# RISK EVALUATION TECHNIQUES

# IN A

# GENERAL INSURANCE ENVIRONMENT

by

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## Abstract

This dissertation considers the management of risks in a general insurance environment.

This dissertation first aims to set a framework whereby business strategies can be evaluated in order to ensure that the business enterprise functions optimally. The process is set in motion through the use of a control cycle:
Understand the risk framework
Specify the problem. These problems will vary.
Develop a solution for the problem. Test the solution and implement the optimal solution.
Monitor the experience to ensure that the solution is indeed optimal.

As all business strategies function in a framework of risk and eventually lead to a balance between risk and return, this study concerns itself with risk evaluation techniques.

The results can give managers and regulators alike additional insight how to optimise business procedures. An example of this would be to optimise the return on shareholders' capital given the contraints laid down by regulators.

As a starting point the general economic and commercial environment in which a general insurance company operates is considered. This is done from the basis of a model insurance company. The business procedures within an insurance company are replicated in the model insurance company. Certain key risk areas relating to such a company are then emphasised and methods appropriate to evaluate risks pertaining to these areas are considered.

A collective risk model is considered whereby the performance of the model insurance company can be compared to benchmarks set within the general economic and commercial environment. Subsequently, the model office can be applied to solve the problem of maximising shareholders' return. This is an iterative process which involves the evaluation of a change in business procedures. Optimal business procedures are identified to meet the criteria specified.

Due to the ever changing nature of insurance business, the general insurer would need to be monitored on a continuing basis in order to ensure that the business procedures remain optimal. The emphasis is on risk evaluation and quantification rather than audit procedures though these two disciplines do overlap.

Once this framework has been set up to ensure proper risk evaluation techniques, the framework can be made available to the relevant stakeholders.

"*We are striving for a framework whose underlying goals and broad strategies can remain relatively fixed, but within which changes in application can be made as both bankers and supervisors learn more. It is the framework we must get right. The application might initially be bare-boned but over time can become more sophisticated.*" – Alan Greenspan, October 1999

Though not the banking industry, this drive is the basis for this study.

**Keywords**
*Optimisation, Return on Capital, General Insurance, Model office*

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# 0. Preface

In an effort to ensure a causal understanding of this dissertation, the following diagram has been included together with an explanation pertaining to the content of each chapter.

Consider the general economic and commercial business environment.

All business strategies are founded in this environment.

An appraisal of the areas of risk and uncertainty is required.

The risks must be evaluated and quantified.

Specify the problem.

A framework for testing business strategies is required.

A methodology to test business strategies is required.

An understanding of the boundaries is required.

Develop the solution.

The framework is developed.

The methodology to test business strategies is developed.

The practical application of the methodology is considered.

Monitor the experience

The arrows indicate the interdependence between the different processes.

The introductory chapter considers the interaction of all the sections. The control cycle is considered, a specific control cycle is proposed and aspects pertaining to each section are considered. In particular the different elements of risk and uncertainty are considered.

In the second chapter the model office framework required to apply the control cycle is considered. This framework is based on three sections namely:

1. The business structure. The business structure is derived from the departmental structure of the company.
2. The business result. The business result following from the departmental structure is reflected in the income statement.
3. Consolidation of the business result. The consolidation over time is reflected in the balance sheet. The balance sheet provides an indication of the value of the company.

An algorithm proposed by other practitioners is also considered.

In the third chapter the methodology required to apply to the framework is considered. This is done by considering the most important areas of risk and uncertainty and proposing and providing a synthesis of the risks that need to be evaluated and the algorithm that will be appropriate for this purpose.

Note that the evaluations specified in these two chapters focus on both the problem of setting an appropriate framework as well as the solution to setting an appropriate framework. The most important areas of risk and uncertainty, based on empirical studies are then considered namely:

1. Pricing risk
2. Reserving risk
3. Asset risk
4. Credit risk
5. Management risk

Chapter 5 considers both pricing and reserving risk under the heading of liability risk because these elements are intertwined with a proper evaluation of the liabilities being taken on (pricing) or the existing liabilities (reserving)

Chapter 6 considers asset risk and how this can be quantified.

Chapter 7 considers credit and expense risk.

Chapter 8 considers corporate governance. The intention here is to ensure that if corporate governance is properly enforced, then the risk pertaining to poor management will be reduced.

Chapter 9 consolidates the ideas into one working framework.

Chapter 10 provides a glossary of terms.

Chapter 11 provides references. The references are important because this dissertation cannot provide the full detail of all the research it is founded on.

Chapter 12 is the annexure which provides a practical application of the ideas set out in this dissertation. It is based on a simple example to illustrate the ideas. The annexure is, however, very important in that it provides the reader with a simple example to intuitively grasp the framework and methodology set out in this dissertation.

# 1. Introduction

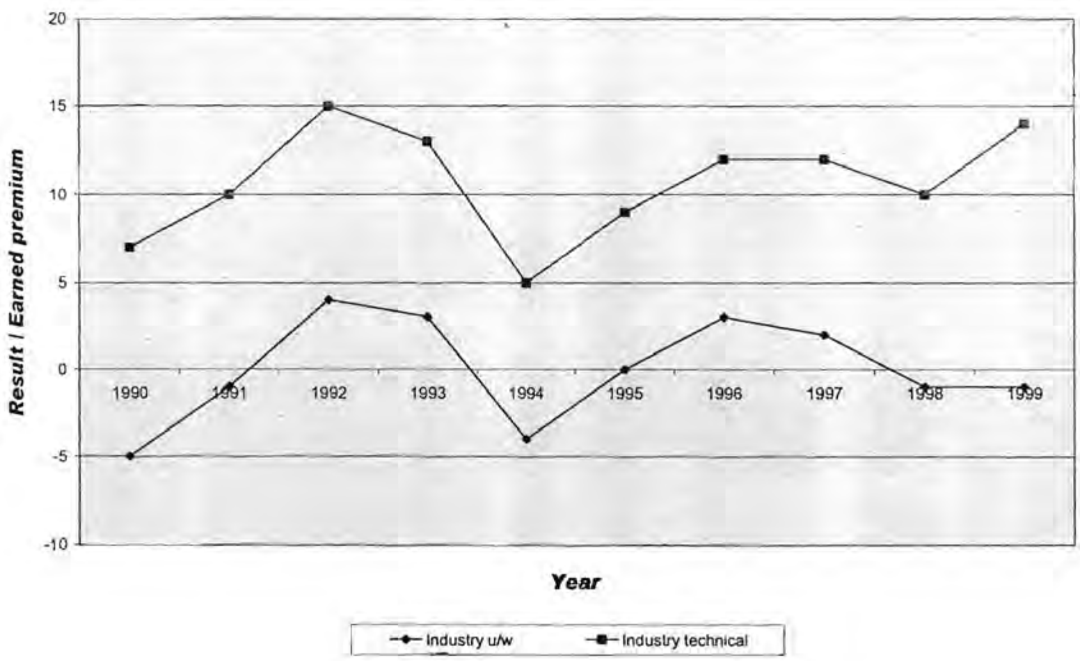
## 1.1 The reasons for this study

In recent years the general insurance industry has been exposed to a variety of events which have adversely affected the entire industry in one way or another. Lloyds of London has been exposed to an aggregation of exposure leading to a spiral of claims. This, combined with adverse underwriting results, has led to the total restructuring of that industry. In South Africa the short-term insurance company IGI has been placed under curatorship due to insolvency. Cycles of underwriting results, namely periods of losses followed by periods of profits[1] have become more pronounced. However, it is getting all the more difficult to adequately increase premium rates to cover past losses in the face of growing competition.

These reasons have led to the realisation that a better assessment of the risks to which a company is exposed is of paramount importance. To use the analogy of a production company: In a normal production company the company first incurs its outgo and then receives its income on the product. For the insurance industry the reverse is true as the company first receives premiums and subsequently pays claims. As a result some managers have, in the past, been relaxed about

[1] Information provided by the Financial Services Board, the regulating authority of insurance companies within South Africa, shows the following graph. The graph clearly shows the results of the entire short-term insurance industry over time from 1990 to 1999. A cycle is apparent in the underwriting results.

***Underwriting cycle comparison***



the assessment of the expected claims cost of a policy as the problem could be solved by writing more policies. It is clear however, that this approach of cash-flow underwriting is neither equitable nor durable. These managers soon realised the problems in this approach as competition for business increased and margins were reduced. The end result was that the necessity for proper risk management, in particular proper and pro-active evaluation of risks, was confirmed.

The field of information technology has also seen tremendous development in recent years. This development has placed very complex modelling techniques within the grasp of many analysts (BRIGHAM 1985). Complex structures can now be modelled using numerical techniques. This approach only recently came to the fore due to the increased computing capability which is now available.

This study has originated from the above two factors namely the need for better analytical[2] risk evaluation and information technology capacity. The aim of the study is to provide a guide for the analytical based methodology as part of risk management to be used in a general insurance environment. This methodology will enable management to obtain a better understanding of the business procedures to the extent that management will be able to gauge the expected variability in the business. Consequently proper risk management principles can then be applied to reduce the cost of risk and optimise the return to the insurer's shareholders.

## 1.2 A structured approach to risk management

Webster's dictionary provides the following definition of risk:

"a hazard; a peril; exposure to injury; the possibility of suffering a loss"

In this dissertation risk will be considered in accordance with the benchmark used to measure risk. When not stated explicitly, risk is assumed to be the possibility of suffering a loss.

Risk can broadly be categorised as follows:

[2] Analytical risk evaluation refers to the process of quantifying risk rather than describing risk in a qualitative manner. Details of such techniques are widely available. DAYKIN, PENTIKAINEN and PESONEN 1994 provide a solid framework.

**Figure 1: An Overview of Risk Categories**

Risk Overview


There is definite overlap between the macro and micro environment as any micro environment is a subset of the macro environment.

Risk management involves the following (CARNEGIE MELLON SOFTWARE ENGINEERING INSTITUTE 2000):
1. Identification: Search for and locate risks.
2. Analysis: Transform risk data into decision-making information.
3. Evaluation: Consider the impact, probability and time frame of the risks identified.
4. Plan: Translate risk information into decisions and mitigating strategies.
5. Implementation of the strategies.
6. Tracking: Monitor and evaluate the strategies. This involves the comparison of actual versus expected experience.
7. Control: Correct deviations from the risk mitigating strategies.

MICCOLIS[3] (2000) considers the assessment of risk to be the first step in the process of enterprise risk management. He states that risk should be described as fully as possible, taking the following into account:
1. Casual factors and consequences.
2. Timing of risk. For example short term or long term or seasonal effects.
3. Correlation with other risks[4], including whether a given risk could trigger or be triggered by other risks and whether certain risks are negatively correlated and therefore present natural hedges against each other.
4. Current risk mitigation strategies and their effectiveness to date.

[3] MICCOLIS is a contributor to the expert commentary published by the International Risk Management Institute www.irmi.com and works for Tellinghast-Towers Perrin. He has published a series of articles on Enterprise risk management which can be obtained from this web site.

[4] Correlation of risks can be regarded as an item that should be treated with extreme caution. EMBRECHTS (2000) shows empirically that correlation matrices cannot be assumed to be fixed over time.

In particular he considers the extreme case of stock market crashes and finds that normal correlation structures change completely. As a result collective modelling approaches cannot be used on a stand alone basis but should rather be considered on a scenario basis in the extreme.

5. Either historical data on or expert assessment of a given risk's impact on financial performance.

This dissertation will consider the application of risk management in a general insurance environment. The approach consists of

1. An appraisal of areas of risk and uncertainty.
2. Identification and specification of the problem.
3. Developing and implementing the solution.
4. Monitoring and control of the solution.

The first component is the same as the first proposed by CMSEI. The second provides a consolidation of components 2 to 4 proposed by CMSEI. The third component is the same as the fifth proposed by CMSEI and the fourth consolidates the sixth and the seventh components proposed by CMSEI.

INSTITUTE OF ACTUARIES AND FACULTY OF ACTUARIES core reading on study material (1995 to 1998) for subject 403 also provide this approach as the basis for its syllabus.

## 1.3 Appraisal of areas of risk and uncertainty.

As indicated in Figure 1: An Overview of Risk Categories, risk is identifiable on a macro and a micro level. Setting all the risk factors without a detailed analysis into a specific insurance operation is not possible as risk on a micro level is dependent on the business operation under investigation. Some of the areas of risk and uncertainty are listed below. This dissertation also includes areas of risk and uncertainty identified by other institutions. These are mentioned in later chapters.

### *1.3.1 Macro risks: The general economic and commercial environment*

Before setting the problem specification of this study, the general economic and commercial environment in which a general insurer operates must be considered and understood. A variety of different risks may arise from this environment. A list of some of the factors in the environment that may have an impact on business operations is provided below. This list is not exhaustive.

#### 1.3.1.1 Statutory and regulatory requirements

The state of the country of domicile of the insurance company may impose certain constraints on insurance companies in order to protect the public interest. In particular the state will be concerned that all companies are run by fit and proper persons and that all companies are sufficiently capitalised in order to protect policyholders' interests. Possible capital requirements will ensure that the company is able to pay claims in adverse circumstances.

A change in statutory and regulatory requirements can have a serious impact on business operations. For example a change in capital requirements might lead to the liquidation of an insurance company. For example, in the instance of increasing capital requirements, some companies with low levels of solvency may subsequently find that they are unable to meet the new requirements. As a result the regulator may require that the company be put in liquidation.

#### 1.3.1.2 Legal or legislative requirements

Legal requirements may be imposed on the insurer. This can range from compulsory insurance to employment policy for staff. The impact of such requirements can enhance or constrain business procedures.

#### 1.3.1.3 Reporting requirements

One aspect that has a tremendous impact on shareholders' appreciation of return is the manner in which results are presented. In a general insurance environment, results may be more difficult to assess due to uncertainties in the business. As a result any changes in reporting requirements may change shareholders' perception of return and the most appropriate manner in which to maximise this return[5].

#### 1.3.1.4 Tax

The fiscal policy of the state will have a direct impact on the return to shareholders. Any change in tax structure can have a direct impact on return. Where tax is not applied consistently to all classes of business, this will lead to changes in business profiles in order to enhance return by paying lower levels of tax.

#### 1.3.1.5 Demographic environment

Changes in the demographic profile of a country and also of the policyholders of an insurer can have an important impact on the results. For example, the current trend in the ageing of populations worldwide is expected to lead to higher costs of medical expenses.

#### 1.3.1.6 Social environment

Social changes, though often unforeseen, can have a devastating effect on the return of shareholders. Recent court action as a result of increased knowledge relating to latent claims, for example asbestosis claims, has lead to the demise of many insurance companies and names in the Lloyds market. In addition increased consumer awareness also leads to increasing numbers of legal actions brought against insurers. This also has a direct impact on shareholders' return.

[5] For some readers this may not be intuitively obvious. Due to the inefficiency of markets and the lack of proper information pertaining to business, estimates are often required in order to present results. The lower the level of information or the more variable the business the less exact the results presented will be.

1.3.1.7 Political environment

Changes in the political dispensation of the country of domicile can impact on the operation of an insurer. In addition political risk will affect the availability of capital in a country of domicile and will therefore affect shareholders' requirements. Changes in the political paradigm may even lead to the closure of an insurer.

1.3.1.8 Economy

The state of the economy will impact on the investment strategy and also the return on investments achieved by the insurer. This has a direct impact on return on capital. Changes can pertain to investment returns, levels of inflation, economic activity, currency rates, cost of labour, borrowing requirements or levels of investment, to name a few. Shareholders' requirements cannot be set without regard for the level of risk inherent in the economy.

1.3.1.9 Technology

Advances in technology have made it possible to present an insurer as a collective risk model. In addition technology also allows insurers to manage their risks quicker through regular reporting. Advances in technology have also made the application of the methodology set out in this dissertation possible.

### *1.3.2 Micro risks: The financial and operational environment of the insurer*

MICCOLIS provides the following list for which business strategies can be developed in order to mitigate risk. Some of these items will be considered in detail later in this study. The first five factors are financial risks while the remainder are operational risks.

1.3.2.1 Pricing

Most insurance companies are in the business of accepting premium in return for the indemnification of insureds in the event of a loss for any or all insureds. Setting a price on the insurance provided is of critical importance for the future existence of the company. If business is written at inadequate rates the company will eventually fail. A proper appraisal of future risks as well as risk inherent in business already sold is important and is the topic of a future chapter.

1.3.2.2 Investment strategy[6]

Investment return is used to enhance premium rates as well as shareholders' returns. A proper understanding of investment opportunities and threats as well as the interaction with liabilities is as important as the appraisal of liabilities themselves. The graph on the first page of this chapter also shows the relative importance of investment returns. They are critical for the future existence of any short-term insurance company.

[6] GOLDMAN, SACHS & Co. and SWISS BANK CORPORATION 1998 provide a thorough non-technical overview of risk management in investment strategy.

Investment risk can be investigated according to the risk inherent in income generated. The expected variability as well as the likelihood of default has to be considered. Similarly assets' capital values need to be considered. A sudden plunge in the value of assets may affect the solvency of the company.

The third aspect pertaining to investment strategy is the extent to which asset cash flows match those of liabilities. Any correlations between these cash flows also need to be considered at this stage.

Asset risk and the models appropriate to quantify this are also covered separately in a future chapter.

#### 1.3.2.3 Capital structure

The most important problem specification considered later is that of optimal capital utilisation. This involves the consideration of the entire business portfolio and subsequently optimising the business process. MICCOLIS (2000) states that the optimisation of return on capital consists of three components:
1. Optimisation of growth of business
2. Optimisation of return on capital
3. Stability of earnings

#### 1.3.2.4 Product mix and marketing strategy

The product structure and distribution channel is arguably one of the most important areas of risk and uncertainty. Furthermore the elasticity of demand for products need to be considered in order to allow for growth expectations relating to business volume as well as earnings. Marketing information for both accepted and declined contracts is required to properly manage this risk.

#### 1.3.2.5 Reinsurance[7]

Reinsurance can function as a substitute for capital. As a result strategies pertaining to reinsurance are important in the optimisation of shareholders' return. Examples of reinsurance strategies will be addressed later in this study.

#### 1.3.2.6 Human resource risks

These risks pertain to the recruitment, training and incentive schemes provided for staff. Human capital is of critical importance but the mitigation of these risks often lie in managerial strategies.

MICCOLIS (2000) differentiates between manageable and strategic risk mitigation. Manageable risks are those that the organisation can address with existing capabilities. These risks might include such things as weak contingency planning in critical facilities or midlevel employees dissatisfied with opportunities for advancement. The proper response to manageable risks is simply to use the

[7] COUTTS and THOMAS 1997 provide further insight in their paper "Modelling the impact of reinsurance on financial strength" This paper is highly recommended.

existing organisational capabilities in mitigating them by assigning them to the appropriate managerial level.

Strategic risk factors are those that have to be addressed with substantial expenditures and / or a change in strategic direction. These can arise when an organisation enters unfamiliar business territory because of a major acquisition, or when a new competitor emerges, or when customers change their buying preferences.

#### 1.3.2.7 Internal controls

These pertain to manageable risks. For an insurance company two areas of internal control that are extremely important are those of credit control and claims control.

Credit control refers to the collection of premium. For a direct insurance company this is less of a burden as the premium will be provided through direct debit order via a bank. For companies using brokers this is, however, not the situation. Broker balances can be substantial and if proper details are not kept then the insurer may very well be on risk for more or less policies than for which premiums are received. Policy records therefore have to be properly audited.

Claim control is also critical. Many insurance claims are fraudulent. The extent of the problem varies but is often cited by market players (R OTTO of OUTSURANCE Insurance Company). As a result it is of the utmost importance that claims that do not appear to be in line with general trends be investigated further and that the number of claims per policy also be investigated. Furthermore companies achieve great savings through incentives to service providers. Some of these strategies will be discussed again in later chapters.

#### 1.3.2.8 Mergers and acquisitions

Mergers and acquisitions require integration of systems and change management. Often the cultures of companies or the operating systems are incompatible. This may render sub optimal returns if it is not managed properly.

#### 1.3.2.9 Technology

Technology has also been identified in the macro risk environment. Technology is extremely important as it has a direct or indirect impact on almost all aspects of the business. A proper appraisal of technology is therefore critical.

#### 1.3.2.10 Customer service

As with technology, customer service is advancing rapidly. Such changes in business processes need to be identified and managed. They do, however, not present risks which should be modelled. These risks are less quantifiable and their mitigation is conducted through proper management.

## 1.4 Identification and specification of the problem

The problem specification of this study is two-fold:

1. A framework or model structure is required to test business practices.
2. The methodology on the assessment and quantification of the different types of risks inherent in this model is required.

A framework is required whereby risk managers can evaluate the risks to which a general insurance enterprise is exposed. This framework must enable managers to quantify the impact of different risks and test different business strategies. Following the tests, managers must be able to optimise their objectives.

Once the problem specified has been discussed, a methodology will be available to test and solve the problems faced by management. Management have to meet their objectives set by shareholders. For this dissertation the primary set of objectives of management is to optimise the level of return on shareholders' capital, optimise the level of growth and ensure a stable return of earnings over time.

The framework will also allow for other objectives and subsets of these objectives. This dissertation will consider the framework on the primary set of objectives specified above but the reader should bear in mind that this is not the sole purpose of the methodology laid out in this study.

The problem specification will be considered further below with reference to the required framework and the optimisation of shareholders' returns.

### *1.4.1 Setting the correct framework*

This study deals primarily with the appropriate framework required to test and implement appropriate business strategies to optimise shareholders' returns. This framework will consider the following:

1.4.1.1 The structure of an insurance company[8]

The business framework of an insurance company must be set up on a simulation basis. This requires that all operational and financial processes be identified and modelled collectively. An evaluation of the financial statements and the areas of risk and uncertainty are required. Through these evaluations the financial model can be determined as well as those risks that are of primary importance.

[8] This dissertation originally started from the work conducted by CHRISTOFIDES et.al. on GISMO (General Insurance Stochastic Model Office) – Short-term modelling for management decisions. This paper was presented at the General Insurance Convention at Stratford-upon-Avon, October 1996.

The paper considers the modelling of all business processes of a general insurance company thereby enabling managers to adopt more appropriate decisions. The paper is based on asset liability modelling techniques and provides excellent reading material for this subject.

The structure is set as follows (and explained in detail in the next chapter):
1. The balance sheet is first considered. This will render the embedded value of the company.
2. The financial statement is then considered. The consolidated movement on the income statement for a certain specified period of time will render the retained earnings available to shareholders after the period of projection.
3. These retained earnings can then be compared to the objectives set and different business strategies can be tested on the same criteria thereby identifying the most appropriate strategies.

#### 1.4.1.2 The structure of the collective risk model

Once the structure of the insurance company has been determined the extent to which different elements of risk and uncertainty will be modelled on a collective basis must be determined. A balance will be required between the cost of such development and the possible return that management can obtain from additional development.

This structure will consider areas such as
1. Liability risk
2. Asset risk
3. Expense risk
4. Capital risk

These items are considered in depth in later chapters.

#### 1.4.1.3 The quality of information

A model of an insurance operation has certain shortcomings as it is never possible to accurately predict the future. It is important to realise that the methodology developed in this study relies on the following:
1. As detailed information as possible on past experience as well as existing and future business structures.
2. Detailed knowledge of business procedures.

These items are combined to form the best estimate of future expected experience. It is important to note that these estimates may be wrong and as a result continual monitoring of expected experience versus actual experience is important to ensure that possible problem areas can be identified and resolved as soon as possible.

BERNSTEIN (1996) warns against applying the techniques of risk management blindly. It is important that the framework be tested regularly and that management is aware of the possible pitfalls inherent in a defective framework.

Once the appropriate framework has been set an objective can be tested in the framework.

### 1.4.2 Evaluation of the primary objective: Optimising business strategies

In order to maximise the return, a benchmark of risk versus return is required. Even though the methodology set out in this study will allow for the investigation into a variety of definitions of risk, the same information might not be available in the market. As a result it may be difficult to set an appropriate benchmark.

For example risk might be defined as the possibility of not paying a dividend in a year. Though quantifiable for the insurer, it may not be quantifiable for other investment opportunities. As a result, risk will initially be considered to be variability in return even though better than expected experience would not normally be considered as risky.]

BRIGHAM (1985) describes three methods to set market benchmarks for the cost of capital.

#### 1.4.2.1 The Capital Asset Pricing Model

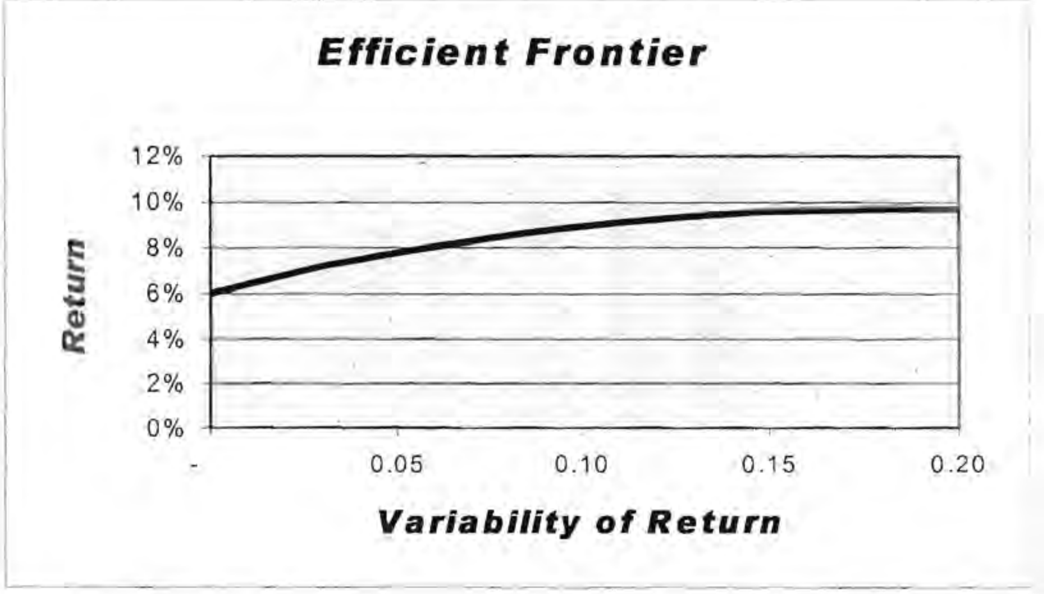
The first method makes use of the capital asset pricing model. This model relies on the efficient frontier[9]. The efficient frontier is the curve presenting the highest yielding portfolio of assets at different levels of risk. The risk is calculated as the standard deviations of returns over a specified period of time. Depending on the level of risk in a certain investment, it is then possible to determine the required return on capital.

[9] A computational procedure for determining the efficient set of portfolios was developed by MARKOWITZ and first reported in his article "Portfolio selection" in the Journal of Finance March 1952, 27 – 91. In this article MARKOWITZ develops the basic concepts of portfolio theory.

The essence of portfolio theory is based on the following ideas:
If an investor were to invest in a single company, the investor would find that by investing in another company with returns independent of the first company, the investor would be able to reduce the variability of return on the portfolio but yet maintain the same level of risk.

As all investors are rational and want to optimise their return at the lowest level of risk, there will exist in the market a range of fully diversified investments with different levels of risk and return where the return will increase with the risk. The range will exist because not all investors have the same view of risk and some may be more risk averse than others.

**Figure 2: An example of the efficient frontier**




For example assume Figure 2 above represents the efficient frontier for the real levels of return[10] in a certain market. Given the expected level and variability of return for a certain investment, it will be possible to determine whether or not that investment is better or worse than an investment in the assets backing the efficient frontier. Clearly, shareholders would only be prepared to invest in a certain investment if it can yield the same or better return than the assets backing the efficient frontier.

Economic theory further holds that the market will quickly eradicate such arbitrage opportunities, where one company has a competitive advantage enabling it to perform better than other investments. It is hence unlikely that any one insurer will be able to perform above the efficient frontier for a significant duration of time.

The problem set is therefore to adjust the business operation in such a fashion to be in line or better than the efficient frontier.

Note that the efficient frontier according to the Capital Asset Pricing Model is given by a straight line. As this model assumes that money can be borrowed and invested at the same rate of interest, the efficient frontier has been adjusted to allow for this incorrect assumption.

It is important to bear in mind that the efficient frontier is not necessarily appropriate as it is based on past returns that may not be achievable in future

[10] Real rate of returns are net of inflation.

again. An allowance for the changes in the general economic and commercial environment will be critical in order to ensure that an appropriate analysis is conducted.

#### 1.4.2.2 Discounted cash flow methods

The second method involves the use of discounted cash flow. Given the price and expected returns expected on a similar investment, a suitable rate of return required for such an investment can be obtained. Shareholders of the company may then require a similar return. Therefore the rate of return required is given by the internal rate of return on similar types of investment.

The internal rate of return is the rate at which the present value of future profits is equal to the market price of the investment at the date of calculation.

$$NPV = \sum_{t=0}^{\infty} Cashflow_t v^t \qquad (1.1)$$

NPV is the net present value of the future cash flows and will take the market value.
t is the time of the possible cash flows from the date of calculation to infinity.
v is the discount factor evaluated at the internal rate of return at time t.

The internal rate of return is obtained through an iterative process until the present value of the cash flows equals the market value.

#### 1.4.2.3 Determine an appropriate risk premium

The third method involves the setting of a risk premium. According to this method the rate of return required will consist of a risk free rate of return plus additional charges for different elements of risk.

Required rate of return = risk free + inflation + risk premium (1.2)

The risk free rate of return does not exist in any market but is approximated by the real returns on long term sovereign debt[11].

Inflation is the long term expectation for future inflation. Note that the first two elements on equation (1.2) are given by the redemption yields of long term sovereign debt issued by the country of domicile.

The risk premium consists of the risk premium applicable to an investment in the particular company. This risk premium consists of a risk component for the relevant industry as well as the relevant company. In order to obtain these risk premiums reference is again required to the second method explained above.

[11] Sovereign debt is debt issued by the government of the country of domicile.

#### 1.4.2.4 Rating agencies

Rating agencies have over time distinguished themselves in identifying the risks inherent in an enterprise. Their risk grades have been quantified and as such managers may be able to use these ratings as a benchmark instead of one of the three methods mentioned above. Given the high value shareholders put on the opinion of rating agencies, this is a very likely scenario.

A comparison of rating agency ratings to the expected probability of insolvency in 10 years is given below (MAHER 2001):

**Table 1: Quantification of Rating Agencies' Rates**

| Moody's | S & P | Expected Probability of ruin in 10 years |
|---|---|---|
| Aaa | AAA | 0.00% |
| Aa1 | AA+ | 0.10% |
| Aa2 | AA | 0.10% |
| Aa3 | AA- | 0.20% |
| A1 | A+ | 0.40% |
| A2 | A | 0.70% |
| A3 | A- | 1.00% |
| Baa1 | BBB+ | 1.40% |
| Baa2 | BBB | 2.00% |
| Baa3 | BBB- | 3.40% |
| Ba1 | BB+ | 5.20% |
| Ba2 | BB | 7.40% |
| Ba3 | BB- | 9.70% |
| B1 | B+ | 12.20% |
| B2 | B | 15.00% |
| B3 | B- | 19.20% |
| Caa | CCC+ | 35.80% |

Using one of the approaches above, the requirement for optimising the business process can be set.

### 1.5 Developing and implementing the solution

This dissertation considers a methodology that can be applied to quantify both the expected return and variability of return of an entire business operation. As a result it will be possible to determine whether or not the business strategies can achieve the results required. In addition the methodology can also be applied to test a variety of business ventures and, based on the results, to set an appropriate course of action for the company to optimise shareholders' return.

The solution revolves around the establishment of a model office. This is similar to a simulation of the entire office given a specified business plan. It is not possible to model all aspects affecting a business operation. The model office will, however, facilitate in the identification of problem areas and aspects which clearly inhibit shareholders' return.

### *1.5.1 Business strategies*

Once the framework of the model office has been established, strategies of business excellence have to be tested. Due to resource limitations, it is not possible to model each and every eventuality. Instead business strategies based on financial and operational issues have to be developed and tested against the requirements of optimal returns, growth and stability.

These strategies will apply to one or more of the following:

#### 1.5.1.1 Business mix strategy

The volume of business by type of business must be considered. Every type of business will have a different risk profile and will have different correlation to the existing business. As a result the impact of writing more business in certain classes will have a different impact to other classes even when their individual risk profiles are similar. This impact can only be determined through a collective model.

Furthermore, the marketing risk must also be considered. Depending on the level of competition in the market and the type of product, the elasticity of demand will vary and as a result business acquisition will come at a varying range of prices for different types of products.

For every type of business the claims management procedures and market custom differ. Often companies will not be able to implement their strategies due to market reluctance to implement the strategies.

#### 1.5.1.2 Managerial strategies

Credit control and claim management are two important aspects to proper insurance governance. By implementing arrangements with assessors or salvage companies or premium collectors, management might be able to enhance the business operation considerably. These strategies will depend on the class of business. Some of the following provide an indication of possible strategies:

1. Scrutinise claims for fraud.
2. Assess claims independently and audit all payments made.
3. Maintain detailed inventories of items on risk in order to check claims at the time of claim.
4. Where necessary require a police investigation.
5. Promote recoveries through salvage.
6. Make use of subrogation rights to sue third parties.
7. Maintain independent record of cover provided and do not pay claims in the event of premium not paid.
8. Ensure that accounting information pertaining to premium conforms to exposure information.
9. Minimise management expenses.

#### 1.5.1.3 Distribution channel strategy

The distribution of business has a critical impact on the success of insurance business. For every type of distribution, the cost of distribution compared to the sale of products needs to be considered. Furthermore, clients' needs are critical. When a distribution channel does not meet the needs of clients, take up following a marketing exercise will be very low.

ROOS (2001) presented a case study to the General Insurance Mini Convention presented by the Actuarial Society of South Africa on the appropriateness of different media for a direct insurance company. This is similar to testing different distribution channels as the take up and the cost of take up is considered in both instances.

Following the advances made in information technology, the number of distribution channels has increased. Also, clients are demanding better service. These developments have led to the necessity for well designed and speedy information systems. Data requirements are discussed in a later chapter.

#### 1.5.1.4 Capital allocation strategy

CLARK[12] (2000) traces the origins of capital allocation back to the banking sector. The essence of such allocation is to ensure that riskier opportunities are allocated more capital to compensate for the higher levels of risk. Such higher allocation also leads to a higher capital charge. This is similar to a risk premium idea but instead of changing the risk premium, the amount of capital allocated to a line of business is changed.

An important idea surfaces from this discussion. The capital requirement is set on a certain specified business structure. This capital requirement is then allocated to the different types of business according to some capital allocation measure. But were the business volumes or business structure to change the capital requirement might also change and as a result new requirements would be set. It is important to note this iterative process.

Capital allocation is critical to ensure that internal management procedures recognise the variability inherent in different classes of business. This ensures that the correct charge is given to the different lines of business. This issue is considered in detail in a later chapter.

#### 1.5.1.5 Investment strategy

The matching of investment income with claims and expense outgo is an important issue that has received extensive coverage in financial papers. CLARK

[12] CLARK is a contributor to the expert commentary published by the International Risk Management Institute www.irmi.com and works for Winterthur International ART & Financial Lines. He has published a series of articles on Enterprise risk management which can be obtained from this web site.

(2000) cites the Savings and Loan crises in the United States in the nineteen eighties as an example of incorrect matching of assets to liabilities.

EMBRECHTS (2000) cites the Long Term Capital Management bankruptcy as an example of incorrect asset liability management. Though not an insurance company, LTCM was also a manager of risks.

Proper asset management is therefore critical and cannot be conducted without a comprehensive understanding of all possible items of outgo. This is a requirement that will only be met through collective risk modelling.

1.5.1.6 Reinsurance strategy

COUTTS and THOMAS (1997) provide a detailed structure for setting the most appropriate reinsurance strategies. As reinsurance mitigates excessive risk it can be considered to be a substitute for capital. As a result reinsurance has a very important part to play in maximising the return on shareholders capital.

The managers will need to consider the cost of reinsurance as compared to the expected cost of claims to the reinsurer as well as the cost of capital required to support the business considered.

Using a collective model it will also be possible to investigate reinsurance structures for individual lines of business as well as the entire business operation.

### *1.5.2 Evaluate business strategies*

The business strategies that have been developed then need to be checked against the requirements specified. On the one hand the requirements for return on capital, stability of returns and levels of growth need to be considered and on the other hand the probability of ruin given the outcomes generated need to be considered. This is no easy matter.

This evaluation will be done by generating financial statements for all classes of business culminating in a collective set of financial statements over time. Given the results generated, it will be possible to evaluate the requirements set and compare different business strategies to the requirements set.

Note that all business strategies need to be considered. During the testing new strategies may also come to light. As a result these would also need to be tested to the criteria specified.

As a collective risk model will require extensive computing power, MICCOLS (2000) suggests that for certain strategies where variables do not affect the strategy, the probability distribution of those variables be replaced by their expected values to speed up the process. Also by maintaining all other variables fixed and only altering isolated strategies will indicate the relative impact of such a change in strategy.

A matrix of strategies can be set up and tested in this fashion to ensure that an optimal set of strategies is employed.

The results of the different combinations of business strategies are then tested to the benchmark in order to identify the optimal set of business strategies.

## 1.6 Monitor the experience

Any model is only as strong as the data, assumptions and methodology supporting it. As a result it is important that these elements will be regularly tested to actual experience and updated as and when discrepancies are identified.

The structure developed in setting the solution should be used to monitor the experience.

The information gleaned from such exercises will indicate possible ways in which strategies can be changed and also ways in which the collective model office can be updated.

The cost of implementing different strategies should also be reflected in the model as any subsequent changes might be very costly and therefore the risk of incorrect implementation should be minimised.

The importance of systems was mentioned. It is critical that all the necessary information required to yield the results is captured on a regular basis. Without such structure, the monitoring described here will also not be possible. Risk data has traditionally not been captured as it was not considered important from a financial point of view. This is no longer the case. Details pertaining to data structures and requirements are discussed later in this study.

## 1.7 Summary

This concludes the introduction to this dissertation. In this introduction the following aspects were considered:

1. This dissertation is a complement to risk evaluation in the general insurance industry. All business strategies operate in a structure of risk and uncertainty.
2. The aim of this study is to discuss a framework and methodologies appropriate to quantify the appropriateness of business strategies for meeting objectives such as the optimisation of shareholders' return on capital.
3. The methodology aims to identify risks and opportunities in a general insurance environment through the development of a model office.
4. The control cycle of this risk evaluation was discussed by considering the general economic environment, specifying the problem, providing the solution and monitoring the results.

