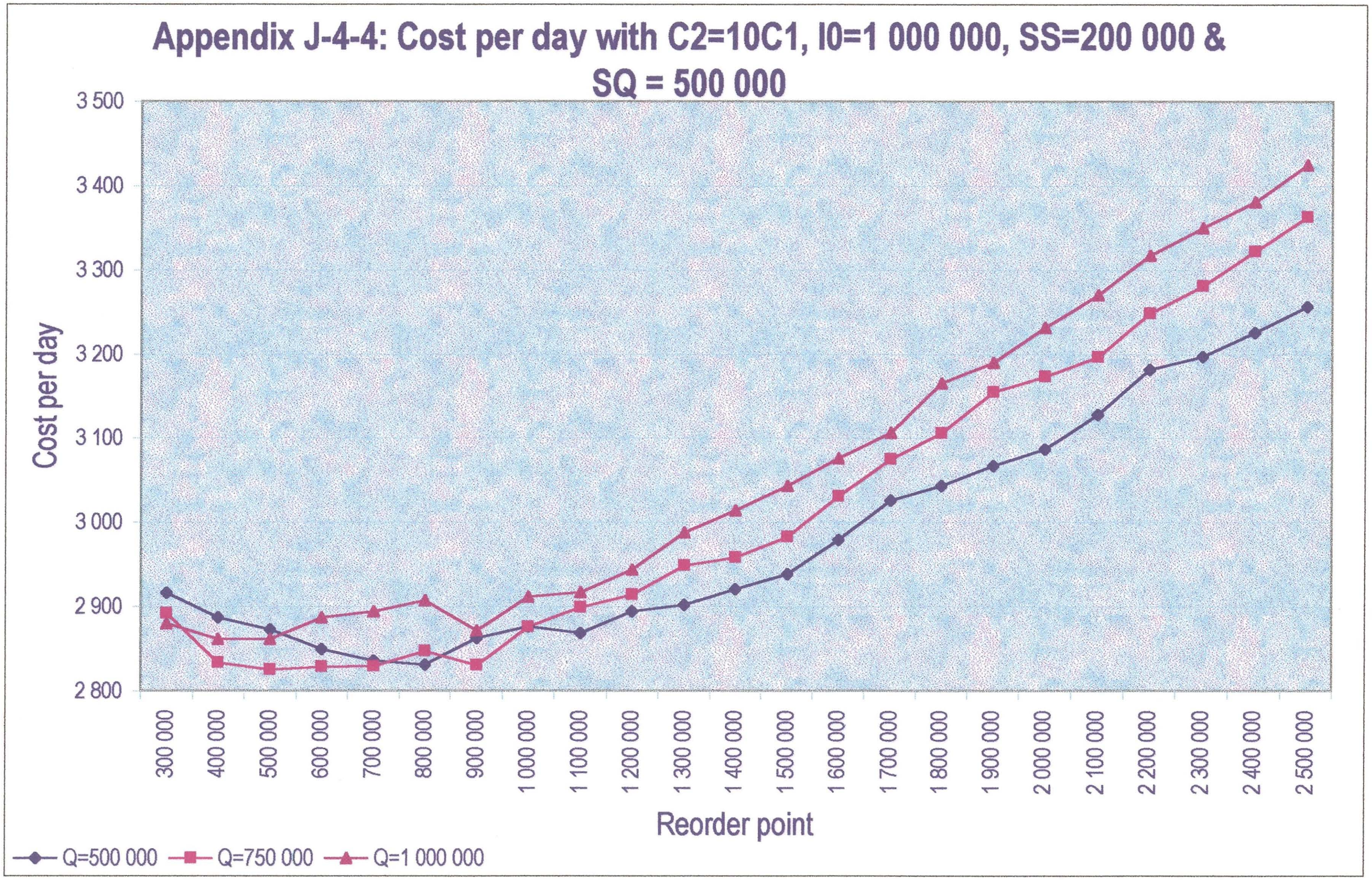
Summary for Q = 750 000, I0 = 1 000 000, c2=10c1, SS = 200 000, SQ = 500 000

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	3 239 567	2 892	172	149	99
400 000	750 000	3 174 254	2 834	183	133	85
500 000	750 000	3 164 530	2 825	194	115	72
600 000	750 000	3 168 976	2 829	198	109	74
700 000	750 000	3 169 449	2 830	207	96	66
800 000	750 000	3 189 324	2 848	217	81	54
900 000	750 000	3 170 689	2 831	230	62	39
1 000 000	750 000	3 221 405	2 876	234	56	35
1 100 000	750 000	3 247 697	2 900	240	47	31
1 200 000	750 000	3 264 272	2 915	244	41	29
1 300 000	750 000	3 302 809	2 949	251	31	24
1 400 000	750 000	3 313 692	2 959	254	26	16
1 500 000	750 000	3 340 619	2 983	258	20	15
1 600 000	750 000	3 394 788	3 031	262	17	14
1 700 000	750 000	3 444 461	3 075	263	13	10
1 800 000	750 000	3 478 986	3 106	262	16	11
1 900 000	750 000	3 533 823	3 155	264	12	10
2 000 000	750 000	3 554 609	3 174	266	9	7
2 100 000	750 000	3 580 323	3 197	268	8	6
2 200 000	750 000	3 638 746	3 249	271	4	4
2 300 000	750 000	3 675 172	3 281	271	4	3
2 400 000	750 000	3 721 438	3 323	270	4	2
2 500 000	750 000	3 766 883	3 363	272	3	2

Appendix J-4-3

Summary for Q = 1 000 000, I0 = 1 000 000, c2=10c1, SS = 200 000, SQ = 500 000

ROP	Q=1 000 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	1 000 000	3 225 996	2 880	143	122	83
400 000	1 000 000	3 205 702	2 862	148	111	81
500 000	1 000 000	3 205 212	2 862	155	98	72
600 000	1 000 000	3 234 403	2 888	161	86	58
700 000	1 000 000	3 241 956	2 895	167	74	48
800 000	1 000 000	3 256 882	2 908	170	68	47
900 000	1 000 000	3 217 005	2 872	176	56	38
1 000 000	1 000 000	3 261 563	2 912	180	48	30
1 100 000	1 000 000	3 267 573	2 917	187	34	23
1 200 000	1 000 000	3 297 559	2 944	190	28	22
1 300 000	1 000 000	3 346 745	2 988	189	30	20
1 400 000	1 000 000	3 376 074	3 014	194	20	14
1 500 000	1 000 000	3 409 048	3 044	194	20	13
1 600 000	1 000 000	3 445 652	3 076	196	16	10
1 700 000	1 000 000	3 479 423	3 107	199	11	9
1 800 000	1 000 000	3 545 907	3 166	200	9	8
1 900 000	1 000 000	3 573 293	3 190	200	9	7
2 000 000	1 000 000	3 619 818	3 232	202	5	5
2 100 000	1 000 000	3 662 967	3 271	202	6	6
2 200 000	1 000 000	3 715 836	3 318	203	4	4
2 300 000	1 000 000	3 752 782	3 351	202	5	3
2 400 000	1 000 000	3 787 230	3 381	204	3	2
2 500 000	1 000 000	3 836 501	3 425	204	3	3


Appendix J-4-4: Cost per day with C2=10C1, I0=1 000 000, SS=200 000 & SQ = 500 000
Cost per day
3 500
3 400
3 300
3 200
3 100
3 000
2 900
2 800
300 000
400 000
500 000
600 000
700 000
800 000
900 000
1 000 000
1 100 000
1 200 000
1 300 000
1 400 000
1 500 000
1 600 000
1 700 000
1 800 000
1 900 000
2 000 000
2 100 000
2 200 000
2 300 000
2 400 000
2 500 000
Reorder point
Q=500 000
Q=750 000
Q=1 000 000

Appendix J-5-1

Summary for Q = 500 000, I0 = 1 000 000, c2=10c1, SS = 500 000, SQ = 500 000

ROP	Q=500 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	500 000	3 216 981	2 872	170	235	99
400 000	500 000	3 236 893	2 890	187	218	99
500 000	500 000	3 250 232	2 902	203	203	89
600 000	500 000	3 235 610	2 889	220	186	70
700 000	500 000	3 231 966	2 886	234	172	67
800 000	500 000	3 215 931	2 871	248	158	63
900 000	500 000	3 234 234	2 888	260	146	53
1 000 000	500 000	3 234 183	2 888	269	137	49
1 100 000	500 000	3 241 387	2 894	289	119	34
1 200 000	500 000	3 268 688	2 918	304	103	24
1 300 000	500 000	3 281 541	2 930	316	91	23
1 400 000	500 000	3 302 391	2 949	327	80	20
1 500 000	500 000	3 315 744	2 960	334	73	18
1 600 000	500 000	3 346 489	2 988	352	57	16
1 700 000	500 000	3 395 736	3 032	361	48	13
1 800 000	500 000	3 421 387	3 055	366	43	13
1 900 000	500 000	3 446 518	3 077	376	33	10
2 000 000	500 000	3 473 263	3 101	379	30	11
2 100 000	500 000	3 517 907	3 141	383	27	10
2 200 000	500 000	3 570 504	3 188	390	20	11
2 300 000	500 000	3 589 367	3 205	391	19	9
2 400 000	500 000	3 628 072	3 239	397	13	8
2 500 000	500 000	3 657 748	3 266	397	13	6

Appendix J-5-2

Summary for Q = 750 000, I0 = 1 000 000, c2=10c1, SS = 500 000, SQ = 500 000

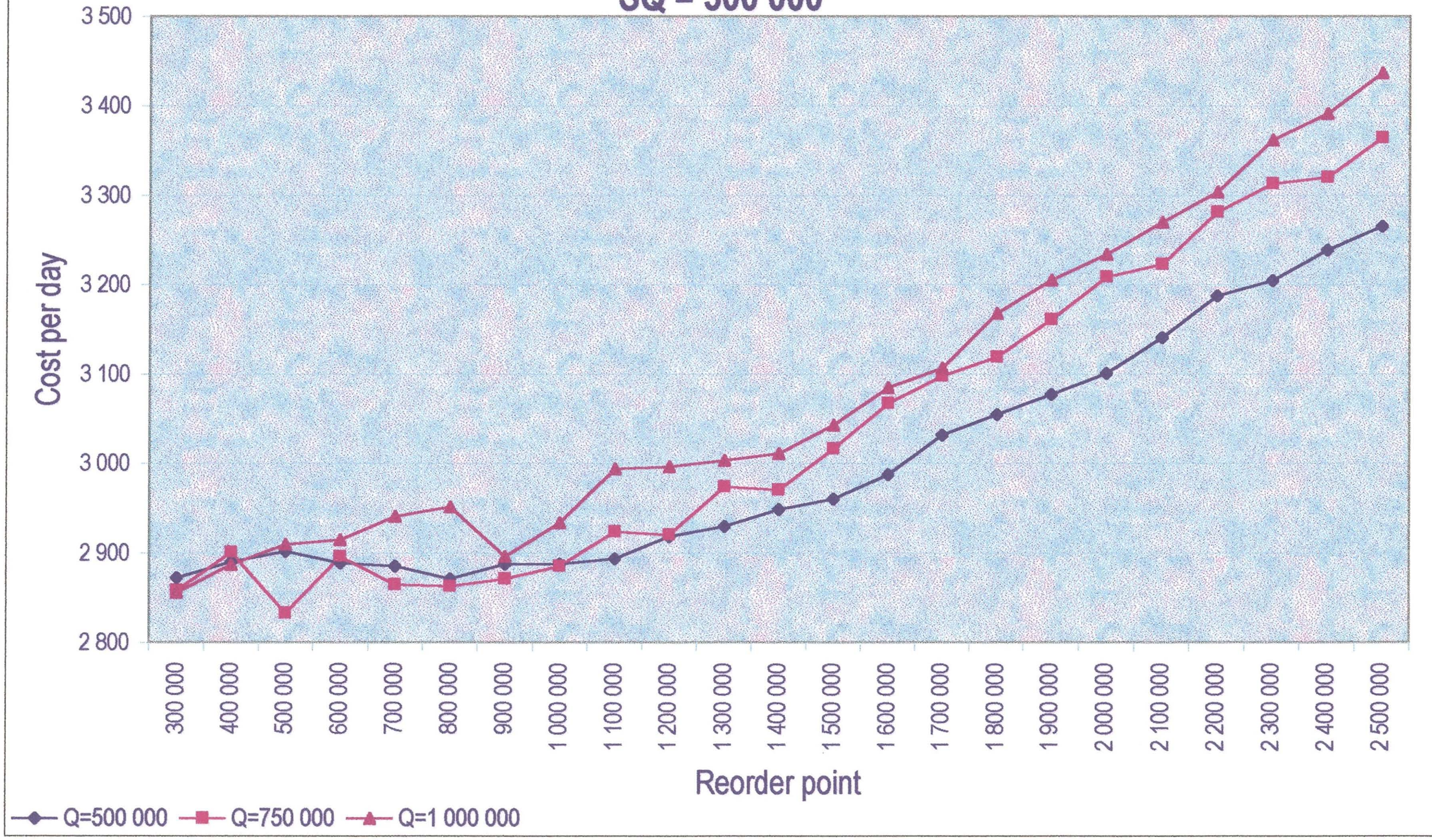
ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	3 201 395	2 858	144	191	76
400 000	750 000	3 249 303	2 901	154	175	68
500 000	750 000	3 172 838	2 833	163	163	54
600 000	750 000	3 243 840	2 896	172	150	53
700 000	750 000	3 208 867	2 865	184	132	44
800 000	750 000	3 206 997	2 863	199	108	34
900 000	750 000	3 216 217	2 872	202	105	32
1 000 000	750 000	3 231 576	2 885	210	92	39
1 100 000	750 000	3 274 660	2 924	216	84	27
1 200 000	750 000	3 270 546	2 920	224	72	19
1 300 000	750 000	3 331 264	2 974	239	49	15
1 400 000	750 000	3 327 308	2 971	238	51	11
1 500 000	750 000	3 378 284	3 016	246	39	12
1 600 000	750 000	3 435 642	3 068	252	30	14
1 700 000	750 000	3 470 152	3 098	252	30	12
1 800 000	750 000	3 493 755	3 119	256	25	10
1 900 000	750 000	3 540 249	3 161	260	18	5
2 000 000	750 000	3 594 269	3 209	262	15	5
2 100 000	750 000	3 609 745	3 223	264	15	6
2 200 000	750 000	3 675 285	3 282	265	11	5
2 300 000	750 000	3 710 855	3 313	266	11	5
2 400 000	750 000	3 718 731	3 320	266	10	3
2 500 000	750 000	3 768 451	3 365	268	9	3

Appendix J-5-3

Summary for Q = 1 000 000, I0 = 1 000 000, c2=10c1, SS = 500 000, SQ = 500 000

ROP	Q=1 000 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	1 000 000	3 198 902	2 856	119	169	62
400 000	1 000 000	3 233 862	2 887	129	149	57
500 000	1 000 000	3 258 890	2 910	136	136	56
600 000	1 000 000	3 265 037	2 915	147	114	46
700 000	1 000 000	3 294 191	2 941	153	130	37
800 000	1 000 000	3 305 641	2 951	155	99	33
900 000	1 000 000	3 243 765	2 896	166	76	30
1 000 000	1 000 000	3 286 289	2 934	167	74	22
1 100 000	1 000 000	3 353 714	2 994	170	68	20
1 200 000	1 000 000	3 356 281	2 997	178	53	14
1 300 000	1 000 000	3 364 301	3 004	182	45	16
1 400 000	1 000 000	3 372 672	3 011	184	41	13
1 500 000	1 000 000	3 408 188	3 043	187	35	11
1 600 000	1 000 000	3 455 135	3 085	190	29	12
1 700 000	1 000 000	3 479 468	3 107	193	23	9
1 800 000	1 000 000	3 548 306	3 168	194	21	8
1 900 000	1 000 000	3 590 213	3 206	197	15	7
2 000 000	1 000 000	3 622 773	3 235	198	13	5
2 100 000	1 000 000	3 663 065	3 271	200	10	6
2 200 000	1 000 000	3 700 433	3 304	201	8	4
2 300 000	1 000 000	3 765 969	3 362	201	8	3
2 400 000	1 000 000	3 799 293	3 392	201	8	2
2 500 000	1 000 000	3 849 730	3 437	203	4	2

Appendix J-5-4: Cost per day with C2=10C1, I0=1 000 000, SS=500 000 & SQ = 500 000



Appendix J-6-1

Summary for Q = 500 000, I0 = 1 000 000, c2=10c1, SS = 1 000 000, SQ = 500 000

ROP	Q=500 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	500 000	3 280 165	2 929	94	312	55
400 000	500 000	3 286 364	2 934	110	296	55
500 000	500 000	3 278 132	2 927	124	282	51
600 000	500 000	3 274 531	2 924	141	265	41
700 000	500 000	3 279 444	2 928	159	247	34
800 000	500 000	3 294 342	2 941	170	236	32
900 000	500 000	3 310 681	2 956	188	218	34
1 000 000	500 000	3 310 252	2 956	203	203	26
1 100 000	500 000	3 359 824	3 000	221	186	22
1 200 000	500 000	3 371 284	3 010	235	172	19
1 300 000	500 000	3 362 055	3 002	249	158	20
1 400 000	500 000	3 394 758	3 031	261	146	15
1 500 000	500 000	3 402 291	3 038	270	137	13
1 600 000	500 000	3 430 579	3 063	290	119	10
1 700 000	500 000	3 466 208	3 095	305	103	10
1 800 000	500 000	3 481 400	3 108	317	91	9
1 900 000	500 000	3 503 640	3 128	329	79	9
2 000 000	500 000	3 524 083	3 147	336	72	7
2 100 000	500 000	3 555 372	3 174	354	56	7
2 200 000	500 000	3 609 687	3 223	363	47	7
2 300 000	500 000	3 632 884	3 244	367	43	8
2 400 000	500 000	3 669 570	3 276	377	33	5
2 500 000	500 000	3 690 878	3 295	380	30	6

Appendix J-6-2

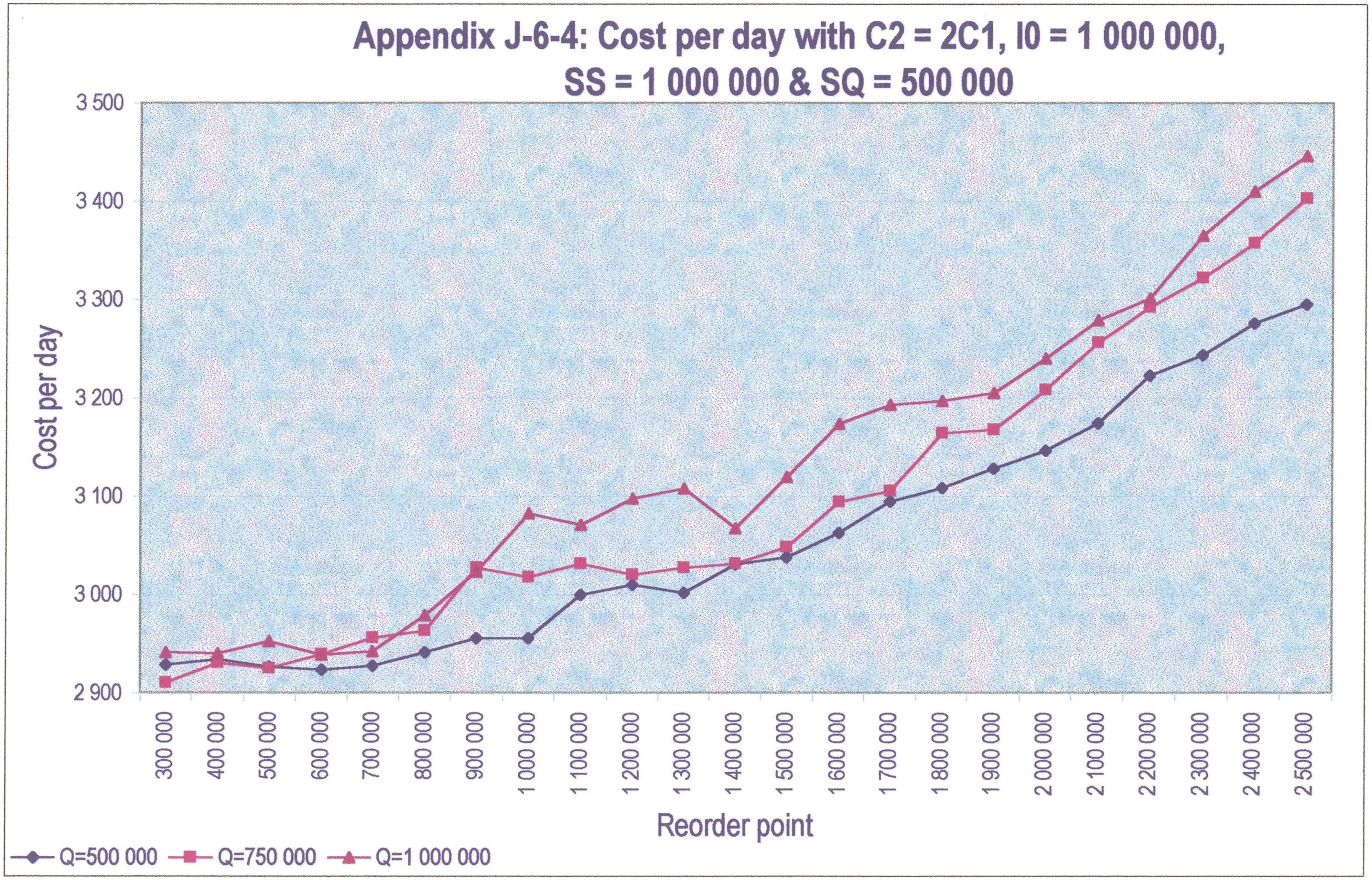
Summary for Q = 750 000, I0 = 1 000 000, c2=10c1, SS = 1 000 000, SQ = 500 000

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	3 260 182	2 911	87	277	36
400 000	750 000	3 282 779	2 931	95	265	43
500 000	750 000	3 276 424	2 925	104	251	41
600 000	750 000	3 292 296	2 940	116	233	34
700 000	750 000	3 310 802	2 956	131	211	25
800 000	750 000	3 318 906	2 963	144	192	18
900 000	750 000	3 390 661	3 027	154	176	22
1 000 000	750 000	3 379 855	3 018	163	163	19
1 100 000	750 000	3 395 110	3 031	174	148	19
1 200 000	750 000	3 382 898	3 020	184	133	16
1 300 000	750 000	3 390 736	3 027	201	106	10
1 400 000	750 000	3 395 201	3 031	204	103	10
1 500 000	750 000	3 413 809	3 048	212	90	8
1 600 000	750 000	3 465 835	3 094	216	85	9
1 700 000	750 000	3 478 320	3 106	226	70	6
1 800 000	750 000	3 544 062	3 164	239	50	5
1 900 000	750 000	3 547 845	3 168	238	52	3
2 000 000	750 000	3 593 636	3 209	246	40	3
2 100 000	750 000	3 647 422	3 257	252	31	4
2 200 000	750 000	3 687 768	3 293	252	31	4
2 300 000	750 000	3 721 448	3 323	256	26	3
2 400 000	750 000	3 760 709	3 358	260	19	2
2 500 000	750 000	3 811 968	3 404	262	16	2

Appendix J-6-3

Summary for Q = 1 000 000, I0 = 1 000 000, c2=10c1, SS = 1 000 000, SQ = 500 000

ROP	Q=1 000 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	1 000 000	3 294 796	2 942	70	268	42
400 000	1 000 000	3 293 604	2 941	78	252	39
500 000	1 000 000	3 306 828	2 953	85	238	35
600 000	1 000 000	3 291 678	2 939	99	210	23
700 000	1 000 000	3 295 607	2 943	109	188	23
800 000	1 000 000	3 336 514	2 979	120	168	18
900 000	1 000 000	3 385 772	3 023	129	150	13
1 000 000	1 000 000	3 452 732	3 083	136	136	15
1 100 000	1 000 000	3 439 517	3 071	147	115	12
1 200 000	1 000 000	3 469 662	3 098	153	104	14
1 300 000	1 000 000	3 480 640	3 108	156	98	8
1 400 000	1 000 000	3 435 530	3 067	167	75	7
1 500 000	1 000 000	3 494 165	3 120	168	73	6
1 600 000	1 000 000	3 554 706	3 174	171	67	9
1 700 000	1 000 000	3 576 123	3 193	179	52	4
1 800 000	1 000 000	3 580 936	3 197	183	44	5
1 900 000	1 000 000	3 589 878	3 205	185	40	3
2 000 000	1 000 000	3 629 257	3 240	188	34	5
2 100 000	1 000 000	3 672 939	3 279	191	28	4
2 200 000	1 000 000	3 698 060	3 302	194	22	4
2 300 000	1 000 000	3 769 233	3 365	195	20	3
2 400 000	1 000 000	3 819 960	3 411	197	16	1
2 500 000	1 000 000	3 860 417	3 447	199	12	1


Appendix J-6-4: Cost per day with C2 = 2C1, I0 = 1 000 000, SS = 1 000 000 & SQ = 500 000
Cost per day
3 500
3 400
3 300
3 200
3 100
3 000
2 900
300 000
400 000
500 000
600 000
700 000
800 000
900 000
1 000 000
1 100 000
1 200 000
1 300 000
1 400 000
1 500 000
1 600 000
1 700 000
1 800 000
1 900 000
2 000 000
2 100 000
2 200 000
2 300 000
2 400 000
2 500 000
Reorder point
Q=500 000
Q=750 000
Q=1 000 000

