CHAPTER 1

INTRODUCTION

1.1 SUPPLY CHAIN MANAGEMENT

1.1.1 Background

A new era has dawned in Supply Chain Management with the advent of globalization. This has led to increased competition and in order to achieve and sustain competitive advantage, companies must be able to respond quickly to customer demand and deliver a high level of customer service. The need for companies to be flexible and to be able to customize their products is also becoming more important. This added pressure on supply chains, coupled with global deregulation, is encouraging many companies to move the sourcing of components and low-value added operations offshore, to lower cost countries (Ross, 2003) - this result in supply chains which increase in distance and complexity.

With global markets and suppliers, companies need to have a supply chain that is lean on inventory and responsive to customer demand. To ensure an efficient supply chain, all aspects of such a supply chain need to be monitored continually and inputs need to be managed in order to anticipate any uncertainty in supply, demand and cost and to ensure that appropriate contingencies are in place.

According to Lakahl et al (2001) companies must concentrate on their core competencies to help sustain competitive advantage. Non-strategic activities that can be performed more effectively by a third party need to be externalized. A company's core competencies depend heavily on its resources and how they are utilized and if a company is able to develop and allocate resources in a way, which creates more value for customers than their competitors can, it creates a sustainable competitive advantage. A superior supply chain strategy maximizes the value added by internal activities while developing solid partnerships leading to high value external activities.

Supply chain management is plagued with conflicting objectives and supply chain managers must make appropriate tradeoffs to ensure optimal functioning of the supply chain. Traditionally inventory was used to ensure compliance with customer demand and to guard against uncertain delivery lead times. Economies of scale is another reason for inventory accumulation - fixed costs are lowered by producing or ordering in large quantities, transportation discounts can be achieved and it guards against uncertainties. The problem with high inventories however is that capital is tied up and high inventory holding costs is incurred. The inability to meet customer demand, in turn, leads to lost profits and in the long run, possibly the loss of clients. Thus the trade off between customer satisfaction and inventory holding costs is one of the most important decisions that a supply chain manager has to make.

The problem of providing customer satisfaction under conditions of demand variability is usually addressed with safety stock. In the literature, safety stock are considered from the traditional inventory theory viewpoint and it fails to address key features of realistic supply chain problems such as multiple products sharing multiple production facilities with capacity constraints and demand originating from multiple customers. Safety stock levels are dependant on factors such as probabilistic distributions of demand, the demand-capacity ratio as well as the dependence of overall customer satisfaction levels on meeting demands for several different products produced at the same facility (Jung et al, 2004).

In order to manage the supply chain, a supply chain manager needs accurate, timely information. To produce corporate planning solutions, one, or a combination of enterprise planning methods are used, these include manual processes, proprietary planning solutions, Enterprise Resource Planning (ERP) and Advanced Planning and Scheduling (APS).

To support the increasingly complex analysis associated with extended supply chains, decision support tools have to lead key strategic, tactical and operational decisions at every stage of the supply chain. These tools have to provide insight into the tradeoffs that have to be made among alternative strategies regarding, for example, site location, transportation strategies, inventory strategies, resource allocation and supply chain operations (Padmos et al, 1999). In addition to this, these tools and the methods that they employ need to take the uncertainties that are characteristic of supply chains (e.g. demand uncertainty), into consideration.

The objective is to have a supply chain were all participants act as if they are part of one entity in an effort to maximize the timely arrival of good quality raw material, minimum lead times and minimum reasonable inventory – this will contribute to a “seamless supply chain” (Kerbache & Smith, 2004).

1.1.2 Literature Review of Supply Chain Optimization

A study was undertaken to consider various supply chain optimization approaches available in literature. Literature with regards to supply chain optimization is abundant and no attempt is made to do a complete review. In agreement with the observation that Kerbache & Smith (2004) made, it is observed that the literature has taken three directions:

1. *Purchasing and supply perspective*: The interest here is directed toward the upstream supply chain.
2. *Transportation and logistics perspective*: Interest focused on the downstream supply chain activities.
3. *Complete supply chain perspective*: Attempts are made to deal with the supply chain as a whole (De Kok & Graves, 2003) .

The interest for this paper was focused on literature that takes the third direction – that is, literature that considers the complete supply chain.

Such literature seemed to be subdivided into three categories:

a. Modelling the supply chain using mathematical programming (Operations Research Techniques) (Stadtler & Kilger, 2002)
b. Modelling the supply chain through simulation modelling
c. Modelling the supply chain using IT-driven techniques – these includes object oriented modelling and intelligent agent technology

A brief discussion of the approach within each of these three groups is provided.

a. Modelling the supply chain using mathematical programming (Operations research techniques)

Operations Research models are either deterministic or stochastic.

(i) Deterministic Programming Models

Deterministic models are used to address strategic and tactical decisions through the use of mixed integer linear programming (MILP) or mixed integer programming (MIP). The objective of these models is usually to maximize after-tax profit or minimize supply chain costs. Because of the complexity of some of the models, heuristics are often used to attain solutions.

Linear programming and mixed integer programming models are developed to address various decisions that have to be made in the supply chain. These solutions give answers to strategic, tactical and operational decisions.

In an effort to make strategic investment decisions easier, for example with regards to alternative products and development projects, Fandel & Stammen (2004) designed a general linear mixed integer model by considering the total product life cycle, including development and recycling. The goal of their approach was to optimize after-tax profit and to fix the product program and the extended supply chain network.

To investigate strategic networking issues, Lakhal et al (2001) developed a large mixed integer programming (MIP) problem that aims to find the networking strategy that maximizes the value added by internal activities of the company (they equate this to maximizing profits). Because of the complexity of the problem, the MIP is relaxed and a heuristic is used to obtain solutions for an illustrative example. The authors however admit that the static nature of the model poses an important limitation, as supply chains are inherently dynamic.

(ii) Stochastic programming models

Stochastic operations research models incorporate multi-objective mixed integer linear programming (MILP) and mixed integer non-linear programming (MINLP) in an attempt to resolve strategic and operational problems. These problems aims to maximize supply chain profit and customer satisfaction. For tactical decision making a non-deterministic (NP) hard problem is used, but because of the complexity a suitable heuristic is develop.

In a multi-objective stochastic MILP problem, Guillén et al (2005) consider strategic and operational decision-making. Decisions such as the capacity and location of plants and warehouses, the amount of products to be made at each plant and the flow of material between each two nodes of the supply chain are addressed in a hypothetical example. The objective of the problem is to maximize supply chain profit and customer satisfaction and also takes uncertainty into account by means of the concept of financial risk. The problem is solved using a standard-constraint method and branch and bound techniques.