MICCOLIS (2000) cites the following advantages to applying this framework:

1. Allows a determination of the necessary capital level for the enterprise, and provides a means to efficiently deploy and improve return on capital.
2. Permits the proper allocation of capital to business segments, thereby improving the performance tracking of those segments.
3. Helps executives evaluate alternative capital structures that leverage returns.
4. Provides a method to ensure that enterprise owners receive proper compensation for the risks they assume.
5. Helps stabilise earnings by identifying and addressing the risks that create the most volatility.
6. Guides the development of an optimal risk financing strategy.
7. Provides better information, which increases negotiating leverage with the enterprises' stakeholders, from shareholders to analysts to regulators to capital markets to merger and acquisition targets.

The next chapter will consider the model insurer. The model insurer is the framework for an insurance company. The model insurer is considered from the basis understandable to shareholders: the financial statements. The financial statements are also used to report the return on capital and are therefore particularly important.

# 2. The model insurer

## 2.1 Introduction

This chapter considers information pertaining to the framework required to evaluate business strategies. This framework is based on a structure that is familiar: The departmental structure of the company is considered. These departments give rise to income and outgo which can be consolidated in an income statement. The result of the income statements over time is reflected in the balance sheet of the company which is subsequently considered. An algorithm suggested by other practitioners is also mentioned. The synthesis of an appropriate algorithm to apply to the framework is considered in the next chapter after the relative importance of the different areas of risk and uncertainty has been considered.

Consider the use of flight simulators in the training of pilots. Flight simulators are very useful, as the trainee is able to experience a variety of circumstances without actually being in a plane. It is therefore possible to simulate the take-off, landing and a variety of situations. Great pains need to be taken, however, to ensure that the simulator will generate the situation as realistically as possible. This normally leads to enormous investment in time and knowledge to ensure that the simulator "flies properly".

The importance of an accurate simulator cannot be over estimated. In the air travel industry people's lives are at stake. Consequently operators of aircraft must take the utmost care to ensure that pilots are properly trained and prepared for any eventuality. The room for error in many situations encountered by a pilot is very small.

The simulator is at the end of the day still just that, a simulator. If the designers of the simulator had failed to foresee certain eventualities or correlations between such eventualities, the pilot may experience such a situation for the first time under real circumstances. This is far from ideal but it has to be borne in mind that a simulator is never perfect.

This example can be used profitably in the context of a general insurance company.

The aim of setting up such a simulator is important. As explained in the introduction, a better appraisal of the risks a company is exposed to is required. Furthermore this appraisal can assist the company in optimising its return on capital.

As the value that will be unlocked using this approach is not always apparent at the outset, care is required to first consider the entire company's framework and subsequently to consider which areas can be modelled and which have to be managed through proper corporate governance. In this study, areas that lend themselves to modelling are considered.

A simulator or a "model insurer" can represent the entire company's operation. The current structure of the company will first be investigated. This can be compared to building the simulator. This process involves identifying all factors affecting the operation of the business and the economic environment in which the business operates. The problem areas are identified and the simulator is constructed to solve the problems identified. Following this, the way the company will "fly" is tested. The next stage would be to test different scenarios i.e. the business strategies mentioned in the introduction. This can be compared to the trainee pilot being subjected to certain conditions and scenarios and monitoring his reactions in such situations. The results seen are then tested to actual experience. This forms part of the monitoring process that is then used to revise the structure of the simulator to provide a better model in future.

As mentioned, room for error in the air travel industry is very limited, hence the quality of the simulator has to be perfect. In the general insurance environment room for error has generally tended to be significant. For instance, market players could change their premiums in the event of poor experience. In recent years these margins have, however, been decreasing. More and more companies are entering the market with new ideas to cut back on expenses, improve service and provide lower rates. The introduction of direct insurance companies in recent years is testimony to this. As a result the companies who are leading the race are those who are able to accurately assess their entire business operation. These are the companies who are willing to invest in the ideas and methodologies described in this study.

As with the simulator, the management of the company will only be able to obtain quality information if the simulator or model insurer is of a high standard. Substantial investment in terms of human resources, time and money in proper data systems and model design is therefore required.

## 2.2 The basic working of a model insurer

### *2.2.1 The aim of implementing a model insurer*

The aim is to develop a stochastic[13] model of the entire business operation to ensure that management can investigate the current business structure as well as the impact of risk management actions on the entire business structure.

[13] Stochastic is often used instead of the word probabilistic to describe a variable or process that has a distribution of possible outcomes each with a possibly different probability of occurring.

### 2.2.2 *The basic operation*[14]

In order to develop a stochastic model of the entire business operation all items affecting cash flows must be investigated. The following considerations should be taken into account:

1. The data available to model expected experience. Data requirements will be considered in a next chapter.
2. The main areas in which optimisation is required. The cost of implementing this exercise can be substantial and therefore the Pareto principle of focusing on 80% of the problems and only incurring 20% of the total cost is relevant.
3. Each and every asset and liability (or obligation) on the books of the insurer must be identified. This is reflected in the balance sheet of the insurance company but due recognition is required for the manner in which the assets and liabilities have been valued. A variety of accounting bases are currently in existence. The important factor here is to realise that a realistic basis is required as the model will be used for internal management purposes[15]. The valuation basis will be discussed further below.
4. Model the expected future cash flows on each asset and liability as well as the expected cash flows on existing and new business for the period of projection. The period of projection can be structured into sub sections as well. The main idea is to set the balance sheet, model the cash flows that occur during the first sub section of the projection period and then set up the new balance sheet at the end of this period. Subsequently, the process will be repeated for the next period and so forth until the company's financial situation has been projected for the entire period of projection.
5. For large companies and for companies with many of the same kind of assets and liabilities, it is not practical to project each and every cash flow. As a result, model points will be chosen to model the expected cash flows. Model points are fictitious assets and liabilities closely representing the actual assets and liabilities by nature of cash flow[16]. One model point can represent several similar assets or liabilities that are actually on the books of the company. The same model point is then used to project the experience of a number of assets, liabilities or policies of the company.

[14] COUTTS and DEVITT (1997) also provide an algorithm that is given in section 2.4.1.

[15] Companies often include margins in the basis used to value assets and liabilities. In a general insurance environment this is done to ensure that the solvency position of the company is not over estimated and also that the profits are not over estimated.

When evaluating the future income streams, the results will be affected by any over or under estimation of assets and liabilities that would become apparent in the evaluation of actual versus expected experience but the reasons for such discrepancies might not be clear.

[16] The nature of cash flows refers to the timing, amount, currency and volatility of cash flows. Model points are also further explained in the glossary.

6. It is important to realise that both assets and liabilities need to be modelled consistently to ensure that the interaction between these can be allowed for. For example, high interest rates will be correlated with higher claims experience in mortgage protection business. High levels of interest rates should also be correlated to higher levels of inflation.
7. Once the model points have been set up, the expected cash flow for each such point can be modelled. Determining the models whereby this can be done will be discussed extensively in other chapters of this dissertation.
8. The cash flows at different durations can then be determined.
9. By simulating this process through the use of Monte Carlo Simulations[17], probability distribution functions can be obtained for a variety of items of cash flow. In this manner a probability distribution function can be determined for the solvency position of the company at each sub period of projection. Similarly a probability distribution function can be determined for the retained earnings in each year. Therefore it is possible to determine a probability distribution function for the return on capital in each sub period as well.

## 2.3 A possible company structure

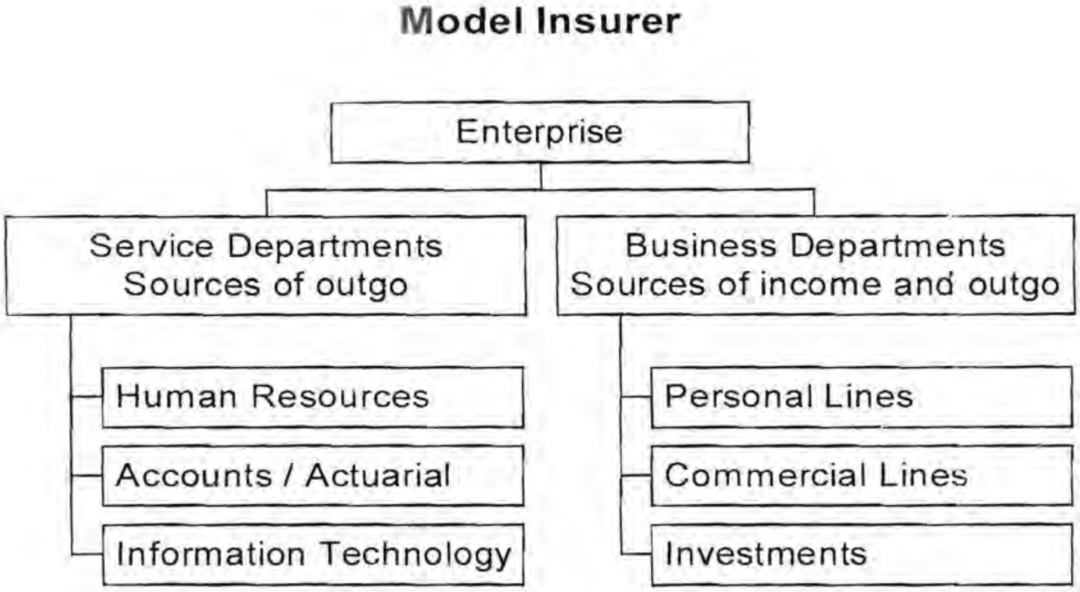
A structure is required whereby the future cash flows can be modelled. The departmental structure of the company as well as the financial statements of the company will provide a guide for the most appropriate structure. Furthermore, it is important to identify which cash flows can be modelled and which not. Where a stochastic approach is not possible the average expected cash flow will be used.

### *2.3.1 Departmental structure*

The model insurer's structure can be represented as follows:

[17] Monte Carlo Simulations refer to the resampling process whereby different possible outcomes, given a specified hypothesis of the distribution of these outcomes, are generated. Each sample is assumed to be independent of any other sample. Simulation techniques are discussed further in later chapters.

**Figure 3: Possible Departmental Structure for a General Insurance Company**




Arguably a variety of different structures may exist. For the purpose of this dissertation it is essential that consideration be given to the structure of the company. This will help the model builder to identify different model points as well as areas of risk and uncertainty affecting these model points. Figure 3 represents a possible layout of a company structure. Each one of these departments will incur expenses and the business departments will receive premium income while the investment department will receive investment income.

The structure of the items of income and expenditure must be scrutinised to determine where these items originate. This is required in order to identify and manage risk areas. In particular the risks associated with the business and investment department are considered in more detail in later chapters of this dissertation.

Accounting information provides a solid framework to combine all cash flows. In particular, the income statement and balance statement must be considered.

In this dissertation consideration is given to accounting information based on a best estimate. Full detail is provided in the Accounting Guide on Short-Term Insurance, THE SOUTH AFRICAN INSTITUTE OF CHARTERED ACCOUNTANTS (2001).

### *2.3.2 Income structure*

The departmental structure outlined above will generate income and outgo during a period of time. This is reflected in the income statement of the company.

## Income statement of a general insurance company

### 2.3.2.1 Evaluation of earned premium[18]

Unearned Risk Reserve[19] at the start of the financial period

less Deferred Acquisition Costs[20] at the start of the financial period

less Unearned Risk Reserve ceded to reinsurers at the start of the financial period

plus Deferred Acquisition Costs ceded to reinsurers at the start of the financial period

Written Premium[21]

less Acquisition Expenses

less Reinsurance Premium ceded

plus Reinsurance Commission

less Unearned Risk Reserve at the end of the financial period

---

[18] The constituents of earned premium can be presented in a variety of circumstances. The structure below considers each item separately.

[19] The unearned risk reserve consists of the unearned premium reserve and the additional unexpired risk reserve. Paragraph 27 of the accounting guide reads: "Earned premium is that that relates to the risks covered during the accounting period. Some policies incepting in the previous (or prior) accounting period will not have expired and the proportion of the written premium relating to the unexpired period of these policies is carried forward as unearned premium to be treated as earned premium of the next accounting period or, where appropriate, future accounting periods. Written premium of the current accounting period is treated as earned premium except to the extent that it relates to unexpired periods of risk at the balance sheet date."

[20] Paragraph 83 and 84 of the accounting guide reads: "A proportion of the acquisition expenses and reinsurance commission revenue is deferred commensurate with the deferral of the related premium revenue and outwards reinsurance premium expense.
Acquisition expenses generally consist of commission paid to intermediaries only."

[21] Paragraph 15 of the accounting guide reads: "Written premiums in an accounting period comprise all premiums relating to policies incepting in the accounting period. They include the premiums for the whole of the period of risk covered by the policies regardless of whether or not these are wholly due for payment in the accounting period. Written premiums also include adjustments to the written premiums of prior accounting periods."
Paragraph 19 reads: "Written premium includes an estimate of pipeline premium"
Paragraph 80 reads: "Portfolio premiums payable are included within premiums for reinsurance outwards in the financial statements of the transferor insurer, but deferred to subsequent accounting periods as appropriate in respect of any unexpired period of risk at the balance sheet date. In the financial statements of the transferee insurer they are included with written premiums with any amount unearned at the balance sheet date being carried forward in the unearned premiums provision.

plus Deferred Acquisition Costs at the end of the financial period

plus Unearned Risk Reserve at the end of the financial period

less Deferred Acquisition Costs ceded to reinsurers at the end of the financial period

**Earned Premium for the financial period**

### 2.3.2.2 Evaluation of incurred claims

Outstanding Claims Reserve[22] at the end of the financial period

plus Incurred But Not Reported Claims Reserve at the end of the financial period

less Reserves ceded to Reinsurers at the end of the financial period

plus Claims Paid[23] during the financial period

less Subrogation Recoveries and Reinsurance Recoveries

less Outstanding Claims Reserve at the start of the financial period

less Incurred But Not Reported Claims Reserves at the start of the financial period

plus Reserves ceded to Reinsurers at the start of the financial period

**Incurred claims for the financial period**

[22] Paragraph 34 of the accounting guide reads: "Provision is made at the balance sheet date for the expected ultimate cost of settlement of all claims incurred in respect of events up to that date, whether reported or not, together with related claims handling expenses, less amounts already paid. If liability exists but there is uncertainty as to its eventual amount, a provision is nevertheless made."
The guide also refers to the necessity of distinguishing between outstanding claim reserves and incurred but not reported claims reserves.
Paragraph 38 reads: "The level of claims provisions is set such that no adverse run-off experience is envisaged" Note: This may not be a best estimate approach and hence the valuation basis used is of critical importance when evaluating the results.

[23] Paragraph 81 reads: "Portfolio claims transfers are accounted for in the financial statements of the transferor insurer as payments in settlement of the claims transferred. For the same reason, the consideration receivable by the transferee insurer is credited to claims payable in the balance sheet.

#### 2.3.2.3 Income statement

| | |
|---|---|
| | Earned premium for the financial period |
| less | Incurred claims for the financial period |
| | **Underwriting result** |
| less | Expenses[24] |
| plus | Investment Income[25] |
| | **Technical Result** |
| Less | Tax |
| Less | Dividends |
| | **Retained income** |

The revenue account and income statement provide a good background to understanding the different areas' activities and therefore the different areas of risk and uncertainty. Given a certain model point for a specific type of policy some of the issues that may be identified are:

1. The premium can be inadequate to meet the future claim payments and expenses.
2. The reserves set aside now may be inadequate to meet the future outgo for which they have been constituted.

[24] Expenses refer to all management expenses that have not been included in the claim-incurred amount. Expenses allocated to claims include allocated and unallocated loss adjustment expenses. These are expenses incurred by the claims department while processing claims.
Paragraph 43 of the accounting guide reads: "Provisions are made at the end of the accounting period for all claims handling expenses to cover the anticipated future costs of negotiating and settling claims that have been incurred, whether reported or not, up to the balance sheet date. Separate provisions are assessed for each category of business.

[25] The allocation of investment income is one of considerable debate: The main line of thinking is that business units have no control over investment performance and therefore investments should be allocated to business units based on a tracking fund that matches the liabilities of the business units. It should be noted also that investment income can only be allocated on cash flow generated and not reserves booked because reserves may be raised without the funds actually being available for investment. This is done in order to assess the profitability of each policy written on an individual or specific class of business grouping basis. No such allocation has been shown in the income statement set out above.

All remaining investment income would be attributed to better than expected investment activity as well as returns on shareholders' capital, both of which should accrue directly to shareholders.

3. The profitable classes of business can be distinguished from the unprofitable classes of business.
4. The impact on the company's experience of reinsurance structures failing can be assessed.
5. Patterns in claim recoveries.
6. Investment income may be inadequate.

This list is not exhaustive.

A company's reserving policy plays a very important role when it comes to the amount of money recognised as profit in a year. This is because reserves are recognised as incurred claims and therefore a conservative policy will lead to a delay in the recognition of profit and an optimistic policy will lead to an acceleration of the recognition of profit but with the greater possibility of recognising unforeseen losses at later durations. Due to this uncertainty, the business needs to be monitored on a regular basis to ensure that estimates remain as accurate as possible.

### *2.3.3 Capital structure*

The balance sheet represents the layout of the value of the company to a certain extent. All factors affecting each of the components of the balance sheet can be modelled in order to project the value of the company in future.

The result of the projection of the income statement will affect the balance sheet. In particular, the retained earnings generated will be transferred to the equity base of the company.

**Balance sheet of a general insurance company**

Ordinary Share Capital[26]

plus Share Premium Account[6]

plus Revaluation Reserve[27]

plus Contingency Reserve[28]

plus Retained Earnings[29]

**Capital and Reserves**

This is the net asset value of the company. Assuming all items are provided at best estimate the net asset value provides the total investment by shareholders to date into the company. As a result this would then be the amount that needs to service shareholders' requirements. That is shareholders expect a return on their capital invested.

The net asset value is represented by the difference between the assets and the liabilities of the company.

2.3.3.1 Evaluation of the assets of the company

Fixed Assets

Property, plant and equipment[30]

Subsidiaries

Investments[31]

[26] Reference will be made to the authorised share capital as well as the issued share capital. The share premium will be calculated on the issued shares.

[27] The revaluation reserve reflects unrealised positions in the assets held by the company. Furthermore, differences in exchange can be reflected in this reserve.

[28] The Contingency Reserve is required in terms of the Short-Term Insurance Act (1998). This constitutes 10% of written premium net of admissible reinsurance.

[29] The build up of retained earnings is reflected in the income statement for a specified period of time. This is discussed further in 2.3.2 below.

[30] In the notes to the financial statements reference will be made to the book value (cost), exchange differences, additions, disposals, depreciation charge, accumulated depreciation and the net book value.

[31] All investments are shown at a best estimate value. That is the value at which the investment can be sold based on market prices.

Current Assets

Receivables and prepayments

Current investments

Bank and cash balances

Technical Assets

Reinsurers' share of technical provisions[32]

Deferred Acquisition Costs

**Total Assets**

The investment strategy might change depending on the results obtained during the year. This will be discussed further in 2.4.1 below. The assets need to be of such a nature that they secure a return and are liquid enough to meet liability outgo as and when required.

2.3.3.2 Evaluation of the liabilities of the company

Technical Liabilities

Unearned Premium Reserves

Outstanding Claims Reserves

Additional Unexpired Risk Reserve

Deferred Acquisition Costs ceded to Reinsurers.

Current Liabilities

Borrowings

Deferred income tax[33]

**Total Liabilities**

[32] Reinsurers will share in the Outstanding Claims Reserve, the Incurred But Not Reported Claims Reserve, the Unearned Premium Reserve as well as other technical reserves that might be set up from time to time. It may well be that no reserve is shown for non-proportional covers and that these are calculated at the outset net of reinsurance.

[33] Note that assessed losses give rise to deferred income tax assets as these losses have to be off-set against future profits.

The balance sheet shown is very condensed and should be expanded upon if a true and fair representation of the company's financial soundness is required. This is done through the notes appended to the financial statements.

An important aspect regarding the risks to which the company is exposed is highlighted in the balance sheet. This is because the company's capital depends on the value of its assets as well as the value of its liabilities. Hence the relationship between these two must be investigated extensively. It is therefore important not only to assess the risks associated with the liabilities (i.e. claims) on the one hand and those with the assets on the other hand, but also the interaction between the assets and the liabilities.

Once the capital base has been determined, the business processes used to secure a return on the capital structure must be evaluated. This is best done by evaluating the returns secured on the different lines of business. These returns are consolidated in the income statement of the company.

Below a generic algorithm is provided, this algorithm can be compiled for any subsection of the business and therefore provides a very powerful base to assess the financial health of the existing business operation.

## 2.4 Combining ideas

Once the model points[34] and the areas of risk have been identified, the next step will be to try to model those areas of risk relevant to the different model points as accurately as possible. This will form a substantial part of the investigation. Once this has been completed the cash flows on each model point can be projected.

This process will be considered in further detail in the next chapter. The following provides an algorithm provided by other practitioners as an example of a possible algorithm suitable for this approach.

### *2.4.1 An algorithm for the business process*

S.M. COUTTS and E.R. DEVITT have suggested the following algorithm for the simulation of the business process of a reinsurance company. The ideas are, however, directly applicable to direct short-term insurers as well:

*Step 1*

Determine the cash inflows for the year in respect of premiums for each line of business. This will relate both to business on the books and to new business and will involve modelling payment patterns which make due allowance for variability and the impact of factors such as commission and reinsurance.

[34] Refer to the glassary of terms for a definition on a model point.

*Step 2*
For each line of business, generate total gross claim payments for the year using assumed probability distributions to generate losses by time and amount for that line of business. This will be obtained from the model points used to represent the liabilities.

*Step 3*
Determine the aggregate cash flows in respect of the insurer's reinsurance programme, making due allowance for bad debts.

*Step 4*
Generate cash inflows in respect of investment income by applying the models for investment yields to the assumed opening asset portfolio. Once again, the model points representing the asset structures are extremely important.

*Step 5*
Aggregate the results of steps 1 to 4 and determine whether cash flow for the year is positive or negative.

*Step 6*
If cash flow is positive the assumed reinvestment strategy is applied. This will lead to a new set of assets at the end of the year. Proceed to step 10.

*Step 7*
If cash flow is negative determine whether there are assets of sufficient value remaining to meet the payments. If there are, proceed to step 8. If there are not, proceed to step 9.

*Step 8*
Apply assumed disinvestment strategy to arrive at a new set of assets at the end of the year. Proceed to step 10.

*Step 9*
Assume sufficient cash is borrowed at some appropriate rate of interest to cover payments for the year. Proceed to step 10.

*Step 10*
Determine whether further claim payments are to be made. If yes, return to step 1 and repeat a further year. If no, proceed to step 11.

*Step 11*
Record the value of the remaining assets. This can be positive or negative.

*Step 12*
Repeat the process 1,000 times (say) from the start, to build up a distribution of possible values of the strength of the company.

This method shows what requirement is placed on capital at different durations. No indication is given as to a possible allocation of capital and the result this will have on the simulated experience. The capital required would, however, be an important consideration for pricing purposes. The proper approach is to set the capital allocated in advance as an item of input. Given the results of the investigation, it would be possible to test different levels of capital allocation.

This algorithm can be adjusted to represent the structure of the general insurance company. The principles underlying the process are analogous.

The modelling of the different structures is a complex matter. This is compounded by the fact that the different cash flows are sometimes correlated. These issues are discussed in further detail in later sections of this study.

The advantages of the algorithm proposed by COUTS and DEVITT according to COUTTS and DEVITT are:

1. It places an obligation on the management to make explicit assumptions about key financial variables.

2. It gives management a better grasp of the essential financial dynamics of their business.

3. It provides a method of looking at capital strength not only in terms of expected results but also in terms of probability.

4. It lets the sensitivity of financial results to various parameters be assessed.

5. It facilitates meaningful investigation into such issues as changes in business mix, different reinsurance protection arrangements and different investment strategies.

6. It enables management to investigate different growth strategies and their associated capital requirements.

7. It allows management to investigate fluctuations in the market in advance so that an assessment of the impact can be made.

8. It permits the effect of investment income on profitability to be appreciated.

## 2.5 Summary

In this chapter the framework required to model different business strategies was touched upon. It was indicated that the structure of the enterprise would need to be considered in terms of the income and outgo generated by every department.

This income and outgo can be represented as an income statement which is subsequently consolidated in the balance sheet. This dissertation will consider further how these items of income and outgo can be projected and which items should be considered the most important. These items will also differ depending on the business strategy that is being applied.

The chapter then continues with an example of an algorithm proposed by COUTTS and DEVITT (not dated).

In the next chapter the different areas of risk and uncertainty will be reconsidered and the most important areas identified. A methodology will be proposed whereby the items of income and outgo can be projected in the framework as discussed above.

Once this framework is in place, the modelling of these cash flows will be considered and subsequently the testing of business strategies and the optimisation of the business process will be considered.

# 3. Setting a framework for the model insurer

## 3.1 Introduction

This study has already covered a variety of aspects required to set appropriate risk evaluation techniques in a general insurance environment.

In the introduction the idea of the control cycle was introduced:

1. Appraise the areas of risk and uncertainty. A structured approach to risk management was introduced and a wide range of risks was introduced. In this chapter the areas of risk will be finalised by considering what other sources have suggested on this topic.
2. Specify the problem. Two problems were identified:
    a. A framework or model structure is required to evaluate business practices. This framework will be finalised in this chapter.
    b. A methodology on the assessment and quantification of the different types of risk inherent in this model is required. This methodology will be discussed in later chapters, starting with an evaluation of data requirements and moving on to the main areas of quantifiable risk.
3. Develop the solution. This requires an investigation into a variety of business strategies and testing these strategies against objectives that have been clearly defined. A number of objectives can be set:
    a. The efficient frontier of returns can be specified.
    b. The minimum level of solvency at any point in time can be specified.
    c. The stability of returns over a specified time horizon can be specified.
4. Once an optimal set has been identified, the experience will need to be monitored to test whether or not the actual experience represents that assumed in the model. As changes become evident the strategy will need to be revisited.

## 3.2 Synthesis of the framework

GOLDMAN, SACHS & Co and SWISS BANK CORPORATION WARBURG DILLON READ (1998) suggest the framework requires the following prerequisites before its implementation can be considered:

1. Commitment from senior management
2. Carefully designed policies and procedures
3. Properly staffed risk analysis and monitoring functions
4. Access to reliable technology
5. High integrity data
6. Validated models
7. Experience and judgement

The framework can be presented as follows:

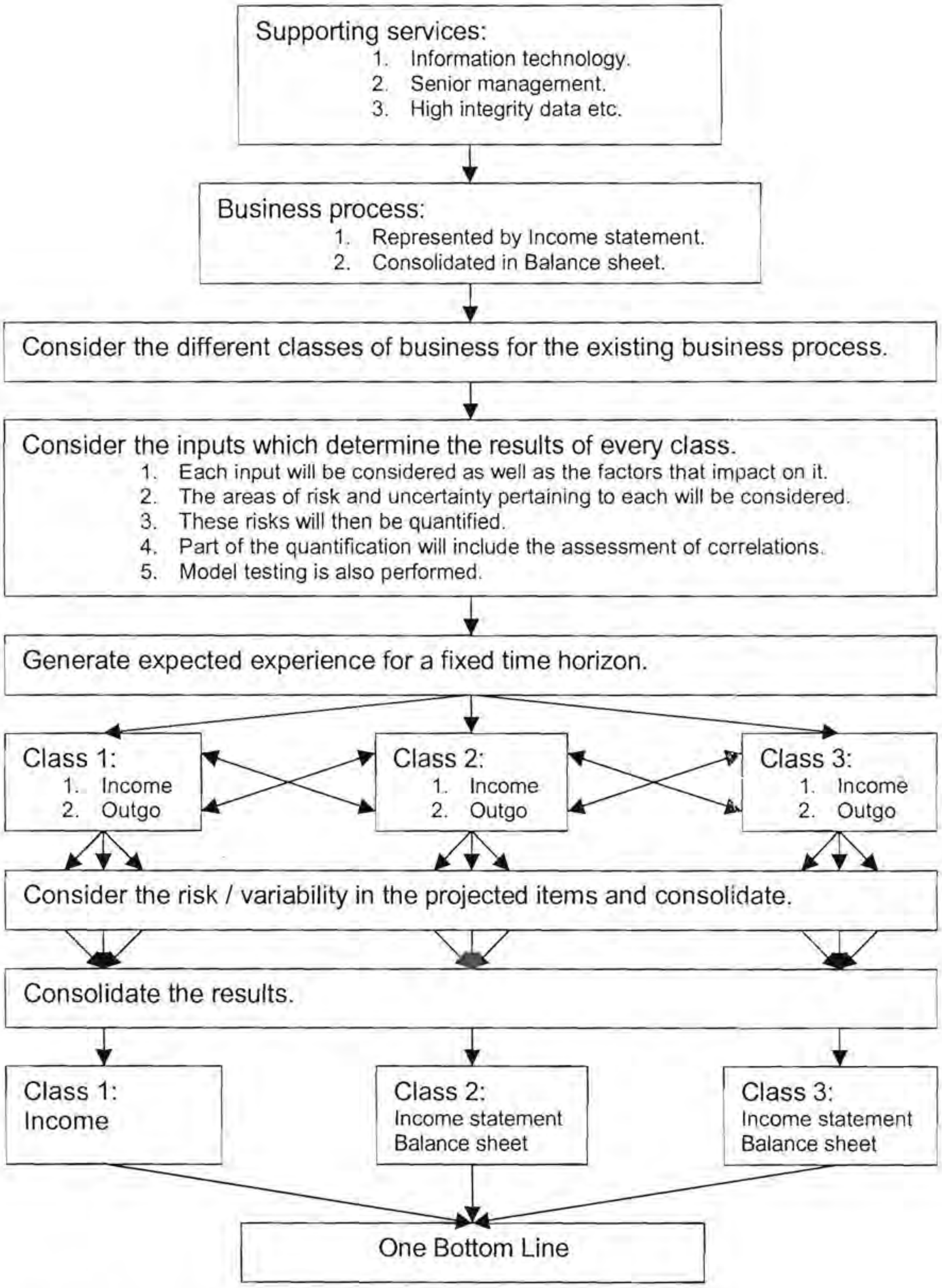



**Figure 4: Framework overview**

This framework is the combination of ideas set out thus far. The commercial environment within the enterprise is considered by evaluating the service and business units. The different classes of business are identified for every business unit.

Recalling the control cycle, the different areas of risk and uncertainty inherent in the business environment must be identified and quantified. This will be done in relation to the existing business strategies.

Proper testing of the model will be required and subsequently the projection will be performed.

This projection is a function of a specified set of business strategies. The starting point will be the existing set of business strategies.

The projection is then considered for each class of business where the income is generated from the business in the department as well as investment income and the outgo is generated based on the business as well as the service departments outgo identified. For each element the risk inherent in the element will be considered and included in the projection. Furthermore the correlations (the cross arrows) will be considered to ensure consistency in the projection.

The projections are then consolidated into revenue accounts and income statements per class of business. These consolidations will include an evaluation of the risk or variability inherent in the projection.

The individual projections will then be consolidated for the entire business enterprise into an income sheet and balance sheet for the entire enterprise.

Both the projections for individual classes of business as well as the projection of the entire enterprise are useful to evaluate business strategies. This evaluation will entail the same control cycle i.e. the identification of problem areas and the testing of solutions to find an optimal solution and the subsequent monitoring of those solutions.

### *3.2.1 A framework or model structure*

In the previous chapter the different structures of the general insurance company were considered i.e. the operation structure, the capital structure and the income structure.

The framework that will be suitable is one that is based on each individual contract and can be consolidated into the different structures as explained previously and illustrated in Figure 4. above

A problem frequently mentioned is that some classes of business are more volatile than others, one such an example being liability business as opposed to private motor business. In such cases, the one class of business may obscure the end result of the other class if the company is viewed on a holistic basis. A dual approach is thus required:

1. The underlying risk structure on each individual line of business must be assessed.
2. The combination of these individual lines of business must be investigated as this will reflect the potential experience of the company.

#### 3.2.1.1 The holistic basis

The holistic basis refers to the fact that the model insurer must include model points of all items in the company giving rise to cash flows. It will not suffice to ignore certain classes as their experience will have an impact on the overall situation. A bottom up approach is therefore required. In particular:

1. For each class of business homogeneous groups of policies must be identified for which the expected experience is similar and for which the assumption can be made that each policy in the group will be identically, independently distributed.
2. Such homogeneous groups will form the model points for the investigation. The risk factors applicable to each model point will be investigated to determine how they influence the expected experience. This will require investigating the risk factors that are expected to have an influence on the frequency and severity distributions of the model points.
3. For assets, the model points will be determined according to the actual investment portfolio. The important aspects to consider will be the expected income stream generated by the asset portfolio as well as the expected level of realised and unrealised asset appreciation or depreciation.
4. When the modelling approach, as discussed in the previous chapter (COUTTS and DEVITT), is implemented, all the model points will be used simultaneously. This approach will assume that each item of cash flow is independent. Where this is not the case, the correlations between the different classes will need to be assessed as well as possible.
5. One aspect that COUTTS and DEVITT failed to address was that the model point structure will change over time. Arguably they did mention that the investments will change. A similar argument holds for the liabilities. The change in exposure over time is a critical requirement to ensure that the future experience is modelled correctly. When setting model points it is important to first evaluate the changes in model points over time. Such changes can be a result of changes in business source or market circumstances.

Before applying the algorithm proposed by COUTTS and DEVITT it is important to project the expected exposure for the period of projection. As COUTTS and

DEVITT rightly pointed out, this change in exposure can also be a result of the actual experience. That is to say the exposure is correlated to the actual experience as is the case with the investment spread.

#### 3.2.1.2 A consolidated algorithm for the proposed framework

Given the ideas set out thus far in this dissertation the consolidated algorithm for the proposed framework can be set out as follows:

##### 3.2.1.2.1 Step 1: Planning

Plan the framework relevant to the business operation. Note that this framework is required to support a variety of investigations from solvency verification to the optimisation of return on capital. The framework is wider than any one objective. In this planning stage the following is required:

1. An understanding of the business process and a realisation that the business result is a reflection of the entire business process. This is critical as optimisation will only be possible when the required changes in business processes can be identified
2. Identification of all the items contributing to the business process.
    a. The actual business on the books or the exposure of business is very important in this regard.
    b. The expected experience on the exposure will determine the business result.
    c. The constraints imposed by management for every type of exposure.
3. Identification of all the areas of risk and uncertainty.
4. Identification of the information requirements necessary to evaluate both the exposure as well as the actual experience on a regular basis.

##### 3.2.1.2.2 Step 2: Understand the needs

Obtain buy-in from all the relevant parties, the most important of which is senior management. Without the active support of senior management, the results identified will not be implemented and as a result the value added will be substantially less. As the framework is holistic, it has to be embraced holistically.

Set up a risk evaluation department with suitably trained personnel to manage the proposed framework.

The framework will be able to support a variety of needs in the company. The interaction with different parties within the organisation needs to be considered:

1. The risk committee who needs to be informed of accumulations of risk and the possible impact thereof.
2. The business units who need to understand the profitability for the different lines of business and the reasons for possible deviations.
3. The finance department who need to produce profit and loss accounts.

4. The regulators who may wish to investigate the model structure and require solvency reports.
5. The auditors.
6. Rating agencies.

Ensure that the necessary information technology structures are in place to provide timely and accurate data. Furthermore the workflow process needs to be audited to ensure that possible sources of data error are eliminated.

#### 3.2.1.2.3 Step 3: Prior investigations

Identify the different sources of risk and uncertainty. Identify those areas that have the greatest probable impact on the business result. This has been discussed in 3.3 above.

Consider how these risks can be modelled and where this is not appropriate how procedures can be put into place to mitigate this risk. This will be discussed further when corporate governance is considered.

For those risks that are quantifiable set up appropriate models and test the models for reasonability. Also conduct sensitivity testing to ensure that the models are robust. It they are not then the constraints on models should be noted and documented. In later chapters this will be discussed at length.

Consider the correlations that exist. Some of these will be correlations between asset classes, correlations between liability classes, correlations between assets and liabilities and correlations between these items and the exposure at any point in time.

#### 3.2.1.2.4 Step 4: Build and document the framework

The exposure structure represented by the model points will be considered first. The starting point will be the existing exposure. On these model points the expected experience will be generated as discussed by COUTTS and DEVITT.

Once one period of experience has been generated the impact of this on the exposure will be considered. This is the change in investment or borrowing that COUTTS and DEVITT referred to but it will also include other independent changes in exposure expected such as a change in policyholder profile.

The framework must be built in such a fashion that full details of all expected model points at different durations as well as expected experience at different durations are recorded.

##### *3.2.1.2.4.1 Step 4.1*

Set up the framework with sufficient flexibility to investigate different time horizons as well as frequency of projections in this time horizon. That is the total duration for which future experience will be projected as well as the number of changes in projections that will be made within this period of projection.

*3.2.1.2.4.2 Step 4.2*

Start with the current exposure. If a different exposure structure needs to be considered, then the manner in which the current structure will change to the proposed needs to be considered. This may be as a result of a marketing drive or change in distribution channel etc.

*3.2.1.2.4.3 Step 4.3*

Based on the models which have been fitted, tested and correlations established, project the expected experience for the first period of projection. This will include the projection of expected claims on the existing exposure, allowance for reinsurance and bad debt, projection of investment income and allowance for all other expenses as noted in the revenue account and income statement. Consolidate the projected experience.

The consolidated experience can be presented by the income and balance sheets discussed previously. All transactions supporting this should also be considered.

Note that the running time can become very slow and therefore an optimisation of the programming structure should also be considered. This will, however, not be discussed further in this dissertation.

*3.2.1.2.4.4 Step 4.4*

Consider the impact of the projected consolidated experience on the exposure of the company. Consider any other expected influences on the exposure as well as the models, if considered necessary. Change the exposure accordingly.

For example, if following the projection, borrowing is required allow for the introduction of a borrowing component in the model points. Or if a change in investment structure is required, allow for this change. Also allow for any other independent expected changes in exposure, for example following the change in distribution channel.

Set up model points for the new set of exposure.

*3.2.1.2.4.5 Step 4.5*

Project the experience for the new set of exposure relevant to the next period of projection and repeat steps 4.3 and 4.4 set out above.