Appendix J-7-1

Summary for Q = 500 000, I0 = 1 000 000, c2=50c1, SS = 200 000, SQ = 500 000

ROP	Q=500 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	500 000	4 557 253	4 069	216	190	133
400 000	500 000	4 331 311	3 867	229	176	130
500 000	500 000	4 208 790	3 758	242	163	117
600 000	500 000	3 971 361	3 546	266	140	99
700 000	500 000	3 786 544	3 381	277	129	83
800 000	500 000	3 708 158	3 311	289	117	76
900 000	500 000	3 726 527	3 327	305	101	68
1 000 000	500 000	3 739 076	3 338	312	94	65
1 100 000	500 000	3 600 380	3 215	334	74	43
1 200 000	500 000	3 587 297	3 203	338	70	40
1 300 000	500 000	3 536 249	3 157	348	60	34
1 400 000	500 000	3 522 668	3 145	357	51	27
1 500 000	500 000	3 511 864	3 136	361	47	25
1 600 000	500 000	3 557 895	3 177	369	40	24
1 700 000	500 000	3 586 801	3 203	374	35	18
1 800 000	500 000	3 591 301	3 207	379	30	16
1 900 000	500 000	3 542 610	3 163	386	23	12
2 000 000	500 000	3 574 021	3 191	388	21	11
2 100 000	500 000	3 611 563	3 225	391	19	11
2 200 000	500 000	3 671 988	3 279	395	15	11
2 300 000	500 000	3 670 562	3 277	396	14	9
2 400 000	500 000	3 672 409	3 279	401	9	8
2 500 000	500 000	3 709 335	3 312	401	9	7

Appendix J-7-2

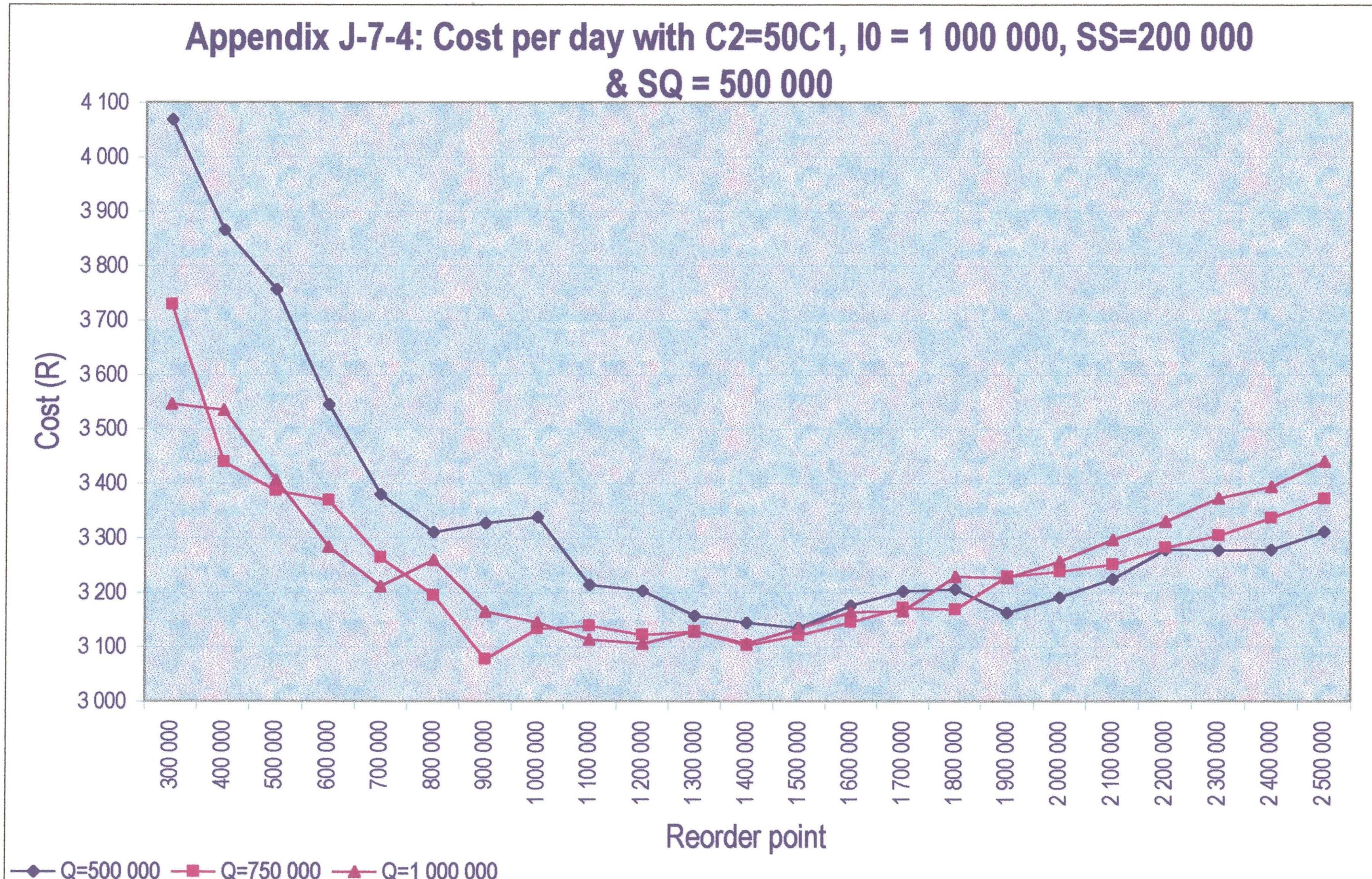
Summary for Q = 750 000, I0 = 1 000 000, c2=50c1, SS = 200 000, SQ = 500 000

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	4 177 500	3 730	172	149	99
400 000	750 000	3 852 508	3 440	183	133	85
500 000	750 000	3 793 158	3 387	194	115	72
600 000	750 000	3 774 138	3 370	198	109	74
700 000	750 000	3 656 031	3 264	207	96	66
800 000	750 000	3 577 866	3 195	217	81	54
900 000	750 000	3 446 967	3 078	230	62	39
1 000 000	750 000	3 509 668	3 134	234	56	35
1 100 000	750 000	3 515 562	3 139	240	47	31
1 200 000	750 000	3 497 568	3 123	244	41	29
1 300 000	750 000	3 504 915	3 129	251	31	24
1 400 000	750 000	3 474 933	3 103	254	26	16
1 500 000	750 000	3 496 058	3 121	258	20	15
1 600 000	750 000	3 522 940	3 145	262	17	14
1 700 000	750 000	3 552 482	3 172	263	13	10
1 800 000	750 000	3 549 555	3 169	262	16	11
1 900 000	750 000	3 616 532	3 229	264	12	10
2 000 000	750 000	3 626 882	3 238	266	9	7
2 100 000	750 000	3 641 737	3 252	268	8	6
2 200 000	750 000	3 676 299	3 282	271	4	4
2 300 000	750 000	3 700 800	3 304	271	4	3
2 400 000	750 000	3 737 736	3 337	270	4	2
2 500 000	750 000	3 776 901	3 372	272	3	2

Appendix J-7-3

Summary for Q = 1 000 000, I0 = 1 000 000, c2=50c1, SS = 200 000, SQ = 500 000

ROP	Q=1 000 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	1 000 000	3 972 270	3 547	143	122	83
400 000	1 000 000	3 960 034	3 536	148	111	81
500 000	1 000 000	3 815 490	3 407	155	98	72
600 000	1 000 000	3 678 566	3 284	161	86	58
700 000	1 000 000	3 597 290	3 212	167	74	48
800 000	1 000 000	3 650 809	3 260	170	68	47
900 000	1 000 000	3 544 971	3 165	176	56	38
1 000 000	1 000 000	3 522 947	3 145	180	48	30
1 100 000	1 000 000	3 487 969	3 114	187	34	23
1 200 000	1 000 000	3 480 052	3 107	190	28	22
1 300 000	1 000 000	3 504 855	3 129	189	30	20
1 400 000	1 000 000	3 479 580	3 107	194	20	14
1 500 000	1 000 000	3 511 240	3 135	194	20	13
1 600 000	1 000 000	3 543 870	3 164	196	16	10
1 700 000	1 000 000	3 545 877	3 166	199	11	9
1 800 000	1 000 000	3 616 493	3 229	200	9	8
1 900 000	1 000 000	3 614 519	3 227	200	9	7
2 000 000	1 000 000	3 647 592	3 257	202	5	5
2 100 000	1 000 000	3 692 554	3 297	202	6	6
2 200 000	1 000 000	3 730 152	3 330	203	4	4
2 300 000	1 000 000	3 778 410	3 374	202	5	3
2 400 000	1 000 000	3 802 006	3 395	204	3	2
2 500 000	1 000 000	3 853 636	3 441	204	3	3


Appendix J-7-4: Cost per day with C2=50C1, I0 = 1 000 000, SS=200 000 & SQ = 500 000
Cost (R)
4 100
4 000
3 900
3 800
3 700
3 600
3 500
3 400
3 300
3 200
3 100
3 000
300 000
400 000
500 000
600 000
700 000
800 000
900 000
1 000 000
1 100 000
1 200 000
1 300 000
1 400 000
1 500 000
1 600 000
1 700 000
1 800 000
1 900 000
2 000 000
2 100 000
2 200 000
2 300 000
2 400 000
2 500 000
Reorder point
Q=500 000
Q=750 000
Q=1 000 000

Appendix J-8-1

Summary for Q = 500 000, I0 = 1 000 000, c2=50c1, SS = 500 000, SQ = 500 000

ROP	Q=500 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	500 000	4 011 654	3 582	170	235	99
400 000	500 000	4 093 962	3 655	187	218	99
500 000	500 000	3 996 601	3 568	203	203	89
600 000	500 000	3 744 303	3 343	220	186	70
700 000	500 000	3 713 484	3 316	234	172	67
800 000	500 000	3 684 667	3 290	248	158	63
900 000	500 000	3 609 128	3 222	260	146	53
1 000 000	500 000	3 567 349	3 185	269	137	49
1 100 000	500 000	3 475 166	3 103	289	119	34
1 200 000	500 000	3 472 180	3 100	304	103	24
1 300 000	500 000	3 474 062	3 102	316	91	23
1 400 000	500 000	3 488 811	3 115	327	80	20
1 500 000	500 000	3 463 846	3 093	334	73	18
1 600 000	500 000	3 492 612	3 118	352	57	16
1 700 000	500 000	3 523 431	3 146	361	48	13
1 800 000	500 000	3 569 609	3 187	366	43	13
1 900 000	500 000	3 536 757	3 158	376	33	10
2 000 000	500 000	3 589 775	3 205	379	30	11
2 100 000	500 000	3 624 232	3 236	383	27	10
2 200 000	500 000	3 678 522	3 284	390	20	11
2 300 000	500 000	3 679 007	3 285	391	19	9
2 400 000	500 000	3 687 149	3 292	397	13	8
2 500 000	500 000	3 717 414	3 319	397	13	6

Appendix J-8-2

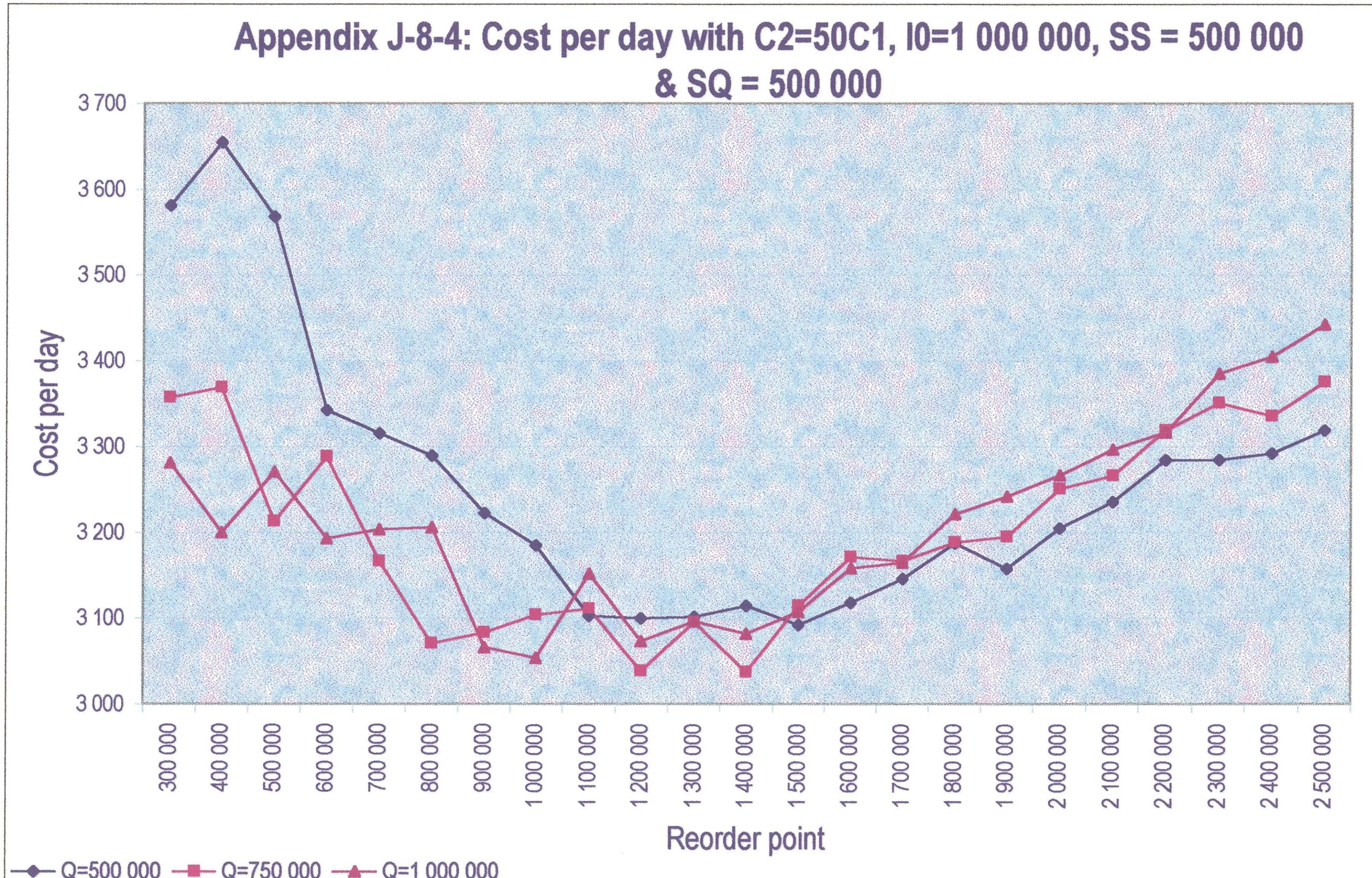
Summary for Q = 750 000, I0 = 1 000 000, c2=50c1, SS = 500 000, SQ = 500 000

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	3 760 259	3 357	144	191	76
400 000	750 000	3 773 750	3 369	154	175	68
500 000	750 000	3 598 989	3 213	163	163	54
600 000	750 000	3 683 184	3 289	172	150	53
700 000	750 000	3 546 524	3 167	184	132	44
800 000	750 000	3 439 943	3 071	199	108	34
900 000	750 000	3 453 403	3 083	202	105	32
1 000 000	750 000	3 476 906	3 104	210	92	39
1 100 000	750 000	3 484 694	3 111	216	84	27
1 200 000	750 000	3 403 725	3 039	224	72	19
1 300 000	750 000	3 467 751	3 096	239	49	15
1 400 000	750 000	3 402 133	3 038	238	51	11
1 500 000	750 000	3 488 946	3 115	246	39	12
1 600 000	750 000	3 552 014	3 171	252	30	14
1 700 000	750 000	3 546 780	3 167	252	30	12
1 800 000	750 000	3 571 123	3 189	256	25	10
1 900 000	750 000	3 578 639	3 195	260	18	5
2 000 000	750 000	3 641 152	3 251	262	15	5
2 100 000	750 000	3 658 440	3 266	264	15	6
2 200 000	750 000	3 717 922	3 320	265	11	5
2 300 000	750 000	3 753 491	3 351	266	11	5
2 400 000	750 000	3 735 866	3 336	266	10	3
2 500 000	750 000	3 781 339	3 376	268	9	3

Appendix J-8-3

Summary for Q = 1 000 000, I0 = 1 000 000, c2=50c1, SS = 500 000, SQ = 500 000

ROP	Q=1 000 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	1 000 000	3 675 529	3 282	119	169	62
400 000	1 000 000	3 584 593	3 201	129	149	57
500 000	1 000 000	3 663 363	3 271	136	136	56
600 000	1 000 000	3 576 451	3 193	147	114	46
700 000	1 000 000	3 588 468	3 204	153	130	37
800 000	1 000 000	3 590 952	3 206	155	99	33
900 000	1 000 000	3 434 865	3 067	166	76	30
1 000 000	1 000 000	3 420 641	3 054	167	74	22
1 100 000	1 000 000	3 529 943	3 152	170	68	20
1 200 000	1 000 000	3 442 213	3 073	178	53	14
1 300 000	1 000 000	3 468 521	3 097	182	45	16
1 400 000	1 000 000	3 452 099	3 082	184	41	13
1 500 000	1 000 000	3 480 629	3 108	187	35	11
1 600 000	1 000 000	3 537 454	3 158	190	29	12
1 700 000	1 000 000	3 545 026	3 165	193	23	9
1 800 000	1 000 000	3 608 020	3 221	194	21	8
1 900 000	1 000 000	3 631 439	3 242	197	15	7
2 000 000	1 000 000	3 659 040	3 267	198	13	5
2 100 000	1 000 000	3 692 651	3 297	200	10	6
2 200 000	1 000 000	3 714 749	3 317	201	8	4
2 300 000	1 000 000	3 791 596	3 385	201	8	3
2 400 000	1 000 000	3 814 070	3 405	201	8	2
2 500 000	1 000 000	3 856 013	3 443	203	4	2


Appendix J-8-4: Cost per day with C2=50C1, I0=1 000 000, SS = 500 000 & SQ = 500 000
Cost per day
3 700
3 600
3 500
3 400
3 300
3 200
3 100
3 000
300 000
400 000
500 000
600 000
700 000
800 000
900 000
1 000 000
1 100 000
1 200 000
1 300 000
1 400 000
1 500 000
1 600 000
1 700 000
1 800 000
1 900 000
2 000 000
2 100 000
2 200 000
2 300 000
2 400 000
2 500 000
Reorder point
Q=500 000
Q=750 000
Q=1 000 000

Appendix J-9-1

Summary for Q = 500 000, I0 = 1 000 000, c2=50c1, SS = 1 000 000, SQ = 500 000

ROP	Q=500 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	500 000	3 704 025	3 307	94	312	55
400 000	500 000	3 691 597	3 296	110	296	55
500 000	500 000	3 620 201	3 232	124	282	51
600 000	500 000	3 557 979	3 177	141	265	41
700 000	500 000	3 516 199	3 139	159	247	34
800 000	500 000	3 495 630	3 121	170	236	32
900 000	500 000	3 550 836	3 170	188	218	34
1 000 000	500 000	3 451 814	3 082	203	203	26
1 100 000	500 000	3 455 046	3 085	221	186	22
1 200 000	500 000	3 494 253	3 120	235	172	19
1 300 000	500 000	3 495 448	3 121	249	158	20
1 400 000	500 000	3 486 675	3 113	261	146	15
1 500 000	500 000	3 480 056	3 107	270	137	13
1 600 000	500 000	3 487 940	3 114	290	119	10
1 700 000	500 000	3 523 570	3 146	305	103	10
1 800 000	500 000	3 537 067	3 158	317	91	9
1 900 000	500 000	3 559 307	3 178	329	79	9
2 000 000	500 000	3 567 215	3 185	336	72	7
2 100 000	500 000	3 598 504	3 213	354	56	7
2 200 000	500 000	3 652 819	3 261	363	47	7
2 300 000	500 000	3 686 569	3 292	367	43	8
2 400 000	500 000	3 707 295	3 310	377	33	5
2 500 000	500 000	3 733 558	3 334	380	30	6

Appendix J-9-2

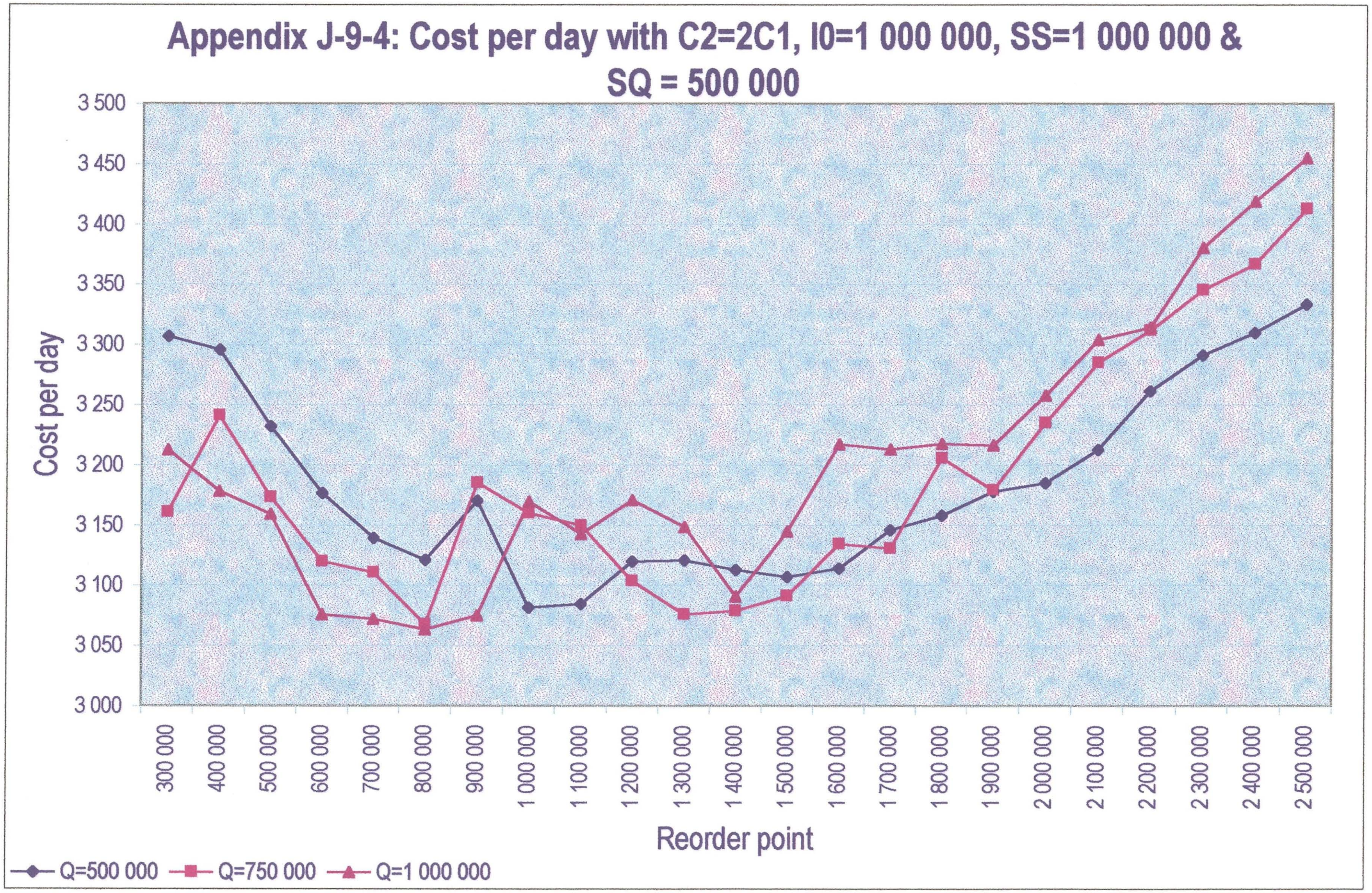
Summary for Q = 750 000, I0 = 1 000 000, c2=50c1, SS = 1 000 000, SQ = 500 000

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	3 540 533	3 161	87	277	36
400 000	750 000	3 630 118	3 241	95	265	43
500 000	750 000	3 554 378	3 174	104	251	41
600 000	750 000	3 494 398	3 120	116	233	34
700 000	750 000	3 484 414	3 111	131	211	25
800 000	750 000	3 436 176	3 068	144	192	18
900 000	750 000	3 567 567	3 185	154	176	22
1 000 000	750 000	3 539 232	3 160	163	163	19
1 100 000	750 000	3 527 656	3 150	174	148	19
1 200 000	750 000	3 476 380	3 104	184	133	16
1 300 000	750 000	3 445 372	3 076	201	106	10
1 400 000	750 000	3 448 755	3 079	204	103	10
1 500 000	750 000	3 462 610	3 092	212	90	8
1 600 000	750 000	3 510 636	3 134	216	85	9
1 700 000	750 000	3 506 694	3 131	226	70	6
1 800 000	750 000	3 590 470	3 206	239	50	5
1 900 000	750 000	3 560 732	3 179	238	52	3
2 000 000	750 000	3 623 510	3 235	246	40	3
2 100 000	750 000	3 679 277	3 285	252	31	4
2 200 000	750 000	3 709 602	3 312	252	31	4
2 300 000	750 000	3 747 076	3 346	256	26	3
2 400 000	750 000	3 771 239	3 367	260	19	2
2 500 000	750 000	3 822 498	3 413	262	16	2

Appendix J-9-3

Summary for Q = 1 000 000, I0 = 1 000 000, c2=50c1, SS = 1 000 000, SQ = 500 000

ROP	Q=1 000 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	1 000 000	3 598 097	3 213	70	268	42
400 000	1 000 000	3 560 016	3 179	78	252	39
500 000	1 000 000	3 538 408	3 159	85	238	35
600 000	1 000 000	3 445 163	3 076	99	210	23
700 000	1 000 000	3 441 024	3 072	109	188	23
800 000	1 000 000	3 431 307	3 064	120	168	18
900 000	1 000 000	3 444 316	3 075	129	150	13
1 000 000	1 000 000	3 549 856	3 170	136	136	15
1 100 000	1 000 000	3 519 970	3 143	147	115	12
1 200 000	1 000 000	3 551 253	3 171	153	104	14
1 300 000	1 000 000	3 526 246	3 148	156	98	8
1 400 000	1 000 000	3 461 791	3 091	167	75	7
1 500 000	1 000 000	3 522 263	3 145	168	73	6
1 600 000	1 000 000	3 603 508	3 217	171	67	9
1 700 000	1 000 000	3 598 541	3 213	179	52	4
1 800 000	1 000 000	3 603 806	3 218	183	44	5
1 900 000	1 000 000	3 602 382	3 216	185	40	3
2 000 000	1 000 000	3 648 825	3 258	188	34	5
2 100 000	1 000 000	3 700 548	3 304	191	28	4
2 200 000	1 000 000	3 712 545	3 315	194	22	4
2 300 000	1 000 000	3 786 368	3 381	195	20	3
2 400 000	1 000 000	3 829 653	3 419	197	16	1
2 500 000	1 000 000	3 870 111	3 455	199	12	1


Appendix J-9-4: Cost per day with C2=2C1, I0=1 000 000, SS=1 000 000 & SQ = 500 000
Cost per day
3 500
3 450
3 400
3 350
3 300
3 250
3 200
3 150
3 100
3 050
3 000
300 000
400 000
500 000
600 000
700 000
800 000
900 000
1 000 000
1 100 000
1 200 000
1 300 000
1 400 000
1 500 000
1 600 000
1 700 000
1 800 000
1 900 000
2 000 000
2 100 000
2 200 000
2 300 000
2 400 000
2 500 000
Reorder point
Q=500 000
Q=750 000
Q=1 000 000

Appendix K

An investigation into further values for safety stock

Explanatory notes

Appendix K provides the results of the investigation into alternate values for safety stock. Appendix K-1 investigates a safety stock level of R0, K-2 investigates a safety stock level of R300 000 and K-3 shows the results for a safety stock level of R750 000.

