Chapter 1

1 INTRODUCTION: OVERVIEW OF GOUGH-STEWART PLATFORMS USED AS MACHINING CENTERS

1.1 Introduction

The main objective of this study is to verify the feasibility, both from a theoretical and practical point of view, of a novel proposed concept of a re-configurable planar Gough-Stewart machining platform.

The selective literature presented in this chapter gives a brief overview of the history of Gough-Stewart platforms, and then focuses on the limited industrial application of this technology to machine tools. The potential of re-configuration is put into perspective, and the existing methods for optimizing the designs of Gough-Stewart platform machine tools are presented. In the concluding Section 1.6, the novel concept proposed in this study is motivated based on the literature survey presented in this chapter.

1.2 History of Gough-Stewart platforms

A robotic manipulator is a mechanical device for the remote handling of objects or materials. Broadly speaking, industrial robotic manipulators may be categorized as either *serial manipulators* or *parallel manipulators*.

A *serial manipulator* consists of a number of links connected one after the other in series. The most well known serial manipulator is in fact the human arm since it fulfills this requirement. Most industrial robotic manipulators in use today are serial manipulators [1, 2]. An explanation for this phenomenon is given in [2]: "As the science and technology of robotics originated with the spirit of developing mechanical systems which would *carry out tasks normally ascribed to human beings*, it is quite natural that the main thrust was towards using *open-loop serial chains* as robot manipulators. Such robot manipulators have the advantage of sweeping workspaces and dextrous maneuverability like the *human arm, ...*".

In spite of the many applications where serial manipulators are used with great success, researchers agree that these manipulators are not ideally suited to deliver high load carrying capacity, good dynamic performance or precise positioning [1, 2]. Many serial manipulators have a *cantilever structure*, which tends to bend under heavy load and therefore inhibits the manipulator's load carrying capacity [2]. To address this problem, bulky links are used for certain applications, but this has a negative influence on the ratio of load capacity to manipulator mass [1].

The intuitive alternative for greater rigidity and superior positional capability is to have the end-effector linked to the base via *several parallel-actuated chains* as illustrated by two very practical examples in [2]:

- human beings use both arms in cooperation to handle heavy loads, and
- for precise work such as writing, three fingers actuated in parallel are used.

More formally, Merlet [1] defines a *generalized parallel manipulator* as a *closed-loop* kinematic chain whose end-effector is linked to the base by several independent chains.

Dasgupta and Mruthyunjaya [2] distinguish between two classifications of robot manipulators (*serial* vs. *parallel*) and (*open-loop* vs. *closed-loop*) and explain that although open-loop manipulators are always serial and parallel ones are always with closed loop(s), it is possible to have closed-loop manipulators which are serial in nature. As an example, they mention that a robot manipulator having single degree-of-freedom (DOF) closed-loop linkages in series is essentially a serial manipulator. They further point out that some robot manipulators have both open and closed kinematic loops and /or complicated series-parallel combinations of actuators, concluding that such manipulators are called *hybrid manipulators*, since they can be *hybrid* in the sense of both classifications.

A particularly important and famous subclass [2, 3] of generalized parallel manipulators is the so-called *Gough-Stewart platforms*. For the purposes of this study, a Gough-Stewart platform is defined as a parallel manipulator consisting of two platforms: a fixed platform (the base) and a moving platform. The moving platform is connected to the base by *six prismatic joints* acting in parallel to control the 6-DOF of the moving platform. Furthermore, all the fixed base joints and the moving platform support joints, respectively, *lie in the same base and platform planes*.

The first working prototype of such a parallel manipulator is the tire test machine of Gough and Whitehall [4] shown in Figure 1.1, and which was operational in 1954-1955 [5].

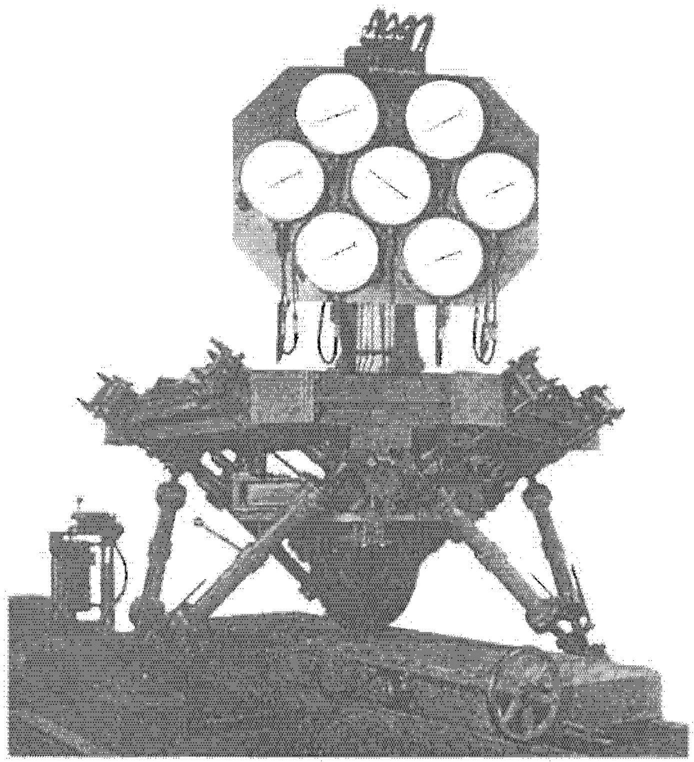

Figure 1.1: The tire test machine of Gough and Whitehall (after [5]).

It is however the classic 1965 paper of Stewart [5] that attracted research attention to the field of parallel manipulators [2]. The mechanism that Stewart proposed as a flight simulator is shown in Figure 1.2. It consists of a triangular platform supported by ball joints over three legs of adjustable lengths and adjustable angular altitudes. The three legs are connected to the ground through two-axis joints (see [2]).

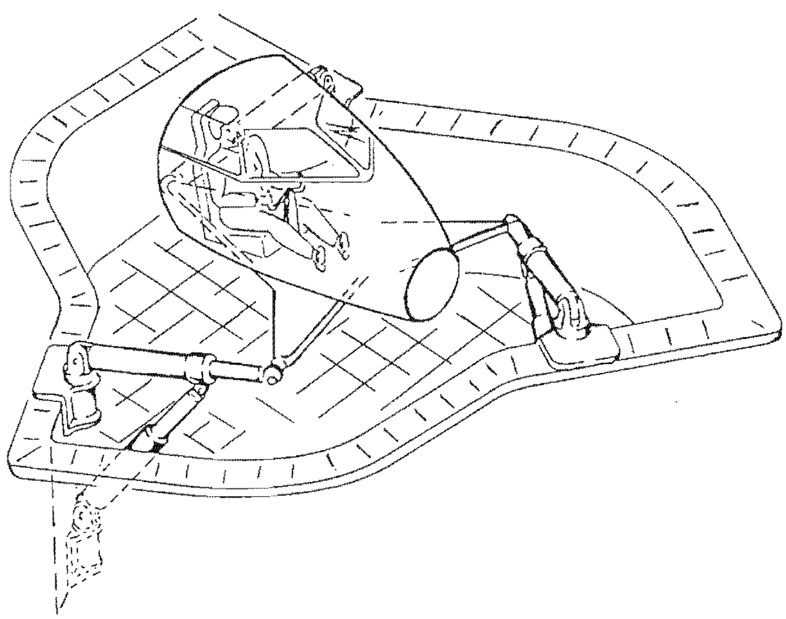

Figure 1.2: Stewart's proposed flight simulator (after [5]).

Note that this particular mechanism cannot be strictly categorized as a Gough-Stewart platform in the sense of the definition given above, because of its leg arrangement ("polar coordinate control leg system") depicted in Figure 1.3. However, in his paper Stewart [5] points out that the moving platform may be controlled in any combination by six "motors" each having a "ground abutment". As a result of this, he describes the use of a "linear coordinate control leg system", (see Figures 15 and 20 in [5]), resulting in a Gough-Stewart platform consistent with the definition used here.

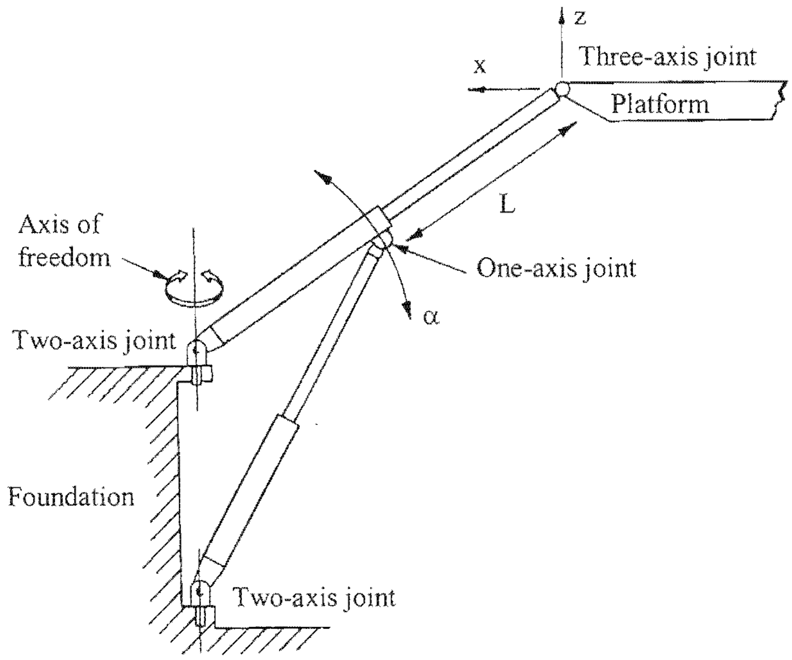

Figure 1.3: Stewart's original platform: "polar coordinate control leg system" (after [5]).

Stewart [5] comments that although his platform fitted with linear control leg systems is very similar to Gough's mechanism, it was independently conceived, and therefore the current usage of the name Gough-Stewart platform to denote such a parallel manipulator.

Researchers agree that parallel manipulators in general evolved into a popular research topic only in the 1980's [2, 6]. This happened after Hunt [7] realized that the stiffness and precise positioning capabilities of parallel manipulators are distinct advantages over serial manipulators, and as such, potential applications of parallel manipulators should be studied in more detail [2].

The systematic study of parallel manipulators in general, and Gough-Stewart platforms in particular, revealed that many theoretical problems that are easily solved for serial manipulators are much more difficult to solve for parallel manipulators, and vice versa. According to Dasgupta and Mruthyunjaya [2], the *generalized 6-DOF Gough-Stewart platform* is the parallel manipulator in which the contrast with respect to serial manipulators is manifested in the most prominent manner, making it the most celebrated manipulator in the entire class.

One specific contrast is the *limited workspace* of a 6-DOF Gough-Stewart platform compared to the *sweeping workspace* and *dextrous maneuverability* of a 6-DOF serial manipulator. The Gough-Stewart platform designs of the 1980's made use of pair wise meeting of the legs on either or both the moving platform and the fixed base. However, researchers of this era soon realized that the coalescence of spherical joints *severely restricts* the mobility of the manipulator [2].

Based on the definition of Gough-Stewart platforms, the most general 6-DOF Gough-Stewart platform would have six distinct leg support joints on *both* the moving platform and fixed base planes (see Figure 1.4).

