10. Glossary of terms

10.1 Introduction

This glossary contains terms that may have different interpretations. The author included only those terms which were queried during proof reading of this dissertation.

10.1.1 Autoregressive

This refers to the process of regressing a variable onto itself, that is to model a certain variable based on the values of that variable at previous durations. This is appropriate in the case of time series modelling.

10.1.2 Bottom up

This technique refers to the most detailed form of modelling. Each element affecting the end result is modelled individually together with the interaction, if any, between the different elements. This method makes use of the most information and aims to model as accurately as possible. The correlation between different elements such as different types of claims are, however, often difficult to quantify leading to certain approximations of a bottom up model. Compare to a top down model.

10.1.3 Business strategies

Business strategies refer to strategies that can be implemented by management in order to change the existing business structure. Such strategies include:

1. Business mix strategies. Deciding on the type of business.
2. Reinsurance strategies. Deciding on the appropriate reinsurance to mitigate risk.
3. Pricing strategies. Deciding on the pricing structure having regard to the business cycle as well as the expected retention of policyholders.
4. Investment strategies. Deciding what the investment policy will be.
5. Management strategies. Implementing structures and procedures that will improve efficiency in the business.

10.1.4 Cost of risk

This may have different interpretations depending on the context in which it is used. It can mean:

1. The funds required to meet the cost of outgo generally referring to claims outgo.
2. The cost of capital as opposed to the cost of reinsurance.

10.1.5 Free reserves

The free reserves are the reserves that will be distributable in the event of wind-up of the insurance company. This is the difference between the present value of all assets and the present value of all liabilities and may include non-distributable reserves if these are distributable in the event of wind-up.

10.1.6 Homoscedastic

This refers to a sample or series with constant variance. When modelling the error structure of such a series or sample the error structure will have constant variance.

10.1.7 Heteroscedastic

This refers to a sample or series where the variance changes over time. This is applicable to time series models and are particularly useful to model the durations of high and low variability that are often seen in economic variables such as exchange rates.

10.1.8 Model point

Suppose the insurer sells only three kinds of policies so that there are

- 10000 identical policies of type A
- 1000 identical policies of type B
- 10 identical policies of type C.

Then it is possible to have three model points to model the entire experience. Each model point must then reflect the expected distribution of the experience of that entire group of policies. This mean that model point A's experience will be simulated 10000 times, model point B's experience will be simulated 1000 times and model point C's experience will be simulated 10 times.

10.1.9 Monte Carlo simulation

This involves obtaining a set of independently simulated results from a specified fitted model under the hypothesis that the fitted model is an accurate representation of the business operation.

10.1.10 Run-off

This refers to the process whereby claims are actually paid after they have been reserved for. At the financial year end a reserve is set up to meet the cost of all outstanding claims for which premium has already been received. These reserves are required in order to ensure that no further strain is placed on capital in future years. That is to say that if the company is solvent today it will be solvent in future given the business model.

Run-off occurs when reserves are released to meet the claims as they fall due. The claims characteristics depend on the manner in which claims are notified, settled and paid and this process can be short or extremely long. Household insurance is said to have an expected run-off shorter than liability business.

The run-off pattern seen in previous years is then also used to decide on the level of reserves required at future durations.

10.1.11 Stochastic

The term stochastic, which is derived from the Greek word "stochos", meaning "guess", is used instead of the term probabilistic.

Stochastic processes refer to a process where the development is based on an error structure that has a probability distribution. For example, the total claims originating from a policy can be seen as a process modelling the time to the next claim for a specified period of time and subsequently considering the "error" in the claim size, that is to say to what extent does the claim size differ from the mean.

10.1.12 Top down approach

This approach uses a model of the entire business operation instead of modelling each component individually. For example, the loss ratio of the entire operation will be modelled using a log-normal distribution. This approach implicitly takes all correlations between different elements into account but does not explicitly model these. As a result a top down approach is a very crude modelling approach. The two approaches are often used together to first recognise the detailed structure of each item of cash flow and secondly to allow for correlations between these items by some approximate means.

10.1.13 Thomson model

This refers to the model developed by R.J THOMSON (1994) for use of modelling returns on a variety of assets in the South African market.

10.1.14 White noise

A term used for a Wiener process to denote normality. Consider a stochastic variable x(t) indexed by continuous time. The family {x(t)} is a Wiener process iif:

- $x(0) = 0$
- $\Delta x = x(t+\Delta t) - x(t)$ are independent variables
- for all $s>=0$, $t>=0$; $x(s+t) - x(s)$ has the normal distribution $N(0,t)$

White noise is also referred to as the Guassian distribution, Brownian motion or normal distribution.

11. References

BAIN L.J. ENGELHARDT M. Introduction to Probability and Mathematical Statistics. Duxbury Press Boston 1987.

BONNARD R. GREENWOOD M. GREYBE S. Bootstrapping Reserve Estimates[106]. Transactions of the Actuarial Society of South Africa. 1999.

BEAUREGARD R.A., FRALEIGH J.B. Linear Algebra Second Edition Addison-Wesley Publishing Company 1990

BERNSTEIN P.L. Against the Gods: The Remarkable Story of Risk. John Wiley & Sons Inc. 1996.

BOWERMAN O'CONNEL Linear Statistical Models An Applied Approach. PWS-KENT Publishing Company 1990.

BRIGHAM E.F Financial Management: Theory and Practice[107]. Holt – Sauners International editions 1985.

---

[106] This paper provides a comprehensive overview of bootstrapping techniques applied to claim reserving methods. All the claim reserving methods are based on triangulation projections.

[107] Brigham provides an overview of financial management. This textbook is often used for MBA study. Due to the wide scope covered in the book the level of detail is limited. The 1999 edition contains the following sections:

INTRODUCTION TO FINANCIAL MANAGEMENT
- Chapter 1 An Overview of Financial Management
- Chapter 2 Financial Statements, Cash Flow, and Taxes
- Chapter 3 Analysis of Financial Statements
- Chapter 4 The Financial Environment: Markets, Institutions, and Interest Rates

FUNDAMENTAL CONCEPTS IN FINANCIAL MANAGEMENT
- Chapter 5 Risk and Rates of Return Appendix 5A Calculating Beta Coefficients
- Chapter 6 Time Value of Money Appendix 6A Continuous Compounding and Discounting

FINANCIAL ASSETS
- Chapter 7 Bonds and Their Valuation Appendix 7A Zero Coupon Bonds Appendix 7B Bankruptcy and Reorganization
- Chapter 8 Stocks and Their Valuation

INVESTING IN LONG-TERM ASSETS: CAPITAL BUDGETING
- Chapter 9 The Cost of Capital
- Chapter 10 The Basics of Capital Budgeting
- Chapter 11 Cash Flow Estimation and Other Topics in Capital Budgeting Appendix 11A Depreciation Appendix 11B Refunding Operations
- Chapter 12 Risk Analysis and the Optimal Capital Budgeting

CAPITAL STRUCTURE AND DIVIDEND POLICY
- Chapter 13 Capital Structure and Leverage Appendix 13A Degree of Leverage
- Chapter 14 Dividend Policy

FINANCIAL PLANNING AND WORKING CAPITAL MANAGEMENT
- Chapter 15 Financial Forecasting
- Chapter 16 Managing Current Assets Appendix 16A The Cash Conversion Cycle
- Chapter 17 Financing Current Assets Appendix 17A Secured Short-Term Financing

Special Topics in Financial Management
- Chapter 18 Multinational Financial Management
- Chapter 19 Derivatives and Risk Management
- Chapter 20 Hybrid Financing: Preferred Stock, Leasing, Warrants, and Convertibles
- Chapter 21 Mergers, LBOs, Divestitures, and Holding Companies

Appendix A Mathematical Tables
Appendix B Solutions to Self-Test Problems
Appendix C Answers to End-of-Chapter Problems
Appendix D Selected Equations and Data

CARNEGIE MELLON SOFTWARE ENGINEERING INSTITUTE Risk management Overview. www.sei.cmu.edu 2000

CLAASSEN M.S., HUBER P.P. Integrating Pension Fund Assets and Liabilities: Techniques for the 1990's[108]. Presented to the Actuarial Society of South Africa 1992.