*3.2.1.2.4.6 Step 4.6*

Repeat the process for all periods of projection for the duration of projection initially specified.

*3.2.1.2.4.7 Step 4.7*

Repeat the process a number of times to establish a distribution of possible outcomes for the purpose of the investigation.

Note that for a sufficiently flexible system, it should be possible to limit the capturing of information to only the relevant items necessary for the purpose of the investigation which will accelerate the running time.

*3.2.1.2.4.8 Step 4.8*

Evaluate the results and provide feedback to the party requesting the results.

This process must be documented for every step of the workflow to ensure proper maintenance and robustness following possible staff movements.

3.2.1.2.5 Step 5: Testing

It is useful to conduct back testing exercises whereby the model is run for previous periods of time and the actual experience compared to the expected.

In similar fashion, the monitoring process will be to compare the future experience as and when it occurs to that projected using the framework and thereby improving the framework. Due to changing business environments, model error is quite likely. The ability to allow for changes in the model is critical as the model is required to be a projection of future expected experience. This is discussed further in subsequent chapters.

The framework can then be used to test business strategies to optimise goals set by shareholders.

3.2.1.3 A note on correlations

This approach is extremely useful as the individual classes can be investigated in detail while the combined influence of all classes together can also be investigated. An area of concern, however, is the level of correlation between the different model points. These correlations will be more pronounced under extreme[35] conditions.

Where correlations are expected, these will be modelled by investigating the sample correlations from investigations and applying the results to the model.

[35] Empirical evidence, such as the fact that all share prices fall in the event of a market crash, is proof that model points, which would normally be considered as independent, are in fact correlated, at least in extreme circumstances.

An analysis of the areas of risk and uncertainty now follows together with an identification of the most important areas of risk and uncertainty. Although the importance will vary from one enterprise to the next, this analysis considers the importance for the general insurance industry as a whole.

## 3.3 Analysis of areas of risk and uncertainty

In the introduction macro and micro risks were identified. The reader is referred back to the introduction for a detailed discussion.

A number of institutions have investigated the areas of risk and uncertainty relevant to general insurance operations. Some of the results of these investigations are given below. It is important to note that such results are not all-inclusive. Furthermore, note that not all these risks can necessarily be quantified.

### *3.3.1 The National Association of Insurance Commissioners (NAIC) in the USA*

The NAIC has established a set of rules whereby a minimum statutory requirement of Risk Based Capital (RBC) can be determined for property and casualty insurance companies in the USA. These rules are applied according to perceived levels of risk in various areas. Different action levels are set upon which the company will face different types of intervention from the regulatory authorities. The aim of this dissertation is not to suggest a statutory basis for calculating a minimum capital requirement or to explain the USA Risk Based Capital system. It is necessary, however, to identify the relevant areas of risk and uncertainty which may be practically used in an internal investigation, can be modelled and which have been highlighted by the NAIC.

1. *Asset Risk.*

   It has been identified that the assets held by the company contribute to the level of risk in the business process. The extent to which asset fluctuations match or differ from liability fluctuations must be investigated. Assets will need to match liabilities by amount, term, nature and currency. This issue is investigated further later in this dissertation.

2. *Credit Risk.*

   Credit risk refers to the possibility of broker insolvency and broker balances being lost or that a reinsurer fails to meet its commitments to the direct writer. In effect, credit risk refers to the possibility of default of any third party insofar as that default has an impact on the financial solvency of the company. Brokers and reinsurers are arguably more important credit risks and should therefore be investigated first.

Assessing credit risk can be difficult. The ideal is that the methods described in this dissertation should be applied to the individual brokers and reinsurers in order to determine their possibility of default.

3. *Growth Risk.*

In their paper to the Casualty Actuarial Society, REDMAN & SCUDELLARI (1992) examined a number of reports concerning the causes of insolvency in the USA. Rate of growth proved to be a significant factor associated with insolvencies. It is debatable whether this may have been due to inadequate pricing but it should still be borne in mind.

Solvency is often measured as a percentage of written premium and for a company expanding at a rapid rate the solvency situation will consequently deteriorate. It is therefore a logical consequence that rapid growth has proved to be a significant factor associated with insolvencies. Given that business written will normally expand free reserves[36] by a ratio smaller than the solvency ratio[37] it follows that the rate at which a company can grow is limited by the following:

1. Its current free reserves.
2. Its minimum solvency requirement.
3. The additional expected contribution to free reserves from the additional business sold.
4. The return achieved on reserves held.

4. *Reserve Risk.*

This is the risk that reserves will prove to be inadequate to meet future outgo on liabilities. Due to the varying nature, term and currency of business sold the office needs to assess the reserve risk per line of business *and* keep in mind that the experience on different lines of business is not necessarily independent. For this reason it is also important to investigate the office holistically.

5. *Underwriting Risk.*

Where the reserve risk stated in 4 above refers to business that has already been sold, this risk refers to business to be sold in future. It is of prime importance to ensure that the premium received for future business will be sufficient to meet the future liability outgo on the business sold unless there is

[36] The free reserves are the assets less liabilities less any non-distributable reserves.
[37] The solvency ratio is defined to be the ratio of free reserves to written premium.

a justified strategy to loss-lead with an associated monitoring system to ensure that the losses do not exceed a pre-determined budget.

The NAIC has suggested certain percentages to be applied to premiums and statutory reserves held by companies in order to determine the company's RBC requirement[38]. These percentages,[39] which reflect the deemed level of risk of each of the items given above, are FINNIS (95):

1. Asset Risk 23%
2. Credit Risk 10%
3. Growth Risk 1%
4. Reserve Risk 43%
5. Underwriting Risk 23%

As the purpose of RBC in the USA is that of a regulatory tool, the calculation of the required capital has had to be restricted to robust methods and hence the split of risk given. The reader will appreciate that other areas of risk also confront the insurance company. Such risks include natural catastrophes and asset liability mismatching[40].

### *3.3.2 The General Insurance Study Group (GISG) in the UK*

A working party was set up by the GISG on Capital Requirements and Risk-Based Capital. This working party produced a comprehensive paper on Risk-Based Capital in General Insurance, HOOKER et al. (1995). The paper stopped short of allocating percentages to each type of risk as it did not suggest ways to quantify each of the risks mentioned. In this paper the following areas of risk were identified:

1. *The uncertainty of claim costs.* These uncertainties vary by line of business and by the trends and cycles to which the business might be exposed in the market.

[38] There is a paradox in this method in that the RBC requirement for a company with prudent reserves is higher than that for a company with liberal reserves.

[39] These percentages were set by the NAIC after investigating the financial results of general insurance companies for the period 1983 - 1993. The factors found to have led to insolvency of companies were investigated. The most frequent factors are given together with their assessed contribution to the possible failure of a general insurance company.

[40] The additional complexity has to be weighed against the additional benefit obtained by including these areas of risk.

2. *Volatility in asset values.* This risk is increased when the liabilities and assets are not matched. The variability in liability outgo makes it extremely difficult to match assets with liabilities.

3. *Inflation.* Several forms of inflationary pressure exist on the outflow of the insurance company. These relate to earnings of personnel, escalation of claim handling expenses and escalation of claim costs. There are a variety of reasons why the claim costs could increase but these reasons depend on the line of business. For example, liability claims will be influenced by cost of living increases as well as by increases in court awards[41].

4. *Exposure to catastrophic losses.* The modelling of this risk is extremely difficult given the low frequency and high severity of these losses.

5. *Credit risks.* This is explained as the risk that brokers' balances will be lost in the event of bankruptcy as well as the risk that reinsurers will be unable to meet contractual obligations or be unable to pay claims promptly.

6. *Covariance.* The interaction between different factors is uncertain and these interactions are quite complex due to the fact that there are many areas of risk and uncertainty.

7. *Growth.* Reference was once again made to the study of REDMAN and SCUDELLARI.

8. *Ownership and corporate structure.* Reference is made to the security of borrowings and the security of a parent.

9. *Management competence.* This is referred to as a "soft" issue due to the fact that it is very difficult to assess.

Different areas of risk and uncertainty will be alluded to in the discussion of the modelling of different items of income and outgo.

### *3.3.3 The General Insurance Board Working Party*

The general insurance board of the Institute of Actuaries have set up a working party to provide a framework for the *Financial Condition Assessment* of general insurance companies. The purpose of this assessment is to ensure that the FINANCIAL SERVICES AUTHORITY, the regulatory body in the UK, can be satisfied that a general insurance company is sufficiently solvent to meet its liability requirements. RYAN et.al. (2001) provides a comprehensive overview of the suggestions made by the working party.

The working party suggests a two tiered approach: The first is an identification of all the different types of risk and the second is the consolidation of all these risks

[41] These awards can also have catastrophic effects but are not quantifiable beforehand.

into one on a holistic basis where the correlation between the different risks can be modelled. The paper mentions two bounds on the risk requirement:

1. If all risks were 100% dependent then the capital requirement would be the sum of the capital requirement for all the individual risks[42].
2. If all risks were 100% independent then the capital requirement would be the square root of the sum of all the squares of the individual capital requirements.

The paper provides the following non-exhaustive appendix of possible areas of risk and uncertainty that will need to be considered in the process of assessing the financial condition of the company.

It considers risk split by underwriting risk, asset risk and other risk:

3.3.3.1 Underwriting risk

1. Market issues
    a. Lack of innovation relative of others
    b. Exposure to market forces
2. Underwriting controls
    a. Inappropriate underwriting strategy
    b. Failure to apply underwriting guidelines
    c. Mis-classification of business
    d. Mis-selling
3. Premiums (growth risk)
    a. Rapid growth as a result of under pricing
    b. Fall in volumes
4. Pricing
    a. Incorrect pricing
        i. Claims
        ii. Expenses
        iii. Reinsurance price
        iv. Earnings
        v. Actuarial / accounting system
    b. Incorrect rate relativities
        i. Methodology
        ii. Data
5. Portfolio management
    a. Changes in business mix
    b. Lack of diversification
    c. Increases in line size

[42] This will be discussed in further detail in chapter 9.

6. Claims
    a. Frequency of claims
    b. Frequency of large claims
    c. Catastrophe claims
    d. Mass tort claims
    e. Latent claims
    f. Unexpected exposures
    g. Inadequate claims reserves
    h. Discounting risk
        i. Interest rate assumption
        ii. Cash flow patterns
    i. Excessive claims reserves
    j. Correlation between claims
7. Expenses
    a. Excessive expenses
    b. Incorrect expense reserve
8. Social
    a. Change in the propensity to claim
9. Currency
    a. Movements in exchange rates
    b. Devaluation
10. Reinsurance
    a. Inappropriate reinsurance programme
        i. Net retentions
        ii. Gaps / coverage
        iii. Nature / basis of cover (not matched to business)
    b. Assessment of exposures
    c. Failure of a reinsurer
    d. General reinsurance market failure
    e. Substantial reinsurance price rise
    f. Unavailability of reinsurance
    g. Reinsurance mis-match
    h. Reinsurance disputes
    i. Financial reinsurance
11. Legal / Legislative
    a. Changes in legal system
    b. Changes in court awards
    c. Policy wording interpretation
12. Political
    a. Taxation
    b. Nationalisation
    c. Change in responsibility of claims

### 3.3.3.2 Asset risk

13. Asset risk
    a. Valuation risk – incorrect asset values
    b. Fall in values
    c. Movement in interest rates
    d. Impaired subsidiary
    e. Default risk (credit risk)
    f. Liquidity
    g. Concentration
        i. Counter party / credit risk
        ii. Diversification / volatility risk
        iii. Diversification (return) risk
        iv. Portfolio management risk
14. Premium reserves
    a. Incorrect premium recognition
    b. Incorrect unexpired risk assessment
15. Financing risk
    a. Dividend commitments / expectations
    b. Return on capital expectations
    c. Repayment
    d. Access to capital debt
    e. Debt interest / repayment commitments (cash flow issues)
16. Other economic
    a. Inflation (claims and expenses)
    b. Economic impact on claims frequency

### 3.3.3.3 Other risk

17. Operational risks
    a. Fraud (management / staff / policyholder)
        i. Inadequate controls
        ii. Inadequate procedures for dealing with fraud
        iii. Inadequate prevention / detection mechanisms
    b. Management risk
        i. Not sound and prudent
        ii. Not fit and proper
        iii. Insufficient development of staff
        iv. Unsuitable / insufficient resources / staff
        v. Over-reliance on key persons
18. Lack of information
    a. Technology risk
        i. Inadequate / outdated IT systems
        ii. Technology failure
        iii. Inadequate backup / disaster recovery
        iv. Inadequate security
        v. Failure of processes

b. Administration risk
    i. Failure of procedures / processes
    ii. Failure of outsourcing (service / reputation / monitoring)
    iii. Inappropriate organisational structure
    iv. Inappropriate reporting structure
    v. Roles and responsibilities linked to firm's mission / objectives
    vi. Inappropriate segregation of duties
c. Planning risk
    i. Inadequate strategic / business / marketing plan
    ii. Business structure / direction
    iii. Market share / competition management
    iv. Investment / underwriting strategy
    v. Mission statement business principles and philosophy
    vi. Identify, measure, manage risk to mission
    vii. Resilience testing
    viii. Budgeting and forecasting
d. Business risk
    i. Expense / cash flow / credit controls
    ii. Market knowledge
    iii. Business knowledge
    iv. Change in market conditions and business environment
    v. Mergers and acquisitions
e. Reputation risk
    i. Moral obligation
    ii. Reputation protection risk
f. Control risk
    i. Inadequate corporate governance
    ii. Inadequate systems and control
g. Regulatory risk
    i. Cost of non-compliance (fines, reputation)
    ii. Cost of compliance UK / EU / International
h. Audit risk
    i. Inadequate internal audit
    ii. Inadequate external audit
    iii. Non-functional audit committee
i. Reporting risk
    i. Reliability and timelines
    ii. Usefulness / relevance
    iii. Data integrity
j. Exposure risk
    i. Product and liability insurance
    ii. Inadequate directors and officers insurance
    iii. Inadequate security procedures
    iv. Inadequate / incorrect legal advice
    v. Inadequate risk management

k. Risk management risk
    i. Inadequate risk identification and controls
    ii. Risk exposure limits
    iii. Inappropriate risk / exposure levels
l. Disaster risk
    i. Fire / power cut / strike action
    ii. Natural disaster
    iii. Unsuitable / insufficient continuity plan
m. Initiative risk
    i. New products / processes
    ii. Financing risk
    iii. Poor understanding of business
    iv. Inadequate systems and controls
    v. Acquisition risk
    vi. Growth risk
    vii. Implementation risk

19. Policyholders' reasonable expectations
    a. Marketing should give fair representation
    b. Management of policies should be consistent with PRE
20. Dependency on others
    a. Underwriting – giving the pen away, binders, follower
    b. Concentration of distribution – reliance on a single broker
    c. Outsourcing
    d. External models
21. Group structure
    a. Impaired parent
    b. Impaired affiliate / subsidiary
    c. Non-insurance activities
    d. Off balance sheet items

The paper also includes the results of the study conducted by A.M. BEST (1999). In this study BEST analysed 683 insolvencies in the USA between 1969 and 1998. The reasons for the company failures are summarised below:

**Table 2: Reasons for company failures in the USA between 1969 and 1998**

| | | | |
|---|---|---|---|
| Underwriting risks | Insufficient reserves / premiums | 22% | |
| | Too rapid growth | 14% | |
| | Catastrophe losses | 6% | 42% |
| Asset risks | Overvalued assets | 6% | |
| | Failure of ceded reinsurance | 3% | 9% |
| Other risks | Subsidiaries | 4% | |
| | Significant change of core business | 4% | |
| | Fraud | 7% | |
| | Miscellaneous | 7% | |
| | Non-identifiable | 27% | 49% |
| | | | 100% |

### 3.4 Synthesis on the areas of risk and uncertainty

The results of the empirical studies by FINNIS (1995) and A.M. BEST (1999) differ in quantum. In particular, non-quantifiable risks form a substantial proportion of the risks identified by A.M. BEST.

The investigation by the NAIC is important. This shows the importance of assessing the liability risk for a company properly. The underwriting and reserving risks both refer to proper appraisal of the liability risk.

The analysis also indicates that there are a wide variety of risks that need to be taken into account. The quantification of these risks is not always intuitively obvious.

In this study, the risks that will be first considered are:

1. The liability risk. This risk includes the risk of setting insufficient rates as well as setting insufficient reserves. As indicated in both studies, for general insurance companies this is the most important risk.
2. The asset risk.
3. Credit risk.
4. Management risk.

In this dissertation the liability risk, asset risk and credit risk will be considered in detail and in particular the quantification of these risks. The other risks are all grouped under management risk and are discussed under the heading of corporate governance. The assumption here is that these risks will be mitigated through the proper implementation of corporate governance.

# 4. Data and information requirements

## 4.1 Introduction

This chapter provides the considerations to be taken into account when setting up the information requirements for a model insurer. The information requirements are very broad and they can be divided into two categories:

1. Data requirements on the individual model points.
2. Information required on the general economic and commercial environment.

It is crucial that sufficient time and other resources are spent to ensure the adequacy and accuracy of the data. Without proper data the model of the insurer will have very little value. The analogy of the flight simulator can be used once again. If the simulator is unable to generate life-like experiences (at least to some degree) the trainee pilot will be unable to call upon his training in a real life situation.

The approach followed in this chapter is to consider all items of information required for costing purposes[43] when considering the individual model points. These requirements will differ by line of business. Private motor business will be used as an example in this chapter. The methodology can, however, be applied in similar fashion to other lines of business.

## 4.2 The methodology underlying the data on individual model points

The overall aim of theoretical costing of risk is to calculate as accurately as possible the cost to the insurance company of writing a policy. To this end it has become evident that the claims experience of a company varies according to the risk characteristics of the insured. There are three generic areas[44] that need to be addressed:

1. The frequency of claims
2. The severity of claims (this will include separate allowance for recoveries)
3. Delays in claim payments

In order to calculate these details, policy and claim information records have to be kept separately with appropriate links to ensure that a claim can be traced back to a certain policy and also to a certain period of cover.

[43] Costing of risk refers to the theoretical assessment of the true cost of risk.

[44] A full explanation and further discussion on these three areas will be given in the next chapter.

This means that extensive record keeping on the underwriting or policy side is also required over time[45]. This will ensure that the office's exposure[46] can be determined at different stages.

## 4.3 The principle of correspondence

A crucial aspect of rating is to ensure that actual experience can be traced back to all the policies from which this experience has originated[47]. If exposure information is not available, testing the experience against the exposure information is of little value. The reason for this is that the experience information cannot be applied to any other exposure information and is therefore of historic interest only.

Many companies in the general insurance arena today do not have the appropriate information to solve this problem. In the sections below an explanation is given of the information requirements for private motor business as an example of the thinking behind the data requirements.

## 4.4 The policy information

The policy information will eventually be consolidated into the different model points. This is done by investigating the different risk characteristics of each policy and subsequently identifying which risk factors have a significant impact on expected experience. The policies are eventually grouped by these risk factors. Details of these investigations are discussed in the next chapter.

The starting point is to consider each policy individually. Where this is not possible, appropriate approximations will need to be made. For each policy the company will need to determine the risk inherent in that policy. This can be done by investigating the risk factors[48] relating to the policy. Risk factors are determined through investigation as well as discussions with underwriters and other persons who carry knowledge of the risks underlying the experience.

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[45] Normal practice in the short-term insurance market tends towards capturing only transaction data over time while policy information is amended and only the current records are kept. This means that changes in policy information cannot be traced. This has a direct implication for the ability of the office to relate its claims experience to its exposure.

[46] The term "exposure" has a variety of meanings in the insurance industry and should always be read in context. It generally refers to the exposure a company has to underwriting risk as explained earlier in this study.

[47] This is, in essence, the principle of correspondence.

[48] A distinction between risk factors and rating factors is often found in actuarial literature. Risk factors often refer to all factors affecting the risk relating to the policy while rating factors refer to the factors used in rating the risk. This distinction is of less concern for the purpose of this study. Risk factors will be taken to mean all *quantifiable* factors affecting the risk inherent in a policy. Whether this factor will then be used in rating at a subsequent stage is a marketing or underwriting consideration and not the prime drive behind this study.

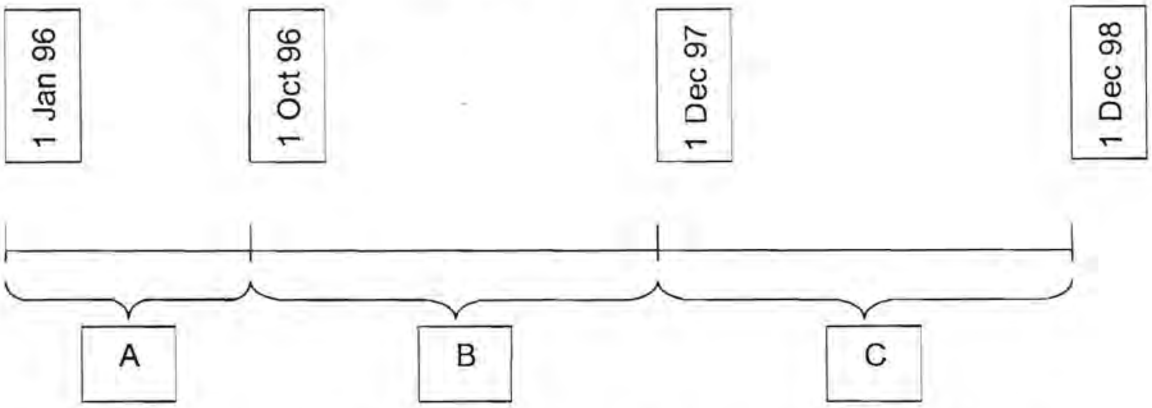
Details regarding the insured and, if different, the drivers of the vehicle, the vehicle itself and the type of cover are required. Furthermore the following list may be considered assuming an investigation of policies with the same type of cover:

- Clarification is required as to who the principle driver of the vehicle is.
- The date of birth of the principle driver.
- The gender of the principle driver.
- The residential address of the principle driver. Postal addresses are more crude in determining location but may be considered instead.
- The date on which cover was first obtained from the company - this is different to the number of claim free years, etc. It has been found that the idea of claim free years is often abused in order to obtain lower rates. This suggestion would need to be investigated.
- Number of claim free years - to test the above-mentioned argument.
- Renewal date of policy.
- Date of cancellation or endorsement.
- Premium.
- Occupation of principle driver.
- Date on which license was obtained.
- Number of speeding fines in the last 5 years.
- Details of excesses[49].
- Private or business use.
- Details of broker.

Note that for each endorsement on a policy a new record must be created. This is to ensure that changes such as change of address are appropriately allowed for. It is also preferable to reflect the number of endorsements and renewals in the policy number but this is not essential. This means that the same policy number is kept but a subsection is allocated to the number of endorsements. The endorsement date is crucial in order to be able to calculate the exposure accurately. The following diagram gives a visual explanation:

[49] It is often found that persons who choose to take a higher excess or deductible on claim payments are more conscious of the risks and their costs. The result of this is that such persons tend to be better risks as they actively apply risk management themselves.

**Figure 5: Evaluating exposure over time**




When a specific policy is considered the characteristics might change. For example in the diagram above the period from 1 October 1996 to 1 December 1997 (period B), the policyholder might have chosen a different excess level or nominated another principle driver or any other change relating to the risk factors might have taken place. As the company will need to compare similar risks it may therefore need to allocate period A, B and C to different risk classes when measuring exposure.

This distinction has to be balanced against the volume of information available.

Details of the vehicle to be insured are also required:

- Age of vehicle.
- Vehicle name and specification.
- Engine capacity.
- Original purchase price.
- Sum insured.
- Security features. Alarm, tracking, gear-lock etc.

It must be stressed that not all factors would necessarily have a significant impact on the cost of risk. The more risk factors that are included, the more scenarios can be tested. It has been found that considerable variation in experience exists between different books of business due to differences in marketing techniques and source[50] of business. This leads to the necessity of investigating all risk factors.

[50] This is based on the author's personal experience in the analyses of such risks.

## 4.5 The claims information[51]

The following is required for each claim:

- The policy number.
- The claim number.
- The date of loss.
- The date of reporting.
- Date of payment.
- The amount paid.
- Status of claim – outstanding or settled.
- The gross cost of claim prior to assessment.
- The gross cost of claim following assessment.
- The excess applicable.
- The type of claim.

If the claims are paid on a one-off basis then no further information will be required. Where settlement takes longer, however, each payment relevant to a specific claim has to be recorded. Outstanding claim records have to be recorded in similar fashion.

As the structure of each claim is different it is important to recognise the different types of claims. The type of claim can be grouped according to the requirements of the company. The following provides a suggestion only:

- Own damage, i.e. claims relating to accidents or damage caused by the insured to the vehicle.
- Third party, i.e. claims caused by the insured to a third party to indemnify the third party against the loss incurred.
- Theft. Theft risk might be further subdivided between theft of vehicles and theft of electronic equipment in the vehicle.
- Hi-jack. This risk can be grouped with theft. Note that the impact of theft and hi-jack is substantial when compared to the other risks as the severity of these claims are for the full sum insured. Consequently careful investigation of the frequency of these risks is required.
- Windscreen

[51] This list applies to other transactions as well, such as recoveries.

- Fire
- Hail

Depending on the level of information, each of the types of claims can be investigated according to the frequency, severity and delay of payment pattern as mentioned earlier. This will be dealt with in the liability valuation section of this study.

In addition, claims experience will be affected by recoveries and reinsurance. Claims will, however, first be considered from ground up and subsequently allowance can be made for all other elements affecting claims.

If claims are captured net of excess payments they will consequently need to be grossed up for any excess paid.

If zero amount claims are not captured then the claim sample will be censored. In order to solve this problem different approaches may be needed such as extrapolating from the distribution fitted to the captured data. The failure to capture zero amount claims impacts significantly on the determination of claims frequency.

If the approach in documenting claims were to change and as a result the frequency of claims were to change, this has to be taken into account. The same holds true for claims management. Hence changes in procedure must be investigated before setting assumptions for modelling.

Full details of recoveries are also required:

- Claim number
- Date of recovery
- Amount of recovery
- Type of recovery for example from salvage, subrogation or reinsurance.

The details set out above provide a framework on which to base the theoretical costing of motor insurance. The lists given are not appropriate to each and every situation but serve as a broad outline. The above should also give the reader an insight into the level and depth of information required to conduct such an analysis.

### 4.6 Data validation

The data available to an insurer is crucial in any investigation regarding past or future experience. As a result the data available must be scrutinised to ensure appropriate and accurate knowledge can be drawn from it. In validating data, potential problem areas include the following:

1. Data specification. Information is often summarised before it is captured. Claims might be captured net of recoveries or net of excess levels. It is important to ensure that full details of each and every transaction are available. This means that details regarding zero claims[52] should also be captured.

2. Incorrect data. This can arise from free form fields or from the incorrect capture of information. It is important to customise a user interface to as great a degree as possible in order to ensure accurate data. It is also important to allocate ownership of the data to a certain department in a company. This is to ensure that the data integrity is maintained at all times.

   Incorrect data can be identified through a variety of tests. Such tests could be spot checks for reasonableness. In addition data reconciliation can be performed by policy numbers, sums insured, premium and duration of cover.

3. Reporting and administrative delays. It is important to track transaction payments over time in order to identify trends in reporting and processing delays. This can be done by capturing the transaction date together with the date of loss or date of reporting.

   It is important to conduct similar investigations by business sources, especially for broker business.

4. Adequate data. This refers to the database design rather than the size of the portfolio. The database must be designed in such a fashion that each and every transaction, once captured, can only be changed by a counter transaction. This will ensure that all details are made available. The system should also be able to accommodate re-opened claims.

5. Exposure details. The principle of correspondence was mentioned earlier. It is crucial to identify the characteristics of the exposure giving rise to claims. Furthermore the underlying structure of homogeneous groupings used in the past may change and as a result inappropriate conclusions may be drawn in future.

6. Missing data. Data may go missing in the extraction process and should be compared to previous data sets and accounting information to ensure all data is considered.

7. Random variation in the data may yield incorrect results. The data may also include abnormal events which are not necessarily an appropriate reflection of future experience.

[52] Zero amount claims can arise from a number of reasons e.g. claims below the deductible, fraudulent claims or claims assessed to be zero. Whatever the reason, it is important to capture this information in order to identify trends and take whatever action is considered necessary.

## 4.7 The volume of data required

Many practitioners, statisticians, actuaries and other interested parties question the level of data required in order to make a proper analysis. The answer to the question lies in the interpretation of what a proper analysis would be.

### *4.7.1 Conventional statistical theory in setting a required sample size*

Conventional statistical theory states that if X(t) is the number of occurrences in the interval [0,t], and $P_n(t)$ in the probability on n occurrences in an interval [0,t] then $P_n(t)$ has a Poisson distribution with parameter $\lambda$ where $\lambda$ is the average number of occurrences in a unit time period[53].

The value of $\lambda$ must be estimated. This is done by investigating the frequency of claims as each claim can be viewed as an occurrence in a Poisson process. The best approach to this is to determine the number of claims and divide it by the exposure of policies that give rise to these claims. The question then arises as to how many claims are required.

The answer to this depends on the underlying distribution of the claims as well as the level of certainty required in estimating the frequency. By using the central limit theorem[54] an approximate number of required claims can be determined:

Assume that the difference between the point estimate and the true population value has to be less than *c* with probability *prob*. If L is the point estimate of $\lambda$ based on the number of claims then the above can be expressed as:

$$P(|L-\lambda| \le c) = prob \tag{4.1}$$

Using the central limit theorem the following expansion can be made:

$$P\left(\frac{-c\sqrt{n}}{\sigma} \le Z \le \frac{c\sqrt{n}}{\sigma}\right) = prob$$

[53] This theorem is subject to the following constraints:
1. X(0) = 0,
2. P[X(t+h) – X(t) = n | X(s) = m] = P[X(t+h) – X(t) = n] for 0 <= s <= t and 0 < h,
3. P[X(t + Δt) – X(t) = 1] = λΔt + o(Δt) for some constant λ > 0, and
4. P[X(t+Δt) – X(t) >= 2] = o(Δt).

This means the following:
1. The possibility of an occurrence in a nil time period is nil.
2. Occurrences are not affected by occurrences in other time periods.
3. The probability of an occurrence is proportional to the time exposed.
4. The probability of more than one claim at a time is nil.

[54] The central limit theorem states that the sum of n independent identically distributed variables with mean $\mu$ and variance $\sigma^2$ will be normally distributed with mean $n\mu$ and variance $n\sigma^2$. Therefore the average has mean $\mu$ and variance $\sigma^2/n$.

as $$Z = \frac{L-\lambda}{\frac{\sigma}{\sqrt{n}}}$$

has a standard normal distribution. For practical purposes σ can be taken to be equal to L as the number of claims follow the Poisson process and n is the exposure in time of the number of policies exposed to the risk of a claim. The value of n can be determined by using the equation

$$\frac{c\sqrt{n}}{\sigma} = \Phi(prob) \tag{4.2}$$

where Φ is the inverse function of the standard normal distribution. It will often be the case in practice that n is known i.e. the number of policies on risk is known and therefore the level of certainty of the test will be determined. For example, where the number of thefts and the exposure of policies during a certain period is known.

The preceding argument shows that lower frequencies are more difficult to model.

### *4.7.2 Conventional practice*

In the past, offices often found themselves with inappropriate data. In these circumstances heavy reliance was often placed on the assessment and opinion of the underwriter or insurance manager[55].

Statistical analyses can now be used to support the views of underwriters and insurance managers. It has been found that tremendous synergy can be had by combining the practical viewpoint of such experts with statistical analysis.

## 4.8 Information requirements on the general economic and commercial environment[56]

Information will be required on the areas of risk stated earlier in this dissertation. The areas to be addressed can be investigated to a great degree of detail. The aim of this study is only to stress the fact that these areas should be taken into account. Details of investigations of such areas are, however, not given.

Some of the information required will include:

1. The statutory requirements imposed by the state of domicile as well as other states in which the general insurance company might operate.

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[55] In Baysian terminology this can be viewed as a prior function regarding the distribution of the parameter sets.

[56] The areas of information mentioned here are extremely important in ensuring proper understanding of the business environment. Substantial investigation may therefore be necessary.

2. The tax environment imposed by the state i.e. detailed tax rules.
3. Government policy regarding regulation of premiums as well as other regulations such as those regarding investments.
4. Current economic outlook. This will be indicative of future growth of business as well as possible moral risks in poor economic situations. In particular, interest rates and rates of inflation must be considered.
5. Past, present and projected levels of economic growth, investment income, asset growth. These factors, together with the factors mentioned in 4. above, are crucial in setting the model points for the assets held by the company. The methodology applied in setting model points for the assets will be discussed at a later stage in this study.
6. The current political situation and any expected changes in future.
7. Demographic movements.
8. The current market situation. This will be indicative of the level of current competition as well as future levels of competition.
9. Current expense structures within the company.
10. Current claims management practices within the company.
11. Current credit control practices within the company.
12. Current reinsurance structures within the company.
13. Credit ratings of external parties.
14. The level of free assets of the company. This will form a crucial part in the analysis of the model office as the solvency of the company is a very important aspect to investigate.
15. The structure of the company with reference to shareholding in the company and the security of the shareholders.
16. The type of business sold by the company.
17. Technological advances made and the impact of these on the business process.
18. The level of adherence to corporate governance and legal compliance.

This list is not exhaustive.

## 4.9 Summary

This chapter has dealt with the considerations of obtaining data for the model insurer investigations. Substantial attention to detail was paid to the structure required on individual model points as this will form the basis for the model insurer.

The data structures required on policy and transaction information were discussed to underline the importance of such information for stochastic modelling purposes. Stochastic modelling will be discussed in the next chapter.

The importance of data validation was stressed and an indication was given of the sample size required to ensure a proper investigation of experience.

Other items of data were also mentioned. In normal business many of these items are certain or fairly predictable and will lead to easier implementation of a cash flow model[57].

In short, quality data is crucial in setting up the model insurer.

[57] Tax structures, for instance, are taken to be fixed. Any other outcome cannot be projected by statistical means.

# 5. Quantifying the liability area of risk

## 5.1 Introduction

This chapter is central to this study. The reason for this centrality is that a fairly new method of quantifying risk is introduced in the form of stochastic simulation[58] and the ideas behind this are then used in a variety of circumstances relevant to quantifying liabilities.

The areas of liability risk to be examined in this chapter are the frequency and severity of occurrences. The severity of occurrences (i.e. claims) is investigated to determine the delay in settlement of claims. The possible differences in parameter sets of the frequency and severity distributions for different classes of business or different homogeneous groups within the same class are also considered. Techniques whereby these parameter sets can be modelled for the different classes are discussed.

## 5.2 Stochastic simulation

Stochastic simulation is the process of generating a random outcome. Consider a stochastic variable X with a certain probability density function f(x) and cumulative distribution function F(x). Stochastic simulation involves generating a random outcome of X given either f(x) or F(x).

The following theorem proves to be very useful when considering F(x):

If X is continuous with CDF F(x), then U = F(x) ~ UNIF(0,1)[59].

This is extremely useful as an outcome can be randomly generated according to the underlying distribution. The following gives an explanation of the process.

1. Generate a random number between 0 and 1.
2. As F(x) is uniformly distributed (0,1) the randomly generated number can be transformed to a random X with distribution f(x).
3. By generating random numbers and calculating the corresponding values of X several times the distribution of f(x) can be determined.[60]

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[58] Stochastic simulation has been mentioned in several publications. DAYKIN, C.D., PENTIKAINEN, T. & PESONEN, M. (1994), *Practical risk theory for actuaries,* is one example. The methodology is, however, fairly new as it relies heavily on extensive numerical techniques which have not been possible in the past due to lack of electronic computing power. Given the developments in the information technology industry this has now changed.

[59] That is to say the cumulative distribution function has a uniform distribution.

[60] This process is, of course, not necessary for a distribution function that can be expressed in closed mathematical form as f(x) can be obtained by differentiating F(x) with reference to x. The process is however very useful to determine f($\underline{x}$) where $\underline{x}$ represents a vector of stochastic variables and F(x) cannot be expressed as a mathematical function.

It can be seen on the graph on the next page that the distribution of X will follow f(x) when F(x) is known. That is to say, very few randomly generated numbers between 0 and 1 will represent the tail values while very many randomly generated numbers will represent the values distributed around the mean.

The dashed line represents a randomly generated number between 0 and 1 which is then used to determine the corresponding x representing P[X<x][61].

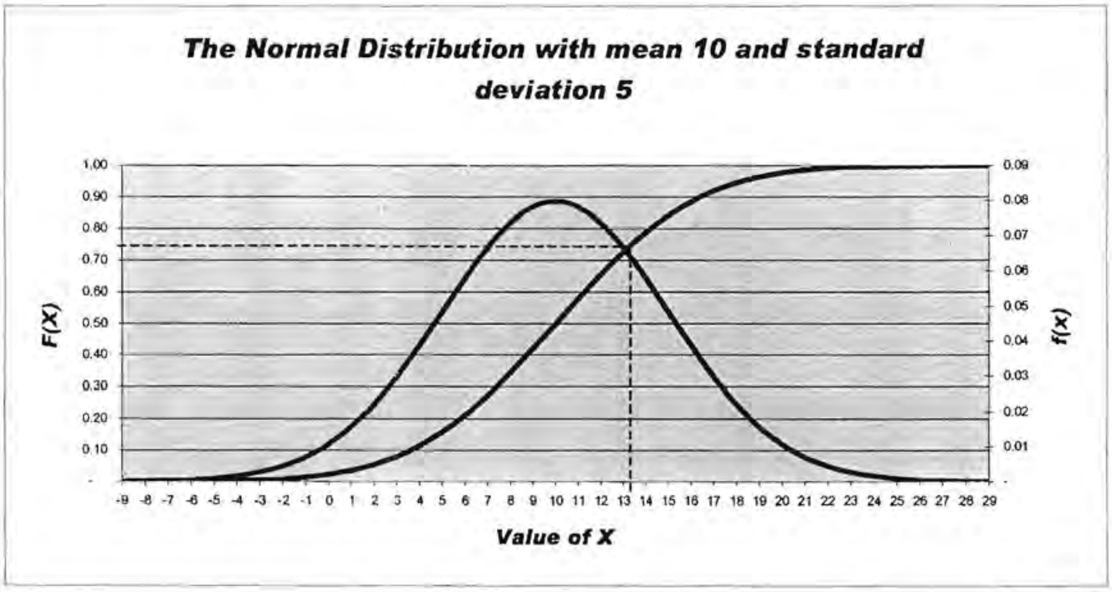


**Figure 6**

Note that as the normal distribution function is not invertible in mathematical form, this approach can only be presented graphically in the case of the normal distribution.