Appendix K-1

Summary for Q = 750 000, I0 = 1 000 000, c2=10c1, SS = 0, SQ = 500 000

ROP	SS = 0	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	0	3 143 022	2 806	199	107	107
400 000	0	3 146 182	2 809	202	104	104
500 000	0	3 168 736	2 829	210	91	91
600 000	0	3 179 439	2 839	214	86	86
700 000	0	3 139 696	2 803	224	71	71
800 000	0	3 175 178	2 835	237	51	51
900 000	0	3 160 098	2 822	238	50	50
1 000 000	0	3 206 743	2 863	244	41	41
1 100 000	0	3 250 708	2 902	250	32	32
1 200 000	0	3 273 648	2 923	250	32	32
1 300 000	0	3 297 987	2 945	256	24	24
1 400 000	0	3 326 025	2 970	260	17	17
1 500 000	0	3 379 686	3 018	262	14	14
1 600 000	0	3 388 382	3 025	264	14	14
1 700 000	0	3 449 645	3 080	265	10	10
1 800 000	0	3 477 312	3 105	265	11	11
1 900 000	0	3 509 358	3 133	266	9	9
2 000 000	0	3 541 637	3 162	268	8	8
2 100 000	0	3 581 520	3 198	270	4	4
2 200 000	0	3 638 746	3 249	271	4	4
2 300 000	0	3 676 217	3 282	271	4	4
2 400 000	0	3 734 954	3 335	272	2	2
2 500 000	0	3 766 485	3 363	272	2	2

Appendix K-2

Summary for SS = 300 000, Q = 750 000, I0 = 1 000 000, c2=10c1,SQ = 500 000

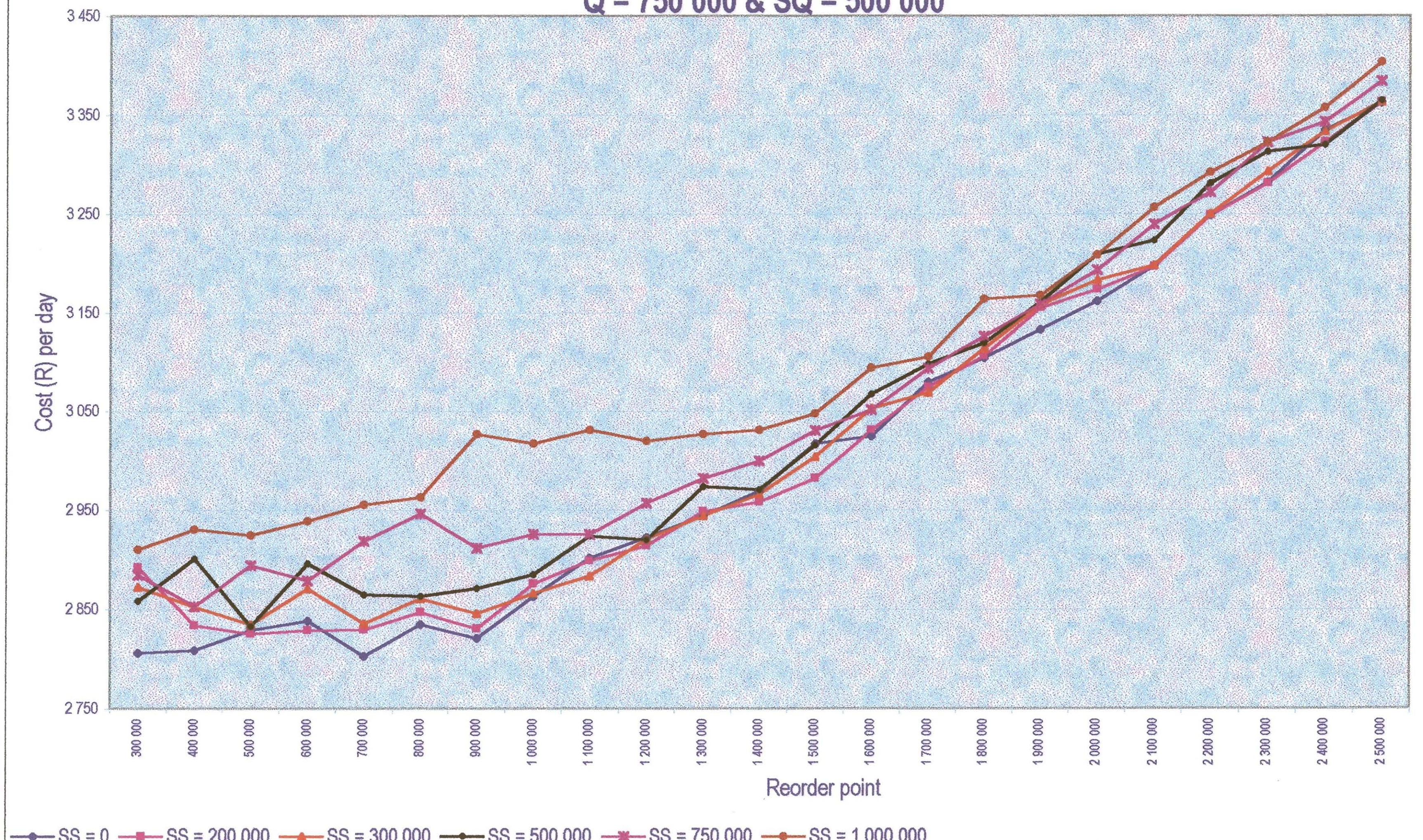
ROP	SS = 300 000	Total cost	Daily cost	No of normal orders	No of special orders	Number of Shortages
300 000	300 000	3 217 804	2 873	163	163	90
400 000	300 000	3 194 955	2 853	172	149	80
500 000	300 000	3 175 546	2 835	182	134	64
600 000	300 000	3 216 036	2 871	190	122	67
700 000	300 000	3 176 604	2 836	205	99	62
800 000	300 000	3 204 483	2 861	213	87	58
900 000	300 000	3 188 166	2 847	220	77	43
1 000 000	300 000	3 210 485	2 867	226	68	32
1 100 000	300 000	3 230 031	2 884	230	62	31
1 200 000	300 000	3 272 068	2 921	239	49	27
1 300 000	300 000	3 298 303	2 945	245	40	20
1 400 000	300 000	3 322 617	2 967	252	29	15
1 500 000	300 000	3 364 936	3 004	254	26	13
1 600 000	300 000	3 420 073	3 054	258	23	12
1 700 000	300 000	3 437 791	3 069	261	16	10
1 800 000	300 000	3 488 053	3 114	261	17	12
1 900 000	300 000	3 537 406	3 158	264	13	10
2 000 000	300 000	3 565 132	3 183	266	10	6
2 100 000	300 000	3 581 755	3 198	266	11	5
2 200 000	300 000	3 640 283	3 250	269	7	6
2 300 000	300 000	3 688 885	3 294	269	7	3
2 400 000	300 000	3 733 806	3 334	270	6	2
2 500 000	300 000	3 767 566	3 364	270	6	2

Appendix K-3

Summary for SS = 750 000, Q = 750 000, I0 = 1 000 000, c2=10c1, SQ = 500 000

ROP	SS = 750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	3 231 581	2 885	111	240	53
400 000	750 000	3 195 393	2 853	124	221	60
500 000	750 000	3 242 309	2 895	140	197	46
600 000	750 000	3 225 130	2 880	146	189	39
700 000	750 000	3 269 843	2 920	158	170	32
800 000	750 000	3 300 303	2 947	166	159	33
900 000	750 000	3 261 609	2 912	178	141	21
1 000 000	750 000	3 277 583	2 926	190	122	22
1 100 000	750 000	3 277 509	2 926	200	107	20
1 200 000	750 000	3 312 481	2 958	209	94	19
1 300 000	750 000	3 340 275	2 982	215	85	15
1 400 000	750 000	3 360 132	3 000	223	74	9
1 500 000	750 000	3 394 888	3 031	232	60	12
1 600 000	750 000	3 418 110	3 052	234	57	7
1 700 000	750 000	3 465 279	3 094	241	47	8
1 800 000	750 000	3 501 395	3 126	245	41	11
1 900 000	750 000	3 537 915	3 159	253	29	5
2 000 000	750 000	3 576 404	3 193	254	27	5
2 100 000	750 000	3 628 376	3 240	259	20	4
2 200 000	750 000	3 664 929	3 272	261	17	4
2 300 000	750 000	3 721 589	3 323	264	14	3
2 400 000	750 000	3 744 653	3 343	264	13	3
2 500 000	750 000	3 790 673	3 385	266	11	2

Appendix K-4: Cost per day with C2=10C1, I0=1 000 000, Q = 750 000 & SQ = 500 000



Appendix L

An investigation into various special order sizes

Explanatory notes

Appendix L provides the results of the investigation into alternate values for the special order size. Appendix L-1 investigates a special order size of R750 000 and L-2 investigates a special order size of R1 000 000.

Appendix L-1-1

Summary for SS = 200 000, Q = 750 000, I0 = 1 000 000, c2=10c1, SQ = 750 000

ROP	SS = 200 000	Total cost	Daily cost	No of normal orders	No of special orders	Number of Shortages
300 000	200 000	3 149 002	2 812	145	127	86
400 000	200 000	3 160 381	2 822	158	144	73
500 000	200 000	3 163 385	2 824	170	102	67
600 000	200 000	3 185 648	2 844	184	88	54
700 000	200 000	3 158 915	2 820	195	77	46
800 000	200 000	3 159 243	2 821	204	68	39
900 000	200 000	3 173 236	2 833	211	61	36
1 000 000	200 000	3 191 630	2 850	217	55	32
1 100 000	200 000	3 213 951	2 870	225	47	27
1 200 000	200 000	3 263 682	2 914	230	42	25
1 300 000	200 000	3 307 757	2 953	241	31	21
1 400 000	200 000	3 343 832	2 986	247	25	17
1 500 000	200 000	3 376 129	3 014	250	22	14
1 600 000	200 000	3 387 165	3 024	254	18	13
1 700 000	200 000	3 428 502	3 061	257	15	11
1 800 000	200 000	3 455 848	3 086	259	13	11
1 900 000	200 000	3 526 535	3 149	262	10	9
2 000 000	200 000	3 582 324	3 199	264	8	7
2 100 000	200 000	3 604 514	3 218	268	6	5
2 200 000	200 000	3 645 838	3 255	268	5	5
2 300 000	200 000	3 694 961	3 299	267	6	5
2 400 000	200 000	3 713 459	3 316	270	4	3
2 500 000	200 000	3 750 859	3 349	271	3	3

Appendix L-1-2

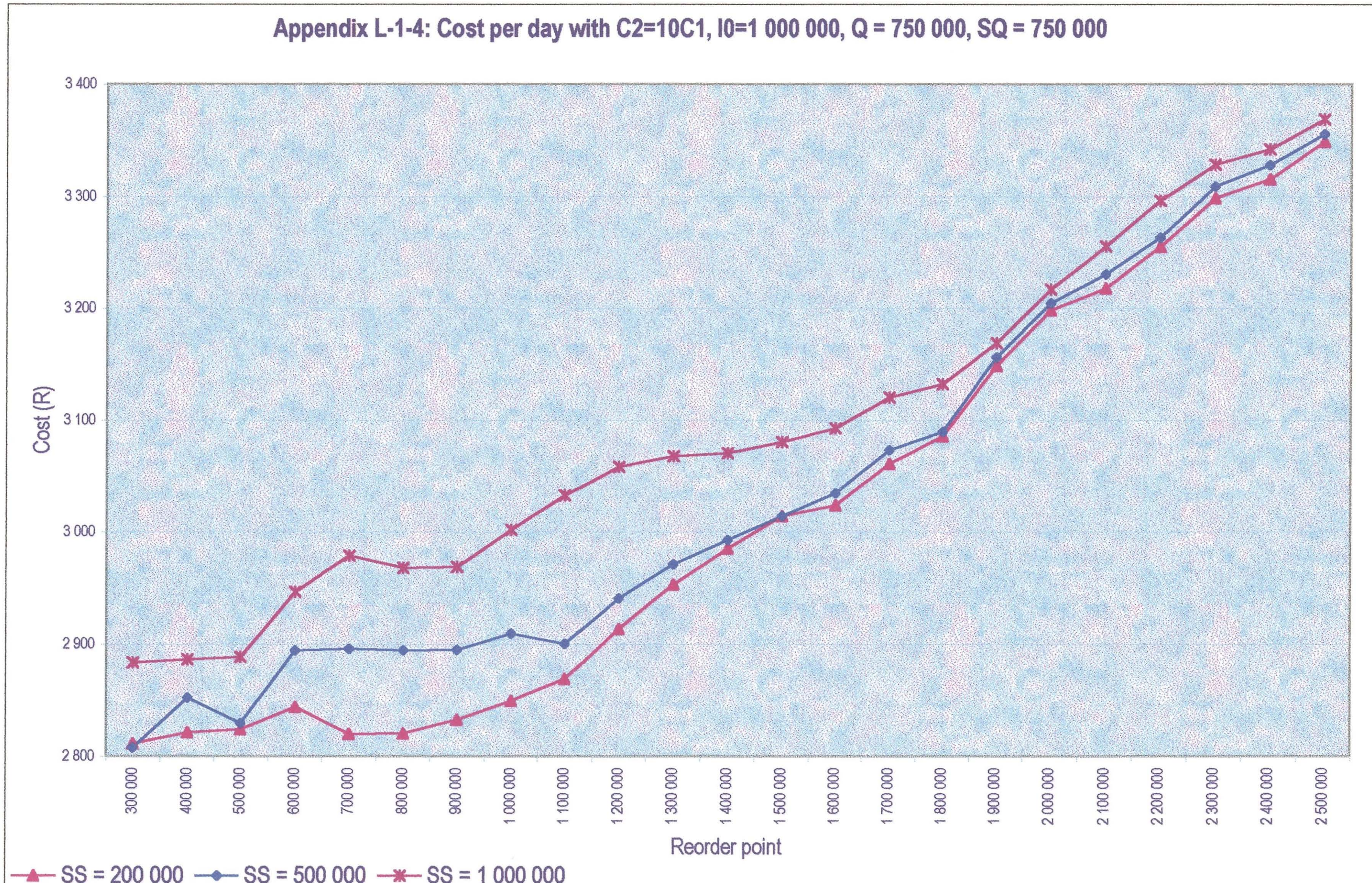
Summary for SS = 500 000, Q = 750 000, I0 = 1 000 000, c2=10c1, SQ = 750 000

ROP	SS = 500 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	500 000	3 145 100	2 808	115	157	58
400 000	500 000	3 194 971	2 853	126	146	61
500 000	500 000	3 169 405	2 830	136	136	44
600 000	500 000	3 241 946	2 895	148	124	43
700 000	500 000	3 243 438	2 896	160	112	34
800 000	500 000	3 241 925	2 895	169	103	33
900 000	500 000	3 242 417	2 895	184	88	31
1 000 000	500 000	3 258 704	2 910	195	77	28
1 100 000	500 000	3 248 809	2 901	205	67	22
1 200 000	500 000	3 293 829	2 941	215	57	18
1 300 000	500 000	3 328 148	2 972	224	48	15
1 400 000	500 000	3 352 370	2 993	232	40	12
1 500 000	500 000	3 376 038	3 014	238	34	11
1 600 000	500 000	3 399 140	3 035	242	30	10
1 700 000	500 000	3 442 068	3 073	246	26	9
1 800 000	500 000	3 460 496	3 090	248	24	9
1 900 000	500 000	3 535 271	3 156	254	18	8
2 000 000	500 000	3 589 782	3 205	258	14	7
2 100 000	500 000	3 618 343	3 231	262	12	5
2 200 000	500 000	3 655 164	3 264	264	9	5
2 300 000	500 000	3 706 653	3 310	264	9	5
2 400 000	500 000	3 727 791	3 328	266	8	2
2 500 000	500 000	3 758 959	3 356	268	6	2

Appendix L-1-3

Summary for SS = 1 000 000, Q = 750 000, I0 = 1 000 000, c2=10c1, SQ = 750 000

ROP	SS = 1 000 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	1 000 000	3 230 120	2 884	52	220	29
400 000	1 000 000	3 233 045	2 887	56	216	27
500 000	1 000 000	3 235 405	2 889	72	200	20
600 000	1 000 000	3 300 456	2 947	81	191	18
700 000	1 000 000	3 336 693	2 979	96	176	16
800 000	1 000 000	3 324 382	2 968	106	166	14
900 000	1 000 000	3 325 404	2 969	123	148	15
1 000 000	1 000 000	3 362 425	3 002	136	136	17
1 100 000	1 000 000	3 397 299	3 033	148	125	14
1 200 000	1 000 000	3 425 317	3 058	160	113	11
1 300 000	1 000 000	3 436 265	3 068	172	101	9
1 400 000	1 000 000	3 439 102	3 071	184	89	7
1 500 000	1 000 000	3 450 538	3 081	193	80	5
1 600 000	1 000 000	3 464 192	3 093	202	71	5
1 700 000	1 000 000	3 494 902	3 120	210	63	5
1 800 000	1 000 000	3 508 101	3 132	218	55	5
1 900 000	1 000 000	3 549 550	3 169	228	45	4
2 000 000	1 000 000	3 603 473	3 217	237	36	3
2 100 000	1 000 000	3 646 517	3 256	242	31	3
2 200 000	1 000 000	3 692 127	3 297	247	26	4
2 300 000	1 000 000	3 728 496	3 329	253	20	3
2 400 000	1 000 000	3 743 677	3 343	257	16	2
2 500 000	1 000 000	3 773 252	3 369	259	14	2


Appendix L-1-4: Cost per day with C2=10C1, I0=1 000 000, Q = 750 000, SQ = 750 000
Cost (R)
3 400
3 300
3 200
3 100
3 000
2 900
2 800
300 000
400 000
500 000
600 000
700 000
800 000
900 000
1 000 000
1 100 000
1 200 000
1 300 000
1 400 000
1 500 000
1 600 000
1 700 000
1 800 000
1 900 000
2 000 000
2 100 000
2 200 000
2 300 000
2 400 000
2 500 000
Reorder point
SS = 200 000
SS = 500 000
SS = 1 000 000

Appendix L-2-1

Summary for SS = 200 000, Q = 750 000, I0 = 1 000 000, c2=10c1, SQ = 1 000 000

ROP	SS = 200 000	Total cost	Daily cost	No of normal orders	No of special orders	Number of Shortages
300 000	200 000	3 119 364	2 785	126	109	66
400 000	200 000	3 149 650	2 812	140	99	70
500 000	200 000	3 178 802	2 838	146	94	65
600 000	200 000	3 137 446	2 801	168	78	52
700 000	200 000	3 152 779	2 815	177	71	42
800 000	200 000	3 175 527	2 835	183	67	39
900 000	200 000	3 185 701	2 844	197	56	32
1 000 000	200 000	3 222 516	2 877	204	51	34
1 100 000	200 000	3 223 072	2 878	220	39	22
1 200 000	200 000	3 259 598	2 910	223	37	29
1 300 000	200 000	3 307 032	2 953	231	31	18
1 400 000	200 000	3 311 617	2 957	240	24	20
1 500 000	200 000	3 361 299	3 001	247	19	11
1 600 000	200 000	3 393 791	3 030	247	19	12
1 700 000	200 000	3 429 416	3 062	257	11	10
1 800 000	200 000	3 476 769	3 104	258	11	10
1 900 000	200 000	3 539 836	3 161	260	9	7
2 000 000	200 000	3 539 312	3 160	262	8	5
2 100 000	200 000	3 600 253	3 215	266	6	3
2 200 000	200 000	3 636 552	3 247	269	3	3
2 300 000	200 000	3 705 865	3 309	268	4	4
2 400 000	200 000	3 751 592	3 350	268	4	2
2 500 000	200 000	3 772 937	3 369	270	3	3

Appendix L-2-2

Summary for SS = 500 000, Q = 750 000, I0 = 1 000 000, c2=10c1, SQ = 1 000 000

ROP	SS = 500 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	500 000	3 148 436	2 811	96	132	55
400 000	500 000	3 143 955	2 807	105	125	49
500 000	500 000	3 183 710	2 843	116	116	38
600 000	500 000	3 191 693	2 850	127	109	38
700 000	500 000	3 185 003	2 844	139	100	36
800 000	500 000	3 230 512	2 884	158	86	35
900 000	500 000	3 267 392	2 917	170	77	22
1 000 000	500 000	3 271 134	2 921	176	72	20
1 100 000	500 000	3 263 728	2 914	184	66	20
1 200 000	500 000	3 279 021	2 928	201	53	17
1 300 000	500 000	3 315 069	2 960	209	47	12
1 400 000	500 000	3 346 224	2 988	218	41	12
1 500 000	500 000	3 399 632	3 035	226	35	11
1 600 000	500 000	3 367 791	3 007	238	26	7
1 700 000	500 000	3 419 006	3 053	233	29	5
1 800 000	500 000	3 473 071	3 101	242	23	7
1 900 000	500 000	3 527 021	3 149	246	20	5
2 000 000	500 000	3 577 027	3 194	256	12	4
2 100 000	500 000	3 612 919	3 226	258	12	4
2 200 000	500 000	3 654 563	3 263	261	9	6
2 300 000	500 000	3 679 120	3 285	261	9	2
2 400 000	500 000	3 741 566	3 341	266	5	2
2 500 000	500 000	3 786 597	3 381	266	5	3

Appendix L-2-3

Summary for SS = 1 000 000, Q = 750 000, I0 = 1 000 000, c2=10c1, SQ = 1 000 000

ROP	SS = 1 000 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	1 000 000	3 238 943	2 892	43	172	16
400 000	1 000 000	3 262 032	2 913	51	166	18
500 000	1 000 000	3 261 527	2 912	53	164	17
600 000	1 000 000	3 279 526	2 928	62	157	14
700 000	1 000 000	3 287 833	2 936	78	146	17
800 000	1 000 000	3 295 637	2 943	94	134	15
900 000	1 000 000	3 320 125	2 964	103	127	11
1 000 000	1 000 000	3 363 609	3 003	117	117	9
1 100 000	1 000 000	3 388 014	3 025	129	108	9
1 200 000	1 000 000	3 391 170	3 028	137	102	9
1 300 000	1 000 000	3 421 475	3 055	160	85	6
1 400 000	1 000 000	3 471 317	3 099	168	79	7
1 500 000	1 000 000	3 489 900	3 116	174	74	5
1 600 000	1 000 000	3 477 704	3 105	182	68	7
1 700 000	1 000 000	3 509 646	3 134	199	55	4
1 800 000	1 000 000	3 533 025	3 154	207	49	5
1 900 000	1 000 000	3 564 891	3 183	220	40	3
2 000 000	1 000 000	3 615 503	3 228	228	34	4
2 100 000	1 000 000	3 595 690	3 210	236	28	1
2 200 000	1 000 000	3 643 796	3 253	235	28	2
2 300 000	1 000 000	3 700 260	3 304	244	22	2
2 400 000	1 000 000	3 752 690	3 351	248	19	2
2 500 000	1 000 000	3 808 274	3 400	258	11	3