CHANDARIA S. et. al. Reinsurance Pricing[109]. Institute and Faculty of Actuaries GISG and ASTIN Colloquium 1998.

CHRISTOFIDES S. Pricing for Risk in Financial Transactions[110]. Institute and Faculty of Actuaries GISG and ASTIN Colloquium 1998.

CHRISTOFIDES S. GISMO (General Insurance Stochastic Model Office) – Short-term modelling for management decisions. Institute and Faculty of Actuaries GISG 1996.

CLARK B. Enterprise Risk – What's Up with That? www.irmi.com 2000

CLARK B. Corporate Risk Finance and the "Internal Economy" of a Company. www.irmi.com 2000

COMMONWEALTH ASSOCIATION FOR CORPORATE GOVERNANCE. 2001 www.combinet.net

COUTTS S.M., DEVITT E.R. (not dated) Simulation Models and the Management of a Reinsurance Company[111].

---

[108] This paper provided the first introduction pertaining to the field of stochastic modelling to the Actuarial Society of South Africa. It is of historic interest only.

[109] This is a paper based on the work of the GISG working party involved with reinsurance pricing. The paper addresses the practical issues pertaining to the pricing of risk. As such it provides an interesting overview into the practicalities behind the risk quantification methodologies which are also dealt with extensively in this study.

[110] An interesting approach to risk rating which was not explored further in this study. This paper received the Brian Hey Prize at the GISG convention for one of the best three papers. The paper considers certain criteria appropriate for risk rating. Given these criteria the approach of loading an expected value of risk with a standard deviation measure is inappropriate. As an alternative, it is suggested to increase the cumulative distribution function thereby moving the probability distribution function higher as well. This increase is based on the market rates currently applicable in the market.

This approach is useful for risk rating but makes the quantification of variability extremely difficult. This author did not see it necessary to pursue this approach further.

[111] A copy of this paper is available from the author. The source of the paper is unknown. The paper provides a very short overview of a practical algorithm required to implement a simulation model for a reinsurance company though it would be equally possible for a general insurance company. This algorithm is provided in this study as well.

COUTTS S.M., THOMAS T.R.H. Modelling the impact of Reinsurance on Financial Strength[112]. Presented to the Institute of Actuaries 1997.

CREDIT SUISSE FINANCIAL PRODUCTS CreditRisk+ A credit Risk Management Framework. 1997

CUMBERWORTH M.P. et.al. Corporate Decisions in General Insurance: Beyond the Frontier[113]. Institute of Actuaries 1999.

CUMMINS J.D., ALLEN F., PHILLIPS R.D. Financial Pricing of Insurance in the Multiple Line Insurance Company[114]. Institute and Faculty of Actuaries GISG and ASTIN Colloquium 1998.

CZEMUSZEWICZ A. et. al. Reserving and Pricing for large claims[115]. Institute and Faculty of Actuaries GISG and ASTIN Colloquium 1998.

DANIEL W. Asset Liability modelling for insurance and reinsurance companies. Article submitted to Global Reinsurance Magazine 1997.

[112] An important paper for this study. This paper provides a framework for the investigation of suitable reinsurance arrangements. For the purpose of this study these ideas can be extended for the implementation of other best business practices as well. This is clearly illustrated in the study as well. Refer to the introduction as well as the appendix.

[113] This paper considers the extent to which the efficient frontier can be used as a benchmark in setting appropriate business strategies. They argue that certain strategies boil down to diversification which an investor can commit to in any event and therefore the practitioner needs to be careful to what extent the strategy proposed actually mitigates specific risk. Specific risk refers to the risk inherent in the company that cannot be eliminated through proper diversification exercises.

[114] This paper received the prize for best paper at the GISG convention. The paper shows that the price of insurance depends on the overall riskiness of the insurance company and not on the riskiness of the individual line of business. The paper introduces the idea that the overall riskiness of the company needs to be established.

This idea is partially shared by the author. The first part of the paper focuses on establishing the likelihood of insolvency for the entire enterprise and suggest that insurance prices should conform to the likelihood i.e. riskier operations should demand higher prices. This conforms to general economic theory as explained in this study. The second conclusion is that once the overall position has been determined, no further allowance for riskiness inherent in the business is necessary. This author does not share this view and argues that the very overall riskiness depends on the volume of different lines of business. As a result the entire process is a dynamic one and cannot be approached in the fashion suggested.

[115] This paper also provides feedback from one of the GISG's working parties. The paper shows some of the problems that can arise in the event of a large claim. Due to the rare occurrence of these claims, they are generally treated on a case estimate basis only. From a reserving point of view they are not permissible according to the international accounting standard. The best approach for smaller companies is to reinsure any exposure to large claims. For large risk writers the debate should continue for some years to come. The main problem arises because the Revenue Services view such reserving practices as tax avoidance.

DAVISON A.C., HINKLEY D.V. Bootstrap Methods and their Applications. Part of the Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press 1997.

DAYKIN, C.D., PENTIKAINEN, T. & PESONEN, M. Practical Risk Theory for Actuaries[116] Prentice Hall 1994

---

[116] An extremely useful reference. The contents is given below:

DOBSON A.J. An Introduction to Generalized Linear Models. Chapman and Hall 1990.

DORRINGTON R.E., ENNIS P.W. A Beginners guide to Asset and Liability modelling. Presented to the Actuarial Society of South Africa 1994.

DVORAK B. Risk Models of Loss. Presentation at the Business to Business Workshop on Enterprise Wide Risk Management. 2001

EMBRECHTS P. Integrated Risk Management[117]. One day Workshop presented at WITS University 2000

11.3 Dividends
12 The insurance process
12.1 Basic equation
12.2 Empirical observations
12.3 Business cycles, analysis of causes and mechanisms
12.4 Simulation of the insurance process
13 Applications to long-term processes
13.1 General features
13.2 Capital requirements of an insurance company
13.3 Evaluation of an insurer's net retention limits
14 Managing uncertainty
14.1 Review of applications
14.2 Basic equations
14.3 The insurer and the market
14.4 Measuring and managing financial strength
14.5 Corporate planning
14.6 Public solvency control
15 Life insurance
15.1 Recapitulation of some basic formulae of life insurance mathematics
15.2 Stochastic cohort approach
15.3 Analysis of the total business
16 Pension schemes
16.1 Pension structures and definitions
16.2 Pension formulae
16.3 Deterministic methods of pension funding
16.4 Stochastic methods for pensions

APPENDICES

A Derivation of the Poisson formula
B Pólya and Gamma distributions
C Asymptotic behaviour of the compound mixed Poisson d.f.
D Numerical calculation of the normal d.f.
E Derivation of the recursion formula for *F*
F Simulation
G Time series
H Portfolio selection
I Solutions to exercises

[117] Professor Embrechts first considered the application of extreme value theory as a risk management tool. Subsequently he considered the evaluation of risk using risk measures such as value at risk and shareholder deficit. These refer to the value of the percentile and the expected value of a claim above the percentile for a certain specified distribution. He went on to consider the correlation between different economic entities, share prices for example, he showed empirically that the correlation matrix in the extreme is different to the correlation matrix generally obtained through normal analysis of variance. This leads to the cautioning of all practitioners who are using correlation matrices for risk evaluation purposes.

FINANCIAL SERVICES BOARD Annual Report 2000. www.fsb.co.za

FINNIS D. Risk Based Capital – Implications for Solvency and Profit measurement for General Insurers. Presented at the Institute of Actuaries of Australia Conference at Coolum 1995.

FRANKEN C.J., KRUGER N.A.S. An Overview of the New Profit-Reporting Basis for Life Insurers in South Africa[118]. Presented to the Actuarial Society of South Africa 1997.

GOLDMAN, SACHS & Co ans SWISS BANK CORPORATION WARBURG DILLON READ The Practice of Risk Management. Euromoney Books 1998.

