CHAPTER 1

INTRODUCTION

1.1 Introduction

Reliability theory is one of the most important branches of Operations Research and Systems Engineering. Any systems analysis in order to be complete, must give due consideration to system reliability. With remarkable advances made in electronics engineering, military and communication systems have become more sophisticated and when such systems fail, very serious situations arise. Thus in the present day context, high system reliability has become very important from the view point of both makers and the users.

A system designer is often faced with problems of determining the various system measures like reliability, availability and interval reliability etc. He also has to suggest ways by which the efficiency of a given system can be improved. Due to the nature of the subject, the methods of Probability Theory and Mathematical Statistics are necessary to study and solve the problems that arise in reliability theory.

Many mathematical models have been proposed to evaluate various measures of system performance and methods of improving them. These models, which describe the various operational characteristics of the system taking into account its essential features, can be studied only with the help of probability theory. The present work is a study of some mathematical models representing the behaviour of a few complex systems. Introduction of redundancy and repair maintenance are two important methods of improving system reliability.

The manufacturing of tools and special equipment is part of human nature. At first experience, faults and accidents were the only schools for learning to make safer and more reliable equipment. Before structural design became an engineering science, the reliability of a bridge was tested with a team of elephants. It is collapsed, a stronger bridge was built and tested again! Obviously, these methods could not continue and is human skills developed a wide variety of very reliable items and structures were designed and

manufactured. One example is the undersea telephone cables built by Bell Telephone Laboratories.

Man's earliest preoccupation with reliability was undoubtedly related to weaponry. Interest as a result of the terrible non-reliability of electronic weapons systems used during World War II. Increasingly complex systems, such as the first missiles, also emphasized the importance of successful operation of equipment in a specific environment during a certain time period. The V-1 missile, developed in Germany with high quality parts and careful attention, was catastrophic: the first 10 missiles either exploded on the launching pad, or landed short of their targets.

Technological developments lead to an increase in the number of complicated systems as well as an increase in the complexity of the systems themselves. With remarkable advancements made in electronics and communications, systems became more and more sophisticated. Because of their varied nature, these problems have attracted the attention of scientists from various disciplines especially the systems engineers, software engineers and the applied probabilists. An overall scientific discipline, called *reliability theory,* that deals with the methods and techniques to ensure the maximum effectiveness of systems (from known qualities of their component parts) has developed. '*Reliability theory introduces quantitative indices of the quality of production' (*Gnedenko et al. (1969)) and these are carried through from the design and subsequent manufacturing process to the use and storage of technological devices.

Engineers, Scientists and Government leaders are all concerned with increasing the reliability of manufactured goods and operating systems. As '*Unreliability has consequences in cost, time wasted, the psychological effect of inconvenience, and in certain instances personal and national security'* (Lloyd & Lipow (1962)). In 1963 the first journal on reliability, IEEE-Transactions on Reliability saw the light.

Due to the very nature of the subject, the methods of Probability theory and Mathematical statistics (information theory, queuing theory, linear and nonlinear programming, mathematical logic, the methods of statistical simulation on electronic computers, demography, manufacturing, etc.), play an important role in the problem solving of reliability theory. Other areas include contemporary medicine, reliable software systems, geoastronomy, irregularities in neuronal activity, interactions of physiological growth, fluctuations in business investments, and many more. In human behaviour mathematical models based on probability theory and stochastic process are helpful in rendering realistic modelling for social mobility of individuals, industrial mobility of labour, educational advancements, diffusion of information and social networks. In the biological sciences stochastic models were first used by Watson and Galton (1874) in a study of extinction of families. Research on population genetics, branching process, birth and death process, recovery, relapse, cell survival after irradiation, the flow of particles through organs, etc, then followed. In business management, analytical models evolved for the purchasing behaviour of the individual consumer, credit risk and term structure. Income determination under uncertainty and more related subjects. Traffic flow theory is a well known field for stochastic models and studies have been developed for traffic of pedestrians, freeways, parking lots, intersections, etc. (Erasmus,2005)

Problems encountered in the design of highly reliable technical systems have led to the development of high accuracy methods of reliability analysis. Two major problems can be identified, namely:

- Creating classes of probability-statistical models that can be used in the description of the reliability behaviour of the systems, and
- Developing mathematical models for the examination of the reliability characteristic of a class of systems.