[61] P[X<x] is the randomly generated number.

In order to generate a normally distributed value with an algorithm a different approach would need to be followed[62].

Stochastic simulation can also be applied to discrete distributions. This will often

$$P[N = 0] = e^{-\mu}$$

be the case with frequency distributions as they tend to have a Poisson distribution[63]. Assume that the number of claims N, generated by a certain insurance policy has a Poisson distribution with mean $\mu$. Then

$$P[N = 1] = e^{-\mu}\mu$$

$$P[N = 2] = \frac{e^{-\mu}\mu^2}{2!}$$

etc.

By considering P[N < n] it is therefore possible to group the number of claims into bands of predetermined probability width. For example, all probabilities in the range [0;exp(-$\mu$)] can be associated with a zero claim outcome.

### 5.3 Fitting distributions to claims information

As mentioned earlier several aspects regarding claims information have to be considered namely:

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[62] In one of the workshops presented by the General Insurance Study Group the following code in Visual Basic was presented as an algorithm to generate a normally distributed value with mean 0 and variance 1. This is the work of Andrew Smith:

```
Function normgen() as double
'Return sample from N(0,1) distribution
Static got_one as Boolean, stored as Double
Dim x1 as Double, x2 as Double, r as Double

If got_one then
        normgen() – stored
        got_one = false
Else 'Generate a point on the unit square
Do
        x1 = 2 * Rnd – 1
        x2 = 2 * Rnd – 1
        r = x1^2 + x2^2
'reject unless inside unit circle
Loop While r >= 1 or r = 0
r = Sqr(-2 * log(r)/r)
stored = x1 * r
normgen() = x2 * r
got_one = true
End if
End Function
```

[63] The constraints regarding the Poisson distribution were mentioned in the previous chapter.

1. The frequency distribution of claims.
2. The severity distribution of claims.
3. The payment pattern of claims.
4. The correlations between claims and other claims or other economic variables.

It is important first to consider the different methods of curve fitting. To fully understand the process the practitioner should have a sound understanding of loss distributions[64].

### *5.3.1 The method of moments*

This method involves equating the sample moments M' to the moments of the distribution $E[X^j]$ used in order to solve for the parameter set of the distribution to be used. This method therefore aims to find a distribution with the same mean, variance and kurtosis (depending on the number of variables in the parameter set) as that of the sample.

$$M'_j = \frac{\sum_{i=1}^{n} X_i^j}{n} \tag{5.1}$$

denotes the $j^{th}$ moment of the sample and provides estimators for the mean and variance:

[64] For an extensive discussion on loss distributions the reader is referred to HOGG R.V. & KLUGMAN S.A.(1976).

$$\hat{\mu} = \overline{X} \tag{5.2}$$

and

$$\hat{\sigma}^2 = \sum_{i=1}^{n} \frac{X_i^2}{n} - \overline{X}^2 = \sum_{i=1}^{n} \frac{\left(X_i - \overline{X}\right)^2}{n} \tag{5.3}$$

which is closely related to the sample variance $S^2$ as it converges to the sample variance as n increases,

$$\hat{\sigma}^2 = \left[\frac{n-1}{n}\right] S^2.$$

It is important to note the underlying reasoning behind this method. Considerable emphasis is placed on the mean and variance of the sample. This has to be borne in mind if selecting this approach to curve fitting.

### *5.3.2 Method of maximum likelihood estimation*

This method fits the distribution most likely to reproduce the sample from which the distribution is fitted. Let $L(\underline{\theta}) = f(x_1,\ldots,x_n;\underline{\theta})$ represent the joint density function of n random variables $X_1,\ldots,X_n$ evaluated at the set $x_1,\ldots,x_n$. $L(\underline{\theta})$ then gives the likelihood of that set of values occurring as a function of $\underline{\theta}$. Therefore, by maximising $L(\underline{\theta})$ it will be possible to find the vector set $\underline{\theta}$ most likely to reproduce the sample for a given distribution function.

Maximising $L(\underline{\theta})$ is achieved by considering either the likelihood or the log likelihood function of $L(\underline{\theta})$ and differentiating with reference to $\underline{\theta}$.

$$L(\underline{\theta}) = \prod_{i=1}^{n} f(x_i;\underline{\theta}) \tag{5.4}$$

and then solve

$$\frac{\partial}{\partial \theta_i} \ln L(\underline{\theta}) = 0 \tag{5.5}$$

representing the $i^{th}$ element of the vector $\underline{\theta}$.

Maximum likelihood estimators for large samples have the following asymptotic properties:

1. The estimator exists and is unique;

2. the estimator is a consistent estimator of $\underline{\theta}$, which means that for large samples the difference between the estimator and $\underline{\theta}$ becomes negligible;
3. the estimator is asymptotically normal with asymptotic mean $\underline{\theta}$ and variance

$$1/nE\left[\frac{\partial}{\partial\theta}\ln f(X;\underline{\theta})\right]^2$$

4. the estimator is asymptotically efficient which means that the expected value of $\underline{\theta}$ is equal to the estimator for the specific element of $\underline{\theta}$.

These qualities have lead practitioners to believe that this is, invariably, the best method to use in estimating the underlying distribution. Care must be taken as this method will often produce a distribution with variance smaller than the sample variance due to the fact that the most likely distribution is found. Furthermore the mean found by the maximum likelihood estimation may be vastly different to the sample mean for the same reason.

### *5.3.3 Method of least squares*

The aim of this method is to find the distribution for the dependent variable which minimises the sum of the squared errors between the actual dependent variables and the expected dependent variables. This can be done by considering the probability density function. That is to say, consider the actual number of claims in specific bands compared to the estimated number of claims in specific bands according to the probability density function form that is to be fitted. The parameters of the probability density function can then be adjusted to find the minimum sum of squared errors.

Therefore the following entity is minimised with reference to $\underline{\theta}$:

$$\text{Sum Squared Errors} = \sum_{band=1}^{n}\left(y_{band} - (\hat{y};\underline{\theta})_{band}\right)^2 \tag{5.6}$$

where the first term represents the actual value observed and the second term in the summation represents the expected value of the dependent variable given the parameter set $\underline{\theta}$.

The squared errors may differ in size if the expected number of claims is not the same in each band. To alleviate this problem the errors can be standardised by dividing the squared error by the expected value of the dependent variable[65].

[65] Note that if the number of expected claims within each band has a Poisson distribution then the mean and variance of this distribution is the same.

The standard error divided by the square root of the expected number of claims can therefore be approximated by a normal distribution. This would also be the case for other distributions where the mean and the variance can be approximated by the same value.

$$\text{Sum Standard Squared Errors} = \sum_{band=1}^{n} \frac{\left(y_{band} - (\hat{y};\underline{\theta})_{band}\right)^2}{(\hat{y};\underline{\theta})_{band}} \tag{5.7}$$

### *5.3.4 Some useful distributions*

The aim of this section is to give the reader an insight into some useful distributions which can be applied in the modelling of liability outgo. For every distribution provided below the method of moments and method of maximum likelihood estimators are also given.

5.3.4.1 The Poisson distribution

This distribution is used in the modelling of claim frequencies.

$$f(X;\theta) = \frac{e^{-\theta}\theta^{x}}{x!} \tag{5.8}$$

where $\theta > 0$ and $x = 0,1,2\ldots$

The mean and variance of this distribution are both $\theta$.

The method of moments estimator and the maximum likelihood estimator for $\theta$ is the same:

$$\hat{\theta} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

where N is the exposure of policies[66] and $x_i$ is the number of claims incurred by the $i^{th}$ policy.

In order to randomly generate values for the Poisson distribution, calculate the probabilities $P[X<x]$. Generate a random value between 0 and 1. Find the value

The sum of the squared errors divided by the expected number of claims therefore has a Chi-square distribution. This is so because a standardised normal variable squared has a Chi-square distribution with one degree of freedom and the sum of Chi-square distributions once again has a Chi-square distribution but the degrees of freedom will depend on the number of variables summed as well as the approximations made in determining each Chi-square distribution.

The least squares estimation method therefore develops into the Chi-square estimation method in this instance and for this specific class of distribution the fit with the smallest Chi-square value will therefore be found.

[66] N is not the number of policies as not all policies may be on risk for the same period of time which could skew the results. It is therefore important to calculate the exposure of each individual policy as explained in chapter 4.

of x for which the randomly generated value is the closest to, but smaller than P[X<x].

The example below should clarify this process:

Assume a claim frequency distribution has a Poisson distribution with mean of 5 claims per period.

| | | | |
|---|---|---|---|
| P[X=0] = | 0.006738 | P[X<=0] = | 0.006738 |
| P[X=1] = | 0.033690 | P[X<=1] = | 0.040428 |
| P[X=2] = | 0.084224 | P[X<=2] = | 0.124652 |
| P[X=3] = | 0.140374 | P[X<=3] = | 0.265026 |
| P[X=4] = | 0.175467 | P[X<=4] = | 0.440493 |
| P[X=5] = | 0.175467 | P[X<=5] = | 0.615961 |
| P[X=6] = | 0.146223 | P[X<=6] = | 0.762183 |
| P[X=7] = | 0.104445 | P[X<=7] = | 0.866628 |
| P[X=8] = | 0.065278 | P[X<=8] = | 0.931906 |
| P[X=9] = | 0.036266 | P[X<=9] = | 0.968172 |

If the randomly generated number were to lie between 0.615961 and 0.762183 this would equate to a realisation of 6 claims in the period.

#### 5.3.4.2 The Normal (Gaussian) distribution

This distribution provides an even spread of outcomes and provides the distribution for an individual outcome as the sample size increases to infinity[67]. The probability density function is given by

$$f(X;\mu,\sigma)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2} \tag{5.9}$$

and $\sigma^2 > 0$.

The mean of this distribution is $\mu$ and the variance is $\sigma^2$.

The method of moments estimators and the maximum likelihood estimators for $\mu$ and $\sigma$ are approximately the same, namely

$$\hat{\mu}=\frac{1}{N}\sum_{i=1}^{N}x_i$$ and

$$\sigma^2=\frac{1}{n-1}\sum(x_i-\hat{\mu})^2 \quad .$$

[67] As per the central limit theorem.

In order to generate a value on the normal distribution the code provided in footnote 62 can be utilised or the distribution function of a spreadsheet can be used. The standard normal distribution can be adjusted to the relevant normal distribution by the linear scaling norm_gen()*$\sigma$ + $\mu$.

5.3.4.3 The Log-Normal distribution

This distribution is often used to fit claims severity or loss ratio severity. It is positively skewed and hence provides an appropriate starting point when investigating a claim distribution. The probability density function is given by

$$f(X;\mu,\sigma)=\frac{1}{\sqrt{2\pi}\sigma x}e^{-\frac{1}{2}\left[\frac{\ln(x)-\mu}{\sigma}\right]^2} \tag{5.10}$$

The mean of this distribution is $\exp(\mu + \sigma^2/2)$ and the variance is $\exp(2\mu + \sigma^2)(\exp(\sigma^2) - 1)$.

This distribution can be easily assessed as the natural logarithm of the outcomes has a normal distribution. Hence, to fit this distribution, calculate the natural logarithms of the actual observations and proceed as with the normal distribution described above.

5.3.4.4 The Exponential distribution

This distribution has a thicker tail than the Log-Normal distribution, is integratable in closed form and has only one parameter, hence it is an easy alternative to other distributions. It is also appropriate to model claim severity.

The distribution has the following form:

$$f(X;\theta)=\frac{1}{\theta}e^{\frac{-x}{\theta}} \tag{5.11}$$

The mean of this distribution is $\theta$ and the variance is $\theta^2$.

The method of moments estimator and the method of maximum likelihood estimator is the same:

$$\hat{\theta}=\tfrac{1}{N}\sum_{i=1}^{N}x_i \tag{5.12}$$

$$F(X;\theta)=1-e^{\frac{-x}{\theta}} \tag{5.13}$$

where N is the number of claims.

In order to generate a value on this distribution consider the cumulative distribution function F(X) = P[x<X]. As the cumulative distribution function has a uniform(0,1) distribution a claim value can be generated.

$$x = -\theta \ln(1 - F(X;\theta))$$

5.3.4.5 The Pareto distribution

This distribution can be used to generate a very thick tail and as a result is very suitable for claims experience with wide variability such as liability claims. The Pareto distribution is more flexible than the exponential distribution as it has two parameters as well as being integratable in closed form. As a result it is a very popular distribution to apply in practice and is often found in pricing models.

The distribution has the following form:

$$f(X;\alpha,\lambda) = \frac{\alpha\lambda^{\alpha}}{(\lambda + x)^{\alpha+1}} \tag{5.14}$$

$X > 0$, $\alpha > 0$, $\lambda > 0$.

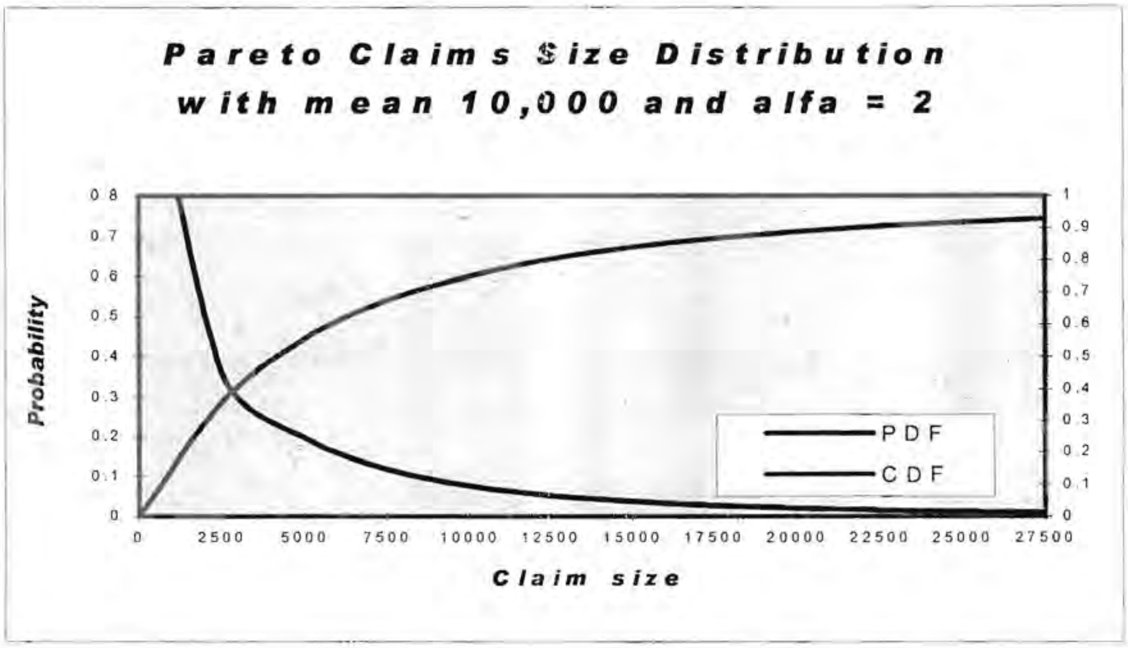
The distribution has a mean of

$$\frac{\lambda}{\alpha - 1}$$

and variance of

$$\frac{\alpha\lambda^2}{(\alpha-1)^2(\alpha-2)} .$$

The reader should note from the above that the mean and variance may not be defined for all x. In such instances, the mean and variance over a range of x values can still be obtained.




For the Pareto distribution the method of moments estimators are:

$$\hat{\lambda} = (\hat{\alpha} - 1)\bar{x}$$

where s is the standard deviation of the sample and

$$\hat{\alpha} = \frac{2s^2}{s^2 - \bar{x}^2} \quad .$$

The estimators for the Pareto distribution found by means of maximum likelihood estimation are found by solving the equation $h(\lambda) = 0$,

$$h(\lambda) = \frac{\sum \frac{1}{\lambda + x_i}}{\sum \frac{x_i}{\lambda(\lambda + x_i)}} - \frac{n}{\sum \ln\left(1 + \frac{x_i}{\lambda}\right)} \tag{5.15}$$

The result is then used to solve the equation

$$\hat{\alpha} = \frac{\sum \frac{1}{\hat{\lambda} + x_i}}{\sum \frac{x_i}{\hat{\lambda}\left(\hat{\lambda} + x_i\right)}} \tag{5.16}$$

In order to generate a value on this distribution consider the cumulative distribution function again:

$$F(X;\alpha,\lambda)=1-\left(\frac{\lambda}{\lambda+x}\right)^{\alpha} \tag{5.17}$$

A claim value can therefore be generated using the equation

$$x=\left(\frac{\lambda}{(1-F(X;\alpha,\lambda))^{\frac{1}{\alpha}}}\right)-\lambda \tag{5.18}$$

5.3.4.6 The Gamma distribution

The Gamma distribution is a very flexible distribution. It is, however, not manipulable in closed form and as a result is best used with a computer package.

The distribution has the following form:

$$f(X;\alpha,\lambda)=\frac{e^{-\lambda x}}{\Gamma(\alpha)}\lambda(\lambda x)^{\alpha-1} \tag{5.19}$$

$X > 0$, $\alpha > 0$.

It is also known as a Pearson's Type III distribution. If $\alpha = 1$ the exponential distribution results. If $\alpha$ is a positive integer it is termed an Erlang distribution. If $\lambda = 0.5$ and $\alpha = 0.5\nu$ then it is termed a chi-squared distribution with $\nu$ degrees of freedom.

The distribution has a mean of

$$\frac{\alpha}{\lambda}$$

and variance of

$$\frac{\alpha}{\lambda^2}.$$

The parameter set can therefore easily be determined by the method of moments. Maximum likelihood estimation requires numeric integration.

### *5.3.5 Testing the fit*

Having decided on a distribution to fit to the data, it is subsequently necessary to test the fit. This is normally done first by a visual inspection of the data and subsequently by investigating the sum of standardised squared errors as explained above. This means a Chi-square test is conducted on the fit.

Judgement plays a very important role in testing the suitability of the fit. It has already been alluded to that a fit may not be appropriate even if the fit appears to be good. The reason for this is that the data representing claims may be considered to be a *less than perfect* representation of the actual underlying population. In these instances consultation with experts in the field needs to be considered. This is especially true regarding very large claims as past structures and practices may not be indicative of future experience.

At all times it is therefore necessary to keep the following in mind:

1. Past information is used as a starting point to find a possible trend in future experience.
2. This assumption is flawed in that it assumes that past, present and future structures regarding business operations, claims management, economic scenarios and a variety of other variables remain the same. This includes past trends which are assumed to exhibit themselves in similar fashion in future.
3. As the model builder may be aware of new trends these have to be allowed for as well. The model builder may also make use of experts such as underwriters to determine whether or not the distribution fit is appropriate[68].

Bearing in mind that the aim is to set a model for future experience, which is uncertain, too great a reliance on past data is also problematic as past experience might not necessarily be repeated in future. The model builder will then graduate the distribution function according to all the information at hand to set a best estimate of future expected experience.

An extension of this would be to build a model for each variable in the $\underline{\theta}$ variable set.

## 5.4 Generalised linear modelling

The crux of liability modelling lies in identifying and quantifying the impact of different risk factors on the parameters used in the model. As an example, the Poisson parameter $\theta$, which is the number of occurrences in a period will depend on the risk characteristics of the pool of policies generating the claims. In estimating $\theta$ it is imperative that all appropriate risk factors are taken into account. This ensures that as accurate an assessment as possible of the underlying risk is obtained. This section will focus on the technique of

[68] A good example of such an approach would be to ask the underwriter his or her assessment of the number of claims for a certain type of business in the different bands for a specific size of business. This will enable the model builder to check whether these answers conform to $P[a < X < b]*N$ where a and b will be the lower and upper limit of each band and N will be the size of the business by policy number, for example.

generalised linear modelling in obtaining a function of the form $f(\theta;\underline{\omega})$ where $\underline{\omega}$ is the vector of risk factors having an influence on $\theta$.

Consider a pool of risks such as policyholders with own damage coverage for their motor cars. These policyholders can be grouped into similar classes according to certain risk criteria. For example, the power to weight ratio of the car, the area in which the car is housed, the age of the driver, etc. Consider the frequency of claims arising from such a risk pool. Each one of these pools will have a maximum likelihood estimator, $\theta$, of the frequency of own damage claims. This estimator will be derived by dividing the number of claims by a suitable exposure unit which in this case would be the duration of cover. Each such estimator $\theta_i$, where i refers to the i'th group of policyholders, will depend on the risk characteristics of the i'th group and is assumed to be independent of the other $\theta_i$'s. These risk characteristics can be represented by either quantitative factors or qualitative factors. These factors are the explanatory variables used in generalised linear modelling. Quantitative factors tend to be rare. Qualitative risk factors can be presented using dummy variables[69].

The aim of generalised linear modelling is to express some function of $\theta_i$, say $g(\theta_i)$[70] as a linear combination of a set of parameters $\beta_1,\ldots\beta_n$ where n < number of classes.

More precisely the following set of equations must be solved

$$g(\theta_i) = X_i^T \beta + \varepsilon_i \tag{5.20}$$

where $\varepsilon_i$ denotes the error term.

This means that when the individual outcome of a claim for any one policy is considered and this outcome can be viewed as the single random variable Y. Y will be from a certain distribution with parameter $\theta$ where $\theta$ is a function of $\underline{\beta}$, which represents the risk characteristics for the specific policy.

In order to apply generalised linear modelling, the distribution of Y must belong to the family of exponential distributions and must depend on a single parameter. This means that it must be of the following form:

$$f(y;\theta) = \exp\left[yb(\theta) + c(\theta) + d(y)\right] \tag{5.21}$$

Given that the distribution of Y can be written in this form it is then possible to solve the coefficient vector $\underline{\beta}$ by maximum likelihood.

[69] For example, consider differentiation by gender. One factor will be required which will be either 0 or 1 depending on the gender.

[70] The function g, termed the link function, must be monotone and differentiable.

Generalised linear modelling is more powerful than ordinary regression as the error structure need not be normal.

In order to facilitate the solving of this problem, the function $g(\theta_i)$ is expressed as $\eta_i$. Note that $g(\theta_i)$ is a monotone differentiable function, referred to as the link function. The link function can be identity, $g(\theta_i) = \theta_i$, natural log function, $g(\theta_i) = \ln(\theta_i)$ or another function which satisfies the criteria of monotonicity and differentiability .

The log likelihood function of all the observations $\underline{y}$ can be presented by

$$l(y;\theta) = \sum y_i b(\theta_i) + \sum c(\theta_i) + \sum d(y_i) \tag{5.22}$$

Solving for $\theta$ is the equivalent of solving for $\beta$. Consider

$$U_j = \frac{\partial l(y;\theta)}{\partial \beta_j} = \sum_{i=1}^{N} \frac{\partial l_i}{\partial \beta_j} \tag{5.23}$$

where $l_i = y_i b(\theta_i) + c(\theta_i) + d(y_i)$ and N is the number of classes of observations

It can be shown that

$$U_j = \sum_{i=1}^{N} \frac{(y_i - \mu_i)x_{ij}}{\mathrm{var}(Y_i)} \left( \frac{\partial \mu_i}{\partial \eta_i} \right) \tag{5.24}$$

where $\mu_i = E(Y_i) = -c'(\theta_i)/b'(\theta_i)$ and $x_{ij}$ is the jth risk characteristic of policy group i. In general the equations $U_j = 0$ have to be solved by numerical iteration. If the Newton-Raphson method is used then the nth approximation is given by

$$\underline{b}^{(n)} = \underline{b}^{(n-1)} - \left[ \frac{\partial^2 l}{\partial \beta_j \partial \beta_k} \right]^{-1}_{\beta = b^{(n-1)}} \underline{U}^{(n-1)} \tag{5.25}$$

where the term in square brackets is the matrix of second derivatives of $l$ evaluated at $\underline{\beta} = \underline{b}^{(n-1)}$ and $\underline{U}^{(n-1)}$ is the vector of first derivatives evaluated at $\underline{\beta} = \underline{b}^{(n-1)}$. It is therefore necessary to start with a randomly or otherwise selected set, $\underline{b}^{(1)}$.

Several commercial applications are available to facilitate the solving of these equations.

### *5.4.1 Special case of generalised linear modelling*

Consider the special case where the individual distribution functions for the frequency of own damage claims for each group of homogeneous policyholders is normally distributed with constant variance but different means.

The likelihood function is given by:

$$l(y,\theta)=\prod_{i=1}^{N}\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{1}{2\sigma^2}(y_i-\theta_i)^2} \tag{5.26}$$

The log-likelihood function is given by:

$$=\frac{-N}{2}\ln(2\pi\sigma^2)-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(y_i-\underline{x}_i^T\ \underline{\beta}\right)^2$$

$$=\frac{-N}{2}\ln(2\pi\sigma^2)-\frac{1}{2\sigma^2}\left(\underline{y}-\underline{X}\underline{\beta}\right)^T\left(\underline{y}-\underline{X}\underline{\beta}\right)$$

where X is the matrix of explanatory variables. Solving for maximum likelihood estimates yields:

$$\frac{\partial l}{\partial \beta_k}=-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\frac{\partial}{\partial \beta_k}\left(y_i-x_i^T\ \beta\right)^2 \tag{5.27}$$

$$\frac{\partial l}{\partial \beta_k}=-\frac{1}{\sigma^2}\sum_{i=1}^{N}\left(y_i-x_i^T\ \beta\right)x_{ik}$$

where k has a maximum value equal to the number of explanatory variables plus 1 for intercept.

$$\frac{\partial l}{\partial \underline{\beta}}=-\frac{1}{\sigma^2}X^T\left(\underline{y}-X\underline{\beta}\right)$$

solving this equation for a maximum yields:

$$\underline{\beta}=\left(X^T X\right)^{-1}X^T\ \underline{y} \tag{5.28}$$

and this is, of course, the standard form of least squares regression. Therefore regression is a special case of generalised linear modelling with the assumptions as set out above.

## 5.5 Delay modelling

The necessary tools required to model the frequency and severity of liabilities have been discussed above. The severity distribution will, however, refer to the present value of liability outgo. It is possible to extend this idea to incorporate the actual cash-flow of the liability outgo. This extension may be more appropriate since liability structures change. The manner in which claims will occur, be notified, administered and settled and possibly re-opened, will have an impact on both the frequency and severity of claims.

In order to model liability outgo accurately it is important to firstly identify the expected frequency and severity and then subsequently to model the outgo by investigating the actual expected emergence of claims over time as well as the actual payment of those claims. It might very well be that the run-off of claims reported later will be different to the run-off of claims reported at an early stage.

The investigation of the run-off of claims is as follows:

### *5.5.1 Step 1*

Claims are grouped into homogeneous groupings or cohorts such as the period in which the claim arose. This grouping is done for the number of claims in that claim period by considering the delay before a specific claim was actually reported. Subsequent to this the payments made in reference to these claims are also grouped, firstly by the duration at which the claim was reported and secondly by the subsequent period of delay before the claim was settled.

It is not always possible to subdivide payments in such a fashion and therefore in certain instances the grouping is considered for the actual amounts paid by reference to a certain grouping period for the incidence of claims.

It is important to consider paid claims in order to assess actual cash flows. It is important to consider incurred claims in order to assess the incurred cash flow position[71]. When modelling liability outgo the incurred position will normally be reflected. It is important to distinguish between that part of the incurred amount which is held as reserve and that part which is actual payment. The level of reserves has a direct impact on the value of the company as reserves will

[71] Incurred claims are claims paid during the year together with any change in reserves set up for the claims (reserves at end of period less reserves at start of period). The reserving policy of a company can distort the apparent cash-flow position of the company. It is therefore important to consider company policy and identify whether this is a best estimate.

generally secure lower investment returns than other business operations. This is because a matched position in assets is normally held for liabilities. These matched positions are often in cash or fixed interest securities which tend to yield a lower return over long durations of time than investments in other business operations.

### *5.5.2 Step 2*

The grouped information must be expressed in such a fashion as to facilitate projection of the experience. Run-off triangles are well known in actuarial literature. In essence the information will be grouped as follows:

**Triangle of reported number of claims.**

| | **Period of reported claim** | | | | |
|---|---|---|---|---|---|
| Period of loss | In same period | One period later | Two periods later | Three periods later | Four periods later |
| First period[72] | $R_{1,1}$ | $R_{1,2}$ | $R_{1,3}$ | $R_{1,4}$ | $R_{1,5}$ |
| Second period | $R_{2,1}$ | $R_{2,2}$ | $R_{2,3}$ | $R_{2,4}$ | |
| Third period | $R_{3,1}$ | $R_{3,2}$ | $R_{3,3}$ | | |
| Fourth period | $R_{4,1}$ | $R_{4,2}$ | | | |
| Fifth period | $R_{5,1}$ | | | | |

**Table 3**

The above table assumes five periods are available for investigation. A period can refer to a week, month, quarter year, half year, year etc. The aim is to determine the portion of claims expected to be reported at each duration in the future given the past experience. This can be done by investigating the development of the cumulative number of reported claims and then applying this development to the experience to date. $R_{i,j}$ refers to claims arising in period I and reported in period j.

[72] This refers to the period for which the run-off of losses are investigated. The first period may be the first month or the first quarter or the first year of the period of investigation.

**Triangle of cumulative reported claims**

| | Period of reported claim | | | | |
|---|---|---|---|---|---|
| Period of loss | In same period | One period later | Two periods later | Three periods later | Four periods later |
| First period | $R_{1,1}$ | $R_{1,1}+R_{1,2}$ | $R_{1,1}+R_{1,2}+R_{1,3}$ | $R_{1,1}+R_{1,2}+R_{1,3}+R_{1,4}$ | $R_{1,1}+R_{1,2}+R_{1,3}+R_{1,4}+R_{1,5}$ |
| Second period | $R_{2,1}$ | $R_{2,1}+R_{2,2}$ | $R_{2,1}+R_{2,2}+R_{2,3}$ | $R_{2,1}+R_{2,2}+R_{2,3}+R_{2,4}$ | |
| Third period | $R_{3,1}$ | $R_{3,1}+R_{3,2}$ | $R_{3,1}+R_{3,2}+R_{3,3}$ | | |
| Fourth period | $R_{4,1}$ | $R_{4,1}+R_{4,2}$ | | | |
| Fifth period | $R_{5,1}$ | | | | |

**Table 4**

The weighted average of the development of the cumulative reported claims at each duration is used to assess a trend. This is an estimate based on the weighted average only. Quantifying the variability within these estimates will be considered at a later stage in this study.

For example, the weighted average of development from the first period after reporting to the second period after reporting will be given by dividing the sum of all the yellow cells in Table 4 by the sum of the blue cells[73]. Such a factor is termed a development factor.

In addition to considering the trend in reported claims, the trend in claim payments needs to be considered if additional delays exist between the time of reporting and the time of actual payment. As mentioned earlier this analysis can also be split for the actual duration of reporting. An illustration is provided assuming the duration at which claim payments are made is different to the duration at which they were reported. Using the notation as in Table 3 the grouping will be as follows:

[73] For an in depth study on triangulation techniques the reader is referred to the core reading published by the Institute and Faculty of Actuaries.

**Triangle of claim payments.**

| | Period of claim payment | | | | |
|---|---|---|---|---|---|
| Period of reporting | In same period | One period later | Two periods later | Three periods later | Four periods later |
| First period ($R_{1,1}$) | $P_{1,1}$ | $P_{1,2}$ | $P_{1,3}$ | $P_{1,4}$ | $P_{1,5}$ |
| Second period ($R_{2,1}+R_{1,2}$) | $P_{2,1}$ | $P_{2,2}$ | $P_{2,3}$ | $P_{2,4}$ | |
| Third period ($R_{3,1}$ + $R_{2,2}$ + $R_{1,3}$) | $P_{3,1}$ | $P_{3,2}$ | $P_{3,3}$ | | |
| Fourth period (etc.) | $P_{4,1}$ | $P_{4,2}$ | | | |
| Fifth period (etc.) | $P_{5,1}$ | | | | |

**Table 5**

$P_{i,j}$ is the amount reported in financial period i and paid in the subsequent $j^{th}$ period. The grouping is by reporting period and therefore the information can be extended to include payments made for claims having the same period of reporting but an earlier period of loss if it can be assumed that the trend in development has remained stable. If this cannot be assumed such a triagle would be set up for each period in which claims arose.

### *5.5.3 Step 3*

For the two types of development (for reporting and for actual payments) development factors are then determined as described above. These factors represent the expected growth in the cumulative claims reported or paid from one period to the next as the case might be[74]. By considering the cumulative effect of these factors from a specific period to the expected ultimate period of development is very useful.

[74] Reported claims and paid claims are not the only entities which can be investigated in this fashion. Claims incurred are often investigated when substantial reserves are required at initial durations and the actual payment patterns therefore provide poor starting points. As the results will be very sensitive to the initial amounts for later years, this feature is often referred to as a corner sensitivity.

**Example: Development factors of claim payments.**

| | Period of claim payment | | | | |
|---|---|---|---|---|---|
| | In same period | One period later | Two periods later | Three periods later | Four periods later |
| **Development factor** | | $\Sigma C_{2,i} / \Sigma C_{1,i}$ (i = 1 to 4) | $\Sigma C_{3,i} / \Sigma C_{2,i}$ (i = 1 to 3) | $\Sigma C_{4,i} / \Sigma C_{3,i}$ (i = 1 to 2) | $\Sigma C_{5,i} / \Sigma C_{4,i}$ (i = 1) |
| **Denoted by** | | $D_1$ | $D_2$ | $D_3$ | $D_4$ |

**Table 6**

Where $C_{i,j} = \Sigma_{(t=1 \text{ to } j)} PC_{i,t}$ is the cumulative claims paid to date grouped by period of loss. $PC_{i,t}$ is the amount of claims paid in development period *t* for period of loss *i*. It therefore allows for both the delay in reporting and the subsequent delay in payment.

The development factors denoted by $D_i$ above yields the weighted average of the individual development ratios. The cumulative development factors are then denoted by:

$CD_1 = D_1 * D_2 * D_3 * D_4$

$CD_2 = D_2 * D_3 * D_4$

$CD_3 = D_3 * D_4$

$CD_4 = D_4$

Given that

cumulative amount to date * the appropriate cumulative development factor = expected ultimate amount (or number of reported claims),

it is possible to determine the proportion of expected ultimate claims to be paid at each future development period.

It is important to apply consistent approaches between the modelling of delays and the modelling of claim frequency and severity. The delay pattern identified for the number of claims reported can be applied to spread the claim frequency. Here it is important that a reported claim will therefore be a claim giving rise to a claim, zero claims cannot be used. For the severity of claims the influence of inflation needs to be considered. The severity will be expressed as an inflation adjusted amount to reflect the present value of expected claims. If, however, these payments are subject to further delays, allowance for further expected increases as a result of inflation will be required.

## 5.6 Bootstrapping

Bootstrapping is a technique involving numerical techniques in the calculation of the variance of a certain set. These techniques are computationally very intensive and require the use of computers. For a detailed discussion the reader is referred to DAVISON and HINKLEY (1997).

*A simple example:*
Take a set of development ratios[75] for a certain class of business:
Assume these development factors are grouped by year of accident and subsequent development for each accident year.

| | Year of development | | | | |
|---|---|---|---|---|---|
| Year of accident | $D_{1,0}$ | $D_{1,1}$ | $D_{1,2}$ | $D_{1,3}$ | $D_{1,4}$ |
| | $D_{2,0}$ | $D_{2,1}$ | $D_{2,2}$ | $D_{2,3}$ | $B_{2,4}$ |
| | $D_{3,0}$ | $D_{3,1}$ | $D_{3,2}$ | $B_{3,3}$ | $B_{3,4}$ |
| | $D_{4,0}$ | $D_{4,1}$ | $B_{4,2}$ | $B_{4,3}$ | $B_{4,4}$ |
| | $D_{5,0}$ | $B_{5,1}$ | $B_{5,2}$ | $B_{5,3}$ | $B_{5,4}$ |
| | ? | ? | ? | ? | ? |

Table 7

$D_{1,0}$ for example is the development of cumulative claims paid in the first accident year from the end of the first period to the end of the second period under investigation.

Past practice has been to investigate such a triangle and to determine development ratios that represent the expected development of cumulative claims between different durations[76]. A development ratio can, however, be viewed as a sample from a certain distribution for the specific duration. Therefore the process of triangulation can be seen as an estimation of the most likely development ratio from a certain unknown distribution of development ratios.

Bootstrapping can be used to gauge the variability of the reserve calculated by means of triangulation[77]. This is done by assuming a certain distribution for the development ratio at each duration. The distribution chosen will depend on the

[75] Development ratios are often used to determine the development of cumulative claims experience based on past data. These ratios are based on the development of cumulative claims from one period to the next for a certain year. The definition of claims by year should be consistent throughout to ensure proper projection.

[76]The last row of Table 7 denotes the development ratios that will be used by the practitioner. In certain instances different development ratios may be used for different years if the practitioner is of the opinion that not all years will exhibit the same development of cumulative claims in future.

[77] The practitioner will need to determine whether or not investment income has to be taken into account.

judgement of the practitioner. A good starting point for a stable[78] book of business will be to assume that the development ratio at each duration has a discrete distribution whereby the development ratio for a certain duration can take any of the ratios shown in Table 7 at the same duration, with equal probability. This assumption can only be made for a book of business that has had a fairly constant size over the period of investigation as a smaller book of business will be subject to a higher level of variability[79].

The calculation process then entails sampling from the triangle to obtain the projected development ratios[80]. This is done by generating a random number and selecting one of the given development ratios for the period of development based on the number generated. A similar process would be followed if a different distribution had been chosen. The selected development ratios are then used to calculate the expected ultimate experience for each year. From this projection all cumulative claims paid to date can be subtracted. The result of this calculation then gives one sample of the approximated distribution of all outstanding experience. Simulating a selected number of these outstanding experiences will give a probability density function of the outstanding experience whereby the variance of future outstanding experience can be approximated.

For a sufficient number of periods of development the result of a bootstrapping exercise can yield a very smooth distribution function. This distribution can be utilised to generate very powerful information. Not only will the practitioner be able to gauge the variability in future experience but the extreme values of the distribution can also be noted.