HARDING J. et. al. The Reserving of Non-Standard Classes of Insurance. Institute and Faculty of Actuaries GISG and ASTIN Colloquium 1998.

HOGG R.V. & KLUGGMAN S.A. Loss Distributions 1976

HOOKER N.D. et al. Risk Based Capital in General Insurance[119]. Presented to the Institute of Actuaries 1995.

HOWIE R.J. Investment Risk and Asset Modelling. Transactions of the Actuarial Society of South Africa 1997.

[118] This paper introduced a financial soundness valuation basis for the Actuarial profession in South Africa. The aim of this basis was to introduce a first tier of excess margins in assumptions which would render a likelihood of insolvency of less than 5% for any company in the Life Insurance Industry. The paper gives some indication of the level of depth to which the ideas described in this dissertation has been implemented in the Life Insurance Industry. Given the peculiarities of that industry the level of investigations are not as in depth mainly because the products have very little or no risk.

[119] A very important paper. This paper was the first attempt to consolidate the ideas of enterprise wide risk management into an actuarial paper. The paper is discussed in this study as well. The reader is referred to chapter 3.

Abstract: This paper looks at the problems of assessing, for solvency purposes, the capital requirements of a non-life insurer in the context of the United Kingdom. It considers how these capital requirements might vary according to the different risks to which an insurer is subject and how this Risk-Based Capital (RBC) might be measured in practice, using as a case study the RBC formula recently introduced in the United States of America. The paper also discusses the application of RBC concepts to the problem of internal capital allocation, to assist in measuring an insurer's rate of return to shareholders by business unit, as well as the more obvious regulatory application. The advantages and disadvantages of a formula-based approach to capital requirements for solvency purposes are discussed in comparison with possible alternative approaches to insurance supervision.

Keywords: general insurance; insurance supervision; risk; risk-based capital; solvency

HITCHCOX A. Risk Based Capital: Used as an Internal Management Tool[120]. Presentation given to the London Insurance and Reinsurance Market Association 1996.

INSTITUTE OF ACTUARIES AND FACULTY OF ACTUARIES. Core reading on study material. Institute and Faculty Education Company. 1995 to 1998. Subject 103, 301, 403.

INSTITUTE OF ACTUARIES AND FACULTY OF ACTUARIES. Manual of Actuarial Practice.

MAHER G. Recent Developments in the International Insurance and Reinsurance Markets. Presented at the General Insurance Mini Convention of the Actuarial Society of South Africa 2001.

MAITLAND J. A Review of Thomson's Stochastic Investment Model. Transactions of the Actuarial Society of South Africa. 1997. www.assa.org.za/Papers/papers96/am/am.htm/[121]

MARKOWITZ H. Portfolio selection. Journal of Finance 1952.

MEHTA S. Pricing of Insurance Risk[122]. Institute and Faculty of Actuaries GISG and ASTIN Colloquium 1998.

MICCOLIS J. Enterprise Risk Management[123]: What's Beyond the Talk? www.irmi.com 2000

MICCOLIS J. Enterprise Risk Management in the Financial Services Industry: Still a long way to go. www.irmi.com 2000

MICCOLIS J. Enterprise Risk management in the Financial Services Industry: From Concept to Management Process www.irmi.com 2000

MÜLLER E. RBC Approaches and Profit Margin Requirements. Presentation at Hannover RE 1997.

[120] This presentation shows how the risk based capital requirements introduced by the National Association of Insurance Commissioners have been implemented by a London Market company. The approach is lacking to the extent that the company may not be exposed to the same risks as all companies combined in the United States.

[121] Maitland's paper provides a detailed overview of the appropriateness of the Thomson model and provides additional insight into the use of ARIMA models.

[122] This paper provides an overview on the integration of portfolio theory and capital allocation with premium rating.

[123] The dissertation considers the process of enterprise risk management. The submissions made by MICCOLIS provide the reader with a very good introduction to the fundamentals pertaining to Enterprise risk management as discussed in this study.

PANJER H.H. WILLMOT G.E. Insurance Risk Models[124]. Society of Actuaries 1992.

ROOS W. A case study on the evaluation of different forms of media for the purpose of obtaining business for a direct company. (Actual title not known) Actuarial Society of South Africa 2001.

RYAN J.P. et.al. Financial Condition Assessment. Institute of Actuaries 2001[125]. www.actuaries.uk.org
SHAH S. Sailing through Rough Winds of Macro Economic Risks. www.irmi.com 2000

SHAH S. Risk Quantification in Rapidly Changing Business Environments www.irmi.com 2000

SHOUGH R. Enterpise wide risk management. Presentation at the Business to Business workshop on enterprise wide risk management. 2001

SIGMA of SWISS-SA Re. The Performance of the Insurance Industry International Comparison: a Risk-Adjusted Analysis. 1995.

SOUTH AFRICAN INSTITUTE OF CHARTERED ACCOUNTANTS Accounting Guide on Short-Term Insurance. 2001

SOUTH AFRICAN INSTITUTE OF CHARTERED ACCOUNTANTS Auditing Guide on Short-Term Insurance. 2001

SMITH A.D. Workshop on Stochastic Asset Models[126]. Presented to the General Insurance Convention 1995.

STEYN A.G.W., SMIT C.F., DU TOIT S.H.C. Moderne Statistiek vir die Praktyk J.L. van Schaick 1987

TEAKER D., UPSON J., WRENN S. Benchmarking. Institute and Faculty of Actuaries GISG and ASTIN Colloquium 1998.

THE ACTUARY MAGAZINE. May 1998.

THOMSON R.J. A stochastic Investment Model for Actuarial Use in South Africa. Transactions of the Actuarial Society of South Africa 1994.

[124] A very useful guide on probability distributions as well as collective risk models.

[125] This paper provides further follow up following the paper by HOOKER 1998. The aim of the paper is to introduce a framework to the FSA whereby the solvency of general insurance companies can be monitored. The content of the paper is encouraging as it provides a confirmation of the the approach set out in this study as a suitable one to address the issue of solvency. As mentioned, in this study, the scope of this framework goes beyond any one business strategy.

[126] This paper is extremely useful as a reference for asset models. Smith covers a variety of asset models in great detail and also provides the visual basic code for these models. The content is considered further in this study in the assessment of asset risk.

VAN DEN HEEVER R.J. Risk Adjusted Capital in a General Insurance Environment. Transactions of the Actuarial Society of South Africa. 1999.

VENTER G. Asset Modelling – Empirical Tests of Yield Curve Generators. Institute and Faculty of Actuaries GISG and ASTIN Colloquium 1998.

VENTER G. Liability Modelling – Empirical Tests of Loss Emergence Generators. Institute and Faculty of Actuaries GISG and ASTIN Colloquium 1998.

WASZINK H. & VAN DER WARDT M. A stochastic model to determine IBNR-Reserves. Institute and Faculty of Actuaries GISG and ASTIN Colloquium 1998.

YAKOUBOV Y.H., TEEGER M.H. and DUVAL D.B[127]. The TY Model - A stochastic Investment Model for Asset and Liability Management. Presented to AFIR and ASTIN colloqia in Tokyo 1999. www.sias.org.uk/papers/model.pdf

[127] This paper provides a very thorough overview of investment models and is definitely worth reading. The authors propose (and this is a biased opinion) the following qualities:

1. Clarity of Design
2. Synthesis of the Economic Framework, Investment Practice and Historic data.
3. Unique approach to equity risk – an explicit market sentiment component is introduced.
4. Grounded in reality.

The paper considers a cascaded asset model similar to the Thomson model and provides the results of the investigation for

1. Price inflation (which is the base)
2. Salary inflation
3. Fixed interest government bonds
4. Cash
5. Index linked government bonds
6. UK equities (evaluated by income, growth and change in rating) this is a wonderful development because this approach allows for the integration of market and credit risk.
7. Overseas equities

12. Annexure

12.1 Introduction

The aim of this annexure is to provide a practical example of the ideas discussed in this dissertation. This example is not based on empirical evidence and serves merely as an illustration of the ideas put forward. Furthermore, this model assumes a predetermined model for liabilities and assets as explained below.