Considering only redundant systems the classical examples are the models of Markov processes with a finite set of states (in particular birth and death processes) (Gnedenko et al. (1969)), Barlow (1984), Gertbakh (1989) and

Kovalenko et al. (1997)), the renewal process method (Cox (1962)), the semi-Markov process method and its generalisations (Cinlar (1975a, b)), generalized semi-Markov process (GSMP) method (Rubenstein (1981)), special models for coherent systems (Aven (1966)) and systems in random and variable environment (Ozekici (1996)) and Finkelstein (1999a, b, c)), van Schoor (2005), Muller (2005).

Depending on the nature of the research, the applicable form of reliability theory can be introduced to each. A stochastic analysis is made based on some good probability characteristics. It is, however, not simply a case of changing terminology in standard probability theory (say, "random variable" changes to "lifetime"), but reliability distinguishes itself by providing answers and solutions to a series of new problems not solved in the "standard" probability theory framework. Gertbakh (1989) points out that reliability,

- of a system is based on the information regarding the reliability of the system's components

- gives a mathematical description of the ageing process with the introduction of several formal notations of ageing (failure rate, etc.)
- introduces well-developed techniques of *renewal theory*
- introduces *redundancy* to achieve optimal use of standby components (an excellent introduction to redundant systems is given in Gnedenko et al. (1969))
- includes the theory of optimal preventative maintenance (Beichelt and Fischer (1980))
- is a study of statistical inference (often from censored data)

Generally, the mathematical problems of lifetime studies of technical objects (reliability theory) and of biological entities (survival analysis) are similar, differing only in the notation. The term "lifetime" therefore does not apply to lifetimes in the strictest literal sense, but can be used in the figurative sense. The idea is that the statistical analysis done in this thesis should be true in any of the applicable disciplines, although the notation is mostly as for engineering
(systems, components, units, etc). With minor modifications the discipline can be changed to biological, or financial, or any other disciplines.

1.2 FAILURE

'*A failure is the result of a joint action of many unpredictable, random processes going on inside the operating system as well as in the environment in which the system is operating.'* (Gertbakh (1989)). Functioning is therefore seriously impeded or completely stopped at a certain moment in time and all failures have a stochastic nature. In some cases the time of failure is easily observed. But if units deteriorate continuously, determination of the moment of failure is not an easy task. In this study we assume that failure of a unit can be obtained exactly.

Failure of a system is called a *disappointment* or a *death* and failure results in the system being in the down state. This can also be referred to as a breakdown (Finkelstein (1999a)).

Zacks (1992) points out that there are two types of data to consider, namely:

- data from continuous monitoring of a unit until failure is observed
- data from observations made at discrete time points, therefore failure counts

Villermeur (1992) gives an extensive list of possible failures and inter-dependent failures. There are catastrophic failures, determined by a sharp change in the parameters and drift failures (the result of wear or fatique), arising as a result of gradual change in the values of the parameters. (Muller, 2005).

1.3 REDUNDANCY AND DIFFERENT TYPES OF REDUNDANT SYSTEMS

In a *redundant system* more units are built into it than is actually necessary for proper system performance. Redundancy can be applied in more than one way

and a definite distinction can be made between *parallel* and *standby* (sequential) redundancy. In parallel redundancy the redundant units form part of the system from the start, whereas in a standby system, the redundant units do not form part of the system from the start (until they are needed).

1.3.1 Parallel systems

A parallel redundant system with n units is one in which all units operate simultaneously, although system operation requires at least one unit to be in operation. Hence a system failure only occurs when all the components have failed.

Let k be a non-negative integer, such that k < n, counting the number of units in an n-unit system. It is customary to refer to such a system as k-out-of-n system.

1.3.2 k-out-of-n: F system

If k-out-of-n system fails, when k units fail, it is called an F-system. The functioning of a minimum number of units ensures that the system is up (Sfakianakis and Papastavridis (1993)).