In a novel approach, Seferlis & Giannelos (2004) uses a two-layer optimization-based control approach for use in operational decision-making. The control strategy applies multivariable model-predictive control principles to the entire supply chain. This is done whilst safety inventory levels are maintained through the use of dedicated feedback controllers for every product and storage node. These inventory controllers are embedded in the optimization framework as additional equality constraints. The optimization-based controller aims to satisfy multiple objectives: that is to maximize customer satisfaction and minimize operating costs. It is not clear from the source which operational research method is employed although extensive detailed equations, assumptions and constraints are described. Illustrative simulations are used to demonstrate that the model can accommodate supply chain networks of realistic size under a variety of stochastic and deterministic disturbances.

(iii) Queuing network models

Using a queuing network, Arda and Hennet (2004) represent a simple two-level supply chain. With this network, the producer uses a base-stock inventory control policy that keeps the inventory position level (current inventory plus pending replenishment orders) constant. The decision variables are the reference inventory position level and the percentages of orders sent to the different suppliers. In the model, the percentages of orders are implemented as Bernoulli branching parameters. The expected cost is obtained as a complex non-linear function of the decision variables. A centralized inventory control model is incorporated to combine supply and demand randomness in the queuing network model. Because of the complexity of the problem, a decomposed approach is proposed for solving the optimization problem in an approximate manner. When applied to a test case, the approximate solutions' quality is evaluated when it is compared with the numerically computed values. This can however only be done for simple cases and the main drawback of this model is its simplicity in that it can only show you the economic advantages for the producer of using several suppliers instead of just one.

Kerbache and Smith (2004) also use queuing network systems to model and analyze supply chains. They focus on using closed queuing network systems to evaluate performance measures such as throughput, cycle time and WIP. The methodology employ analytical queuing networks coupled with nonlinear optimization in order to maximize the throughput of the system offset by the cost of providing the service. A case study is used to demonstrate the use of the model and to show that it provides a useful tool with which to analyze congestion problems and to evaluate the performance of the network.

b. Modelling the supply chain using simulation

Supply chain modelling with simulation can be divided into descriptive and normative/optimization models. Simulation proves to be problematic as that experts are needed to construct realistic models. This is time consuming and even if a realistic model is constructed it is even more problematic and time consuming to gather the input data for the model (Bansal 2002).

Notwithstanding these problems, simulation is still used in supply chain optimization. A discrete-event simulation model, which have a linear programming model embedded in it, is used to minimize costs, maximize customer satisfaction and sustain acceptable inventory levels.

Kalasky (1996) presents an application of discrete-event simulation in modelling the supply chain for consumer products. The author employs a linear program (LP) to provide for cost models of the supply chain. The objective of the LP is to satisfy multiple objectives, namely minimize costs, maximize customer service levels and sustain acceptable inventory levels. The combined technologies of simulation and optimization provide a viable and useful tool for planning and operation of supply chains.

c. Modelling the supply chain using IT-driven techniques

IT-driven approaches suggested to optimize and model supply chains, are object oriented modelling and intelligent agent technology. Object oriented modelling employs generic building blocks in a simulation model. Operations research techniques (LP and MIP) are embedded in the object-oriented model to help with strategic, tactical and operational decision-making.

1.2 INVENTORY OPTIMIZATION

1.2.1 Inventory Management

Managing inventory within the supply chain is a key aspect of almost any business, that is the ability to provide the right goods or materials at the right price, place and time. Inventory is one of the most visible and tangible aspects of doing business and, as a result, all the problems of a business often end up in inventory. The role of inventory management is to coordinate the actions of sales, marketing, production and purchasing to ensure that the correct level of stocks are held to satisfy customers demand at the lowest possible cost. Inventory management aims to balance the supply and demand equation by regulating the supply of goods to affect their availability in such a way that they match demand conditions as closely as possible (Wheller, 2004). Inventory management involves methods or processes and is a fundamental requirement prior to considering inventory or supply chain optimization.

1.2.2 Inventory Optimization in Software Applications

"Few supply-chain problems have proved as difficult as inventory optimization" according to Murphy (2003:1). He compares managing inventory levels across the supply chain, so as to consistently meet customer requirements at the least possible cost, to squeezing a balloon: air that gets pressed out in one place pops up somewhere else. One reason is that functional solutions tend to optimize a single point in the chain without taking into account the impact of these changes on other areas. Moreover, determining just the right amount of each product to make, how much to place where, when to re-order and in what quantities, are very hard problems to solve. Supply and demand variability precludes the use of linear algorithms that is used to optimize other areas of the supply chain.

A report by Aberdeen Group found that more than 60% of companies use overly simplistic inventory management methods, such as ABC inventory policies or simple weeks-of-supply rules for products. These companies frequently have 15-30 % more inventory than they need and lower service levels. Less that 5 % of companies surveyed are factoring in total supply chain variability when determining inventory policies (Enslow 2004:1-17).

According to Murphy (2003:1) companies are trying to deal with the inventory problem from an execution perspective. They use visibility and alerting tools to get an early view of where the plan is wrong in order to ensure that corrective action can be taken. While helpful, this approach is not a substitute for optimizing inventory levels across the chain. Execution tools can go a long way toward solving supply disruptions, but often resolution is not responsive enough, resulting in more buffer inventories.

Academic research has resulted in significant breakthroughs in stochastic modelling (problems with a high degree of variability). Mathematical algorithms invented in the 1990's and tested over several years at individual companies, are now coming to the market in the form of new Inventory Optimization products. These solutions promise to change the way companies set policies on safety stock, not just for finished goods, but across entire supply chains, with huge potential for savings. These optimization engines are highly sophisticated with algorithms that consider consumption, supply, various lead-times and then determine the amount of inventory required at different location. "It's a myth to think anyone will ever get to zero inventory, but inventory optimization engines are the next step in that direction," according to Mary Haigis of Clarkston Consulting, Durham, N.C. (Murphy 2003:1).

The Aberdeen benchmark study found that companies using new optimization methods commonly drove 20-30 % reductions in on-hand inventory and 10-20 % improvements in time to market (Enslow 2004:1-17). The study also found that nearly half of respondents have shifted away from purchase orders or release notices for some of their suppliers. Instead, these companies are setting a minimum and maximum inventory target level for an item at a plant or other company location, and then ask the supplier to take responsibility for ensuring that inventory is maintained within that range – in essence, Vendor Managed Inventory. Inventory reduction of 30 % and more has been realized in these enterprises and stock-outs have been drastically reduced. New supplier collaboration technology is helping companies execute these min/max replenishment strategies in a way that enables suppliers to also reduce their own inventories. Companies need to be much more aggressive in using the new generation of multi-echelon inventory optimization technology and inventory collaboration technology.