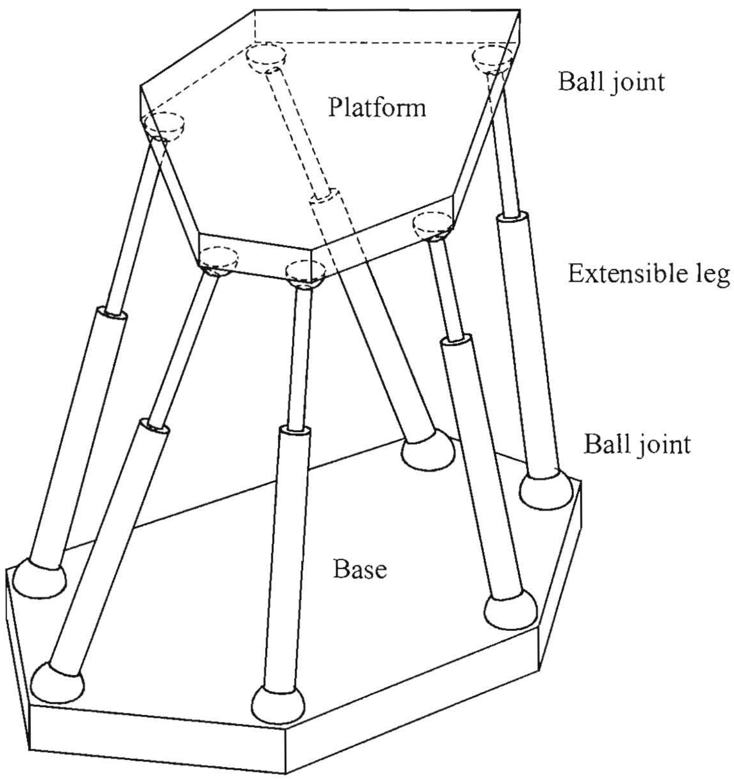

Figure 1.4: Schematic representations of a general 6-DOF Gough-Stewart platform (after [2]).

Over the past two decades, there has been an ever-increasing research interest in the field of parallel manipulators [1, 2]. In their recent review article, with an extensive list of more than 200 references, Dasgupta and Mruthyunjaya [2] present a state-of-the-art review of the literature on Gough-Stewart platforms with critical examination of solved and unsolved problems in various aspects of kinematics, dynamics and design. According to them, and with regard to Gough-Stewart platforms in particular, three of the main areas in which open problems exists are:

- dynamics and control,
- workspace and singularity analysis, and
- design.

More specifically, Dasgupta and Mruthyunjaya [2] state that there are very few works on the *systematic design* of Gough-Stewart platforms and emphasize the importance of further research in this direction for the enhancement and realization of the mechanism's potential.

With reference to parallel manipulators in general, one of the concluding remarks in the Gough-Stewart platform review [2] is that the different nature of parallel manipulators, compared to their conventional serial counterparts, calls for *unconventional strategies* and *novel concepts* for *analysis* and *design*. This is in agreement with one of the main conclusions reached by Merlet [1] in his recent comprehensive book on parallel robots and in which more than 600 literature references are cited. He states that: "Among the open research fields are *synthesis*, *design* and *optimal design*".

1.3 Gough-Stewart platforms as machining centers

1.3.1 6-DOF Gough-Stewart machining platforms

In reaction to Stewart's paper [5], researchers immediately realized the potential application of Gough-Stewart platforms as machine tools. For instance, in the communications on Stewart's article [5], Tindale presents an artistic impression of a "universal mill" based on the platform Stewart proposed as a flight simulator. In his accompanying description, Tindale explains that such a milling machine could be used to machine complicated shapes (such as propellers) with simple cutters. He adds that the economically viability of such a machine tool would require a period of expensive study and development.

In 1966 Lewis [8] also gave a very detailed description of how such a machine tool could be applied in practice. In spite of this, it was only 28 years later that "...two American machine tool companies, Giddings & Lewis and Ingersoll, *surprised* the world with the presentation of a *new type of machine tool* at the 1994 International Manufacturing Technology Show (IMTS) in Chicago". This quotation is taken from Pritschow's [9] presentation on "Research and development in the field of parallel kinematic systems in Europe" at the first "European-American Forum on Parallel Kinematic Machines: Theoretical Aspects and Industrial Requirements" that was held in Milan, Italy in 1998 [10].

The machine tools that were presented in Chicago in 1994 were the "Variax Hexacenter" by Giddings and Lewis [11] shown in Figure 1.5, and the "Octahedral Hexapod" machine tool from the Ingersoll Milling Machine Company [12] shown in Figure 1.6.

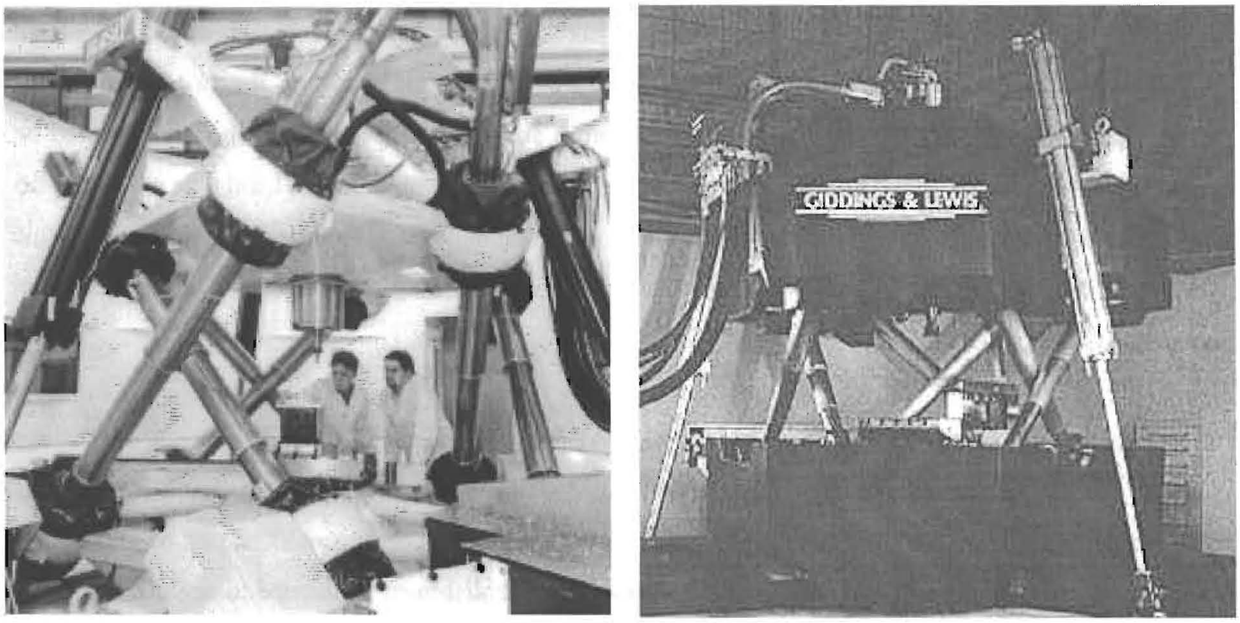

Figure 1.5: The "Variax Hexacenter".

Gindy et al. [13] explain that the "Variax" structure consists of a triangulated arrangement of three pairs of crossed legs. The prismatic legs of the "Variax" are all based on a "simple ball screw design, each powered by a separate servomotor". By inspection of the left hand photograph in Figure 1.5, the fixed base joints and the moving platform support joints all lie in the same base and moving platform planes. The additional cylinders that can be seen in the right hand photograph in Figure 1.5, are the three "counterbalance cylinders" that "support the weight of the upper platform so that the ball screws can perform the singular task of moving the machine" [13]. In spite of these additional cylinders, the "Variax Hexacenter" is categorized as a general Gough-Stewart platform.

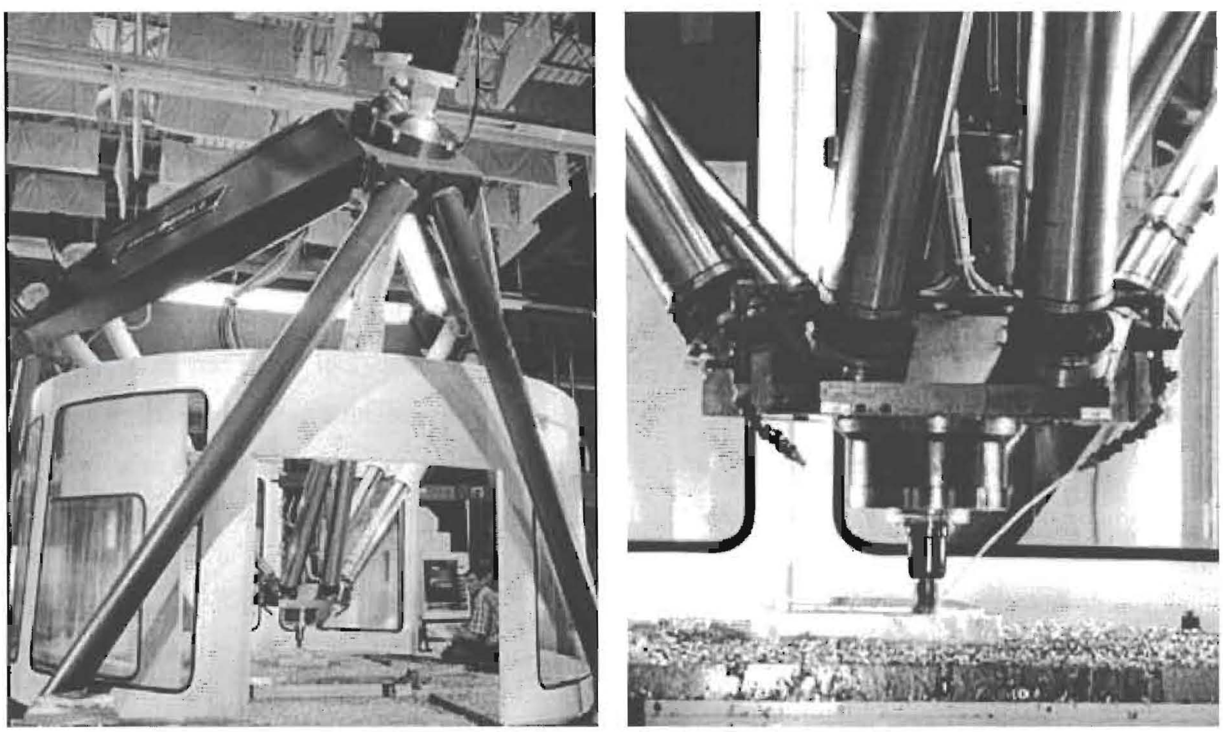

Figure 1.6: The "Octahedral Hexapod" (after [12]).

From Figure 1.6 it is evident that the pairs of base joints (each pair consisting of three joints), of the "Octahedral Hexapod" lie in two separate parallel planes. The offset between the two parallel base planes appears to be very small compared to the overall size of the machine, which is reported to be approximately 5 m tall [12]. Visual inspection of the enlarged view on the right hand side of Figure 1.6 shows that the moving platform support joints all lie in the same plane. Hence, for the purposes of this overview, the "Octahedral Hexapod" may also be considered a general Gough-Stewart platform according to the definition given in Section 1.2.

Interestingly, both these Gough-Stewart platform machine tools were installed at research institutions shortly after their introduction. The Department of Mechanical Engineering and Operations Management at The University of Nottingham purchased the "Variax Hexacenter" as part of a four-year research initiative, and in doing so, was the first research establishment in Europe to acquire a parallel manipulator type machine tool [11]. The Ingersoll Milling Machine Company installed its "Octahedral Hexapod" at the American Department of Commerce, National Institute for Standards and Technology (NIST) in May 1995 [12].

By 1998 the respective research institutions concluded that:

- "The research into these machines (parallel manipulator machine tools) is in its early stages and much work is still required in their design, optimization and control" [11], and
- "Parallel kinematic machine tools continue to look promising, and yet some very interesting and difficult challenges remain" [12].

Apart from the above less than promising conclusions, the unveiling of these Gough-Stewart platform machine tools in 1994 triggered the search for *new improved kinematic structures for machine tools* [9].

Conventional machine tools are largely constructed as serial kinematic chains connecting the workpiece to the tool. By far the largest majority of machine tools are of the Cartesian type, with two or three linear slides arranged in a mutually perpendicular fashion [14] such that the lower axis carries the one above it [15]. Ziegert et al. [14] comment that this basic type of machine tool has been in widespread development and use for nearly 200 years. During that time, this machine configuration has become well understood and now represents a very mature technology. Continuous improvements in technology and manufacturing methods have led to the high levels of performance expected of "modern" machine tools.

In spite of the success of this technology, increasingly challenging requirements with regard to productivity, economy and flexibility in manufacturing increasingly revealed the limitations of conventional machine tools:

- the machine structure is subject to bending loads, causing deformations,
- the structure of the machine requires large masses to be moved, and
- there is an accumulation of errors due to the in series arrangement of the axes [15].