1.3.3 k-out-of-n: G-system

A G-system is operational if and only if at least k units of the system are operational. Recent work related to this topic can be seen in Zhang and Lorn (1998) and Liu (1998). Suppose a radar network has n radar control stations covering a certain area: the system can be operable if and only if at least k of these stations are operable. In other words, to ensure functioning of the system it is essential that a minimum number of units, k, are functioning.

Lately attention moved to load-sharing k-out-of-n: G systems, where

- the serving units share the load
- the failure rate of a component is affected by the magnitude of the load it shares.

1.3.4 n-out-of-n: G system

A series that consists of *n* units and when the failure of any one unit causes the system to fail. Although this type of system is not redundant system, as all the units are in series and have to be operational, it can still be considered as a special case of a k-out-of-n system. There are many papers on the reliability of these systems. Scheuer (1988) studied reliability for *shared-load k-out-of-n:* G systems, where there is an increasing failure rate in survivors, assuming identically distributed components with constant failure rates. Shao and Lamberson (1991) considered the same scenario, but with imperfect switching. Then Huamin (1998) published a paper on the influence of work-load sharing in non-identical, non-repairable components, each having an arbitrary failure time distribution. He assumed that the failure time distribution of the components can be represented by the accelerated failure time model, which is also a proportional hazards model when base-line reliability is Weibull. (Muller, 2005)

1.4 REPAIRABLE SYSTEM

In order to increase the system reliability, failed units may be replaced by new ones. However when this proves to be very expensive, resort is made to repair the failed units. On failure, a unit is sent to a repair facility. If the repair facility is not free, failed units queue up for repair. The life time of a unit while online, while in standby and the repair time are all independent random variables. It is assumed that the distribution functions of these random variables are known and that they have probability density functions.

Barlow (1962) had considered some repairman problems and they have much in common with queuing problems. Rau (1964) had discussed the problem of finding the optimum value of m in an m out of n: G system for maximizing reliability.

Ascher (1968) has pointed out some inconsistencies in the modelling of repairable systems by renewal theory. Several authors, notably Barlow and Proschan (1965), Sandler (1963), Shooman (1968), Buzacott (1970) and Doyon and Berssenbrugge (1968) have used continuous time discrete state Markov renewal process model for describing the behaviour of a repairable system.

These conceptionally simple methods are not practically feasible for systems with large number of states. Gaver (1963), Gnedenko et al (1969), Osaki (1969, 70 a, b) and Srinivasan (1966) have employed the techniques of Semi-Markov processes for finding the reliability of a system with exceptional failures. By the use of Semi-Markov processes, Kumagi (1971) studied the effect of different failure distributions on the availability through numerical calculations. Branson and Shah (1971) studied repairable systems with arbitrary failure distributions using Semi-Markov Processes. Srinivasan and Subramanian (1977), Venkatakrishnan

(1975), Ravichandran (1979), Natarajan (1980), Sarma (1982), Botha (2001), Muller (2005) have used

regeneration point technique to analyse repairable systems with many, though not all, arbitrary distributions. More references in related topics can be found in the review papers by Subba Rao and Natarajan (1970), Osaki and Nakagawa (1976), Pierskalla and Voelker (1976) and Lie, Wang and Tillman (1977) and Kumar and Agarwal (1980), Gopalan (2004).

1.5 SYSTEMS WITH NON-INSTANTANEOUS SWITCHOVER

In the study of redundant systems it is generally assumed that when the unit operating online fails, the unit in standby is automatically switched online and the switchover from the standby state to online state is instantaneous. Srinivasan (1968), Osaki (1972), Khalil (1977), Subramanian and Ravichandran (1978 a), Gopalan and Marathe (1978, 80), Singh et al (1979) and Kalpakam and Shahul Hameed (1980), Subramanian and Sarma (1982) have studied redundant systems incorporating non-negligible switchover times.

1.6 SYSTEMS WITH IMPEREFCT SWITCH

To transfer a unit from the standby state to the online state, a device known as 'switching device' is required. Generally we assume that the switching device is perfect in the sense that it does not fail. However; there are practical situations where the switching device can also fail. This has been pointed out by Gnedenko et al (1969). Such systems in which the switching device can fail are called systems with imperfect switch.