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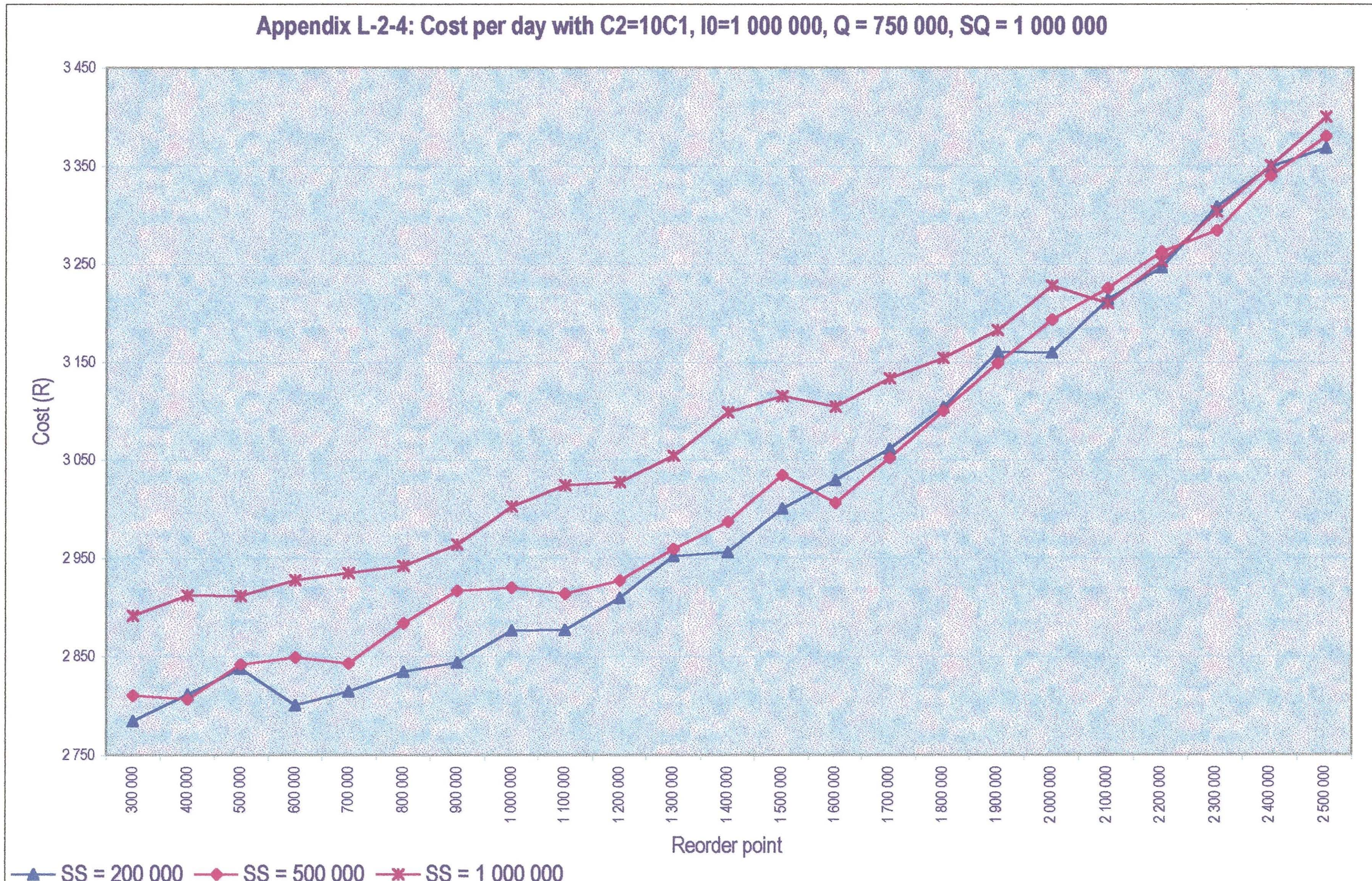

Appendix M

An investigation into returning excessive cash amounts

Explanatory notes

Appendix M combines a return amount with an upper limit for the amount of cash held at the branch. Appendix M-1 uses a return amount of R200 000 whereas Appendix M-2 uses a return amount of R500 000 in turn each combined with upper limits of R2 000 000 and R2 500 000.

Appendix M-1-1

Summary for Q = 750 000, I0 = 1 000 000, c2=10c1, SS = 200 000, SQ = 500 000
Return amount R200 000 and upper limit R2 000 000

ROP	UL = 2 000 000	Total cost	Cost per day	No of normal orders	No of special orders	No of Shortages	No of Returns
300 000	2 000 000	3 220 129	2 875	213	128	129	97
400 000	2 000 000	3 238 495	2 892	227	175	122	112
500 000	2 000 000	3 263 836	2 914	248	160	113	129
600 000	2 000 000	3 229 975	2 884	260	142	93	129
700 000	2 000 000	3 230 994	2 885	271	134	93	136
800 000	2 000 000	3 234 211	2 888	287	122	87	148
900 000	2 000 000	3 254 159	2 905	310	104	78	165
1 000 000	2 000 000	3 285 838	2 934	330	103	69	194
1 100 000	2 000 000	3 306 540	2 952	351	94	63	216
1 200 000	2 000 000	3 348 608	2 990	370	88	64	240
1 300 000	2 000 000	3 401 655	3 037	396	86	61	277
1 400 000	2 000 000	3 423 664	3 057	418	80	50	304
1 500 000	2 000 000	3 444 111	3 075	437	69	49	320
1 600 000	2 000 000	3 512 183	3 136	461	65	47	352
1 700 000	2 000 000	3 543 378	3 164	474	76	46	373
1 800 000	2 000 000	3 564 886	3 183	493	49	33	382
1 900 000	2 000 000	3 598 623	3 213	508	53	37	408
2 000 000	2 000 000	3 650 423	3 259	525	53	36	434

Appendix M-1-2

Summary for Q = 750 000, I0 = 1 000 000, c2=10c1, SS = 200 000, SQ = 500 000
Return amount R200 000 and upper limit R2 500 000

ROP	UL = 2 500 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of shortage	No of returns
300 000	2 500 000	3 195 796	2 853	194	168	111	54
400 000	2 500 000	3 167 425	2 828	210	146	107	56
500 000	2 500 000	3 158 372	2 820	214	139	90	55
600 000	2 500 000	3 124 398	2 790	228	118	73	55
700 000	2 500 000	3 150 243	2 813	239	108	78	62
800 000	2 500 000	3 149 474	2 812	251	102	63	74
900 000	2 500 000	3 155 004	2 817	272	87	59	90
1 000 000	2 500 000	3 178 042	2 838	288	75	54	102
1 100 000	2 500 000	3 199 009	2 856	304	59	42	110
1 200 000	2 500 000	3 215 247	2 871	306	60	36	114
1 300 000	2 500 000	3 251 631	2 903	328	54	37	141
1 400 000	2 500 000	3 319 349	2 964	348	50	38	167
1 500 000	2 500 000	3 346 993	2 988	366	43	30	187
1 600 000	2 500 000	3 368 883	3 008	389	34	25	212
1 700 000	2 500 000	3 443 821	3 075	413	40	29	255
1 800 000	2 500 000	3 471 422	3 099	421	36	22	263
1 900 000	2 500 000	3 525 949	3 148	440	33	26	288
2 000 000	2 500 000	3 543 865	3 164	455	31	20	308

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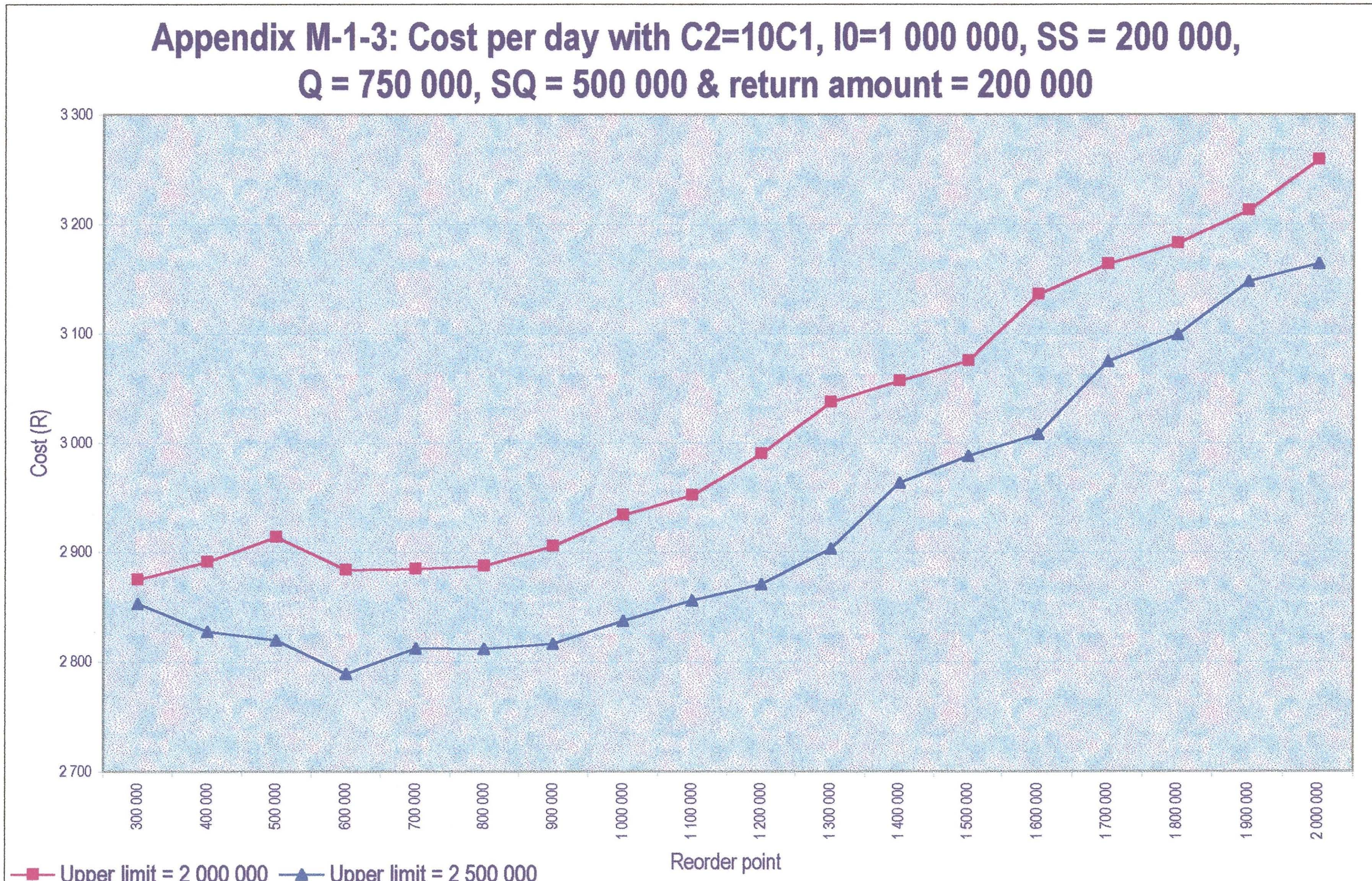


Appendix M-2-1

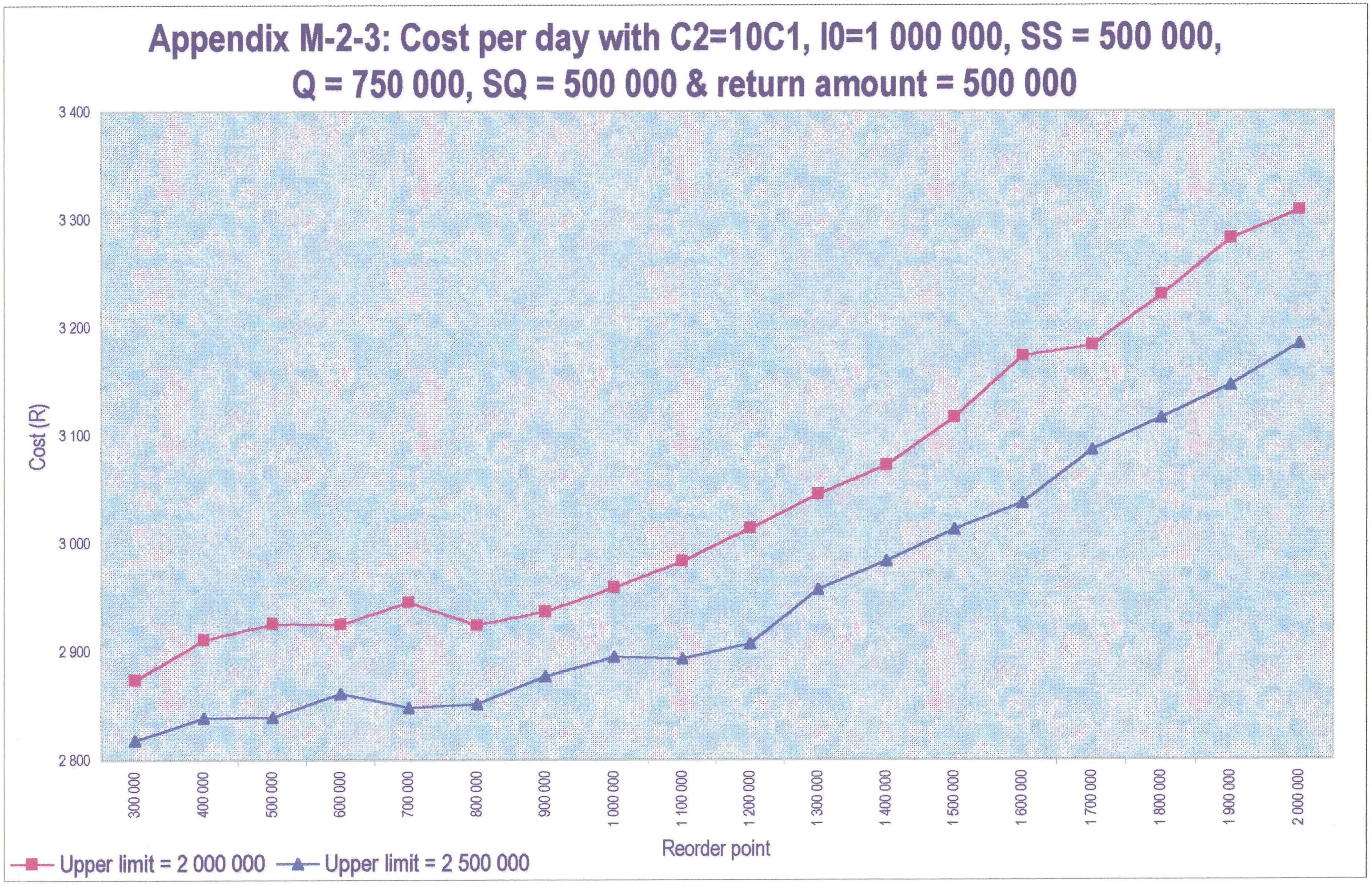
Summary for Q = 750 000, I0 = 1 000 000, c2=10c1, SS = 500 000, SQ = 500 000
Return amount R500 000 and upper limit R2 000 000

ROP	UL = 2 000 000	Total cost	Cost per day	No of normal orders	No of special orders	No of Shortages	No of returns
300 000	2 000 000	3 218 940	2 874	278	144	94	107
400 000	2 000 000	3 260 467	2 911	194	228	98	116
500 000	2 000 000	3 277 097	2 926	217	217	92	138
600 000	2 000 000	3 276 627	2 926	230	202	86	143
700 000	2 000 000	3 299 286	2 946	252	183	84	158
800 000	2 000 000	3 275 349	2 924	268	167	78	166
900 000	2 000 000	3 289 691	2 937	288	157	62	186
1 000 000	2 000 000	3 314 334	2 959	306	145	58	201
1 100 000	2 000 000	3 341 304	2 983	335	132	49	230
1 200 000	2 000 000	3 375 321	3 014	352	125	55	250
1 300 000	2 000 000	3 410 871	3 045	374	121	53	278
1 400 000	2 000 000	3 440 745	3 072	396	109	48	300
1 500 000	2 000 000	3 490 648	3 117	416	113	45	334
1 600 000	2 000 000	3 554 573	3 174	439	109	44	363
1 700 000	2 000 000	3 565 323	3 183	459	98	44	382
1 800 000	2 000 000	3 617 846	3 230	483	83	36	401
1 900 000	2 000 000	3 676 124	3 282	492	94	40	428
2 000 000	2 000 000	3 705 946	3 309	512	84	36	447

Appendix M-2-2

Summary for Q = 750 000, I0 = 1 000 000, c2=10c1, SS = 500 000, SQ = 500 000
Return amount R500 000 and upper limit R2 500 000

ROP	UL = 2 500 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of shortages	No of returns
300 000	2 500 000	3 156 646	2 818	156	221	82	50
400 000	2 500 000	3 179 966	2 839	181	207	83	74
500 000	2 500 000	3 180 825	2 840	189	189	78	68
600 000	2 500 000	3 205 119	2 862	208	174	74	81
700 000	2 500 000	3 190 517	2 849	221	152	64	78
800 000	2 500 000	3 194 299	2 852	237	142	52	92
900 000	2 500 000	3 222 435	2 877	248	134	46	102
1 000 000	2 500 000	3 242 825	2 895	262	125	49	114
1 100 000	2 500 000	3 240 614	2 893	268	113	38	111
1 200 000	2 500 000	3 256 319	2 907	289	97	31	125
1 300 000	2 500 000	3 312 147	2 957	311	92	29	153
1 400 000	2 500 000	3 342 071	2 984	328	80	30	167
1 500 000	2 500 000	3 374 325	3 013	348	74	30	191
1 600 000	2 500 000	3 401 830	3 037	361	71	30	207
1 700 000	2 500 000	3 456 929	3 087	386	62	27	237
1 800 000	2 500 000	3 489 996	3 116	410	55	24	266
1 900 000	2 500 000	3 524 443	3 147	426	48	17	282
2 000 000	2 500 000	3 567 569	3 185	445	48	19	310


Appendix M-2-3: Cost per day with C2=10C1, I0=1 000 000, SS = 500 000, Q = 750 000, SQ = 500 000 & return amount = 500 000
Cost (R)
3 400
3 300
3 200
3 100
3 000
2 900
2 800
300 000
400 000
500 000
600 000
700 000
800 000
900 000
1 000 000
1 100 000
1 200 000
1 300 000
1 400 000
1 500 000
1 600 000
1 700 000
1 800 000
1 900 000
2 000 000
Reorder point
Upper limit = 2 000 000
Upper limit = 2 500 000

Appendix N

An analysis of the sensitivity of the actual patterns to the proposed "best" policies

Explanatory notes

Appendix N investigates the behaviour of actual withdrawal and deposit amounts when the "best" policies determined in Chapter 6 are applied.

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Appendix N-1

Summary for actual amounts where Q = 750 000, c2=10c1, SS = 200 000, SQ = 500 000 Return amount R500 000 and upper limit R2 000 000

ROP	UL = 2 000 000	SS = 200 000	Total cost	Cost per day	No of normal orders	No of special orders	No of Shortages	No of returns
0	2 000 000	200 000	226 560	2 490	8	14	8	0
100 000	2 000 000	200 000	217 447	2 390	9	13	6	1
200 000	2 000 000	200 000	215 949	2 373	10	11	5	1
300 000	2 000 000	200 000	222 175	2 441	12	8	3	4
400 000	2 000 000	200 000	221 624	2 435	13	7	4	2
500 000	2 000 000	200 000	221 015	2 429	15	6	1	3
600 000	2 000 000	200 000	220 908	2 428	15	5	1	2
700 000	2 000 000	200 000	223 323	2 454	15	6	1	2
800 000	2 000 000	200 000	224 278	2 465	15	6	1	2
900 000	2 000 000	200 000	$\boxed{231\ 186}$	$\boxed{2\ 541}$	22	2	0	9
1 000 000	2 000 000	200 000	235 241	2 585	24	2	0	12
1 100 000	2 000 000	200 000	241 473	2 654	27	0	0	14
1 200 000	2 000 000	200 000	244 148	2 683	29	0	0	17
1 300 000	2 000 000	200 000	249 654	2 743	30	1	0	19
1 400 000	2 000 000	200 000	252 131	2 771	31	0	0	20
1 500 000	2 000 000	200 000	255 541	2 808	32	0	0	22
1 600 000	2 000 000	200 000	258 853	2 845	34	0	0	25
1 700 000	2 000 000	200 000	261 507	2 874	34	0	0	25
1 800 000	2 000 000	200 000	268 191	2 947	35	1	1	26
1 900 000	2 000 000	200 000	271 728	2 986	37	0	0	29
2 000 000	2 000 000	200 000	280 195	3 079	39	2	1	33

Appendix N-2

Summary for actual amounts where Q = 750 000, c2=10c1, SS = 200 000, SQ = 500 000 Return amount R500 000 and upper limit R2 500 000

ROP	UL = 2 500 000	SS = 200 000	Total cost	Cost per day	No of normal orders	No of special orders	No of Shortages	No of returns
0	2 500 000	200 000	226 560	2 490	8	14	8	0
100 000	2 500 000	200 000	220 876	2 427	9	13	5	0
200 000	2 500 000	200 000	215 949	2 373	10	11	5	0
300 000	2 500 000	200 000	227 962	2 505	11	10	4	0
400 000	2 500 000	200 000	222 031	2 440	13	7	2	0
500 000	2 500 000	200 000	217 023	2 385	14	5	1	0
600 000	2 500 000	200 000	220 603	2 424	14	5	2	0
700 000	2 500 000	200 000	223 418	2 455	15	3	1	0
800 000	2 500 000	200 000	225 545	2 479	16	3	1	0
900 000	2 500 000	200 000	224 435	2 466	16	2	0	0
1 000 000	2 500 000	200 000	227 620	2 501	16	2	0	0
1 100 000	2 500 000	200 000	233 533	2 566	19	0	0	2
1 200 000	2 500 000	200 000	234 268	2 574	18	0	0	1
1 300 000	2 500 000	200 000	238 098	2 616	19	0	0	2
1 400 000	2 500 000	200 000	250 888	2 757	24	0	0	9
1 500 000	2 500 000	200 000	253 019	2 780	25	0	0	10
1 600 000	2 500 000	200 000	261 109	2 869	27	0	0	13
1 700 000	2 500 000	200 000	263 877	2 900	28	0	0	14
1 800 000	2 500 000	200 000	266 119	2 924	29	0	0	16
1 900 000	2 500 000	200 000	270 387	2 971	31	0	0	19
2 000 000	2 500 000	200 000	274 221	3 013	32	0	0	21

Appendix N-3

Summary for actual amounts where Q = 750 000, c2=10c1, SS = 500 000, SQ = 500 000
Return amount R500 000 and upper limit R2 000 000