Consequently, after the "surprise" unveiling in 1994, many joint research efforts and consortiums were formed by industry and universities worldwide, which were aimed specifically at applying parallel manipulators as machine tools [1, 9, 11, 12, 16]. In order to further stimulate the exchange of ideas and findings in this regard, a biannual international conference is organized for this research community. The first gathering was at the 1998 "European-American Forum on Parallel Kinematic Machines" mentioned earlier, and the second was the "Year 2000 Parallel Kinematic Machines International Conference", held in Ann Arbor, Michigan USA [17].

With reference to 6-DOF Gough-Stewart platforms, the conclusive outcome of this intensified research effort is that there are limitations prohibiting their application as production machine tools [16]:

- the unfavorable ratio of manipulator size to manipulator workspace,
- limited dexterity and tilting angles ($15^\circ - 30^\circ$),
- inherent danger of strut collision, and
- singularities inside the workspace.

It is therefore no surprise that one of the more successful parallel manipulator type machine tools used by industry is *not* a Gough-Stewart platform. Instead it is a 3-DOF parallel manipulator with fixed leg lengths, and actuated base joints. The patented "Z^3-head", developed by DS Technologie Gmbh (DST), is shown in Figure 1.7. It has two rotational DOF with *tilting angles* of $\pm 40^\circ$ *within* a 370 mm stroke length of the translational DOF. The *maximum stroke length* of the translational DOF is 670 mm [18].

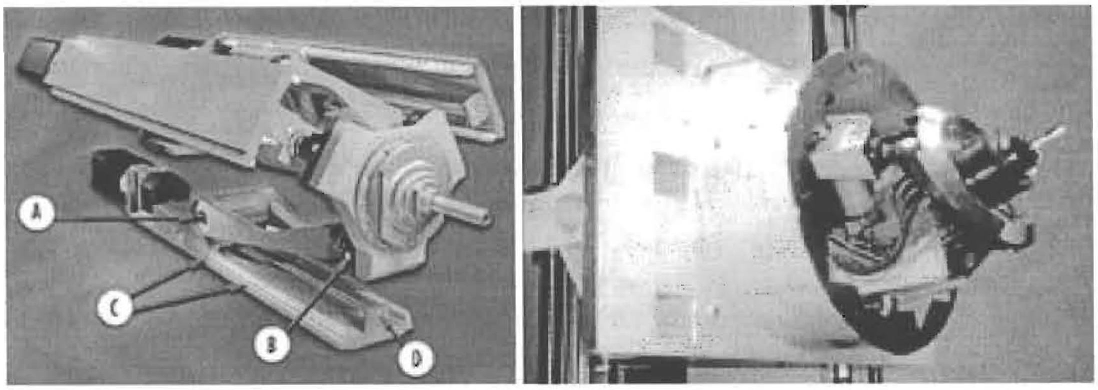

Figure 1.7: The "Z^3-head" (after [18]).

The "Z^3-head" accommodates a motorized spindle that holds the cutting tool. It forms part of a five-axis *hybrid* machine tool [19], with the parallel manipulator head mounted on a two-axis Cartesian base.

This machine tool is marketed in the aerospace industry through the alliance formed by Cincinnati Machine and DS Technologie [18].

1.3.2 Planar Gough-Stewart machining platforms

1.3.2.1 The "Smartcuts" planar Gough-Stewart platform

Dasgupta and Mruthyunjaya [2] also review the so-called "lower dimensional parallel manipulating structures" that are similar to the 6-DOF Gough-Stewart platform, in that they are also equipped with linear actuators. One such sub-class is the "planar 3-DOF parallel manipulator" schematically shown in Figure 1.8.

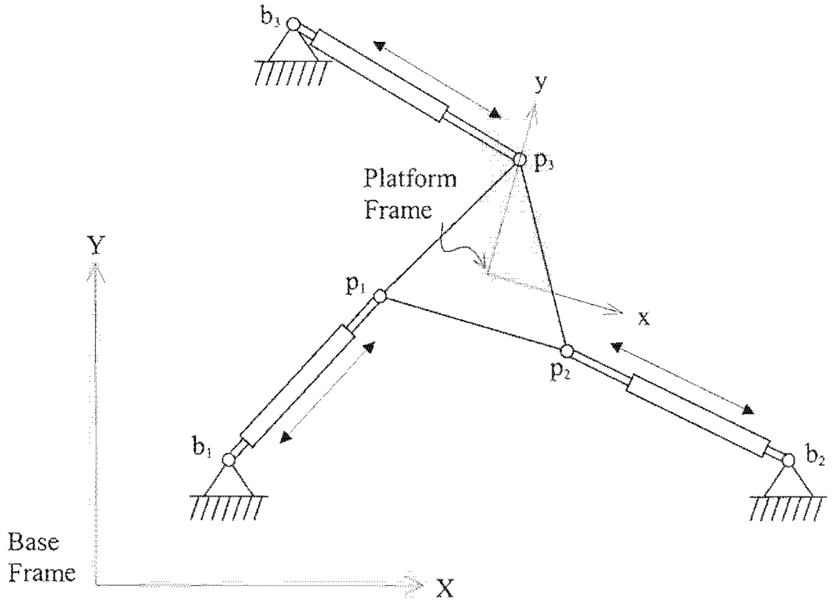

Figure 1.8: Planar 3-DOF parallel manipulator (after [2]).

The research relevance of this mechanism is evident from the respective overviews given in [1] and [2]. More specifically, the book by Duffy [20] presents the kinematic analysis of several planar parallel mechanisms. Many other authors have studied planar parallel mechanisms in a context of robotics. For instants, the inverse and direct kinematic problems have been solved [21, 22, 23, 24, 25], dynamic models have been developed [26], the singularities have been studied [27] and the kinematic design has been addressed [28, 29, 30].

For the purposes of this study a *planar 3-DOF Gough-Stewart platform* is considered a subclass of the (general) planar 3-DOF parallel manipulator shown in Figure 1.8. More specifically (and in correspondence with the Gough-Stewart platform definition given in Section 1.2), the revolute joints connecting the three linear actuator legs of the *planar 3-DOF Gough-Stewart platform* to the moving platform and base, respectively lie in the in the same base and moving platform *lines*.

Of particular importance with reference to this study, is that Satya et al. [31] have proposed a planar 3-DOF Gough-Stewart platform as an alternative to the "6-DOF Gough-Stewart platform" type machine tool in 1995. They also constructed a prototype 3-DOF platform as part of the "Smartcuts" research project of the University of Illinois at Urbana-Champaign [32]. A schematic representation of the "Smartcuts" planar manipulator, showing its three DOF, is given in Figure 1.9.

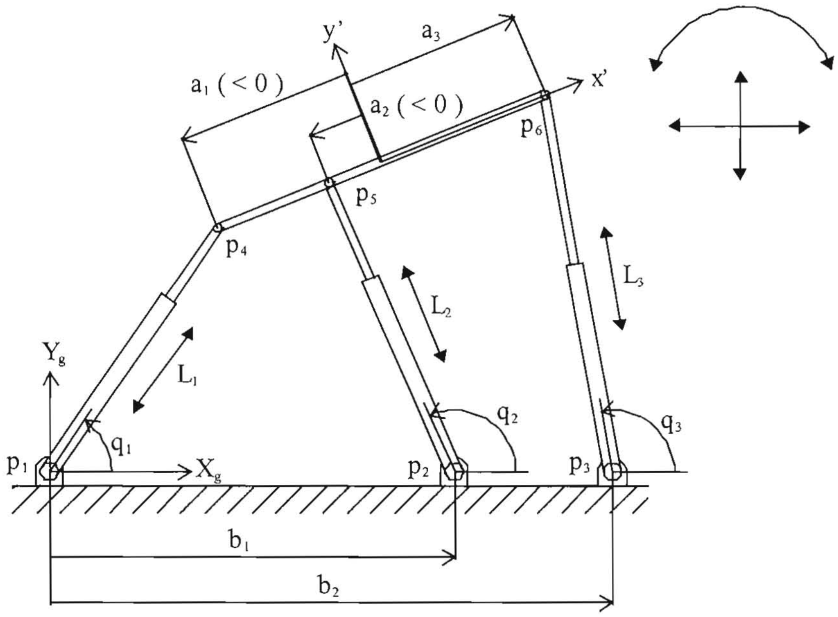

Figure 1.9: Schematic of the "Smartcuts" planar Gough-Stewart platform (after [31]).

Satya et al. [31] acknowledge that in order for a machine tool to perform any task, it should have five DOF (three orthogonal translations with rotations about two of these axes), and hence propose a hybrid serial-parallel scheme with two of the "Smartcuts" planar platforms (see Figure 1.10). The simultaneous control of both mechanisms shown in Figure 1.10 is required for five axis machining. In particular, if the spindle carrying the cutting tool is attached to one of the planar platforms, and the workpiece to the other, the two rotational DOF of the hybrid machine are about two orthogonal translational axes, both of which are also orthogonal to the third translational axis.

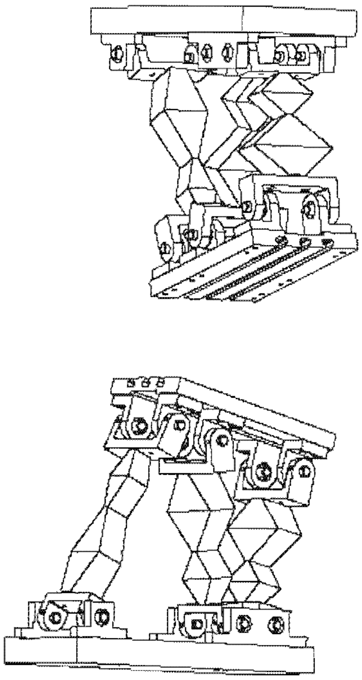

Figure 1.10: Schematic of two "Smartcuts" mechanisms in a series-parallel hybrid 5-axis machine tool (after [31, 33]).

Besides the fact that a simplified Gough-Stewart platform is easier to analyze than the 6-DOF version, another noteworthy advantage of the suggested series-parallel hybrid approach over purely parallel schemes [33] is that the use of 3-DOF systems obviously results in *simplified mechanical construction.*

In spite of its promising features, the use of the "Smartcuts" planar Gough-Stewart platform is inhibited by two potential drawbacks: *small workspace* and *insufficient lateral stiffness* as discussed below.

- In another related paper, El-Khasawneh and Ferreira [34] determine the reachable workspace of a *modified* "Smartcuts" planar Gough-Stewart platform. This specific mechanism has the three "moving platform leg joints" in line, but not the "base leg joints". Although it is not explicitly stated, from the results presented it is clear that the associated *workspace* of this planar parallel manipulator is very small compared to the physical size of the manipulator. In correspondence with the disadvantages of the 6-DOF Gough-Stewart platform (Section 1.3.1), an unfavorable ratio of manipulator size to manipulator workspace is considered a severe limitation for the practical application of the mechanism as a machine tool.
- Although a planar Gough-Stewart platform has as an inherent characteristic a good stiffness in the plane of motion, the *lateral stiffness* (perpendicular to the plane of motion of the moving platform) is in general dependent on the bending stiffness of the "beam-like" actuator legs. The problem is that a cantilever beam in general shows large deflections under a bending moment load [35], and hence insufficient lateral stiffness of the moving platform can be expected.

1.3.2.2 The "Dyna-M" and "Honda HVS-5000" machine tools

Both the above drawbacks associated with the "Smartcuts" platform have been addressed in the similarly designed "Dyna-M" [36] and "Honda HVS-5000" [37] planar parallel manipulator type machine tools. Both machines are *hybrid* 3-axis machine tools with the three orthogonal translations of a Cartesian coordinate system as the three axes of motion. More specifically, the "ram" [36] or "head" [37] of the machine tool is positioned in the xy-plane by a planar 2-DOF parallel manipulator consisting of two linear actuators, the respective *stationary ends* of which are connected to the base via two revolute joints, and the respective *extendable ends* of which are connected to each other and to the ram / head via fork-type revolute joint. The ram / head is a *serial mechanism* which moves in the z-direction, and carries the tool.