Chow (1971), Osaki (1972), Nakagawa and Osaki (1975 a), Nakagawa (1977), Venkatakrishnan (1975), Prakash and Kumar (1970), Srinivasan and Subramanian (1980) and Subramanian and Natarajan (1980), Subramanian & Sarma (1984) have considered models where the switching device can also fail.

1.7 INTERMITTENTLY USED SYSTEMS

In almost all the models of redundant systems studied so far, it is assumed that the system under consideration is needed all the time. But in some practical

situations continuous failure free performance may not be necessary. In such cases we have to take into consideration the fact that the system can be in down state during certain intervals without any real consequence. In this case the probability that the system is in the up state is not an important measure; what is really important is the probability that the system is available when it is needed. Gaver (1964) pointed out that is pessimistic to evaluate the performance of an intermittently used system solely on the basis of the distribution of the time to failure. Srinivasan (1966), Nakagawa et al (1976), Srinivasan and Bhaskar (1979 a, b, c), Kapur and Kapoor (1978, 80) extended Gaver's results for two-unit systems. Detailed study of an n-unit intermittently used system is made. The statistical inference of some of these models has been studied recently by Yadavalli et al (2000, 2001).

1.8 MEASURES OF SYSTEM PERFORMANCE

The previous sections briefly describe the various types of redundant systems discussed in the literature. In this section some of the important measures of system performance useful in different contexts are discussed (Barlow and Proschan (1965), Gnedenko et al (1969)).

(a) Reliability:

Reliability is the probability that the system will perform satisfactorily for a given period of time in its intended application. Let $\{\xi(t), t \geq 0\}$ be the performance process of the system; then for a fixed t, $\xi(t)$ is a binary random variable which takes the value 1 if the system operates satisfactorily at a given time t, and takes the value 0 otherwise.

Then the reliability R (t) is given by

$$\begin{aligned} R(t) &= \Pr[\text{system is up in } (0, t]] \\ &= \Pr[\xi(u) = 1; \text{ for all } u \text{ such that } 0 \leq u \leq t] \end{aligned}$$

The expectation of the random variable representing the duration of time measured from the point the system starts operating till the instant it fails for the first time is called Mean time to System Failure (MTSF).It can be obtained from R (t) from the relation

$$\text{MTSF} = \int_0^\infty R(u)\,du$$

(b) Pointwise Availability:

This is defined as the 'probability that the system is able to operate within the tolerances at a given instant of time'. In symbols:

$$\text{Pointwise availability } A(t) = \Pr[\xi(t) = 1]$$

(c) Asymptotic or Steady-State Availability:

Steady-state availability $A_\infty = \lim_{t \to \infty} A(t)$.

It can be shown (Barlow and Proschan (1975)) that this is equal to the expected fraction per unit time in the long run that the system operates satisfactorily.

(d) Interval Reliability:

The interval reliability R (t, x) is the probability that the system is up in the interval $[t, t + x]$.

Hence:

R (t, x) = Pr [ξ (u) = 1, for all u such that t ≤ u ≤ t + x]

We observe that the reliability R(x) and the pointwise availability *A (t)* can be obtained from the interval reliability R (t, x) by putting t = 0 and x = 0 respectively.

(e) Limiting interval reliability:

This is defined as the limit of R (t, x) as t→∞, and hence is denoted by $R_\infty(x)$, which is the ordinary reliability function.

(f) Mean number of events in (0, t):

Let N(x, t) denote the number of particular type of event (like break down etc.) in $\left(x, x+t\right]$.Then the mean number of events in (0, t) is given by

$$E\,[N\,(0, t)] = \int_0^t h_1(u)du$$

where $h_1(t)$ is the first order product density of the events (product densities are defined in a subsequent section in this chapter) .The stationary rate of occurrence of those events is given by:

$$E\,[N] = \lim_{t \to \infty} \frac{E[N(0,t)]}{t}$$

1.9 TECHNIQUES USED IN THE ANALYSIS OF REDUNDANT SYSTEMS.

This section is a compilation of the techniques used in the analysis of redundant repairable systems.

1.9.1 Renewal Theory

Renewal theory forms an important in the study of stochastic processes and applied probability models, and is extensively used by many to study specific reliability problems. Feller (1968) made significant contributions to renewal theory giving the proper lead.