Before progressing to the actual model the key steps in the modelling process need to be considered[128]:

1. Develop a well defined set of objectives. This includes a reference to the duration over which the model will be considered, the definition and the probability limits of ruin as well as the definition of the objectives.
2. Plan the model process and how the model will be validated. The data requirements will be considered as well as the other information requirements necessary to set a model appropriate to simulate future expected experience. Testing the model to past data and expert opinion also needs to be considered.
3. Collect the data. Be wary of possible data errors and rectify these. Also recognise that past experience is not necessarily a good guide of the future. Consider the general economic and commercial environment as discussed in the introduction.
4. Define the model through setting the requirements of the real world that it is required to simulate. Refinement of the model can come through the monitoring process.
5. Consult experts in the different areas. It is not possible to have full knowledge on all the areas and considerable interaction will be required with economists, underwriters, industry leaders etc.
6. Consider the platform on which the model will be built. A statistically reliable random generator is required that will be suitable to the objectives of the models. Remember that the model is better used to investigate deviations than extremes merely because it is based on numerical techniques. For the extremes analytical tools based on extreme value theory will be more appropriate[129].
7. Write the computer program for the model.

[128] These are the steps included in the INSTITUTE AND FACULTY OF ACTUARIES Core Reading on subject 103.

[129] EMBRECHTS, KLUPPELBERG, MIKOSCH Modelling of Extremal Events for Insurance and Finance (1997) In which the distribution of maxima of samples are considered. This is a limiting behaviour similar to the central limit theorem.

8. Debug the program to make sure it performs the intended operation in the model definition.
9. Test the reasonableness of the output from the model.
10. Review and carefully consider the appropriateness of the model in the light of small changes in input parameters.
11. Analyse the output from the model.
12. Communicate and document the results and the model.

This process is then integrated into the control cycle which was discussed in the introduction.

This chapter will cover the details of the model used, the VISUAL BASIC FOR APPLICATIONS code used to model the experience and the results of the model will be presented briefly.

The main objective will be to apply the principles of optimisation to the simple example presented here. In order to do this a suitable benchmark for an efficient frontier is required. As the purpose of this appendix is a presentation of the working of the ideas set out thus far, the efficient frontier set out in chapter 9 of this dissertation shall be applied.

12.2 Details of the model

The ideas are applied to a fictitious insurance company. This company writes only motor insurance and only invests its funds in either cash or alternatively in equities. Furthermore it is assumed that the company has just started writing business (or alternatively a new class of business is investigated). In order to properly present the cash flows turn back to chapter 2. The first step is to evaluate the income statement and generate a model for the income as it is expected to incur in practice in the year to come. Allowance is therefore required for the income in premium and investment returns and the outgo in the form of claims. This model can be extended to include all the transaction types illustrated in the income statement.

Assume that the company had access to data and expertise which enabled it to set up models which can accurately reflect the expected future experience.

12.2.1 The liability model

Firstly consider the variability in the claims outgo. Details of the liability model are given below. The model considers only motor business. Two perils are considered namely the loss of a vehicle through theft and accidental damage to the vehicle.

For both types of peril a frequency and severity distribution has been obtained. These distributions will be obtained through the techniques explained in chapter

5. That is to say past exposure grouped by risk factor will be investigated, the significant risk factors identified and regression techniques applied to the data broken down by risk factor for each peril.

In addition the parameters of these distributions are assumed to be affected by risk factors. In this example only two risk factors have been used; namely age and area. The table below shows the impact these risk factors are expected to have on the frequency and severity distributions of thefts and accidents respectively:

Theft frequency **Poisson** **1%**

Class	**Risk Factor 1**	**Value**	**Risk Factor 2**	**Value**
Motor	Age1	1	Area1	1.5
	Age2	1	Area2	1
			Area3	0.8

Theft severity is the Sum Insured

It can be shown that the frequency of independent occurrences will follow a Poisson process hence both the theft and the accident frequencies are assumed to have Poisson distributions.

Accident frequency **Poisson** **15%**

Class	**Risk Factor 1**	**Value**	**Risk Factor 2**	**Value**
Motor	Age1	1.3	Area1	0.9
	Age2	0.8	Area2	1.1
			Area3	1.2

Accident severity **Pareto** **20%**

Class	**Risk Factor 1**	**Value**	**Risk Factor 2**	**Value**
Motor	Age1	1.1	Area1	1
	Age2	1	Area2	1
			Area3	1

It can be seen from the figures that the area risk factor is not expected to affect the severity of accidents and similarly the age risk factor is not expected to affect the frequency of thefts.

The severity of accidents is assumed to represent the severity in the mid term of the policy term and no allowance is made for inflationary movements on claims.

The frequency distributions of both perils are assumed to be Poisson and the Pareto distribution is assumed to have an alfa parameter of 2. With proper empirical testing these assumptions will be based on the estimates described in chapter 5.

12.2.2 The asset model

The asset model has been split into two sections. One section has been compiled for cash and the other for equity investments. The cash model is a simple discrete model whereby the monthly cash return on any funds is assumed to have a starting value, a minimum and a maximum with a distribution of movement from one month to the next. This is shown below.

Cash

Mean return per month		0.50%
Probability	Cumulative	Monthly Movement
0.1	0.1	0.50%
0.15	0.25	0.20%
0.15	0.4	0.10%
0.3	0.7	0%
0.2	0.9	-0.10%
0.05	0.95	-0.20%
0.05	1	-0.50%
Min		0.20%
Max		1.10%

Note that this model is very simplistic. The model builder may decide to investigate the past movements of money market rates and subsequently set a similar model or alternatively apply an auto-regressive model as set out in chapter 6.

The asset model for shares is shown below. This has the working of a simple auto-regressive model with a white noise element as the error term.

Shares
Value of index for the last 6 months

555
547
560
578
599
596

Error structure:	Mean error	0.5% of index at time t
	Standard Deviation	10% of index at time t

Model

Index(t) = 0.6*Index(t-1) + 0.2*Index(t-2) - 0.3Index(t-3) +0.4Index(t-4) + 0.2Index(t-6) + Error term

The model set out above might be over parameterised. This model may be based on another model or on forward market rates or some other sources of information. The simplistic nature of this model does, however, illustrate the working of such a model.

It is therefore possible to project the growth of funds invested in either cash or equities. These models have not been based on empirical evidence and are for the purpose of illustration only.

12.2.3 Exposure details

Once the area of liability and asset uncertainty has been addressed, the model then needs to consider other areas of uncertainty that may apply in the company. Clearly many of these will be very difficult to quantify. It will be possible to check whether or not the company maintains an appropriate level of solvency given growth projections, for example.

For the purpose of this model I do not allow for any further growth but consider the experience on one book of business written at the beginning of the period. The model is applied to a book of motor business. All premiums are expected annually in advance. Details of the exposure are shown below.

Liability model points

Class	Sum Insured	Risk Factor 1	Risk Factor 2	Num	Theft Freq	Acc Freq	Acc Sev	Risk Premium	Office Premium	Total Office
Motor	40000	Age1	Area1	50	0.015	0.1755	0.22	107220	3000	150000
Motor	30000	Age1	Area2	100	0.01	0.2145	0.22	171570	2500	250000
Motor	30000	Age1	Area3	100	0.008	0.234	0.22	178440	2500	250000
Motor	80000	Age2	Area1	500	0.015	0.108	0.2	1464000	400	2000000
Motor	70000	Age2	Area2	400	0.01	0.132	0.2	1019200	3500	1400000
Motor	70000	Age2	Area3	500	0.008	0.144	0.2	1288000	3500	1750000
								4228430		5800000

Asset model points

Class	Value
Cash	50%
Shares	50%

12.2.4 Capital requirements

The business will be supported by capital to ensure that all claims will be met within certain bounds of certainty or alternatively the capital requirement can be set based on statutory requirements. The Risk Based Capital methodology applied by the National Association of Insurance Commissioners is an example of such an approach. Alternatively, a percentage of premium or claims can be used by the regulator to set a capital requirement.