The xyz-translational workspace of the "Dyna-M" is reported to be $630\text{ mm} \times 630\text{ mm} \times 500\text{ mm}$ with the projected area of the machine $3\text{ m} \times 6\text{ m}$ [36]. A three-dimensional schematic representation of the "Dyna-M", and its associated workspace is shown on the left-hand side of Figure 1.11. The right-hand side of Figure 1.11 is a photograph of the "Dyna-M" prototype machine tool.

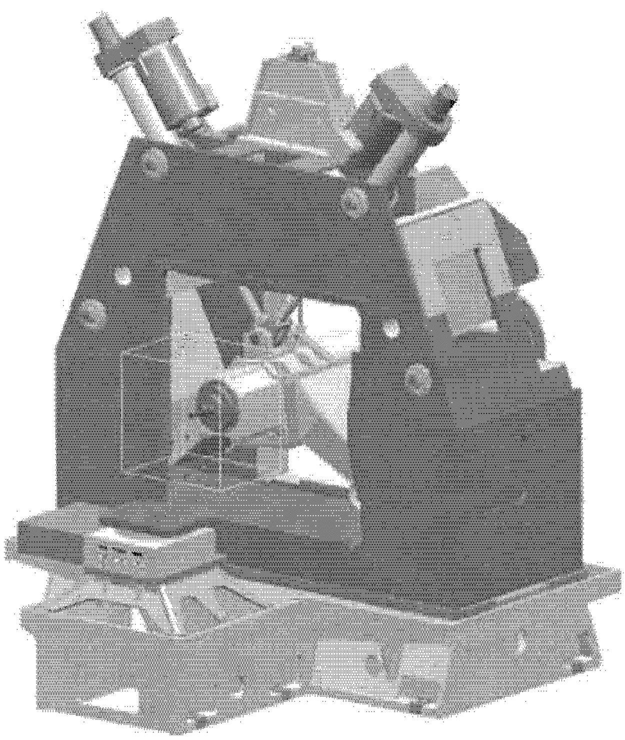
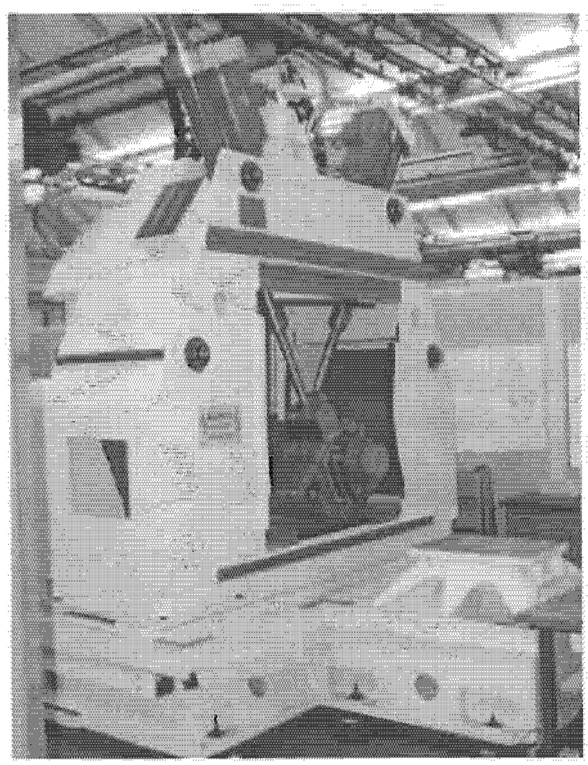

Figure 1.11: The "Dyna-M" (after [36]).

The "Dyna-M" shows a slight improvement in terms of the workspace to "projected machine area" relation over the 6-DOF Gough-Stewart platform type machine tools. For example, according to Tönshoff and Grendel [16], the 6-DOF "Ingersoll HOH-600" has a workspace cube in the xyz-Cartesian space of $600\text{ mm} \times 600\text{ mm} \times 800\text{ mm}$, with a projected machine area of $6.7\text{ m} \times 5.6\text{ m}$.

Furthermore, once an angular tilt is specified for the moving platform, the associated workspace volume of the spatial Gough-Stewart platform decreases significantly. Researchers at the University of Nottingham specify the angular capabilities of the "Variax Hexacenter" in terms of *cone angles*, where the perpendicular axis to the moving platform is tilted from the vertical by a fixed angle but in any direction [38]. The "Variax Hexacenter" reportedly has a 5° angular capability within a 630 mm work cube [39]. However, the maximum angular tilt *changes* depending on the position within the workspace, and hence Whittingham et al. [39] express the need for additional analysis tools to define exactly what these angular limits of the machine are throughout the workspace. In this regard, Du Plessis and Snyman [40] have recently proposed a new numerical method for determining workspaces and their results, for both planar and spatial Gough-Stewart platforms, confirm the earlier statement about the decreasing workspace size associated with larger angular orientation specifications.

Evidently, the "Dyna-M" and the "Honda HVS-5000" machine tools are not affected by this orientation limitation, since they only have three translational DOF. Furthermore, Moriwaki [37] reports that the "Honda HVS-5000" has a more compact structure compared to the "Dyna-M", which would further improve the workspace to "projected machine area" ratio.

In terms of the lateral stiffness of these machine tools, the ram / head is connected to the frame with two additional chain links. Each of these additional chain links consist of two pivoting bodies connected to both the base and ram / head via separate revolute joints. Figure 1.12 (a) shows an assembled view of the "Dyna-M" machine tool. Two isolated views of the "Dyna-M" ram as positioned by the two linear actuators, and supported laterally by the stabilizing chain links are shown for illustrative purposes in Figure 1.12 (b) and (c). The middle cylinder attached to the ram and shown in Figure 1.12 (b) and (c), is presumably a measuring device.

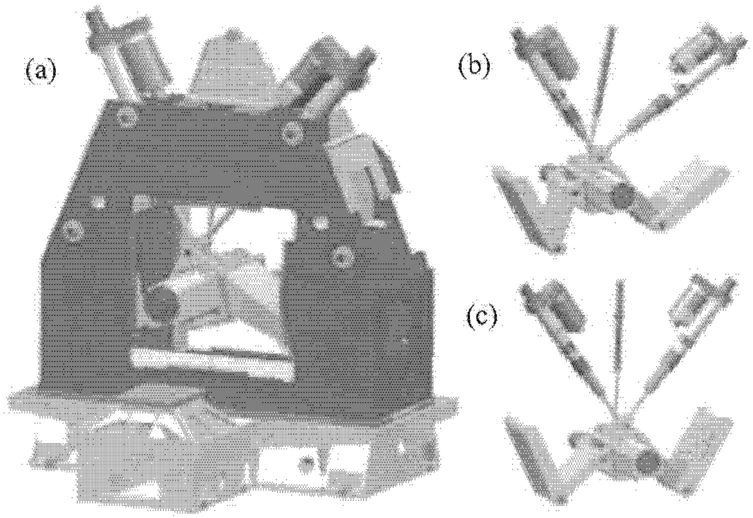

Figure 1.12: (a) The "Dyna-M". (b) and (c) The "Dyna-M" ram as positioned by the two linear actuators and supported laterally by two stabilizing chain links (after [36]).

The minimum lateral stiffness of the "Dyna-M" prototype is $60\ \mathrm{N/\mu m}$, which compares well with the minimum stiffness of $39.4\ \mathrm{N/\mu m}$ ($2.25 \times 10^5\ \mathrm{lb/in}$) reported by El-Khasawneh and Ferreira [35] for the specific spatial Gough-Stewart platform they studied. Note that for a single leg of the latter mechanism, the stiffness in average is about $175\ \mathrm{N/\mu m}$ ($1.0 \times 10^6\ \mathrm{lb/in}$). The minimum stiffness in the x- and y-directions of the "Dyna-M" is $30\ \mathrm{N/\mu m}$, which is of course dependent on the linear stiffness of the two linear actuators.

It is of interest that the "Dyna-M" is a *prototype* 3-axis machine tool intended for application in the automotive industry. It is reported to have a maximum velocity of $90\ \mathrm{m/min}$ and a maximum acceleration of $15\ \mathrm{m/s^2}$ in all three axes [36].

The "Honda HVS-5000" is presumably also a prototype, since it is *intended* to replace existing transfer machines in the automotive industry for the machining of automobile cylinder heads and cylinder blocks. The stabilizing links are made of aluminum, and it is also equipped with an automatic tool changer that requires only $0.5\ \mathrm{s}$ for a tool change. The positional accuracy of the HVS-5000 is $0.01\ \mathrm{mm}$ ($10\ \mathrm{\mu m}$), and the "accuracy of drilling is $\pm 0.05\ \mathrm{mm}$ ($\pm 50\ \mathrm{\mu m}$)" [37].

1.4 The re-configurable concept

Looking at machine tools from a different angle, the recent trend in manufacturing systems is *re-configurability*. Gopalakrishnan et al. [19] explain that "*re-configurable machine tools* assembled from machine modules such as spindles, slides and worktables (see also [41]) are designed to be *easily re-configured to accommodate new machining requirements*". They elaborate, stating that these systems are required in order to quickly respond to changes in market demand and the resulting product design changes. Furthermore, the essential characteristics of re-configurable machine tools are listed in [19] as modularity, flexibility, convertibility and cost effectiveness. Finally, Gopalakrishnan et al. state that "the goal of *re-configurable machining systems*, composed of *re-configurable machine tools* and other types of machines, is to provide *exactly* the capacity and functionality, *exactly* when needed". According to Koren [42] a recent report of the US National Research Council (Visionary 2020) mentioned that *adaptive re-configurable manufacturing* is considered as *first priority* for future (manufacturing) systems. As a result 2001 saw the "CIRP 1st International Conference on Agile, Re-configurable Manufacturing" [42] as a communication forum for this important issue.

Researchers have come to realize that Gough-Stewart platforms in particular, have unique features allowing for re-configurability, thus making these manipulators applicable for consideration as re-configurable machine tools. The two main approaches that have been proposed for the re-configuration of Gough-Stewart platforms are *modular design* and a *variable geometry*.

1.4.1 Modular Gough-Stewart platforms

The prototype re-configurable 6-DOF Gough-Stewart platform, of the Department of Mechanical Engineering at the New Jersey Institute of Technology [43, 44], is discussed here as an illustrative example. Its re-configuration is achieved through *modular design* such that any of the leg modules can be replaced with a different range of motion, and can be placed on the mobile platform and the base at any desired location and orientation.

A previous study by Ji [45] has shown that the *moving range* of the legs and the *placement* of the legs have a great effect on the *shape* and *size* of the *workspace*. Ji and Song [44] further comment that since the workspace of a Gough-Stewart platform is difficult to visualize, and usually *limited*, such a mechanism should be *re-configurable to allow for the specification of different task requirements*, especially the workspace requirement.

Given a set of legs, the Ji [43, 44] approach to re-configuration involves the determination of the position and orientation of joints on the mobile platform and the base for a specified task. When legs of different ranges are available, the re-configuration must also consider what combination of legs to use.

Ji and Leu [43] explain that they use a discretization procedure to determine the so-called foot-placement space for a given desired workspace of the mobile platform. The foot-placement space is a set of all base locations where the foot of the leg can be placed to ensure the required workspace. Here, having *chosen* the position of the leg joint on the moving platform, the motion limits of the upper spherical joint and the minimum and maximum leg length limits are also taken into consideration. If the resultant foot-placement space is the null space, then the desired workspace cannot be obtained no matter where the foot is placed. One then has to *choose* a different location for the leg joint on the moving platform, or use another leg of different range. The same process has to be applied to all six legs to obtain six foot placement spaces, one for each leg.

In essence, this is a trial-and-error methodology to re-configuration. Ji and Song [44] conclude by saying that the idea is to develop an inventory of standardized leg modules and customized mobile and base platforms, so that their modular Gough-Stewart platform can be custom-configured, portable and easy to repair.