Smith (1958) gave an extensive review and highlighted the applications of renewal theory to a variety of problems. A lucid account of renewal theory is given by Cox (1962).

Definition 1.1

A renewal process is a sequence of independent, non-negative and identically distributed random variables $\{Y_i$, i = 1, 2,...} which are not all zero with probability one.

We assume that these random variables are defined on the same probability space and have finite mean μ.A renewal process is completely determined by means of f (.), the pdf of X_i. Associated with a renewal process is a r.v *N (t)* which represents the number of renewals in the time interval (0, t]; *N (t)* is also known as the renewal counting process (Parzen, 1962, Beichelt and Fatti (2002).

If policy 0 is the practical background of a renewal process, then Y_i denotes the time between the $(i-1)-th$ and the $i-th$ renewal. If at time $t=0$ policy 0 has already been in effect for a while, then Y_1 is a residual lifetime in the sense of section 1.2.3. However, the age of the

system working at time $t=0$ need not to be known. But if at time $t=0$ a new system started working, then all the random variables $Y_1, Y_2,...$ are identically distributed.

Let the random variables $Y_2, Y_3,...$ be identically distributed as Y with distribution function $F(t)=P(Y \leq t)$, whereas Y_1 has distribution function $F_1(t)=P(Y_1 \leq t)$.

Definition 1.2 (see Beichelt and Fatti, 2002)

A renewal process is called delayed if $F_1(t) \not\equiv F(t)$ and ordinary if $F_1(t) \equiv F(t)$.

Since, by assumption, the renewal occur in negligible time, T_n defined by

$$T_n = \sum_{i=1}^{n} Y_i \text{ ; n = 1,2,…;}$$

is the time point at which the nth failure (renewal) takes place. Hence, T_n is called a renewal time. The time intervals between two neighbouring renewals are called renewal cycles.

Let the renewal counting process $\{N(t), t \geq 0\}$ be defined by

$$N(t) = \begin{cases} \max(n;\ T_n \leq t) \\ 0 \quad for \quad t < T_1 \end{cases}$$

$N(t)$ is the random number of renewals occurring in $(0, t]$.Since $N(t) \geq n$ if and only if $T_n \leq t$,

$$F_{T_n}(t) = P(T_n \leq t) = P(N(t) \geq n),$$

where, because of the independence of the $Y_i, F_{T_n}(t)$ is the convolution of F_1 with the $(n-1)-th$ convolution power of F.

$$F_{T_n}(t) = F_1 © F^{(n-1)}(t),\ F^{(0)}(t) \equiv 1,\ t \geq 0;\ n = 1,\ 2,...$$

If the densities $f_1(t) = F_1'(t)$ and $f(t) = F'(t)$ exist, then the density of T_n is

$$f_{T_n}(t) = f_1 \copyright f^{(n-1)}(t)\ ,\ f^{(0)}(t) \equiv 1,\ t \geq 0; n = 1,2,\ldots$$

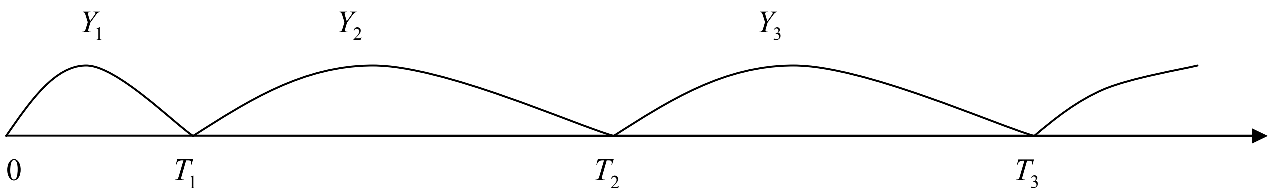

Figure: illustration of a renewal process

Definition 1.3

The expected value of *N (t)* is called the renewal function and is denoted by $H(t)$. The derivative of $H(t)$, if it exists, is denoted by $h(t)$ and is called the renewal density. The quantity $h(t)dt$ is the probability that a renewal occurs in $(t, t+dt)$.