The premiums include the loading for the cost of capital.

12.2.5 The collective risk model

An evaluation of the expected value of claim payments for this group of business reveals the following distribution for the collective claim outgo. Note that this distribution assumes no inflationary influences on claim amount during the year.

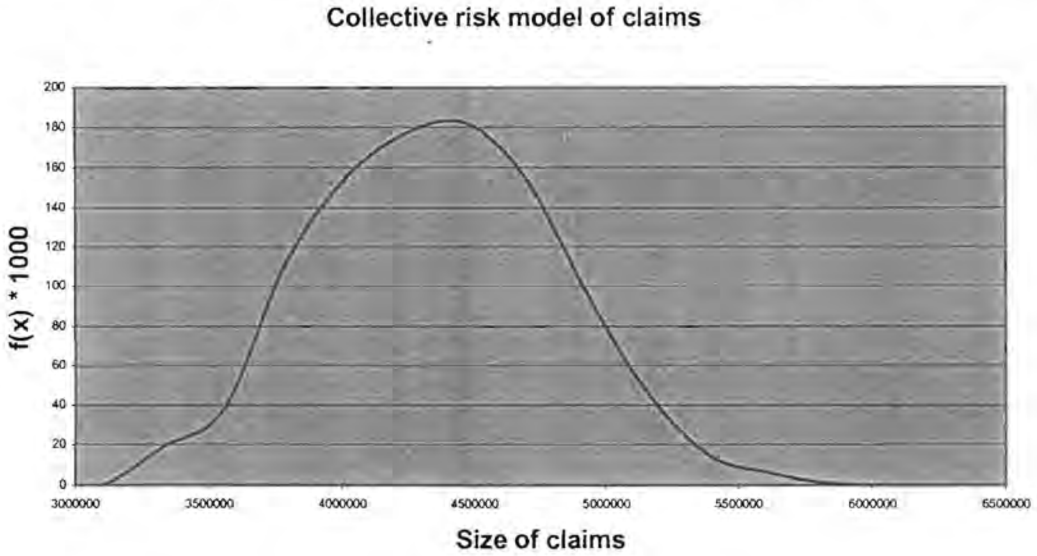

Figure 15: Expected outgo in the year for the collective motor model

Assuming the owners of the company require a 95% certainty of not needing to put up any additional capital during the year to meet claim payments, inspecting the curve above this means that funds of approximately R5.5million will be required. As the expected office premium is R5.8 million, no additional capital will be required to write the business.

If, however, the majority of the office premium is intended to meet other items of expenditure, then some additional capital may be required. In any event, the regulatory authorities in a country may set a minimum capital requirement. For sake of argument I shall assume a capital requirement of 10% of premium even though statistical evidence may indicate that this is not required. Hence capital of R0.58 is required to support the business.

From the discussion it should already be clear that the country of domicile of an insurance company has a significant effect on the return on capital of that insurance company.

12.3 Identifying the problem

Once all the elements required to model future outgo have been determined, a clear objective for the simulation exercise must be set. For the sake of example, assume that the objective is to optimise the return on capital given the model details as set out above. This optimisation will be subject to the probability of the return being less than 5% per annum to be less than 1%.

Clearly a variety of changes in the business procedure is possible in order to achieve this objective. The company may decide to market only to certain types of policyholders, or alternatively address the business procedure in order to change the risk premiums or change the investment structure. Different business strategies can be tested here to find the optimal fit.

In order to illustrate the working of the model consider a change in the investment structure. Arguably other adjustments can also be made.

It will be important to set a timeframe for the experience to be investigated. The longer the timeframe the more uncertain the projection will become.

It may also be necessary to generate further outcomes if the curve does not appear to be smooth. The number of simulations will depend on the purpose of the model. In general the techniques will have more merit in investigating the variability of experience than extreme outcomes of experience due to the variability in simulations in the extreme.

12.4 Developing the solution

Detailed Visual Basic Code is included to facilitate the understanding of the techniques described thus far. This is included in section 12.6 at the end of this appendix.

The approach applied is to assume that all premium is received and invested at the start of the year. Claims are then incurred during the year and paid from the available funds. The approach of disinvestment is to assume that there are no transaction costs as a result of disinvestment and that cash investment is first utilised before equity investment to pay claims.

The level of funds available at the end of the year is then compared to the capital which was made available to support the business in order to evaluate the objectives and in particular the return on capital.

12.5 Example of results

The model explained above is best viewed by means of a series of graphs. Each graph will depict the development of a specific entity throughout the year.

The funds received are initially invested in equal proportions in cash and equities as per the model explained above.

A random sample of six simulations has been drawn and the results shown below.

12.5.1 Development of cash position

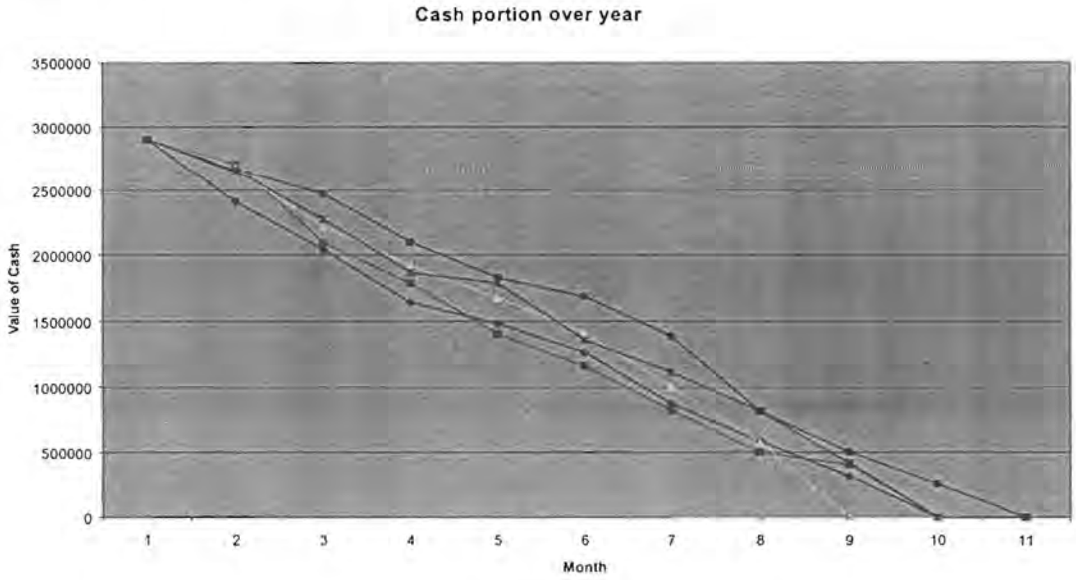

Figure 16: Development of cash held – reducing as claims are paid

The graph above shows the run-off of the cash position over time. Bearing in mind that cash is first used before equities are realised and that the cash position is not changed during the year, it is expected that the cash position will gradually worsen as shown. The cash position is, however, affected by the incidence of claims and not only by the return secured on cash investment.

It can be seen that the cash is exhausted for all six random samples that were drawn from the simulation results. This is because only half of the premium was invested in cash at the start of the year and the expected risk premium was substantially higher than half of the office premium. The risk premium refers to the amount that is expected to be paid out in respect of the risks covered. The office premium is the amount that is received in respect of each policy by the office.

12.5.2 Development in the claims experience

The variability of claims experience is apparent from Figure 15. This graph shows the random nature of the claim structure of a general insurer for the class of business described in the liability model above.