This method makes allowance for underwriting risk and reserve risk only. Additional allowance will be required for asset risk, credit risk and growth risk through the modelling of all these risks together.

The approach explained above is very simplistic. It is also possible to assume a standard run-off model and then to investigate the error terms generated through a comparison of the model and the actual past experience. These errors can be standardised and used in a distribution as a sampling basis to project future development.

[78] The stability of a book of business will depend on the underlying variability of that business. General rules of thumb may be applied (such as a minimum number of 3000 claims per year of accident). Such rules are crude approximations based on a normality assumption. Data sets are, however, often limited. In such instances it will have to be accepted that the results of a bootstrapping exercise will be crude.

[79] Using development ratios based on smaller books of business in the past (for a growing company) will result in the variability of total experience being overestimated. This may be seen as a prudent approach but should be handled with caution.

[80] These are the $B_{x,y}$ values shown in Table 7.

BONNARD R. GREENWOOD M. GREYBE S. (1999) provides a practical illustration of a variety of these techniques appropriate for modelling future run-off of claim payments.

## 5.7 Model building

The process of stochastic simulation and distribution fitting has been considered. The area of model building is now discussed. In his presentation to the Actuarial Society of South Africa's General Insurance Mini Convention (1999), KARL MURPHY of ENGLISH MATTHEW AND BROCKMAN stated that model building is not only a science but also an art. This author seconds this view.

To put this in perspective, the objective of building a liability model must be borne in mind: The intention is to develop a model that gives as accurate a representation of expected future experience as possible. In order to do this past experience is investigated and assumptions are made as to which characteristics future experience might exhibit. This process relies on a considerable element of judgement or "gut feeling". It is here that the 'art of the science' comes into play.

The following provides a simple approach to model building:

### *5.7.1 Step 1*

Identify the class of business for which the model is to be built. As the risk details differ by class of business it is important to develop different models for different homogeneous groups of business. A balance needs to be struck between the grouping of business and the volume of business as too many subdivisions will lead to spurious accuracy or alternatively too small samples to make any credible conclusions.

### *5.7.2 Step 2*

For the class of business identify the different perils for which a model is required. All classes of peril may be combined or subdivisions may be made. For example, for motor business the subdivision might be for theft, third party, own damage and other.

### *5.7.3 Step 3*

For each peril a frequency distribution will be required and subsequently the delay pattern appropriate to the actual payment pattern will be required.

#### 5.7.3.1 Step 3.1

In order to determine the frequency distribution the first step is to divide the observed number of occurrences by the appropriate measure of exposure, that is the time for which the policies observed were exposed to the possibility of actually incurring a claim.

Now for this frequency distribution, say for example the frequency of own damage claims, the risk factors affecting the distribution need to be identified[81]. As risk factors are not always captured due to their sometimes unquantifiable nature, rating factors are used instead[82].

Rating factors identified might be the age of the driver, the gender of the driver, the duration for which the driver has been insured, the power to weight ratio of the vehicle and whether or not the vehicle is equipped with safety features such as ABS braking systems.

In order to assess the appropriateness of each factor this author proposes an initial one-way analysis of each rating factor. This will involve grouping the entire data set by the one factor and testing whether or not the means of the different groupings are statistically different.

In order to test whether or not two groupings are statistically different the standardised difference in the means can be considered. This is calculated in the following way:

$$Z = \frac{\frac{1}{n_1}\sum_{i=1}^{n_1} X_i - \frac{1}{n_2}\sum_{j=1}^{n_2} X_j}{\sqrt{\frac{1}{n_1}\sum_{i=1}^{n_1}\left(X_i - \overline{X}_1\right) + \frac{1}{n_2}\sum_{j=1}^{n_2}\left(X_j - \overline{X}_2\right)}} \tag{5.29}$$

where $n_1$ and $n_2$[83] are the exposure durations of the first and second group and the X's refer to the number of observations in each group. Z will have a standard normal distribution[84] and therefore be tested on the grounds of the probability of a certain observation actually having a standard normal distribution. Significant differences will normally be assumed for $|Z| > 1,96$.

[81] It is important to note that the investigations regarding rating factors are required before any business has been written at the system development stage. Preferably more factors rather than fewer need to be captured as this will facilitate the future analysis of experience. Not all fields captured may have a direct bearing on the risk but their impact needs to be tested over time. Applying a structured thought process to the actual risk is very useful in identifying possible risk factors.

[82] The distinction between risk factors and rating factors is common in the actuarial literature. In essence, risk factors are factors which have a direct impact on the risk to be evaluated. For example the mileage of the vehicle. As the mileage is not always readily available and difficult to verify a rating factor may be used instead, for example, the use of the vehicle (business, leisure, etc.).

[83] In an earlier chapter data considerations were considered as well as the volume of data necessary to form a statistically significant opinion regarding the experience.

[84] For sufficiently large sizes of $n_1$ and $n_2$

For example, when considering the gender of the insured, data will be grouped by male or female and a hypothesis test conducted to ascertain whether the means of the two groupings are significantly different.

This one-way analysis will give the practitioner an initial feel as to which risk factors have a significant impact on the risk. In choosing which factors to test, the practitioner should rely on expert opinion in the market or the company.

5.7.3.2 Step 3.3

Once the rating factors to be used have been considered, possible correlation between the rating factors needs to be considered. As an example consider age and gender. The influence of gender on age will not be the same for all ages. Experience of young males will tend to show a greater difference to young females than mature males to mature females. This means that the rating factors are correlated.

In order to allow for this feature, the age and gender should be combined as one factor. That is to say, instead of using a gender differential and a separate age differential, use an age-gender differential. Alternatively a correlation factor could be included.

The correlation between two factors is determined as follows:

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_y}$$

Where $Cov(X,Y) = \frac{1}{n}\sum(x_i - \mu_x)(y_i - \mu_y)$

i.e. the correlation is calculated by dividing the sample covariance by the product of the standard deviation of the two parameters investigated.

This approach can, however, only be applied where sufficient volumes of data exist in order to justify such a grouping.

5.7.3.3 Step 3.4

When the factors affecting the parameter of the frequency function have been determined generalised linear modelling techniques can be applied to obtain a function that will yield the mean value, given the risk characteristics.

The purpose of this exercise is to determine a function of rating factors, based on past experience, such that the mean frequency of a certain homogeneous group is determined accurately by applying the function.

*f*(risk factors) = mean parameter.

In order to determine the most appropriate set of risk factors it should be borne in mind that the higher the number of risk factors, the higher the likelihood will be of reducing the unexplained variation of the model. The analogy to this is the fitting of a polynomial function to a certain set of points. The higher the rank of the polynomial, the higher the number of points that will lie on the function.

The aim is therefore to obtain the best possible fit with the least number of variables.

This will involve decreasing the unexplained variation

$$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2,$$

in comparison to the total variation of the model,

$$\sum_{i=1}^{n} (y_i - \bar{y})^2 .$$

The multiple coefficient of determination, $R^2$, is then defined as

*1 – (Unexplained variation / Total variation)* (5.30)

A variety of approaches optimising this problem set are discussed in statistical literature. In essence these techniques entail choosing parameters that contribute the most to declaring the total variation. One method is testing one factor at a time and then choosing the factor with the biggest reduction in variation. Once the first factor is determined the second will be tested and this process will continue until further additional factors have no additional benefit by way of declaring variation. This process is often referred to as Forward Selection. Most statistical applications allow for the automatic calculation of this process. A similar approach could be to start with all parameters and then test the impact of eliminating each factor one at a time and subsequently obtain the model with the highest multiple coefficient of determination and the lowest number of parameters.

#### 5.7.3.4 Step 3.5

Once the model building has been completed a frequency model per peril for a certain class of business will be available to model future expected experience for certain model policies given the risk characteristics of the policies.

### *5.7.4 Step 4*

The next step is to fit a severity function to the claims distribution. As risk factors will have a direct impact on the severity of a claim it will be necessary to apply the same methodology as for frequency calculations to severity calculations. Note that different risk factors will be applicable for the severity distribution as opposed to the frequency distribution.

Severity of claims have a larger spread than frequency of claims and as a result parameter error is more likely for a subdivision by rating factor than would be the case for frequency distributions.

This author proposes that the practitioner first find an appropriate severity distribution for the peril involved using the techniques explained earlier in this chapter. Subsequent to this, the same methodology as used to calculate the average frequency for a certain set of risk characteristics can be used to calculate the average severity and the distribution applied can then be scaled accordingly.

This approach would, however, not allow for changes in the variability of claims per peril by risk factor. For substantial volumes of data proper allowance could be made by fitting a separate distribution for each set of risk characteristics.

The severity calculated per claim will be calculated as the present value of the expected claim. For long tail classes of business this claim can be spread according to the run-off pattern identified for the claim. This process may introduce spurious accuracy for shorter classes of business and may be ignored for such classes.

## 5.8 Summary

This unit contains a detailed investigation into the methodology required to quantify the liability area of risk. It includes the following:

1. A detailed discussion on the topic of stochastic simulation.
2. Reference to appropriate stochastic distribution for use in a general insurance operation.
3. A discussion on the manner in which to obtain the appropriate distributions.
4. A discussion on generalised linear modelling.
5. A discussion on the determination of cash-flow development trends in claim payments.
6. The assimilation of all concepts in the model building process.

In the next chapter the area of asset risk is considered.

# 6. Quantifying the asset area of risk

## 6.1 Introduction

In the previous chapter the liability area of risk was considered for claim outgo only. This chapter considers the modelling of asset proceeds as well as the stability of asset values over time. This is often also referred to as market risk.

Asset proceeds consist of income and capital gain. Each element may be dependent on other factors or may be modelled based only on past data using time series techniques. Asset returns by asset type tend to be more dependent than liability outgo due to the fact that all returns are generally obtained in the same economy. To allow for this feature, asset proceeds are often modelled as a function of a certain base index.

Some of the techniques and processes used for asset modelling will firstly be considered and subsequent to this some of the approaches required for model building will be discussed.

## 6.2 Stochastic modelling

For the purpose of this study the assets' proceeds will also be modelled stochastically. This will ensure consistency to the modelling of liability outgo. Details regarding the process of stochastic modelling have already been discussed in the previous chapter.

## 6.3 The modelling of asset proceeds[85]

Research on the modelling of asset proceeds numbers many volumes. Due to the short-term nature of asset returns investigated in this study the results will tend to be more variable than for business of a long term nature such as a pension fund. The result of this is that too much refinement in the investment model is considered spurious and this chapter has been prepared with this in mind[86].

### *6.3.1 Random walk models*

This is a simple model whereby the original asset value can either move up or down by a certain specified amount over a certain specified time period with a certain specified probability. It is, of course, possible to expand the range of outcomes per time period. In the example below Bernoulli trials are used but other probability distributions will be just as appropriate.

[85] For a more detailed discussion the reader is referred to a paper prepared by A. D. Smith concerning a workshop on stochastic Asset Models held at the General Insurance Convention: 18-21 October 1995

[86] Short-term asset movements are driven by sentiment rather than fundamental evaluation of future investment proceeds. As a result the movements can be erratic and extremely unpredictable.

*Example:*

Let *h* represent time.

Let X be a stochastic variable normally distributed with mean $\mu h$ and variance $\sigma^2 h$ representing the force of asset growth.

Let $Y_t$ be the asset value at time t, then:

$$Y_{t+h} = Y_t \exp(X) \tag{6.1}$$

Different relationships between $Y_{t+h}$ and $Y_t$ can be tested as well as different distributions for the random variable X. Authors such as S. COUTTS have suggested this model due to its simplicity and ease of application.

The graph given below shows a random walk for an asset portfolio with expected mean return of 10% per annum and standard deviation of 7%. The asset return is expected to be normally distributed.


**Figure 7: Simulated asset growth**

In order to obtain such a model the following process is required:

1. Obtain the growth for period *h* by considering the ratio of $Y_{t+h}/Y_t$ for all available t.
2. Take the natural logarithm of these ratios in order to determine the force of asset growth of the period *h*, $i_{t+h}$.
3. Obtain the mean and the standard deviation of all the observed $i_{t+h}$. This provides the mean growth for the period *h* as well as the standard deviation of growth.
4. Test the results for different periods of duration *h*.

In order to generate the growth over time consider the current value of assets. This value should preferably be smoothed in order to ensure a stable starting point. In order to project the asset value, generate[87] an expected asset growth rate for period *h* by assuming the asset growth rate distribution estimated as described above.

This distribution will be assumed to be normal with mean and standard deviation as obtained. This assumption can be tested given the information set obtained as described above. Such a test will entail grouping the observations in ranges based on the mean and standard deviation as calculated. For example, observations less than three standard deviations below the mean, observations between two and three standard deviations below the mean and so forth. The actual number of observations can then be compared to the expected, assuming the normal distribution as indicated. A chi-square test can be applied to test the reasonableness of the fit.

The process is repeated over different time periods *h* in order to simulate the growth of asset proceeds over time. Note that in this instance the proceeds combine both income and capital gains. A specific total time period will be chosen, say one year, for which the asset values will be available at every duration *h*.

By repeating this projection on a random basis a distribution of the cumulative growth in asset values for the period specified as well as at every duration will be obtainable.

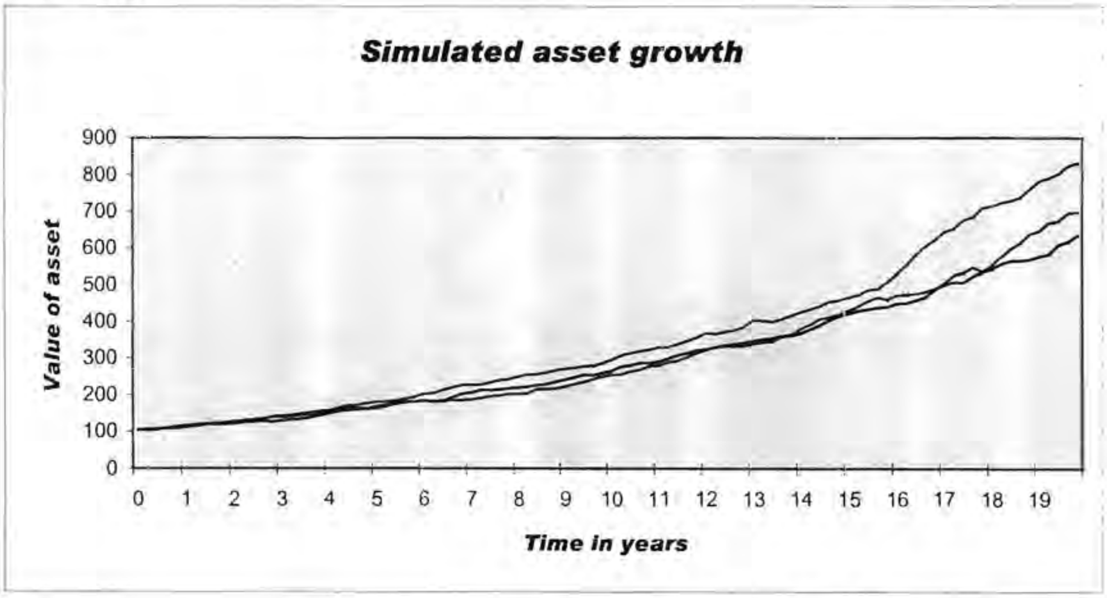



**Figure 8: Simulated asset growth examples**

[87] Details regarding the generation of normal distribution function values are discussed in the previous chapter.

The graph above shows three additional trials of the model used in the previous graph. This clearly shows the variability in the simulated results. By extending the number of simulated outcomes a distribution of asset growth can be determined at each time period.

#### 6.3.1.1 Example

In order to make this approach more apparent to the reader, the JSE All share index was considered on a monthly basis for the period May 1981 to August 1998. The graph of the index value[88] over this period is shown below:


**Figure 9: The JSE All Share index**

For the sake of this example the entire data set was used to fit a model. It must be recognised that such a model is only likely to be appropriate if the macro economic environment were to repeat itself in the next 20 year period. This is extremely unlikely and therefore consistency between asset growth and inflation assumptions is critical when setting an appropriate model.

Fitting a normal distribution to the force of interest (i.e. the force on capital growth reflected in the index) the following results were obtained:

[88] The index value was calculated as the average of the highest and lowest values of the index during the month.

**Table 8: Annualised returns**

| Summary Period | Average | Standard Deviation |
|---|---|---|
| Monthly | 14.5% | 18.6% |
| Quarterly | 15.3% | 22.5% |
| Half-Yearly | 15.2% | 23.7% |
| Annually | 15.9% | 21.4% |

Table 8 shows the degree of variability inherent in fitting a normal distribution based on monthly, quarterly, half-yearly and annual growth. This reinforces the idea that great care is required to project future expected experience.

The density of monthly returns are shown below:

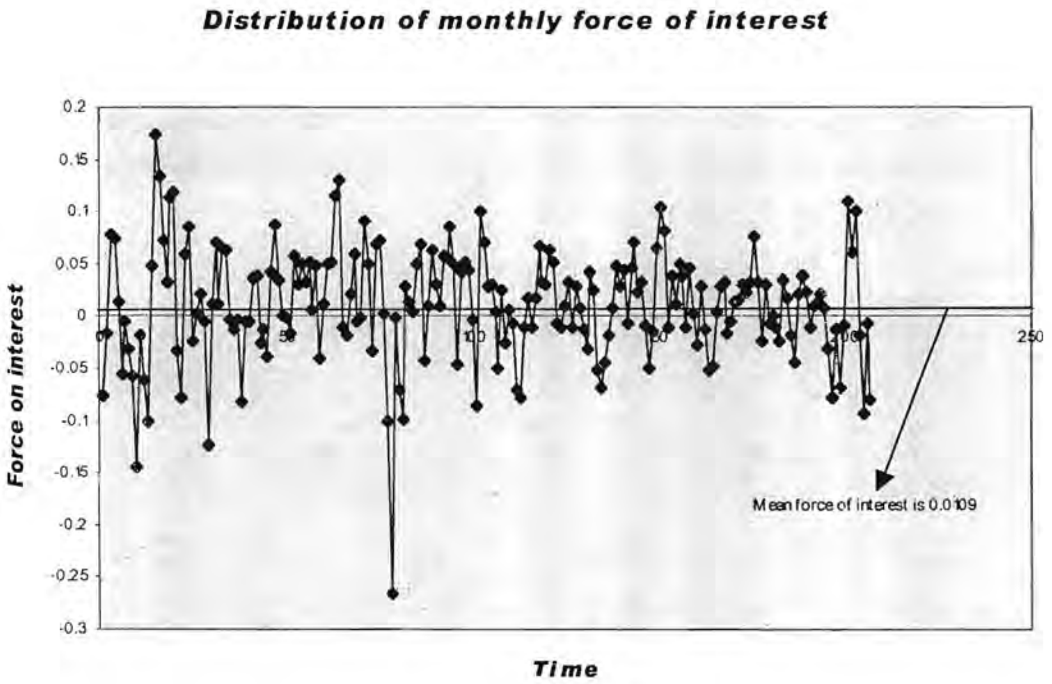



**Figure 10: JSE monthly force of interest**

In order to apply a random walk model to this data, the monthly force of interest is assumed to have a normal distribution. A random sample is then drawn from this distribution and this is used to project the future capital values of the index. Note that this model does not include the dividend yield which should be allowed for separately.

Comparing the actual density to the expected density yields the following:

***Distribution of the monthly force of interest***

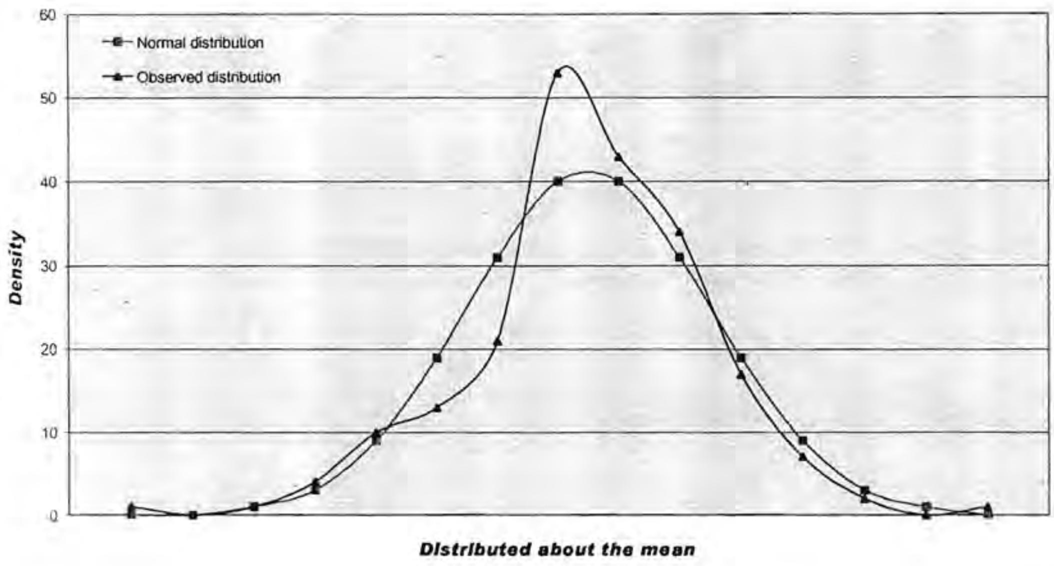



**Figure 11: Distribution of monthly force of interest**

It is clear that assuming a normal distribution is a crude assumption. Nevertheless, this model provides a simple basis and as the tail ends of the expected normal distribution are similar to the actual observations, this model will be appropriate to assess variability in solvency, for example.

### *6.3.2 Auto-Regressive Models*

The term auto-regressive means that a regression is applied to the same data set i.e. regression onto itself. This is appropriate in time series modelling where the future value is a function of some of the previous values in the series. Auto-regression is an additional way in which to set a relationship between a dependent and explanatory variables with the focus on the correlations between the same series at different time intervals. Note that a series can also consist of a multivariate series[89].

These models can take a variety of forms. Some of the models converge to a long-term mean while others are based on the experience of the variable over a certain time period. Some models are therefore assuming that past experience is indicative of future experience while others use this assumption together with the assumption that the variable will converge to a certain mean over time.

[89] Multivariate series are not discussed in this study.

For a detailed discussion on auto-regressive models the reader is referred to the core reading of the INSTITUTE AND FACULTY OF ACTUARIES subject 103.

6.3.2.1 Background to auto-regressive models

The aim of time series modelling is to identify, isolate and remove all possible deterministic trends in a time series through differencing and to end up with a stationary process[90]. Trends can include:

1. Seasonal trends over every year in which case differencing on a yearly basis will be required.
2. Cyclical trends over time. The duration of the cycle would be required before differencing could be applied.
3. A trend over time longer than the period of the study. By differencing this trend can also be eliminated. Consider the growth in an index. By differencing the yield per period can be obtained. This yield may have the same distribution over time thereby allowing the investigation of a stationary process.
4. Random fluctuations. These will not be eliminated through differencing and the aim of the investigation would be to model the randomness in the time series[91].

6.3.2.2 The Box and Jenkins[92] methodology

$\{Y_t\}$ is an Auto-regressive integrated moving average ARIMA (p,d,q) process if

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} + Z_t + \beta_1 Z_{t-1} + \ldots + \beta_q Z_{t-q} \qquad (6.2)$$

where $X_t = (1-B)^d Y_t$ and

$Y_t$ is the variable under investigation at time t and

$\alpha_i$ is a correlation factor between $X_t$ and $X_{t-i}$.

$Z_t$ is the error term at time t which has a standard normal distribution.

$\beta_t$ is a correlation factor between $X_t$ and $Z_t$ and B is a backward shift operator. therefore $(1\text{-}B)$[93] is a backwards differencing operator which means that the

[90] A stochastic process is said to be stationary if the joint distributions of $X_t, X_{t+1},,,,, X_{t+n}$ and $X_{k+t}, X_{k+t+1},,,,, X_{k+t+n}$ are identical for all t and k+t. This means that the statistical properties of the process remain unchanged as time elapses.

[91] This was already referred to in the previous section where the error distribution was considered.

[92] This methodology is referred to in the Core Reading on applied statistics (Subject 103) issued by the Institute and Faculty of Actuaries.

difference between the $t^{th}$ and the $(t-1)^{th}$ element is taken. The d denotes the number of times the backward shift operator is applied. $\{Y_t\}$ is said to be integrated because $\{X_t\}$ has been obtained through differencing and therefore $\{Y_t\}$ will be obtained through integration (summation).

The autocovariance and the autocorrelation of a time series are important because of the necessity to determine the correlation between future values of a time series and past values.

The covariance of any $X_r$ and $X_s$ from a stationary sequence depends on r and s only through the difference r-s. The autocovariance is defined as follows:

$$\gamma(r-s) = \operatorname{cov}(X_s, X_r) = E(X_s X_r) - E(X_s)E(X_r)$$

The autocorrelation function for elements of the time series with lag n is defined by:

$$\rho_n = \frac{\gamma(n)}{\gamma(0)} \tag{6.3}$$

The autocorrelation is often plotted for different lag periods. Where the autocorrelation tapers off to zero slowly, this indicates a correlation to previous values in the time series and therefore an autoregressive component is required. Where the autocorrelation tapers off to zero quickly, this indicates limited correlation to previous values in the time series and therefore a moving average component would seem more appropriate. For other trends a different structure of autoregressive components as well as moving average components may be required.

In order to set the autoregressive coefficients, the partial autocorrelation is used. The partial autocorrelation is defined by

$$\phi(k) = \frac{\det P_k^*}{\det P_k} \tag{6.4}$$

where $P_k$ is the k x k autocorrelation matrix

[93] (1-B) is often referred to in texts as $\nabla$.

$$P_k = \begin{pmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{k-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{k-2} \\ \rho_2 & \rho_1 & 1 & \cdots & \rho_{k-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{k-1} & \rho_{k-2} & \rho_{k-3} & \cdots & 1 \end{pmatrix}$$

and $P^*_k$ is $P_k$ with the last column replaced by $(\rho_1, \rho_2, \ldots, \rho_k)^T$.

Note that the partial autocorrelations provide the solutions to the vector $\underline{\alpha}$ in the equation

$$\underline{\rho} = P_k \underline{\alpha} \quad (6.5)$$

[94].

Therefore, once the structure of the model has been tested the coefficients will be obtained through investigation of the partial autocorrelations.

This model does not necessarily converge to a long-term mean as is evident from the formula given in (6.2) above. In some instances it may, however, be required to construct the model such that the variable under investigation converges to a long term mean. In this instance the model given above will be of the following form:

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} + \left(1 - \sum_{i=1}^{p} \alpha_i\right)\mu + Z_t + \beta_1 Z_{t-1} + \ldots + \beta_q Z_{t-q} \quad (6.6)$$

where $\mu$ is the long-term mean and $\sum_{i=1}^{p} \alpha_i < 1$.

[94] For example, consider an autoregressive process ARIMA(2,d,0) of the form

$$X_n = \alpha_1 X_{n-1} + \alpha_2 X_{n-2} + e_n.$$

Considering the covariance for n = 1 and n = 2, we have the following two equations:

$$\gamma_1 = \alpha_1 \gamma_0 + \alpha_2 \gamma_1$$
$$\gamma_2 = \alpha_1 \gamma_1 + \alpha_2 \gamma_0$$

As the covariance for lag 1 and lag –1 is the same thing. By dividing by $\gamma_0$ this yields

$$\rho_1 = \alpha_1 + \alpha_2 \rho_1$$
$$\rho_2 = \alpha_1 \rho_1 + \alpha_2$$

And these equations can be presented in the matrix notation of (6.5) above.

6.3.2.3 Example

The autoregressive model can be estimated using some standard characteristics.

The mean of the stationary model can be estimated by the sample mean

$$\hat{\mu} = \tfrac{1}{T}\sum_{t=1}^{T} x_t$$

The autocovariance function $\gamma(t)$ can be estimated using the sample autocovariances

$$\hat{\gamma}_t = \tfrac{1}{T-t}\sum_{t=t+1}^{T-t}(x_t - \hat{\mu})(x_{t-t} - \hat{\mu})$$

The autocorrelation $\rho_t$ can be estimated using

$$\hat{\rho}_t = \frac{\hat{\gamma}_t}{\hat{\gamma}_0}$$

and the partial autocorrelation $\phi(t)$ can be estimated using

$$\hat{\phi}(t) = \frac{\det \hat{P}_k^*}{\det \hat{P}_k}.$$

Recalling Figure 10: JSE monthly force of interest an autoregressive model can be fit to this process, which appears to be stationary.

An investigation into the correlgraph, the autocorrelation at different lags yield the following:

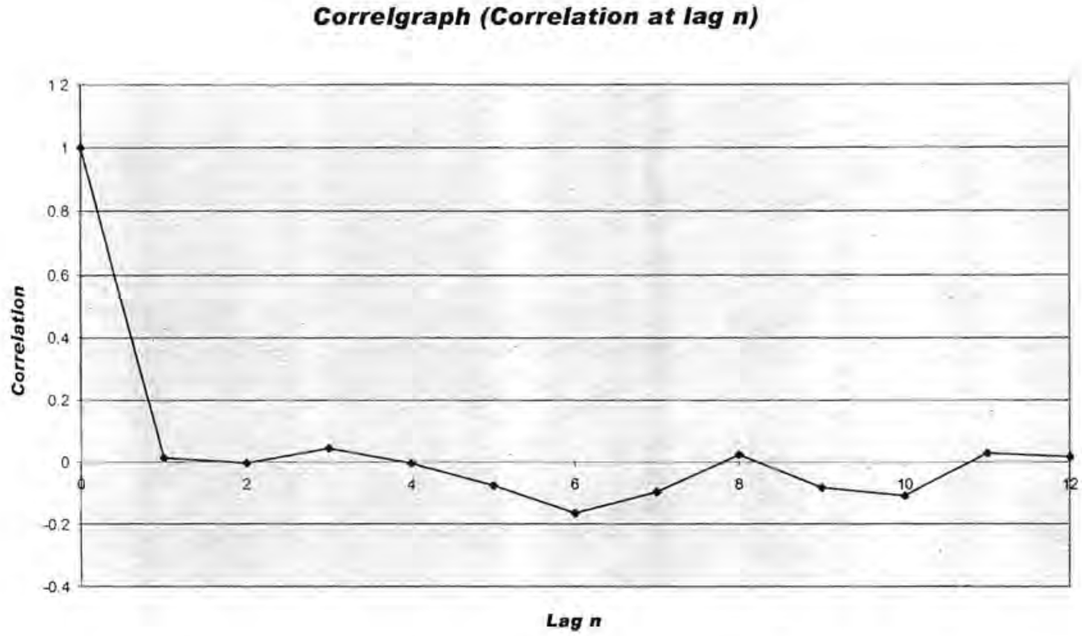



**Figure 12: Correlgraph for the force of growth of the JSE All-Share index**

The correlgraph shows no clear correlation between the force of growth at the different lag periods. This leads to the conclusion that a random walk model is probably a better model for this process than an autoregressive model.

6.3.2.4 An asset model for use in South Africa

THOMSON (1994) developed an asset model for use in South Africa. The model was originally set up for asset liability matching purposes for the life and pension fund industries but as short-term insurance companies often apply a long-term investment philosophy and this model might therefore sometimes be considered appropriate.

The model considers a variety of asset classes namely,

1. Equity dividend growth and dividend yields. (i.e. the total return on equities).
2. Inflation is also considered as a base for the other asset classes
3. Property unit trust dividend growth and dividend yields.
4. Direct property investment dividend growth and dividend yields.
5. Long-term interest rates.
6. Short-term interest rates.

MAITLAND (1996)[95] provides a review of the Thomson model.

[95] Maitland provides the following summary of the Thomson model (only the equity, inflation and short-term interest rate models are mentioned below):

The model is an integrated autoregressive model and therefore included in this section.

#### 6.3.2.5 Dealing with varying variance in autoregressive functions

The functions considered up to this stage all make use of error structures that have distributions with constant variance. The treatment of changes in the mean value has already been considered through the differencing of a series.

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The variables $\eta_t$ are each independently and identically distributed N(0,1) random variables.

The equity dividend yields and dividend growth rates are modelled as $EQDG_t$ and $EQDY_t$ respectively where $EQDG_t$ is the mean force of equity dividend growth in year t and $EQDY_t$ is the natural logarithm of the All Share Index Dividend Yield at time t per cent.

The equations for equities are as follows:

$EQDG_t = 0.093 + 0.116\eta_t + 0.076\eta_{t-1}$

$EQDY_t = 0.310 + 0.810EQDY_{t-1} + 0.198\eta_t$

The intended model for inflation is

$INFL_t \approx 0.008 + 0.899INFL_{t-1} + 0.088EQDG_t - 0.079EQDG_{t-1} + 0.077EQDG_{t-2} - 0.069EQDG_{t-3} + 0.020\eta_t$

where $INFL_t$ is the mean force of inflation in year t.

The interdependence in the model is clear. Inflation is modelled as a function of dividend growth, which is similar to the growth considered in the share index values.

Short-term interest rates are modelled as the sum of a unit gain function $Z_{MINT,t}$, and an error process $MINTZ_t$. The variable $MINT_t$ is the annual force of interest on money-market instruments as measured by the GINSBERG MALAN & CARSONS Money-market Index.

The equations for short-term interest rates are as follows:

$Z_{MINT,t} = 0.004 + 0.141INFL_t + 0.85Z_{MINT,t-1}$

$MINTZ_t = 0.008 - 0.091EQDG_t + 0.885LINTZ_t + 0.019\eta_t + 0.010\eta_{t-1}$

$MINT_t = Z_{MINT,t} + MINTZ_t$

In order to start the simulation, the one-step-ahead residuals for both MA(q) and ARMA(p,q) processes are needed. They are -0.04587 for *EQDG* and -0.00552 for *MINTZ*. These translate into the following N(0,1) errors which can be used directly in the difference equations:

$\eta_{EQDG,1993} = -0.3954$
$\eta_{MINTZ,1993} = -0.2905$

It is quite likely that the variance of a process will not be constant over time but will change and have periods of very stable variance followed by periods of very unstable variance. This is intuitively obvious as stock markets are known for durations of great variations and durations of fair stability.

ENGLE (1982) developed autoregressive models with conditional heteroscedasticity (ARCH). Later BOLLERSLEV (1986) developed these models into generalised autoregressive models with conditional heteroscedasticity (GARCH).

Normal autoregressive models of the form

$$X_t = \alpha_1 X_{t-1} + Z_t$$

have been considered above. $Z_t$ was considered to have a certain distribution with mean $\mu$ and variance $\sigma^2$.

The generalised autoregressive function with conditional heteroscedasticy considers the model for $\sigma^2$. This function is presented as

$$\sigma_t^2 = \alpha_0 + \sum_{k=1}^{p} \alpha_k \left(X_{t-k} - \mu\right)^2 + \sum_{l=1}^{q} \beta_l \sigma_{t-l}^2$$

with constraints: $\alpha_0 > 0$ and the sum of all other coefficients smaller than 1. This ensures that the function is mean reverting. Note that the smaller the other coefficients the sooner the variance will revert back to the mean. This mean reversion process captures the clustering of variability that was mentioned earlier SHAH (2001).

Where the last term is not considered, this is said to be an ARCH(p) process and the general case is referred to as a GARCH(p,q) process.

### *6.3.3 Fractal models*

Consider a graph of asset growth where the size of growth over time is not stated. Some practitioners are of the opinion that it is not possible to discern the actual period of time for which the asset growth pattern was investigated. As a result asset returns can be based on a Poisson process. That is to say, irrespective of the time period considered between consecutive asset values a Poisson process can be applied.

Fractal models are based on the assumption that the same probability distribution will hold irrespective of the time span considered and as such are very similar to a compound claims process with the main difference that downward adjustments should also be allowed for. (For the collective risk model claims are always positive.) Asset changes therefore follow a Poisson process and the severity of the changes can follow a variety of distributions. The actual quantum of the severity will be a function of the time period considered between consecutive asset values.

This study does not consider fractal models in any further detail. The reader is referred to the work of A.D. SMITH (1994) contained in the proceedings of the GISG convention of 1994 for further information regarding fractal models.

### *6.3.4 Chaotic models*

Chaos theory has become a buzzword in the modelling of various enterprises. The reason for this is probably attributable to the interesting graphical patterns generated using chaos theory.

These models are based on non-linear relationships between variables at different times. The models are termed "chaotic" because the interrelationships over time are very complex due to the non-linear relationship between the explanatory and dependent variables. These models have the drawback of producing the same values if the starting set is exactly the same. Depending on the nature of the starting set of variable, however, this may never occur in practice, assuming of course that the model is sufficiently complex.

Due to the complexities and level of judgement required in estimating the interrelationships between the explanatory and dependant variables, these models are not considered further.

### *6.3.5 Neural networks*

Neural networks are designed to replicate the operation of the human brain. That is to say, for a certain parameter set as input the network will generate a certain outcome. Time series modelling and regression analysis can therefore be obtained from neural networks as well.

Neural networks provide more flexibility in that the manner of linking a set of explanatory variables to a dependant variable is based on a weighting system. This allows a variety of combinations of explanatory variables to be used in generating a result based on some function of the explanatory variables. This result can again be applied in subsequent functions until the dependent variable is determined. Software packages are available to generate the best fit. These fits are, however, not always understandable as a result of the complexities of the network.

Neural networks provide an additional tool to practitioners to investigate experience. Given the level of uncertainty in asset returns and the fact that the human brain has not yet identified a suitable relationship between asset growth and explanatory variables, the topic of neural networks is not considered further in this study.

## 6.4 Fundamentals of Asset Models

An extensive array of models exists to model asset proceeds. The assumptions underlying each group of models as well as their level of complexity may be different but the following factors should also be borne in mind when determining appropriate models (SMITH (1994)):

1. *Practical Issues*

The model must be based on available information. Furthermore, the cost of development should be weighted against the added benefit of a more accurate assessment of the asset proceeds.

2. *Theoretical justification*

The model must conform to sound economic and actuarial principles and should always be checked for reasonableness. This is not a very strict requirement as there are many theories as to how the market behaves. Reasonableness is, however, crucial to avoid non-sensical predictions for the future.

3. *Fit to historic data*

The model must be able to conform to previous data sets. If the model is unable to produce reasonable estimates of past experience it will surely be unable to predict future experience as well. Auto-regressive analysis lends itself perfectly to the testing of models against past data. As a result, a considerable amount of literature is available on auto-regressive asset models.