The Inventory Optimization Tool marketplace is a niche market since all software vendors present in this market also deliver other software components such as Forecasting, Supply Chain Network Design, Enterprise Resource Planning (ERP), Retail software or Advance Planning Systems (APS). It is also interesting to note that all of the Inventory Optimization software vendors also deliver forecasting software, which is often closely linked to the Inventory Optimization functionalities, that is optimization of inventory levels based on future forecasting data (Cap Gemini Ernst & Young 2003:4).

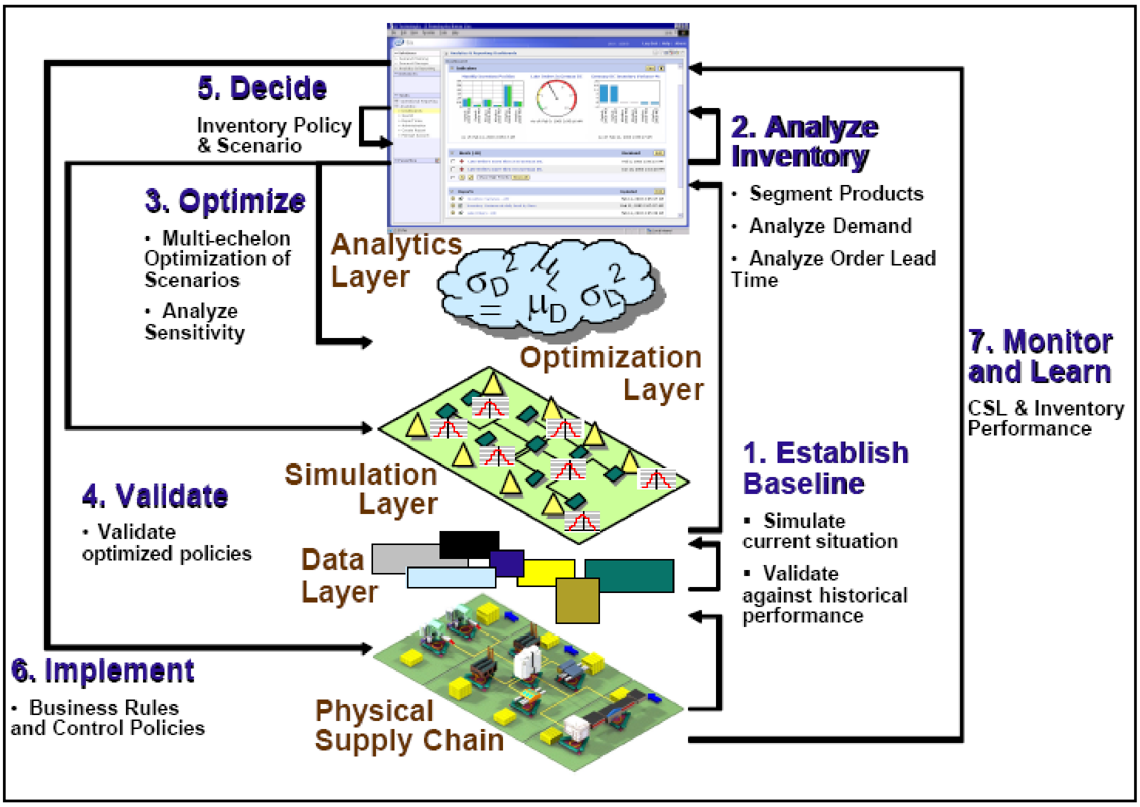
Inventory Optimization tools typically contain functionalities such as:

- Calculation of optimal safety stock levels based on customer service level parameters
- Calculation of ABC classifications
- Determination of the best ordering methodology
- Best before date management and optimal lot size calculations
- Analysis for stock / non-stock decisions
- Dynamic safety stock level management

Software vendors are increasingly realizing that the new direction in supply chain management will require them to have an inventory optimization module. In reaction to this optimization packages are emerging in two forms: Specialized Optimization Packages or Inventory Optimization modules as an addition to ERP systems.

1.2.3 *i2* Technology Seven Step Approach

As an example of an approach used in commercial information technology, ***i2*** Technology, a leading supply chain optimization solution provider, deploys inventory optimization through a seven-step process as depicted in Figure 1.2 below (***i2*** Technology, 2003).



Source: i2 Technology Inc. White Paper 2003:26)

Figure 1.2: The i2 Technology Seven Step Approach to Inventory Optimization

The ***i2*** Seven Step Approach is described below:

a. Establish Baseline – Simulate the current situation and validate against history

i2 starts the process by building a valid simulation model of the supply chain based on historical data.

b. Analyze Inventory – Segment products and analyze demand

Concurrently with the establishment of the baseline and through close cooperation with business leaders, an understanding of the companies' business priorities, the market environment and the customers are obtained. This leads to a profile of customers' buying behaviour and lead-time expectations. These insights are used to design an appropriate segmentation and stratification strategy for the companies' customers and products, driven by the companies' customer expectations and business priorities. Items may be analyzed for demand patterns, demand volumes, service criticality, product lifecycle, product structure similarities, lead times, and competitive posture.

c. Optimize scenarios and measure sensitivity

Candidate "to-be" scenarios are identified in cooperation with the companies' business leaders. Based on ***i2***'s optimization technology, which takes into account anticipated demand and the companies' supply chain constraints and business priorities, calculations are made in order to determine how much of what inventories must be carried and where and in what form, it should be.

d. Validate optimized policies with simulation

The optimized recommendations are then validated with a simulation run. The simulation allows ***i2*** to get more detailed expected performance metrics pertinent to the supply chain for each scenario.

e. Select best inventory policy scenario for business

i2's solution includes an analytics framework that provides metrics based on the SCOR model across the supply chain. The metrics can be tailored according to part, location, customer and time hierarchies and comparison of metrics at any level for part, location, customer and time, between scenarios or within each scenario can be made. This analysis helps the company to decide which "to-be" scenario is best for the business.

f. Implement business rules and inventory policies in the supply chain

i2's solution provides direct integration to supply chain planning business processes through standard API and ***i2***'s Supply Chain Operating System (SCOS) architecture.

g. Monitor service levels and inventory performance

i2 provides a structured framework for continuous learning and process improvement using Six Sigma concepts. Standard reports are provided to monitor actual performance against plans. These reports will also help the company understand if the assumptions that plans were based on, were valid or not. The analysis framework provides guided analysis paths that help to quickly identify root causes of execution problems.

i2 bases their inventory management technologies on the concept of response buffers. The response buffer is the inventory point from which material is consumed to fulfil a customer order. In the retail environment, for instance, the primary response buffer is at the customer facing location. The retail store's shelf is the response buffer. If the customer fails to find the item he wants on the shelf, he simply goes elsewhere and the store loses the order. On the other hand, in a manufacturing environment with component inventories, assemblies, and finished goods, the response buffer can be anywhere in the supply chain, predicated by the business model. In a make-to-stock setting for instance, the response buffer is downstream in the supply chain similar to the retail store. In contrast, in a build-to-order environment, the response buffer may be upstream in the form of raw material.