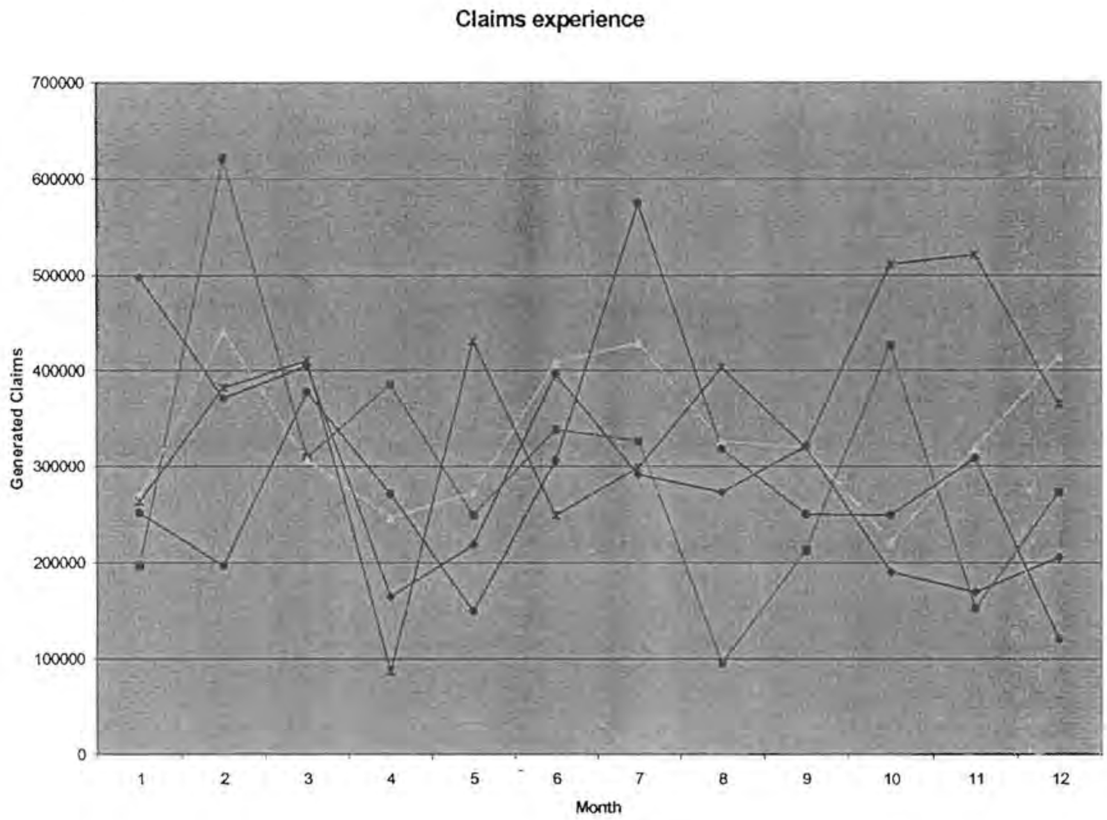

Figure 17: Projected claims experience for the year

It can be clearly seen that the monthly experience is extremely volatile. This is a feature of the assumptions chosen. Note that the model did not allow for any cyclical movements in experience though this would be possible.

Furthermore, delay patterns in claims experience has been ignored. This results in the projected return being underestimated. For short tail classes of business where investment returns are lower due to the quick turnabout time between premiums received and claims paid this does not have as a significant impact as long tail classes of business. The development of delay patterns is discussed in chapter 5.

12.5.3 Development in equity experience

Once the cash has been exhausted the equity funds are realised to meet claims outgo. Note that due to the increased level of variability in equity investment this may mean that the insurer needs to realise equity investment at an inappropriate time.

Details of the growth in the fund are shown below.

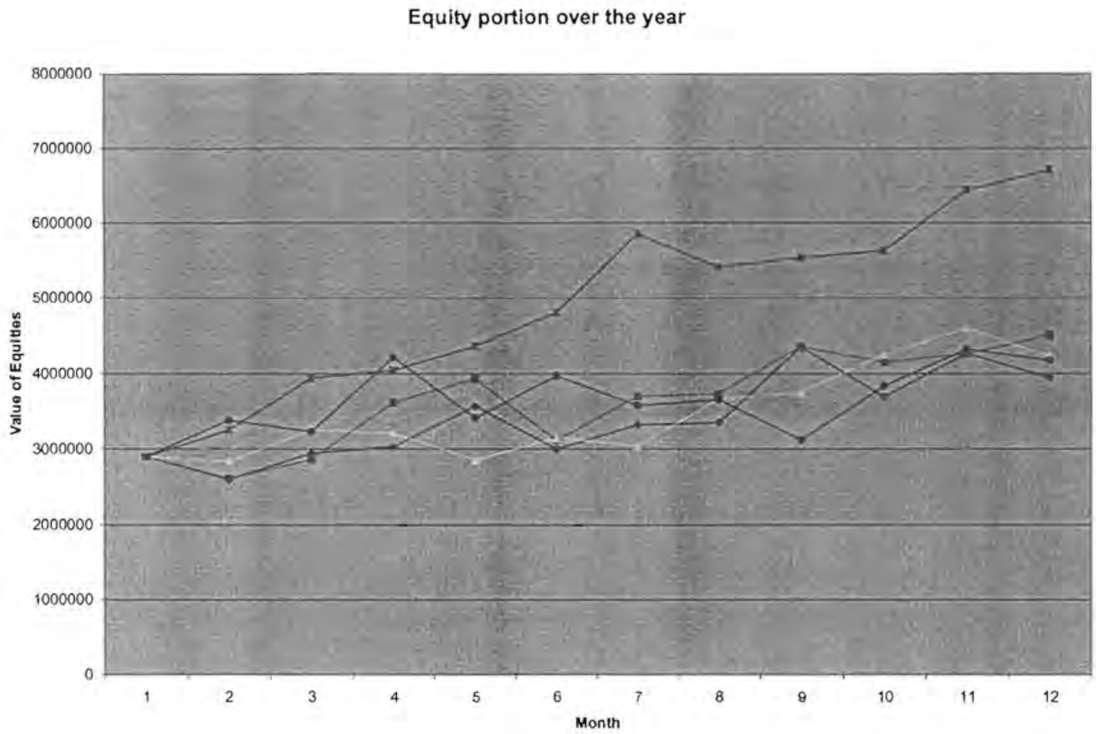

Figure 18: Development of equity position

Note the higher level of variability in the growth of equity investment as opposed to cash investment. This is also intuitive from the assumptions set out above.

12.5.4 Development of the fund

These three elements are then taken together to yield the value of the book of business at the end of the year.

The year-end value of the book of business also has a density function.

The progress of the value of the fund supporting the business is shown below.