1.4.2 Variable geometry Gough-Stewart platforms

In their 1993 layman evaluation of (Gough)-Stewart platforms for manufacturing, Fitzgerald and Lewis [46] recognized that a *variable geometry base* would *improve the practical working volume* of the manipulator "so *singularities can be moved relative to the workpiece*". They explain that one of the problems with a Gough-Stewart platform is that it can collapse and may not be able to recover under its own power when it loses control near singularities. Correspondingly, a "straight forward" solution is proposed: "*Stay away from singularities*, which effectively are regions of non-performance in the robot's space. *Predict where they will be and plan paths around them*; many applications do not require operating near singularities, and *more flexible applications will depend on routine generation of large path sequences that will require new, more intelligent path-planning methods.*"

At the time, Fitzgerald was the Program Coordinator, and Lewis the Technical Leader of the Advanced Controls and Sensors Group, Automation and Robotics Research Institute (ARRI). This research institute, which is affiliated with the University of Texas at Arlington, built a prototype Gough-Stewart platform with a variable base geometry. Figure 1.13 shows a photograph of this manipulator, from which it is clear that each of the three pairs of base joints are individually adjustable.

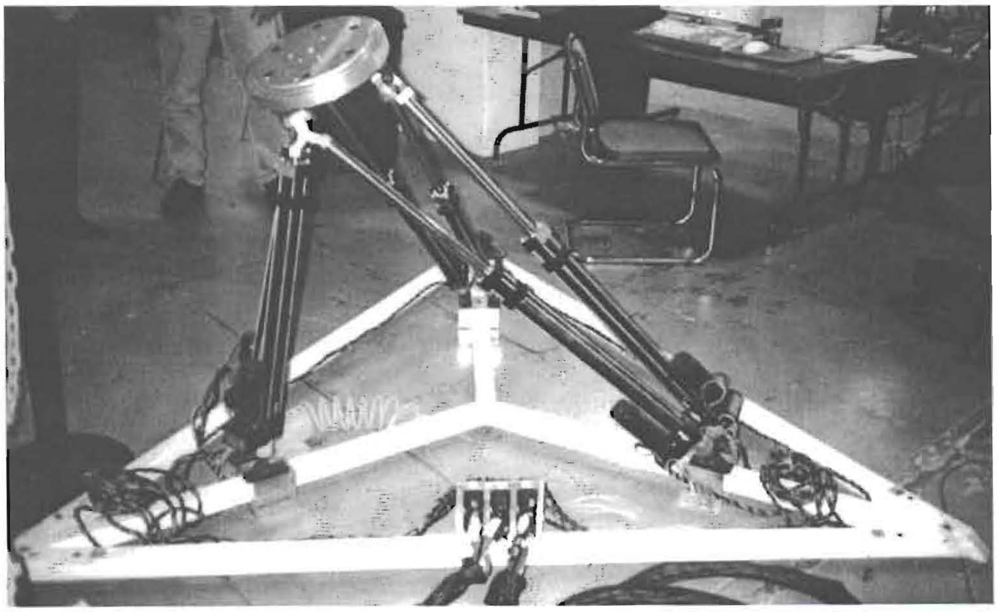

Figure 1.13: Photograph of the ARRI-(variable geometry base) Gough-Stewart platform.

The specially designed moving platform joints (gimbals) of the ARRI-Gough-Stewart platform are however not adjustable (see Figure 1.14).

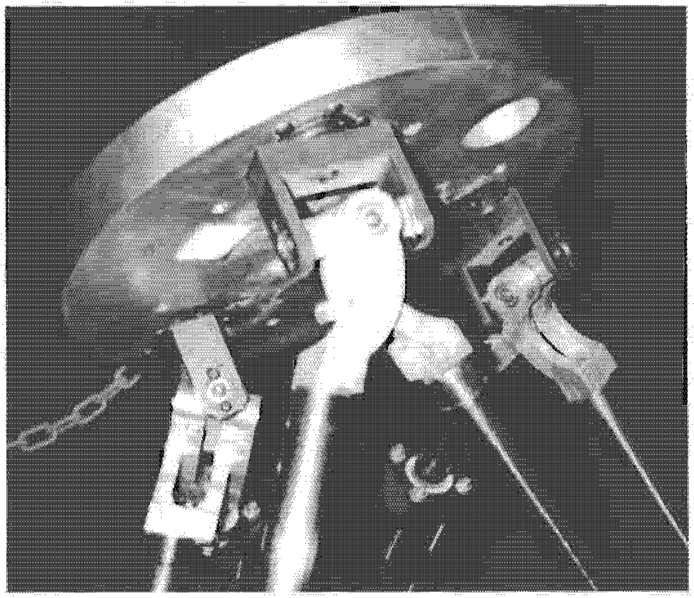

Figure 1.14: Photograph of the moving platform gimbals of the ARRI- Gough-Stewart platform.

Machine tools are continually being reprogrammed to move along different prescribed paths, implying that the corresponding workspace requirements of the machine tool is also continually changing. Therefore, a re-configurable Gough-Stewart platform type machine tool, where re-configuration is achieved via a variable geometry, may potentially overcome the workspace limitations that has been hindering its application as a machine tool. Experience with such adjustable tools will also contribute to the important research field of re-configurable manufacturing systems.

The potential improvement that the variable geometry capability could have on the practical working volume of Gough-Stewart platform type machine tools can, however, only be realized if its adjustable capability is combined with an *efficient methodology* for determining the *optimum geometry* for the machining task at hand. This observation follows simply from the fact that, if each leg joint can be adjusted separately and in a continuous manner, then infinitely many possible combinations exist. Thus if such a capability could be found it would be far superior to the trail-and-error selection method in situations where the task requirement varies.

1.5 The optimum design of Gough-Stewart platforms

Although to date re-configurable geometry Gough-Stewart platforms have not received much attention in literature, the related and extremely important subject of the *optimum design* of (fixed configuration) parallel manipulators has indeed been a very important issue.

Merlet [1] explains that the (optimal) *design* of a general parallel manipulator essentially is "*the determination of the dimensions of the manipulator so that it complies as closely as possible with the performance needed for the task at hand*".

Speaking in very broad terms, parallel manipulator research is conducted via two fundamentally different (but complementary) approaches, namely the *analytical approach*, and the *numerical approach*.

1.5.1 The analytical approach

In his review paper of the optimization of multi-DOF mechanisms, Chedmail [47], distinguishes between the *analysis phase* ("given a set of design variables of a mechanism, which is its mechanical behavior?"), and the *synthesis phase* of a mechanism ("given an expected mechanical behavior of a mechanism, define its design variables").

By far the most popular choice when it comes to the *analysis* of parallel manipulators, the *analytical approach* would be to find an analytical relationship between any given set of design parameters, and the mechanical behavior of the manipulator. The two very recent reviews by Merlet [1] and Dasgupta and Mruthyunjaya [2] respectively, give comprehensive and detailed accounts of the work done to date in this regard.

Some of the very successful analytical results that were obtained are based on the "monumental theory of screws of Ball" [2] that was developed over a *century* ago. Dasgupta and Mruthyunjaya [2], explain that Ball's theory of screws provides an elegant framework for the analytical representation and *analysis* of mechanical systems.

Merlet [1] should be credited, not only for presenting an extensive overview of the research done on parallel manipulators, but also for contributing towards the *analytical approach* for the analysis of these mechanisms.

The inverse of the analysis process is the synthesis (design) process, and indeed, if an analytical relationship – between any required *performance criterion / criteria* and the chosen *design parameter(s)* – exists, such that *analytical closed-form mathematical equations* may be formulated, then the *optimum* values of the design parameter(s) may be *determined exactly* and *very efficiently*, using *algebraic* methods. Unfortunately, it is a very challenging task in general to formulate such closed-form mathematical solutions. This may explain why the leading authority, Merlet [1], states that in spite of all the research that has been published in this field, there is still no answer to the question of determining the best parallel manipulator for realizing a given task.

1.5.2 The numerical approach

The use of (numerical) optimization techniques in mechanical engineering is becoming increasingly more popular, due to the sustained increase of computer power [47].

As far as the *optimum design* of *structures* is concerned, *numerical techniques* are currently in widespread use [48]. Chedmail [47] mentions, for example, that it is now possible to (numerically) optimize subsets of complex products such as the wings of an airplane, and hence conclude that (numerical) optimization is one of the possible approaches to mechanism synthesis.

Following the explanation given in [1], the typical layout of the *numerical approach* to the design and optimum design of parallel manipulators involves

- the *selection* of a specific *mechanical architecture* for the parallel manipulator (Gough-Stewart platform or any other type of *general* parallel manipulator), and
- the *computer simulation* of the specific architecture for *determining* the *physical and geometrical characteristics* (values of the design variables or parameters) of the mechanism that are best suited for the prescribed task. Two general techniques are listed for *utilizing* the *simulation output*:
  - the simulation output may be used directly by the user to *select* values of the design parameters via *trial-and-error*, or
  - the simulation output can be used to construct a *cost-function*, and one of several available *numerical optimization techniques* may be applied to *determine* the *optimum* values of the design variables through the *minimization of the cost-function.*

1.5.2.1 Genetic Algorithms

Due to the inherent characteristics (such as non-linearity, discontinuity and the presence of local minima) of typical cost-functions formulated for parallel manipulators, it is no surprise that numerical optimization using *genetic algorithms* is preferred by most researchers attempting to optimize a parallel manipulator design through the minimizing of a cost-function. Genetic algorithms are easy to program, and are able to take into account any type of variable (discrete or continuous) [47].

Typical of work done using a genetic algorithm in the optimal design of parallel manipulator machine tools is that of Zhang and Gosselin [49]. They optimized the "Tricept" machine tool with respect to its global stiffness, using a genetic algorithm, and explain that genetic algorithms are *powerful* and *broadly applicable stochastic* search and optimization techniques based on the evolutionary principle of natural chromosomes. The evolution of chromosomes due to the operation of crossover, mutation and natural selection, is based on Darwin's survival-of-the-fittest principles, and is artificially simulated to constitute a *robust search* and optimization procedure.

The "Tricept" machine tool is a special type of parallel manipulator, although it has similarities to the Gough-Stewart platform, with prismatic actuators connecting the moving platform to the base. It is also equipped with a *passive constraining leg* between the moving platform and the base. The specific degrees of freedom of this type of parallel manipulator are determined by the specific degrees of freedom of the passive leg. In particular, the "Tricept" machine tool has three DOF (one translational and two rotational). The respective leg joints on the moving platform and the base coincide with the vertices of two respective equilateral triangles, one on the moving platform and one on the base. The radii of the respective circles circumscribing the respective equilateral triangles are referred to as the *radius of the base platform* (R_b), and the *radius of the moving platform* (R_p).

In order to obtain the maximum global stiffness of the "Tricept" machine tool, three *architectural parameters* are considered as optimization variables. They are R_b, R_p and the height of the moving platform relative to the base (z). Zhang and Gosselin [49] comment that using these three parameters, it is *very difficult* obtain the *analytical expressions* for each of the six stiffness elements of the moving platform (also see Section 1.5.1). The six stiffness elements are related to the 6-DOF of a rigid body in three-dimensional space. They further comment that *traditional* numerical optimization methods can be expected to experience convergence problems when faced with these types of cost-functions they consider.

In searching for an optimal design, the feasible ranges of the three architectural parameters of the "Tricept" machine tool may be expressed as inequality constraints: $200 \leq R_p \leq 300$, $400 \leq R_b \leq 600$ and $900 \leq z \leq 1500$, where all extreme values are given in mm.

Fixing the two rotational DOF of the moving platform, Zhang and Gosselin [49] maximize the sum of the six stiffness elements, starting with an *initial design* given by: $R_p = 225\,\mathrm{mm}$, $R_b = 500\,\mathrm{mm}$ and $z = 1300\,\mathrm{mm}$. The sum of the six stiffness elements for the initial design is 0.0078189. The *optimal design* found after 100 generations of the genetic algorithm, is given by $R_p = 300\,\mathrm{mm}$ (maximum allowable), $R_b = 600\,\mathrm{mm}$ (maximum allowable) and $z = 900\,\mathrm{mm}$ (minimum allowable). The sum of the six stiffness elements of the *optimal design* is 0.0153369.