Response buffers play a fundamental role in inventory optimization strategies. ***i2*** has identified four fundamental strategies that define, according to them, world-class inventory management. These strategies are:

- *Optimized segmentation* **-** stratification of products based on common inventory characteristics and similar response buffer strategies
- *Optimized postponement* **-** deals with decisions around which echelon (node in the supply chain) to position the response buffers in the supply chain
- *Optimized inventory levels* **-** drives decision on how much inventory to carry in the response buffers
- *Continuous learning for process improvement* - enables ongoing, incremental improvement of the inventory management process

In addition to these strategies, ***i2*** describe three variables fundamental in performing inventory optimization:

- ***Demand distribution***

  Demand distributions reflect the expected volume and variability for demand Normal distributions are typically used for medium and high volume demand streams. Poisson distributions are typically good to represent low volume or intermittent demand streams. The system will automatically choose the appropriate distribution based on demand data.

- ***Order Lead Time distribution***

  The order lead-time (OLT) is the time between the last change-order date and customer request date (CRD).

- ***Supply Lead Time distribution***

  The supply path of an end-item has lead times for each of the upstream echelons of the supply chain. Sometimes the lead-time may be insignificant but typically this can range anywhere from a few days to a few weeks.

Added to the fundamental variables, ***i2*** believes that there are two key policy inputs to inventory optimization, namely:

- *Target Customer Service Level (CSL)*
- *Minimum Offered Lead Time*

  This is the minimum lead-time the planner would like to offer for a particular sub-scope of the supply chain. This means that the inventory optimization will plan for at least this much lead time regardless of the customer request date.

With these concepts in mind, ***i2*** then goes on to define an objective function and summarizes the inputs, outputs and decision outputs as follows (i2 Inventory Optimization User Manual 2005:6-10):

- *Objective Function*

  Minimize total expected inventory cost while meeting target CSL
- *Inputs*

  • CSL or delinquency target for end item buffers

  • Demand rate by time period

  • Demand variability

  • Cycle time & cycle time variability for each arc in the network
- *Key Outputs*

  • Inventory targets for all buffers

  • Inventory Turns

  • Revenue (R)

  • Inventory carrying costs (R)

  • Delinquency (R)

  The primary outputs of inventory optimization are the optimized inventory targets for every buffer in the supply chain. These targets are passed on to

Supply Chain or Replenishment Planning. Data is obtained from current ERP or legacy systems employed by the company.

1.3 INVENTORY MODELS

A storage point into and out of which commodities move or flow is termed an inventory system. The inflow is characterised by replenishment from production sources and demand processes induce the outflow. The net flow generates a cascade of problems pertaining to the control and maintenance of inventory systems. There are numerous factors pertaining to the functioning of an inventory system and considering only a small number of factors in the formulation of an inventory model can result in a very complex model. Accordingly, it is quite impossible to obtain a traitable mathematical model that will truly reflect the behaviour of an inventory system. However, several nearly realistic models have been proposed and studied extensively in the past giving importance to the inherent stochastic nature of these systems. Most of these models assume that the organisations maintaining the inventory have control in determining when and in what quantity the inventory have to be replenished, but have no control over the demand process. A systematic account of the early analyses of stochastic inventory systems can be found in Arrow et al (1951, 1958), Beckmann (1961) and Hadley and Whitin (1963). As the study of these systems progressed over time, several reviews have appeared to highlight the state-of-art (for example, see Aggarwal (1974), Nahmias (1978), and Raafat (1991). A review and critique of inventory problems that have been effectively solved is provided by Silver (1981), who also suggested some problems for future research. Girlich (1984) executed a survey of dynamic inventory problems and models that can be implemented.

1.3.1 Types of Inventory Models

The various models of stochastic analysis of inventory systems are broadly classified into two types namely, periodic review systems and continuous review systems. In periodic review systems the state of the system is examined only at specific time intervals at equally spaced points in time and decisions such as placing of orders and the quantity to be added to the inventory are made only at these review points. In continuous review systems, on the other hand, all events associated with the time evolution of the inventory are recorded and the stock level is reviewed continuously at the occurrence of each demand for the product in inventory. Continuous review systems have occupied a wider scope for application since failure of review of the inventory level even at a single time point may prove disastrous for organisations in the defence and medical industries. Inventory systems are also classified as either single product inventory systems or multi product inventory systems, based on the consideration of a single product or a variety of products in interaction.

1.3.2 Single Product Inventory Systems

Several models for single product inventory systems have been proposed. Optimal ordering policies have been developed and studied extensively in the past by several researchers both for periodic and continuous review cases. For example, see Beckmann (1961), Dirickx and Koevoets (1977), Wijngaard and Winkel (1974), Kalpakam and Arivarignan (1985, 88), Horrowitz and Doganso (1986), Beckmann and Srinivasan (1987), Ramanarayanan and Jacob (1987), Ravichandran (1988), Weiss (1988), Srinivasan (1989), Krishnamoorthy and Laxmy (1990), Kalpakam & Sapna (1996), Hargreaves (2002) and Krishnamoorthy and Manoharan (1990).

1.3.3 Multi-product Inventory Systems

Many real life situations exist in which multi-product inventories are required. For example a pharmacist keeps a number of medicines of different brands, a ready-made clothing shop keeps dresses of different designs, colours, and sizes, a shoe store stocks shoes of various styles and sizes. Hence the study of multi- product inventory models has drawn special attention recently. Page and Paul (1976) and Chakravarthy (1981), Sung and Chang (1986), Oneiva and Larraneta (1987), Aksoy and Erengue (1988), Amiya and Martin (1988), Goyal (1988) and Correnu (1990) have analysed multi-product inventory systems.

a. Ordering Policies

In a multi-product inventory system the inventory control policies and the nature of demands may be different from that of a single product system. First we consider inventory control policies. The inventory of each product may be controllable independently or there may exist an interaction among the items and a joint control of the inventory may be required. For example demand for tyres for off-road vehicles will not affect the demand for truck tyres available at the same dealership. Inventory of such items can be controlled individually. The demand for new and retreads of trucks may be highly dependent and need to be controlled jointly. Hence we may have the following two types of re ordering policies for the control of inventory on products:

(i) Individual order policy

This policy determines that each item is ordered according to its own single item policy.