4. *Market assessment of future trends*

There are also implicit assumptions to be made regarding future asset proceeds available in the market. These assumptions are reflected in the yield curve, market prices as well as in the prices of derivative instruments. The inclusion of parameters relating to these entities will, therefore, prove useful.

## 6.5 Model building

This author proposes the use of auto-regressive models given the level of flexibility that can be applied, the ease of understanding of these models, the consistency with liability models, the direct link to historical trends and the allowance for future experience. As these methods are also applied extensively in other disciplines, it should be easier for the reader to follow the reasoning below.

The main aim is to find a model which provides an appropriate fit to past experience as well as sensible future projections. The following approach can be applied:

#### 6.5.1.1 Step 1

Consider the economic factor to be modelled. This can be the total return on an asset, the income stream, or the capital appreciation on a certain asset class. Furthermore as the return may depend on other factors, it would be appropriate to model these factors first. R. Thompson (1994) has developed an asset model for use in long-term asset projections such as pension projection. This model, similar to the Wilkie model, is based on a base model[96], in this case real dividend growth, and subsequently inflation and other economic factors can be linked to this model.

This approach is appropriate in allowing for correlations between different economic factors.

#### 6.5.1.2 Step 2

Consider the elements which could have an influence on the projected value of the dependent variable, in particular those elements which can be considered to influence the current value of the dependent variable. This can be either the value of the factor some time ago (or at various durations), a moving average over a certain period of time, a mean base value that is assumed to be appropriate for the period of projection or some other variable such as a forward rate derived from current yield curves.

The elements which are the most important will be identified through autoregressive techniques explained above. The aim is to find the smallest set of explanatory variables that declares the most of the variability.

A variety of factors will need to be tested to ensure that an appropriate model is found.

Considering the Box-Jenkins methodology explained earlier, the model can either be investigated for a specific index or the difference operator can be applied to consider the rate of growth. The difference operator stabilises the index and eliminates any seasonal trends. Furthermore, the Box-Jenkins methodology is applied to series that appear to be stationary.

#### 6.5.1.3 Step 3

Once the appropriate explanatory elements have been found it is important to test the model against past experience and, in particular, to consider the error structure over time. It may be necessary to allow for a GARCH(p,q) model if the error structure does not have constant variability over time. Furthermore, the error structure might not have a normal distribution and this would need to be allowed for.

#### 6.5.1.4 Step 4

The only step remaining is to test the model against randomly selected periods of past data to ensure that the model will yield appropriate results. It must be stressed that this process is part art, part science and that the practitioner will

[96] The base model is the model of an economic variable on which some or all of the other economic variables are built. For example a model on inflation that will affect all other real return assets.

often be confronted with situations where it is extremely difficult to form an opinion as to the validity of the model.

This approach is based on short-term variability and as a result may not recognise the long-term theoretical structure of asset movements but rather aim to replicate the volatility exhibited in the market. This is intentional.

Once a model has been set this can then be applied in similar fashion as was done to liability models in projecting future income.

## 6.6 Summary

In this chapter asset modelling was considered. The following issues were addressed:

1. The different types of techniques available to model asset proceeds were considered. Considerable focus was placed on random walk models as well as autoregressive models. Other techniques were mentioned and the interested reader is referred to the references given.
2. The considerations required before setting an asset model were considered.
3. The process of fitting an asset model was considered. The techniques are similar to those applied in fitting liability distributions though the basic model may be completely different. This process is visited in further detail in the annexure which provides an example of the application of the ideas set out in this dissertation.

In the next chapter other areas of quantifiable risk will be considered.

# 7. Quantifying the other areas of risk

## 7.1 Introduction

The previous two chapters considered the techniques required to model the liability and asset area of risk. This chapter aims to give some background to the other areas of risk mentioned in the introduction. It goes on to consider how all these techniques are used to form a basis to build the model insurer.

## 7.2 Other areas of risk

### *7.2.1 Credit risk*

Credit risk refers to the possibility that monies receivable from brokers, reinsurers or other counter parties will not be forthcoming as a result of insolvency of the third party. In addition, credit risk can also refer to the risk of default on asset investments.

There is no short way in which to consider this risk. A starting point will be the credit rating of third parties. In order to model this area of risk the different sources of premium income need to be considered and with this the possibility that the provider of the premium will default and that the premium will not be forthcoming or that balances will be lost. Similar evaluations will be required for reinsurance recoverables.

This has been identified as a substantial area of risk given past market experience. As full details regarding third parties are always readily available, this risk must be managed using proper risk management principles e.g. not entering into a relationship with risky partners. In addition, a contingency loading should be included based on past market experience.

The contingency loading can be modelled by assuming a probability that a certain section of premium will not be forthcoming or that a reinsurer will default.

7.2.1.1 A structured approach to credit risk quantification

CREDIT SUISSE (1997) recommended the following structure in order to consider the default of a counter party:

1. Determine the size of the exposure to the counter party.
2. Determine the probability of default.
3. Determine the possible recoveries to be made from the counter party.

The approach is exactly the same as that required to model liability risk and explained previously in this study. Again the frequency of losses is considered and in the event of a loss the size of the loss is determined through the size of the exposure less possible recoveries. Hence the detail explained in the chapter on the modelling of the liability area of risk is relevant here as well.

The model is often applied as either a continuous or discreet approach. In the continuous instance, probability distributions are used whereas in the discreet approach deterministic probabilities and transition matrices are used.

7.2.1.1.1 Determining the size of exposure

This needs to be done on a continuous basis and is as relevant for credit risk as it is for liability risk. The maximum possible loss from any one counter party or any class of business needs to be considered at all times. Banks and financial institutions often place limits and the level of credit exposure that they will allow the institution to carry.

In similar fashion insurance companies have limits on the levels of exposure that they are prepared to take on certain perils for example catastrophes. Credit exposure is much more volatile than insurance exposure and banks are required to monitor these exposures on a daily basis and are also regulated on this basis.

The BASLE accord stipulates the regulatory requirements for banks and relies on the internal models of banks to regulate the industry. Continuous evaluation of exposure is critical in this regard.

7.2.1.1.2 Determining the probability of default

The probability of default has been suggested to be linked to the credit rating of the counter party[97]. It is, however, important to realise that these credit ratings are not static and can change on a regular basis. As a result transition matrices

[97] Recall MAHER (2001):

**Table: Quantification of Rating Agencies' Rates**

| Moody's | S & P | Expected Probability of ruin in 10 years |
|---|---|---|
| Aaa | AAA | 0.00% |
| Aa1 | AA+ | 0.10% |
| Aa2 | AA | 0.10% |
| Aa3 | AA- | 0.20% |
| A1 | A+ | 0.40% |
| A2 | A | 0.70% |
| A3 | A- | 1.00% |
| Baa1 | BBB+ | 1.40% |
| Baa2 | BBB | 2.00% |
| Baa3 | BBB- | 3.40% |
| Ba1 | BB+ | 5.20% |
| Ba2 | BB | 7.40% |
| Ba3 | BB- | 9.70% |
| B1 | B+ | 12.20% |
| B2 | B | 15.00% |
| B3 | B- | 19.20% |
| Caa | CCC+ | 35.80% |

have been developed to model the transition of credit ratings and therefore the probability of default over time. The following provides a transition matrix compiled by STANDARD & POOR'S (1996)

**Table 9: Example of credit rating transition matrix**

| Initial | Rating at year-end | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Rating | AAA | AA | A | BBB | BB | B | CCC | Default |
| AAA | 90.81 | 8.33 | 0.68 | 0.06 | 0.12 | 0 | 0 | 0 |
| AA | 0.7 | 90.65 | 7.79 | 0.64 | 0.06 | 0.14 | 0.02 | 0 |
| A | 0.09 | 2.27 | 91.05 | 5.52 | 0.74 | 0.26 | 0.01 | 0.06 |
| BBB | 0.02 | 0.33 | 5.95 | 86.93 | 5.3 | 1.17 | 0.12 | 0.18 |
| BB | 0.03 | 0.14 | 0.67 | 7.73 | 80.53 | 8.84 | 1.00 | 1.06 |
| B | 0 | 0.11 | 0.24 | 0.43 | 0.48 | 83.46 | 4.07 | 5.20 |
| CCC | 0.22 | 0 | 0.22 | 1.3 | 2.38 | 11.24 | 64.86 | 19.79 |

Such matrices need to be evaluated on a regular basis. Clearly the basis is not static and would need to allow for other external variables such as economic cycles. MCKINSEY consultants have developed a CREDIT PORTFOLIO VIEW model that allows for such external economic variables.

Credit rating and transition matrices have also come under substantial criticism because of the fact that credit ratings are only issued after substantial time delays due to the investigations required to set a credit rating.

DVORAK (2001) argues that the KMV model provides a better approach. This model is based on a company's share price and argues that the market is aware of credit problems long before the credit agencies. The model uses the share price as a proxy for the market value of its assets and as the solvency of a company deteriorates i.e. as the market value of assets converges to the value of liabilities of the company, the model allocates a probability of default to the company.

DVORAK cites an investigation conducted by KMV and notes the following:

1. The credit markets recognise that not all companies rated the same are equally risky.
2. The range of expected default within a broad rating grade is usually very wide.
3. In an AA- rating group with default probability of 0.2% the default probability calculated by KMV was between 0.02% and 19.7% for companies with the same grading.
4. Therefore in some cases the credit risk was over stated ten times and in others under stated a 100 times.

The approach applied by KMV is also applied by other practitioners. The TY model developed by YAKOUBOV Y.H., TEEGER M.H. and DUVAL D.B (1999) applies a similar approach to the setting of the equity model.

7.2.1.1.3 Determining the possible recoveries to be made from a counter party

The recovery to be made depends on the ranking of the debt to the counter party as well as the net asset value structure of the counter party. It is extremely difficult to model these items appropriately as the detail of all debt issued to the counter party would be required before the recovery could be considered. A prudent approach is therefore preferably in evaluating recoveries i.e. rather recognise recoveries when they are booked than making a projection of expected recoveries. This approach will lead to an overestimation of credit risk but it is more prudent.

Loans can rank in the following manner and this must be borne in mind:

1. Secured loans
2. Unsecured loans
3. Preferential shares
4. Ordinary shares

Credit risk is very important to banks as this constitutes the biggest share of their exposure. General insurance companies have neglected this area of risk and at their own peril as indicated by the research of FINNIS (1995) which indicates that 10% of all insolvencies of US general insurance companies are as a result of credit risk.

### *7.2.2 Growth risk*

Growth risk refers to the risk that volume increases too quickly and as a result capital support for the business is inadequate or, alternatively, the risk that volume reduces and as a result the marginal cost of writing business becomes too high.

The relative sizes of policies are also important as growth in small policies leads to more expenses than growth in large policies as a proportion of the premium income.

Growth risk is modelled by applying sensitivity tests to the model insurer. As these items are seen as management decisions they are not considered to be stochastic variables. Instead, a proper expense investigation is required to assess the costs involved in writing business and hence the impact of a change in policy size or policy volume.

Through the use of financing arrangements, general insurance companies are able to alleviate this risk through the use of quota share reinsurance arrangements[98] as long as the additional business is profitable.

### 7.2.3 *Expense risk*

An expense analysis entails the allocation of all expenses to different sources of income as described below.

#### 7.2.3.1 Step 1

Consider all the expenses incurred by the general insurance enterprise. The different departments in the company incur these expenses. This will consist of personnel expenses, accommodation expenses, marketing expenses, investment expenses and other overheads.

Overheads will include all other expenses reflected in the revenue account.

The aim is to allocate the expenses as accurately as possible in order to ensure an equitable charge to the different classes of business. The crucial aspect is, however, that all costs need to be covered.

#### 7.2.3.2 Step 2

Consider each type of expense and allocate the expense to a specific function in the enterprise. The expenses will relate either to business or service functions. The business functions should relate to the business flow and should therefore refer to

1. acquisition expenses
2. policy capturing expenses
3. maintenance expenses
4. claim settlement expenses or
5. investment expenses.

The service function costs need to be allocated in similar fashion. This is required to allocate the different expenses to the different line functions.

It must also be considered whether or not the expenses are directly related to the business sold or are only indirectly linked. Direct expenses will be expected to change as business volume changes whereas indirect expenses will be expected to remain fairly stable.

#### 7.2.3.3 Step 3

The different expenses then need to be allocated to different classes of business. This allocation can be done in a variety of ways depending on the expense.

[98] A company which experiences capital strain will be able to cede a proportion of its business to a reinsurance company. This company will then share in the experience in exactly the same manner as the ceding company. Often the reinsurance company will also provide a profit commission which will aid the ceding company. By taking the business off its books the ceding company is able to protect itself against the risk of excessive growth.

Certain classes of expense are quite clear such as commission which is simply a certain percentage of premium. Computer expenses, which will be service costs as well as the amortisation cost of the computer investment, will be allocated according to computer usage.

Another way of allocating expenses is to conduct a time analysis of the functions of personnel. For example within a certain department a percentage of time will be spent on each element of the line function. This expense can be either direct or indirect. By investigating this time allocation for each department it is possible to split the cost by line function and class of business. This approach can be used to allocate personnel costs. Alternatively, such an exercise might already have been completed in the past or the expertise of external consultants can be obtained in order to set a relative cost for each function. The number of times the function is carried out can then be multiplied by its relative cost in order to obtain weights for each type of function whereby the total cost apportioned to those functions can be split. This is referred to as functional costing.

Rent can be allocated to different departments and can subsequently be included with personnel costs.

#### 7.2.3.4 Step 4

Once the allocation by department, by line function and by type of business has been completed the type of expense can be considered further. The expense can be expressed as

1. a percentage of premium
2. a fixed amount per policy
3. a percentage of sum insured or
4. a combination of the above.

Bear in mind that some expenses are direct and some are not. As a result, fixed expenses should be split further between the direct costs which will increase linearly with an increase in volume and the indirect expenses which will reduce with an increase in volume.

The aim is still to obtain as accurate and equitable an allocation of expenses as possible. Therefore the costs should not be adjusted to accommodate marketing preferences e.g. small policies will have a higher relative cost than larger policies.

#### 7.2.3.5 Step 5

Once this exercise has been completed it is possible to model the expected expenses per line of business and for a certain volume of business. It will also be possible to test the impact of a change in volume and a change in average policy size on the overall profitability of the company.

It is important to apply consistency checks to ensure that the model used to project expenses actually projects current experience

### 7.2.4 *Other risks*

Some of the other risks that can be considered are risks involving

1. a change in tax structures,
2. management risk,
3. risk of failure of the owners of capital,
4. political risks,
5. changes in technology,
6. demographic changes,
7. social changes and
8. catastrophe risks.

None of these risks can be easily modelled using stochastic techniques and here a more robust approach will be required. This can be making a rough estimate as to the overall contingency margin that should be allowed for these eventualities. Due to the uncertainty other risks are considered to fall outside the scope of this study.

## 7.3 Summary

This unit contains a discussion regarding other areas of risk. Not all areas of risk were discussed, as the list provided can never be fully comprehensive. The following issues were addressed:

1. The issues pertaining to credit risk were considered and an approach was outlined whereby credit risk can be assessed.
2. Growth risk was considered and the importance of reinsurance was mentioned for the event of growth risk.
3. An expense analysis was considered and the methodology explained. The expense analysis is a crucial component in any premium rating exercise and enables the company to identify the risk in over or under charging because of an inaccurate allocation of expenses to individual policies.

Other areas of risk were mentioned.

The identification of risk is finalised in the next chapter through a consideration of corporate governance.

# 8. Corporate governance

## 8.1 Introduction

This study has considered an appropriate structure to consider risks in a general insurance environment and have paid attention to the quantification of some of the risks. This entire process needs to be supported actively by management in a sound businesses environment. Corporate governance is therefore a critical foundation for proper risk evaluation techniques.

In the banking field of risk management, this risk is evaluated through the evaluation of operational risk. Due to the nature of the risk, a numeric approach is more difficult to apply. A points scoring approach is possible, but should be handled with extreme caution due to possible model risk. An alternative approach to quantifying this risk would be to ensure that all aspects of corporate governance are adhered to.

Without corporate governance, internal control and internal audit, the risk management described in this study would not be sustainable. A stable framework is required wherein the ideas considered so far can be implemented.

The COMMONWEALTH ASSOCIATION FOR CORPORATE GOVERNANCE (1999) cites the following quotes:

*□The proper governance of companies will become as crucial to the world economy as the proper governing of countries."*

James D. Wolfensohn, President of the World Bank c. 1999

*"Capacity should be established in all Commonwealth countries to create or reinforce institutions to promote best practice in corporate governance; in particular, codes of good practice establishing standards of behaviour in the public and private sector should be agreed to secure greater transparency, and to reduce corruption."*

Commonwealth Business Forum Resolution, October 1997, endorsed by the Edinburgh Commonwealth Economic Declaration

*"Corporate Governance is the system by which companies are directed and controlled."*

Cadbury Report c. 1992

The TREADWAY Report (1987) first mentioned the importance of corporate governance in the USA. The CADBURY report (1992) published a "Code of Best Practice" which would lay the foundation for further development in the guidelines for corporate governance. The GREENBURY report (1995) published a report on good practice in determining directors' remuneration.

The HAMPEL Committee was commissioned to evaluate all guidance provided and consolidate this into a "Combined Code". This committee was tasked to

1. Review the Cadbury code to ensure the original purpose was being achieved.
2. Review the role of directors
3. Review the Greenbury recommendations
4. Address the roles of shareholders and auditors in corporate governance.

The TURNBULL report (1999) was issued to give guidance on the implementation of this consolidated code of recommended practice following a request by the Institute of Chartered Accountants in England and Wales and the London Stock Exchange.

The codes were evaluated by the CACG together with codes established in South Africa, Australia, New Zealand and the USA.

### 8.2 The CACG guidelines for corporate governance

The board should:

*Principle 1* – exercise leadership, enterprise, integrity and judgment in directing the corporation so as to achieve continuing prosperity for the corporation and to act in the best interest of the business enterprise in a manner based on transparency, accountability and responsibility.

*Principle 2* – ensure that through a managed and effective process board appointments are made that provide a mix of proficient directors, each of whom is able to add value and to bring independent judgment to bear on the decision-making process.

*Principle 3* – determine the corporation's purpose and values, determine the strategy to achieve its purpose and to implement its values in order to ensure that it survives and thrives, and ensure that procedures and practices are in place that protect the corporation's assets and reputation.

*Principle 4* – monitor and evaluate the implementation of strategies, policies, management performance criteria and business plans.

*Principle 5* – ensure that the corporation complies with all relevant laws, regulations and codes of best business practice.

*Principle 6* – ensure that the corporation communicates with shareholders and other stakeholders effectively.

*Principle 7* – serve the legitimate interests of the shareholders of the corporation and account to them fully.

*Principle 8* – identify the corporation's internal and external stakeholders and agree a policy, or policies, determining how the corporation should relate to them.

*Principle 9* – ensure that no one person or a block of persons has unfettered power and that there is an appropriate balance of power and authority on the board which is, *inter alia*, usually reflected by separating the roles of the chief executive officer and Chairman, and by having a balance between executive and non-executive directors.

*Principle 10* – regularly review processes and procedures to ensure the effectiveness of its internal systems of control, so that its decision-making capability and the accuracy of its reporting and financial results are maintained at a high level at all times.

*Principle 11* – regularly assess its performance and effectiveness as a whole, and that of the individual directors, including the chief executive officer.

*Principle 12* – appoint the chief executive officer and at least participate in the appointment of senior management, ensure the motivation and protection of intellectual capital intrinsic to the corporation, ensure that there is adequate training in the corporation for management and employees, and a succession plan for senior management.

*Principle 13* – ensure that all technology and systems used in the corporation are adequate to properly run the business and for it to remain a meaningful competitor.

*Principle 14* – identify key risk areas and key performance indicators of the business enterprise and monitor these factors.

*Principle 15* – ensure annually that the corporation will continue as a going concern for its next fiscal year.

### *8.2.1 An evaluation of the risk evaluation techniques according to the CACG guidelines.*

The purpose of this section is to show how the techniques considered thus far tie in with the guidelines set by the COMMONWEALTH ASSOCIATION FOR CORPORATE GOVERNANCE.

Principles 1 and 2 are not addressed.

Principles 3, and 4 are addressed in detail. The structure explained thus far will allow the board to evaluate the appropriateness of certain strategies. In the next chapter the optimisation of the return on capital will be considered and it will be shown how the framework considered assists the board in setting objectives and monitoring those objectives for a general insurance company.

Principle 5 will be adhered to by compliance committees and the structure considered is based on the assumption that the company does comply with all laws.

Principle 6 pertains to communication. Understanding the problems at hand is often difficult. The methodology explained in this study provides a base for quantifying certain problems i.e. quantifying the impact of a risk or problem on the solvency of profitability of the company. Once an understanding of the problem has been laid down by the board, the board will be in a better situation to communicate to all the relevant stakeholders on the problems and their solutions.

Principles 7 to 9 are not addressed in this study.

Principle 10 is concerned with the regular monitoring of procedures and processes. The structure explained in this study is not only relevant for the evaluation of risk at a certain point in time but also for the evaluation of risk over time. The ability to generate expected experience over time allows the company to monitor its own state of health on a regular basis. This provides for a very powerful framework to identify any problem at any point in time and not only with substantial hindsight. The algorithm of COUTS and DEVITT (1997) explained earlier provides an insight into the approach that would be used. Such techniques are often referred to as model office simulations.

Principle 11 is concerned with the monitoring of the performance and effectiveness of the operation. This is similar to principle 10 and therefore this study is also relevant in this instance.

The study does not address principles 12 and 13.

At its core this study meets the full requirements of principle 14. The entire structure set out in this study will be appropriate to identify risk factors and key performance measures, to develop solutions for problems identified through these and to monitor the experience following the implementation of these solutions. This was also explained in the introduction to this study.

Principle 15 will follow naturally after the implementation of the aforementioned principles.

### 8.3 Other guidance

In South Africa the KING report on corporate governance was issued to assist companies with governing themselves through the board of directors. The KING report is currently (2000) subject to revision and currently contains reference to the following:

1. Significance for planning
2. Background to corporate governance
3. The board of directors
4. Remuneration
5. Nominating committee
6. Stakeholder communications
7. Auditing
8. Internal controls
9. Audit committee
10. Interim report
11. Going concern
12. Illegal acts
13. Internal auditors
14. The company links
15. Affirmative action
16. Ethics

There exists considerable overlap between the principles set out by CACG and the KING report.

Other guidance includes[99]:

1. Australian Investment Manager's Association
2. Stock Exchange of Hong Kong ("SEHK")
3. Summaries of Selected International Codes
4. Business Roundtable
5. GM Board of Directors
6. Report of the NACD Blue Ribbon Commission on Director Professionalism
7. TIAA-CREF Policy Statement

## 8.4 Enterprise wide risk management

SOUGH (2001) argues that corporate governance, risk management, internal control and internal audit are all part and parcel of enterprise wide risk management.

In order to implement the techniques outlined in this study, this collective thinking pertaining to risk management must be borne in mind.

Recognising the interrelationship between all these aspects leaves some room for confusion. This author suggests that internal audit and corporate governance be viewed as risk mitigating techniques for operational risk. By ensuring regular

[99] All these guidance notes are available on the CACG website.

internal audit, verified by external audit through the adherence to the guidelines set above, it is possible to manage operational risk. The management does not place emphasis on the quantification of these risks but rather ensures that operational risks are negligible and therefore do not need quantification.

HYLTON (2001) argues for the establishment of an independent risk management function solely focused on the implementation of the ideas discussed in this study.

## 8.5 Summary

This chapter considered the importance of corporate governance in enterprise wide risk management. The link between corporate governance and the ideas set out in this study is considered.

The extent to which the techniques in this study support the principles of corporate governance as outlined by the COMMONWEALTH ASSOCIATION FOR CORPORATE GOVERNANCE is considered and the importance of these techniques stated.

In the next chapter all the ideas set out thus far are consolidated into a framework suitable for implementation as an enterprise wide risk management information system.

# 9. Enterprise wide risk management

## 9.1 Introduction

This dissertation has followed the following train of thought:

1. An overview of risk areas, risk evaluation and the uses of risk evaluation were considered in the introduction.
2. The objective was set out two-fold. The first was to consider a framework whereby risk could be evaluated for a general insurance company and the second was to consider how this framework would enable practitioners to optimise the requirements set to them.
3. The framework was discussed based on the income statement and balance sheet of a company. Each item depends on certain factors and correlations exist between these factors.
4. In order to allocate resources in an appropriate manner the most important areas of risk and uncertainty were subsequently considered based on empirical studies. The main areas identified were reserving and pricing risk, asset risk, growth risk and other risks pertaining to management decisions.
5. These risks were then considered in turn. The first two were discussed under the quantification of the liability area of risk. Asset risk quantification was subsequently considered. The importance of a model office was also mentioned whereby all influences can be modelled concurrently. This will enable the practitioner to set limits on the level of growth in business. Expense risk was also considered and the importance of proper quantification highlighted. In order to address the problems that may arise from poor management the issue of corporate governance and the guidelines currently set were considered. The difficulty in trying to quantify this risk was also mentioned. Arguably such quantification is possible through scoring techniques based on corporate governance guidelines.
6. Throughout the dissertation mention was made of a model office and the integration of all these ideas. The model office provides the framework which has been set as an objective at the outset. The optimisation of goals using this framework is now considered in this chapter.

In the introduction, mention was made of different types of business strategies namely:

1. Business mix strategies
2. Managerial strategies
3. Distribution channel strategies
4. Capital allocation strategies
5. Investment strategies
6. Reinsurance strategies

The framework of model office is intended to test these strategies and then to evaluate the outcomes for the objectives set by all the relevant stakeholders. The business strategy to be investigated first will be the current one. This will indicate the solvency of the company as well as the distribution of the expected return for the existing business structure. These results can then be compared against benchmarks or objectives set.

One such an objective is the use of modelled cash flows in order to determine an optimal usage of capital. These ideas are not new to the actuarial profession in South Africa. In the past M.S.Claassen and P.P.Huber (1992) made mention of the use of asset-liability models and these ideas were further discussed by P.W.Ennis and R.E.Dorrington (1994).

N.A.S.Kruger and C.J.Franken (1997) addressed the issue of the probability that the margins on assumptions used in the financial reports of life insurance companies are sufficient. To extend on this idea the concept of probability of ruin is discussed in this dissertation and a suggestion is made to move from a prudent valuation basis to a basis whereby the valuator is, say, 99% confident that reserves will be adequate given the assumptions.

## 9.2 Capital, business structure and the probability of ruin

All work on risk evaluation is based on three interacting areas:

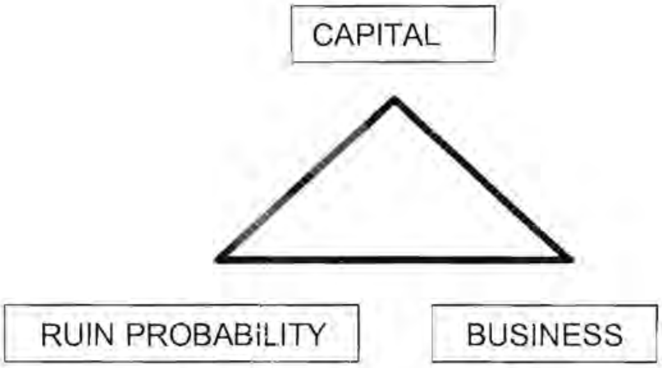



Figure 13: Relation between capital, business and the probability of ruin

The capital block represents the funds of the insurance company. The higher the level of funds available, the lower the probability of ruin will become but at the same time the higher the shareholders' requirements will become for return on capital because of greater amounts invested. As a result, to maintain the same probability of ruin when capital is increased, more business will need to be written which must yield the marginal return required in order to meet shareholders' requirements.

The ruin probability is a measure of risk. It can be limited to a specified probability for example the probability that liabilities will exceed assets in the next year must be less than 2%. This is similar to the value at risk measures commonly found in the banking environment. Alternatively the probability can be associated with a monetary limit as well. For example the probability that liabilities exceed assets by more than 10% and less than 50% must be less than 1%. Alternatively the distribution of the excess of liabilities over assets can be considered. This is similar to the concept of shareholder shortfall found in the banking environment.

The business represents the current business operation, level of business sold and level of profitability. For an increase in business sold given a fixed level of capital allocated to the business environment, the ruin probability will be increased.

Therefore, any change in any one of the elements of the triangle in Figure 13 will mean that at least one of the other elements will change. For example an increase or decrease in capital, given that the business element stays the same will decrease or increase the probability of ruin. Alternatively to decrease the probability of ruin but maintaining the same level of return on capital, more profitable business or more business which is as profitable as the current business must be sold with the available resources.

It is important to note the impact of statutory reserving requirements at this stage. Statutory requirements imposed by regulators aim to protect policyholders and shareholders alike. The requirements also limit the potential increase in risk and as a result *limit the potential return on capital.* The insurance company will not be able to increase risk to enhance return beyond a specific maximum implicitly implied by the statutory requirements.

This framework has drawn the attention of many parties involved in the regulation of the financial services industry. The BASEL II accord for the regulation of banking institutions propose that regulation of these institutions be based on the internal risk management models developed by the bank. This will overcome the problems currently faced by regulators as a result of inappropriate regulatory standards being forced upon the banks as well as the arbitrage applied by banks to overcome the regulatory requirements.

Currently regulators base their requirements on empirical studies for the entire industry, this is not appropriate due to the fact that not one of the banks will have a risk profile similar to that of the entire market. As a result banks may decide to change their business structure to mitigate the penal effect of such regulatory requirements. For example, if the regulatory requirement is a fixed percentage of premium then companies will sell non-proportional insurance covers instead of proportional insurance covers.

This is also the framework that can support the objective mentioned by Alan Greenspan in October 1999 mentioned in the abstract of this dissertation.

## 9.3 Applying the framework

The framework proposed in chapters 2 and 3 has to be set up. Applying this framework it will be possible to obtain the distribution of the expected earnings based on the current business structure for the next year. This distribution can be compared to the capital base of the company to determine the distribution of the return on capital.

As explained in chapter 5, the capital base of the company also has a distribution, this is as a result of the expected fluctuations in both asset and liability values. In order to ensure that the results are sensible, it is important to evaluate the valuation bases used to set the capital value.

### *9.3.1 Optimising the reserving policy of the company*

Certain minimum reserving requirements may already be laid down either through regulations or through accounting principles. The reserving basis will tend to be either conservative or optimistic. A conservative basis is one where the expected run-off of business is set worse than what would be expected on a best estimate basis. That is to say greater amounts are set aside to meet the outstanding liabilities than what would be considered necessary on a best estimate basis i.e. there is more than a 50% chance that there will be funds available when all the liabilities have been met. An optimistic basis is the reverse of a conservative basis.

When the reserving policy is very conservative the probability of ruin will generally reduce and where the reserving policy is optimistic the reverse will generally hold true. If shareholders realise that part of their capital is tied up in liabilities, the insurance company will therefore wish to hold as little reserve as possible in order to reduce the cost of capital and return profits to the providers of capital as soon as possible. This action will, however, lead to situations where additional capital will be required from shareholders and hence the reserving policy will be optimised by comparing the effect of a change in policy to the subsequent expected rate of return and measure of risk.

This author is of the opinion, and it is a subjective opinion, that shareholders have absolutely no idea whether or not reserves are set at conservative or optimistic bases and as a result do not understand how much capital is tied or may be required in addition. This obviously allows companies to present incorrect results to their shareholders through the overstating of liabilities. If shareholders consider these to be appropriate on a best estimate basis and they are, in fact, conservative then the apparent capital base according to the shareholders is less than what is actually the case and the return that can be secured on this base is higher than what would otherwise be the case. In short, shareholders are living in the illusion that they are obtaining returns on their capital higher than is actually the case.

In order to understand the true capital base, a sensible starting point is the embedded value[100] of the company.

### 9.3.2 *Calculating the embedded value*

The embedded value is the expected value of the company on all business written to date. It consists of the following:

The assets of the company (this will be the present value at which the assets can be realised)

*less* the liabilities of the company

*This equals the net asset value of the company*

*plus* any profit on business written to date that has not yet been recognised (margins in reserves) (for general insurance business no renewal of business is expected)

*plus* any unrecognised balances such as outstanding premium balances

*This equals the embedded value of the company*

It is important to recognise the possible sources of fluctuation in the embedded value. In particular, great care should be taken in the evaluation of the assets of the company if the basis of valuation is not consistent with the basis used to value liabilities. The reason is simply that if asset values are depressed, say after a fall in the market, then the net asset value might be unrealistically low when it can be reasonably be expected that the market will recover in the foreseeable future. Evaluating assets based on a projected cash flow basis might actually be more suitable in such situations.

The embedded value can be calculated at future durations as well. Using the models of claim and expense outgo and asset income as described, it is possible to calculate the probability distribution of the embedded value of the company.

This goes a long way from using only one set of assumptions in order to calculate the value of an insurance company.

To optimise the reserving policy, the only element of the model that is then changed is the reserving policy and the effect thereof is determined by comparing the probability density function of the embedded value before and

[100] Many practitioners in the market consider the use of the tenn "Embedded value" to be one only applicable to the life insurance industry. If all bases applied by a general insurance company are best estimate bases i.e. there is a 50% probability of experience being either higher or lower, then this conviction is a correct one. As mentioned earlier, this author is not convinced that this is the case and as a result the concept of an embedded value as described above is also relevant to the general insurance industry.

after the change in the reserving policy. It is important to recognise that different types of investigations will yield different results. For example, a requirement to maintain a minimum level of embedded value over time will yield a different required reserving policy to a requirement to have a minimum embedded value now. This is because the first requirement is subject to more variation.

Another factor to take into account when optimising the reserving policy is the tax requirements of the government. By increasing the reserves at any point in time, this allows for the deferment of tax and subsequently increasing the return on capital. This situation arises because the company is able to secure an income on the funds which would otherwise have been paid to the government as tax.

If the insurance company can invest its reserves in similar investments as that which the investor would otherwise invest in, the investor will prefer to retain the funds in the company as reserves and defer the payment of tax. If the investor, however, has a higher investment requirement, the investor will be prepared to sacrifice some of the return as tax in order to secure the other higher yielding investment opportunities.

When calculating the embedded value it is therefore important to allow for the payment of tax as accurately as possible. The present value of future payments is determined at the risk discount rate. This is the rate of return required by the shareholders. Depending on the shareholders' requirement either a very conservative or very optimistic reserving policy may be suitable.

Using the framework, it is therefore possible to determine the distribution of the return on capital.

## 9.4 Setting a benchmark

It is important to start this section by evaluating the primary assumptions for the optimal utilisation of capital:

1. Markets are efficient. In particular, the price of capital is not influenced by the actions of any one party in the market.
2. Providers of capital are rational. This means that for the same level of risk a shareholder will prefer the investment with the highest level of return.
3. Market players have full knowledge of the level of return versus risk (risk is assumed to be the variability in return)[101] available in the market and these are available for all market instruments.
4. A specific set of assumptions regarding the business operation equates to an expected return on capital and this can be approximated through the techniques mentioned in this dissertation.

[101] From the variability in return the distribution of the downside risk can be assessed and this is, in essence, more important than total variability *per se*.

These assumptions do not hold in all circumstances. This chapter, however, relies on these assumptions in drawing conclusions on the proper utilisation of capital.
Mention has already been made that in the current economic environment, it is unlikely that assumption 3 above is true. It has to be noted, however, that securities and exchange bourses around the world are driving the speedy delivery of information and the International Accounting Standards Committee and others are driving the process of full disclosure to allow investors to make better decisions. The framework proposed will also assist in this regard.

Assuming the assumptions do hold, then every investment opportunity can be expressed in terms of its risk and return. Portfolio theory dictates that the risk and return on any one investment is irrelevant because the risk will be mitigated through diversification. This can only be done if the risk and return of each element within the portfolio is known as well as the correlation between the returns on the different investments within the portfolio.

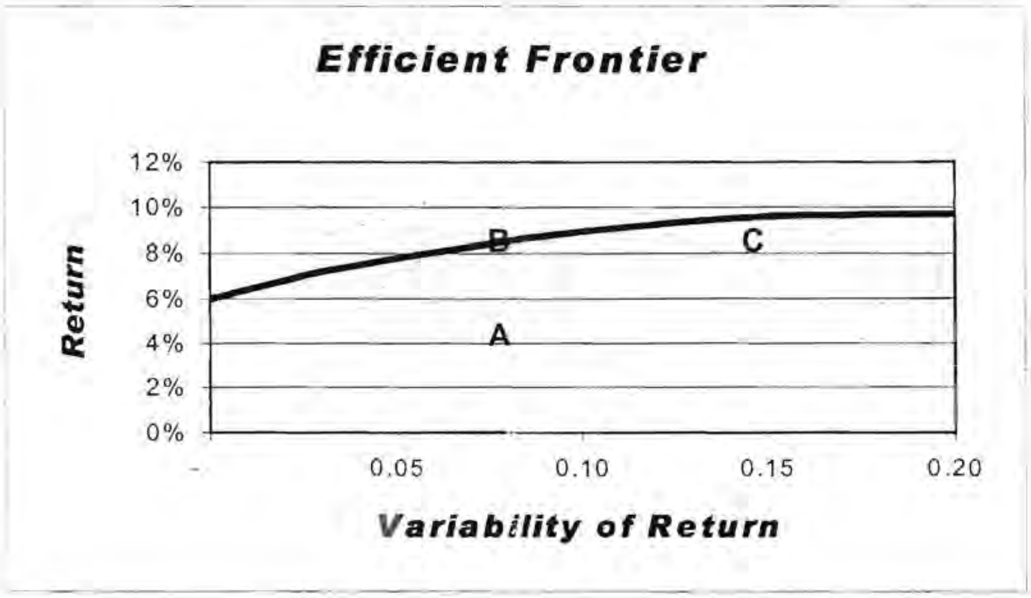
The level of correlation will be determined by the Beta of every investment. That is the correlation of the investment's return to a market index of returns. Investment managers often publish the Beta of shares based on historic experience. This goes beyond the purpose of this dissertation. It suffices to say that if the risk and return of the company are calculated, a portfolio manager will need to consider the Beta of the company and then consolidate the entire portfolio to determine the risk and return of the entire portfolio.

Having said that, portfolio managers are often adamant that the risk and return of an individual investment should be comparable to that of a diversified portfolio, arguing that the investment or business has sufficient diversification within. If this is true, then the risk and return obtained for the company, derived from this proposed framework could be compared to the efficient frontier.