The renewal density satisfies the following famous integral equation, known as the functional equation of renewal theory.

$$h(t) = f(t) + \int_0^t f(u)h(t-u)du$$

The solution of the above equation is:

$$h(t) = \sum_{n=1}^{\infty} f^{(n)}(t)$$

where $f^{(n)}(t)$ is the n-fold convolution of f(t).

We now briefly indicate how renewal theory has been used in the solution of reliability problems. Srinivasan et al (1971) used renewal theory to obtain some operating characteristics of a one unit system. The integral equation of renewal theory was used by Gnedenko et al (1969) to obtain MTSF of a two-unit standby system. Osaki (1970b) applied the integral equation to study several redundant systems. Buzacott (1971) used renewal theoretic arguments to study some priority redundant systems.

1.9.2 SEMI-MARKOV AND MARKOV RENEWAL PROCESS

Now we consider a stochastic process which makes transitions from one state to another in accordance with a Markov chain but the amount of time spent in each state before a transition is probabilistic. Denoting the state space by the set of non-negative integers {0,1, 2….}. Let the transition probabilities be given by p_{ij}, i,j = 0,1,2,…..Let $F_{ij}(t)$, t> 0 be

the conditional distribution function of the sojourn time in state i, given that the next transition will be into state j.

Let

$$Q_{ij}(t) = p_{ij} F_{ij}(t), \text{ i, j} = 0,1,2,\ldots..$$

Then $Q_{ij}(t)$ denotes the probability that the process makes a transition into state j in an amount of time less than or equal to t given that it just entered state i at t = 0.The functions $Q_{ij}(t)$ satisfy the following conditions:

$$Q_{ij}(0) = 0, \qquad Q_{ij}(\infty) = p_{ij};$$

$$Q_{ij}(t) \geq 0, \qquad i,j = 0,1,2,\ldots$$

$$\sum_{j=0}^{\infty} Q_{ij}(t) = 1$$

Let J_0 denote the initial state of the process and J_n (n = 1, 2…) the state of the process after the n-th transition has occurred. Then the process $\{J_n$, n = 0, 1, 2…} is a Markov Chain with transition probabilities P_{ij}.This is called the embedded Markov Chain. Let $N_i(t)$ denote the number of transitions into state i in (0, t] and define

$$N(t) = \sum_{i=0}^{\infty} N_i(t)$$

Now define a stochastic process {Z (t), t≥0} where Z (t) = i, denotes that the process is in state i at time t. Then it is clear that Z (t) = $J_n(t)$

Definition 1.4

The stochastic process {Z (t), t ≥ 0} is called a Semi-Markov process (SMP).

Definition 1.5

The vector stochastic process $\{ N_1(t), N_2(t), \ldots ,t\geq 0\}$ is called a Markov Renewal Process (MRP).

Thus the SMP records the state of the process at each time point, while the MRP is a counting process which keeps track of the number of visits to each state. Denote by X_i the random variable denoting the time interval between two successive visits to state i of the process $\{ Z(t), t \geq 0 \}$.Then we observe that $\{ x_i \}$ is a renewal process for i = 0,1,2,…. Detailed treatments of SMP and MRP can be found in Pyke (1961 a, b), Cinlar (1975 a) and Ross (1970).

The survey by of Cinlar (1975 b) demonstrates the usefulness of the theory of MRP and SMP in applications. Barlow et al (1965) used these processes to determine the MTSF of a two unit system. Srinivasan (1968), Cinlar (1975 b), Osaki (1970 a, 1972). Arora (1976 a, b), Nakagawa and Osaki (1974, 1976), and Nakagawa (1974) have used the theory of SMP to discuss some reliability problems.

1.9.3 STOCHASTIC POINT PROCESSES

Stochastic point processes are more general than those considered in the earlier sections. Since point processes have been studied by many with varying backgrounds, there have been several definitions of the point processes each appearing quite natural from the view point of the particular problem under study. [See for example Bartlett (1966), Bhaba (1950), Harris (1963) and Khinchine (1955)]. A comprehensive definition of point process is due to Moyal (1962) who deals with such processes in a general space which is not necessarily Euclidean.

Roughly speaking a stochastic point process can be defined as continuous time parameter discrete state space stochastic process.