(ii) Joint order policy

This policy determines that all jointly controlled items is ordered whenever an order for specific product order is triggered, irrespective of the inventory level of the other items. That is wherever replenishment occurs; every product is replenished to a specified inventory level.

b. Demand Interaction

Considering the nature of demand, a demand may be for a single product or several products. For example, the inventory of a dealership for new cars, in addition to new vehicles, consists of replacement parts for maintenance and optional accessories such as special trimming. The buyer has the option to take one or more of these accessories. It is also possible that a demand for a particular product during its stock-out may be substituted with another similar product in the inventory. Examples of products having at least partial substitutability include:

- Consumer products such as different brands of toothpastes and different types of pastas or cereals.
- Building products such as different brand of paints and containers of different sizes of the same brand.
- Clothing products such as dresses in the same design and brand but in different colours.
- Electrical products such as fluorescent light bulbs of different makes and ceiling fans of different brands.

When this type of interaction occurs, large stock quantities of a particular product can be avoided, as it is substitutable by another similar product. The available total inventory storage space can be shared optimally as to reduce the lost demand due to unavailability. Kamat (1971) studied substitutability of demands by considering a two substitutable product inventory model with a prescribed order period and obtained a cost function. McGillivray and Silver (1978) investigated the effect of substitutable demands on stock control rules and a heuristic approach for establishing the value of control parameters (the order up to levels) for the case of two products. Parlar and Goyal (1984) considered a model of two substitutable products as an extension of the classical single period *news-boy* problem. They have shown that the optimal order quantities can be found for each product by maximizing the expected profit function, which is strictly concave for the wide range of parameter values. Parlar (1988) used *game-theoretic* concepts (two person continuous game) to analyse an inventory problem with two substitutable products having random demands.

1.3.4 Perishable Product Inventory

Apart from these considerations, the perishability of products also plays a vital role in inventory theory. Several inventory models of perishable products have been proposed and studied extensively. A review of work done on perishable inventory can be found in Nahmias (1982). Further and Weiss (1986), Nahmias and Schmidt (1986), Sarma (1987), Abdel, Malek and Ziegle (1988), Ravichandran (1988), Mandal and Phaujdar (1989) and Perry and Posner (1990) have analysed perishable inventory models. In his survey article, Raafat (1991) has consolidated the work done on continuously deteriorating inventory models. Kalpakam and Sapna (1994, 96) studied a perishable inventory model with (s, S) policy and arbitrary lead times.

a. Demand Interaction

A different type of interaction can occur in the case of perishable inventory. Products such as vegetables, fish, etc. have a short life span and deteriorate in quality due to ageing. The same applies to fashion clothing losing its value due to changing seasons or new trends. In these cases there may also be a demand for an item slightly deteriorated in quality if the cost is reduced compared to the new or fresh product. A multi product perishable inventory system with economic substitution, which deals with a product that perishes in a single period has been proposed and studied by Deuermeyer (1980). Parlar (1985) has also developed a Markov decision model to generate ordering policies for perishable (in two periods) and substitutable products.

1.3.5 Random Environment

In the stochastic analysis of inventory systems, it is generally assumed that the distributions of the random variables, representing the number of demands over a period of time, the life of the product (in case of a perishable product) and the lead-time, remain the same and do not change through the domain of the analysis. However, there are external factors that affect these random variables. Seasonal changes can affect the demand rate, the perishing rate, the selling price and the cost of replenishment. The demand for umbrellas and rain shoes are higher in the rainy season than at other times of the year. The selling price and the cost of replenishment also fluctuate over time due to inflation, non-availability of the products, cost of transport, etc. The state of the environment in which the system is operating may randomly change due to weather, breakdown of storage facilities, etc. Consequently, consideration of the impact of the random environment on such inventory systems is absolutely essential.

1.3.6 Deteriorating Inventory

Balkhi (1999) developed a unified inventory model for integrated production systems with a single product. The production, demand and deterioration rates for the finished product and the deterioration rates for raw materials are assumed to be known functions of time.

The objective of the author is to determine the optimal values of the length of the production stage and the length of the inventory cycle that minimizes the total variable cost of the inventory system. The problem is converted to an unconstrained minimization problem, and when a solution to the underlying inventory system exists, it is the unique global optimal solution. A rigorous mathematical formulation proves the global optimality of the solution. The article is concluded with a numerical example that illustrates the solution procedure.

Rau et al (2003) worked on an integrated inventory model for deteriorating items under a multi-echelon supply chain environment**.** Demand, production and deterioration rate is assumed to be deterministic and constant with production rate greater than demand rate. Only a single supplier, producer, buyer and product are considered**.** A model that gives the optimal joint total cost from an integrated perspective among the supplier, producer and buyer is obtained and Matlab is used to obtain the optimal solution. A numerical example illustrates the use of the model and it shows that an integrated approach results in the lowest joint total cost as compared with the independent decision strategies.

1.3.7 Techniques Used in the Study of Inventory Models

a. Renewal Theory

One of the important types of stochastic processes is the renewal process. Several researchers in the theory of renewal processes have made outstanding contributions, e.g. Feller (1965), Cox and Smith (1958), Smith (1958) and Neuts (1978). A systematic account of renewal theory and its applications to diversified fields can be found in Cox (1962), Parzen (1962), Sahin (1990) and Medhi (1994). A renewal process is a sequence of independent, non-negative and identically distributed random variables, which are not all zero with a probability of one.

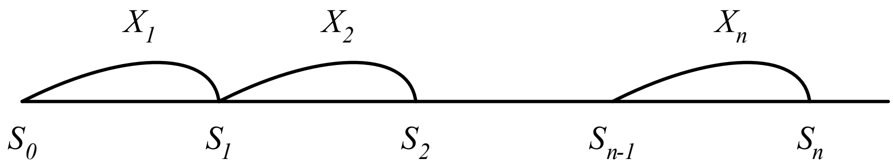
(i) Definition

Let $\{X_n \; ; \; n = 1, 2, \ldots\}$ be a collection of non-negative random variables which are independent and identically distributed. Then $\{X_n\}$ is called a renewal process.

We assume that each of the random variable X_i has a finite meaning. A renewal process is completely determined by $f(\cdot)$, the pdf of X_i. Let

$$S_0 = 0$$

$S_n = X_1 + X_2 + \ldots\ldots + X_n,\ n = 1, 2, \ldots$



$N(t) = \max\{n : S_n \leq t\},\ t > 0$

Then *N (t)* is called the number of renewal up to time *(t)*. The expected value of *N (t)*, namely *E [N (t)]* is called the renewal function and is denoted by *H (t)*. The derivative *H (t)*, whenever it exists, is called the renewal density and is denoted by *h (t)*.