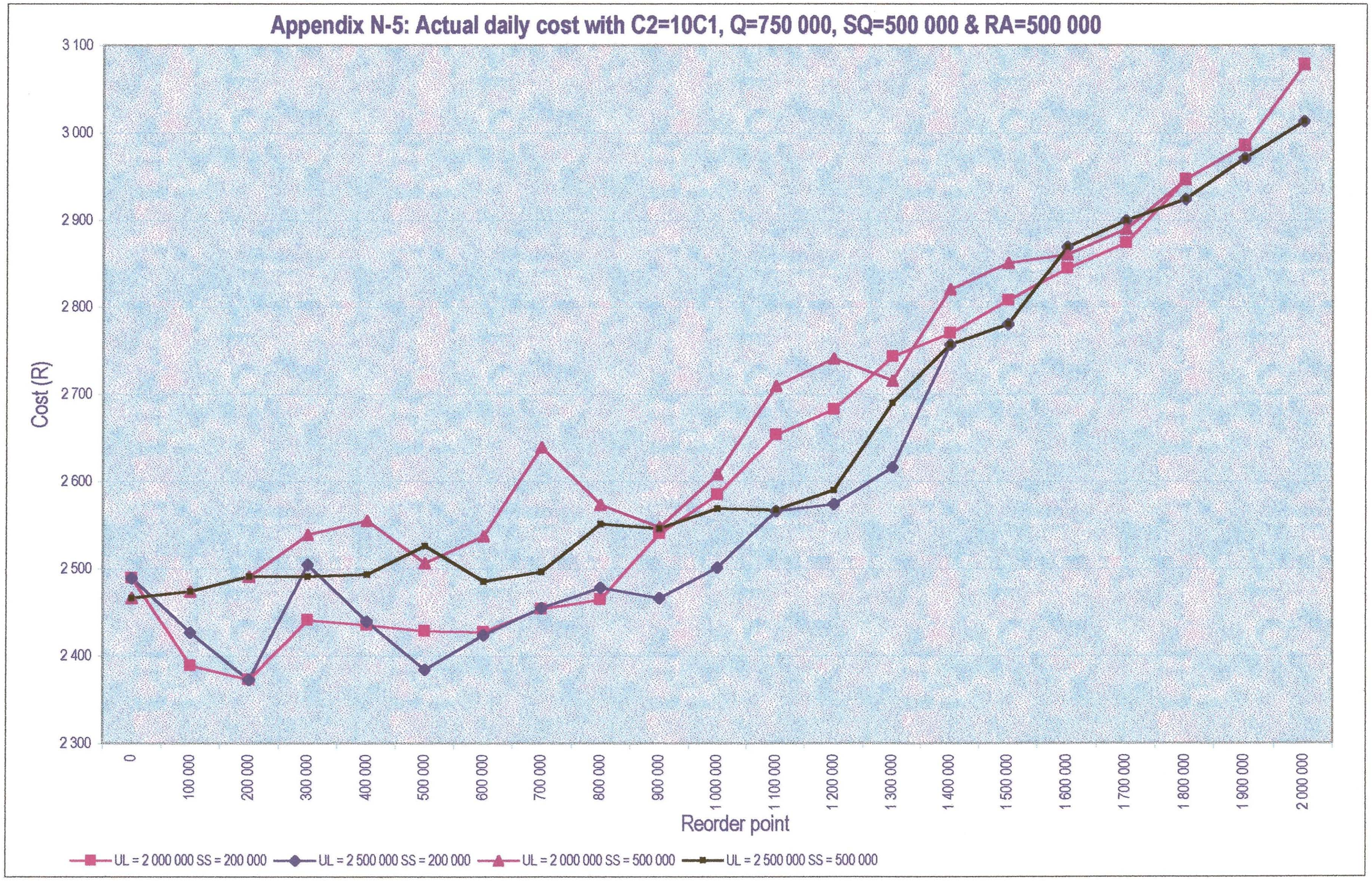
ROP	UL = 2 000 000	SS = 500 000	Total cost	Cost per day	No of normal orders	No of special orders	No of Shortages	No of returns
0	2 000 000	500 000	224 448	2 466	22	2	2	0
100 000	2 000 000	500 000	225 150	2 474	17	5	4	0
200 000	2 000 000	500 000	226 673	2 491	15	7	2	0
300 000	2 000 000	500 000	231 077	2 539	10	14	2	3
400 000	2 000 000	500 000	232 525	2 555	12	14	1	6
500 000	2 000 000	500 000	228 096	2 507	12	12	0	4
600 000	2 000 000	500 000	230 927	2 538	14	10	0	6
700 000	2 000 000	500 000	240 174	2 639	18	11	0	11
800 000	2 000 000	500 000	234 181	2 573	17	8	0	7
900 000	2 000 000	500 000	231 855	2 548	19	4	0	7
1 000 000	2 000 000	500 000	237 378	2 609	22	5	0	11
1 100 000	2 000 000	500 000	246 564	2 709	25	4	0	15
1 200 000	2 000 000	500 000	249 451	2 741	27	4	0	18
1 300 000	2 000 000	500 000	247 160	2 716	27	2	0	16
1 400 000	2 000 000	500 000	256 695	2 821	30	4	0	21
1 500 000	2 000 000	500 000	259 437	2 851	30	5	0	23
1 600 000	2 000 000	500 000	260 312	2 861	34	1	0	25
1 700 000	2 000 000	500 000	262 966	2 890	34	1	0	25
1 800 000	2 000 000	500 000	268 191	2 947	35	1	1	26
1 900 000	2 000 000	500 000	271 728	2 986	37	0	0	29
2 000 000	2 000 000	500 000	280 195	3 079	39	2	1	33

N5

Appendix N-4

Summary for actual amounts where Q = 750 000, c2=10c1, SS = 500 000, SQ = 500 000
Return amount R500 000 and upper limit R2 500 000

ROP	UL = 2 500 000	SS = 500 000	Total cost	Cost per day	No of normal orders	No of special orders	No of Shortages	No of returns
0	2 500 000	500 000	224 448	2 466	22	2	2	0
100 000	2 500 000	500 000	225 150	2 474	5	17	4	0
200 000	2 500 000	500 000	226 673	2 491	7	15	2	0
300 000	2 500 000	500 000	226 703	2 491	8	14	2	1
400 000	2 500 000	500 000	226 885	2 493	10	13	0	2
500 000	2 500 000	500 000	229 852	2 526	11	12	2	2
600 000	2 500 000	500 000	226 157	2 485	12	7	1	0
700 000	2 500 000	500 000	227 170	2 496	13	7	1	0
800 000	2 500 000	500 000	232 137	2 551	13	8	1	1
900 000	2 500 000	500 000	231 672	2 546	14	5	0	0
1 000 000	2 500 000	500 000	233 795	2 569	14	5	0	0
1 100 000	2 500 000	500 000	233 596	2 567	17	1	0	0
1 200 000	2 500 000	500 000	235 727	2 590	18	1	0	1
1 300 000	2 500 000	500 000	244 829	2 690	20	3	0	5
1 400 000	2 500 000	500 000	250 888	2 757	24	0	0	9
1 500 000	2 500 000	500 000	253 019	2 780	25	0	0	10
1 600 000	2 500 000	500 000	261 109	2 869	27	0	0	13
1 700 000	2 500 000	500 000	263 877	2 900	28	0	0	14
1 800 000	2 500 000	500 000	266 119	2 924	29	0	0	16
1 900 000	2 500 000	500 000	270 387	2 971	31	0	0	19
2 000 000	2 500 000	500 000	274 221	3 013	32	0	0	21


Appendix N-5: Actual daily cost with C2=10C1, Q=750 000, SQ=500 000 & RA=500 000
Cost (R)
3 100
3 000
2 900
2 800
2 700
2 600
2 500
2 400
2 300
0
100 000
200 000
300 000
400 000
500 000
600 000
700 000
800 000
900 000
1 000 000
1 100 000
1 200 000
1 300 000
1 400 000
1 500 000
1 600 000
1 700 000
1 800 000
1 900 000
2 000 000
Reorder point
UL = 2 000 000 SS = 200 000
UL = 2 500 000 SS = 200 000
UL = 2 000 000 SS = 500 000
UL = 2 500 000 SS = 500 000

Appendix O

Investigating the impact of lead time on the "best" policies

Explanatory notes

Appendix O uses increased lead times for normal and special orders. The lead time used is two days instead of one day for special orders and three days instead of two days for normal orders.

Appendix O-1

Summary for actual amounts where Q = 750 000, c2=10c1, SS = 200 000, SQ = 500 000
Return amount R500 000 and upper limit R2 500 000, increased lead time

ROP	UL = 2 500 000	SS = 200 000	Total cost	Cost per day	No of normal orders	No of special orders	No of Shortages	No of returns
0	2 500 000	200 000	265 983	2 923	10	12	12	2
100 000	2 500 000	200 000	237 366	2 608	9	12	8	0
200 000	2 500 000	200 000	242 321	2 663	11	12	9	5
300 000	2 500 000	200 000	230 894	2 537	11	10	8	0
400 000	2 500 000	200 000	226 213	2 486	12	9	4	0
500 000	2 500 000	200 000	225 998	2 483	11	9	4	0
600 000	2 500 000	200 000	249 388	2 741	17	8	6	10
700 000	2 500 000	200 000	252 535	2 775	17	10	6	10
800 000	2 500 000	200 000	248 828	2 734	18	7	5	9
900 000	2 500 000	200 000	259 238	2 849	22	8	5	14
1 000 000	2 500 000	200 000	260 194	2 859	22	8	5	14
1 100 000	2 500 000	200 000	276 831	3 042	25	9	4	20
1 200 000	2 500 000	200 000	276 831	3 042	25	9	4	20
1 300 000	2 500 000	200 000	276 831	3 042	25	9	4	20
1 400 000	2 500 000	200 000	267 495	2 940	28	5	1	19
1 500 000	2 500 000	200 000	267 938	2 944	28	3	1	20
1 600 000	2 500 000	200 000	269 386	2 960	31	2	0	21
1 700 000	2 500 000	200 000	273 944	3 010	31	3	1	22
1 800 000	2 500 000	200 000	277 169	3 046	30	4	2	22
1 900 000	2 500 000	200 000	279 673	3 073	36	0	0	26
2 000 000	2 500 000	200 000	279 673	3 073	36	0	0	26

Appendix O-2

Summary for actual amounts where Q = 750 000, c2=10c1, SS = 300 000, SQ = 500 000 Return amount R500 000 and upper limit R2 500 000

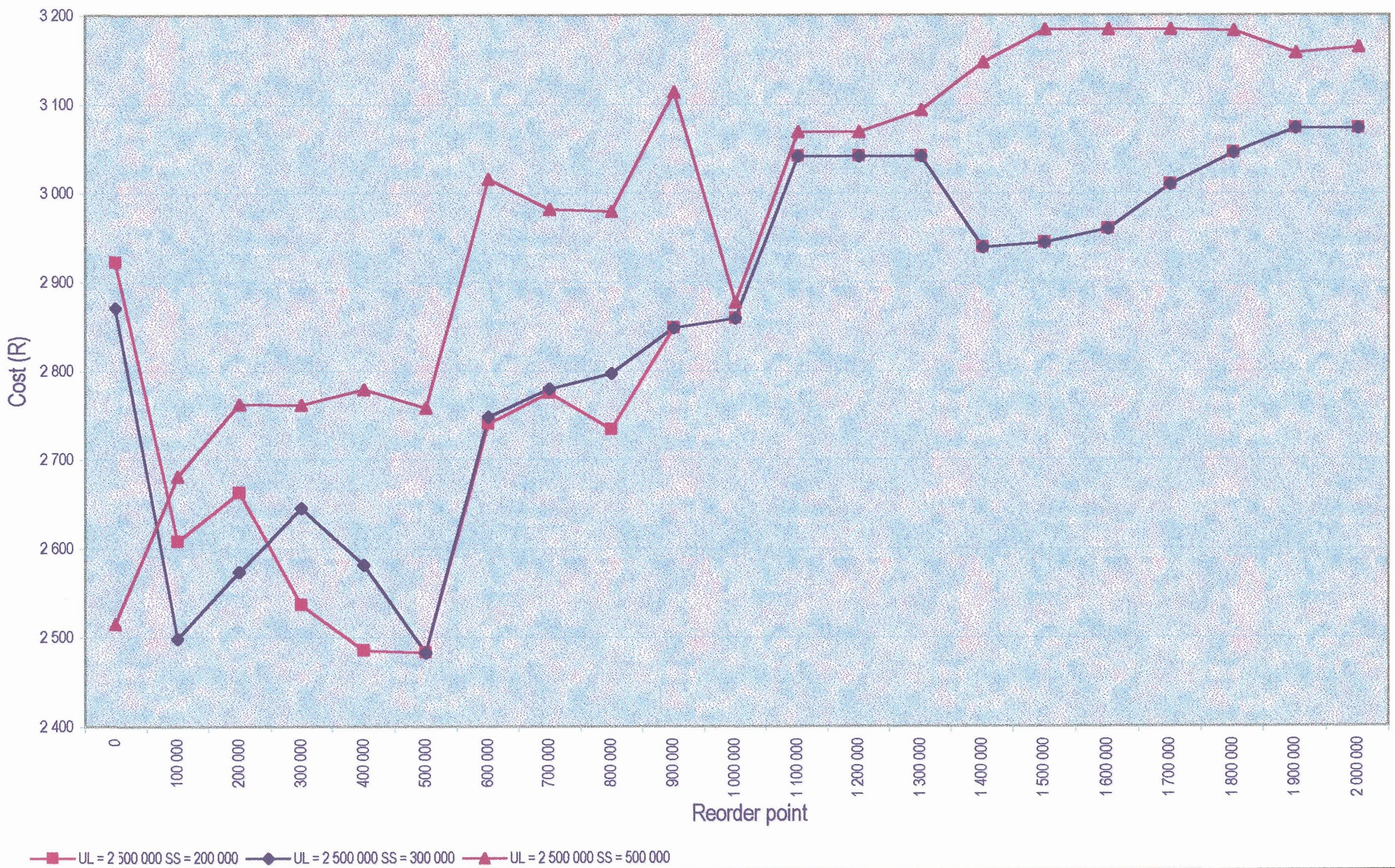
ROP	UL = 2 500 000	SS = 300 000	Total cost	Cost per day	No of normal orders	No of special orders	No of Shortages	No of returns
0	2 500 000	300 000	261 265	2 871	11	14	12	4
100 000	2 500 000	300 000	227 407	2 499	7	15	8	0
200 000	2 500 000	300 000	234 259	2 574	8	13	8	0
300 000	2 500 000	300 000	240 743	2 646	11	12	9	2
400 000	2 500 000	300 000	234 961	2 582	10	10	5	0
500 000	2 500 000	300 000	225 998	2 483	11	9	4	0
600 000	2 500 000	300 000	250 119	2 749	17	10	5	8
700 000	2 500 000	300 000	252 960	2 780	17	10	6	10
800 000	2 500 000	300 000	254 570	2 797	17	9	6	8
900 000	2 500 000	300 000	259 238	2 849	22	8	5	14
1 000 000	2 500 000	300 000	260 194	2 859	22	8	5	14
1 100 000	2 500 000	300 000	276 831	3 042	25	9	4	20
1 200 000	2 500 000	300 000	276 831	3 042	25	9	4	20
1 300 000	2 500 000	300 000	276 831	3 042	25	9	4	20
1 400 000	2 500 000	300 000	267 495	2 940	28	5	1	19
1 500 000	2 500 000	300 000	267 938	2 944	28	3	1	20
1 600 000	2 500 000	300 000	269 386	2 960	31	2	0	21
1 700 000	2 500 000	300 000	273 944	3 010	31	3	1	22
1 800 000	2 500 000	300 000	277 169	3 046	30	4	2	22
1 900 000	2 500 000	300 000	279 673	3 073	36	0	0	26
2 000 000	2 500 000	300 000	279 673	3 073	36	0	0	26

Appendix O-3

Summary for actual amounts where Q = 750 000, c2=10c1, SS = 500 000, SQ = 500 000
Return amount R500 000 and upper limit R2 500 000

ROP	UL = 2 500 000	SS = 500 000	Total cost	Cost per day	No of normal orders	No of special orders	No of Shortages	No of returns
0	2 500 000	500 000	228 931	2 516	4	21	5	1
100 000	2 500 000	500 000	244 008	2 681	8	20	7	6
200 000	2 500 000	500 000	251 402	2 763	11	17	10	8
300 000	2 500 000	500 000	251 330	2 762	12	17	9	8
400 000	2 500 000	500 000	252 908	2 779	14	18	6	13
500 000	2 500 000	500 000	251 048	2 759	15	15	5	13
600 000	2 500 000	500 000	274 457	3 016	21	17	7	21
700 000	2 500 000	500 000	271 374	2 982	20	17	7	21
800 000	2 500 000	500 000	271 195	2 980	20	15	7	21
900 000	2 500 000	500 000	283 405	3 114	24	17	7	27
1 000 000	2 500 000	500 000	261 844	2 877	20	12	6	15
1 100 000	2 500 000	500 000	279 335	3 070	28	14	6	28
1 200 000	2 500 000	500 000	279 335	3 070	28	14	6	28
1 300 000	2 500 000	500 000	281 535	3 094	30	13	6	30
1 400 000	2 500 000	500 000	286 401	3 147	30	12	6	31
1 500 000	2 500 000	500 000	289 803	3 185	33	11	4	34
1 600 000	2 500 000	500 000	289 803	3 185	33	11	4	34
1 700 000	2 500 000	500 000	289 803	3 185	33	11	4	34
1 800 000	2 500 000	500 000	289 673	3 183	34	11	2	34
1 900 000	2 500 000	500 000	287 395	3 158	35	9	1	34
2 000 000	2 500 000	500 000	287 997	3 165	36	9	3	35

Appendix O-4: Actual daily cost with C2=10C1, Q=750 000, SQ=500 000 & RA=500 000



Appendix P

Results of the application of the decision support model assuming that the withdrawal and demand patterns exhibit the same seasonal cycle

Explanatory notes

The decision support model uses the same cycle to forecast demand and withdrawal patterns combined with the order policy which provided the "best" results in Chapter 6. The results of the calculations are shown in Appendices P1, P2, P3 and P4 for the four differing cycles, whereas Appendices P1-5, P2-10, P3-4 and P4-7 compare the daily cost of those approaches that comply with the suitability criteria discussed in Chapter 7.

Appendix P1-1

Seasonality based on a six day cycle

Withdrawals forecast using moving averages and simple seasonal relatives

Deposits forecast using moving averages and simple seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	218 228	2 398	17	0	0
400 000	750 000	218 228	2 398	17	0	0
500 000	750 000	219 502	2 412	17	0	0
600 000	750 000	224 598	2 468	17	0	0
700 000	750 000	225 872	2 482	17	0	0
800 000	750 000	233 198	2 563	17	0	0
900 000	750 000	242 013	2 659	18	0	0
1 000 000	750 000	244 243	2 684	18	0	0

Appendix P1-2

Seasonality based on a six day cycle

Withdrawals forecast using moving averages and simple seasonal relatives

Deposits forecast using Winter's method with a regressed trend & simple seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	217 667	2 392	16	1	0
400 000	750 000	233 171	2 562	17	1	0
500 000	750 000	223 643	2 458	17	0	0
600 000	750 000	225 452	2 477	18	0	0
700 000	750 000	242 013	2 659	18	0	0
800 000	750 000	250 294	2 750	18	0	0
900 000	750 000	250 931	2 757	18	0	0
1 000 000	750 000	249 555	2 742	19	0	0

Appendix P1-3

Seasonality based on a six day cycle

Withdrawals forecast using Winter's method with a regressed trend & simple seasonal relatives

Deposits forecast using moving averages and simple seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	223 182	2 453	15	2	2
400 000	750 000	223 713	2 458	15	2	2
500 000	750 000	229 852	2 526	16	0	1
600 000	750 000	231 763	2 547	16	0	1
700 000	750 000	236 559	2 600	15	2	2
800 000	750 000	234 948	2 582	16	0	1
900 000	750 000	244 503	2 687	16	0	1
1 000 000	750 000	247 369	2 718	16	0	1

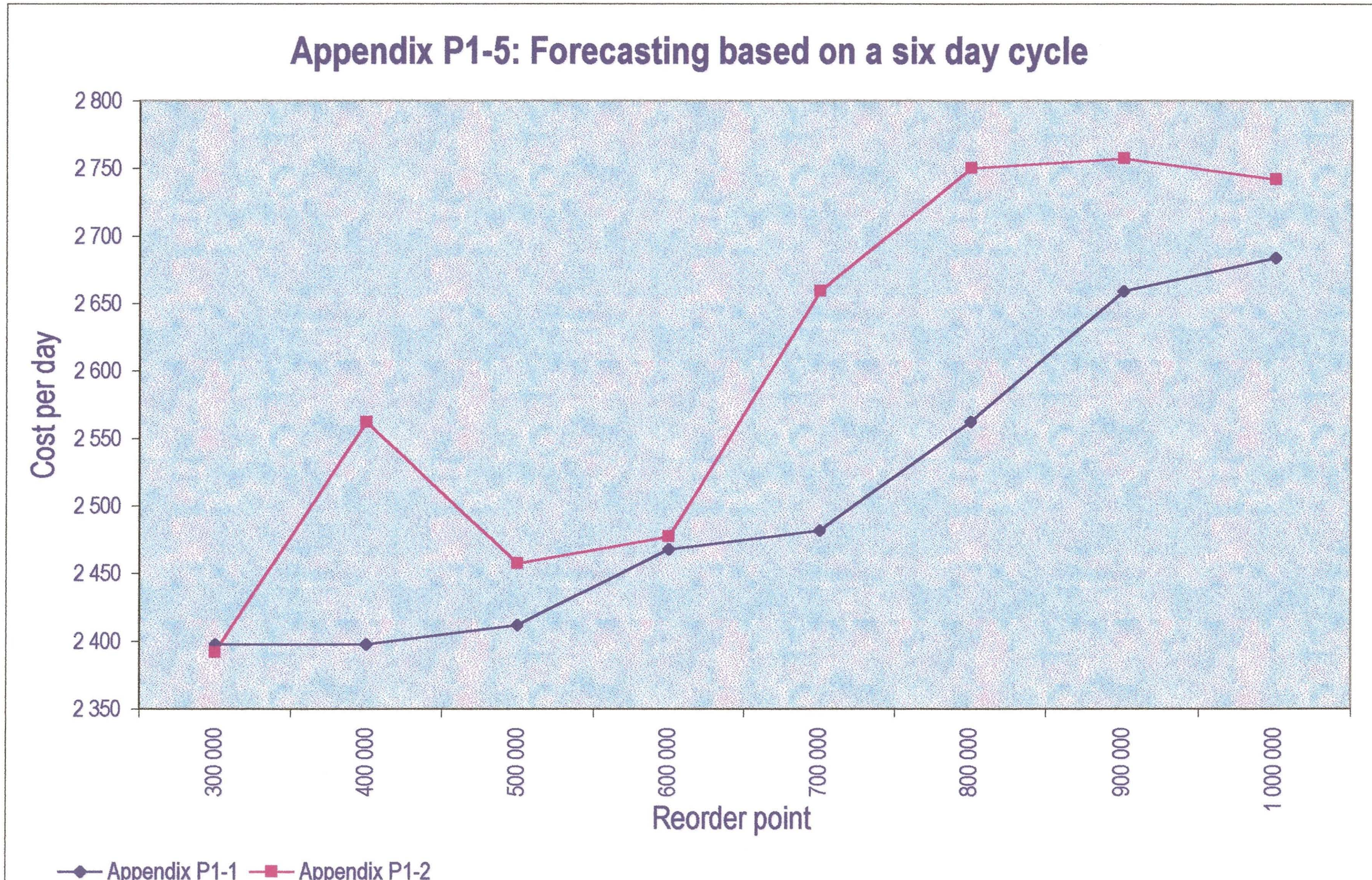
Appendix P1-4

Seasonality based on a six day cycle

Withdrawals forecast using Winter's method with a regressed trend & simple seasonal relatives

Deposits forecast using Winter's method with a regressed trend & simple seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	216 287	2 377	16	1	0
400 000	750 000	222 227	2 442	15	2	2
500 000	750 000	222 227	2 442	15	2	2
600 000	750 000	225 235	2 475	17	0	0
700 000	750 000	231 126	2 540	16	0	1
800 000	750 000	238 293	2 619	17	0	0
900 000	750 000	241 478	2 654	17	0	0
1 000 000	750 000	242 332	2 663	18	0	0


Appendix P1-5: Forecasting based on a six day cycle
Cost per day
2 800
2 750
2 700
2 650
2 600
2 550
2 500
2 450
2 400
2 350
300 000
400 000
500 000
600 000
700 000
800 000
900 000
1 000 000
Reorder point
Appendix P1-1
Appendix P1-2

P4

Appendix P2-1

Seasonality based on a 24 day cycle

Withdrawals forecast using FIT smoothing with a default trend & simple seasonal relatives

Deposits forecast using simple exponential smoothing & moving seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	219 754	2 415	14	3	2
400 000	750 000	225 629	2 479	16	1	0
500 000	750 000	221 413	2 433	17	0	0
600 000	750 000	228 739	2 514	17	0	0
700 000	750 000	236 064	2 594	17	0	0
800 000	750 000	241 318	2 652	16	0	1
900 000	750 000	241 160	2 650	17	0	0
1 000 000	750 000	246 893	2 713	17	0	0

Appendix P2-2

Seasonality based on a 24 day cycle

Withdrawals forecast using FIT smoothing with a default trend & simple seasonal relatives

Deposits forecast using FIT smoothing with a regressed trend & moving seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	217 267	2 388	15	2	1
400 000	750 000	224 886	2 471	16	1	0
500 000	750 000	222 444	2 444	16	1	0
600 000	750 000	224 917	2 472	17	0	0
700 000	750 000	236 064	2 594	17	0	0
800 000	750 000	237 656	2 612	17	0	0
900 000	750 000	241 160	2 650	17	0	0
1 000 000	750 000	241 160	2 650	17	0	0

Appendix P2-3

Seasonality based on a 24 day cycle

Withdrawals forecast using FIT smoothing with a default trend & simple seasonal relatives

Deposits forecast using simple averages & simple seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	219 921	2 417	15	2	1
400 000	750 000	220 215	2 420	16	1	0
500 000	750 000	224 598	2 468	17	0	0
600 000	750 000	226 509	2 489	17	0	0
700 000	750 000	228 578	2 512	16	0	1
800 000	750 000	233 835	2 570	17	0	0
900 000	750 000	242 013	2 659	18	0	0
1 000 000	750 000	241 695	2 656	18	0	0

Appendix P2-4

Seasonality based on a 24 day cycle

Withdrawals forecast using simple averages and simple seasonal relatives

Deposits forecast using simple exponential smoothing using moving seasonal relatives

ROP	Q = 750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	215 838	2 372	16	0	1
400 000	750 000	230 013	2 528	17	0	0
500 000	750 000	227 146	2 496	17	0	0
600 000	750 000	235 427	2 587	17	0	0
700 000	750 000	237 496	2 610	16	0	1
800 000	750 000	243 229	2 673	16	0	1
900 000	750 000	244 663	2 689	17	0	0
1 000 000	750 000	246 893	2 713	17	0	0

Appendix P2-5

Seasonality based on a 24 day cycle

Withdrawals forecast using simple averages and simple seasonal relatives

Deposits forecast using FIT smoothing with a regressed trend & moving seasonal relatives

ROP	Q = 750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	215 838	2 372	16	0	1
400 000	750 000	229 133	2 518	16	1	0
500 000	750 000	227 146	2 496	17	0	0
600 000	750 000	235 427	2 587	17	0	0
700 000	750 000	236 859	2 603	16	0	1
800 000	750 000	241 636	2 655	16	0	1
900 000	750 000	244 184	2 683	16	0	1
1 000 000	750 000	246 893	2 713	17	0	0

Appendix P2-6

Seasonality based on a 24 day cycle

Withdrawals forecast using simple averages and simple seasonal relatives

Deposits forecast using simple exponential smoothing using moving seasonal relatives