The efficient frontier was introduced in the introduction. It provides the combination of risk versus return for all optimal combinations of investment portfolios within a market. That is to say, based on an investor's appetite for risk, the efficient frontier will indicate the expected return.

Whether or not investors understand the concept of risk is an entirely different matter. It is the author's opinion, following the burst of the e-bubble, that the majority of investors did not understand this concept.

Figure 14: An example of the efficient frontier




Given the investigations conducted using the proposed framework, the company may arrive at the conclusion that the moments of the company's return on equity can be plotted as A on the graph above.

This would indicate that the company is currently not performing optimally. The objective is to change the business process to ensure that the company moves to point B. It is important to note, that if the shareholders were to sell the company and invest in a fully diversified portfolio, they would be able to achieve B. As a result it is reasonable to expect this from the company if the company can also be considered as a fully diversified portfolio. This is never the case and as a result it may be reasonable to set the objective at point C. This would mean that with diversification the risk would reduce to B and the return would still be at the level required.

Depending on the shareholder, the focus might be increasing the return or limiting the risk or a combination of these. Note that there are external constraints as well, for example it is not possible to increase the risk indefinitely. Regulatory requirements may implicitly limit the level of risk that a company can take on. Alternatively, using this framework, the regulator may not accept a level of risk higher than a certain level. The level of risk will, however, not be measured on the variability in return but rather on the probability of insolvency. Note that this can be obtained from the variability in return. The same framework is used to accommodate both types of questions.

It is important to note that the risk measure can change and that the framework will be able to accommodate this. For example, the risk measure might be the probability of not paying a dividend.

Alternatively, the rating scale mentioned in the introduction can be used. Recall that rating agencies provide ratings for listed companies. These ratings can be roughly translated into a probability of ruin in the forthcoming year. This was also elaborated upon in chapter 7 where credit risk was discussed. Using the framework, it is possible to calculate the probability of ruin. This can also be used for comparative purposes using the efficient frontier.

Furthermore, the efficient frontier provides the moments of the return over an indefinite time horizon. When comparing actual returns to expected, this will be allowed for by investigating the confidence interval applicable to the efficient frontier. This will provide an upper and lower bound of return given the level of risk and will appear as an expanding funnel expanding with the increase in risk.

Given the objective, it is possible for management to start testing and subsequently implementing suitable business strategies to achieve the objectives set.

The manner in which this is achieved can be any one of a variety of options:

1. More (profitable) business can be sold with the existing capital base.
2. The existing capital base can be used to purchase additional businesses.
3. The pricing of products can be adjusted to be more representative of the cost of supporting that business.
4. The reserving policy can be changed. This will change the level of risk as well as the level of return. The deferral of the payment of tax must be considered here as well.
5. Those products which are more profitable can be identified through the techniques discussed and the insurance company's market share of those products can be increased.
6. Where problem areas are identified, such as poor claim management, management strategies can be put in place to improve the audit and identify problem areas earlier. For example, the risk rating techniques discussed in chapter 5 can be used to identify fraudulent claims.
7. Similarly, comparison of actual to expected premium will indicate where premium collection might be a problem and this can also be rectified through management strategies.
8. The reinsurance structure can be changed to transfer business with too high levels of risk.
9. The investment policy can be changed to match liability outgo better or to enhance returns further.

## 9.5 Optimising the pricing of products

Optimising the pricing of products entails calculating as accurately as possible the cost of producing the product and then comparing the cost to the market rates and selling those products which are most profitable.

In order to accurately price production cost the following has to be considered:

1. The cost of claims or risk.
2. The cost of expenses.
3. The cost of capital.
4. Investment returns.

The cost of claims risk was discussed in chapter 5 and the cost of expense risk was discussed in chapter 7. It is possible to allow for investment return, if any, or alternatively to allow for the cost of borrowing funds until the premium is received. This cost of borrowing will be at the same rate as the cost of capital if the funds have to be obtained from shareholders. In addition, the profit criteria of shareholders have to be met. It is therefore necessary to allocate capital to the different classes of business in such a manner that it will result in the equitable rating of the different products. There are different approaches possible.

Various methods have been suggested whereby capital can be adjusted according to risk. Insurance Research Letter 5/96 makes mention that opinions on the cost of capital differ. Some suggestions for splitting capital are mentioned below.

Allowance for investment income has to be taken into account. This will be done by modelling all cash flows. If this is not done the variability for long tail classes of business may be over-estimated and, as a result, the charge for capital provided will be too high. Some practitioners may, however, regard such margins as prudent[102].

It has to be realised that some of the techniques are more appropriate for pricing while others lend themselves to assessing solvency.

### *9.5.1 Allocating capital by risk premium*

Capital can be defined to be total funds allocated to shareholders, the embedded value explained earlier. Shareholders' funds will require a certain return, which has been set in advance. These funds can then be split by the risk premium of business sold. A return on capital for premium rating purposes can then be set according to this allocation. The allocation of capital is dependent on the manner in which the risk premium is calculated[103].

Note that many regulatory capital requirements are based on premium. These capital requirements are based on the assumption that the higher the premium

[102] The aim of this paper is to suggest ideas whereby as many factors as possible are modelled as explicitly as possible thereby eliminating the need for a subjective opinion of prudence and rather putting a figure on how prudent the results are e.g. 9 in 10.

[103] As risk premiums are often based on expected average experience, adequate allowance for variability in experience is not made. As a result, this method is considered to be inappropriate.

the riskier the product. This is generally not the case. It is, however, important to realise that a company with capital close to the statutory minimum requirement will require to meet a cost of capital equivalent to service the statutory requirement as there is no cross subsidisation possible with existing capital balances. This illustrates the advantage of economies of scale.

### *9.5.2 Allocating capital by statutory requirement*

Many practitioners base the charge for capital, for pricing purposes, on the statutory capital requirement for that class of business. In his presentation "Risk Based Capital: Used as an Internal Management Tool" (1996), A. Hitchcox suggests the use of the NAIC RBC rules to allocate capital to different classes of business as this can be viewed as the minimum capital requirement for US domiciled insurers.

The US has the most detailed set of statutory capital requirements. Many statutory requirements can be seen as risk adjusted capital requirements imposed by the regulatory authorities. The aim is to ensure that sufficient capital is held to protect the interests of policyholders. Risk adjusted capital provides an added advantage in that the probability of ruin or the probability of the company not being able to meet its liabilities can be quantified for the specific company. This poses a considerable advantage as opposed to applying fixed rules because allowance is made for the risk specific nature of each individual enterprise.

This method is practical but the danger is that the NAIC RBC rules are fixed and therefore do not adequately allow for an assessment of the risks at hand.

### *9.5.3 Allocating capital by variability per line of business*

Mention has been made of the relationship between the capital available, the probability of ruin and the premium rate given a certain requirement for return on capital. When the purpose of the investigation is to determine appropriate charges for capital, the probability of ruin and the capital available will be set in advance and consequently the premium rate will be determined.

The capital to be allocated by line of business will consist of all shareholders' funds. The reserving policy adopted will have a direct impact on the shareholders' funds in that the accumulation of net retained profit is delayed. The practitioner therefore needs to decide on an appropriate valuation approach for both assets and liabilities consistent with the requirements of shareholders.

Once the capital requirement for the company as a whole has been set, the capital can then be allocated per line of business according to the variability in each line.

In order to calculate the variability per line of business, the techniques discussed in chapter 5 are used. An approach may be to calculate the reserve requirement for each class of business by line of business and subsequently to allocate the capital required proportionally.

It is important to first model the entire business operation otherwise this analysis will not be possible.

The variability per line of business has not been defined. This variability can be measured using a variety of definitions for example using the reserve calculated on a line by line basis. The level of sufficiency of reserve can then be used to allocate capital. For example calculate the reserve requirement for each line of business to ensure a 95% sufficiency of reserve. Subsequently, allocate the capital required proportionally.

The criteria may be changed and will result in different capital allocations. Another example will be to allocate capital according to the expected contribution a class of business can make to a possible loss to the company for a 100-year event. For this type of investigation short tail classes may not have any capital charge due to the stability of their results.

Deciding on a proper indication of the variability of a class of business and subsequently the cost of capital is open to argument. A risk measure equivalent to the level of risk of the entire business operation can be used.

It is reasonable to use the standard deviation of experience. Evaluating the overall capital requirement will also be based on the standard deviation of experience. The overall capital requirement can then be proportionally allocated according to the variance of each class of business assuming independence between the classes of business. If the classes are correlated then the covariance must be allocated in equal proportions and subtracted from the variance.

In other words: The capital requirement is normally derived from the standard deviation which is the square root of the experience.

Std dev = $\mathrm{Var(ALL)}^{0.5} = (\mathrm{Var(A)} + \mathrm{Var(B)})^{0.5}$ assuming independence. Therefore a proportional allocation can be made according to the variability in each class.

Or

Std dev = $\mathrm{Var(ALL)}^{0.5} = (\mathrm{Var(A)} + \mathrm{Var(B)} - \mathrm{Cov(A,B)})^{0.5}$ where independence does not exist. In this instance the proportional allocation can only be made once the covariance has been allocated equally to both A and B.

This exercise will indicate which classes are more capital hungry and evaluate all returns on a level playing field. This will allow for the identification of the most profitable classes of business in return on risk adjusted capital terms. Such analysis can then be used to optimise the mix of business sold.

### *9.5.4 Allocating capital by marginal capital allocation*

Another approach applied by some practitioners is to calculate the marginal capital requirement for new business. This can be done by setting up a model office and calculating the capital requirement before a new policy is written and after a new policy is written[104]. This will lead to very low charges for capital as the company benefits from economies of scale.

### *9.5.5 A suitable approach*

This writer is of the opinion that an equitable allocation of capital can only be made if the statutory requirements and risk characteristics of the business are both taken into account. The cost of capital will therefore consist of a statutory minimum charge together with a charge to those lines of business which are expected to affect the free reserves of the company. Any subsequent changes to this charge will reflect a marketing decision to reflect cross subsidies in the cost of capital.

Note that given recent developments in the approach to setting regulatory capital requirements i.e. the BASEL II accord which encourages internal models and approval of these models by the regulator, the approach to capital allocation will be able to change to a true risk allocation. The regulator will be concerned about the accuracy of the models used and will rely on backdating tests to be carried out on a regular basis. This is the same requirement as the monitoring of experience suggested in the framework. In addition though, the regulator may penalise the company if the models were to exhibit levels of error higher than can reasonably be expected.

### *9.5.6 Practical Application of Risk Adjusted Capital*

#### 9.5.6.1 Variability of experience

In order to properly manage the ideas set forth thus far, a model office based on the framework set out thus far must be set up. This requires substantial initial investment.

The model office will form the base of the calculations. In order to assess the capital required, the variability of experience will be determined. This transforms the information available on the entire business operation into knowledge that can be conveyed to management.

[104] Assuming that the expected experience which will result from writing the business can be assessed.

Knowledge on the variability of experience is a very powerful management tool as it aids the identification of problem areas and the timeous setting up of appropriate, remediable action plans.

#### 9.5.6.2 Internal Solvency

The split of capital according to the variability inherent in each line of business has been considered. Assessing the variability of experience for each line of business means that all the areas of risk need to be appropriately modelled for each line of business.

Once the probability density function of the experience for each line of business can be modelled, the capital required to support that business could be assessed. This capital requirement will be based on the average expected future experience plus a contingency loading. The methodology discussed thus far will enable the practitioner to equate the contingency margin to a specific probability of ruin. This is a powerful result.

It is important to note that the calculation of capital requirements per line of business will over estimate the capital requirement for the whole business operation, as all classes of business are not fully dependent. When assessing internal solvency for the company it will therefore be necessary to model all processes concurrently to allow for correlations.

#### 9.5.6.3 Probability of ruin

General insurance markets tend to follow a cycle of oscillating underwriting results. This means that practitioners will generally not be able to price products based on risk cost plus an appropriate capital charge plus all other loadings but will have to make do with the premium rates available in the market. The insurer who can beat the market cycle, inter alia by better pro-active pricing, stands to be more profitable in the long run. Using the methods of allocating capital according to risk inherent in the business sold, the relationship between capital and premium rates can be used to calculate the probability of ruin for specific premium rates. Management and underwriters can then use this information to decide on the volume of business to be sold as well as the lines of business to be marketed.

It has been found through the author's own experience that the techniques discussed are more appropriately used to test the profitability of market premiums than they are to set premium rates as profit requirements tend to be set at levels that are often not attainable in practice. The techniques set out can therefore be used for risk arbitrage[105].

[105] This means that the practitioner will use the techniques to identify areas of business which are less risky than other areas for the same level of return. By shifting the company's marketing to the less risky business arbitrage is achieved by obtaining the same level of return for a reduced level of risk.

#### 9.5.6.4 Pricing

A number of authors have made mention of the application of risk adjusted capital in the pricing of insurance contracts. This approach is very much dependent on the market to be considered. The capital charge in pricing is based on the capital allocated to each contract as calculated by methods mentioned above.

When applying these ideas to pricing, the run-off of claims must also be considered to ensure that appropriate credit is given for investment income earned on reserves.

The method used, as described above, will be to determine the ratio of capital allocated to premium volume. The overall charge for capital can then be adjusted according to this ratio and subsequently used in the pricing of the business.

## 9.6 Optimising the investment policy

The previous section considered the optimising of the pricing of insurance products. Prior to that mention was also made of optimising the reserving policy. Apart from pricing and reserving risk, empirical studies cited in chapter 3 i.e. Ryan et.al. (2001) indicated that asset risk is also a great contributor to the risk inherent in the business operation.

In order to evaluate the asset risk, the business operation will be assumed to remain fixed. For the projected business, the distribution of return for a specified asset mix will be considered. Clearly the risk will depend on the extent to which assets and liabilities are matched. For short-term policies, studies may show that the investment policy has to be limited to cash to limit the variability, for example.

The approach will be to consider the different types of investment policies that are possible. For each policy a split of assets will be considered by type, term and currency. For example 20% cash, 50% fixed interest securities, 30% equities. Each of these might be split further for example the equities may be limited to certain industries. For this asset split, the expected experience will then be generated as explained in the framework. The income statements will be consolidated and the variability of return will be modelled for a certain period. A year of projection is normally appropriate as this coincides with the financial period of the company.

For different investment policies the results will be evaluated and based on this analysis an optimal investment structure can be determined based on the shareholders requirements and possible constraints that have been set initially.

An example of this approach will be considered in the appendix.

## 9.7 Optimising the operation risk of the company

Operational risk is difficult to quantify. The reason is that empirical studies cannot always draw a clear cause and effect line. The operational risk that gave rise to the problem is often overlooked. Instead of attempting to model this directly, a better approach would be to have a scoring technique. This technique would work as follows:

Approach management and senior management and list all the requirements of proper corporate governance as outlined in chapter 8. Ask management to indicate how important they consider the different components to be for the business operation. Rank the requirements based on the opinions of management after careful scrutiny of the results presented.

Subsequently ask management how they perceive the company actually ranks in each of the components introduced in the previous exercise. These outcomes can now be weighted according to the rankings obtained from the previous outcomes. Consideration should then be had as to what level of compliance indicates towards what level of operational risk. Some margin will be required to increase the variability of results as a result of non-compliance to these requirements. The extent of this margin can be established through similar studies across the industry and a subsequent evaluation of the credit ratings of these companies. Similarly cases of companies that went into liquidation can be considered and the rating they would have received in the year prior to going into liquidation.

It is not the purpose of this study to set this framework. The author proposes that management ensures that all the principles cited are adhered to rather than trying to assess the level of compliance. In other words comply fully which will mean that this issue is not a risk anymore.

## 9.8 The optimisation process

Note that the processes discussed above are iterative in nature. That is to say the optimal investment policy is only optimal while all other factors remain the same. Therefore if the business mix were to change then the investment structure might need to change as well.

In this regard it is important to note the importance of the different areas of risk. These were mentioned earlier. Based on the level of importance, i.e. those risks that have been the highest contributors to company insolvencies, the optimisation should be considered.

Therefore the pricing of business should be considered firstly. This includes the evaluation of the reserving policy. Once the reserving policy and the pricing structures have been set, the business mix can be evaluated. This will be by type of business as well as source of business.

Once the insurance operation has been evaluated, the impact of reinsurance can be considered. This will have a direct impact on the variability of expected returns and is often also viewed as a substitute for capital.

Subsequently the investment policy can be considered. In this instance the different investment policies can be considered.

Allowance will need to be made for credit risk, but this can be done on a deterministic basis. As mentioned previously, not all risks can be properly quantified and where this is the case proper management of the risks provides a suitable solution.

The results can be evaluated for every set of processes and the optimal processes identified. These processes can then be communicated to management and implemented.

## 9.9 Conclusion

The framework discussed in this dissertation aims to quantify all risks to which the company is exposed. Z. Khorasanee, in one of his editorials in 'The Actuary' magazine, stated that actuaries should translate their knowledge into understandable ratios for their clients. The aim of this study is to further this process by moving away from the "prudent" assumption base to the "99 in a 100" assumption base. The key to this lies in assessing variability based on the framework as discussed.

The current developments on the regulatory side in banking as well as other financial services companies provide this author with the confidence that this framework will eventually be standard practice for all financial institutions.

## 9.10 Summary

This chapter consolidates the ideas set out in this study. The framework discussed initially is considered again. The methodology used to set up the framework and quantify the different areas of risk and uncertainty is consolidated in the evaluation of the distribution of the return of the business operation.

This return can be expressed as a return on capital.

This return can be used to determine the probability of ruin. I.e. the return required to put the business in an insolvent position can be established as well as the likelihood as well as the distribution of all returns exceeding this return. This links to the value at risk and shareholder shortfall concepts found in the banking arena.

The distribution can then be compared to a benchmark. The efficient frontier is discussed as a benchmark and problem areas current in the market are mentioned.

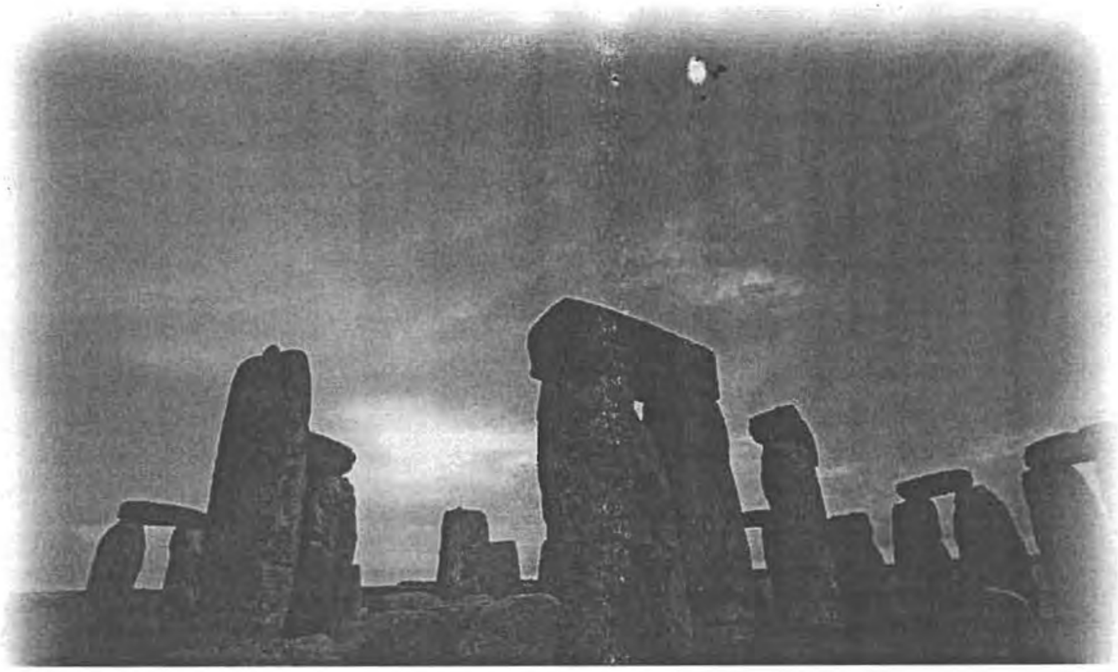
The idea of applying different business strategies to optimise returns is then considered. A number of different business strategies are mentioned.

The pricing strategy of the company and the proper allocation of capital are considered in detail due to the importance of this risk.

The interlinking between these ideas for internal control and regulatory control is mentioned.

The actuarial profession is in the fortunate position that it has the necessary skills to implement these ideas. The area of application is not limited to general insurance and can be applied to other fields as well.

The opportunity exists to translate complex business structures and valuation bases to a simple probability of ruin. As realistic a basis as possible will allow the practitioner to move away from being simply "prudent" to being "99% confident" given the valuation basis used.



# 10. Glossary of terms

## 10.1 Introduction

This glossary contains terms that may have different interpretations. The author included only those terms which were queried during proof reading of this dissertation.

### *10.1.1 Autoregressive*

This refers to the process of regressing a variable onto itself, that is to model a certain variable based on the values of that variable at previous durations. This is appropriate in the case of time series modelling.

### *10.1.2 Bottom up*

This technique refers to the most detailed form of modelling. Each element affecting the end result is modelled individually together with the interaction, if any, between the different elements. This method makes use of the most information and aims to model as accurately as possible. The correlation between different elements such as different types of claims are, however, often difficult to quantify leading to certain approximations of a bottom up model. Compare to a top down model.

### *10.1.3 Business strategies*

Business strategies refer to strategies that can be implemented by management in order to change the existing business structure. Such strategies include:

1. Business mix strategies. Deciding on the type of business.
2. Reinsurance strategies. Deciding on the appropriate reinsurance to mitigate risk.
3. Pricing strategies. Deciding on the pricing structure having regard to the business cycle as well as the expected retention of policyholders.
4. Investment strategies. Deciding what the investment policy will be.
5. Management strategies. Implementing structures and procedures that will improve efficiency in the business.

### *10.1.4 Cost of risk*

This may have different interpretations depending on the context in which it is used. It can mean:

1. The funds required to meet the cost of outgo generally referring to claims outgo.
2. The cost of capital as opposed to the cost of reinsurance.

### *10.1.5 Free reserves*

The free reserves are the reserves that will be distributable in the event of wind-up of the insurance company. This is the difference between the present value of all assets and the present value of all liabilities and may include non-distributable reserves if these are distributable in the event of wind-up.

### *10.1.6 Homoscedastic*

This refers to a sample or series with constant variance. When modelling the error structure of such a series or sample the error structure will have constant variance.

### *10.1.7 Heteroscedastic*

This refers to a sample or series where the variance changes over time. This is applicable to time series models and are particularly useful to model the durations of high and low variability that are often seen in economic variables such as exchange rates.

### *10.1.8 Model point*

Suppose the insurer sells only three kinds of policies so that there are

- 10000 identical policies of type A
- 1000 identical policies of type B
- 10 identical policies of type C.

Then it is possible to have three model points to model the entire experience. Each model point must then reflect the expected distribution of the experience of that entire group of policies. This mean that model point A's experience will be simulated 10000 times, model point B's experience will be simulated 1000 times and model point C's experience will be simulated 10 times.

### *10.1.9 Monte Carlo simulation*

This involves obtaining a set of independently simulated results from a specified fitted model under the hypothesis that the fitted model is an accurate representation of the business operation.

### *10.1.10 Run-off*

This refers to the process whereby claims are actually paid after they have been reserved for. At the financial year end a reserve is set up to meet the cost of all outstanding claims for which premium has already been received. These reserves are required in order to ensure that no further strain is placed on capital in future years. That is to say that if the company is solvent today it will be solvent in future given the business model.

Run-off occurs when reserves are released to meet the claims as they fall due. The claims characteristics depend on the manner in which claims are notified, settled and paid and this process can be short or extremely long. Household insurance is said to have an expected run-off shorter than liability business.

The run-off pattern seen in previous years is then also used to decide on the level of reserves required at future durations.

### *10.1.11 Stochastic*

The term stochastic, which is derived from the Greek word "stochos", meaning "guess", is used instead of the term probabilistic.

Stochastic processes refer to a process where the development is based on an error structure that has a probability distribution. For example, the total claims originating from a policy can be seen as a process modelling the time to the next claim for a specified period of time and subsequently considering the "error" in the claim size, that is to say to what extent does the claim size differ from the mean.

### *10.1.12 Top down approach*

This approach uses a model of the entire business operation instead of modelling each component individually. For example, the loss ratio of the entire operation will be modelled using a log-normal distribution. This approach implicitly takes all correlations between different elements into account but does not explicitly model these. As a result a top down approach is a very crude modelling approach. The two approaches are often used together to first recognise the detailed structure of each item of cash flow and secondly to allow for correlations between these items by some approximate means.

### *10.1.13 Thomson model*

This refers to the model developed by R.J THOMSON (1994) for use of modelling returns on a variety of assets in the South African market.

### *10.1.14 White noise*

A term used for a Wiener process to denote normality. Consider a stochastic variable x(t) indexed by continuous time. The family {x(t)} is a Wiener process iif:

- $x(0) = 0$
- $\Delta x = x(t+\Delta t) - x(t)$ are independent variables
- for all $s>=0$, $t>=0$; $x(s+t) - x(s)$ has the normal distribution $N(0,t)$

White noise is also referred to as the Guassian distribution, Brownian motion or normal distribution.

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[106] This paper provides a comprehensive overview of bootstrapping techniques applied to claim reserving methods. All the claim reserving methods are based on triangulation projections.

[107] Brigham provides an overview of financial management. This textbook is often used for MBA study. Due to the wide scope covered in the book the level of detail is limited. The 1999 edition contains the following sections:

**INTRODUCTION TO FINANCIAL MANAGEMENT**
- Chapter 1 An Overview of Financial Management
- Chapter 2 Financial Statements, Cash Flow, and Taxes
- Chapter 3 Analysis of Financial Statements
- Chapter 4 The Financial Environment: Markets, Institutions, and Interest Rates

**FUNDAMENTAL CONCEPTS IN FINANCIAL MANAGEMENT**
- Chapter 5 Risk and Rates of Return Appendix 5A Calculating Beta Coefficients
- Chapter 6 Time Value of Money Appendix 6A Continuous Compounding and Discounting

**FINANCIAL ASSETS**
- Chapter 7 Bonds and Their Valuation Appendix 7A Zero Coupon Bonds Appendix 7B Bankruptcy and Reorganization
- Chapter 8 Stocks and Their Valuation

**INVESTING IN LONG-TERM ASSETS: CAPITAL BUDGETING**
- Chapter 9 The Cost of Capital
- Chapter 10 The Basics of Capital Budgeting
- Chapter 11 Cash Flow Estimation and Other Topics in Capital Budgeting Appendix 11A Depreciation Appendix 11B Refunding Operations
- Chapter 12 Risk Analysis and the Optimal Capital Budgeting

**CAPITAL STRUCTURE AND DIVIDEND POLICY**
- Chapter 13 Capital Structure and Leverage Appendix 13A Degree of Leverage
- Chapter 14 Dividend Policy

**FINANCIAL PLANNING AND WORKING CAPITAL MANAGEMENT**
- Chapter 15 Financial Forecasting
- Chapter 16 Managing Current Assets Appendix 16A The Cash Conversion Cycle
- Chapter 17 Financing Current Assets Appendix 17A Secured Short-Term Financing

**Special Topics in Financial Management**
- Chapter 18 Multinational Financial Management
- Chapter 19 Derivatives and Risk Management
- Chapter 20 Hybrid Financing: Preferred Stock, Leasing, Warrants, and Convertibles
- Chapter 21 Mergers, LBOs, Divestitures, and Holding Companies

Appendix A Mathematical Tables
Appendix B Solutions to Self-Test Problems
Appendix C Answers to End-of-Chapter Problems
Appendix D Selected Equations and Data

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CHANDARIA S. et. al. Reinsurance Pricing[109]. Institute and Faculty of Actuaries GISG and ASTIN Colloquium 1998.

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COUTTS S.M., DEVITT E.R. (not dated) Simulation Models and the Management of a Reinsurance Company[111].

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[108] This paper provided the first introduction pertaining to the field of stochastic modelling to the Actuarial Society of South Africa. It is of historic interest only.

[109] This is a paper based on the work of the GISG working party involved with reinsurance pricing. The paper addresses the practical issues pertaining to the pricing of risk. As such it provides an interesting overview into the practicalities behind the risk quantification methodologies which are also dealt with extensively in this study.

[110] An interesting approach to risk rating which was not explored further in this study. This paper received the Brian Hey Prize at the GISG convention for one of the best three papers. The paper considers certain criteria appropriate for risk rating. Given these criteria the approach of loading an expected value of risk with a standard deviation measure is inappropriate. As an alternative, it is suggested to increase the cumulative distribution function thereby moving the probability distribution function higher as well. This increase is based on the market rates currently applicable in the market.

This approach is useful for risk rating but makes the quantification of variability extremely difficult. This author did not see it necessary to pursue this approach further.

[111] A copy of this paper is available from the author. The source of the paper is unknown. The paper provides a very short overview of a practical algorithm required to implement a simulation model for a reinsurance company though it would be equally possible for a general insurance company. This algorithm is provided in this study as well.

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DANIEL W. Asset Liability modelling for insurance and reinsurance companies. Article submitted to Global Reinsurance Magazine 1997.

[112] An important paper for this study. This paper provides a framework for the investigation of suitable reinsurance arrangements. For the purpose of this study these ideas can be extended for the implementation of other best business practices as well. This is clearly illustrated in the study as well. Refer to the introduction as well as the appendix.

[113] This paper considers the extent to which the efficient frontier can be used as a benchmark in setting appropriate business strategies. They argue that certain strategies boil down to diversification which an investor can commit to in any event and therefore the practitioner needs to be careful to what extent the strategy proposed actually mitigates specific risk. Specific risk refers to the risk inherent in the company that cannot be eliminated through proper diversification exercises.

[114] This paper received the prize for best paper at the GISG convention. The paper shows that the price of insurance depends on the overall riskiness of the insurance company and not on the riskiness of the individual line of business. The paper introduces the idea that the overall riskiness of the company needs to be established.

This idea is partially shared by the author. The first part of the paper focuses on establishing the likelihood of insolvency for the entire enterprise and suggest that insurance prices should conform to the likelihood i.e. riskier operations should demand higher prices. This conforms to general economic theory as explained in this study. The second conclusion is that once the overall position has been determined, no further allowance for riskiness inherent in the business is necessary. This author does not share this view and argues that the very overall riskiness depends on the volume of different lines of business. As a result the entire process is a dynamic one and cannot be approached in the fashion suggested.

[115] This paper also provides feedback from one of the GISG's working parties. The paper shows some of the problems that can arise in the event of a large claim. Due to the rare occurrence of these claims, they are generally treated on a case estimate basis only. From a reserving point of view they are not permissible according to the international accounting standard. The best approach for smaller companies is to reinsure any exposure to large claims. For large risk writers the debate should continue for some years to come. The main problem arises because the Revenue Services view such reserving practices as tax avoidance.

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DAYKIN, C.D., PENTIKAINEN, T. & PESONEN, M. Practical Risk Theory for Actuaries[116] Prentice Hall 1994

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[116] An extremely useful reference. The contents is given below:

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**PART TWO - STOCHASTIC ANALYSIS OF INSURANCE BUSINESS**

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DVORAK B. Risk Models of Loss. Presentation at the Business to Business Workshop on Enterprise Wide Risk Management. 2001

EMBRECHTS P. Integrated Risk Management[117]. One day Workshop presented at WITS University 2000

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11.3 Dividends
**12 The insurance process**
12.1 Basic equation
12.2 Empirical observations
12.3 Business cycles, analysis of causes and mechanisms
12.4 Simulation of the insurance process
**13 Applications to long-term processes**
13.1 General features
13.2 Capital requirements of an insurance company
13.3 Evaluation of an insurer's net retention limits
**14 Managing uncertainty**
14.1 Review of applications
14.2 Basic equations
14.3 The insurer and the market
14.4 Measuring and managing financial strength
14.5 Corporate planning
14.6 Public solvency control
**15 Life insurance**
15.1 Recapitulation of some basic formulae of life insurance mathematics
15.2 Stochastic cohort approach
15.3 Analysis of the total business
**16 Pension schemes**
16.1 Pension structures and definitions
16.2 Pension formulae
16.3 Deterministic methods of pension funding
16.4 Stochastic methods for pensions

**APPENDICES**

**A Derivation of the Poisson formula**
**B Pólya and Gamma distributions**
**C Asymptotic behaviour of the compound mixed Poisson d.f.**
**D Numerical calculation of the normal d.f.**
**E Derivation of the recursion formula for *F***
**F Simulation**
**G Time series**
**H Portfolio selection**
**I Solutions to exercises**

[117] Professor Embrechts first considered the application of extreme value theory as a risk management tool. Subsequently he considered the evaluation of risk using risk measures such as value at risk and shareholder deficit. These refer to the value of the percentile and the expected value of a claim above the percentile for a certain specified distribution. He went on to consider the correlation between different economic entities, share prices for example, he showed empirically that the correlation matrix in the extreme is different to the correlation matrix generally obtained through normal analysis of variance. This leads to the cautioning of all practitioners who are using correlation matrices for risk evaluation purposes.

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HOGG R.V. & KLUGGMAN S.A. Loss Distributions 1976

HOOKER N.D. et al. Risk Based Capital in General Insurance[119]. Presented to the Institute of Actuaries 1995.

HOWIE R.J. Investment Risk and Asset Modelling. Transactions of the Actuarial Society of South Africa 1997.

[118] This paper introduced a financial soundness valuation basis for the Actuarial profession in South Africa. The aim of this basis was to introduce a first tier of excess margins in assumptions which would render a likelihood of insolvency of less than 5% for any company in the Life Insurance Industry. The paper gives some indication of the level of depth to which the ideas described in this dissertation has been implemented in the Life Insurance Industry. Given the peculiarities of that industry the level of investigations are not as in depth mainly because the products have very little or no risk.

[119] A very important paper. This paper was the first attempt to consolidate the ideas of enterprise wide risk management into an actuarial paper. The paper is discussed in this study as well. The reader is referred to chapter 3.

Abstract: This paper looks at the problems of assessing, for solvency purposes, the capital requirements of a non-life insurer in the context of the United Kingdom. It considers how these capital requirements might vary according to the different risks to which an insurer is subject and how this Risk-Based Capital (RBC) might be measured in practice, using as a case study the RBC formula recently introduced in the United States of America. The paper also discusses the application of RBC concepts to the problem of internal capital allocation, to assist in measuring an insurer's rate of return to shareholders by business unit, as well as the more obvious regulatory application. The advantages and disadvantages of a formula-based approach to capital requirements for solvency purposes are discussed in comparison with possible alternative approaches to insurance supervision.

Keywords: general insurance; insurance supervision; risk; risk-based capital; solvency

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[120] This presentation shows how the risk based capital requirements introduced by the National Association of Insurance Commissioners have been implemented by a London Market company. The approach is lacking to the extent that the company may not be exposed to the same risks as all companies combined in the United States.

[121] Maitland's paper provides a detailed overview of the appropriateness of the Thomson model and provides additional insight into the use of ARIMA models.

[122] This paper provides an overview on the integration of portfolio theory and capital allocation with premium rating.

[123] The dissertation considers the process of enterprise risk management. The submissions made by MICCOLIS provide the reader with a very good introduction to the fundamentals pertaining to Enterprise risk management as discussed in this study.

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[124] A very useful guide on probability distributions as well as collective risk models.
[125] This paper provides further follow up following the paper by HOOKER 1998. The aim of the paper is to introduce a framework to the FSA whereby the solvency of general insurance companies can be monitored. The content of the paper is encouraging as it provides a confirmation of the the approach set out in this study as a suitable one to address the issue of solvency. As mentioned, in this study, the scope of this framework goes beyond any one business strategy.

[126] This paper is extremely useful as a reference for asset models. Smith covers a variety of asset models in great detail and also provides the visual basic code for these models. The content is considered further in this study in the assessment of asset risk.

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YAKOUBOV Y.H., TEEGER M.H. and DUVAL D.B[127]. The TY Model - A stochastic Investment Model for Asset and Liability Management. Presented to AFIR and ASTIN colloqia in Tokyo 1999. www.sias.org.uk/papers/model.pdf

[127] This paper provides a very thorough overview of investment models and is definitely worth reading. The authors propose (and this is a biased opinion) the following qualities:

1. Clarity of Design
2. Synthesis of the Economic Framework, Investment Practice and Historic data.
3. Unique approach to equity risk – an explicit market sentiment component is introduced.
4. Grounded in reality.

The paper considers a cascaded asset model similar to the Thomson model and provides the results of the investigation for

1. Price inflation (which is the base)
2. Salary inflation
3. Fixed interest government bonds
4. Cash
5. Index linked government bonds
6. UK equities (evaluated by income, growth and change in rating) this is a wonderful development because this approach allows for the integration of market and credit risk.
7. Overseas equities

# 12. Annexure

## 12.1 Introduction

The aim of this annexure is to provide a practical example of the ideas discussed in this dissertation. This example is not based on empirical evidence and serves merely as an illustration of the ideas put forward. Furthermore, this model assumes a predetermined model for liabilities and assets as explained below.

Before progressing to the actual model the key steps in the modelling process need to be considered[128]:

1. Develop a well defined set of objectives. This includes a reference to the duration over which the model will be considered, the definition and the probability limits of ruin as well as the definition of the objectives.

2. Plan the model process and how the model will be validated. The data requirements will be considered as well as the other information requirements necessary to set a model appropriate to simulate future expected experience. Testing the model to past data and expert opinion also needs to be considered.

3. Collect the data. Be wary of possible data errors and rectify these. Also recognise that past experience is not necessarily a good guide of the future. Consider the general economic and commercial environment as discussed in the introduction.

4. Define the model through setting the requirements of the real world that it is required to simulate. Refinement of the model can come through the monitoring process.

5. Consult experts in the different areas. It is not possible to have full knowledge on all the areas and considerable interaction will be required with economists, underwriters, industry leaders etc.

6. Consider the platform on which the model will be built. A statistically reliable random generator is required that will be suitable to the objectives of the models. Remember that the model is better used to investigate deviations than extremes merely because it is based on numerical techniques. For the extremes analytical tools based on extreme value theory will be more appropriate[129].

7. Write the computer program for the model.

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[128] These are the steps included in the INSTITUTE AND FACULTY OF ACTUARIES Core Reading on subject 103.

[129] EMBRECHTS, KLUPPELBERG, MIKOSCH Modelling of Extremal Events for Insurance and Finance (1997) In which the distribution of maxima of samples are considered. This is a limiting behaviour similar to the central limit theorem.