(ii) Renewal Equation

The quantity of *h(t) dt* has the probabilistic interpretation that it denotes probability that the renewal occurs in the interval *(t, t + dt)*. Since this renewal may be either the first or the subsequent renewal, the function *h (t)* satisfies the equation.

$$h(t) = f(t) + \int_0^t h(u) f(t-u) du$$

This equation is called the renewal equation.

(iii) Key Renewal Theorem

Let *Q(t)* be non-negative and non-increasing for *t > 0* such that

$$\int_0^t Q(u)du < \infty$$

then

$$\lim_{t\to\infty}\int_0^t Q(u-x)dH(x) = \frac{1}{\mu}\int_0^{co} Q(u)du$$

where $\mu = E[X_i]$

b. Markov Renewal Processes

These stochastic processes are generalisations of renewal processes and have become indispensable in inventory applications. A systematic and in depth study can be found in Pyke (1961a,b), Cinlar (1975a,b) and Medhi (1994).

Let E be a finite set, N be the set of non-negative integers and $R_+ = [0,\infty]$. Suppose we have on a probability space (Ω, X, P), random variables,

$$X_n : \Omega \to E, T_n : \Omega \to R_+$$

defined for each $n \in N$, so that

$$0 = T_0 \leq T_1 \leq \ \ldots \leq T_n$$

Definition 1: The stochastic process $(X,T) = \{X_n, T_n; n \in N\}$ is said to be a Markov renewal process with state space E provided that

$$P[X_{n+1} = j,\ T_{n+1} - T_n \leq t \mid X_0, X_1, \ldots, X_n; T_0, T_1, \ldots, T_n]$$

$$= P[X_{n+1} = j,\ T_{n+1} - T_n \leq t \mid X_n]$$

for all $n \in N$, $j \in N$ and $t \in R_+$

Assuming that *(X, T)* is time homogeneous, that is, for any $i, j \in E$ and $t \in R_+$,

$$P[X_{n+1} = j,\ T_{n+1} - T_n \leq t \mid X_n = i] = Q\,(i, j, t) \text{ is independent of } n.$$

The family of probabilities

$$\boldsymbol{Q} = \{Q(i,j,t); i,j \in E, t \in R_+\}$$

is called a semi-Markov Kernel over *E*. We assume that $Q(i,j,t) = 0$ for all *i, j* in *E*.

For each pair *(i,j)* the function $t \to Q(i,j,t)$ has all properties of a distribution function except that

$$P(i,j) = \lim_{t \to \infty} Q(i,j,t)$$

is not necessarily 1. It can be seen that

$$P(i,j) \geq 0$$

$$\sum P(i,j) = 1, j \in E$$

That is, $P(i,j)$ are the transition probabilities for some Markov chain with state space *E*. It follows from Definition 1 and the above that

$$P\left[X_{n+1} = j \middle| X_0, X_1, ..., X_n; T_0, T_1, ..., T_n\right] = P\left(X_n, j\right)$$

for all $n \in N, j \in E$. This implies that

$$X = \left\{X_n; n \in N\right\}$$

is a Markov chain with state space *E* and transition matrix *P*.

We write $P_i\left(A\right)$ for the conditional probability $P\left(A \mid X_0 = i\right)$ and similarly $E_i\left(X\right)$ for the conditional expectation of X given $\left\{X_0 = i\right\}$. We also assume that $P_i\left[T_0 = T_1 = T_2 = ... = 0\right] = 0$. We define

$$Q^n\left(i, j, t\right) = P_i\left[X_n = j, T_n = t\right]; \qquad i, t \in E, t \in R_+$$

for all $n \in N$. Then

$$Q^0\left(i, j, t\right) = \delta_{ij} = \begin{cases} 1\, if \;\; i = j \\ 0\; otherwise \end{cases}$$

for all $t \geq 0$; and $n \geq 0$, we have the recursive relation. δ_{ij} is the Kronecker's delta function.

$$Q^{n+1}\left(i, k, t\right) = \sum_{j \in E} \int_0^t Q\left(i, j, du\right) Q^n\left(j, k, t - u\right)$$

where the integration is over $\left(0, t\right]$

The expression $R(i,j,t)$, which gives the expected number of renewals of the position j in the interval $(0,t]$, is given by

$$R(i,j,t) = \sum_{n=0}^{\infty} Q^n(i,j,t)$$

This is finite for any $i,j \in N$ and $t < \infty$. The $R(i,j,t)$ are called Markov renewal functions and the collection $\boldsymbol{R} = \{R(i,j,t); i,j \in E, t \in R_+\}$ of these functions is called the Markov renewal Kernel corresponding to Q. We note that for fixed $i,j \in E$ the function $t \to R(i,j,t)$ is a renewal function.

We can easily see from the various expressions above that $R_\alpha = [I - Q_\alpha]^{-1}$ where I is the unit matrix, and

$$Q_\alpha(i,j) = \int_0^\infty e^{-\alpha t} Q(i,j,t)\,dt; \qquad \alpha > 0$$

$$R_\alpha(i,j) = \int_0^\infty e^{-\alpha t} R(i,j,t)\,dt; \qquad \alpha > 0$$

The class B of functions which we will be working with, is the set of all functions $f : E \times R_+ \to R$ such that for every $i \in E$, the function $t \to f(i,t)$ is Borel measurable and bounded over finite intervals, and for every fixed $j \in E$, the functions $(i,j) \to Q^n(i,j,t)$ and $(i,j) \to R(i,j,t)$ both belong to B.

For any function of $f \in B$, the function $Q © f$ defined by

$$Q © f(i,t) = \sum_{j\in E} \int_0^t Q(i,j,ds) f(t,t-s)$$

is well defined and $Q © P$ belongs to B again. Hence the operation can be repeated, and the n^th^ iteration is given by

$$Q^n © f(i,t) = \sum_{j\in E} \int_0^t Q^n(i,t,ds) f(j,t-s)$$

We can replace Q by R and note that $R © f$ is again a well defined function; that is $f \in B$

$$R © f = \sum_{j\in E} \int_0^t R(i,j,ds) f(j,t-s)$$

A function $f \in B$ is said to satisfy a Markov renewal equation if for all $i \in E$ and $t \in R_+$

$$f(i,t) = g(i,t) + \sum_{j\in E} \int_0^t Q(i,j,ds) f(j,t-s)$$

for some function of $g \in B$.