Zhang and Gosselin [49] thus *improved* the sum of the stiffness elements by a factor of 1.96 using a genetic algorithm. In practical terms their approach may be used not only for the optimal design of a machine tool, but also for the optimum placement of the workpiece relative to the base. This placement is an important issue as explained by Chrisp and Gindy [38], who studied the component (workpiece) positioning for the "Variax Hexacenter", mentioned in Section 1.3. Another recent paper on this subject is the one by Wang et al. [50].

Although the solution of the problem posed by Zhang and Gosselin [49] is an important achievement, the particular design optimization problem they considered is incomplete. In their problem the stiffness of the moving platform is optimized for a single position inside the workspace of the manipulator, and with the moving platform fixed at a specific orientation. Machine tools are, however, normally required to have good stiffness characteristics over the complete workspace.

Kirchner and Neugebauer [51] emphasize that a parallel manipulator machine tool cannot be optimized by considering a single performance criterion. Also using a genetic algorithm, they consider multiple design criteria, such as the "velocity relationship" between the moving platform and the actuator legs, the influence of actuator leg errors on the accuracy of the moving platform, actuator forces, stiffness as well as a singularity-free workspace. These specified design criteria are summarized into three discrete objectives (cost-functions) related to the Jacobian matrix of the manipulator:

- *maximize* the minimum singular value of the Jacobian matrix over the workspace,
- *minimize* the maximum singular value of the Jacobian matrix over the workspace, and
- *maximize* the inverse condition number over the workspace.

The size of the workspace, and the rotational capability of the moving platform inside the workspace are additional design criteria, i.e. the rectangular shaped workspace should be as large as possible, with a maximum rotational capability of the moving platform inside the workspace [51].

Kirchner and Neugebauer [51] use 13 architectural design parameters in the simulation of their six-DOF Gough-Stewart platform machine tool.

As an alternative to solving the optimization problem by formulating a weighted multi-criteria objective function, the so-called "Pareto optimal-region" is determined. The number of criteria in the multi-criteria objective function determines the dimension of the Pareto optimal region. If only two criteria are optimized for, the associated Pareto-optimal region should be a curve representing all the optimum designs, and showing how the respective criteria weigh up with one another [52]. Once the Pareto optimal region is determined, the user evaluates the individual criteria against each other, and selects a design based on the compromise reached between the different criteria [51].

Some specific disadvantages associated with the use of genetic algorithms are [47]:

- the *stochastic exploration* of the space of design variables is very expensive in terms of CPU time,
- it is necessary to *experimentally predetermine* the mutation and cross over parameters,
- there is *no proof of convergence*, and
- compared with a pure random approach, the gain is rarely greater than a factor of 5.

1.5.2.2 The "Democrat" design methodology

Merlet [1, 53] lists some disadvantages of the classical approach to optimizing a parallel manipulator design through the minimization of a *cost-function*:

- the weights given to the various criteria of a multi-criteria cost-function strongly influences the results that are obtained by numerical optimization procedure,
- a single criterion objective function, such as for example maximizing the workspace, does not always account for "hidden criteria" such as singularity considerations throughout the workspace,
- non-continuous cost-functions are difficult to handle for most numerical optimization techniques. In addition to this difficulty, the cost-function may have numerous local minima and consequently the minimization procedure may have difficulty to locate the global minima, and
- the computational time may be excessive if the evaluation of the cost-function requires computer simulations of the performance of the manipulator over the whole workspace. This is considered a serious drawback for most numerical optimization methods requiring frequent evaluations of the cost-function.

As an *alternative* to the cost-function approach, Merlet [1, 53] proposes the so-called "Democrat" design methodology for the optimum design of parallel manipulators, where a *specified set of performance requirements are considered* to determine the optimum design.

This design methodology is based on the concept of the *parameter space*, where each dimension of this space represents a design parameter of the parallel manipulator. It works in two phases: during the *cutting phase* different *analytical design criteria* are mapped as *criterion regions* in the parameter space. The subset of all the criterion regions in the parameter space where all specified criteria are satisfied, is isolated and referred to as the *search region.* Finally, during the *refining phase*, the search space is *sampled* at regular intervals for evaluation against the specified set of *performance requirements*, to obtain the optimum parallel manipulator design(s).

1.5.2.2.1 Democrat: the cutting phase

For a general parallel manipulator with six in-parallel links, and under the assumptions made in [1] and [53], six architectural parameters represent the positions of the respective six leg joints on the base, relative to the base coordinate frame. An additional six architectural parameters represent the respective positions of the six leg joints on the moving platform, relative to the moving platform coordinate frame. These twelve parameters are in fact radii of twelve circles, six of which are centered at the base coordinate frame, and six of which are centered at the moving platform coordinate frame. The respective *heights*, and respective *orientation angles* of the moving platform and base joints are assumed to be known *relative* to the respective *coordinate frames*. The twelve architectural parameters specified result in a twelve-dimensional *parameter space*.

The following two *criterion regions* are considered in [1]:

- The *prescribed workspace criterion* is associated with known minimum and maximum values of the respective actuator legs. The user defines line segments inside the prescribed workspace for the moving platform to trace with a specified fixed orientation. The "workspace criterion region" in the twelve dimensional parameter space indicates all the allowable designs of the parallel manipulator, i.e. all the designs that would allow the parallel manipulator to follow the prescribed line segments without violating the extreme leg lengths.

  At any time instant, each of the six leg lengths only depend on the position and orientation of the moving platform along the specified line segment, and the respective positions of the two leg joints (moving platform and base) of that specific leg. Hence, the twelve dimensional parameter space is decomposed into six different *parameter planes*. For a "circular" 6-DOF Gough-Stewart platform with the respective moving platform and base joints spaced at known angular intervals on two circles, the twelve dimensional parameter space reduces to a single parameter plane, since the respective radii of the two circles are the only two architectural parameters needed to describe the design of the manipulator.

  The *analytical* workspace criterion that Merlet [1] formulates, allows him to trace the "workspace criterion region" in the parameter plane in approximately $500\,\text{ms}$. Furthermore, for the 6-DOF "circular" Gough-Stewart platform described above, Merlet [54] shows that interferences between the actuator links may easily be included in the analytical workspace criterion.

- The second criterion considered by Merlet [1] deals with constraints on *articular velocities* of the in-parallel links of the parallel manipulator. Here, the requirement is that a specified point on the moving platform be able to reach a specified velocity (speed and direction), at all locations in the desired workspace, without the articular velocities violating the allowable extreme values. The desired workspace is again approximated by a set of line segments, and the parameter space is again decomposed into parameter planes.

The *analytical* "articular velocities criterion" is used to trace the "articular velocities criterion region" in the parameter plane in typically 2.5 s. Merlet [1] points out that this region is not necessarily closed. Hence mapping the "articular velocities criterion region" requires the specification of the maximum values on both parameters of each parameter plane.

In [53] Merlet reports that the mapping of each criterion region can be as quick as 100 ms, or can take a few minutes, depending on the number of line segments analyzed inside the workspace.

In each parameter plane, the two-dimensional *search region* is then isolated as the intersection of the "workspace criterion region" and the "articular velocities criterion region". For the general parallel manipulator considered by Merlet [1, 53], the six two-dimensional search regions constitute the twelve dimensional search region. Six points, one in each two-dimensional search region, are required to define a unique geometry. For the 6-DOF "circular" Gough-Stewart platform, there is only a single two-dimensional search region, and any point in this search region defines a unique geometry that satisfies both the workspace and articular velocities *design criteria*. Some user interaction is required to isolate the search region [53].

1.5.2.2.2 Democrat: the refining phase

Once all the *feasible geometrical designs* are isolated, the fully automated [53] *refining phase* discretizes the search region, and compares each feasible design based on a set of performance criteria deemed necessary for that application, in search for the optimal design.

A *high-level computer language* was developed for the evaluation of specific parallel manipulator performance criteria in a modular fashion. As an example, Merlet [1] shows that the *absence of singularities* inside the prescribed workspace, monitoring of *positioning errors*, as well as *stiffness consideration* may readily be incorporated as performance criteria. The high-level computer language also allows for the evaluation of any cost-function that would normally be defined for a numerical optimization procedure.

Note that the performance criteria are evaluated for *all positions* of the moving platform in the specified volume – the "translation workspace" [53]. The evaluation is done *without discretizing* the translational workspace because of the ability of the *high-level computer language* to treat specific types of "translational workspaces". In particular, the translational workspace can be a normal cube, or it can have a complex shape (see Figure 1.15), in which case it will be defined by a set of two-dimensional cross-sections in three-dimensional Cartesian space.

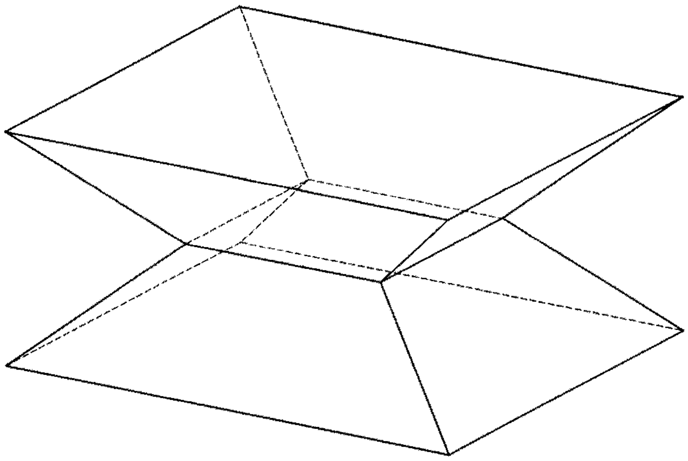

Figure 1.15: An example of a translational workspace volume that can be treated by the algorithms in Democrat (after [53]).

The volume can also be specified in the high-level computer language as a prescribed "hypercube" in the "articular space". The number of articulated in-parallel links of a parallel manipulator determines the number of dimensions of the articular space. For a 6-DOF Gough-Stewart platform with six arbitrary spaced actuator legs, a six dimensional "articular space" is required to define the "hypercube" $\ell_i^{\min} \leq \ell_i \leq \ell_i^{\max}$, $i = 1,2,\ldots,6$; with $\ell_i^{\min}$ and $\ell_i^{\max}$ respectively the minimum and maximum allowable leg lengths of legs $i = 1,2,\ldots,6$.

Merlet [53] distinguishes between a *translation workspace* as described above, and a *general workspace* which, apart from the specified Cartesian volume, also includes specified *ranges for the three orientation angles* of the moving platform. In the latter case, the high-level computer language "continuously" evaluates the performance criteria for the specified three-dimensional displacement volume, but the "three-dimensional *orientation* volume" is discretized during the evaluation process.

As a specific example of how the high-level computer language works, Merlet [53] explains the instruction:

%VO = minimal stiffness in cube center 0 0 30, 0 10 10 10

This instruction *commands* the computation of the "minimal values of the diagonal of the stiffness matrix of the parallel manipulator" for *all positions* of the moving platform in the specified cubic volume ($10 \times 10 \times 10$), centered at $(\mathrm{x}, \mathrm{y}, \mathrm{z}) = (0, 0, 30)$. The returned minimal values are stored in the array **VO**.

The user specifies *allowable* minimal values as the "stiffness performance requirement", which is then used in the evaluation of the different *feasible* designs given by the discretized search region. In particular, the high-level computer language returns "0" if the feasible design does not fulfill the user's requirement, "1" if it fulfills the requirement and "2" if it fulfills the requirement and is better than the previous solution [53].

It is reported that the computational time of this final stage of the proposed design methodology is dependent on the size (and dimension) of the search region, and the efficiency with which the performance criteria is evaluated [1].

1.5.2.2.3 Democrat: Optimizing the "HFM2" 6-DOF Gough-Stewart platform design

The "HFM2" 6-DOF "circular" Gough-Stewart platform "meant to be used for fine motions of heavy loads ($850\,\text{kg}$) in a relatively small workspace", is presented in [1] and [53] as a case study for the "Democrat" design methodology.