300 000	750 000	220 616	2 424	16	0	1
400 000	750 000	220 776	2 426	17	0	0
500 000	750 000	226 509	2 489	17	0	0
600 000	750 000	229 057	2 517	17	0	0
700 000	750 000	230 968	2 538	17	0	0
800 000	750 000	238 451	2 620	16	0	1
900 000	750 000	240 841	2 647	17	0	0
1 000 000	750 000	246 574	2 710	17	0	0

Appendix P2-7

Seasonality based on a 24 day cycle

Withdrawals forecast using simple averages moving seasonal relatives

Deposits forecast using simple exponential smoothing using moving seasonal relatives

300 000	750 000	222 232	2 442	16	1	0
400 000	750 000	226 691	2 491	16	1	0
500 000	750 000	229 533	2 522	16	0	1
600 000	750 000	236 701	2 601	17	0	0
700 000	750 000	236 540	2 599	16	0	1
800 000	750 000	241 797	2 657	17	0	0
900 000	750 000	247 109	2 715	18	0	0
1 000 000	750 000	247 428	2 719	18	0	0

Appendix P2-8

Seasonality based on a 24 day cycle

Withdrawals forecast using simple averages moving seasonal relatives

Deposits forecast using FIT smoothing with a regressed trend & moving seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	220 321	2 421	16	1	0
400 000	750 000	226 054	2 484	16	1	0
500 000	750 000	227 941	2 505	16	0	1
600 000	750 000	235 109	2 584	17	0	0
700 000	750 000	237 656	2 612	17	0	0
800 000	750 000	240 681	2 645	16	0	1
900 000	750 000	247 109	2 715	18	0	0
1 000 000	750 000	246 893	2 713	17	0	0

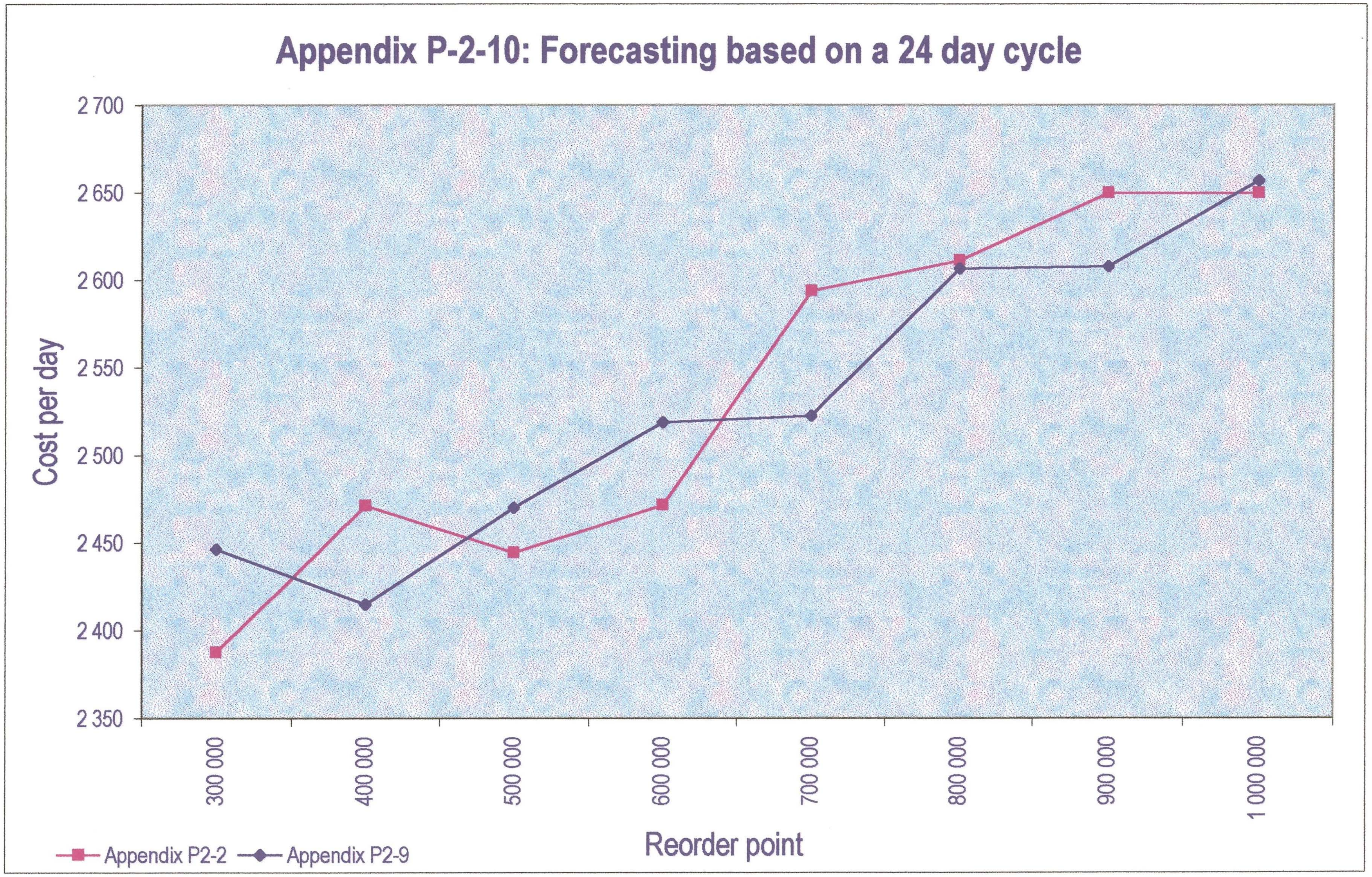
Appendix P2-9

Seasonality based on a 24 day cycle

Withdrawals forecast using simple averages moving seasonal relatives

Deposits forecast using simple exponential smoothing using moving seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	222 627	2 446	16	1	1
400 000	750 000	219 760	2 415	16	1	1
500 000	750 000	224 780	2 470	16	1	0
600 000	750 000	229 239	2 519	16	1	0
700 000	750 000	229 557	2 523	16	1	0
800 000	750 000	237 236	2 607	18	0	0
900 000	750 000	237 338	2 608	17	0	0
1 000 000	750 000	241 797	2 657	17	0	0


Appendix P-2-10: Forecasting based on a 24 day cycle
Cost per day
2 700
2 650
2 600
2 550
2 500
2 450
2 400
2 350
300 000
400 000
500 000
600 000
700 000
800 000
900 000
1 000 000
Reorder point
Appendix P2-2
Appendix P2-9

Appendix P3-1

Seasonality based on a 26 day cycle

Withdrawals forecast using simple averages and simple seasonal relatives

Deposits forecast using simple averages and simple seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	221 630	2 435	18	0	0
400 000	750 000	221 630	2 435	18	0	0
500 000	750 000	227 681	2 502	18	0	0
600 000	750 000	228 318	2 509	18	0	0
700 000	750 000	228 955	2 516	18	0	0
800 000	750 000	241 695	2 656	18	0	0
900 000	750 000	243 822	2 679	19	0	0
1 000 000	750 000	250 829	2 756	19	0	0

Appendix P3-2

Seasonality based on a 26 day cycle

Withdrawals forecast using moving averages and simple seasonal relatives

Deposits forecast using Winter's method with a regressed trend & simple seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	220 356	2 421	18	0	0
400 000	750 000	225 452	2 477	18	0	0
500 000	750 000	227 363	2 498	18	0	0
600 000	750 000	237 236	2 607	18	0	0
700 000	750 000	231 503	2 544	18	0	0
800 000	750 000	241 058	2 649	18	0	0
900 000	750 000	244 243	2 684	18	0	0
1 000 000	750 000	248 281	2 728	19	0	0

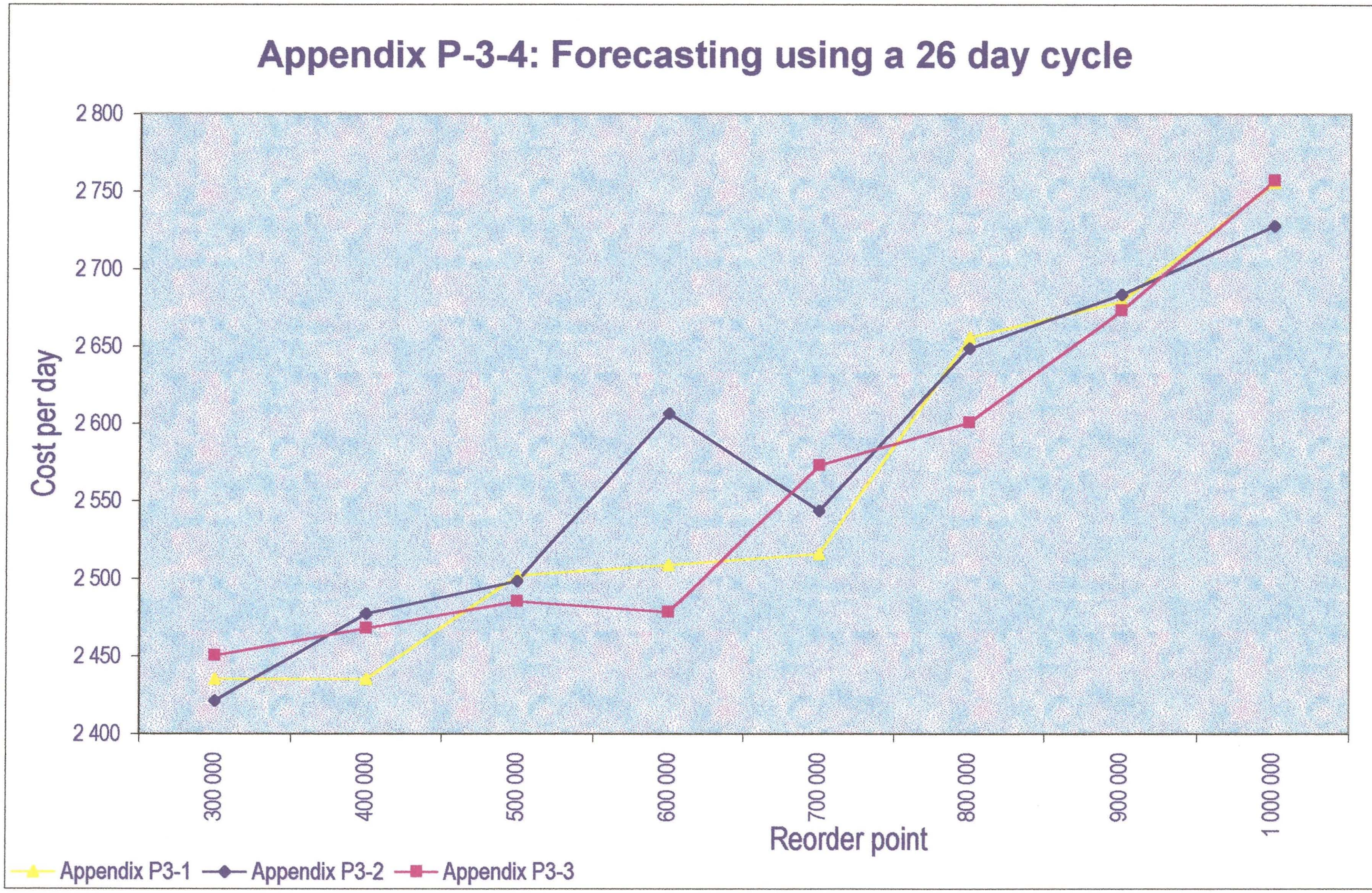
Appendix P3-3

Seasonality based on a six day cycle

Withdrawals forecast using moving averages and simple seasonal relatives

Deposits forecast using Winter's method with a regressed trend & moving seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	223 006	2 451	17	0	0
400 000	750 000	224 598	2 468	17	0	0
500 000	750 000	226 191	2 486	17	0	0
600 000	750 000	225 554	2 479	17	0	0
700 000	750 000	234 153	2 573	17	0	0
800 000	750 000	236 701	2 601	17	0	0
900 000	750 000	243 287	2 673	18	0	0
1 000 000	750 000	250 931	2 757	18	0	0


Appendix P-3-4: Forecasting using a 26 day cycle
Cost per day
2 800
2 750
2 700
2 650
2 600
2 550
2 500
2 450
2 400
300 000
400 000
500 000
600 000
700 000
800 000
900 000
1 000 000
Reorder point
Appendix P3-1
Appendix P3-2
Appendix P3-3

Appendix P4-1

Seasonality based on a 30 day cycle

Withdrawals forecast using FIT smoothing with a default trend and simple seasonal relatives

Deposits forecast using simple exponential smoothing and simple seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	215 756	2 371	16	1	0
400 000	750 000	219 502	2 412	17	0	0
500 000	750 000	223 006	2 451	17	0	0
600 000	750 000	233 198	2 563	17	0	0
700 000	750 000	237 873	2 614	18	0	0
800 000	750 000	234 370	2 575	18	0	0
900 000	750 000	242 650	2 666	18	0	0
1 000 000	750 000	246 472	2 708	18	0	0

Appendix P4-2

Seasonality based on a 30 day cycle

Withdrawals forecast using moving averages and simple seasonal relatives

Deposits forecast using Winter's method with a regressed trend & simple seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	217 685	2 392	17	1	1
400 000	750 000	220 260	2 420	17	0	1
500 000	750 000	223 404	2 455	17	1	0
600 000	750 000	234 870	2 581	17	1	0
700 000	750 000	233 733	2 568	18	0	0
800 000	750 000	233 733	2 568	18	0	0
900 000	750 000	247 007	2 714	19	0	0
1 000 000	750 000	243 822	2 679	19	0	0

Appendix P4-3

Seasonality based on a 30 day cycle

Withdrawals forecast using moving averages and simple seasonal relatives

Deposits forecast using Winter's method with a regressed trend & moving seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	215 756	2 371	16	1	0
400 000	750 000	219 502	2 412	17	0	0
500 000	750 000	227 863	2 504	17	1	0
600 000	750 000	226 509	2 489	17	0	0
700 000	750 000	234 370	2 575	18	0	0
800 000	750 000	236 599	2 600	18	0	0
900 000	750 000	241 695	2 656	18	0	0
1 000 000	750 000	242 013	2 659	18	0	0

Appendix P4-4

Seasonality based on a 30 day cycle

Withdrawals forecast using FIT smoothing with a default trend and simple seasonal relatives

Deposits forecast using simple exponential smoothing and simple seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	225 093	2 474	15	2	2
400 000	750 000	220 458	2 423	17	0	0
500 000	750 000	223 400	2 455	16	1	0
600 000	750 000	233 910	2 570	16	1	0
700 000	750 000	233 591	2 567	16	1	0
800 000	750 000	242 191	2 661	16	1	0
900 000	750 000	244 026	2 682	17	0	0
1 000 000	750 000	243 924	2 680	18	0	0

Appendix P4-5

Seasonality based on a 30 day cycle

Withdrawals forecast using moving averages and simple seasonal relatives

Deposits forecast using Winter's method with a regressed trend & simple seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	221 271	2 432	15	2	2
400 000	750 000	227 757	2 503	17	1	0
500 000	750 000	234 092	2 572	16	1	1
600 000	750 000	223 324	2 454	17	0	0
700 000	750 000	238 612	2 622	17	0	0
800 000	750 000	240 523	2 643	17	0	0
900 000	750 000	241 695	2 656	18	0	0
1 000 000	750 000	242 969	2 670	18	0	0

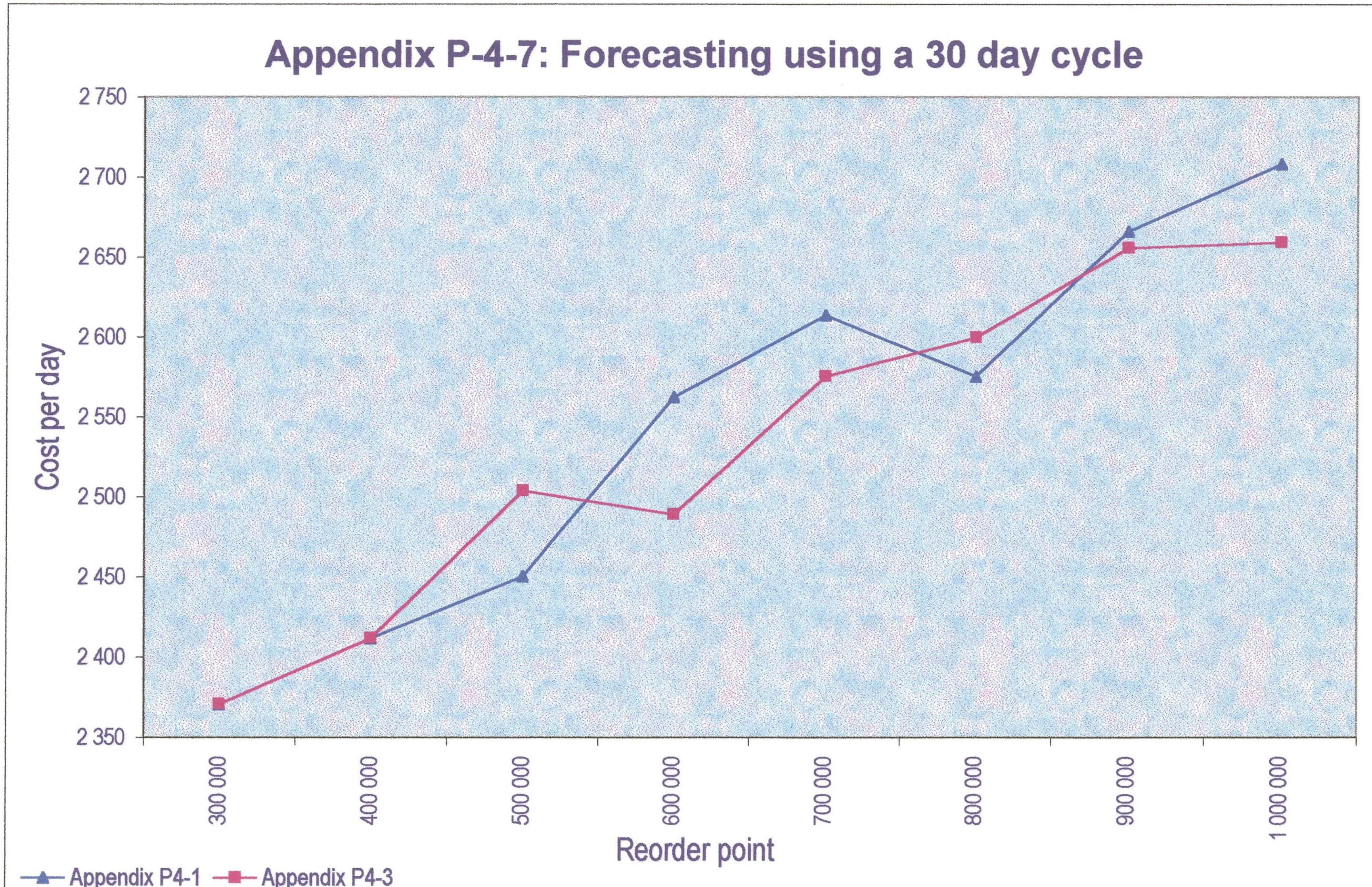
Appendix P4-6

Seasonality based on a 30 day cycle

Withdrawals forecast using moving averages and simple seasonal relatives

Deposits forecast using Winter's method with a regressed trend & moving seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	216 706	2 381	15	2	2
400 000	750 000	225 417	2 477	16	1	0
500 000	750 000	222 050	2 440	17	0	0
600 000	750 000	233 198	2 563	17	0	0
700 000	750 000	233 198	2 563	17	0	0
800 000	750 000	240 917	2 647	16	1	0
900 000	750 000	240 523	2 643	17	0	0
1 000 000	750 000	245 198	2 694	18	0	0


Appendix P-4-7: Forecasting using a 30 day cycle
Cost per day
2 750
2 700
2 650
2 600
2 550
2 500
2 450
2 400
2 350
300 000
400 000
500 000
600 000
700 000
800 000
900 000
1 000 000
Reorder point
Appendix P4-1
Appendix P4-3

Appendix Q

Results of the application of the decision support model based on the minimisation of forecast error

Explanatory notes

The decision support model uses the methods suggested by the minimisation of three different measures of forecast error. The results of the calculations are shown in Appendices Q1 and Q4.

Appendix Q1-1

Seasonality of deposits based on a 24 day cycle & withdrawals on 26 day cycle

Withdrawals forecast using simple averages & simple seasonal relatives

Deposits forecast using simple exponential smoothing & moving seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	225 452	2 477	18	0	0
400 000	750 000	227 681	2 502	18	0	0
500 000	750 000	229 911	2 526	18	0	0
600 000	750 000	229 911	2 526	18	0	0
700 000	750 000	236 781	2 602	17	1	0
800 000	750 000	240 102	2 638	18	0	0
900 000	750 000	245 517	2 698	18	0	0
1 000 000	750 000	247 746	2 722	18	0	0

Appendix Q1-2

Seasonality of deposits based on a 24 day cycle & withdrawals on 26 day cycle

Withdrawals forecast using simple averages & simple seasonal relatives

Deposits forecast using simple averages & simple seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	220 993	2 428	18	0	0
400 000	750 000	222 267	2 442	18	0	0
500 000	750 000	223 541	2 456	18	0	0
600 000	750 000	226 089	2 484	18	0	0
700 000	750 000	232 777	2 558	18	0	0
800 000	750 000	240 739	2 645	18	0	0
900 000	750 000	244 459	2 686	19	0	0
1 000 000	750 000	248 600	2 732	19	0	0

Appendix Q1-3

Seasonality of deposits based on a 24 day cycle & withdrawals on 26 day cycle

Withdrawals forecast using simple averages & simple seasonal relatives

Deposits forecast using FIT smoothing with a regressed trend & moving seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	221 918	2 439	17	1	0
400 000	750 000	227 681	2 502	18	0	0
500 000	750 000	227 681	2 502	18	0	0
600 000	750 000	229 911	2 526	18	0	0
700 000	750 000	234 370	2 575	18	0	0
800 000	750 000	239 784	2 635	18	0	0
900 000	750 000	246 791	2 712	18	0	0
1 000 000	750 000	246 154	2 705	18	0	0

Appendix Q2-1

Seasonality of deposits based on a 24 day cycle & withdrawals on 30 day cycle

Withdrawals forecast using FIT smoothing with a default trend & simple seasonal relatives

Deposits forecast using simple exponential smoothing & moving seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	224 328	2 465	16	2	0
400 000	750 000	226 270	2 486	17	1	0
500 000	750 000	233 414	2 565	18	0	0
600 000	750 000	234 472	2 577	17	0	0
700 000	750 000	241 058	2 649	18	0	0
800 000	750 000	251 250	2 761	18	0	0
900 000	750 000	253 161	2 782	18	0	0
1 000 000	750 000	254 435	2 796	18	0	0

Appendix Q2-2

Seasonality of deposits based on a 24 day cycle & withdrawals on 30 day cycle

Withdrawals forecast using FIT smoothing with a default trend & simple seasonal relatives

Deposits forecast using simple averages & simple seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	230 035	2 528	16	3	0
400 000	750 000	231 791	2 547	17	1	0
500 000	750 000	227 465	2 500	17	0	0
600 000	750 000	233 414	2 565	18	0	0
700 000	750 000	245 517	2 698	18	0	0
800 000	750 000	248 702	2 733	18	0	0
900 000	750 000	255 811	2 811	17	0	0
1 000 000	750 000	257 938	2 834	18	0	0

Appendix Q2-3

Seasonality of deposits based on a 24 day cycle & withdrawals on 30 day cycle

Withdrawals forecast using FIT smoothing with a default trend & simple seasonal relatives

Deposits forecast using FIT smoothing with a regressed trend & moving seasonal relatives

ROP	Q=750 000	Total cost	Cost per day	No of normal orders	No of special orders	Number of Shortages
300 000	750 000	224 328	2 465	16	2	0
400 000	750 000	225 098	2 474	16	1	0
500 000	750 000	233 914	2 570	17	1	0
600 000	750 000	234 370	2 575	18	0	0
700 000	750 000	239 784	2 635	18	0	0
800 000	750 000	244 561	2 687	18	0	0
900 000	750 000	253 161	2 782	18	0	0
1 000 000	750 000	252 842	2 778	18	0	0

Appendix H

Results of forecasting methods investigated

Explanatory notes

Appendix H summarises the various forecasting techniques investigated for the deposit and withdrawal patterns for differing seasons regarded as possibilities when forecasting these patterns. The methodology is described in detail in Chapter 5.

