Chapter 6

6 CONCLUSION

In the optimization procedure developed in this study for the optimum operation of a re-configurable machining platform, the evaluation of the objective and constraint functions was done through the computer simulation of the dynamics of the device. For the actual optimization the simulation was coupled to a relatively novel and particularly robust optimization method (LFOPC). The resulting methodology was successful in yielding operational geometries for the re-configurable platform which are not only feasible in accommodating various specified trajectories, but also optimal with respect to the actuator forces required to produce the associated prescribed kinematics.

The validity of the operational designs obtained via theoretical modeling, computer simulation and numerical optimization, was also physically verified through the execution of the non-trivial paths using an actual physically re-configurable device, designed and constructed for the purposes of the current study. The successful execution of the paths demonstrates the practical potential of the device since the paths represent complicated, and free-form planar machining tasks.

In the light of the above, it is clear that the main objective of this study, namely the verification of the feasibility, both from a theoretical and practical point of view, of a novel proposed concept of a re-configurable planar Gough-Stewart machining platform, has been achieved. The remainder of this chapter takes on the form of a detailed review and assessment of the accomplishments of this study.

6.1 Computer simulation

A special purpose computer program was developed in **Chapter 2**, that models the kinematics and the associated dynamical behavior of a planar Gough-Stewart platform. Coupled to this program, the trajectory-planning algorithm developed in **Chapter 3,** allows for the realistic simulation of the continuous motion of the platform.

6.1.1 *Kinematic and kinetic modeling*

The above mentioned special purpose program, with which the kinematic and kinetic modeling of a planar Gough-Stewart platform may be done, was developed using the Newton-Euler approach as presented by Haug [66] and Nikravesh [65].

This dedicated program yields closed-form solutions for the inverse kinematics (positions, velocities and accelerations) of the seven moving bodies comprising the mechanism. With the positions, velocities and accelerations known, the special purpose program also provides closed-form solutions for the inverse dynamics of the mechanism. Here, the weight and inertia of each moving body, as well as the single external force acting on the mechanism, are taken into consideration to determine in the required input actuator forces.

The single external force referred to is the cutting force resulting from the contact between the cutting tool and the workpiece during material removal. Here the program was successfully adapted to simulate either of two possible machining scenarios, i.e. either with the workpiece or the cutting tool externally fixed.

6.1.2 *OCAS trajectory-planning algorithm*

Although the OCAS trajectory-planning algorithm was specifically developed for implementation in conjunction with the above mentioned simulation program, its application is not limited to the planar Gough-Stewart platform under consideration. In fact, since all three planar DOF are catered for, the motion of the end-effector of any other planar mechanism can similarly be described.

The results presented in Section 3.5 show that planar motion along test paths described by analytical non-linear functions may realistically be simulated, without compromising on the positional or orientational accuracy. Note that slight positional and orientational inaccuracies are inevitable since the approximating cubic splines are fitted in the time domain, while the nodal points, representing the prescribed path, are specified in the two-dimensional Cartesian domain. For all practical purposes, these discrepancies are negligible.

Apart from specifying the kinematics along test paths described by analytical non-linear functions, the OCAS-algorithm was also successfully used for trajectory-planning along a non-analytical test curve. The treble clef test curve was generated using a Non-Uniform Rational B-Spline (NURBS) with commercial Computer Aided Design (CAD) software. Here it was shown that an adequately refined set of nodal points along this curve is required to ensure sufficient accuracy for the kinematics. For the example, while the NURBS test curve was generated with 42 nodal points, the OCAS-algorithm requires

49 nodal points on this curve for a representation accurate enough to generate the associated kinematic description.

For all test curves considered, continuous position, velocity and acceleration curves as well as the continuous orientation angle, orientation angular velocity and orientation angular acceleration curves are generated using the OCAS trajectory-planning algorithm. These continuous curves representing the kinematics, are essential input data to the simulation program modeling the kinematics of the planar Gough-Stewart platform. As a result of the continuous acceleration curves, the determined input actuator forces required for manipulating the moving platform along a prescribed path, are also continuous.

Although the application of the OCAS trajectory-planning algorithm was limited here to computer *simulation* of planar motion, the increased research interest in bridging the gap between free-form design and *actual machining*, requires further investigation of its applicability to actual physical control. Since a planar Gough-Stewart machining platform is ideally suited to execute non-linear paths, it can theoretically be applied as a machining device for free-form trajectories. The real-time control of such an actual planar Gough-Stewart machining platform could be done by the OCAS-algorithm implemented as part of an open architecture controller.

It should be added that, in principle, the OCAS trajectory-planning algorithm can also be extended to general three-dimensional motion-planning.

6.2 Formulation of the constrained design optimization problem

This study shows that an optimum relative *positioning* for a prescribed trajectory, as well as the associated planar Gough-Stewart platform *geometry* may be determined by the careful *formulation* and *solution* of an appropriate constrained optimization problem.

In particular, the single criterion *cost-function* introduced here is the minimization of the overall maximum magnitude actuator force, as the prescribed trajectory is traced. It is evident from the results presented in **Chapter 5** that the use of this objective function successfully prevents the mechanism from encountering singular or near singular configurations with associated infinitely large actuator forces. The success of this approach is borne out by the moderate optimum objective function values, $F(\mathbf{X}^*)$, obtained for all the prescribed trajectories.