8. Debug the program to make sure it performs the intended operation in the model definition.
9. Test the reasonableness of the output from the model.
10. Review and carefully consider the appropriateness of the model in the light of small changes in input parameters.
11. Analyse the output from the model.
12. Communicate and document the results and the model.

This process is then integrated into the control cycle which was discussed in the introduction.

This chapter will cover the details of the model used, the VISUAL BASIC FOR APPLICATIONS code used to model the experience and the results of the model will be presented briefly.

The main objective will be to apply the principles of optimisation to the simple example presented here. In order to do this a suitable benchmark for an efficient frontier is required. As the purpose of this appendix is a presentation of the working of the ideas set out thus far, the efficient frontier set out in chapter 9 of this dissertation shall be applied.

## 12.2 Details of the model

The ideas are applied to a fictitious insurance company. This company writes only motor insurance and only invests its funds in either cash or alternatively in equities. Furthermore it is assumed that the company has just started writing business (or alternatively a new class of business is investigated). In order to properly present the cash flows turn back to chapter 2. The first step is to evaluate the income statement and generate a model for the income as it is expected to incur in practice in the year to come. Allowance is therefore required for the income in premium and investment returns and the outgo in the form of claims. This model can be extended to include all the transaction types illustrated in the income statement.

Assume that the company had access to data and expertise which enabled it to set up models which can accurately reflect the expected future experience.

### *12.2.1 The liability model*

Firstly consider the variability in the claims outgo. Details of the liability model are given below. The model considers only motor business. Two perils are considered namely the loss of a vehicle through theft and accidental damage to the vehicle.

For both types of peril a frequency and severity distribution has been obtained. These distributions will be obtained through the techniques explained in chapter

5. That is to say past exposure grouped by risk factor will be investigated, the significant risk factors identified and regression techniques applied to the data broken down by risk factor for each peril.

In addition the parameters of these distributions are assumed to be affected by risk factors. In this example only two risk factors have been used; namely age and area. The table below shows the impact these risk factors are expected to have on the frequency and severity distributions of thefts and accidents respectively:

**Theft frequency** **Poisson** **1%**

| **Class** | **Risk Factor 1** | **Value** | **Risk Factor 2** | **Value** |
|---|---|---|---|---|
| *Motor* | Age1 | 1 | Area1 | 1.5 |
| | Age2 | 1 | Area2 | 1 |
| | | | Area3 | 0.8 |

**Theft severity is the Sum Insured**

It can be shown that the frequency of independent occurrences will follow a Poisson process hence both the theft and the accident frequencies are assumed to have Poisson distributions.

**Accident frequency** **Poisson** **15%**

| **Class** | **Risk Factor 1** | **Value** | **Risk Factor 2** | **Value** |
|---|---|---|---|---|
| *Motor* | Age1 | 1.3 | Area1 | 0.9 |
| | Age2 | 0.8 | Area2 | 1.1 |
| | | | Area3 | 1.2 |

**Accident severity** **Pareto** **20%**

| **Class** | **Risk Factor 1** | **Value** | **Risk Factor 2** | **Value** |
|---|---|---|---|---|
| *Motor* | Age1 | 1.1 | Area1 | 1 |
| | Age2 | 1 | Area2 | 1 |
| | | | Area3 | 1 |

It can be seen from the figures that the area risk factor is not expected to affect the severity of accidents and similarly the age risk factor is not expected to affect the frequency of thefts.

The severity of accidents is assumed to represent the severity in the mid term of the policy term and no allowance is made for inflationary movements on claims.

The frequency distributions of both perils are assumed to be Poisson and the Pareto distribution is assumed to have an alfa parameter of 2. With proper empirical testing these assumptions will be based on the estimates described in chapter 5.

### *12.2.2 The asset model*

The asset model has been split into two sections. One section has been compiled for cash and the other for equity investments. The cash model is a simple discrete model whereby the monthly cash return on any funds is assumed to have a starting value, a minimum and a maximum with a distribution of movement from one month to the next. This is shown below.

**Cash**

| | | |
|---|---|---|
| Mean return per month | | 0.50% |
| Probability | Cumulative | Monthly Movement |
| 0.1 | 0.1 | 0.50% |
| 0.15 | 0.25 | 0.20% |
| 0.15 | 0.4 | 0.10% |
| 0.3 | 0.7 | 0% |
| 0.2 | 0.9 | -0.10% |
| 0.05 | 0.95 | -0.20% |
| 0.05 | 1 | -0.50% |
| Min | | 0.20% |
| Max | | 1.10% |

Note that this model is very simplistic. The model builder may decide to investigate the past movements of money market rates and subsequently set a similar model or alternatively apply an auto-regressive model as set out in chapter 6.

The asset model for shares is shown below. This has the working of a simple auto-regressive model with a white noise element as the error term.

**Shares**
Value of index for the last 6 months

555
547
560
578
599
596

| | | |
|---|---|---|
| Error structure: | Mean error | 0.5% of index at time t |
| | Standard Deviation | 10% of index at time t |

Model

Index(t) = 0.6*Index(t-1) + 0.2*Index(t-2) - 0.3Index(t-3) +0.4Index(t-4) + 0.2Index(t-6) + Error term

The model set out above might be over parameterised. This model may be based on another model or on forward market rates or some other sources of information. The simplistic nature of this model does, however, illustrate the working of such a model.

It is therefore possible to project the growth of funds invested in either cash or equities. These models have not been based on empirical evidence and are for the purpose of illustration only.

### *12.2.3 Exposure details*

Once the area of liability and asset uncertainty has been addressed, the model then needs to consider other areas of uncertainty that may apply in the company. Clearly many of these will be very difficult to quantify. It will be possible to check whether or not the company maintains an appropriate level of solvency given growth projections, for example.

For the purpose of this model I do not allow for any further growth but consider the experience on one book of business written at the beginning of the period. The model is applied to a book of motor business. All premiums are expected annually in advance. Details of the exposure are shown below.

**Liability model points**

| Class | Sum Insured | Risk Factor 1 | Risk Factor 2 | Num | Theft Freq | Acc Freq | Acc Sev | Risk Premium | Office Premium | Total Office |
|---|---|---|---|---|---|---|---|---|---|---|
| *Motor* | 40000 | Age1 | Area1 | 50 | 0.015 | 0.1755 | 0.22 | 107220 | 3000 | 150000 |
| *Motor* | 30000 | Age1 | Area2 | 100 | 0.01 | 0.2145 | 0.22 | 171570 | 2500 | 250000 |
| *Motor* | 30000 | Age1 | Area3 | 100 | 0.008 | 0.234 | 0.22 | 178440 | 2500 | 250000 |
| *Motor* | 80000 | Age2 | Area1 | 500 | 0.015 | 0.108 | 0.2 | 1464000 | 400 | 2000000 |
| *Motor* | 70000 | Age2 | Area2 | 400 | 0.01 | 0.132 | 0.2 | 1019200 | 3500 | 1400000 |
| *Motor* | 70000 | Age2 | Area3 | 500 | 0.008 | 0.144 | 0.2 | 1288000 | 3500 | 1750000 |
| | | | | | | | | 4228430 | | 5800000 |

**Asset model points**

| Class | Value |
|---|---|
| Cash | 50% |
| Shares | 50% |

### *12.2.4 Capital requirements*

The business will be supported by capital to ensure that all claims will be met within certain bounds of certainty or alternatively the capital requirement can be set based on statutory requirements. The Risk Based Capital methodology applied by the National Association of Insurance Commissioners is an example of such an approach. Alternatively, a percentage of premium or claims can be used by the regulator to set a capital requirement.

The premiums include the loading for the cost of capital.

### *12.2.5 The collective risk model*

An evaluation of the expected value of claim payments for this group of business reveals the following distribution for the collective claim outgo. Note that this distribution assumes no inflationary influences on claim amount during the year.

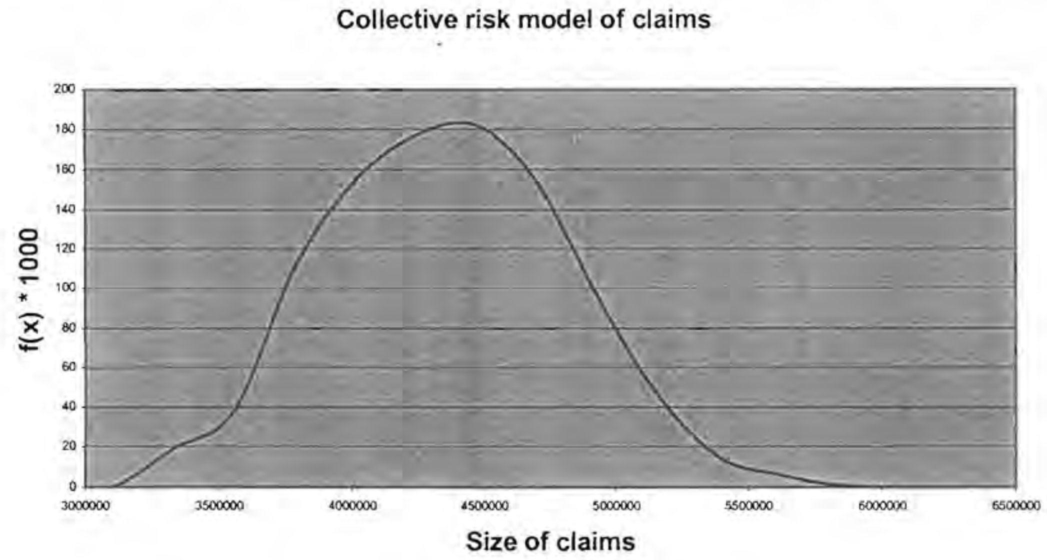


Figure 15: Expected outgo in the year for the collective motor model

Assuming the owners of the company require a 95% certainty of not needing to put up any additional capital during the year to meet claim payments, inspecting the curve above this means that funds of approximately R5.5million will be required. As the expected office premium is R5.8 million, no additional capital will be required to write the business.

If, however, the majority of the office premium is intended to meet other items of expenditure, then some additional capital may be required. In any event, the regulatory authorities in a country may set a minimum capital requirement. For sake of argument I shall assume a capital requirement of 10% of premium even though statistical evidence may indicate that this is not required. Hence capital of R0.58 is required to support the business.

From the discussion it should already be clear that the country of domicile of an insurance company has a significant effect on the return on capital of that insurance company.

## 12.3 Identifying the problem

Once all the elements required to model future outgo have been determined, a clear objective for the simulation exercise must be set. For the sake of example, assume that the objective is to optimise the return on capital given the model details as set out above. This optimisation will be subject to the probability of the return being less than 5% per annum to be less than 1%.

Clearly a variety of changes in the business procedure is possible in order to achieve this objective. The company may decide to market only to certain types of policyholders, or alternatively address the business procedure in order to change the risk premiums or change the investment structure. Different business strategies can be tested here to find the optimal fit.

In order to illustrate the working of the model consider a change in the investment structure. Arguably other adjustments can also be made.

It will be important to set a timeframe for the experience to be investigated. The longer the timeframe the more uncertain the projection will become.

It may also be necessary to generate further outcomes if the curve does not appear to be smooth. The number of simulations will depend on the purpose of the model. In general the techniques will have more merit in investigating the variability of experience than extreme outcomes of experience due to the variability in simulations in the extreme.

## 12.4 Developing the solution

Detailed Visual Basic Code is included to facilitate the understanding of the techniques described thus far. This is included in section 12.6 at the end of this appendix.

The approach applied is to assume that all premium is received and invested at the start of the year. Claims are then incurred during the year and paid from the available funds. The approach of disinvestment is to assume that there are no transaction costs as a result of disinvestment and that cash investment is first utilised before equity investment to pay claims.

The level of funds available at the end of the year is then compared to the capital which was made available to support the business in order to evaluate the objectives and in particular the return on capital.

## 12.5 Example of results

The model explained above is best viewed by means of a series of graphs. Each graph will depict the development of a specific entity throughout the year.

The funds received are initially invested in equal proportions in cash and equities as per the model explained above.

A random sample of six simulations has been drawn and the results shown below.

### 12.5.1 Development of cash position

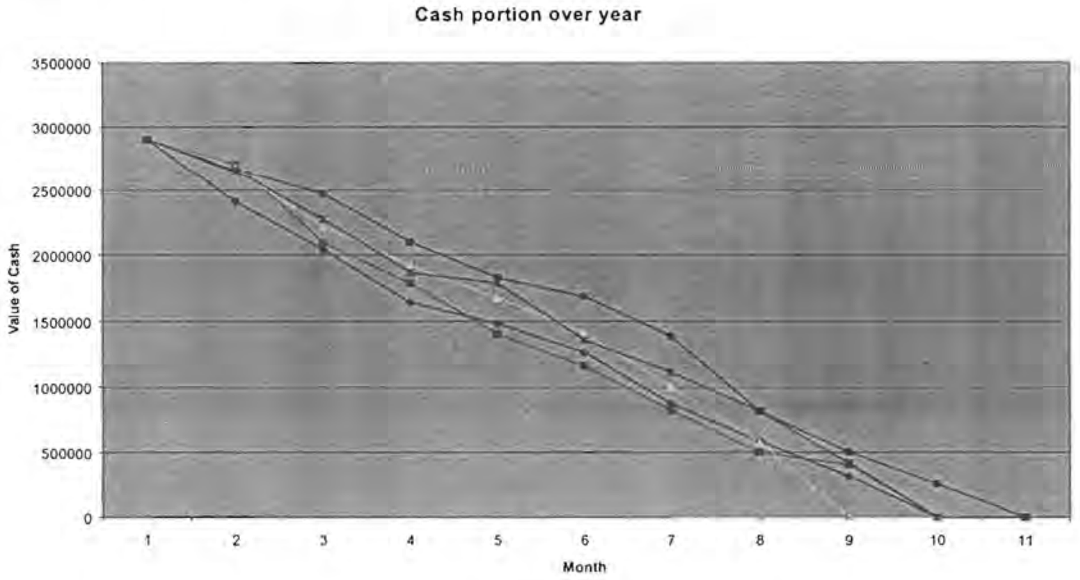


Figure 16: Development of cash held – reducing as claims are paid

The graph above shows the run-off of the cash position over time. Bearing in mind that cash is first used before equities are realised and that the cash position is not changed during the year, it is expected that the cash position will gradually worsen as shown. The cash position is, however, affected by the incidence of claims and not only by the return secured on cash investment.

It can be seen that the cash is exhausted for all six random samples that were drawn from the simulation results. This is because only half of the premium was invested in cash at the start of the year and the expected risk premium was substantially higher than half of the office premium. The risk premium refers to the amount that is expected to be paid out in respect of the risks covered. The office premium is the amount that is received in respect of each policy by the office.

### 12.5.2 Development in the claims experience

The variability of claims experience is apparent from Figure 15. This graph shows the random nature of the claim structure of a general insurer for the class of business described in the liability model above.

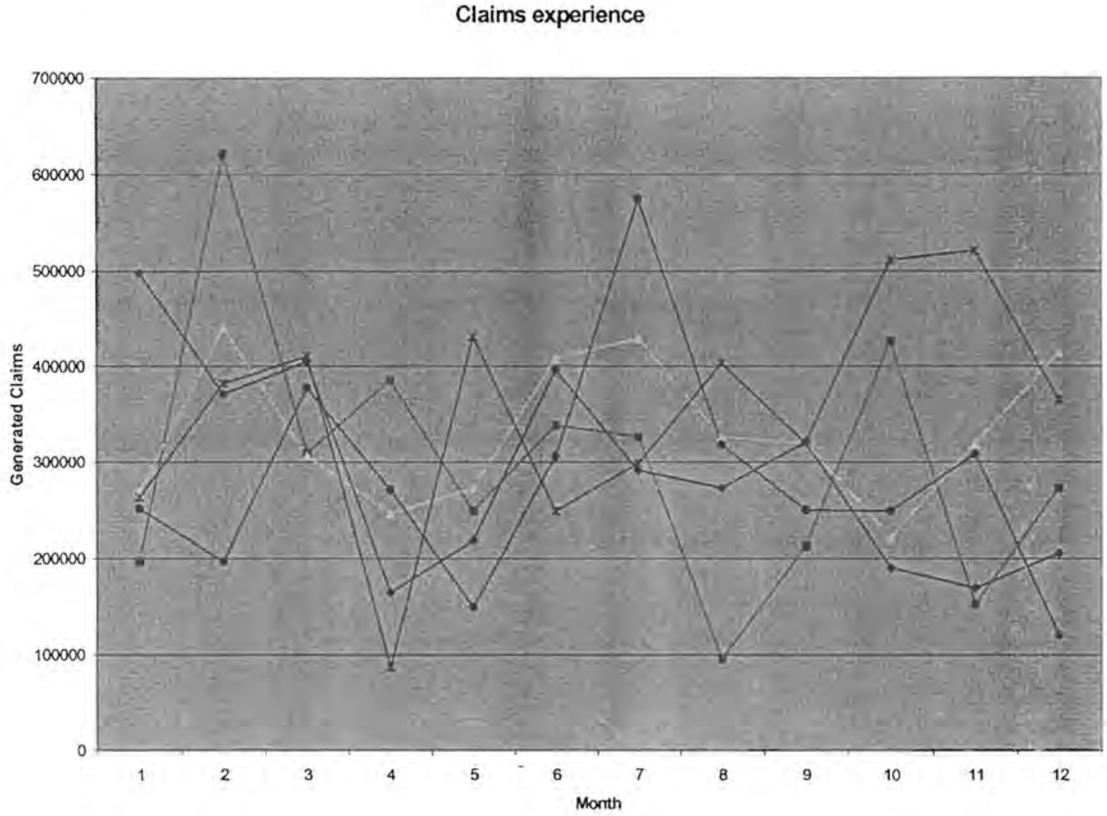


Figure 17: Projected claims experience for the year

It can be clearly seen that the monthly experience is extremely volatile. This is a feature of the assumptions chosen. Note that the model did not allow for any cyclical movements in experience though this would be possible.

Furthermore, delay patterns in claims experience has been ignored. This results in the projected return being underestimated. For short tail classes of business where investment returns are lower due to the quick turnabout time between premiums received and claims paid this does not have as a significant impact as long tail classes of business. The development of delay patterns is discussed in chapter 5.

### 12.5.3 Development in equity experience

Once the cash has been exhausted the equity funds are realised to meet claims outgo. Note that due to the increased level of variability in equity investment this may mean that the insurer needs to realise equity investment at an inappropriate time.

Details of the growth in the fund are shown below.

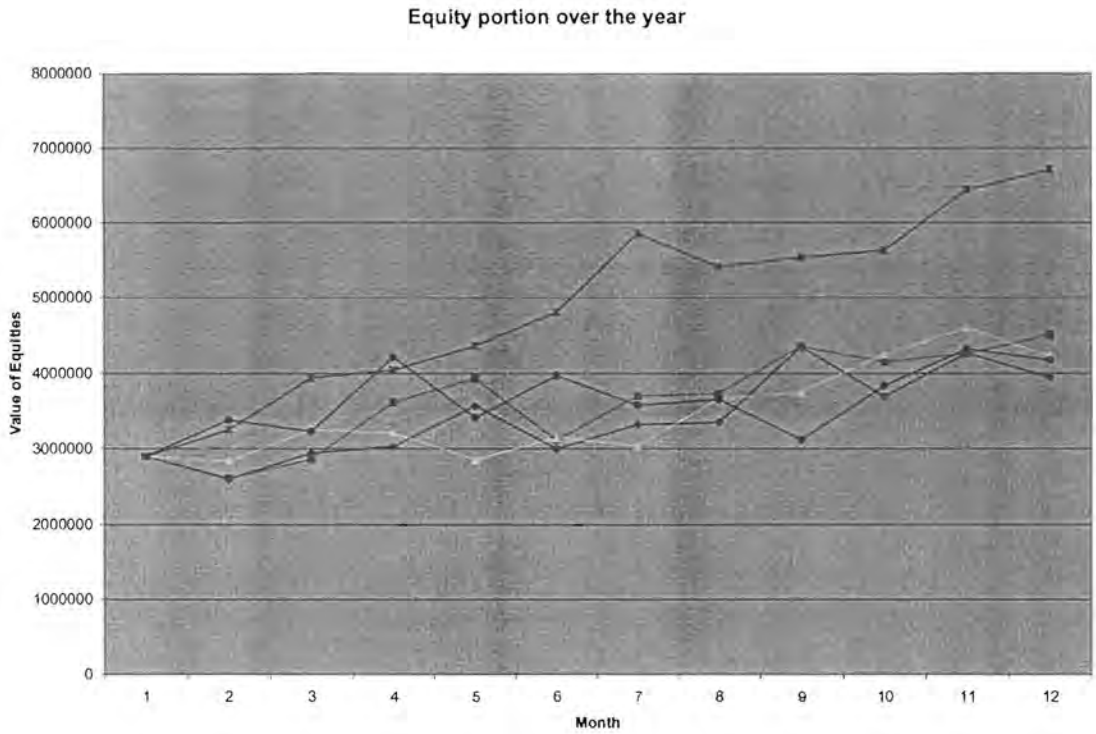


Figure 18: Development of equity position

Note the higher level of variability in the growth of equity investment as opposed to cash investment. This is also intuitive from the assumptions set out above.

### 12.5.4 Development of the fund

These three elements are then taken together to yield the value of the book of business at the end of the year.

The year-end value of the book of business also has a density function.

The progress of the value of the fund supporting the business is shown below.

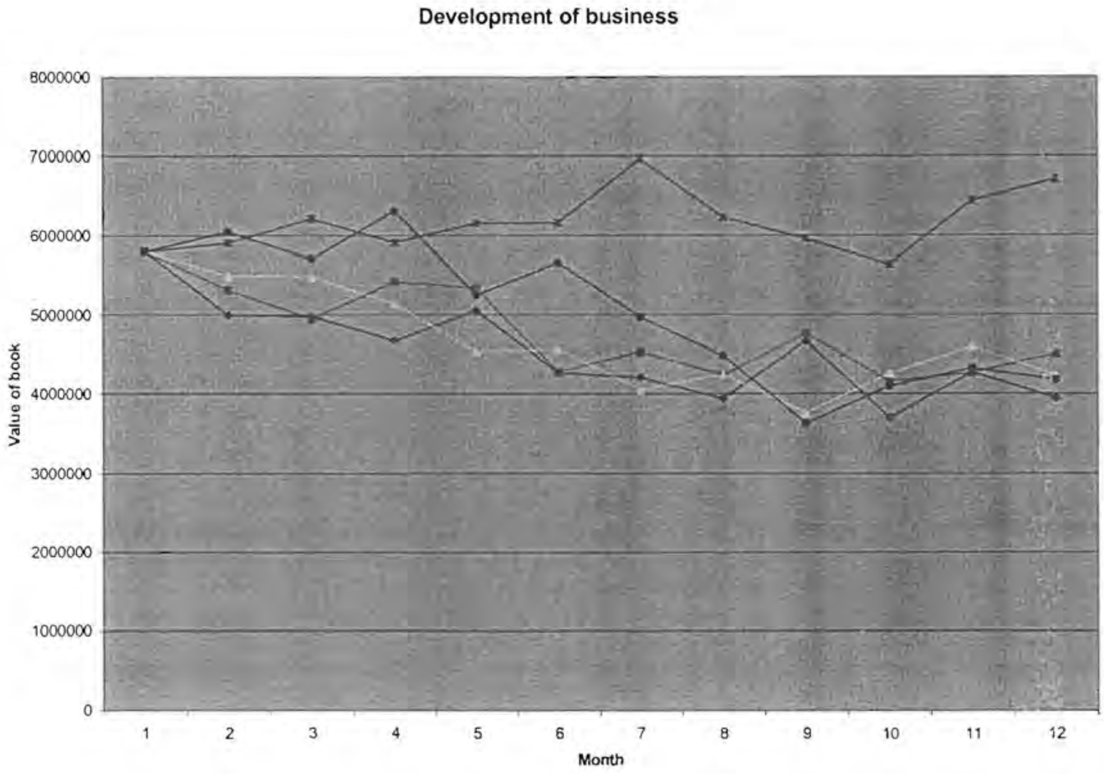


Figure 20: Projection of business develoment

It can be seen from the projection of year-end values that the influence of investment return as assumed in the valuation model is substantial. As can be seen from the graph above, the influence of equity investment is such that in one instance the value of the fund remained almost stable. This may be indicative that the asset model is too optimistic or that sufficient allowance has not been made for possible claim payment increases during the period.

### *12.5.5 Investigating the objective*

The value of the fund at the end of the year is now investigated. This value can be expressed as a percentage of capital in order to determine whether or not the returns will meet shareholders expectation.

The value as at duration twelve has the following probability density function.

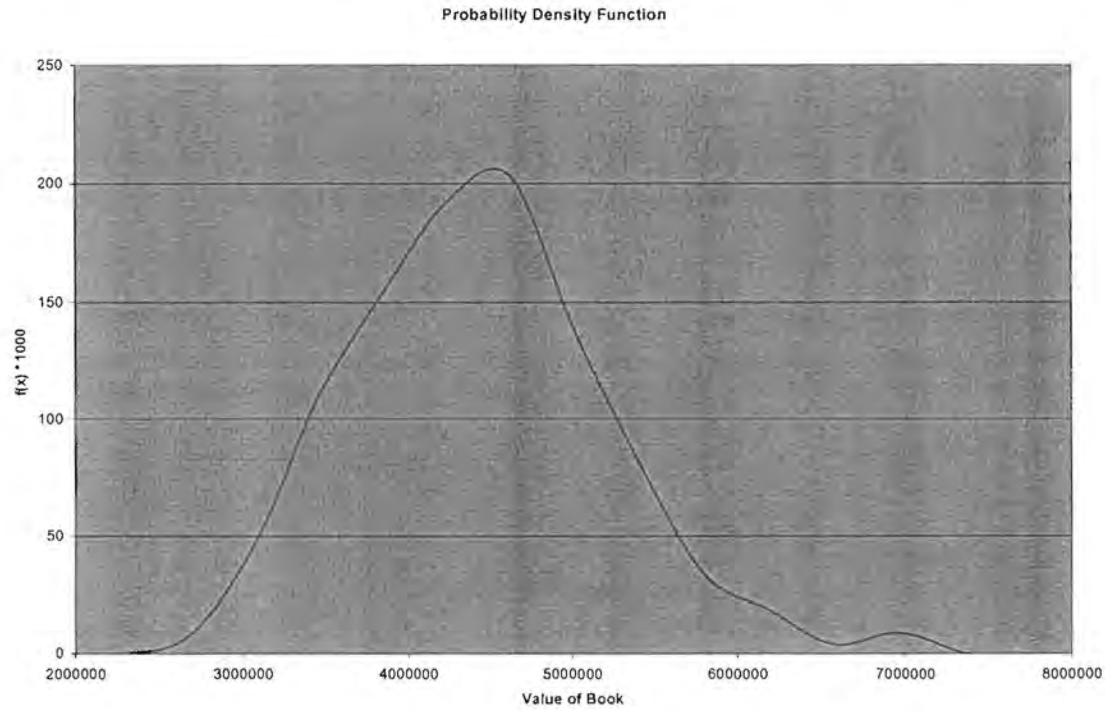


Figure 21: PDF of the value of the fund based on the model

This distributions has an average of R4.2million and a standard deviation of R700,000. It is therefore clear that the investment returns have been substantial. This would yield a phenomenal rate of return.

When comparing the return to the efficient frontier, it is clear that this investment will be a substantially better investment than any currently available in the market.

It is, however, possible to optimise the return even further by investigating different investment strategies or other risk management alternatives.

For example, assume a investment change from an equal split between cash and equities to a 80% allocation to cash and a 20% allocation to equities. The resulting year-end value of the book of business would change. This change is shown below.

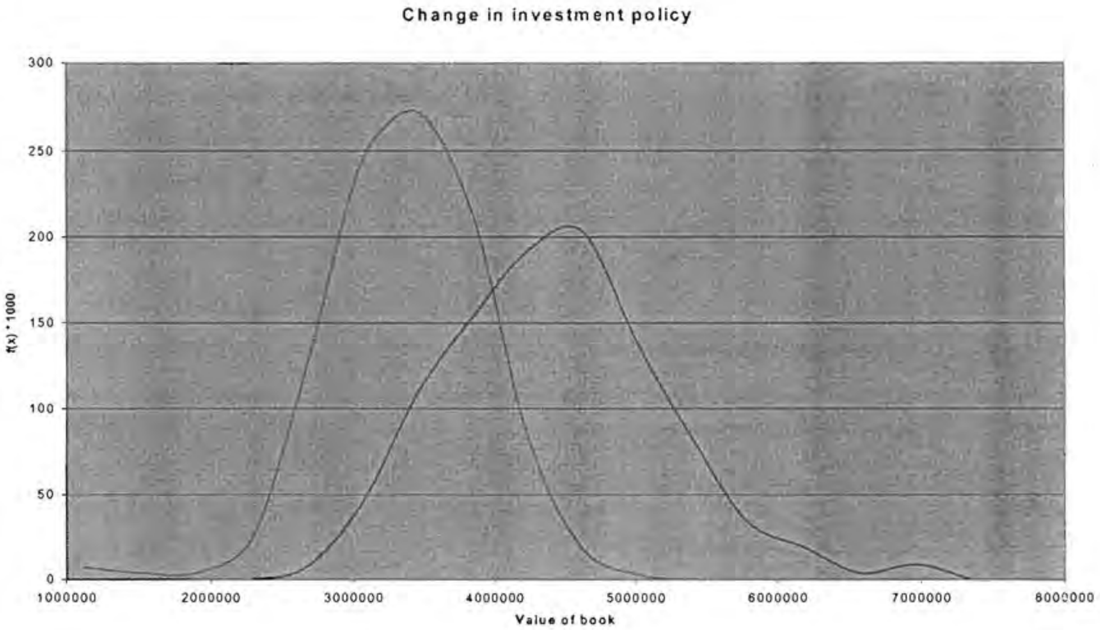


Figure 22: Result of a change in investment policy on the fund value after 12 months

The effect of changing the investment policy is a reduction in variability, as would be expected but also a reduction in the expected value of the book of business at the end of the year. The deduction in variability then needs to be compared to the cost of the variability. The model used does not illustrate the cost of variability accurately as the down side of equity investment is not severe enough. In this example an entire investment in equities would be the best approach.

### 12.6 Visual Basic Code

The following Visual Basic for Applications Code is meant to provide some additional explanation to the methodology discussed in "Model Office techniques in a General Insurance Environment".

```
'The following code specifies the public variables used in the code.

Option Explicit
Option Base 1


Dim got_one As Boolean
Dim z, x1, x2, r, stored As Variant
```

**Function norm_gen()**

```
'This function creates a value from a standard normal distribution.
'The code has been taken from the GISG 1995 Asset Models working group.

If got_one Then
   z = stored
   got_one = False
Else
   Do
      x1 = 2 * Rnd - 1
      x2 = 2 * Rnd - 1
      r = x1 ^ 2 + x2 ^ 2
   Loop While r >= 1 Or r = 0
   r = Sqr(-2 * Log(r) / r)
   stored = x1 * r
   z = x2 * r
   got_one = True
End If
norm_gen = z
End Function
```

**Sub main()**

```
'This procedure generates the experience for the model points as specified under the exposure details.

'The aim of this procedure is to model the expected worth of the model book of business after one year.
```

```
'Note that the objective of this dissertation is to consider model office techniques
and not to provide empirically justified asset and liability models.

'A very simplistic motor portfolio is used.
'There are six model points - 2 age categories and 3 area categories
'The model points are only exposed to the perils of theft and accident
'Details of the frequency and severity of these perils are included in the liability
model description.

'There are 6 model points and for each the following must be read:
'1. The average sum insured
'2. The number of policies for the model point
'3. The theft frequency
'4. The accident frequency
'5. The accident severity

Dim exp_details(6, 5) As Variant
Dim loop1, loop2 As Integer

For loop1 = 1 To 6
   'The average sum insured is obtained
   exp_details(loop1, 1) = Worksheets("Exposure details").Cells(loop1 + 5, 2)
Next

For loop1 = 1 To 6
   For loop2 = 2 To 5
      'The other model detail is obtained
      exp_details(loop1, loop2) = Worksheets("Exposure details"). _
      Cells(loop1 + 5, loop2 + 3)
   Next
Next

'The model assumes only one theft and/or accident claim is possible in
'a month. Furthermore the frequencies obtained are assumed to be annual
'frequencies.


'A thousand Monte Carlo simulations of a year's experience will be run

Dim sim, model_points As Integer

'For the purpose of this example the premium is assumed to be received up front
and invested as specified in the asset model points. It should be clear that it is
possible to extent this approach to allow for a different flow of premium income.
```

```
Dim funds, cash, equities As Variant

'Details of the asset model is required to project the cash flow over time.

Dim share_index(18) As Variant
'The past months' six values of the share index are obtained

For loop1 = 1 To 6
   share_index(loop1) = Worksheets("Asset model").Cells(loop1 + 21, 1)
Next

Dim cash_distribution(7, 2) As Variant
'The movement in cash returns with their associated probabilities are obtained.
The first column is the cumulative probability and the second is the size of
change from one month to the next.

For loop1 = 1 To 7
   cash_distribution(loop1, 1) = Worksheets("Asset model").Cells(loop1 + 6, 2)
   cash_distribution(loop1, 2) = Worksheets("Asset model").Cells(loop1 + 7, 3)
Next

Dim mean_cash, min_cash, max_cash As Variant

min_cash = Worksheets("Asset model").Cells(16, 3)
max_cash = Worksheets("Asset model").Cells(17, 3)

Dim months

Dim claim, total_claims
'These are used to generate a claim and to calculate the total balance of claims
in a month

Dim prob
'This is used to generate a random variable

Dim error_term

got_one = False

For sim = 1 To 1000
'A thousand years' experience will be generated

'First the projected share index is calculated

funds = Worksheets("Exposure details").Range("Toffice")
cash = funds * Worksheets("Exposure details").Range("cash_per")
```

```
equities = funds * Worksheets("Exposure details").Range("shares_per")
mean_cash = Worksheets("Asset model").Cells(4, 4)

For loop1 = 7 To 18
   share_index(loop1) = 0.6 * share_index(loop1 - 1) + _
   0.2 * share_index(loop1 - 2) - 0.3 * share_index(loop1 - 3) + _
   0.4 * share_index(loop1 - 4) + 0.2 * share_index(loop1 - 6)

   error_term = (norm_gen() * 10 + 0.5) / 100 * share_index(loop1)

   share_index(loop1) = share_index(loop1) + error_term
Next


For months = 1 To 12
   'The mean cash return for the month must be generated
   total_claims = 0
   Randomize

   Worksheets("Workings").Cells(sim, months) = cash
   Worksheets("Workings").Cells(sim, months + 13) = equities
   Worksheets("Workings").Cells(sim, months + 26) = funds

   prob = Rnd

   loop1 = 1

   If prob < cash_distribution(7, 1) Then
   While prob > cash_distribution(loop1, 1)
      loop1 = loop1 + 1
   Wend
   Else
      loop1 = 7
   End If



   mean_cash = mean_cash + cash_distribution(loop1, 2)
   If mean_cash > max_cash Then mean_cash = max_cash
   If mean_cash < min_cash Then mean_cash = min_cash

   cash = cash * (1 + mean_cash)
   equities = equities * share_index(months + 6) / share_index(months + 5)
   funds = cash + equities
```

```
For model_points = 1 To 6
   For loop1 = 1 To Int(exp_details(model_points, 2))

      'Determine whether or not there was a theft claim

      If Rnd < exp_details(model_points, 3) / 12 Then
         claim = exp_details(model_points, 1)
         'i.e. the loss is for the entire sum insured

      total_claims = total_claims + claim
      End If

      'Determine whether or not there was an accident claim

      If Rnd < exp_details(model_points, 4) / 12 Then

      'The percentage of the sum insured loss will now be calculated
      'assuming a pareto distribution with mean as obtained from
      'the Exposure details and alfa parameter = 2.

         claim = (exp_details(model_points, 5) / _
         ((1 - Rnd) ^ (0.5))) - exp_details(model_points, 5)

         If claim > 1 Then claim = 1

      claim = claim * exp_details(model_points, 1)
      total_claims = total_claims + claim
      End If

   Next 'loop1
Next 'model_point

Worksheets("Workings").Cells(sim, months + 39) = total_claims

'A variety of investment options can now be tested i.e. where will
'the cost of claims be recouped from - cash or equities

If cash > total_claims Then
   cash = cash - total_claims
End If

If cash > 0 And total_claims > cash Then
   cash = 0
   equities = equities - (total_claims - cash)
```

```
  End If

  If cash < 0 Then
     equities = equities - total_claims
  End If

  funds = cash + equities
Next 'month
Next 'simulation of a year's experience

End Sub
```

## 12.7 Conclusion

This annexure provides an outline of a model similar to what might be used in practice.

The model considers the exposure of the company, the expected claims outgo as well as the possible return from investment. The model is then applied to project the expected cash flows to form the accounting entries normally found in the income statement.

The profit or loss generated on the income statement is presented as a probability density function. This function's moments can be considered for comparison to the efficient frontier. As indicated in the example, due to its simplicity, the return on capital is unrealistically high.

A change in investment policy is then considered as a business strategy. Any of the business strategies discussed could be considered.

A comparison is made between the probability density function of returns given the two different investment policies. In similar fashion a variety of strategies can be tested.

Based on the results, the shareholders can decide which set of strategies meet their needs the best.

### 3.5 Summary

In this chapter a synthesis of the framework required for risk and business strategy evaluation techniques was considered.

The framework for setting risk evaluation techniques is as follows:

1. Model points are identified for each class of business and asset class.
2. The expected cash flow on these model points is considered to the extent that all the items in the income statement are covered.
3. An income statement is then generated for each class of business incorporating the projection model for each element contributing to the cash flow of that class of business.
4. The different income statements can be consolidated for the group.
5. The expected change in the capital base can be evaluated over time using the models.
6. Different business strategies can be tested given the model and compared to the constraints set.
7. The optimal business strategy can be implemented.

Subsequently an analysis of the different areas of risk and uncertainty was considered. This followed on from the analysis in the introduction of this dissertation. A synthesis of the areas of risk and uncertainty revealed that the most weight in analysis should be placed on:

1. The liability risk
2. Asset risk
3. Credit risk
4. Corporate governance

In the next chapter the data required for further investigations as well as problem areas pertaining to investigations will be considered.