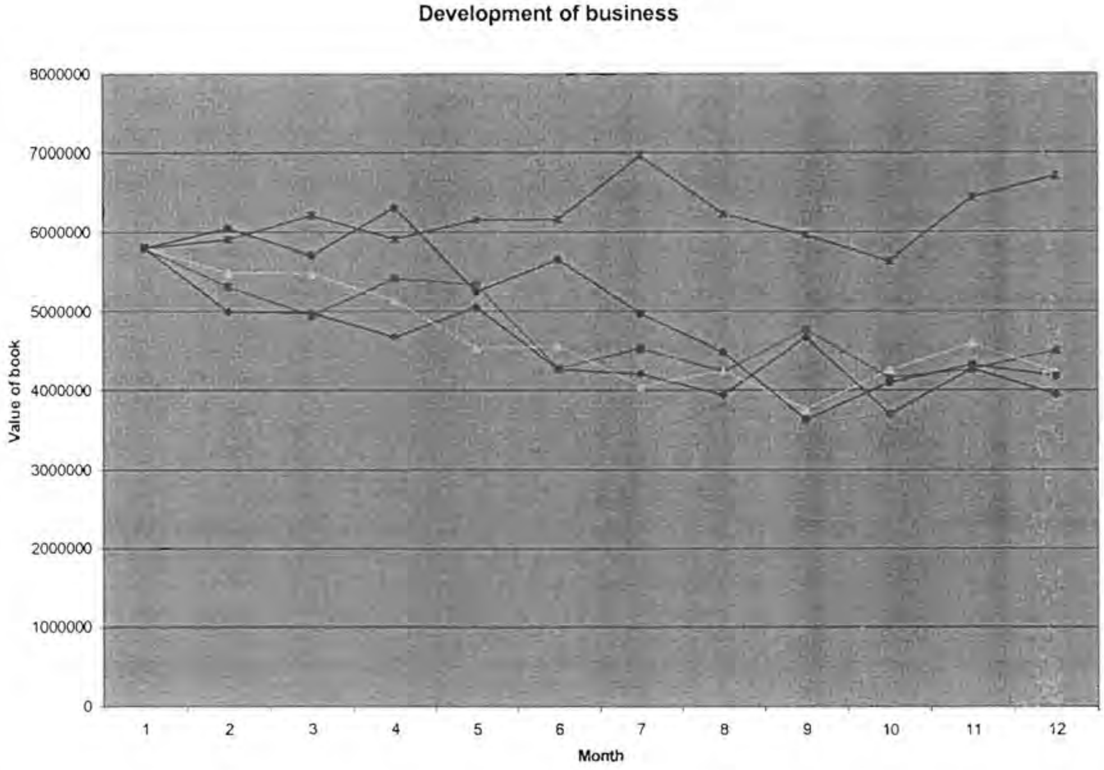

Figure 20: Projection of business develoment

It can be seen from the projection of year-end values that the influence of investment return as assumed in the valuation model is substantial. As can be seen from the graph above, the influence of equity investment is such that in one instance the value of the fund remained almost stable. This may be indicative that the asset model is too optimistic or that sufficient allowance has not been made for possible claim payment increases during the period.

12.5.5 Investigating the objective

The value of the fund at the end of the year is now investigated. This value can be expressed as a percentage of capital in order to determine whether or not the returns will meet shareholders expectation.

The value as at duration twelve has the following probability density function.

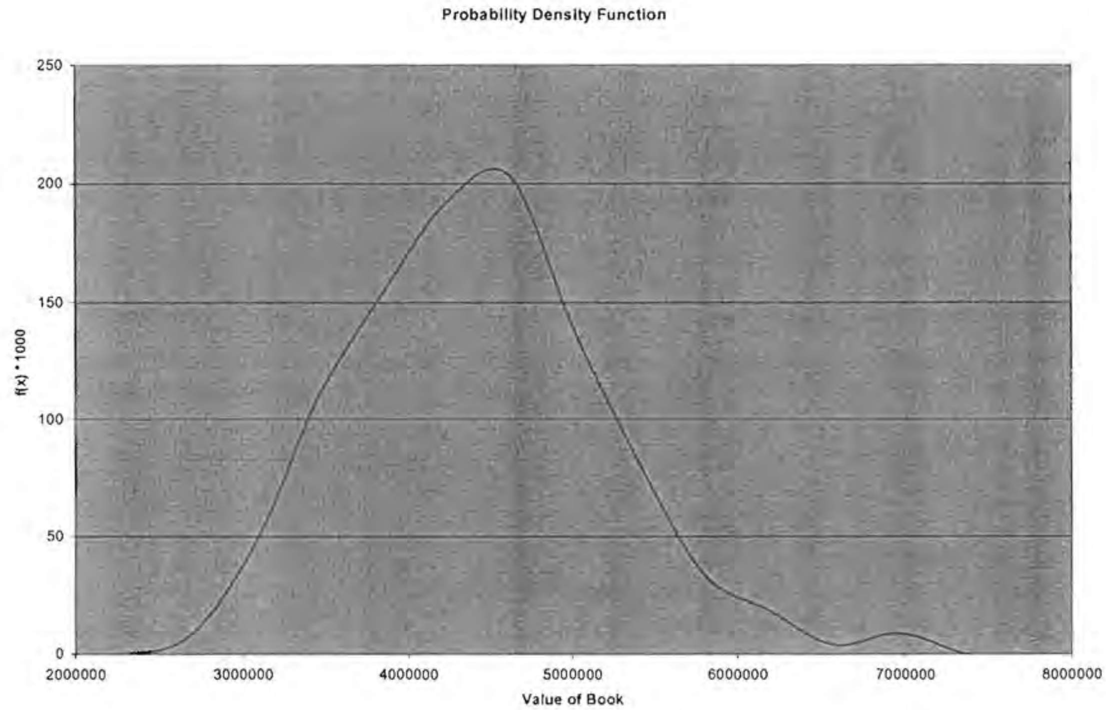

Figure 21: PDF of the value of the fund based on the model

This distributions has an average of R4.2million and a standard deviation of R700,000. It is therefore clear that the investment returns have been substantial. This would yield a phenomenal rate of return.

When comparing the return to the efficient frontier, it is clear that this investment will be a substantially better investment than any currently available in the market.

It is, however, possible to optimise the return even further by investigating different investment strategies or other risk management alternatives.

For example, assume a investment change from an equal split between cash and equities to a 80% allocation to cash and a 20% allocation to equities. The resulting year-end value of the book of business would change. This change is shown below.

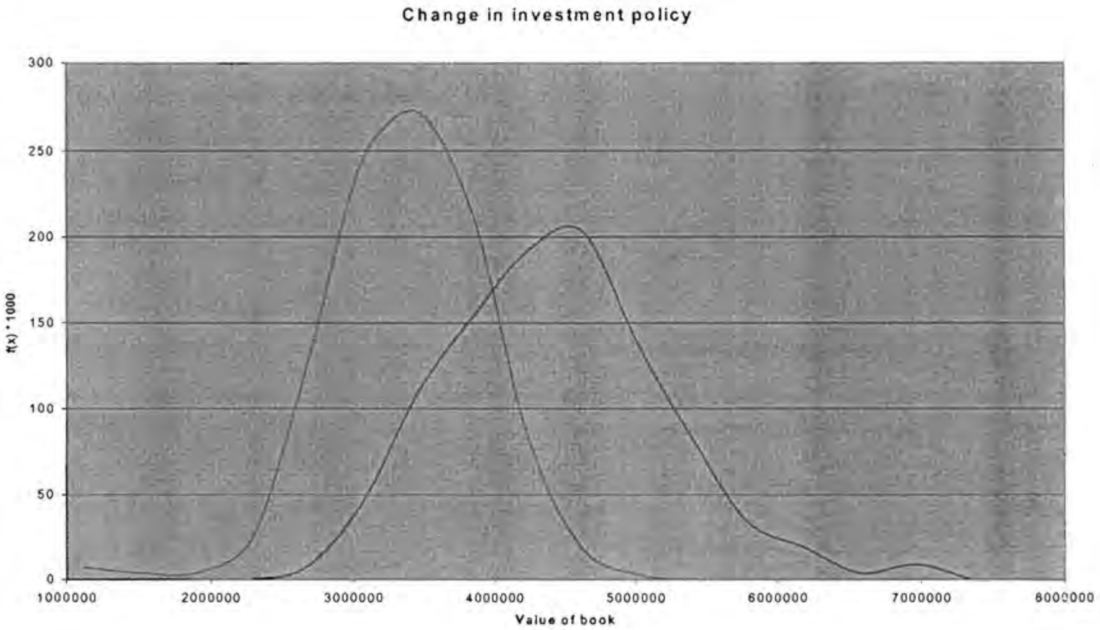

Figure 22: Result of a change in investment policy on the fund value after 12 months

The effect of changing the investment policy is a reduction in variability, as would be expected but also a reduction in the expected value of the book of business at the end of the year. The deduction in variability then needs to be compared to the cost of the variability. The model used does not illustrate the cost of variability accurately as the down side of equity investment is not severe enough. In this example an entire investment in equities would be the best approach.