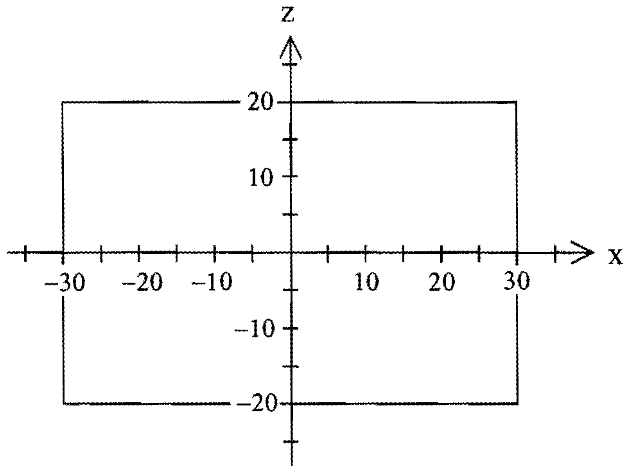

Figure 1.16: Rectangle (scale 1:1) showing the x and z workspace constraints of the "HFM2" [1] 6-DOF "circular" Gough-Stewart platform.

Figure 1.16 shows a rectangle in the $x-z$ plane, where the position of the coordinate system is chosen to represent the x and z workspace "constraints" as given Merlet [1, 53] for the HFM2 manipulator:

x (mm)	y (mm)	z (mm)
± 30	–	± 20

The remaining workspace constraints listed for the HFM2 platform are

θ_x (mrad)	θ_y (mrad)	θ_z (mrad)
± 5	± 5	$0-10$

which may be interpreted as follows:

The three respective orientation angles of the moving platform, θ_x, θ_y and θ_z, are required to assume *all* values in the respective ranges $[(-0.2865^\circ)-(0.2865^\circ)]$, $[(-0.2865^\circ)-(0.2865^\circ)]$ and

$[(0^\circ)-(0.573^\circ)]$, at *any* point inside the rectangle shown in Figure 1.16. Such a workspace, where positional and rotational requirements are specified, is formally known as a *dextrous workspace* [40].

Other than the workspace constraints, Merlet [1, 53] specifies positional and accuracy requirements, the most stringent of which are $\pm 0.01\,\text{mm}$ positioning accuracy in the x-direction, and $\pm 0.05\,\text{mrad}$ ($\pm 0.002865^\circ$) rotation accuracy about the z-axis (θ_z). Optimization of the manipulator should be done firstly with regard to maximizing the "rotational stiffness" about the z-axis, and secondly with regard to maximizing the "positional stiffness" in the x-direction.

One of the issues in determining the optimum design of the manipulator is, of course to determine the position of the prescribed dextrous workspace and manipulator base relative to each other. This *"base / required workspace"-position* introduces *additional parameters* to the two "leg joint position" parameters that are required for describing the design of the 6-DOF "circular" Gough-Stewart platform. They should also be considered during the optimization procedure.

Merlet [1, 53] does not mention this "base / required workspace" position as such, but in requiring the use of linear actuators with known and fixed stroke lengths, he indirectly addresses the positioning problem by *determining* a minimum actuator leg length for all six actuator legs. In essence, for any *specified* "base / *required* workspace"-position, the "base / *actual* workspace"-position may be *adjusted*, until it coincides with the "base / *required* workspace"-position. In practice this adjustment is made possible in one of two ways:

- extensions may be added to the lower ends of all six actuators to lengthen the minimum actuator leg length of all six actuators, and hence *lifting* the "base / actual workspace" (see the illustrative 2-DOF example in Figure 1.17), or

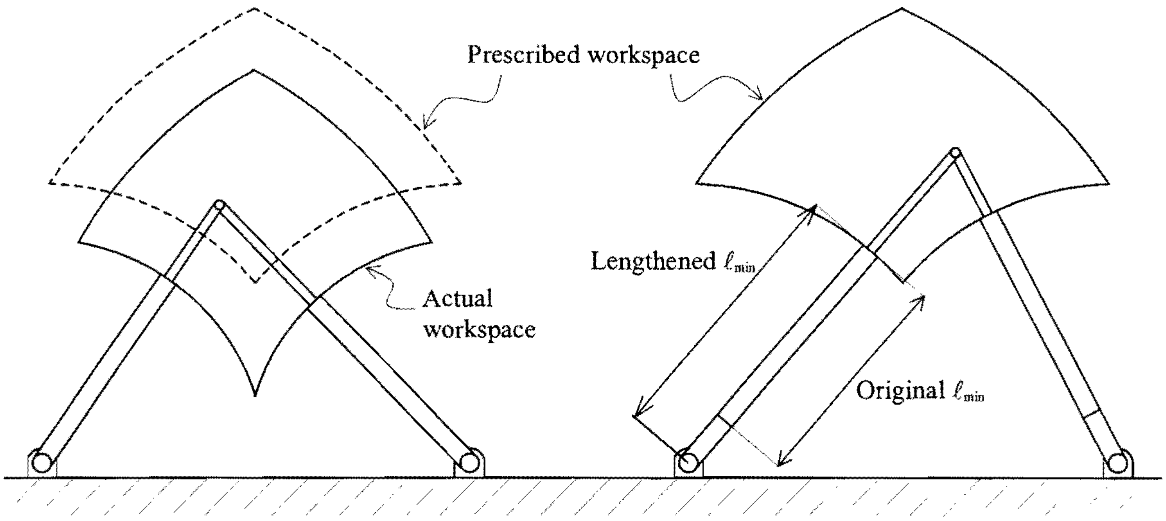

Figure 1.17: The adjustment of the base / *actual* workspace position by lengthening the minimum actuator lengths.

- shortening the minimum actuator leg length of all six actuators by mounting the actuator leg base joint at the required location along the casing of, for example, a hydraulic actuator (see the illustrative 2-DOF example in Figure 1.18).

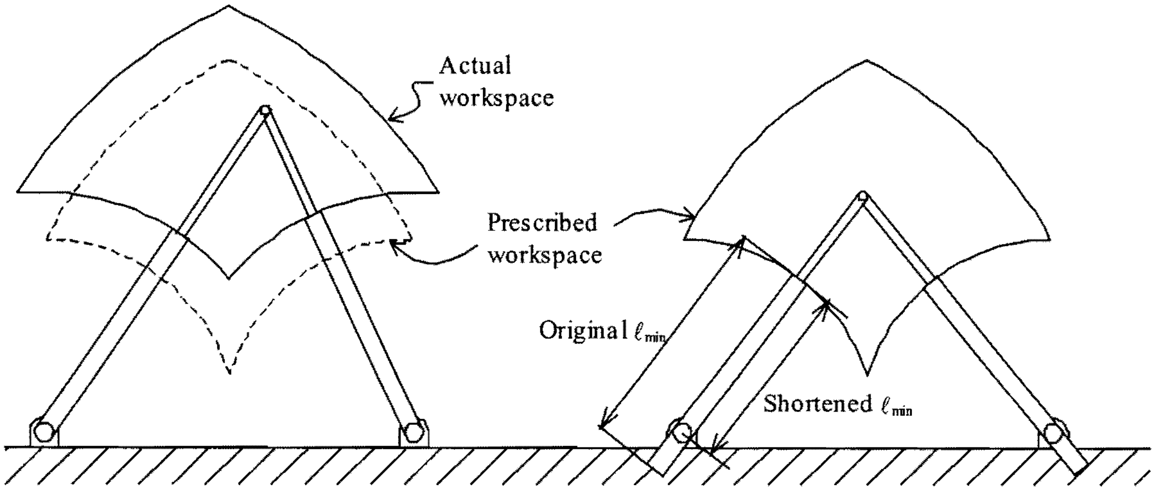

Figure 1.18: The adjustment of the base / *actual* workspace position by shortening the minimum actuator lengths.

For the "HFM2" manipulator, Merlet [1, 53] defines 19 line segments to analytically represent the prescribed dextrous workspace, and then calculates the "area of the search region" as a function of the "minimum actuator leg length". This is presumably done by choosing different "minimum actuator leg length" values, and calculating the corresponding "area of the search region" value. This being the case, the discrete data points could be represented on a graph, either by connecting them using straight-lines, or by fitting an approximation polynomial through them [55].

With the best value of the "minimum actuator leg length" ($750\,\text{mm}$) determined through a "systematic search" involving "various trials" in the domain plotted ($590\,\text{mm} - 835\,\text{mm}$), Merlet [1, 53] finally shows the associated search plane from which the optimum HFM2 "circular" Gough-Stewart platform is to be determined, using the high-level computer language algorithm. Merlet [1, 53] comments that the optimum manipulator geometry in terms of the "rotational stiffness" about the z-axis, is to be fitted with sensors capable of a $\pm 2\,\mu\text{m}$ accuracy in order to comply with the specified manipulator accuracy.

Without *giving specific parameter values or reporting on the computational effort*, a photograph of the prototype "HFM2" manipulator that was built according to the optimum design parameters, is shown in [53]. Merlet [53] reports that the repeatability of the prototype under a load of $230\,\text{kg}$ is estimated to be better than $0.1\,\mu\text{m}$, and that 10 other prototypes have subsequently been built.

1.5.2.2.4 Democrat: Optimizing the "HDM1" 6-DOF Gough-Stewart platform design

In a second example, which has a similarly small dextrous workspace requirement (see Figure 1.19), and exactly the same accuracy requirements as before, Merlet [1] attempts to optimize another "circular" Gough-Stewart platform ("HDM1"), firstly in terms of the "rotational stiffness" about the z-axis, and secondly in terms of the positional stiffness in the x-direction. As additional constraints, the respective radii of the base and moving platforms are also limited.

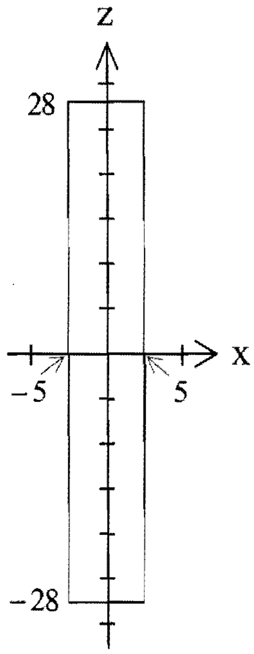

Figure 1.19: Rectangle (scale 1:1) showing the x and z workspace constraints of the "HDM1" [1] 6-DOF "circular" Gough-Stewart platform.

For this case study, Merlet [1, 53] considers the "rotational stiffness" about the z-axis, as a function of two *additional design parameters*: the angle between two adjacent joint centers on the moving platform, and the angle between two adjacent joint centers on the base. Note that for the 6-DOF "circular" Gough-Stewart platform, the three pairs of adjacent joints on the moving platform, as well as the three pairs of adjacent joints on the base, are equally spaced at 120° angular intervals. The *minimum limits* imposed on the respective angles are 10° for the angle between two adjacent joint centers on the base, and 20° for the angle between two adjacent joint centers on the moving platform.

Subject to the above constraints, different values of the two angles are *iteratively* chosen [53]. For each choice of angles, Merlet [1]:

- determines a best value for the minimum actuator leg length, which is associated with a maximal possible "rotational stiffness" in the z-direction for the manipulator in its nominal position (the six linear actuator legs in the middle of their respective ranges),
- calculates the associated search plane, and

- utilizes a special "procedure" in the high-level computer language, to discretize and analyze the search plane, in search of a manipulator design with which the required specified accuracies may be obtained with the least stringent sensor accuracy.

No information is given regarding the number of different choices of angle-pairs evaluated. Furthermore, *without giving any specific parameter values or reporting on the computational effort*, Merlet [1] comments that the two best solutions, in terms of least stringent sensor accuracies, are 2.4 μm and 2.79 μm respectively.

As a conclusion to the discussion of the Democrat design methodology, some of its reported advantages and disadvantages are listed here. Merlet [1, 53] points to the advantageous modularity and versatility with which the high-level computer language can evaluate almost any type of performance requirement. Furthermore, although the reduction of the parameter space into a search region is considered as an advantage in limiting the required computational time, the constraints imposed on the criterion regions (Section 1.5.2.2.1) and consequently also on search region, admittedly, limit the number of feasible designs when searching for an optimum parallel manipulator design.

1.6 Motivation for the present study

In conclusion to, and as part of the literature review presented here, the concept of a novel re-configurable planar Gough-Stewart machining platform will now be motivated. In doing so the scope of the present study will also be outlined.