Appendix H1-1
A comparison of forecasting techniques applied to total withdrawals
Season = 6 days (using all available data)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	α = 0.3015	Simple seasonal relatives	389 330	52.03%	269 769
Simple exponential smoothing	α = 0.3155	Moving seasonal relatives	396 126	52.34%	271 859
FIT smoothing (trend = default)	α = 0.3125 $\delta = 3.052\times10^{-5}$	Simple seasonal relatives	389 950	52.00%	269 544
FIT smoothing (trend = regressed)	α = 0.3235 $\delta = 3.052\times10^{-5}$	Simple seasonal relatives	389 661	52.40%	269 166
FIT smoothing (trend = default)	α = 0.3164 $\delta = 3.052\times10^{-5}$	Moving seasonal relatives	396 202	52.34%	271 863
FIT smoothing (trend = regressed)	α = 0.3135 $\delta = 3.052\times10^{-5}$	Moving seasonal relatives	395 867	52.80%	272 142
Trend regressed exponential smoothing	α = 0.2987	Simple seasonal relatives	388 852	52.45%	269 396
Trend regressed exponential smoothing	α = 0.3126	Moving seasonal relatives	395 769	52.79%	272 068
Simple average	-	Simple seasonal relatives	374 509	54.69%	271 982
Simple average	-	Moving seasonal relatives	376 408	54.73%	273 307
Moving average	Step = 5	Simple seasonal relatives	384 732	50.20%	258 837
Moving average	Step = 5	Moving seasonal relatives	390 139	50.50%	261 718
Winter's method (trend = default)	α = 0.2 δ = 0 γ = 0	Simple seasonal relatives	390 028	53.91%	280 178
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Simple seasonal relatives	370 683	55.04%	270 157
Winter's method (trend = default)	α = 0.2 δ = 0 γ = 0	Moving seasonal relatives	394 359	54.04%	282 228
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Moving seasonal relatives	372 196	54.41%	268 584

Appendix H1-2
A comparison of forecasting techniques applied to total withdrawals
Season = 24 days (using all available data)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	α = 0.0739	Simple seasonal relatives	302 008	38.58%	219 938
Simple exponential smoothing	α = 0.0447	Moving seasonal relatives	310 584	38.71%	218 934
FIT smoothing (trend = default)	α = 0.0625 δ = 0.0021	Simple seasonal relatives	301 406	38.54%	220 001
FIT smoothing (trend = regressed)	α = 0.0234 δ = 0.0157	Simple seasonal relatives	299 225	38.91%	220 478
FIT smoothing (trend = default)	α = 0.0312 δ = 0.0157	Moving seasonal relatives	310 535	39.48%	221 144
FIT smoothing (trend = regressed)	α = 0.0234 δ = 0.0157	Moving seasonal relatives	310 609	39.47%	222 111
Trend regressed exponential smoothing	α = 0.0512	Simple seasonal relatives	299 050	38.84%	218 993
Trend regressed exponential smoothing	α = 0.0387	Moving seasonal relatives	309 959	38.85%	218 807
Simple average	-	Simple seasonal relatives	298 723	39.97%	218 769
Simple average	-	Moving seasonal relatives	316 419	40.88%	218 248
Moving average	Step = 5	Simple seasonal relatives	323 195	38.96%	221 802
Moving average	Step = 5	Moving seasonal relatives	328 715	40.55%	227 278
Winter's method (trend = default)	α = 0.1 δ = 0 γ = 0	Simple seasonal relatives	343 496	44.92%	245 103
Winter's method (trend = regressed)	α = 0 δ = 0.25 γ = 0	Simple seasonal relatives	302 954	41.61%	221 002
Winter's method (trend = default)	α = 0.1 δ = 0 γ = 0	Moving seasonal relatives	365 742	45.93%	244 373
Winter's method (trend = regressed)	α = 0 δ = 0.15 γ = 0	Moving seasonal relatives	319 130	42.36%	226 724

Appendix H1-3
A comparison of forecasting techniques applied to total withdrawals
Season = 26 days (using all available data)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	$\alpha = 0.0590$	Simple seasonal relatives	306 436	48.43%	239 701
Simple exponential smoothing	$\alpha = 0.0856$	Moving seasonal relatives	351 434	52.71%	265 701
FIT smoothing (trend = default)	$\alpha = 0.0468$ $\delta = 0.0117$	Simple seasonal relatives	303 955	47.56%	238 901
FIT smoothing (trend = regressed)	$\alpha = 0.0468$ $\delta = 0.0117$	Simple seasonal relatives	303 960	47.57%	238 899
FIT smoothing (trend = default)	$\alpha = 0.0771$ $\delta = 0.0625$	Moving seasonal relatives	358 854	50.03%	267 282
FIT smoothing (trend = regressed)	$\alpha = 0.0776$ $\delta = 0.0625$	Moving seasonal relatives	358 489	50.19%	267 412
Trend regressed exponential smoothing	$\alpha = 0.0591$	Simple seasonal relatives	306 452	48.44%	239 710
Trend regressed exponential smoothing	$\alpha = 0.0907$	Moving seasonal relatives	353 808	53.32%	267 078
Simple average	-	Simple seasonal relatives	291 437	44.49%	226 927
Simple average	-	Moving seasonal relatives	326 687	47.69%	238 418
Moving average	Step = 5	Simple seasonal relatives	347 014	49.07%	261 032
Moving average	Step = 5	Moving seasonal relatives	391 174	51.28%	274 856
Winter's method (trend = default)	$\alpha = 0.1$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	313 340	48.45%	245 566
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0.05$ $\gamma = 0$	Simple seasonal relatives	296 798	45.36%	230 338
Winter's method (trend = default)	$\alpha = 0.1$ $\delta = 0$ $\gamma = 0$	Moving seasonal relatives	348 403	51.13%	261 373
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0.05$ $\gamma = 0$	Moving seasonal relatives	331 910	47.12%	236 580

Appendix H1-4
A comparison of forecasting techniques applied to total withdrawals
Season = 30 days (using all available data)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	α = 0.0992	Simple seasonal relatives	337 682	40.24%	210 795
Simple exponential smoothing	α = 3.052×10^{-5}	Moving seasonal relatives	412 515	57.96%	261 629
FIT smoothing (trend = default)	α = 0.1250 δ = 3.052×10^{-5}	Simple seasonal relatives	338 206	39.90%	209 357
FIT smoothing (trend = regressed)	α = 0.1250 δ = 3.052×10^{-5}	Simple seasonal relatives	338 443	41.33%	211 689
FIT smoothing (trend = default)	α = 3.052×10^{-5} δ = 3.052×10^{-5}	Moving seasonal relatives	412 525	57.96%	261 629
FIT smoothing (trend = regressed)	α = 0.0234 δ = 3.052×10^{-5}	Moving seasonal relatives	423 579	60.12%	272 043
Trend regressed exponential smoothing	α = 0.0858	Simple seasonal relatives	337 942	42.55%	214 794
Trend regressed exponential smoothing	α = 0.0257	Moving seasonal relatives	420 144	59.18%	268 490
Simple average	-	Simple seasonal relatives	333 285	43.83%	223 189
Simple average	-	Moving seasonal relatives	392 757	52.39%	242 212
Moving average	Step = 5	Simple seasonal relatives	354 927	40.87%	219 510
Moving average	Step = 5	Moving seasonal relatives	484 344	53.07%	293 756
Winter's method (trend = default)	α = 0.1 δ = 0 γ = 0	Simple seasonal relatives	342 335	42.67%	222 082
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Simple seasonal relatives	334 110	43.30%	223 706
Winter's method (trend = default)	α = 0.1 δ = 0 γ = 0.05	Moving seasonal relatives	403 032	50.43%	246 547
Winter's method (trend = regressed)	α = 0 δ = 0.05 γ = 0	Moving seasonal relatives	371 368	44.74%	221 923

Appendix H1-5
A comparison of forecasting techniques applied to total withdrawals
Season = 6 days (using most recent 56 data points)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	α = 0.3123	Simple seasonal relatives	433 783	56.61%	290 818
Simple exponential smoothing	α = 0.3000	Moving seasonal relatives	449 878	56.42%	291 382
FIT smoothing (trend = default)	α = 0.3125 δ = 3.052×10^{-5}	Simple seasonal relatives	433 806	56.61%	290 815
FIT smoothing (trend = regressed)	α = 0.3128 δ = 3.052×10^{-5}	Simple seasonal relatives	433 864	56.54%	290 795
FIT smoothing (trend = default)	α = 0.3125 δ = 3.052×10^{-5}	Moving seasonal relatives	451 469	56.52%	291 787
FIT smoothing (trend = regressed)	α = 0.3125 δ = 3.052×10^{-5}	Moving seasonal relatives	451 489	56.39%	291 769
Trend regressed exponential smoothing	α = 0.3126	Simple seasonal relatives	433 839	56.54%	290 785
Trend regressed exponential smoothing	α = 0.3003	Moving seasonal relatives	449 981	56.29%	291 360
Simple average	-	Simple seasonal relatives	415 451	62.51%	308 940
Simple average	-	Moving seasonal relatives	420 123	61.38%	307 313
Moving average	Step = 5	Simple seasonal relatives	333 210	45.65%	224 409
Moving average	Step = 5	Moving seasonal relatives	321 613	44.08%	221 423
Winter's method (trend = default)	α = 0 δ = 0 γ = 0	Simple seasonal relatives	415 478	62.66%	309 175
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Simple seasonal relatives	415 426	62.00%	309 464
Winter's method (trend = default)	α = 0 δ = 0 γ = 0	Moving seasonal relatives	420 083	61.28%	307 157
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Moving seasonal relatives	419 981	60.66%	307 722

Appendix H1-6
A comparison of forecasting techniques applied to total withdrawals
Season = 24 days (using 56 most recent data points)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	$\alpha = 3.052 \times 10^{-5}$	Simple seasonal relatives	289 205	33.94%	201 055
Simple exponential smoothing	$\alpha = 0.0043$	Moving seasonal relatives	391 171	36.38%	212 947
FIT smoothing (trend = default)	$\alpha = 3.052 \times 10^{-5}$ $\delta = 3.052 \times 10^{-5}$	Simple seasonal relatives	289 205	33.94%	201 055
FIT smoothing (trend = regressed)	$\alpha = 3.052 \times 10^{-5}$ $\delta = 0.0078$	Simple seasonal relatives	289 139	33.01%	200 002
FIT smoothing (trend = default)	$\alpha = 0.0043$ $\delta = 0.0156$	Moving seasonal relatives	391 229	36.64%	213 542
FIT smoothing (trend = regressed)	$\alpha = 0.0167$ $\delta = 0.0547$	Moving seasonal relatives	394 524	36.50%	214 149
Trend regressed exponential smoothing	$\alpha = 3.052 \times 10^{-5}$	Simple seasonal relatives	289 082	33.03%	199 930
Trend regressed exponential smoothing	$\alpha = 0.0326$	Moving seasonal relatives	399 563	35.52%	212 361
Simple average	-	Simple seasonal relatives	289 157	34.32%	201 528
Simple average	-	Moving seasonal relatives	394 934	41.45%	232 109
Moving average	Step = 5	Simple seasonal relatives	286 826	30.98%	190 958
Moving average	Step = 5	Moving seasonal relatives	305 407	31.45%	183 103
Winter's method (trend = default)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	294 185	37.03%	207 551
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	291 331	36.48%	206 214
Winter's method (trend = default)	$\alpha = 0$ $\delta = 0$ $\gamma = 0.2$	Moving seasonal relatives	389 034	44.78%	243 614
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0.2$	Moving seasonal relatives	388 286	43.93%	239 382

Appendix H1-7
A comparison of forecasting techniques applied to total withdrawals
Season = 26 days (using 56 most recent data points)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	$\alpha = 0.0324$	Simple seasonal relatives	329 690	51.58%	260 070
Simple exponential smoothing	$\alpha = 3.052\text{x}10^{-5}$	Moving seasonal relatives	516 250	80.24%	385 390
FIT smoothing (trend = default)	$\alpha = 0.0198$ $\delta = 0.0625$	Simple seasonal relatives	324 878	50.61%	259 087
FIT smoothing (trend = regressed)	$\alpha = 0.0071$ $\delta = 0.0312$	Simple seasonal relatives	320 555	50.05%	255 043
FIT smoothing (trend = default)	$\alpha = 3.052\text{x}10^{-5}$ $\delta = 3.052\text{x}10^{-5}$	Moving seasonal relatives	516 250	80.24%	385 390
FIT smoothing (trend = regressed)	$\alpha = 0.0068$ $\delta = 3.052\text{x}10^{-5}$	Moving seasonal relatives	521 318	81.09%	390 202
Trend regressed exponential smoothing	$\alpha = 0.0073$	Simple seasonal relatives	321 933	50.46%	255 285
Trend regressed exponential smoothing	$\alpha = 0.0066$	Moving seasonal relatives	521 256	81.09%	390 108
Simple average	-	Simple seasonal relatives	318 145	47.66%	253 211
Simple average	-	Moving seasonal relatives	471 847	68.66%	336 619
Moving average	Step = 5	Simple seasonal relatives	348 129	50.53%	263 522
Moving average	Step = 5	Moving seasonal relatives	503 497	63.57%	291 590
Winter's method (trend = default)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	322 562	49.85%	255 380
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0.3$ $\gamma = 0$	Simple seasonal relatives	318 422	49.17%	253 526
Winter's method (trend = default)	$\alpha = 0$ $\delta = 0$ $\gamma = 0.1$	Moving seasonal relatives	443 489	59.34%	296 608
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0.15$	Moving seasonal relatives	440 629	58.81%	294 033

Appendix H1-8
A comparison of forecasting techniques applied to total withdrawals
Season = 30 days (using 56 most recent data points)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	α = 0.0146	Simple seasonal relatives	448 444	46.56%	197 987
FIT smoothing (trend = default)	α = 0.0153 δ = 0.0625	Simple seasonal relatives	447 901	46.52%	196 393
FIT smoothing (trend = regressed)	α = 0.0144 δ = 0.0117	Simple seasonal relatives	604 661	77.36%	320 977
Trend regressed exponential smoothing	α = 0.0150	Simple seasonal relatives	605 731	77.40%	319 083
Simple average	-	Simple seasonal relatives	512 397	61.88%	275 501
Moving average	Step = 5	Simple seasonal relatives	533 851	51.45%	267 328
Winter's method (trend = default)	α = 0 δ = 0 γ = 0	Simple seasonal relatives	444 092	47.67%	216 910
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0.95	Simple seasonal relatives	439 816	49.51%	211 868

Appendix H2-1
A comparison of forecasting techniques applied to deposits
Season = 6 days (using all available data)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	α = 0.0157	Simple seasonal relatives	289 298	48.29%	200 066
Simple exponential smoothing	α = 0.0497	Moving seasonal relatives	290 137	49.01%	202 102
FIT smoothing (trend = default)	α = 0.0312 δ = 0.0235	Simple seasonal relatives	289 489	49.62%	201 713
FIT smoothing (trend = regressed)	α = 0.0061 δ = 0.0156	Simple seasonal relatives	283 766	55.48%	208 743
FIT smoothing (trend = default)	α = 0.0312 δ = 0.0157	Moving seasonal relatives	289 984	49.21%	203 805
FIT smoothing (trend = regressed)	α = 0.0081 δ = 0.0312	Moving seasonal relatives	285 521	56.17%	211 154
Trend regressed exponential smoothing	α = 0.0053	Simple seasonal relatives	283 879	55.62%	208 906
Trend regressed exponential smoothing	α = 0.0121	Moving seasonal relatives	286 382	56.33%	211 465
Simple average	-	Simple seasonal relatives	284 526	54.49%	209 582
Simple average	-	Moving seasonal relatives	285 434	54.71%	209 800
Moving average	Step = 5	Simple seasonal relatives	299 207	47.91%	194 297
Moving average	Step = 5	Moving seasonal relatives	302 022	47.96%	195 476
Winter's method (trend = default)	α = 0 δ = 0 γ = 0	Simple seasonal relatives	286 969	59.46%	217 488
Winter's method (trend = regressed)	α = 0 δ = 0.05 γ = 0	Simple seasonal relatives	281 227	52.10%	205 376
Winter's method (trend = default)	α = 0 δ = 0 γ = 0	Moving seasonal relatives	288 010	59.49%	217 252
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Moving seasonal relatives	282 251	52.26%	206 208

Appendix H2-2

A comparison of forecasting techniques applied to deposits
Season = 24 days (using all available data)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	$\alpha = 0.1070$	Simple seasonal relatives	247 897	42.01%	171 837
Simple exponential smoothing	$\alpha = 0.0985$	Moving seasonal relatives	246 084	41.02%	171 956
FIT smoothing (trend = default)	$\alpha = 0.1035$ $\delta = 0.0042$	Simple seasonal relatives	248 250	42.17%	171 556
FIT smoothing (trend = regressed)	$\alpha = 0.0782$ $\delta = 3.052 \times 10^{-5}$	Simple seasonal relatives	246 275	42.67%	169 970
FIT smoothing (trend = default)	$\alpha = 0.0927$ $\delta = 0.0078$	Moving seasonal relatives	247 422	41.37%	171 419
FIT smoothing (trend = regressed)	$\alpha = 0.0703$ $\delta = 3.052 \times 10^{-5}$	Moving seasonal relatives	245 944	41.60%	168 784
Trend regressed exponential smoothing	$\alpha = 0.0782$	Simple seasonal relatives	246 355	42.63%	169 999
Trend regressed exponential smoothing	$\alpha = 0.0686$	Moving seasonal relatives	246 109	41.50%	169 800
Simple average	-	Simple seasonal relatives	239 617	47.38%	174 917
Simple average	-	Moving seasonal relatives	243 389	46.33%	172 069
Moving average	Step = 5	Simple seasonal relatives	259 097	43.96%	170 837
Moving average	Step = 5	Moving seasonal relatives	252 281	43.08%	172 429
Winter's method (trend = default)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	244 884	51.80%	177 029
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	240 935	46.09%	172 659
Winter's method (trend = default)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Moving seasonal relatives	249 823	50.66%	176 412
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Moving seasonal relatives	243 095	44.74%	171 061

Appendix H2-3
A comparison of forecasting techniques applied to deposits
Season = 26 days (using all available data)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	$\alpha = 3.052\text{x}10^{-5}$	Simple seasonal relatives	269 709	60.69%	198 165
Simple exponential smoothing	$\alpha = 3.052\text{x}10^{-5}$	Moving seasonal relatives	310 031	64.49%	214 801
FIT smoothing (trend = default)	$\alpha = 3.052\text{x}10^{-5}$ $\delta = 3.052\text{x}10^{-5}$	Simple seasonal relatives	269 709	60.69%	198 165
FIT smoothing (trend = regressed)	$\alpha = 0.0176$ $\delta = 0.0312$	Simple seasonal relatives	274 652	61.87%	201 466
FIT smoothing (trend = default)	$\alpha = 0.0312$ $\delta = 3.052\text{x}10^{-5}$	Moving seasonal relatives	310 754	59.77%	214 328
FIT smoothing (trend = regressed)	$\alpha = 0.0263$ $\delta = 0.0235$	Moving seasonal relatives	323 581	66.26%	220 620
Trend regressed exponential smoothing	$\alpha = 0.0386$	Simple seasonal relatives	279 056	62.24%	204 045
Trend regressed exponential smoothing	$\alpha = 0.0400$	Moving seasonal relatives	337 346	68.26%	227 668
Simple average	-	Simple seasonal relatives	268 642	58.08%	197 049
Simple average	-	Moving seasonal relatives	300 737	60.35%	210 346
Moving average	Step = 5	Simple seasonal relatives	306 489	61.13%	216 121
Moving average	Step = 5	Moving seasonal relatives	425 587	63.42%	256 960
Winter's method (trend = default)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	271 298	62.33%	199 408
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	270 903	56.22%	195 660
Winter's method (trend = default)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Moving seasonal relatives	300 831	60.40%	210 397
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Moving seasonal relatives	300 479	54.46%	204 932

Appendix H2-4

A comparison of forecasting techniques applied to deposits
Season = 30 days (using all available data)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	α = 0.0530	Simple seasonal relatives	284 331	43.81%	193 144
Simple exponential smoothing	α = 0.0341	Moving seasonal relatives	308 166	47.91%	200 555
FIT smoothing (trend = default)	α = 0.0312 δ = 0.0468	Simple seasonal relatives	287 720	44.93%	196 366
FIT smoothing (trend = regressed)	α = 0.0053 δ = 0.0156	Simple seasonal relatives	289 100	50.83%	198 830
FIT smoothing (trend = default)	α = 0.0189 δ = 0.0625	Moving seasonal relatives	314 911	49.45%	203 407
FIT smoothing (trend = regressed)	$\alpha = 3.052\text{x}10^{-5}$ $\delta = 3.052\text{x}10^{-5}$	Moving seasonal relatives	356 194	60.94%	229 930
Trend regressed exponential smoothing	α = 0.0030	Simple seasonal relatives	290 447	51.14%	199 954
Trend regressed exponential smoothing	$\alpha = 3.052\text{x}10^{-5}$	Moving seasonal relatives	353 853	60.10%	227 430
Simple average	-	Simple seasonal relatives	271 597	49.18%	193 199
Simple average	-	Moving seasonal relatives	321 547	58.51%	207 582
Moving average	Step = 5	Simple seasonal relatives	322 968	47.09%	202 033
Moving average	Step = 5	Moving seasonal relatives	432 587	55.25%	239 110
Winter's method (trend = default)	α = 0 δ = 0 γ = 0	Simple seasonal relatives	272 934	50.13%	194 179
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Simple seasonal relatives	274 551	45.35%	192 724
Winter's method (trend = default)	α = 0 δ = 0 γ = 0	Moving seasonal relatives	309 540	54.24%	200 702
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Moving seasonal relatives	307 072	49.69%	202 899

Appendix H3-1

A comparison of forecasting techniques applied to face-to-face withdrawals (using all available data)

Season = 6 days

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	$\alpha = 0.3012$	Simple seasonal relatives	359 031	74.83%	248 351
Simple exponential smoothing	$\alpha = 0.3199$	Moving seasonal relatives	366 527	74.17%	252 905
FIT smoothing (trend = default)	$\alpha = 0.3125$ $\delta = 3.052\text{x}10^{-5}$	Simple seasonal relatives	359 666	74.78%	248 523
FIT smoothing (trend = regressed)	$\alpha = 0.3125$ $\delta = 3.052\text{x}10^{-5}$	Simple seasonal relatives	359 000	76.45%	248 246
FIT smoothing (trend = default)	$\alpha = 0.3203$ $\delta = 3.052\text{x}10^{-5}$	Moving seasonal relatives	366 356	75.17%	252 917
FIT smoothing (trend = regressed)	$\alpha = 0.3125$ $\delta = 3.052\text{x}10^{-5}$	Moving seasonal relatives	365 771	76.90%	253 140
Trend regressed exponential smoothing	$\alpha = 0.2931$	Simple seasonal relatives	357 856	76.62%	248 283
Trend regressed exponential smoothing	$\alpha = 0.3116$	Moving seasonal relatives	365 670	76.87%	253 034
Simple average	-	Simple seasonal relatives	353 079	87.42%	251 869
Simple average	-	Moving seasonal relatives	355 418	87.54%	257 929
Moving average	Step = 5	Simple seasonal relatives	355 101	71.96%	234 168
Moving average	Step = 5	Moving seasonal relatives	362 198	72.53%	241 345
Winter's method (trend = default)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	353 956	92.13%	257 083
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	342 664	83.60%	243 184
Winter's method (trend = default)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Moving seasonal relatives	356 613	91.21%	262 350
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0.05$ $\gamma = 0$	Moving seasonal relatives	344 622	82.11%	244 036

Appendix H3-2
A comparison of forecasting techniques applied to face-to-face withdrawals (using all available data)
Season = 24 days

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	α = 0.1532	Simple seasonal relatives	286 986	53.90%	202 392
Simple exponential smoothing	α = 0.0972	Moving seasonal relatives	284 840	55.01%	200 761
FIT smoothing (trend = default)	α = 0.1484 δ = 0.0006	Simple seasonal relatives	286 673	53.91%	202 190
FIT smoothing (trend = regressed)	α = 0.1211 δ = 3.052×10^{-5}	Simple seasonal relatives	282 122	55.30%	199 264
FIT smoothing (trend = default)	α = 0.0948 δ = 0.0039	Moving seasonal relatives	284 436	55.53%	201 076
FIT smoothing (trend = regressed)	α = 0.0786 δ = 3.052×10^{-5}	Moving seasonal relatives	282 567	56.58%	201 246
Trend regressed exponential smoothing	α = 0.1205	Simple seasonal relatives	282 061	55.21%	199 124
Trend regressed exponential smoothing	α = 0.0793	Moving seasonal relatives	282 503	56.51%	201 132
Simple average	-	Simple seasonal relatives	290 168	64.37%	210 516
Simple average	-	Moving seasonal relatives	301 970	67.77%	210 338
Moving average	Step = 5	Simple seasonal relatives	300 909	54.17%	203 846
Moving average	Step = 5	Moving seasonal relatives	295 933	58.09%	206 460
Winter's method (trend = default)	α = 0 δ = 0 γ = 0	Simple seasonal relatives	292 947	70.42%	216 145
Winter's method (trend = regressed)	α = 0 δ = 0.3 γ = 0	Simple seasonal relatives	280 954	62.94%	206 505
Winter's method (trend = default)	α = 0 δ = 0 γ = 0	Moving seasonal relatives	307 087	73.02%	214 572
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Moving seasonal relatives	291 804	64.87%	209 543

Appendix H3-3
A comparison of forecasting techniques applied to face-to-face withdrawals (using all available data)
Season = 26 days