1.9.4 PRODUCT DENSITIES

One of the ways of characterizing a general stochastic point process is through product densities (Ramakrishnan (1950, 1958), Srinivasan (1974)).These densities are analogous of the renewal density in the case of non-renewal processes.

Let N (x, t) denote the random variable representing the number of events in the interval (t, t + x), d_x N(x , t) the events in the interval (t + x, t + x +dx) and p(n , x , t) = Pr[N (x , t) = n].

The product density of order n is defined as:

$$h_n(x_1, x_2, \ldots, x_n) = \lim_{\Delta_1, \Delta_2, \ldots, \Delta_n \to 0} \frac{E\left[\prod_{i=1}^{n} N(\Delta_1, x_i)\right]}{\Delta_1 \Delta_2 \ldots \Delta_n}$$

$$x_1 \neq x_2 \neq \ldots x_n .$$

A process is said to be regular if the probability of occurrence of more than one event in an interval of length is o (Δ). For such process we have:

$$h_n(x_1, x_2, \ldots, x_n) = \lim \frac{\Pr\left[N(\Delta_i, x_i) \geq 1, i = 1,2,\ldots,n\right]}{\Delta_1 \Delta_2 \ldots \Delta_n}, x_1 \neq x_2 \neq \ldots \neq x_n$$

These densities represent the probability of an event in each of the intervals $(x_{1,}x_1+\Delta_1)$, $(x_2, x_2+\Delta_2), \ldots, (x_n, x_n+\Delta_n)$.

Even though the functions $h_n(.,.,.,..)$ are called densities, it is important to note that their integration will not give probabilities but will yield the factorial
moments. The ordinary moments can be obtained by relaxing the condition that all x_i are distinct.

1.9.5 REGENERATIVE STOCHASTIC PROCESSES

The idea of regeneration point was first introduced by Bellman and Harris (1948) while studying population point processes. Feller (1949), in the theory of recurrent events, dealt with a special case of regeneration points. Later on, Smith (1955) generalized Feller's results and dealt with more general stochastic point processes possessing such regeneration points, familiarity known as regenerative processes. A formal theory of such processes has been developed by Kingman (1964).

A regenerative event R of a stochastic process {X (t)} is an event that is characterized by the property that if it is known that R happens at $t=t_1$, then the knowledge of the history of the process prior to t_1 loses its predictive value. In some special cases, the event R is the only characteristic, so that the process regenerates itself with each occurrence of R.

In more general cases, in addition to the occurrence of R, knowledge of X (t) is necessary for the prediction of the process. The renewal process can be thought of as a general point process in which each point at which the event R occurs is a regeneration point. The occurrence of an event at

$t = t_1$ uniquely determines the distribution of events from any collection of segments of points $t \geq t_1$.If we further specialize to the case when the intervals between successive events are exponentially distributed, we notice that any point (not necessarily a point where an event occurs) on the time axis is a regeneration point. Gnedenko (1964), Srinivasan and Gopalan (1973 a, b), Birolini (1974, 75), Srinivasan and Subramanian (1977), Hines (1987), Hargreaves (2002),

Botha (2001), Muller (2005) have used such regenerative events to study some reliability problems.

1.9.6 CONCLUDING REMARKS AND SCOPE OF WORK

Reliability theory is a very important branch of systems engineering and operations and deals with general method of evaluating the various measures of performance of a system that may be subject to gradual deterioration. Several models of redundant systems have been studied in the literature and the following are some of the typical assumptions made in analyzing such systems:

(i) the repair times are assumed to be exponential

(ii) the estimated study of the system measures has not been made.

(iii) the system is available continuously

(iv) Environmental factors not affecting the system

(v) The failures take place in one mode

(vi) The switching device is perfect

(vii) System reliability evaluated for given chance constraints

(viii) The switchover time required to transfer a unit from the standby state to online stage is negligible.

(ix) the failures and repairs are assumed to be independent.

However, we frequently come across systems in which one or more of these assumptions have to be dropped and hence there is an increasing need for studying models in which at least some of these assumptions could be relaxed. That is the motivation for the detailed study of the models presented in this thesis. This thesis is a study of some redundant repairable systems with ‘rest period’ for the operator, non-instantaneous switchover, imperfect switch, intermittent use and optimization study.