1.6.1 The concept of a re-configurable planar Gough-Stewart machining platform

1.6.1.1 Mechanical feasibility

Although to date the concept of a *re-configurable* planar Gough-Stewart machine has not been satisfactorily demonstrated, researchers have recently shown an increased interest in such re-configurable platforms. This renewed interest is stimulated by the desire to overcome the workspace and singularity limitations (see Section 1.4), which have been inhibiting the practical application of conventional Gough-Stewart platforms as machine tools. The case studies presented by Merlet [1, 53] reconfirm the fact that the conventional 6-DOF Gough-Stewart platforms have very small usable workspaces (see Figure 1.16 and Figure 1.19).

The *simplified mechanical construction* of the planar 3-DOF Gough-Stewart platform (see Figure 1.9) to be studied here, makes it well suited for the implementation of re-configuration. This is so because its variable geometry allows for the easy adjustment of the relative positions of the base and moving platform revolute joints as shown in **Appendix D**.

Furthermore, the existing "Dyna-M" and "Honda HVS-5000" machine tools (Section 1.3.2) prove that a planar parallel manipulator can be constructed in such a way that sufficient lateral stiffness is provided for hybrid serial-parallel machining operations.

The above indicates that the successful implementation of a planar re-configurable platform as a machine tool is not so much limited by its mechanical design, but rather by the availability of a suitable *operating system*. Here the operating system should ensure that any reasonably specified trajectory is feasible and can accurately be followed. In particular, the operating system should be able to a priori *simulate* the motion of the mechanism along the prescribed trajectory. Based on the simulation the system should be capable of deciding on the *necessary adjustments of the variable geometry* so that the prescribed trajectory can accurately and optimally be followed. The first part of the current study is therefore the development of a reliable and efficient dynamic simulation module for the "overall operating system".

1.6.1.2 Simulation of a planar Gough-Stewart platform

1.6.1.2.1 Inverse Dynamic simulation

Shamblin and Wiens [56] characterize the dynamics of two 6-DOF Gough-Stewart machining platforms for which they derive the equations of motion with inclusion of the strut masses. They state that in order to capture dynamics (i.e. determine the actuator forces), a *motion trajectory must be specified*, along which the mechanism's dynamical behavior is *simulated*. Accordingly **Chapter 2** of this study shows how the *inverse dynamic analysis* of a planar Gough-Stewart platform may be performed so as to give *closed-form expressions* for the *required actuator forces* necessary for the execution of a *specified trajectory*. This inverse dynamic analysis is specifically developed for implementation on a computer in near real time, hence the need for closed-form mathematical solutions to the forces at discrete and *appropriately chosen time instants* along the path.

The advantage of the inverse dynamic analysis is that for different *adjustable parameter values*, which give rise to different *mechanism geometries* and different *relative positions of the prescribed trajectory*, the corresponding motions may be analyzed and compared with each other.

The output of the inverse dynamic analysis is a set of actuator forces at discrete time instants. The usefulness of this information lies in the fact that if the prescribed trajectory is positioned such that the simulation shows that the Gough-Stewart platform will move through or near a *singular configuration* in tracing the trajectory, then this will be evident from the *near infinitely large actuator forces* in the simulation output at certain time instants. By comparing the discrete computed actuator forces at the discrete time instants for any specific prescribed trajectory, the computer simulation can be utilized to isolate the "maximum magnitude actuator force" for the *specific positioning* of the prescribed trajectory and the given mechanism *geometry*. This information can in turn be utilized to *determine* an appropriate relative *positioning* for the prescribed trajectory, as well as an appropriate mechanism *geometry*, such that a large "maximum magnitude actuator force" resulting from passing through or near a singular configuration, may be avoided.

In their related investigation, Shamblin and Wiens [56] specify a trajectory to "simulate a chamfering and deburring operation along the edge of a workpiece as well as to show the dominant forces under a variety of conditions". It follows that a further function to be performed by the operating system being developed here, is that of *kinematic trajectory-planning*. This subject will be dealt with in the next sub section.

1.6.1.2.2 Trajectory-planning

Many researchers have studied trajectory-planning from the point of view that, given an initial and final pose of the manipulator end-effector, it is required to *determine* how the manipulator should be actuated in between these two poses (see for instance [57]). With specific reference to Gough-Stewart platforms, this approach is popular, since it allows for the *avoidance of singularities inside the workspace* of the manipulator [58]. To avoid singularities, Merlet [59] proposes a trajectory verifier and indicates analytically which part of the specified trajectory is outside the reachable workspace of the parallel manipulator, and whether the specified trajectory will lead to a singular configuration. The application of this trajectory verifier is limited to a 6-DOF Gough-Stewart platform, although it is claimed to be easily extendable to general parallel manipulators.

Trajectory-planning as defined by Wolovich [60] is the specification of desired time-dependent paths in either Cartesian or link space. In terms of performing the *inverse dynamic analysis* of a planar machining platform, the tool trajectory must be specified in Cartesian space. The inertia forces in the dynamic analysis of the motion of a machine are of course dependent on the manner in which the Cartesian path is specified in the *time domain* [56]. If the trajectory is specified in such a way that the resulting accelerations are discontinuous, then the inertia forces will also be discontinuous.

With specific reference to trajectory-planning for existing Computer Numerical Control (CNC) machine tools, Zhang and Greenway [61] state that CNC systems typically only support motion along straight-line and circular paths. However, *free-form* design and machining have become important in a variety of applications in the automotive-, aerospace-, and ship building industries. Specific examples are the design and machining of dies and molds, as well as propeller and impeller blades [62]. The consensus seems to be that free-form surfaces can easily be modeled in 3-D space, but that the manufacturing of free-form surfaces has been a difficulty up to now.

The difference between various *representation schemes* with which free-form surfaces are modeled in 3-D space, lies in the utilization of different geometrical and polynomial properties required to control and modify the desired geometrical shapes [62]. More specifically, Non-Uniform Rational B-Splines (NURBS) have long been favored in Computer-Aided Design (CAD) systems, since "they offer exact uniform representation of both analytical and free-form parametric curves" [61].

Bahr et al. [62] explain that a typical way to *machine* parts with spline surfaces (including NURBS) on a CNC machine tool is converting or transforming the surfaces to linear or circular segments according to a prescribed error tolerance, so that the CNC machines can reproduce the parts. For many applications, these conversions or transformations will produce a large amount of data. Furthermore, with the path divided into straight-line segments, in current five-axis machining with off-line programming, the *tool orientation is maintained constant during each segment*. This implies that the *orientation of the tool must be changed abruptly between two segments*, which according to Kim et al. [63] can produce an unpredictable reaction at the point of contact with the surface and *prevent a smooth finish*.

Kim et al. [63] acknowledge the value of a real-time NURBS curve interpolator for a 6-axis robot developed by Zhang and Greenway [61]. They state that real-time parametric interpolators reduce the memory requirement and communication load in guaranteeing continuity in the first-order and second-order properties of the tool position. They emphasize however, that *the most significant problem in the generation and control of a five-axis NC trajectory is a continuous and smooth description of the tool orientation that will change smoothly along the contour surface.* Therefore, an important area of research is to generate a control algorithm that will accommodate a continuous and sufficiently smooth description of the orientation of the tool.

Kim et al. [63] focus on the fact that the tool tip and a unit line vector attached to the tool generate a ruled surface. The curvature theory of a ruled surface, which is a study of the differential motion of the ruled surface, is then used to provide the properties of the tool motion in a *strictly mathematical manner.* When the surface to be machined is a free-formed surface and cannot be represented by an analytical closed-form equation, Kim et al. [63] use the Ferguson curve model to *geometrically* represent the ruled surfaces for the tool trajectory.

In **Chapter 3**, an alternative *trajectory-planning interpolation algorithm* is proposed and developed with which a user may specify the desired path to be followed by any *planar* industrial robot, and therefore in particular also by a planar Gough-Stewart platform. Given specified points along the path, an interpolation curve is fitted in such a way that *continuous* displacement, velocity and acceleration curves are generated in the time-domain. The user-specified information is also used to determine how the end-effector orientation angle should vary along the specified curve, and in particular, generates *continuous* orientation angle, orientation angular velocity and orientation angular acceleration time curves.

In terms of the current research, which is focused on a planar Gough-Stewart machining platform, the relevance of the proposed free-form trajectory-planning algorithm lies in the fact this machine tool is ideally suited to machine along non-linear curves. Powell et al. [11] explain that for a conventional machine tool, based on serial kinematic chains, the simplest movements are linear motions along the orthogonal axes (x, y and z). To provide more complex motion requires the synchronized movement of all three of the axes. With the Gough-Stewart platform type machine tools, all motion is derived from the simultaneous motion of all the actuator legs, hence the moving platform orientation can also be varied in a continuous manner.

Of particular importance here, with reference to the machining problems previously experienced and outlined above, is that the algorithm proposed in this study allows for the generation of a kinematically smooth trajectory. The resulting beneficial effect is that the inertia forces in the actuators, as well as the orientation of the tool will vary in a continuous manner. This should ensure smooth finishing during the machining operation.

It should be noted here that for the actual motion of the physical machine tool to correspond with its simulated motion, the proposed trajectory-planning algorithm can not simply be loaded on a conventional CNC controller. In fact, Kim et al. [63] explain that the implementation of any extended algorithm that allows for interpolated motion beyond straight lines and circles, requires an open architecture controller, which is considered a new concept in CNC machining. Although this practical aspect is very important, it falls beyond the scope of this study. However, the ability to accurately simulate the continuous kinematics and associated dynamical behavior of the motion of the planar Gough-Stewart platform along non-trivial prescribed paths, is imperative in determining the optimum mechanism geometry for any prescribed path.

1.6.1.3 Optimal adjustment of the variable geometry

With reference to Sections 1.5.2.1 and 1.5.2.2, Merlet [1, 53] and Kirchner and Neugebauer [51] agree that a single performance objective criterion cannot be used to optimize a Gough-Stewart machining platform. In particular, Merlet [1, 53] points out that there are "hidden criteria" such as singularity considerations that are not considered when, for example, the workspace is maximized.

In spite of the above reservations, a single criterion cost-function will nevertheless be used in this study to *determine* an appropriate relative *positioning* for the prescribed trajectory, as well as an appropriate planar Gough-Stewart platform *geometry* for different machining tasks. In particular, the cost-function to be minimized here will be the "maximum magnitude actuator force" mentioned in Section 1.6.1.2.1. The rationale is that if the "maximum magnitude actuator force" is *as small as possible*, the corresponding relative *positioning* for the prescribed trajectory and the particular mechanism *geometry* will be such that the prescribed trajectory will successfully be traced. This will be so since the planar manipulator will be *"as far as possible"* from any singular configurations.

Many numerical optimization techniques also allow for non-trivial *inequality* and *equality constraints* to be specified. The careful formulation of such constraints extends the value of the solutions to the corresponding constrained optimization problems, beyond that where only simple limitations are imposed on the minimum and maximum allowable design variable (parameter) values. For example, the minimum and maximum allowable actuator leg lengths of a planar Gough-Stewart platform may be incorporated as inequality constraints, to ensure that the design parameters are adjusted so that the prescribed trajectory lies inside the mechanism's workspace.

In **Chapter 4** it will be shown that, in spite of the non-smooth nature of the "maximum magnitude actuator force" cost-function and "actuator leg length" inequality constraints, the gradient-based mathematical programming LFOPC optimization algorithm [64] used in this study, successfully solves the comprehensively constrained optimization problem. Indeed, LFOPC has in the past been successfully applied to many engineering optimization problems where noise and *discontinuities* were present in the objective and constraint functions [64].

1.6.2 The concept verification: a re-configurable planar Gough-Stewart platform test-model

In **Chapter 5** the ultimate task of designing, constructing and putting into operation a re-configurable planar Gough-Stewart platform test-model is tackled. The chapter shows in particular how the simulation and optimization processes are integrated in an operating system, that allows for the set-up of the machine and the execution of the prescribed tasks.

The constructed test-model may be seen as a technology demonstrator rather than a prototype. The value of this demonstrator lies in the fact that it enables a practical assessment of the feasibility and potential of the re-configurable device, and associated operating system, as a practical machining center. This evaluation is presented in the concluding **Chapter 6,** in which suggestions for future research are also made.