In conjunction with the formulated objective function, non-trivial *inequality constraint functions* allow for the incorporation of additional design requirements. Hence, apart from the limitations imposed on the values of the design variables, inequality constraint functions are successfully used to limit the actuator leg lengths to within their allowable ranges. Furthermore, the complicated issue of prohibiting mechanical interference of the physical machine while in motion, is also achieved by means of inequality constraint functions. The effective formulation of the latter inequality constraints was made possible by the availability of closed-form solutions for the instantaneous positions of the individual bodies comprising the machining platform (see Section 6.1.1).

A sound foundation having been laid for the trajectory-based constrained optimization problem considered in this study, and a logical next step, to be pursued in the near future, is to optimize the adjustable geometry and placement of a planar Gough-Stewart platform for different prescribed *workspace requirements*.

6.3 The LFOPC-algorithm

The study also served to reinforce confidence in the LFOPC optimization algorithm as a method to solve optimization problems of practical engineering importance.

The optimization results have confirmed the LFOPC-algorithm's reputation as a robust method, capable of yielding accurate and reliable results despite the presence of numerical noise, the use of finite difference approximations, and discontinuities in the objective and constraint functions. In particular the discontinuities in the slope of the objective function, that occurred in this study due to switching of the maximum actuator force between the legs, presented no problem and reliable results were obtained with reasonable computational economy. The respective prescribed trajectories investigated in this study required an average of approximately 6 minutes computational time on a Pentium IV 1.5 GHz computer with 640 MB DDRAM before the LFOPC-algorithm converged to the respective optimum operational geometries. In each case, the optimum solution corresponded to the specification of very accurate convergence tolerances. The algorithm was successful in each application, handling the five variables and up to 22 physical constraints with ease, even when starting from a severely infeasible initial design.

Since a standard initial design $\mathbf{X}^0$ was used for all the prescribed tool paths investigated, it cannot be stated with certainty that the determined *local* optimum design is indeed the *global optimum* for all cases. Although the determination of the *global* optimum may be important from a theoretical point of view, the *fact that the proposed methodology provides local optimum solutions that are indeed feasibly* *executable, and correspond to acceptably low objective function values, is considered the most important contribution of this study.*

The "analysis of convergence to the optimum" performed for the first five prescribed tasks was important, because it shed light on the functioning of the algorithm when applied to the class of design optimization problems considered in this study. Of particular importance was the finding that sufficiently accurate "engineering solutions" may be obtained by only applying phase 0 of the LFOPC algorithm. Considering the prescribed tasks investigated here, the percentage computational time that can be saved by using only phase 0, varies between $0\,\%$ and $62\,\%$ with an average of $30\,\%$. Indeed, termination of the LFOPC-algorithm after phase 0, will produce a practically feasible and near optimum design.

The robustness and effectiveness with which the LFOPC-optimization algorithm handles the formulated constrained optimization problem is best illustrated by the case of the "bigger parabolic tool path", investigated in Section 5.7. Here the prescribed tool path cannot be continuously traced because of the physical limitations of the planar test-model. As a result, the LFOPC-algorithm yields a best compromised design. The compromised design, although infeasible, is invaluable in pointing out which inequality constraints are violated, and to what extent. It was shown for the "bigger parabolic tool path" that by analyzing the relevant information provided by the compromised solution, a rational piece-wise execution strategy can be determined for the successful and optimal execution of the complete path.

Consideration of the above mentioned factors, as well as previous successful experience in the LFOPC-algorithm [64], motivated its usage in solving the currently formulated constrained optimization problems. No other optimization algorithms were tested or applied in this study.

6.4 The adjustable geometry planar Gough-Stewart platform test-model

The adjustable capability required for the proposed concept was successfully implemented in the design of the test-model. The test-model's demonstration capability is however limited to tracing different prescribed tool paths on an externally fixed workpiece. This machining option is considered more likely to be used in practice, since the alternative option of mounting the workpiece to the moving platform is limited by the size and weight of the workpiece that can realistically be mounted on the moving platform.

The prescribed trajectories were successfully traced simply by controlling the required variation of the actuator leg lengths. These lengths are found by solving the three closed-form inverse kinematic equations relating the position and orientation of the moving platform to the actuator leg lengths. The position and orientation of the moving platform follows directly from the prescribed trajectory, and hence the required actuator leg lengths may easily be determined.

To further extend the practical application of the proposed planar Gough-Stewart machining platform, its (open architecture) control software will have to be adapted to enable the direct manipulation of the three DOF of the moving platform. Such a capability would however require knowledge of the maximum possible workspace. This raises the possibility of incorporating the *ranges of configurability* as design variables in an extended optimization methodology. Hence, the foreseen extended optimization system, in which the *geometry of the workspace may be prescribed*, should eventually also include the maximum ranges of re-configurability capabilities of the mechanism as possible additional design variables.

Even though the test-model was not calibrated, visual inspection shows that the actual executed paths closely resemble the prescribed trajectories. The positional error of the moving platform is evidently very small compared to the overall size of the respective paths. This shows that a practical adjustable machine tool, corresponding to the design proposed and demonstrated in this study, may confidently be applied to the execution of machining trajectories intended for the rough material removal in the manufacturing of moulds.

In order to eventually fully exploit the inherent accuracy characteristics of the Gough-Stewart platform type machine tool, a calibration strategy remains to be developed, especially if the manipulator is frequently to be re-configured in order to accommodate differently specified trajectories.