Limiting ourselves to function $f, g \in B$, which are non negatives and denoting this set by B_+, the Markov renewal equation now becomes

$$f = g + Q © f \; ; \; f, g \in B_+$$

This Markov renewal equation has a solution $R © g$. Every solution f is of the form

$$f = R \copyright g + h$$

where h satisfies

$$h = Q \copyright h, h \in B_+$$

c. Semi-Markov Processes

Let *(X,T)* be a Markov renewal process with state space E and semi-Markov Kernel Q. Define $L = \underset{n}{Sup}\, T_n$. Then L is the lifetime of *(X,T)*. If E is finite or if X is irreducible recurrent, then $L=+\infty$ almost surely. By weeding out those $\omega \in \Omega$ for which $\underset{n}{Sup}\, T_n(\omega) < \infty$, we assume that $\underset{n}{Sup}\, T_n(\omega) = +\infty$ for all ω. Then for any $\omega \in \Omega$ and $t \in R$, there is some integer $n \in N$. Such that $T_n(\omega) \leq t < T_{n+1}(\omega)$. We can therefore define a continuous time parameter $Y = (Y_t)_{t \in R_+}$ which stake space E by putting $Y_t = X_n$ on $\{T_n \leq t < T_{n+1}\}$. The process $Y = (Y_t)_{t \in R_+}$ so defined is called a Semi-Markov process with state space E and Semi Markov transition Kernel $Q = \{Q(i, j, t)\}$.

d. Semi-Regenerative Processes

Let a stochastic process $Z = (Z_t)_{t \in R_+}$ be a stochastic process with a topological state space F, and suppose that the function $t \rightarrow Z_t(\omega)$ is right continuous and has left-hand limits for almost all $\omega \in \Omega$. A random variable $T : \Omega \rightarrow [0,\infty)$ is called stopping time for Z provided that for any $t \in R_+$, the occurrence or non occurrence of the event $\{T \leq t\}$ can be determined once the history $H_t = \sigma(Z_u : u \leq t)$ of Z before t is known. If T is the stopping time for Z, then we denote by H the history of Z

before T. The process $Z= \{Z_t \; ; \; t \geq 0\}$ is called a regenerative if there exists a sequence S_0, S_1, S_2 .... of stopping times such that

(i) $S = \{S_n; n \in N\}$ is a renewal process

(ii) For any $n,\ m \in N; t_1, t_2, ..., t_n \in R_+$ and any bounded function f defined on E^n

$$E\left[f\left(Z_{s_m+t_1}, Z_{s_m+t_2}, ..., Z_{s_m+t_m}\right) | Z_u; u \leq S_m \right] = E\left[f\left(Z_{t_1}, Z_{t_2}, ..., Z_{t_n}\right)\right]$$

Definition 2: Let $Z = (Z_t)_{t \in R_+}$ be a stochastic process topological state space F, and suppose that the function $t \rightarrow Z(\omega)$ is right continuous and has left hand limits for almost all ω. The process Z is said to be semi-regenerative if there exists a Markov renewal process (X,T) with infinite lifetime satisfying the following:

i) for each $n \in N, T_n$ is a stopping time for Z

ii) for each $n \in N, X_n$ is determined by $\{Z_u : u \leq T_n\}$

iii) for each $n \in N, M \geq 1, 0 \leq t_1 < t_2 < ... < t_m$ and function f defined on F^m

$$E_i\left[f\left(Z_{T_n+t_1}, Z_{T_n+t_2}, ..., Z_{T_n+t_m}\right) | Z_u; u \leq T_m \right] = E_j\left[f\left(Z_{t_1}, Z_{t_2}, ..., Z_{t_m}\right)\right] \text{ on } [X_n = j]$$

In this definition E_i and E_j refer to the expectations given the initial state for the Markov chain X.

Detailed treatment and MRP can be found in Pyke (1961a,b), Levy (1954), Cinlar (1975b) and Ross (1970). The survey of Cinlar (1975a) demonstrates the usefulness of the theory MRP and SMP in applications.

e. Stochastic Point Processes

Stochastic point processes form a class of processes more general than those considered in the previous sections. Since point processes have been more studied by many with varying backgrounds there have been several definitions of the point processes each appearing quite natural from the viewpoint of the particular problem under study. [See for example Bhabha (1950), Khinchine (1960), Harris (1963) and Bartlett (1966)]. A stochastic point process is the mathematical abstraction, which arises from considering such phenomena as randomly located population or a sequence of events in time. Typically there is envisaged a state space X and a set of points X_n, from X representing the locations of the different members of the population or the times at which the events occur. Because a realization (or sample path) of any of these phenomena is just a set of points in time or space, a family of such realizations has come to be called a point process. (Daley and Vere-Jones, 1971)

A comprehensive definition of point process is due to Moyal (1962) who deals with such processes in a general space, which is not necessarily Euclidian. Consider a set of objects, each of whose state is described by a point x of a fixed set X of points. Such a collection of objects, which we may call a population, may be stochastic if there exists a well-defined probability distribution P on σ some field β of subsets of the space Φ of all states. We shall assume that members of the population are indistinguishable from one another. The state of the population is defined as an unordered set $x^n = \{x_1, x_2, \ldots , x_n\}$ representing the situation where the population has n members with one each in the states $x_1, x_2, \ldots , x_n$. Thus the population state space Φ is the collection of all x^n with $n = 0,1,2\ldots$where x^0 denotes the empty population. A point process is defined to be the triplet (Φ, β, P). For a detailed treatment of stochastic point processes with special reference to their applications, refer to Srinivasan (1974). A point process is called a regular point process if the probability of occurrence of more than one event *(0, Δ)* is *0(Δ)*, where *Δ* is very small.

(i) Product Densities

One of the ways of characterising a general stochastic point process is enough product densities (Ramakrishnan 1950, 1958) and Srinivasan (1974). These densities are analogous to the renewal density in the case of non-renewal processes.