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	$\alpha = 3.052 \times 10^{-5}$	Simple seasonal relatives	282 232	70.24%	223 087
Simple exponential smoothing	$\alpha = 3.052 \times 10^{-5}$	Moving seasonal relatives	318 368	80.11%	240 271
FIT smoothing (trend = default)	$\alpha = 0.0625$ $\delta = 3.052 \times 10^{-5}$	Simple seasonal relatives	301 150	70.02%	232 830
FIT smoothing (trend = regressed)	$\alpha = 0.0176$ $\delta = 0.0234$	Simple seasonal relatives	292 628	74.21%	229 129
FIT smoothing (trend = default)	$\alpha = 3.052 \times 10^{-5}$ $\delta = 3.052 \times 10^{-5}$	Moving seasonal relatives	318 368	80.11%	240 271
FIT smoothing (trend = regressed)	$\alpha = 0.0235$ $\delta = 0.0273$	Moving seasonal relatives	332 338	83.08%	245 939
Trend regressed exponential smoothing	$\alpha = 0.0367$	Simple seasonal relatives	298 744	74.33%	231 014
Trend regressed exponential smoothing	$\alpha = 0.0384$	Moving seasonal relatives	343 258	84.63%	249 542
Simple average	-	Simple seasonal relatives	281 946	68.64%	222 329
Simple average	-	Moving seasonal relatives	315 550	77.37%	238 402
Moving average	Step = 5	Simple seasonal relatives	331 641	71.34%	247 318
Moving average	Step = 5	Moving seasonal relatives	381 454	74.06%	258 388
Winter's method (trend = default)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	283 196	72.14%	224 550
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	293 777	67.90%	227 237
Winter's method (trend = default)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Moving seasonal relatives	316 916	78.84%	239 361
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Moving seasonal relatives	329 150	73.28%	236 267

Appendix H3-4

A comparison of forecasting techniques applied to face-to-face withdrawals (using all available data)

Season = 30 days

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	α = 0.1265	Simple seasonal relatives	322 704	59.98%	199 049
Simple exponential smoothing	α = 0.0273	Moving seasonal relatives	338 918	69.75%	198 369
FIT smoothing (trend = default)	α = 0.1259 δ = 3.052×10^{-5}	Simple seasonal relatives	322 704	59.99%	199 073
FIT smoothing (trend = regressed)	α = 0.1250 δ = 3.052×10^{-5}	Simple seasonal relatives	324 044	64 82%	203 779
FIT smoothing (trend = default)	α = 0.0157 δ = 0.0625	Moving seasonal relatives	342 986	72.37%	197 722
FIT smoothing (trend = regressed)	α = 3.052×10^{-5} δ = 3.052×10^{-5}	Moving seasonal relatives	465 911	118.26%	323 155
Trend regressed exponential smoothing	α = 0.0350	Simple seasonal relatives	326 617	72.20%	209 427
Trend regressed exponential smoothing	α = 3.052×10^{-5}	Moving seasonal relatives	460 694	116.28%	317 478
Simple average	-	Simple seasonal relatives	315 001	70.75%	214 553
Simple average	-	Moving seasonal relatives	383 268	100.90%	249 132
Moving average	Step = 5	Simple seasonal relatives	340 150	60.84%	213 442
Moving average	Step = 5	Moving seasonal relatives	619 447	90.28%	331 723
Winter's method (trend = default)	α = 0 δ = 0 γ = 0	Simple seasonal relatives	314 276	69.15%	212 685
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Simple seasonal relatives	314 289	63.88%	203 353
Winter's method (trend = default)	α = 0 δ = 0 γ = 0	Moving seasonal relatives	345 442	78.52%	212 800
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Moving seasonal relatives	342 680	70.67%	199 770

Appendix H4-1
A comparison of forecasting techniques applied to ATM withdrawals
Season = 6 days (using all available data points)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	α = 0.3183	Simple seasonal relatives	70 346	49.18%	54 162
Simple exponential smoothing	α = 0.3290	Moving seasonal relatives	73 356	48.59%	54 460
FIT smoothing (trend = default)	α = 0.2969 δ = 0.0576	Simple seasonal relatives	70 506	44.77%	53 180
FIT smoothing (trend = regressed)	α = 0.2500 δ = 0.0655	Simple seasonal relatives	70 400	43.82%	52 895
FIT smoothing (trend = default)	α = 0.3047 δ = 0.0586	Moving seasonal relatives	73 002	44.24%	53 579
FIT smoothing (trend = regressed)	α = 0.2974 δ = 0.0585	Moving seasonal relatives	72 653	43.81%	53 282
Trend regressed exponential smoothing	α = 0.3085	Simple seasonal relatives	70 192	47.51%	53 791
Trend regressed exponential smoothing	α = 0.3192	Moving seasonal relatives	73 058	47.08%	54 127
Simple average	-	Simple seasonal relatives	79 433	62.22%	65 256
Simple average	-	Moving seasonal relatives	79 809	61.78%	65 350
Moving average	Step = 5	Simple seasonal relatives	64 835	44.81%	47 724
Moving average	Step = 5	Moving seasonal relatives	65 544	44.06%	47 204
Winter's method (trend = default)	α = 0.3 δ = 0 γ = 0	Simple seasonal relatives	69 725	49.17%	53 957
Winter's method (trend = regressed)	α = 0.2 δ = 0 γ = 0	Simple seasonal relatives	67 523	46.41%	50 945
Winter's method (trend = default)	α = 0.3 δ = 0.1 γ = 0	Moving seasonal relatives	71 169	44.51%	53 142
Winter's method (trend = regressed)	α = 0.2 δ = 0 γ = 0	Moving seasonal relatives	68 537	46.10%	51 604

Appendix H4-2
A comparison of forecasting techniques applied to ATM withdrawals
Season = 24 days (using all available data points)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	α = 0.2512	Simple seasonal relatives	65 903	44.37%	48 435
Simple exponential smoothing	α = 0.3326	Moving seasonal relatives	100 060	45.29%	61 042
FIT smoothing (trend = default)	α = 0.2500 δ = 3.052x10^{-5}	Simple seasonal relatives	65 903	44.38%	48 439
FIT smoothing (trend = regressed)	α = 0.2354 δ = 3.052x10^{-5}	Simple seasonal relatives	65 391	42.89%	48 037
FIT smoothing (trend = default)	α = 0.2969 δ = 0.0605	Moving seasonal relatives	98 485	41.75%	59 560
FIT smoothing (trend = regressed)	α = 0.2969 δ = 0.0584	Moving seasonal relatives	97 819	41.54%	59 343
Trend regressed exponential smoothing	α = 0.2361	Simple seasonal relatives	65 492	42.90%	48 051
Trend regressed exponential smoothing	α = 0.3223	Moving seasonal relatives	99 195	43.91%	60 435
Simple average	-	Simple seasonal relatives	71 736	54.53%	58 942
Simple average	-	Moving seasonal relatives	78 624	54.21%	61 423
Moving average	Step = 5	Simple seasonal relatives	58 723	41.68%	44 564
Moving average	Step = 5	Moving seasonal relatives	66 208	39.34%	48 372
Winter's method (trend = default)	α = 0.3 δ = 0 γ = 0	Simple seasonal relatives	73 614	45.23%	51 834
Winter's method (trend = regressed)	α = 0.2 δ = 0 γ = 0	Simple seasonal relatives	61 687	42.36%	46 798
Winter's method (trend = default)	α = 0.2 δ = 0.3 γ = 0	Moving seasonal relatives	78 680	41.21%	56 553
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Moving seasonal relatives	67 158	46.04%	50 964

Appendix H4-3
A comparison of forecasting techniques applied to ATM withdrawals
Season = 26 days (using all available data points)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	α = 0.3001	Simple seasonal relatives	56 143	37.45%	43 633
Simple exponential smoothing	α = 0.4102	Moving seasonal relatives	77 698	39.75%	55 609
FIT smoothing (trend = default)	α = 0.2450 δ = 0.0625	Simple seasonal relatives	57 417	34.70%	43 517
FIT smoothing (trend = regressed)	α = 0.2450 δ = 0.0391	Simple seasonal relatives	56 779	34.35%	42 937
FIT smoothing (trend = default)	α = 0.3672 δ = 0.0625	Moving seasonal relatives	76 099	37.54%	55 412
FIT smoothing (trend = regressed)	α = 0.3593 δ = 0.0625	Moving seasonal relatives	75 535	37.28%	55 105
Trend regressed exponential smoothing	α = 0.2871	Simple seasonal relatives	55 758	35.91%	42 862
Trend regressed exponential smoothing	α = 0.3984	Moving seasonal relatives	76 326	38.59%	54 825
Simple average	-	Simple seasonal relatives	71 570	51.02%	59 516
Simple average	-	Moving seasonal relatives	80 814	52.12%	62 373
Moving average	Step = 4	Simple seasonal relatives	54 825	35.42%	40 642
Moving average	Step = 4	Moving seasonal relatives	67 611	35.76%	47 516
Winter's method (trend = default)	α = 0.3 δ = 0 γ = 0	Simple seasonal relatives	58 168	38.08%	45 566
Winter's method (trend = regressed)	α = 0.3 δ = 0 γ = 0	Simple seasonal relatives	55 550	35.94%	43 083
Winter's method (trend = default)	α = 0.1 δ = 0.55 γ = 0	Moving seasonal relatives	67 692	34.58%	49 647
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Moving seasonal relatives	65 096	41.16%	49 352

Appendix H4-4

A comparison of forecasting techniques applied to ATM withdrawals

Season = 30 days (using all available data points)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	α = 0.2453	Simple seasonal relatives	66 157	44.44%	49 356
Simple exponential smoothing	α = 0.1408	Moving seasonal relatives	206 757	70.89%	106 798
FIT smoothing (trend = default)	α = 0.2452 δ = 3.052×10^{-5}	Simple seasonal relatives	66 156	44.43%	49 355
FIT smoothing (trend = regressed)	α = 0.2344 δ = 3.052×10^{-5}	Simple seasonal relatives	65 462	42.80%	48 739
FIT smoothing (trend = default)	α = 0.1250 δ = 0.0938	Moving seasonal relatives	207 673	77.59%	118 555
FIT smoothing (trend = regressed)	α = 0.1172 δ = 0.0938	Moving seasonal relatives	201 647	77.41%	117 460
Trend regressed exponential smoothing	α = 0.2344	Simple seasonal relatives	65 483	42.80%	48 719
Trend regressed exponential smoothing	α = 0.1248	Moving seasonal relatives	195 661	64.27%	100 444
Simple average	-	Simple seasonal relatives	74 942	56.95%	60 131
Simple average	-	Moving seasonal relatives	113 312	72.61%	92 319
Moving average	Step = 5	Simple seasonal relatives	65 698	40.85%	47 530
Moving average	Step = 5	Moving seasonal relatives	224 216	61.50%	87 263
Winter's method (trend = default)	α = 0.3 δ = 0 γ = 0	Simple seasonal relatives	72 177	45.64%	54 870
Winter's method (trend = regressed)	α = 0.1 δ = 0 γ = 0	Simple seasonal relatives	65 794	45.54%	51 812
Winter's method (trend = default)	α = 0.1 δ = 0 γ = 0.05	Moving seasonal relatives	170 590	68.78%	98 229
Winter's method (trend = regressed)	α = 0 δ = 0.05 γ = 0.15	Moving seasonal relatives	80 911	46.45%	62 237

Appendix H4-5

A comparison of forecasting techniques applied to ATM withdrawals
Season = 6 days (using most recent 56 data points)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	$\alpha = 0.2187$	Simple seasonal relatives	67 235	50.27%	50 012
Simple exponential smoothing	$\alpha = 0.2164$	Moving seasonal relatives	67 815	50.00%	50 564
FIT smoothing (trend = default)	$\alpha = 0.2500$ $\delta = 0.2812$	Simple seasonal relatives	69 180	47.32%	50 062
FIT smoothing (trend = regressed)	$\alpha = 0.2500$ $\delta = 0.2812$	Simple seasonal relatives	69 208	47.34%	50 083
FIT smoothing (trend = default)	$\alpha = 0.2500$ $\delta = 0.2812$	Moving seasonal relatives	70 576	47.65%	51 129
FIT smoothing (trend = regressed)	$\alpha = 0.2500$ $\delta = 0.2812$	Moving seasonal relatives	70 611	47.68%	51 155
Trend regressed exponential smoothing	$\alpha = 0.2170$	Simple seasonal relatives	67 177	50.65%	50 099
Trend regressed exponential smoothing	$\alpha = 0.2140$	Moving seasonal relatives	67 750	50.45%	50 715
Simple average	-	Simple seasonal relatives	64 567	52.98%	49 249
Simple average	-	Moving seasonal relatives	64 839	51.89%	49 430
Moving average	Step = 5	Simple seasonal relatives	54 159	40.20%	39 428
Moving average	Step = 5	Moving seasonal relatives	54 526	39.99%	39 911
Winter's method (trend = default)	$\alpha = 0.2$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	67 935	52.42%	51 631
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	64 617	51.59%	49 392
Winter's method (trend = default)	$\alpha = 0.2$ $\delta = 0$ $\gamma = 0$	Moving seasonal relatives	68 787	52.50%	52 515
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Moving seasonal relatives	65 073	51.33%	49 851

Appendix H4-6
A comparison of forecasting techniques applied to ATM withdrawals
Season = 24 days (using most recent 56 data points)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	$\alpha = 0.0774$	Simple seasonal relatives	58 075	35.98%	41 391
Simple exponential smoothing	$\alpha = 0.0777$	Moving seasonal relatives	61 720	38.25%	39 378
FIT smoothing (trend = default)	$\alpha = 0.0391$ $\delta = 0.0468$	Simple seasonal relatives	58 641	36.91%	42 194
FIT smoothing (trend = regressed)	$\alpha = 0.0085$ $\delta = 0.0312$	Simple seasonal relatives	57 825	36.84%	41 522
FIT smoothing (trend = default)	$\alpha = 0.0352$ $\delta = 0.0468$	Moving seasonal relatives	61 752	38.18%	38 988
FIT smoothing (trend = regressed)	$\alpha = 0.0312$ $\delta = 0.0351$	Moving seasonal relatives	61 592	38.12%	38 787
Trend regressed exponential smoothing	$\alpha = 0.0118$	Simple seasonal relatives	57 718	36.50%	41 362
Trend regressed exponential smoothing	$\alpha = 0.0585$	Moving seasonal relatives	61 368	37.97%	38 883
Simple average	-	Simple seasonal relatives	55 652	37.79%	39 823
Simple average	-	Moving seasonal relatives	60 495	40.00%	37 423
Moving average	Step = 5	Simple seasonal relatives	53 923	32.22%	35 638
Moving average	Step = 5	Moving seasonal relatives	49 108	31.67%	30 153
Winter's method (trend = default)	$\alpha = 0.2$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	60 705	39.73%	42 524
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	58 583	39.58%	42 066
Winter's method (trend = default)	$\alpha = 0.1$ $\delta = 0$ $\gamma = 0$	Moving seasonal relatives	70 616	46.93%	43 085
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Moving seasonal relatives	63 326	40.67%	39 162

Appendix H4-7
A comparison of forecasting techniques applied to ATM withdrawals
Season = 26 days (using most recent 56 data points)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	α = 0.0771	Simple seasonal relatives	49 938	33.49%	35 660
Simple exponential smoothing	α = 0.1960	Moving seasonal relatives	74 431	42.47%	41 782
FIT smoothing (trend = default)	α = 0.0761 $\delta = 3.052\text{x}10^{-5}$	Simple seasonal relatives	49 920	33.49%	35 638
FIT smoothing (trend = regressed)	α = 0.0899 δ = 0.0056	Simple seasonal relatives	50 448	33.90%	36 203
FIT smoothing (trend = default)	α = 0.1960 $\delta = 3.052\text{x}10^{-5}$	Moving seasonal relatives	74 428	42.47%	41 781
FIT smoothing (trend = regressed)	α = 0.2011 $\delta = 3.052\text{x}10^{-5}$	Moving seasonal relatives	75 314	43.31%	42 480
Trend regressed exponential smoothing	α = 0.0937	Simple seasonal relatives	50 687	34.21%	36 488
Trend regressed exponential smoothing	α = 0.2009	Moving seasonal relatives	75 248	43.29%	42 452
Simple average	-	Simple seasonal relatives	46 900	30.43%	33 194
Simple average	-	Moving seasonal relatives	62 187	36.47%	35 384
Moving average	Step = 4	Simple seasonal relatives	52 291	31.22%	35 787
Moving average	Step = 5	Moving seasonal relatives	59 056	32.74%	31 036
Winter's method (trend = default)	α = 0.1 δ = 0 γ = 0	Simple seasonal relatives	52 636	35.45%	37 681
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Simple seasonal relatives	47 525	31.73%	33 975
Winter's method (trend = default)	α = 0.1 δ = 0.05 γ = 0	Moving seasonal relatives	66 098	40.27%	40 791
Winter's method (trend = regressed)	α = 0 δ = 0 γ = 0	Moving seasonal relatives	61 022	34.61%	33 647

Appendix H4-8
A comparison of forecasting techniques applied to ATM withdrawals
Season = 30 days (using most recent 56 data points)

Forecasting method	Smoothing constants	Seasonality	Measures of forecast error		
			RSME	MAPE	MAD
Simple exponential smoothing	$\alpha = 0.0690$	Simple seasonal relatives	73 596	34.38%	35 025
FIT smoothing (trend = default)	$\alpha = 0.0625$ $\delta = 0.1250$	Simple seasonal relatives	76 148	35.75%	36 368
FIT smoothing (trend = regressed)	$\alpha = 0.0234$ $\delta = 0.0157$	Simple seasonal relatives	91 961	50.97%	55 725
Trend regressed exponential smoothing	$\alpha = 0.0304$	Simple seasonal relatives	92 138	50.56%	54 842
Simple average	-	Simple seasonal relatives	77 954	43.65%	49 631
Moving average	Step = 5	Simple seasonal relatives	95 795	44.80%	42 461
Winter's method (trend = default)	$\alpha = 0.1$ $\delta = 0.05$ $\gamma = 0$	Simple seasonal relatives	78 931	40.97%	44 069
Winter's method (trend = regressed)	$\alpha = 0$ $\delta = 0$ $\gamma = 0$	Simple seasonal relatives	66 367	31.79%	35 682

Appendix I

An investigation into the application of the existing order policy as formulated by the branch

April to June 1998

Explanatory note

Appendix I investigates to what extent the order policy as formulated by the branch is indeed applied when ordering cash. MSB denotes main safe balance, CB refers to the cashiers' daily balance, NTD denotes a normal trading day, WE a week end and ME a month end. The investigation is based on Figure 4.2.

Appendix I:
Testing the existing order policy: April 1998

Date	Days (holidays)	MSB	CB	Total on hand excl ATM	Planning for ...	Order schedule as from policy		Actual orders
31-Mar-98	Tuesday	865 935	244 889	1 110 824	NTD	MSB>500 000		
1-Apr-98	Wednesday	530 935	214 321	745 256	WE	MSB<1 000 000	place order	866 000
2	Thursday	234 135	241 078	475 213	NTD	MSB+CB<500 000		
3	Friday	542 535	260 170	802 705	NTD	MSB>500 000		866 000
4	Saturday	158 535	338 714	497 249	NTD	MSB+CB<500 000	place order	
5	Sunday							
6	Monday	587 535	329 220	916 755	NTD	MSB>500 000		1 300 000
7	Tuesday	910 535	217 858	1 128 393	WE	MSB>1 000 000		
8	Wednesday	1 963 635	189 248	2 152 883	NTD	MSB>500 000		1 300 000
9	Thursday	969 635	294 696	1 264 331	NTD	MSB>500 000		
10	Good Friday							
11	Saturday	568 555	375 737	944 292	NTD	MSB>500 000		
12	Sunday							
13	Family day							
14	Tuesday	1 455 555	185 665	1 641 220	NTD	MSB>500 000		600 000
15	Wednesday	1 273 880	253 593	1 527 473	WE	MSB>1 000 000		
16	Thursday	1 614 920	195 262	1 810 182	NTD	MSB>500 000		600 000
17	Friday	1 202 320	285 454	1 487 774	NTD	MSB>500 000		
18	Saturday	1 036 320	358 986	1 395 306	NTD	MSB>500 000		
19	Sunday							
20	Monday	1 526 400	179 439	1 705 839	NTD	MSB>500 000		
21	Tuesday	1 389 400	145 763	1 535 163	NTD	MSB>500 000		
22	Wednesday	1 389 400	212 390	1 601 790	ME	MSB+CB>1 500 00		
23	Thursday	1 124 400	142 530	1 266 930	ME	MSB+CB<1 500 000	place order	
24	Friday	255 640	193 994	449 634	ME	MSB+CB<1 500 000		
25	Saturday	319 640	182 286	501 926	ME	MSB+CB<1 500 000	place order	1 000 000
26	Sunday							
27	Freedom day							
28	Tuesday	229 640	371 010	600 650	ME	MSB+CB<1 500 000	place order	
29	Wednesday	1 096 640	224 676	1 321 316	NTD	MSB>500 000		1 000 000
30	Thursday	93 200	303 859	397 059	NTD	MSB+CB<500 000	place order	

Appendix I:
Testing the existing order policy: May 1998

Date	Days (holidays)	MSB	CB	Total on hand excl ATM	Planning for ...	Order schedule as from policy		Actual orders
1-May-98	Workers' day							
2	Saturday	93 000	244 052	337 052	NTD	MSB+CB<500 000	place order	
3	Sunday							
4	Monday	186 060	378 590	564 650	NTD	MSB+CB>500 000		1 000 000
5	Tuesday	148 220	383 759	531 979	NTD	MSB+CB>500 000		
6	Wednesday	1 220 220	435 105	1 655 325	WE	MSB>1 000 000		1 000 000
7	Thursday	1 083 020	282 705	1 365 725	NTD	MSB>500 000		
8	Friday	767 020	551 500	1 318 520	NTD	MSB>500 000		
9	Saturday	767 020	552 680	1 319 700	NTD	MSB>500 000		
10	Sunday							
11	Monday	982 020	357 135	1 339 155	NTD	MSB>500 000		
12	Tuesday	125 060	350 348	475 408	NTD	MSB+CB<500 000	place order	
13	Wednesday	1 356 370	338 214	1 694 584	WE	MSB>1 000 000		
14	Thursday	1 424 370	309 699	1 734 069	NTD	MSB>500 000		
15	Friday	895 930	498 160	1 394 090	NTD	MSB>500 000		
16	Saturday	895 930	442 554	1 338 484	NTD	MSB>500 000		
17	Sunday							
18	Monday	1 182 420	463 393	1 645 813	NTD	MSB>500 000		
19	Tuesday	1 304 420	412 546	1 716 966	NTD	MSB>500 000		
20	Wednesday	1 375 420	426 158	1 801 578	WE	MSB>1 000 000		
21	Thursday	1 149 420	334 170	1 483 590	NTD	MSB>500 000		
22	Friday	902 420	556 737	1 459 157	NTD	MSB>500 000		
23	Saturday	902 420	619 860	1 522 280	ME	MSB+CB>1 500 000		500 000
24	Sunday							
25	Monday	711 420	512 644	1 224 064	ME	MSB+CB<1 500 000	place order	
26	Tuesday	1 285 020	228 622	1 513 642	ME	MSB+CB>1 500 000		500 000
27	Wednesday	1 179 020	309 803	1 488 823	ME	MSB+CB<1 500 000	place order	500 000
28	Thursday	670 500	319 798	990 298	ME	MSB+CB<1 500 000		
29	Friday	514 500	349 490	863 990	NTD	MSB>500 000		500 000
30	Saturday	154 500	402 942	557 442	NTD	MSB+CB>500 000		500 000
31	Sunday							

Appendix I:
Testing the existing order policy: June 1998

Date	Days (holidays)	MSB	CB	Total on hand excl ATM	Planning for ...	Order schedule as from policy		Actual orders
1-Jun-98	Monday	132 500	492 282	624 782	NTD	MSB+CB>500 000		
2	Tuesday	969 500	331 943	1 301 443	NTD	MSB>500 000		500 000
3	Wednesday	922 500	484 720	1 407 220	WE	MSB+CB>1 000 000		
4	Thursday	1 174 540	414 880	1 589 420	NTD	MSB>500 000		
5	Friday	583 540	575 445	1 158 985	NTD	MSB>500 000		
6	Saturday	583 540	588 893	1 172 433	NTD	MSB>500 000		
7	Sunday							
8	Monday	1 018 940	371 165	1 390 105	NTD	MSB>500 000		
9	Tuesday	1 070 620	430 245	1 500 865	NTD	MSB>500 000		
10	Wednesday	1 143 700	329 038	1 472 738	WE	MSB>1 000 000		
11	Thursday	1 204 500	285 273	1 489 773	NTD	MSB>500 000		
12	Friday	700 500	467 873	1 168 373	NTD	MSB>500 000		
13	Saturday	580 460	526 568	1 107 028	NTD	MSB>500 000		
14	Sunday							
15	Monday	902 460	296 004	1 198 464	NTD	MSB>500 000		
16	Youth day							
17	Wednesday	1 252 460	337 831	1 590 291	WE	MSB>1 000 000		
18	Thursday	1 351 900	246 847	1 598 747	NTD	MSB>500 000		
19	Friday	633 800	544 716	1 178 516	NTD	MSB>500 000		
20	Saturday	633 800	544 878	1 178 678	NTD	MSB>500 000		
21	Sunday							
22	Monday	977 020	271 740	1 248 760	NTD	MSB>500 000		
23	Tuesday	939 060	301 787	1 240 847	ME	MSB+CB<1 500 000	place order	650 000
24	Wednesday	721 300	246 854	968 154	ME	MSB+CB<1 500 000		300 000
25	Thursday	1 170 320	330 445	1 500 765	ME	MSB+CB>1 500 000		650 000
26	Friday	780 900	392 402	1 173 302	ME	MSB+CB<1 500 000	place order	300 000
27	Saturday	730 900	286 719	1 017 619	ME	MSB+CB<1 500 000		
28	Sunday							
29	Monday	461 100	387 836	848 936	NTD	MSB+CB>500 000		
30	Tuesday	240 560	473 335	713 895	NTD	MSB+CB>500 000		

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