12.6 Visual Basic Code

The following Visual Basic for Applications Code is meant to provide some additional explanation to the methodology discussed in "Model Office techniques in a General Insurance Environment".

```
'The following code specifies the public variables used in the code.

Option Explicit
Option Base 1


Dim got_one As Boolean
Dim z, x1, x2, r, stored As Variant
```

Function norm_gen()

```
'This function creates a value from a standard normal distribution.
'The code has been taken from the GISG 1995 Asset Models working group.

If got_one Then
   z = stored
   got_one = False
Else
   Do
      x1 = 2 * Rnd - 1
      x2 = 2 * Rnd - 1
      r = x1 ^ 2 + x2 ^ 2
   Loop While r >= 1 Or r = 0
   r = Sqr(-2 * Log(r) / r)
   stored = x1 * r
   z = x2 * r
   got_one = True
End If
norm_gen = z
End Function
```

Sub main()

'This procedure generates the experience for the model points as specified under the exposure details.

'The aim of this procedure is to model the expected worth of the model book of business after one year.

```
'Note that the objective of this dissertation is to consider model office techniques
and not to provide empirically justified asset and liability models.

'A very simplistic motor portfolio is used.
'There are six model points - 2 age categories and 3 area categories
'The model points are only exposed to the perils of theft and accident
'Details of the frequency and severity of these perils are included in the liability
model description.

'There are 6 model points and for each the following must be read:
'1. The average sum insured
'2. The number of policies for the model point
'3. The theft frequency
'4. The accident frequency
'5. The accident severity

Dim exp_details(6, 5) As Variant
Dim loop1, loop2 As Integer

For loop1 = 1 To 6
   'The average sum insured is obtained
   exp_details(loop1, 1) = Worksheets("Exposure details").Cells(loop1 + 5, 2)
Next

For loop1 = 1 To 6
   For loop2 = 2 To 5
      'The other model detail is obtained
      exp_details(loop1, loop2) = Worksheets("Exposure details"). _
      Cells(loop1 + 5, loop2 + 3)
   Next
Next

'The model assumes only one theft and/or accident claim is possible in
'a month. Furthermore the frequencies obtained are assumed to be annual
'frequencies.


'A thousand Monte Carlo simulations of a year's experience will be run

Dim sim, model_points As Integer

'For the purpose of this example the premium is assumed to be received up front
and invested as specified in the asset model points. It should be clear that it is
possible to extent this approach to allow for a different flow of premium income.
```

```
Dim funds, cash, equities As Variant

'Details of the asset model is required to project the cash flow over time.

Dim share_index(18) As Variant
'The past months' six values of the share index are obtained

For loop1 = 1 To 6
   share_index(loop1) = Worksheets("Asset model").Cells(loop1 + 21, 1)
Next

Dim cash_distribution(7, 2) As Variant
'The movement in cash returns with their associated probabilities are obtained. The first column is the cumulative probability and the second is the size of change from one month to the next.

For loop1 = 1 To 7
   cash_distribution(loop1, 1) = Worksheets("Asset model").Cells(loop1 + 6, 2)
   cash_distribution(loop1, 2) = Worksheets("Asset model").Cells(loop1 + 7, 3)
Next

Dim mean_cash, min_cash, max_cash As Variant

min_cash = Worksheets("Asset model").Cells(16, 3)
max_cash = Worksheets("Asset model").Cells(17, 3)

Dim months

Dim claim, total_claims
'These are used to generate a claim and to calculate the total balance of claims in a month

Dim prob
'This is used to generate a random variable

Dim error_term

got_one = False

For sim = 1 To 1000
'A thousand years' experience will be generated

'First the projected share index is calculated

funds = Worksheets("Exposure details").Range("Toffice")
cash = funds * Worksheets("Exposure details").Range("cash_per")
```

```
equities = funds * Worksheets("Exposure details").Range("shares_per")
mean_cash = Worksheets("Asset model").Cells(4, 4)

For loop1 = 7 To 18
    share_index(loop1) = 0.6 * share_index(loop1 - 1) + _
    0.2 * share_index(loop1 - 2) - 0.3 * share_index(loop1 - 3) + _
    0.4 * share_index(loop1 - 4) + 0.2 * share_index(loop1 - 6)

    error_term = (norm_gen() * 10 + 0.5) / 100 * share_index(loop1)

    share_index(loop1) = share_index(loop1) + error_term
Next


For months = 1 To 12
    'The mean cash return for the month must be generated
    total_claims = 0
    Randomize

    Worksheets("Workings").Cells(sim, months) = cash
    Worksheets("Workings").Cells(sim, months + 13) = equities
    Worksheets("Workings").Cells(sim, months + 26) = funds

    prob = Rnd

    loop1 = 1

    If prob < cash_distribution(7, 1) Then
    While prob > cash_distribution(loop1, 1)
        loop1 = loop1 + 1
    Wend
    Else
        loop1 = 7
    End If


    mean_cash = mean_cash + cash_distribution(loop1, 2)
    If mean_cash > max_cash Then mean_cash = max_cash
    If mean_cash < min_cash Then mean_cash = min_cash

    cash = cash * (1 + mean_cash)
    equities = equities * share_index(months + 6) / share_index(months + 5)
    funds = cash + equities
```

```
For model_points = 1 To 6
   For loop1 = 1 To Int(exp_details(model_points, 2))

      'Determine whether or not there was a theft claim

      If Rnd < exp_details(model_points, 3) / 12 Then
         claim = exp_details(model_points, 1)
         'i.e. the loss is for the entire sum insured

      total_claims = total_claims + claim
      End If

      'Determine whether or not there was an accident claim

      If Rnd < exp_details(model_points, 4) / 12 Then

      'The percentage of the sum insured loss will now be calculated
      'assuming a pareto distribution with mean as obtained from
      'the Exposure details and alfa parameter = 2.

         claim = (exp_details(model_points, 5) / _
         ((1 - Rnd) ^ (0.5))) - exp_details(model_points, 5)

         If claim > 1 Then claim = 1

      claim = claim * exp_details(model_points, 1)
      total_claims = total_claims + claim
      End If

   Next 'loop1
Next 'model_point

Worksheets("Workings").Cells(sim, months + 39) = total_claims

'A variety of investment options can now be tested i.e. where will
'the cost of claims be recouped from - cash or equities

If cash > total_claims Then
   cash = cash - total_claims
End If

If cash > 0 And total_claims > cash Then
   cash = 0
   equities = equities - (total_claims - cash)
```

```
    End If

    If cash < 0 Then
       equities = equities - total_claims
    End If

    funds = cash + equities
Next 'month
Next 'simulation of a year's experience

End Sub
```

12.7 Conclusion

This annexure provides an outline of a model similar to what might be used in practice.

The model considers the exposure of the company, the expected claims outgo as well as the possible return from investment. The model is then applied to project the expected cash flows to form the accounting entries normally found in the income statement.

The profit or loss generated on the income statement is presented as a probability density function. This function's moments can be considered for comparison to the efficient frontier. As indicated in the example, due to its simplicity, the return on capital is unrealistically high.

A change in investment policy is then considered as a business strategy. Any of the business strategies discussed could be considered.

A comparison is made between the probability density function of returns given the two different investment policies. In similar fashion a variety of strategies can be tested.

Based on the results, the shareholders can decide which set of strategies meet their needs the best.

3.5 Summary

In this chapter a synthesis of the framework required for risk and business strategy evaluation techniques was considered.

The framework for setting risk evaluation techniques is as follows:

1. Model points are identified for each class of business and asset class.
2. The expected cash flow on these model points is considered to the extent that all the items in the income statement are covered.
3. An income statement is then generated for each class of business incorporating the projection model for each element contributing to the cash flow of that class of business.
4. The different income statements can be consolidated for the group.
5. The expected change in the capital base can be evaluated over time using the models.
6. Different business strategies can be tested given the model and compared to the constraints set.
7. The optimal business strategy can be implemented.

Subsequently an analysis of the different areas of risk and uncertainty was considered. This followed on from the analysis in the introduction of this dissertation. A synthesis of the areas of risk and uncertainty revealed that the most weight in analysis should be placed on:

1. The liability risk
2. Asset risk
3. Credit risk
4. Corporate governance

In the next chapter the data required for further investigations as well as problem areas pertaining to investigations will be considered.