Let $N(t,x)$ denote the random variable representing the number of events in the interval $(t, t+x)$, $d_xN(t,x)$ the events in the interval $(t+x, t+x+dx)$ and $P_n(n,t,x)= P\,[N\,(t,x) = n]$. The product density of order n is defined as:

$$h_n\left(x_1, x_2, \ldots, x_n\right) = \lim_{\Delta_1, \Delta_2, \ldots, \Delta_n \to 0} P\left[\frac{N\left(x_i, \Delta_i\right) \geq 1, i = 1, 2, \ldots, n}{\Delta_1 \Delta_2 \ldots \Delta_n}\right]$$

where $x_1 \neq x_2 \neq \ldots \neq x_n$,

or equivocally for a regular process

$$h_n\left(x_1, x_2, \ldots, x_n\right) = \lim_{\Delta_1, \Delta_2, \ldots, \Delta_n \to 0} E\left[\frac{\prod_{i=1}^{n} N\left(x_i, \Delta_i\right)}{\Delta_1 \Delta_2 \ldots \Delta_n}\right]$$

where $x_1 \neq x_2 \neq \ldots \neq x_n$

These densities represent the probability of an event in each of the intervals $(x_1, x_1 + \Delta x_1)$, $(x_2, x_2+\Delta x_2)$, … , $(x_n , x_n+\Delta x_n)$. Even though the functions h_n $(x_1,$

$x_2, \ldots, x_n$) are called density, it is important to note that their integrates will not give probabilities, but will yield the factorial moments. The stationary moments can be obtained by relaxing the condition that all x_i are different.

1.3.8 Measures of System Performance

In this section some of the important measures of inventory systems are explained. Let $I(t)$ be the inventory level at time t and S be the maximum capacity of the inventory. Then the next inventory level distribution $P(i,t|k)$ at any time t is given by

$$P(i,t|k) = P[I(t) = i|I(0) = k];\ i,\ k = 0,\ 1,\ \ldots,\ S$$

The limiting distribution $P(i)$, if it exists, is defined as:

$$P(i) = \lim_{t\to\infty} P(i,t|k)$$

For a two product system let the state of the system be represented by the ordered pair $(X(t), Y(t))$, where $X(t)$ is the inventory level of product 1 and $Y(t)$ is the inventory level of product 2 . Then the inventory level distribution $P(i,j,t|k,l)$ at time t is given by

$$P\left[i,j,t|k,l\right] = P\left[\left(X(t),Y(t)\right) = (i,j)\middle|\left(X(0),Y(0)\right) = (k,l)\right]$$

$$i\ ;\ k = 0,1,2,\ldots,S_1;\ j;\ \ l = 0,1,\ldots,S_2$$

where S_1 and S_2 are the maximum inventory levels of product 1 and product 2 respectively. The limiting of distribution $P(i,j)$, if it exists, is defined as:

$$P(i,j) = \lim_{t\to\infty} P(i,j,t|k,l)$$

The expected stock on hand of mean inventory level *E(L)*, at any time for a single product system in the steady state is given by:

$$E(L) = \sum_{i=0}^{S} iP(i)$$

In any inventory model, apart from the distribution of the inventory level, the mean number of re orders places, replenishments made, demand satisfied demands lost in an arbitrary interval of time are also some of the important measures.

In the context of a multi-product system allowing substitution, the number of demands for a particular product satisfied by a different product deserves considerations. The stationary roles of these events are used in the cost analysis of the system. To find these measures, we follow the procedure given below.

Let $N(\eta,t)$ denote the number of a specific type of event η (like re-orders, replenishment, demand for a product satisfied by the same product, demand for a product satisfied by another product, demands lost, etc.) in $(o,t]$. Then the expected number of n events in $(o,t]$ is given by:

$$E\left[N(\eta,t)\right] = \int_{0}^{t} h(u)\,du$$

Where *h(u)* is the first order product density corresponding to the event under consideration. In the long term, the stationary role of η events is given by:

$$E(\eta) = \lim_{t\to\infty} \frac{E\left[N(\eta,t)\right]}{t} = \lim_{t\to\infty} h(t)$$

1.3.9 Cost Analysis

a. Inventory Related Costs

We consider the following costs in the analysis of the inventory models.

(i) Holding Costs

This not only includes the expenses incurred by storage facilities but also the amount invested that could have earned a return on investment elsewhere. This cost at any time depends upon the level of stock on hand.

(ii) Re-ordering Costs

When the stock in hand comes down to a level where re-order is necessary, an order is placed. This involves additional expenses with regard to transactions, paperwork, inspection and material handling costs.

(iii) Cost for Demand Lost

When demand is not met and also not backordered, the profit that would have been made is lost together with some goodwill.

(iv) Procurement Cost

This is the price at which the items are bought either from a manufacturer or from the market. Most inventory control procedures recognise price fluctuations, and they are treated accordingly in this study.

b. Cost Optimization

There are a number of objectives that may be sought after by inventory managers. These usually involve the minimisation (maximisation) of costs (profits) function, which could be either discounted or undiscounted. The planning period of horizon may be finite or infinite, In stochastic models the mean value of costs are measured and the criterion consists in the minimisation of the total expected cost pen unit time or of the expected discount cost over a finite or infinite horizon. The cost function will, in general, consist of the additive contribution of the procurement cost, the holding cost and the storage cost.

Under the *(S, s)* policy, the objective function will, in general, be expressible as a function of two variables *S* and *s.* The resultant optimization problem consists in determining the optimal values of *S* and *s* to achieve the selected extension. For a multi-product system the maximum inventory levels of the various products and the re-order levels can be considered as variables for optimization.

In this regard, it should be pointed out that there are two distinct approaches in formulating and solving the stochastic inventory problems both in theory and in practise.

In the first approach the system is viewed as a multi-stage decision process and the technique of dynamic programming is employed in finding the optimal policy that minimises the total expected cost over the duration of the process.

The following second approach is often used when the duration process is infinite: an ordering policy of a given type is chosen and the stationary behaviour of the inventory levels is analysed without reference to the cost structure of the problem. Such entities as the expected frequencies of orders and the expected quantity on hand, etc. are computed. A cost structure is then imposed on the system and the stationary total expected cost rate for operating the inventory system is minimised. In this thesis, the stationary approach is adopted for optimal analysis.

If $C(t)$ represents the total cost in [$0,\ t$], then the expected cost rate, $E(C)$, is given by:

$$E(C) = \lim_{t \to \infty} \frac{E\left[C(t)\right]}{t}$$

Notation:

λ_i : Demand rate of product i, i = 1,2

μ_i : Perishable rate of product i, i = 1,2

S_i : Maximum inventory level of product i, i = 1,2

s_i : Re-order level of product i, i = 1,2

S_i-s_i : Quantity of product i re-ordered, i = 1,2

d_i : Event that a demand for product i is satisfied with product i, i = 1,2

g : Event that a demand for product 1 is satisfied by product 2

l_i : Event that a demand for product i is lost, i = 1,2

$N(\eta,t)$: Number of η events in interval (0,t]

δ_{ij} : Kronecker's delta function

$H(\bullet)$: Heaviside function

© : Convolution Symbol

$\xi^*(s)$: Laplace transform of $\xi(t)$

$f^{(n)}(t)$: n-fold convolution of $f(t)$

$$\bar{F}(t) = 1 - F(t) = 1 - \int_0^t f(u)du$$